NUCLEAR PHYSICS (22)

1) THE MEAN-FIELD POTENTIAL:

HARTREE-FOCK,

SKYRME & GOGNY INTERACTIONS

2) EXOTIC NUCLEAR STRUCTURES:

HALO NUCLEI, α-CLUSTERS,

HYPERNUCLEI

RECAP | There were two issues with the SHELL MODEL :

1) How to find a good V^{HF}? ←

1.a) Convenency
(e.g. harmonic oscillator)

1.b) Other more theoretical means
(e.g. Hartree-Fock)

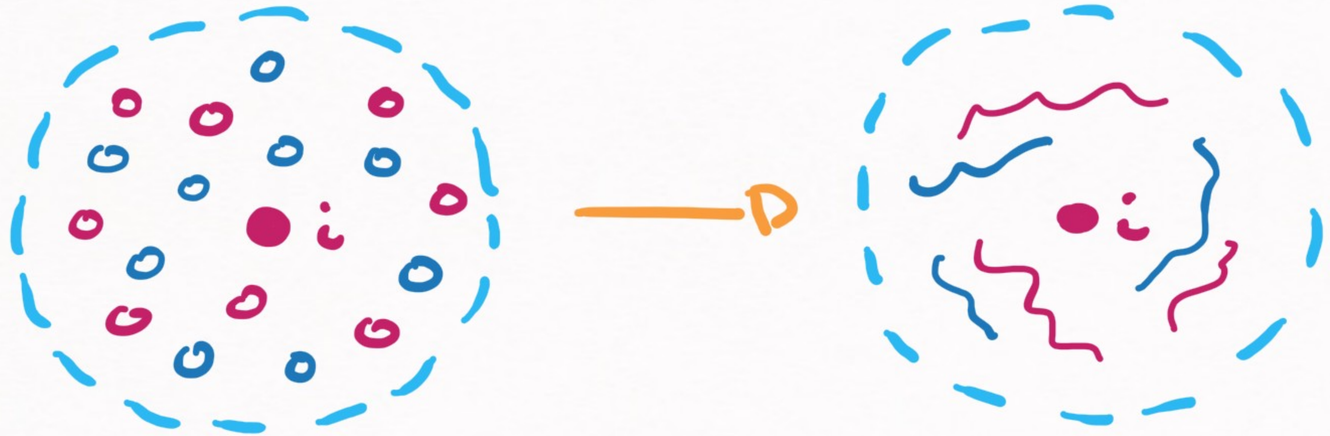
} Todays lesson ⇝ (Hartree-Fock)

2) How to deal with residual interactions?

⇒ Past lesson ⇝

[THE MEAN-FIELD POTENTIAL] (1)

=D The basic idea:



1) Each particle affects each other:

$$V = \sum_{ij} v_{ij}^{2B} + \dots$$

2) Each particle will feel the average potential:

$$V = \sum_{i} v_i^{MF} \rightarrow$$ the mean field potential

[THE MEAN-FIELD POTENTIAL] ②

⇒ How do we derive this potential?

Strategy A: GOD'S VIEW

A.1) We begin w/ the A-body Schrödinger equation:

$$\left(\sum_i T_i + \sum_{ij} V_{ij}\right)\Psi(\vec{r}_1, \ldots \vec{r}_A) = E_A \Psi(\vec{r}_1, \ldots, \vec{r}_A)$$

We will solve it for some V_{ij} and obtain the A-body w.f.

(Simplification: we will momentarily only consider 2-body forces)

[THE MEAN-FIELD POTENTIAL] (3)

Strategy A: God's view

A.2) For each particle, we will get the average pot:

$$V_i^{HF}(\vec{r}_i) = \sum_{j\neq i} \int \left(\prod_{j\neq i} d\vec{r}_j\right) |\Psi_0(\dots \vec{r}_i \dots \vec{r}_j \dots)|^2 \, V(\vec{r}_i - \vec{r}_j)$$

↳ The average of the effects of all other particles once we know the complete w.f..

[THE MEAN-FIELD POTENTIAL] (4)

Strategy A: God's view

A.2) Or, more elegantly as:

$$\hat{V}_i(\vec{r}) = \delta(\vec{r} - \vec{r}_i) \sum_{j \neq i} V(\vec{r}_i - \vec{r}_j)$$

$$\overset{\text{MF}}{\Rightarrow} \quad V_i(\vec{r}) = \langle \psi_A | \hat{V}_i(\vec{r}) | \psi_A \rangle$$

the point is that if we have $|\psi_A\rangle$, it is easy to obtain the mean-field potential

[THE MEAN-FIELD POTENTIAL] (5)

Strategy A: God's view

A.3) Now we solve the monoparticular w.f.s:

$$\left[-\frac{\nabla_i^2}{2m_i} + V_i^{MF}(r_i)\right]\phi_i(r_i) = \epsilon_i \phi_i(r_i)$$

A.4) Find the mean-field Δ-body w.f.:

$$\Psi_\Delta^{MF} = \prod_i \phi_i(\vec{r}_i)$$ (or its antisymmetrization)

(for simplicity, I will assume distinguishable particles)

[THE MEAN-FIELD POTENTIAL] (6)

Strategy A: God's view

A.1) + A.2) + A.3) + A.4) $\Rightarrow$ this assumes that I know everything & that I can calculate everything

$\Rightarrow$ IF I could calculate everything, why would I try to find a mean-field?

[THE MEAN-FIELD POTENTIAL] (7)

⇒ It is more probable than not that

I can't solve the A-body problem

↳ Strategy B

B.1) I begin w/ a two-body pot: $V_{ij} = V(\vec{r}_i, \vec{r}_j)$

But now I cannot solve the A-body Schrödinger eq:

$$\left(\sum_i T_i + \sum_{ij} V_{ij}\right) \Psi_A = E_A \Psi_A$$

[THE MEAN-FIELD POTENTIAL] (8)

Strategy B : the (probable) real situation

B.2) So... I will try to invent some one-body "mean-field" potential

$$v_i^{(0)} = v^{(0)}(\vec{r}_i)$$

$$(T_i + v_i^{(0)})\, \varphi_i^{(0)} = \epsilon_i^{(0)}\, \varphi_i^{(0)}$$

[THE MEAN-FIELD POTENTIAL] (9)

Strategy B: the (probable) real situation

B.3) Now, I use the HF wavefunctions to obtain a new HF potential:

$\Psi_A^{(0)} = \prod_i \phi_i^{(0)}(\vec{r}_i)$ (again, I assume distinguishable particles)

↳ $V_i^{(1)}(\vec{r}) = \langle \Psi_A^{(0)} | \underbrace{V_i(\vec{r})}_{= \delta(\vec{r}-\vec{r}_i) \sum_{j \neq i} v(\vec{r}_i - \vec{r}_j)} | \Psi_A^{(0)} \rangle = \textcircled{*}$

$$\textcircled{*} = \sum_{j \neq i} \int |\phi_j^{(0)}(\vec{r}_j)|^2 v(\vec{r} - \vec{r}_j)\, d^3\vec{r}_j$$

