NUCLEAR PHYSICS 21

THE SHELL MODEL:

RESIDUAL INTERACTIONS

RECAP [THE SHELL MODEL]

1) Explains the existence of magic numbers ✓

⇒ This requires a spin-orbit interaction

2) Explains the J^P of the ground & excited states of nuclei with N, Z close to a magic number

3) However, it fails if N, Z are far away of a magic number

⇒ $\exists$ other nuclear models for this (collective model)

RECAP! ASSUMPTIONS OF THE SHELL MODEL (I)

1) We don't have to solve the full Hamiltonian with all the interactions

$\Rightarrow$ Instead, we can assume the existence of an average field

$$H = \sum_i T_i + \sum_{ij} V_{ij}^{2B} + \sum_{ijk} V_{ijk}^{3B} + \dots$$

Residual interaction

$$\Rightarrow H = \sum_i \left(T_i + V_i^{MF}\right) + \underbrace{\Delta V}$$

We assume that this is small

RECAP! ASSUMPTIONS OF THE SHELL MODEL (II)

2) Nucleons are fermions ✓

$\Rightarrow$ We simply fill the shells

Example: $V^{MF} = \frac{1}{2} m \omega^2 r^2 - \xi \vec{\ell} \cdot \vec{s}$ ←

→ Lowest energy level: $1s_{1/2}$, $\epsilon = \frac{3}{2}\omega$ $\Rightarrow$ 2 neutrons, 2 protons

→ Next energy level: $1p_{3/2}$, $\epsilon = \frac{5}{2}\omega - \frac{\xi}{2}$

$\Rightarrow$ 6 neutrons, 6 protons

(and so on)

[BUT... $\exists$ TWO OPEN PROBLEMS HERE:] 1

1) How do we find out a suitable V^{MF}?

Previously, we simply assumed a V^{MF}

$$V^{MF} = \frac{1}{2}m\omega^2 r^2 - \xi \vec{\ell}\cdot\vec{s}$$

this works pretty well (at least at explaining the orbitals)

$\Rightarrow$ This V^{MF} is purely phenomenological

Q: Are there better ways to find V^{MF}?

A: Hartree-Fock, Skyrme/Gogny interactions (next lesson)

[BUT... ∃ TWO OPEN PROBLEMS HERE:] 2

2) How do we deal with $\underline{\underline{\Delta V}}$?

⟹ Previously, we have learned about the pairing interaction (a type of ΔV)

$$\langle jj(JM)|V_{pairing}|jj(JM)\rangle = -\frac{1}{2}g(j+1)\delta_{J0}\delta_{M0}$$

$(g>0)$

(also phenomenological)

⟹ But, we would like a more systematic treatment of ΔV

(today)

[BUT... $\exists$ TWO OPEN PROBLEMS HERE:] ③

1) Defining a mean field $\Rightarrow$ Next (exact) lesson

2) Residual interactions $\Rightarrow$ This (today's) lesson

$\Downarrow$

First thing: we need to find a strategy for dealing with ΔV

[SHELL MODEL: DEFINING A BASIS] 1

⇒ Let's review (again) the ingredients of the shell model:

1) Mean field potential ⇒ $H = \sum_i h_i + \Delta V$, $h_i = T_i + V_i^{MF}$

2) Monoparticular wave functions: (solutions of h_i)

$$h_i \phi^{(n)} = \epsilon_n \phi^{(n)}$$

3) Total wave function is obtained by antisymmetrization:

$$\Psi_a = \mathcal{A}\left[\prod_{i=1}^{A} \phi_i\right]$$

→ We are going to reinterpret this in a different way

[SHELL MODEL: DEFINING A BASIS] 2

⇒ Detail: the set of all Φ_a's defines a basis of the A-body Hilbert space ←

Example: $A = 1$, harmonic oscillator as the $V^{MF}(x)$ in one dimension

$$H|\Phi\rangle = E|\Phi\rangle \Rightarrow -\frac{1}{2m}\psi''(x) + V(x)\psi(x) = E\psi(x)$$

We are assuming that the mean field is:

$$V = V(x) = \frac{1}{2}m\omega^2x^2 + \boxed{\left(V(x) - \frac{1}{2}m\omega^2x^2\right)} = V^{MF} + \boxed{\Delta V}$$

[SHELL MODEL : DEFINING A BASIS] 3

⇒ So far we have :

$H|\Psi\rangle = E|\Psi\rangle$ with $H = T + V$

Then we write: $V = \underbrace{\frac{1}{2}m\omega^2x^2}_{V^{MF}} + \underbrace{\left(V(x) - \frac{1}{2}m\omega^2x^2\right)}_{\Delta V}$

→ $H^{MF} = T + V^{MF}$

The solution for a harmonic oscillator:

$H^{MF}|n\rangle = \hbar\omega(n + 1/2)|n\rangle$ (check your QM textbook)

For the moment this is the same as we did with the shell model

[SHELL MODEL: DEFINING A BASIS] ④

$\Rightarrow$ Then we notice that the oscillator basis is indeed a basis

$$\mathbb{1} = \sum_n |n\rangle\langle n| \longleftrightarrow \mathbb{1} = \int \frac{dp}{2\pi} |p\rangle\langle p|$$