[THE MEAN-FIELD POTENTIAL] (10)

Strategy B : the (probable) real situation

B.4) I solve the monoparticular w.f.'s again:

$$[T_i + V_i^{(1)}(\vec{r})]\phi_i^{(1)}(\vec{r}) = \epsilon_i^{(1)}\phi_i^{(1)}(\vec{r})$$

B.5) Check whether there is convergence:

$$\phi_i^{(1)}(\vec{r}) \leq \phi_i^{(0)}(\vec{r})$$

If not, repeat again and obtain $\phi_i^{(0')}(\vec{r})$

[THE MEAN-FIELD POTENTIAL] (13)

Strategy B : the (probable) real situation

=> Basically, this is an iterative algorithm :

$$[T_i + V_i^{(n)}]\,\phi_i^{(n)} = \epsilon_i^{(n)}\,\phi_i^{(n)}$$

$$V_i^{(n+1)}(\vec{r}) = \sum_{j \neq i} \int d^3\vec{r}_j \, |\phi_j^{(n)}|^2 V(\vec{r} - \vec{r}_j)$$

$$[T_i + V_i^{(n+1)}]\,\phi_i^{(n+1)} = \epsilon_i^{(n+1)}\,\phi_i^{(n+1)}$$

If not convergent

We repeat until : $\phi_i^{(n)} \simeq \phi_i^{(n+1)}$ →

This is really hard

(still easier than the A-body Schrö)

[THE MEAN-FIELD POTENTIAL] (12)

Strategy B : the (probable) real situation

=D When $\phi_i^{(n+1)} = \phi_i^{(n)}$, then we say that
the calculation is self-consistent

But, previously we made the simplification that we are dealing with distinguishable particles

=D Things get more complicated with fermions & bosons

[THE MEAN-FIELD POTENTIAL] (12)

$\Rightarrow$ Identical particles:

$$\Psi_A = \prod_j \phi_j(\vec{r}_j) \Longrightarrow \Psi_D = \mathcal{A}\left[\prod_j \phi_j(\vec{r}_j)\right] \text{ (Fermions)}$$

$$= S\left[\prod_j \phi_j(\vec{r}_j)\right] \text{ (bosons)}$$

$\Rightarrow$ The mean-field potential will change

[THE MEAN-FIELD POTENTIAL] (§3)

⇒ Identical particles: mean-field potential

$$\Big[T_i + \sum_{j\neq i} \int d^3\vec{r}_j \, |\phi_j(\vec{r}_j)|^2 \, V(\vec{r}-\vec{r}_j) \, \phi_i(\vec{r}) \Big]$$

Hartree term

$$\pm \sum_{j\neq i} \int d^3\vec{r}_i d^3\vec{r}_j \, \phi_j^*(\vec{r}_j) \phi_j(\vec{r}) V(\vec{r}-\vec{r}_j) \phi_i(\vec{r}_j) \Big] = \epsilon_i \phi_i(\vec{r})$$

Fock term

(identical particle)

+ : bosons

− : fermions

[THE MEAN-FIELD POTENTIAL] (33)

⇒ What we have obtained here is a non-local potential:

Reminder → $\left[-\frac{\nabla^2}{2m}\phi(\vec{r}) + \int d^3\vec{r}\, V(\vec{r},\vec{r}')\,\phi(\vec{r}')\right] = \epsilon\,\phi(\vec{r})$

non-local potential

$V(\vec{r},\vec{r}') = \delta^{(3)}(\vec{r}-\vec{r}')\,V(\vec{r})$ → local potential

(especial case of all possible potentials)

[THE MEAN-FIELD POTENTIAL] (14)

⇒ In the form $\int V(\vec{r},\vec{r}')\phi(\vec{r}')$,

the Hartree-Fock potential looks like:

$$V_i(\vec{r},\vec{r}') = \delta^{(3)}(\vec{r}-\vec{r}')\sum_j \int d^3\vec{r}'\,|\phi_j(\vec{r}')|^2 V(\vec{r}-\vec{r}')$$

Hartree-term: local

$$\pm \sum_j V(\vec{r}-\vec{r}')\,\phi_j^*(\vec{r}')\,\phi_j(\vec{r})$$

Fock term: non-local

[THE MEAN-FIELD POTENTIAL] (15)

$\Rightarrow$ Now, all that is left is to iterate:

$$\psi^{(0)} \to V^{(1)} \to \psi^{(1)} \to V^{(2)} \to \psi^{(2)} \to \dots \to \psi^{(n)} \to V^{(n)} \to \psi^{(n+1)}$$

$\Rightarrow$ Problem: still very computationally demanding

$\psi^{(n)} \simeq \psi^{(n+1)}$ and now we can stop $\lightning$

[THE HARTREE-FOCK METHOD]

⇒ Ideally, we should try to simplify the previous methods

$|\Psi_A\rangle = \mathcal{A}[\prod_j (\phi_j \sigma_j)]$ → this is not the full w.f. (but an approximation) ✓

How to choose a good approx. w.f. in the first place

↓

Variational principle

[THE HARTREE-FOCK METHOD]

⇒ Variational principle:

$$\delta\left(\frac{\langle\Psi_A|H|\Psi_A\rangle}{\langle\Psi_A|\Psi_A\rangle}\right) = 0$$

→ this condition defines a wave function that minimizes the energy of the system

RECAP

① Let's imagine that I know the true w.f.

$$E_A^{true} = \frac{\langle\Psi_A^{true}|H|\Psi_A^{true}\rangle}{\langle\Psi_A^{true}|\Psi_A^{true}\rangle}$$

② Now, I compute the same matrix element with an arbitrary w.f. $|\Psi_A^{trial}\rangle$

$$E_A^{true} \leq E_A^{trial} = \frac{\langle\Psi_A^{trial}|H|\Psi_A^{trial}\rangle}{\langle\Psi_A^{trial}|\Psi_A^{trial}\rangle}$$

[THE HARTREE-FOCK METHOD]

⇒ The key idea of the variational principle is that

the exact wave function $|\Psi_A^{true}\rangle$ minimizes E_A $\quad (E_A^{min} = E_A^{true})$

⇒ Thus, by asking the condition:

$$\delta\left(\frac{\langle\Psi_A|H|\Psi_A\rangle}{\langle\Psi_A|\Psi_A\rangle}\right) = 0$$

what we will obtain is the best approx. to E_A^{true} that is possible with our family of trial w.f.

e.g. $E_A^{trial} = A[\prod_j \phi_j(r_j)]$

[THE HARTREE-FOCK METHOD]

⇒ But how can we solve $\delta\left(\frac{\langle\psi_A|H|\psi_A\rangle}{\langle\psi_A|\psi_A\rangle}\right) = 0$?

⇒ We use Lagrange mult. pliers

a) We minimize $\langle\psi_A|H|\psi_A\rangle$

b) with the condition $\langle\psi_A|\psi_A\rangle = 1$

Method of Lagrange multiplier ↑

PROBLEM: min of $f(x)$ / $g(x) = 0$

SOLUTION: define $\mathcal{L}(x,\lambda) = f(x) - \lambda g(x)$ and then minimize $\mathcal{L}(x,\lambda)$ instead

[THE HARTREE-FOCK METHOD]

⇒ Lagrange multipliers

$$\left[\min \text{ of } f(x) / g(x)=0 \right\} \Rightarrow \min \text{ of } \mathcal{L}(x,\lambda) = f(x) - \lambda g(x) \Big]$$

For HF we will have:

$$\mathcal{L}(\phi_i,\lambda_i) = \langle \psi_0 | H | \psi_0 \rangle - \sum \lambda_i [\langle \phi_i | \phi_i \rangle - 1]$$

and then we minimize...

[THE HARTREE-FOCK METHOD]

$$\mathcal{L}(\phi_i, \lambda_i) = \langle \psi_0 | H | \psi_0 \rangle - \sum \lambda_i (\langle \phi_i | \phi_i \rangle - 1)$$

↳ This is minimized as follows:

$$\frac{\delta}{\delta \phi_j^*} \left[\langle \psi_0 | H | \psi_0 \rangle - \sum \lambda_i \langle \phi_i | \phi_i \rangle \right] = 0$$

we drop the "-1" bc $\frac{\delta}{\delta \phi_j^*}(-1) = 0$

Remember that:

$$\langle \psi_0 | H | \psi_0 \rangle = \sum_i \langle \phi_i | \Pi_i | \phi_i \rangle + \frac{1}{2} \sum_{ij} \left(\langle \phi_i \phi_j | v | \phi_i \phi_j \rangle - \langle \phi_i \phi_j | v | \phi_j \phi_i \rangle \right)$$

(assumes fermions) ⟶

[THE HARTREE-FOCK METHOD]

a) We have: $\frac{\delta}{\delta \phi_j^*}\left[\langle \psi_A | H | \psi_A \rangle - \sum \lambda_i \langle \phi_i | \phi_i \rangle\right] = 0$

with: $\langle \psi_A | H | \psi_A \rangle = \sum_i \langle \phi_i | T_i | \phi_i \rangle + \frac{1}{2}\sum \langle \phi_i \phi_j | V | \phi_i \phi_j \rangle$

$$- \frac{1}{2}\sum \langle \phi_i \phi_j | V | \phi_j \phi_i \rangle$$

b) This leads to:

$$T_i|\phi_i\rangle + \sum_j \int d^3\vec{r}_j \, |\phi_j(\vec{r}_j)|^2 V(\vec{r}-\vec{r}_j)\,\phi_i(\vec{r})$$

$$- \sum_j \int d^3\vec{r}_j \, \phi_j^*(\vec{r}_j)\,\phi_j(\vec{r})\,V(\vec{r}-\vec{r}_j)\,\phi_i(\vec{r}_j) = \lambda_i \phi_i(\vec{r})$$

with $\lambda_i = \epsilon_i$ $\Rightarrow$ minimizing has resulted in the Schrödinger eq.

[THE HARTREE-FOCK METHOD]

⇒ But this is the same result as before,
what have we learned?

⇒ Solving HF should already gives us a decent description of the wave function

Yet, the main problem is the choice of $V(\vec{r}_i - \vec{r}_j)$ (this sets how difficult it will be to solve HF)

initial 2B pot

[HARTREE-FOCK AND CONTACT POTENTIALS]

⇒ As usual, there is an extremely convenient choice of potential: $V(\vec{r}-\vec{r}') = C\,\delta^{(3)}(\vec{r}-\vec{r}')$ (contact-range pot.)

Why is it convenient?

$$V^{Fock}(\vec{r}) = \pm \sum_j V(\vec{r}-\vec{r}')\,\phi_j^*(\vec{r}')\,\phi_j(\vec{r}')$$

$$= \pm C\,\delta(\vec{r}-\vec{r}') \sum_j |\phi_j(\vec{r})|^2$$

This choice will be popular

⇒ The Fock potential becomes local

[HARTREE-FOCK AND CONTACT POTENTIALS]

Example: a one-level system + contact-range
+ fermions

For fermions, only
the Hartree-term survives:

$$T\phi(\vec{r}) + \int d^3\vec{r}\,' |\phi(\vec{r}\,')|^2 \underbrace{V(\vec{r}-\vec{r}\,')}_{C\,\delta^{(3)}(\vec{r}-\vec{r}\,')} \phi(\vec{r}) = \epsilon\phi(\vec{r})$$

$$-\frac{\nabla^2}{2m}\phi(\vec{r}) + C\,\rho(\vec{r})\,\phi(\vec{r}) = \epsilon\phi(\vec{r})$$

$\rho(\vec{r}) = |\phi(\vec{r})|^2$ → density-dependent potentials

[HARTREE-FOCK AND CONTACT POTENTIALS]

⇒ A possible toy model: A-boson system with one-level
(now ∃ a Fock term w/ + sign and equal to
the Hartree term)

$$-\frac{\vec{\nabla}^2}{2m}\phi(\vec{r}) + 2C(A-1)|\phi(\vec{r})|^2\phi(\vec{r}) = \epsilon\phi(\vec{r})$$

⇒ This model should still work for nuclei
with $\Delta \leq 4$ (^{4}He) → nucleons have
4 states
($p\uparrow$ $p\downarrow$ $n\uparrow$ $n\downarrow$)

[HARTREE-FOCK AND CONTACT POTENTIALS]

⇒ A possible toy model: A-boson system with one-level

$$\left[-\frac{\vec{\nabla}^2}{2m}\phi(\vec{r}) + 2C\binom{A}{2}|\phi(\vec{r})|^2\phi(\vec{r})\right] = \epsilon\phi(\vec{r})$$

1) Fix C to the deuteron binding energy

⇒ $B(^4He) \sim 12\,MeV$

2) Fix C to the triton binding energy

⇒ $B(^4He) \sim 17\,MeV$

$B_{exp}(^4He) \sim 28\,MeV$

This means that this still example works rather good

[SKYRME INTERACTIONS]

=> In general, contact-range forces are really practical for Hartree-Fock

a) Skyrme forces

b) Gogny forces

a) => $V_{2B}(\vec{r}_1-\vec{r}_2) = t_0(1+x_0P_\sigma)\,\delta^{(3)}(\vec{r}_1-\vec{r}_2) + \frac{1}{2}t_1\left\{\delta^{(3)}(\vec{r}_1-\vec{r}_2),(-\vec{\nabla})^2\right\}$

$+t_2\,\vec{\nabla}\,\delta^{(3)}(\vec{r}_1-\vec{r}_2)\cdot\vec{\nabla} + iW_0(\vec{\sigma}_1+\vec{\sigma}_2)\cdot\left[\vec{\nabla}_1(\delta^{(3)}(\vec{r}_1-\vec{r}_2)\vec{\nabla}\right]$