(analogous) or $\mathbb{1} = \int dx\, |x\rangle\langle x|$

This means that we can write:

$$|\psi\rangle = \mathbb{1}|\psi\rangle = \sum_n |n\rangle\langle n|\psi\rangle = \sum_n \psi_n |n\rangle$$

$$\left(|\psi\rangle = \int dx\, \psi(x)\, |x\rangle \right.$$
$$\left. \text{or } |\psi\rangle = \int \frac{dp}{2\pi} \psi(p)\, |p\rangle \right)$$

[SHELL MODEL : DEFINING A BASIS] 5

⇒ And we write Schrödinger in the oscillator basis:

$$|\psi\rangle = \sum_n |n\rangle\langle n|\psi\rangle = \sum_n \psi_n |n\rangle$$

And we have: $H|\psi\rangle = E|\psi\rangle$ (full hamiltonian)

$$H_{mn} = \langle m|H|n\rangle$$

$$\left[\sum_n H_{mn}\psi_n = E\psi_m\right]$$

(with H_{mn} a infinitely large matrix)

[SHELL MODEL : DEFINING A BASIS] 6

$\Rightarrow$ Notice that this is not new : $H|\psi\rangle = E|\psi\rangle$

1) $\langle x|H|\psi\rangle = \int dx' \langle x|H|x'\rangle\langle x'|\psi\rangle$

$|\psi\rangle = \int dx\, |x\rangle\langle x|\psi\rangle$

$\langle x|H|x'\rangle = \left(-\frac{1}{2m}\frac{d^2}{dx^2} + U(x)\right)\delta(x-x')$

$\Rightarrow \left(-\frac{1}{2m}\frac{d^2}{dx^2} + U(x)\right)\psi(x) = E\psi(x)$

$H_{mn}\psi_n = E\psi_m$

same concept, different basis

[SHELL MODEL: DEFINING A BASIS] ⑦

$\Rightarrow$ Notice that this is not new: $H|\psi\rangle = E|\psi\rangle$

1) $|\psi\rangle = \int dx\, |x\rangle \psi(x) \Rightarrow \left[-\frac{1}{2m}\frac{d^2}{dx^2} + V(x)\right]\psi(x) = E\psi(x)$

2) $|\psi\rangle = \sum_n |n\rangle \psi_n \Rightarrow H_{mn}\psi_n = E\psi_m$

(infinitely-dimensional matrix equation)

[SHELL MODEL : DEFINING A BASIS] 8

⇒ Basic idea: you can solve in whatever basis you like

Schrödinger in r-space

Equivalent

Schrödinger in oscillator space

$$\left(-\frac{1}{2m}\frac{d^2}{dr^2}+V(r)\right)\psi(r) = E\psi(r)$$

$$H_{mn}\psi_n = E\psi_n$$

[SHELL MODEL: DEFINING A BASIS] 9

=> Now, we only have to extend this idea from 1-dimension and 1-particle to 3-dimensions and A-particles (conceptually simple, technically complicated)

1) $H = \sum_i T_i + \sum_{ij} V_{ij}^{2B} + \sum_{ijk} V_{ijk}^{3B} + \ldots = \sum_i (T_i + V_i^{HF}) + \Delta V$

$\hookrightarrow$ $H_{\alpha\beta} \Psi_\beta = E \Psi_\beta$ (the same idea)

[SHELL MODEL : DEFINING A BASIS] 10

1) $H = \sum_i T_i + \sum_{ij} V^{2B}_{ij} + \sum_{ijk} V^{3B}_{ijk} + \ldots = \sum_i (T_i + V^{HF}_i) + \Delta V$

2) $H_{\alpha\beta} \Phi_\beta = E \Phi_\alpha$ w/ $\Phi_\alpha = \mathcal{A}\left[\prod_{i=1}^{A} \phi_i\right]$

Each α is now a set of harmonic oscillator quantum numbers for particles $i = 1, 2, \ldots, A$

[SHELL MODEL: TRUNCATING THE BASIS] 1

⇒ There is an obvious problem:

infinitely-dimensional basis

⇓

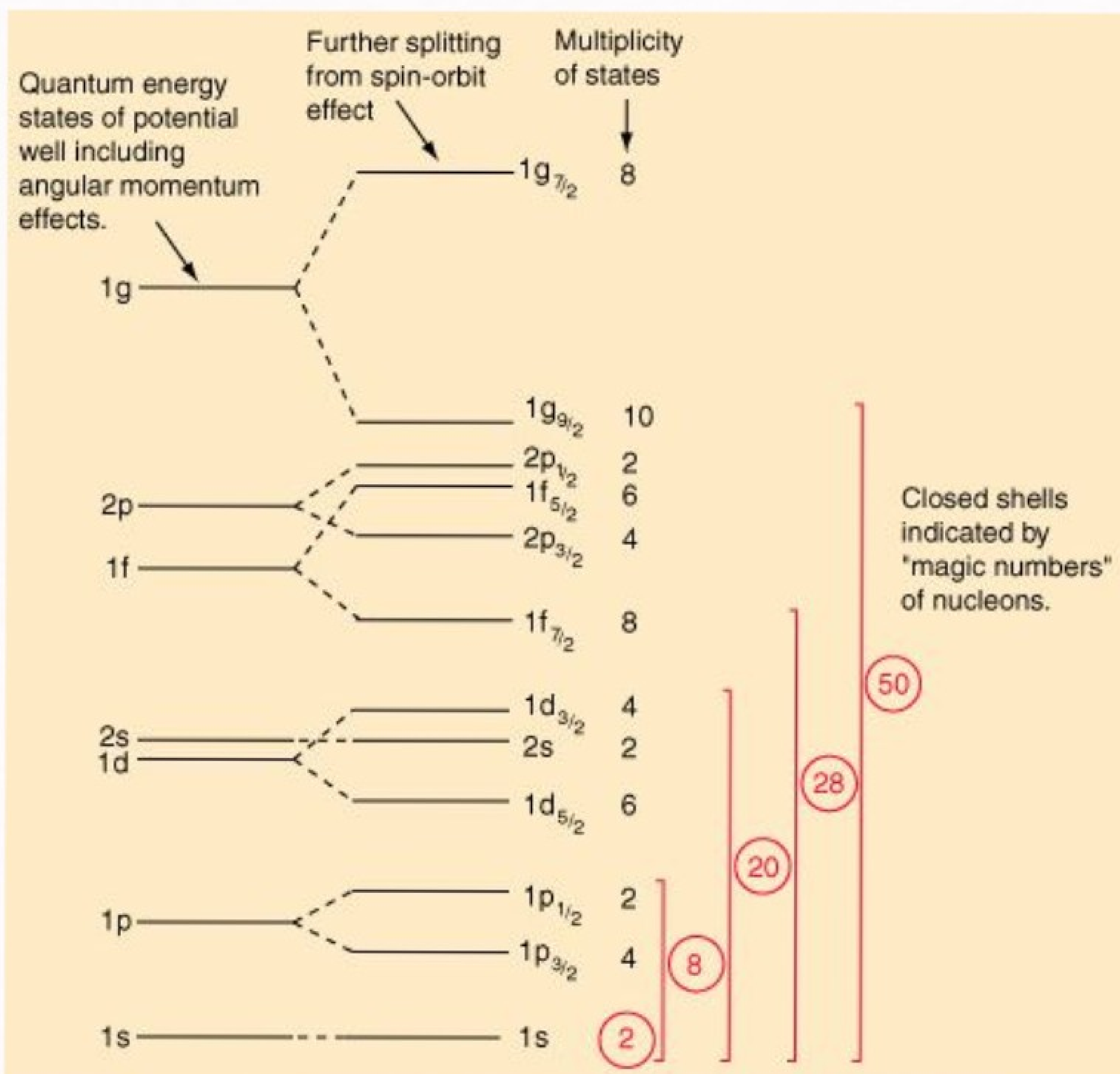
If we want to be able to do calculations,
we have to truncate the basis

⇓

We have to restrict ourselves to
the shells that are important

[SHELL MODEL: TRUNCATING THE BASIS] 2

⇒ But we already know that the shell-model works



→ The ordering indicates that:

1) Not all shells are equally important

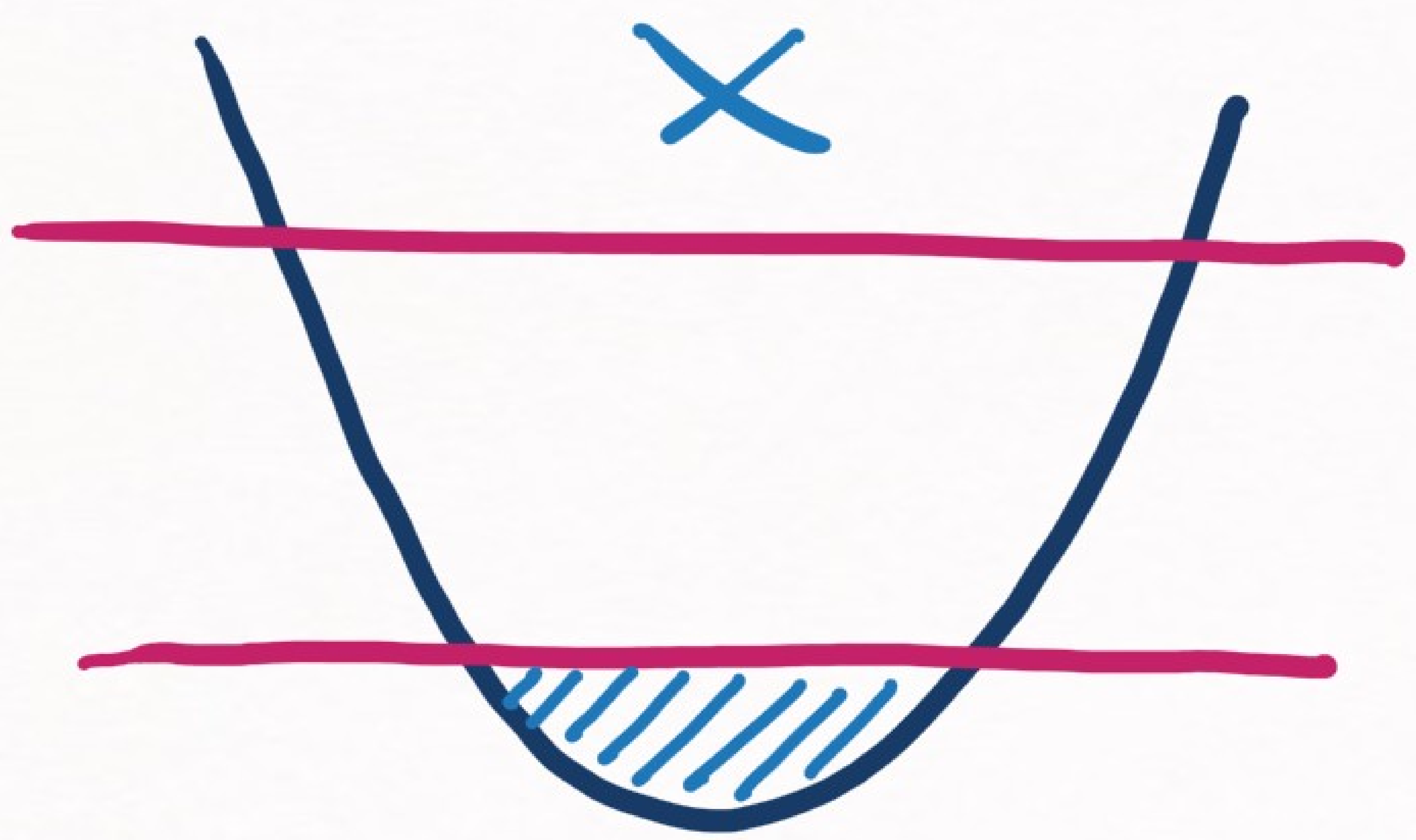
higher shells → rarely occupied

lower shells → usually full

2) By ignoring the higher/lower shells we will end up with a finite-dimensional problem

[SHELL MODEL: TRUNCATING THE BASIS] 3

⇒ The most standard set-up is:



External → levels that are unlikely to be occupied (ignore them)

Valence → what matters

Core → levels that are usually fully occupied (assume that they are fully occupied)

Example: ^{18}O = ^{16}O core$^{(*)}$ + two valence neutrons

(*) → ^{16}O is doubly magical → superstable → always full

[SHELL MODEL: TRUNCATING THE BASIS] 4

We divide the shell space into:

1) Core → fully occupied levels
2) Valance → what matters
3) External → forbidden levels

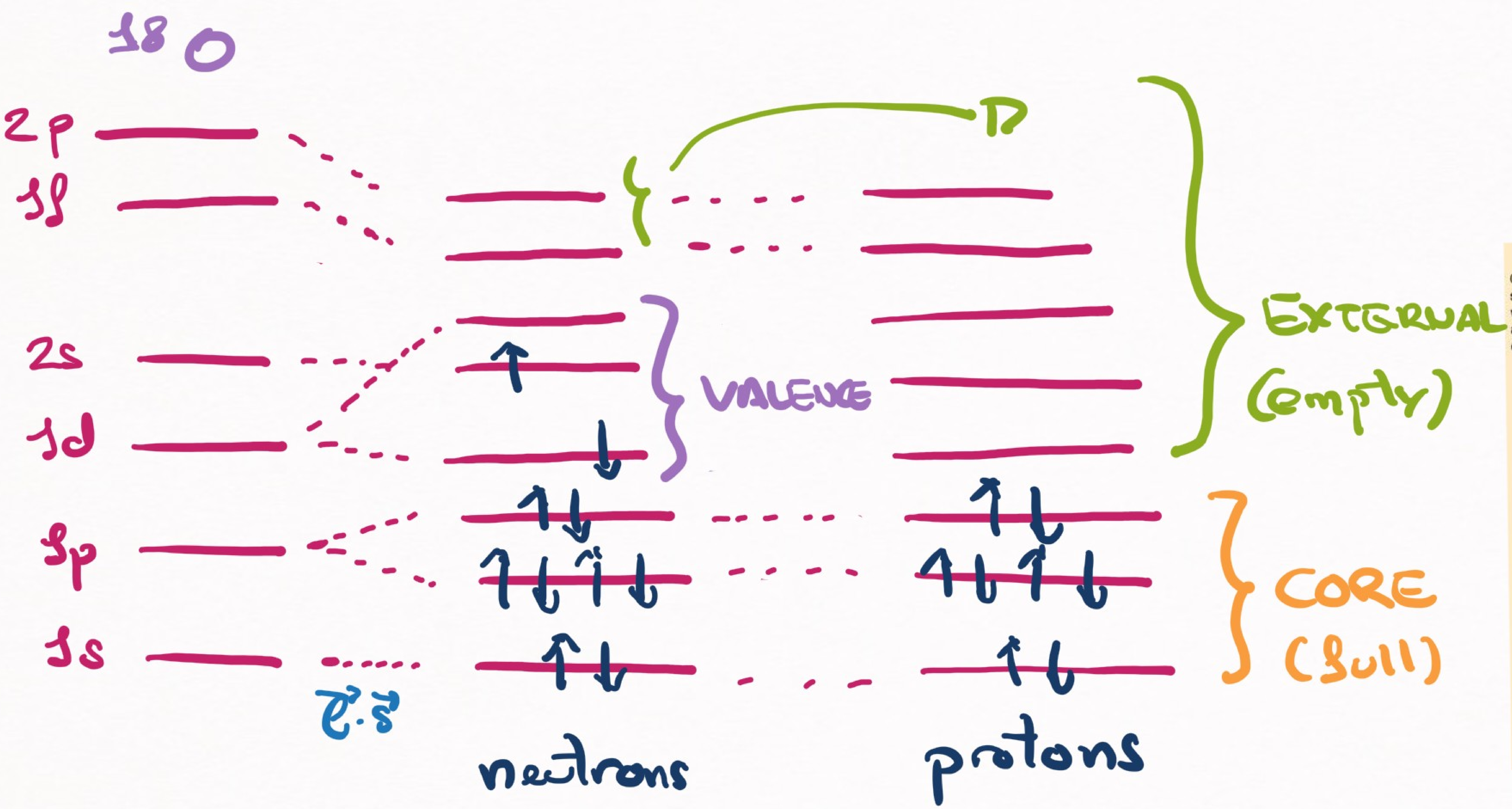
What is importance is that the Valence is a finite-dimensional space

$\underline{\underline{H_{\alpha\beta}}} \psi_\beta = E \psi_\alpha$

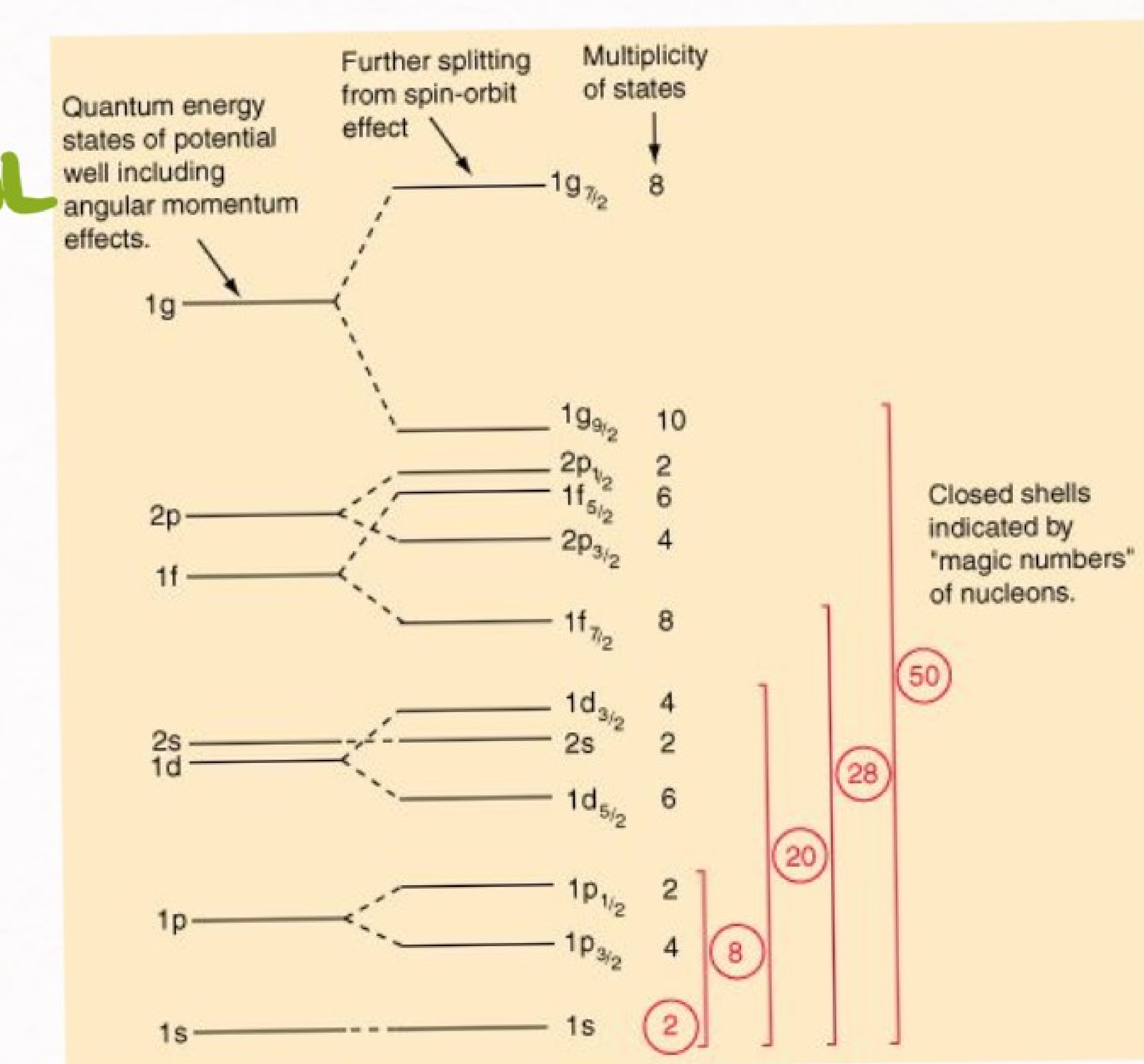
finite-dim in Valence

[SHELL MODEL: TRUNCATING THE BASIS] 5

⇒ In this case, we can have the following:



Reminder:



[SHELL MODEL: TRUNCATING THE BASIS] 6

⇒ Continuing with this example (^{18}O):

1) Core: 1s, 1p orbitals for both neutrons & protons

2) Valence: 1d, 2s orbitals for neutrons

3) External: everything else

[SHELL MODEL: TRUNCATING THE BASIS] 8

$\Rightarrow$ How do we solve $^{38}\bigcirc$ then?