$V_{3B} = t_3\,\delta^{(3)}(\vec{r}_1-\vec{r}_2)\,\delta^{(3)}(\vec{r}_2-\vec{r}_3)$

[SKYRME INTERACTIONS]

Comments: a.1) $t_0, x_0, t_1, t_2, w_0, t_3$ are parameters

a.2) $P_\sigma = \frac{1}{2}(1 + \vec{\sigma}_1 \cdot \vec{\sigma}_2)$, $P_\tau = \frac{1}{2}(1 + \vec{\tau}_1 \cdot \vec{\tau}_2)$

a.3) $\exists$ a lot of parametrization for Skyrme (~ 240)

a) $\Rightarrow$ $V_{2B}(\vec{r}_1 - \vec{r}_2) = t_0(1 + x_0 P_\sigma)\,\delta^{(3)}(\vec{r}_1 - \vec{r}_2) + \frac{1}{2} t_1 \left\{ \delta^{(3)}(\vec{r}_1 - \vec{r}_2), (-\vec{\nabla})^2 \right\}$

$+ t_2 \vec{\nabla}\,\delta^{(3)}(\vec{r}_1 - \vec{r}_2) \cdot \vec{\nabla} + i w_0 (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \left[\vec{\nabla}_1 \left(\delta^{(3)}(\vec{r}_1 - \vec{r}_2) \vec{\nabla} \right) \right]$

$V_{3B} = t_3\,\delta^{(3)}(\vec{r}_1 - \vec{r}_2)\,\delta^{(3)}(\vec{r}_2 - \vec{r}_3)$

[GOGNY FORCES]

=D Similar to Skyrme, but adds a few finite-range effects:

$$V(\vec{r}_1-\vec{r}_2) = \sum_{j=1}^{2} e^{-(\vec{r}_1-\vec{r}_2)^2/\mu_j^2}(W_j + B_j P_\sigma - H_j P_\tau - M_j P_\sigma P_\tau)$$

$$+ t_3(1+x_0 P_\sigma)\,\delta^{(3)}(\vec{r}_1-\vec{r}_2)\left[\rho\left(\frac{\vec{r}_1+\vec{r}_2}{2}\right)\right]^\alpha$$

$$+ i W_{LS}\,\vec{\nabla}\delta^{(3)}(\vec{r}_1-\vec{r}_2)\times\vec{\nabla}\cdot(\vec{\sigma}_1\cdot\vec{\sigma}_2)$$

W_j, B_j, H_j, μ_j → parameters

RECAP

1) Shell model → Depends on a choice of mean-field pot

1.a) Phenomenological Choice → We propose a potential

1.b) Theory-based → We devise some method to get it

2) Self consistent approach: $V_{HF}^{(0)} \to \phi^{(0)} \to V_{HF}^{(1)} \to \phi^{(1)}$ (repeat until convergence)

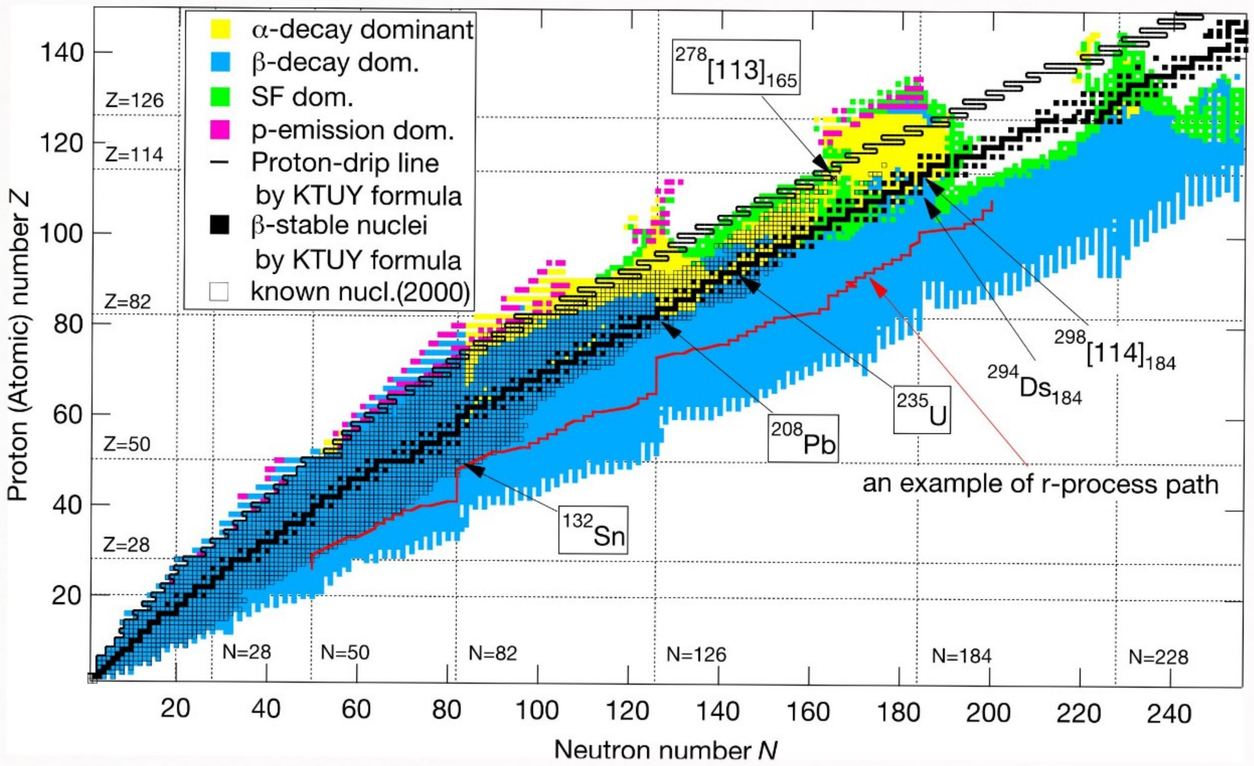
3) Hartree-Fock approach:

$\delta(\langle\phi_A|H|\phi_A\rangle / \langle\phi_A|\phi_A\rangle) = 0$ → Explains success of mean-field pots

4) Contact interactions → Skyrme & Gogny

[Exotic Nuclear Structures]

=> Nuclear structure: extremely complex



a) $A \leq (10\text{-}12)$ ab-initio (i.e. solving Schrödinger) is possible

b) For the rest of nuclei => use nuclear models

[Exotic Nuclear Structures]

=> But a few exceptional cases exist

a) Nuclei that do not fit into conventional models

b) Sometimes they allow for simplifying assumptions

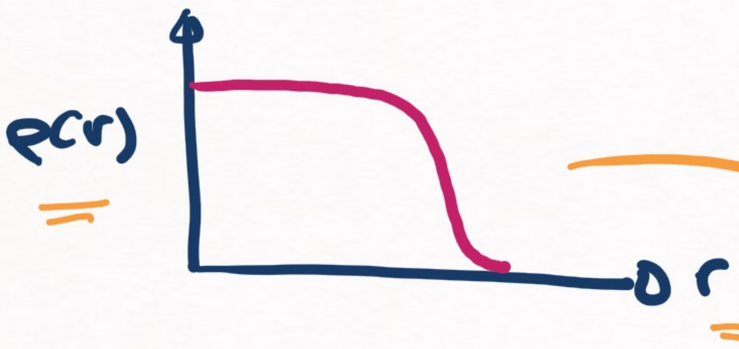
Examples: 1) Halo nuclei:

2) Alpha cluster (^{8}Be, ^{12}C)

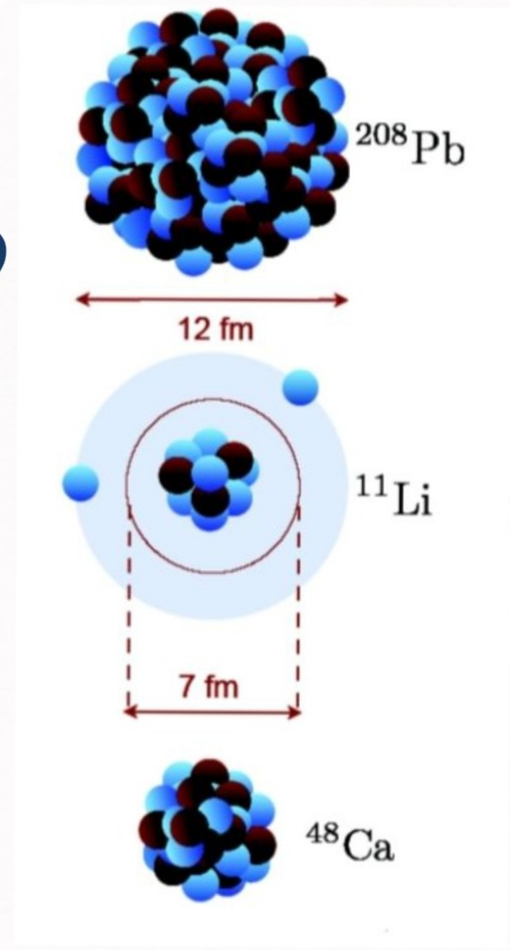
⇝

[HALO NUCLEI]

1) Usual nuclei → almost constant density (and a rapid fall afterwards)



→ Woods-Saxon parametrization



2) Halo nuclei

↳ core (normal nuclear density)

\+

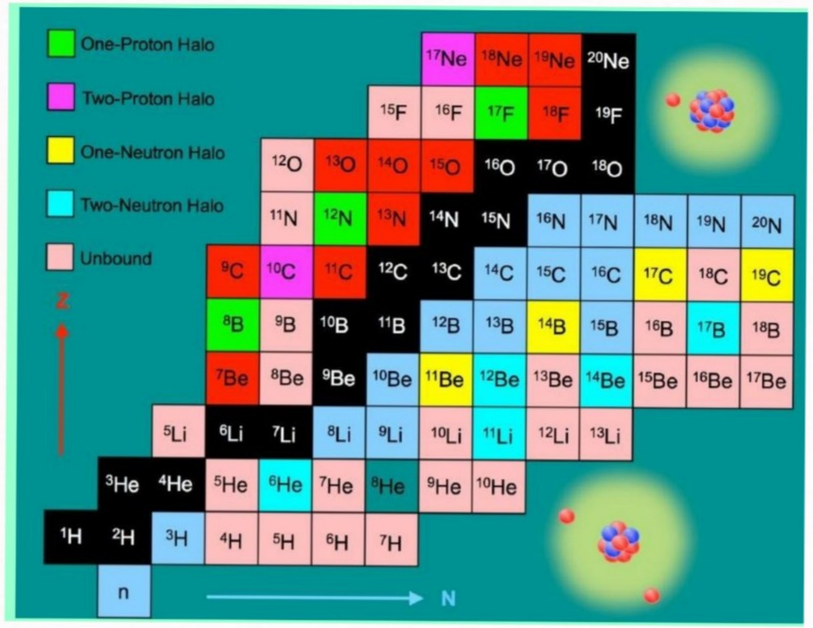
halo (large + low density)

^{11}Li ⇆ ^{9}Li + 2n

(core) (halo)

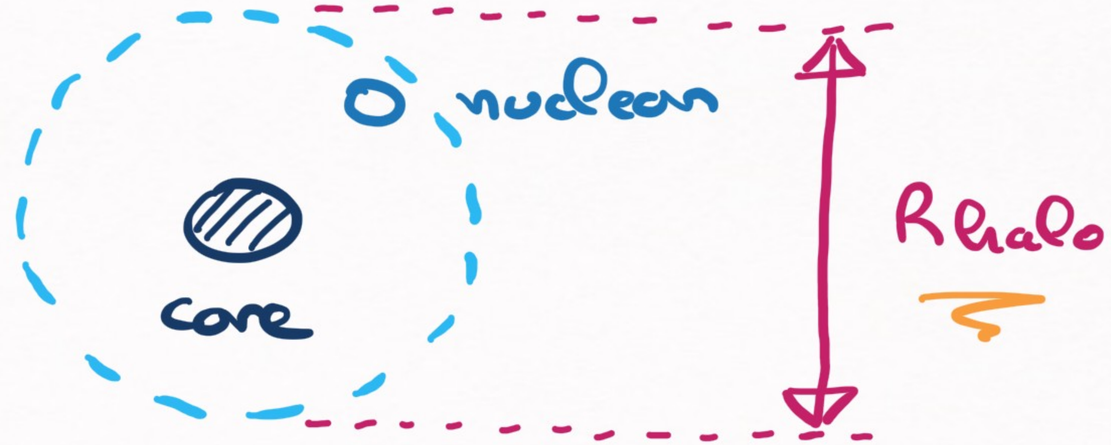
[HALO NUCLEI]

⇒ A few examples:



⇒ Description:

⇒ One nucleon halo:

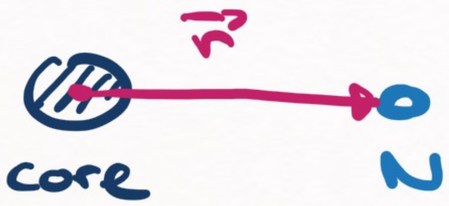


$R_{halo} \gg R_{core}$ ⇒ ☺

☺ ⇒ This means that we can treat the core as a point particle (and use two-body techniques)

[HALO NUCLEI]

3) One nucleon halo ⇒ two body problem w/ a contact-range potential



$V_{core-N}(\vec{r}) = C\,\delta^{(3)}(\vec{r})$

$^{8}B = {}^{7}Be + p$ (p-wave)

$^{8}Li = {}^{7}Li + n$ (p-wave)

$\psi(r) \to A_p\left(1+\frac{1}{\beta r}\right)e^{-\beta r}$

→ The same as we saw in previous lessons

(contacts, regularization, renormalization, etc.)