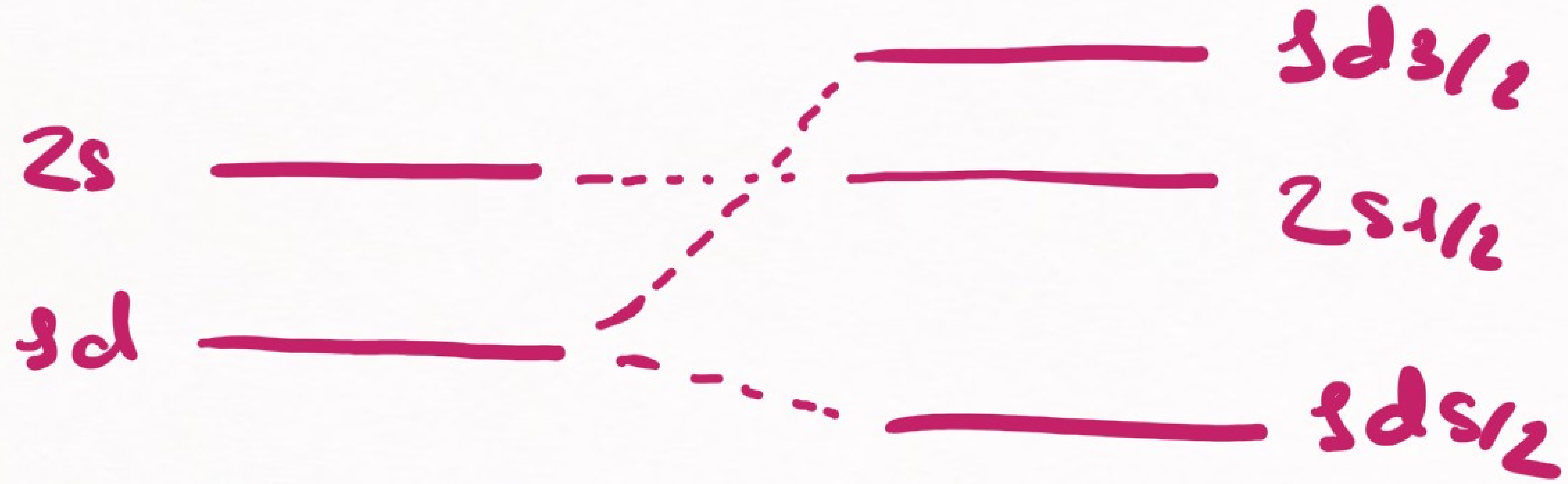
1) Write the basis states:

$$|\Psi_\alpha\rangle = \underbrace{|\Psi_\alpha^{core}\rangle}_{\text{fixed}} \times \underbrace{|\Psi_\alpha^{valence}\rangle}_{\text{non-trivial part}}$$

2) Find the dimension of $|\Psi_a^{valence}\rangle$

[SHELL MODEL: TRUNCATING THE BASIS] 9

⇒ How do we find the dimension of $|\Psi_\alpha^{\text{valence}}\rangle$?



(4 state)
(2 state)
(6 states)

$\left.\begin{matrix}(4\text{ state})\\(2\text{ state})\\(6\text{ states})\end{matrix}\right\}$ 12 states

1) One fermion in this valence space → dim = 12

2) Two fermions in this valence space

→ $\text{dim} = \dfrac{12 \cdot 11}{2} = 66$ $\binom{12}{2} = 66$ (binomial coefficients)

[SHELL MODEL: TRUNCATING THE BASIS] 10

$\Rightarrow$ How do we find the dimension of $|\Psi_\alpha^{valence}\rangle$?

a) One fermion (^{17}O) $\Rightarrow$ Dim = 12

b) Two fermions (^{18}O) $\Rightarrow$ Dim $= \binom{12}{2} = 66$

# of combinations
without repetitions

[SHELL MODEL: TRUNCATING THE BASIS] 11

⇒ So for ^{18}O we have a 66-dim valence basis:

$$|\Psi_\alpha^{\text{valence}}\rangle \,, \alpha = 1, 2, \ldots, 66$$

$$\{\alpha\} = \left.\begin{array}{l} \{ \; |1d_{5/2}(+5/2)\rangle |1d_{5/2}(+3/2)\rangle , \\ \quad |1d_{5/2}(+5/2)\rangle |1d_{5/2}(+1/2)\rangle , \\ \quad \vdots \\ \quad |1d_{5/2}(-3/2)\rangle |1d_{5/2}(-5/2)\rangle , \\ \quad |1d_{5/2}(+5/2)\rangle |1s_{1/2}(+1/2)\rangle , \\ \quad \vdots \\ \quad \} \end{array}\right\} \text{66 states}$$

[SHELL MODEL: TRUNCATING THE BASIS] 32

⇒ Once we build the basis,
we find the matrix elements:

$$H \longrightarrow H_{\alpha\beta} = \begin{pmatrix} H_{11} & H_{12} & \cdots & H_{1,66} \\ H_{21} & H_{22} & \cdots & H_{2,66} \\ \vdots & \vdots & \ddots & \vdots \\ H_{66,1} & H_{66,2} & \cdots & H_{66,66} \end{pmatrix}$$

This is difficult to do operations with,
but still much better than $\dim = \infty$

[SHELL MODEL: TRUNCATING THE BASIS] 13

⇒ And finally, once we have the matrices we diagonalize

$$H_{\alpha\beta} \Psi_\beta = \Psi_\alpha$$

Really easy to understand

(and if you have a lot of computing power, it will also be easy to calculate)

[SHELL MODEL: TRUNCATING THE BASIS] 14

⇒ PROBLEM: the dimension of the basis grows quickly

Example: ${}^{60}_{30}\mathrm{Zn}_{30} \rightarrow {}^{40}_{20}\mathrm{Ca}_{20}$ core (doubly magical)

+ $\begin{matrix} 10\,p \\ 10\,n \end{matrix}$ in $\underbrace{p/f \text{ shells}}_{\text{the valence space}}$

the valence space

→

20 valence states

$\mathrm{Dim}({}^{60}_{30}\mathrm{Zn}) = \binom{20}{10}\binom{20}{10}$

$\sim 3.4 \cdot 10^{10}$

(somewhat large dimension)

[SHELL MODEL: TRUNCATING THE BASIS] 15

⇒ In fact, the interacting shell model quickly becomes completely unwieldy

$$\mathrm{Dim} = \binom{\Omega_p}{N_p}\binom{\Omega_n}{N_n}$$

Ω → dimension of the valence space

N → number of proton/neutrons within this valence space

$\binom{a}{b}$ → binomial coefficient

[SHELL MODEL: TRUNCATING THE BASIS] 16

⇒ Luckily, there are further simplification: ✓

We can further reduce the dimension of this valence space by concentrating on nuclei with a given J^P

$$\mathrm{Dim}(J^P) \ll \binom{\Omega_p}{N_p}\binom{\Omega_n}{N_n}$$

⇒ By concentrating on the orbitals giving us J^P we will greatly simplify this problem

[SHELL MODEL: TRUNCATING THE BASIS] (37)

$\Rightarrow$ Trivial example: the deuteron

$1s_{1/2}$ — ↑ — — ↑ — $1s_{1/2}$ } Valence

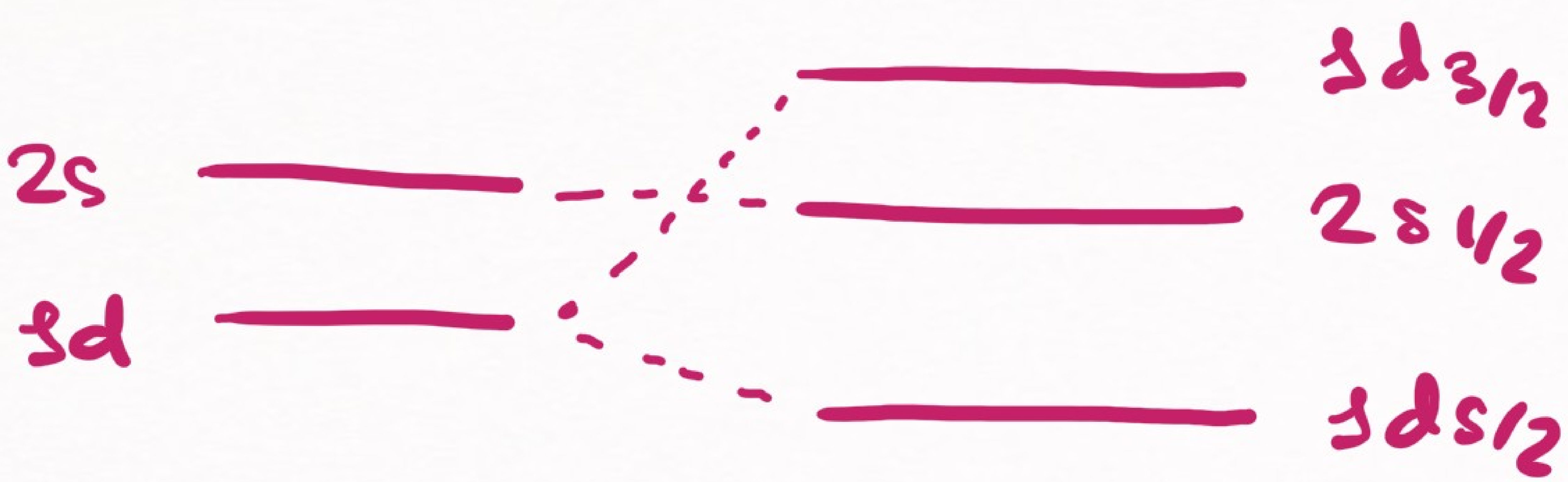
$$\text{Dim(valence)} = \binom{2}{1}\binom{2}{1} = \underline{\underline{4}} \Rightarrow \text{But: } \text{Dim}(1^+) = 3^{(*)}, \quad \text{Dim}(0^+) = 1$$

(*) We can reduce it further by concentrating a specific $|J\,M\rangle$ state: $|J\,J\rangle = |1\,1\rangle \rightarrow \underline{\underline{\text{Dim}(|1\,1\rangle) = 1}}$

[SHELL MODEL: TRUNCATING THE BASIS] (18)