[HALO NUCLEI]

⇒ Typical calculations w/ Halo nuclei:

a) $p + {}^{7}Be \rightarrow {}^{8}B + \gamma$: very important for solar neutrino production (intermediate step)

b) $n + {}^{7}Li \rightarrow {}^{8}Li + \gamma$: isospin mirror of previous reaction (does not contain Coulomb)

⇒ These calculations are in general easy

[HALO NUCLEI]

2) Two-nucleon halo: easy 3-body problem

(3 body + contacts → easy)

Examples: ^{6}He, ^{6}Li, ^{8}Be

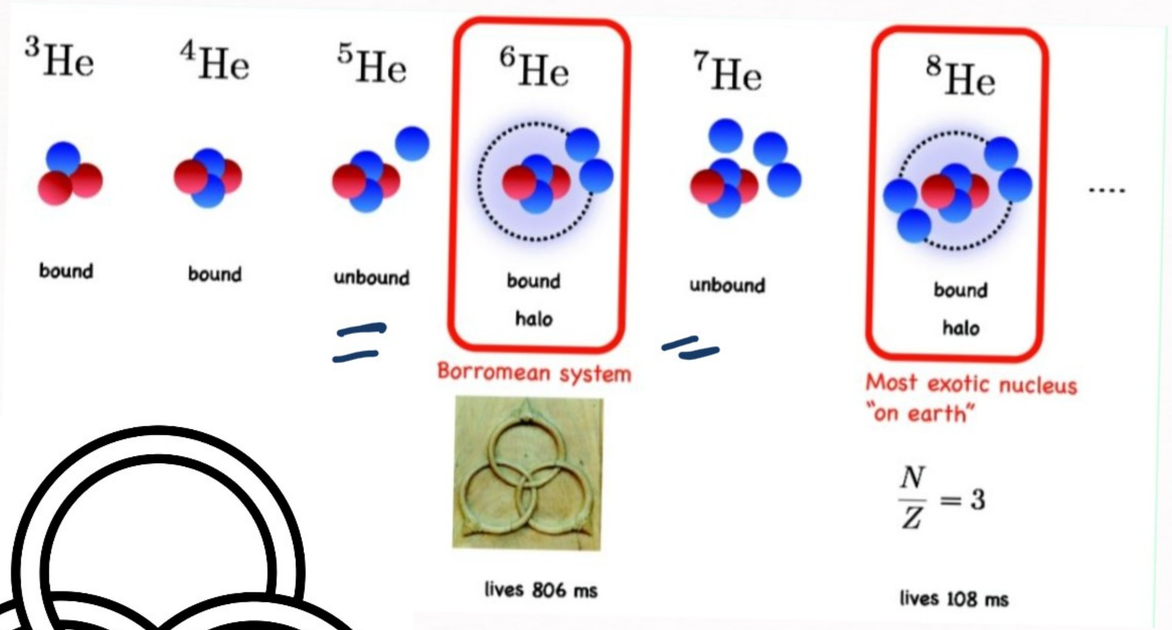
^{4}He (core) + nn / np / pp

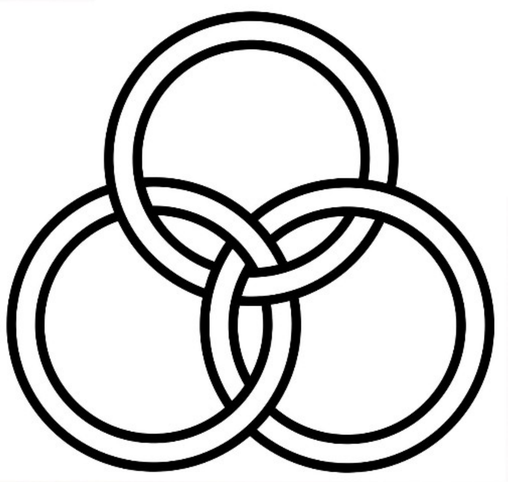
Observation: ^{4}He + n has no bound state

(but this is still a p-wave resonance)

[HALO NUCLEI]

2.b) Borromean nuclei: just a special case of a two-nucleon halo





[Core + 1N unbound
Core + 2N bound]

↓

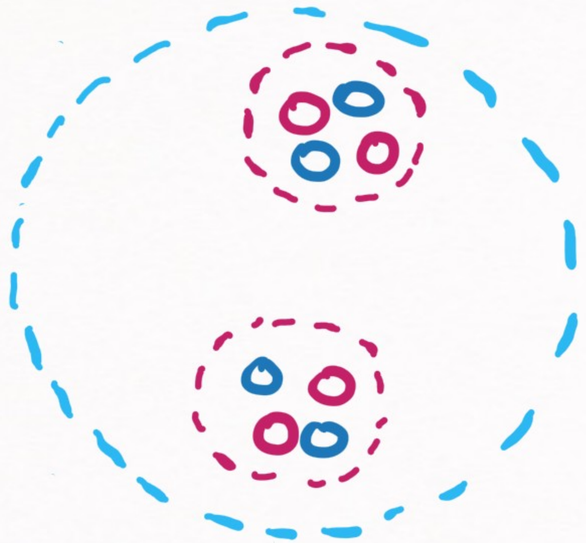
Borromean nucleus

(^{6}He, ^{11}Li, ^{17}B)

[ALPHA CLUSTERS]

⇒ Many nuclei can be described as bound states of α-particles (instead of nucleons)

Reason: α-particle is really stable



→ ^8Be : (a) not bound

(b) but forms a shallow S-wave resonance

[ALPHA CLUSTERS]

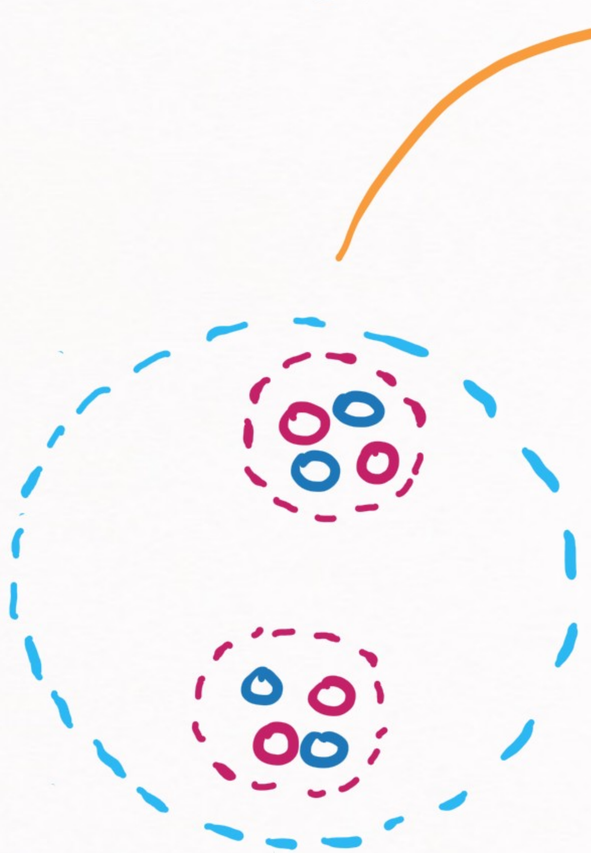
$E(^8Be) = E_R - i\frac{\Gamma_R}{2}$

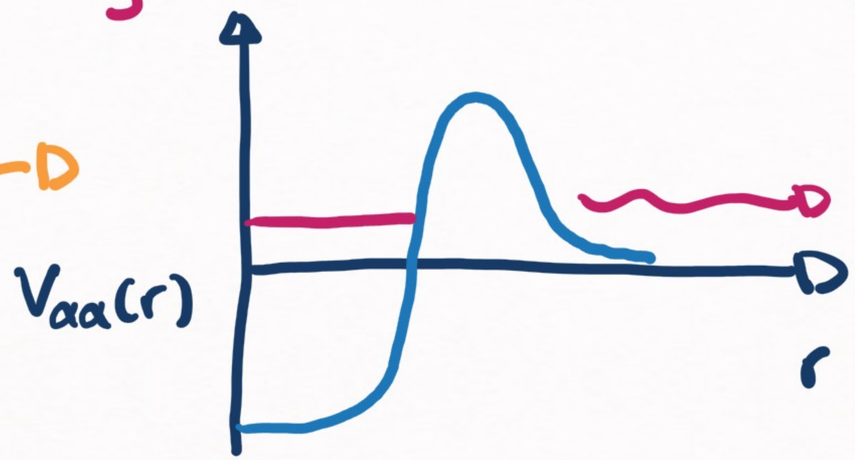
$E_R = 91.8$ KeV

$\Gamma_R = 5.57$ eV

(long-lived)

→ ^8Be:





Short range nuclear attraction

Long-range Coulomb repulsion

HOYLE STATE

⇒ Related with ${}^{8}Be$:

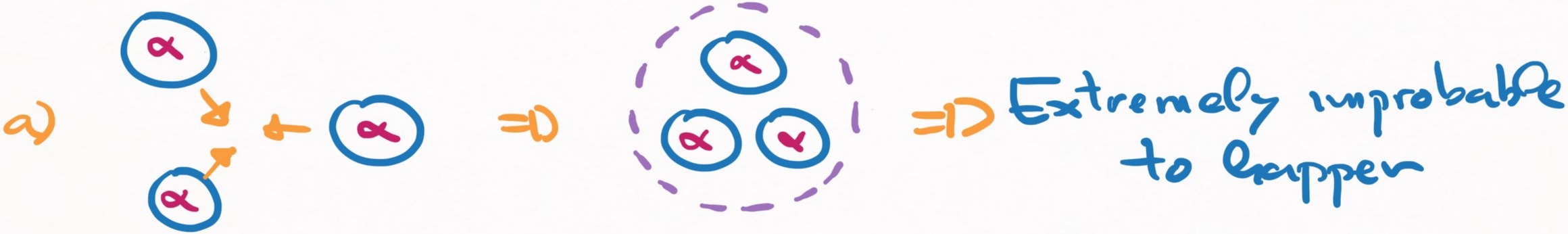
a) $\nexists$ $A = 5, 8$ stable nuclei

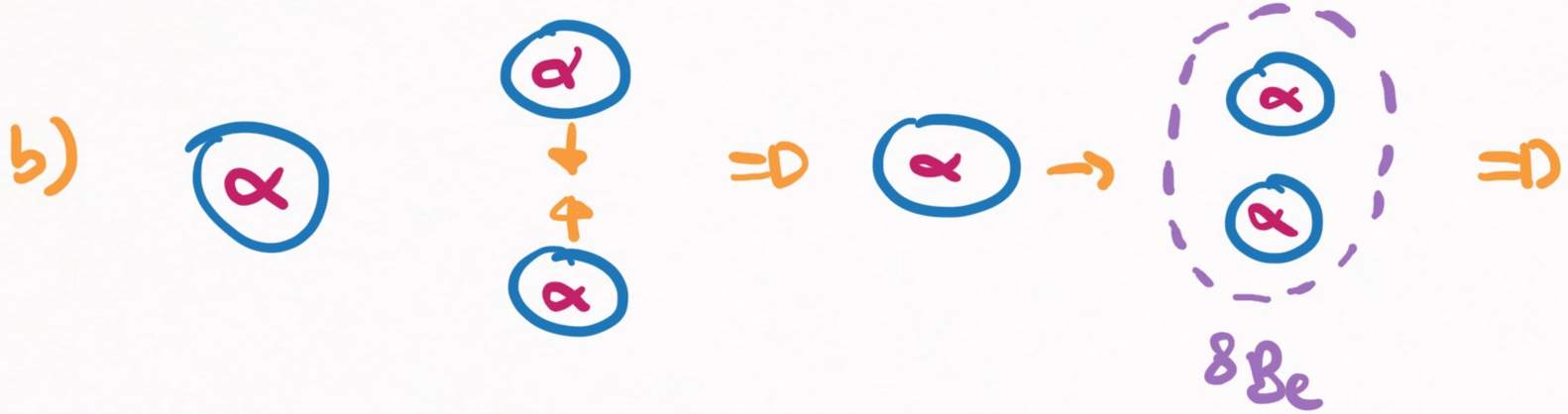
b) But $\exists$ a lot of ${}^{12}C$ in the universe

⇒ Hoyle basically deduced the location of an excited state of ${}^{12}C$ that was required to explain ${}^{12}C$ production

(and later was found in experiment)

[HOYLE STATE : TRIPLE ALPHA PROCESS]

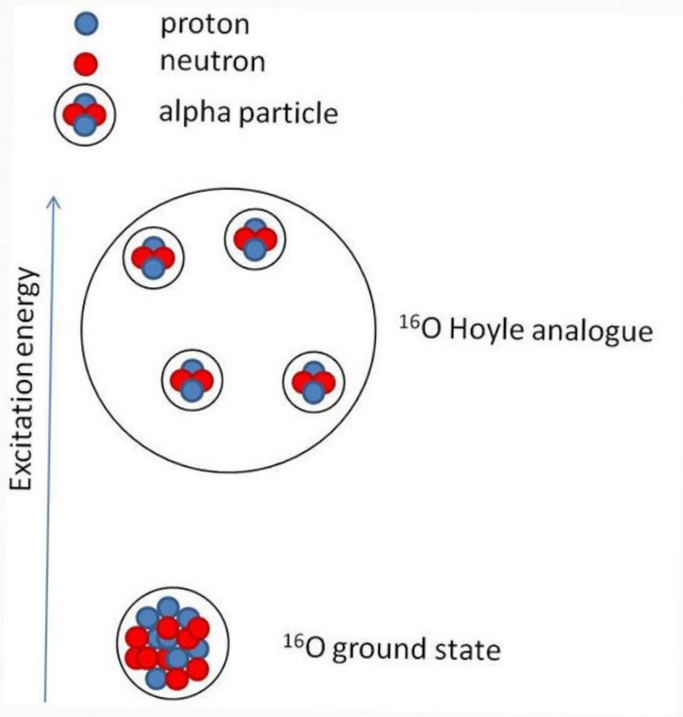
a) 

b) 

⇒ much more probable

[HOYLE STATE : TRIPLE ALPHA PROCESS]

=> The bottom-line : the processes in the previous slide are basically 2- and 3-body processes (easy to calculate)



=> This idea can be extended to heavier nuclei
=>

[HYPERNUCLEI]

a) Normal nuclei: nucleons → $|p\rangle = |uud\rangle$, $|n\rangle = |udd\rangle$

But, in principle it could be possible to have bound states w/ other types of baryons

b) Hypernuclei: we change one or more nucleons by a hyperon (baryon containing an s-quark)

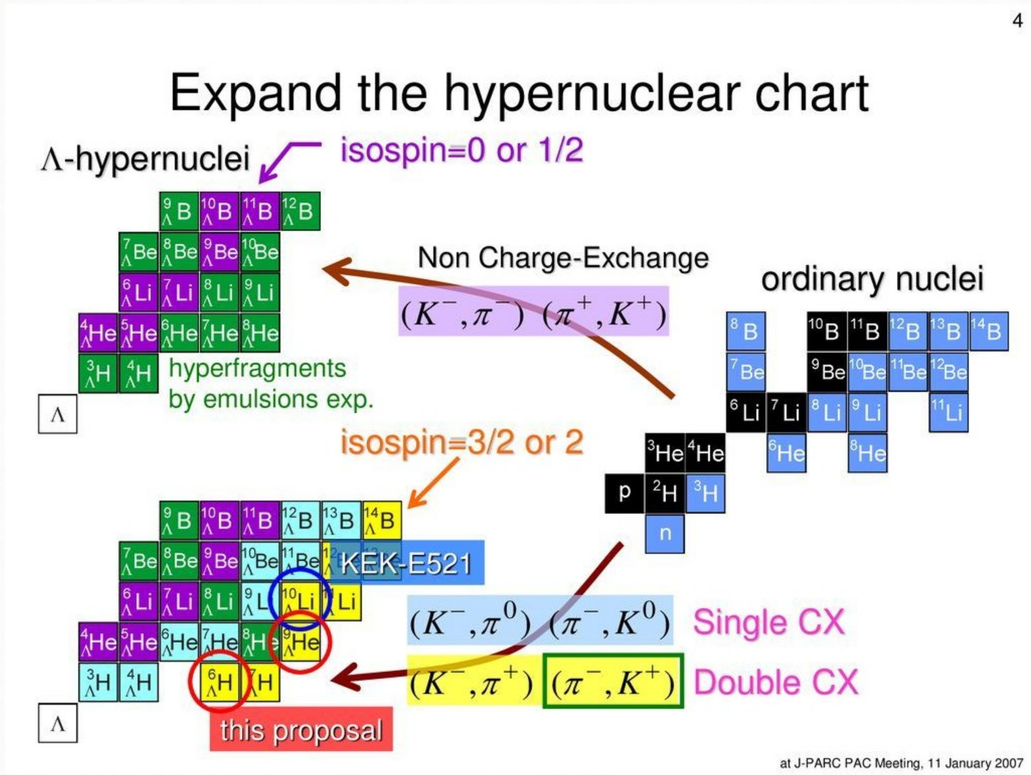
$|\Lambda\rangle = |uds\rangle$, $I=0$, $S=1/2$

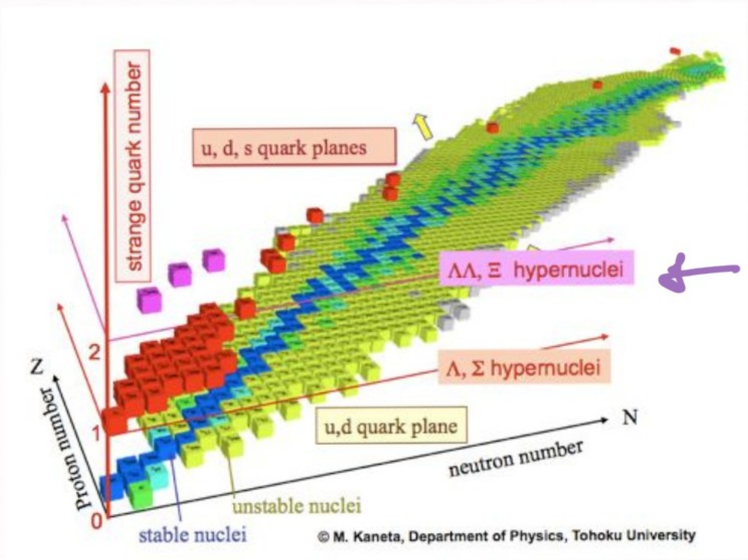
$|\Sigma\rangle = |qqs\rangle$ w/ $q=u,d$ $I=1$, $S=1/2$

$|\Xi\rangle = |qss\rangle$, $I=1/2$, $S=1/2$

[HYPERNUCLEI]

⇒ Basically, it's just an extension of the nuclear chart:





[HYPERNUCLEI]

⇒ Why are hypernuclei so common?

a) Pauli principle does not apply to a single hyperon (is not a nucleon), and this allows the hyperon to be in S-wave.

b) Even if the ΛN interaction is weak (provided it is attractive), if ∃ enough nucleons, the Λ will eventually bind

(Y)

RECAP Besides usual nuclei, there are other nuclear configurations:

a) Halo nuclei ⇒ nuclear core + loosely bound nucleon(s)

b) α-clusters ⇒ help to model many nuclei
⇒ useful for triple-alpha process

c) Hypernuclei ⇒ a possible extension of the nuclear chart

⇒ All of them are good research projects

Problem → difficult exercises (only do one)

139.180.195.74

Nuclear Physics, Spring 2023

In this server, we will compile the lecture notes of the course, the exercise sheets and the slides of the classes.

The lecture notes of this year are under development (actually, they are always under development: the course changes a bit every year), and you should come here from time to time for updates. Currently we have the following notes:

- General introduction
- The two-nucleon system
- Nuclear Structure

Additional materials: SU(3) Clebsch-Gordan coefficients, ChPT Reviews, the OBE model.

This year we will have a problem and a theory exercise sheet: problems are hard-to-solve questions, while theory are easy-to-solve and only require the direct application of some idea seen during the course. Currently they are in development, buy by next Monday they should be finished and you will be able to choose:

wait till Monday →

- Problems (2023)
- Theory (2023)

} → you can begin to check them

Theory → easy exercises

We can have extra sessions for problem solving

We have finished