⇒ Of course, the deuteron is just a silly example

Better example: ^{18}O ⇒ even-even, the ground state will be 0^+



⇓

We might concentrate on configurations giving us $J^P = 0^+$

[SHELL MODEL: TRUNCATING THE BASIS] 59

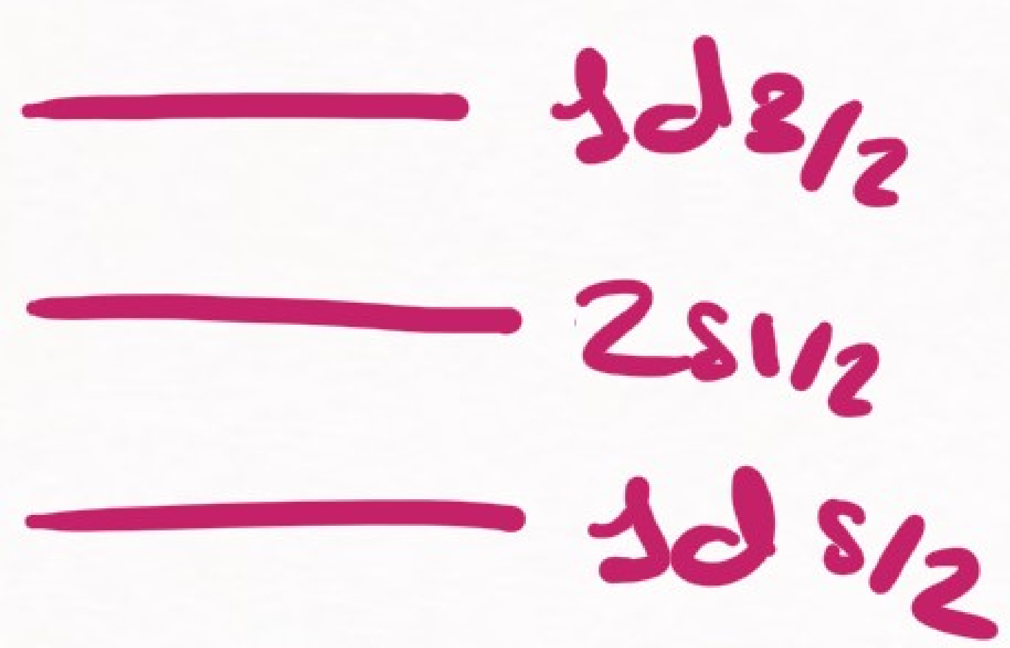
$\Rightarrow$ Now we have : $\underline{\underline{{}^{18}O}} = \underbrace{\underline{\underline{{}^{16}O}}}_{J^P=0^+} + 2n$ in the valence
with $J^P = 0^+$

Q : How many configurations are there
with $J^P = 0^+$?

(Easier than it looks)

[SHELL MODEL: TRUNCATING THE BASIS] (20)

⇒ Valence configurations of ^{18}O (^{16}O core + s/d valence):



$$\frac{3}{2}\otimes\frac{3}{2} = 0\oplus\cdots\oplus 3 \qquad \frac{1}{2}\otimes\frac{3}{2} = 1\oplus 2$$

$$\frac{1}{2}\otimes\frac{1}{2} = 0\oplus 1 \qquad \frac{1}{2}\otimes\frac{5}{2} = 2\oplus 3$$

$$\frac{5}{2}\otimes\frac{5}{2} = \underbrace{0}_{\text{useful}}\oplus\cdots\oplus 5 \qquad \underbrace{\frac{3}{2}\otimes\frac{5}{2} = 1\oplus\cdots\oplus 4}_{\text{not useful for } J^P = 0^+}$$

the only 3 possibilities
(dim(0^+) = 3)

[SHELL MODEL: TRUNCATING THE BASIS] (21)

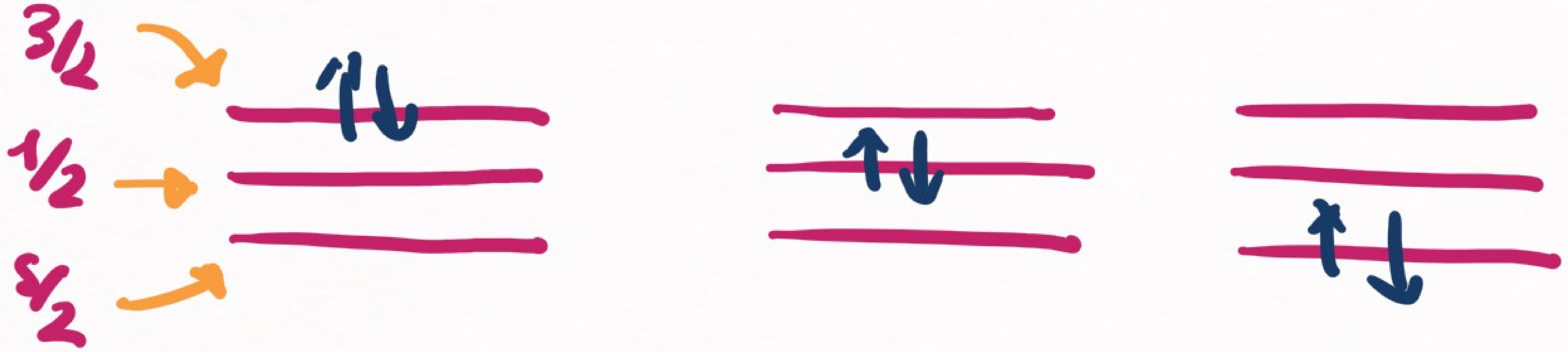
⇒ Valence configurations of ^{18}O (^{16}O core + s/d valence):

Neutrons are fermions: (we also need to check that we are using antisymmetric configs)

$\frac{1}{2}\otimes\frac{1}{2} = 0_A \oplus \cancel{1_S}$ forbidden

$\frac{3}{2}\otimes\frac{3}{2} = 0_A \oplus \cancel{1_S} \oplus 2_A \oplus \cancel{3_S}$

$\frac{5}{2}\otimes\frac{5}{2} = 0_A \oplus \dots \oplus \cancel{5_S}$



no changes (∀ already antisymmetric)

[SHELL MODEL: TRUNCATING THE BASIS] 22

=> To summarize the ^{18}O example:

1) ^{18}O = core (^{16}O) + valence ($2n$ in s/d shells)

dim = 66

2) We impose the condition that J^P is fixed

$J^P = 0^+ \longrightarrow \dim = 3 \ll 66$

3) Conclusion: we obtain a much better dimension (reduced)

[SHELL MODEL : HOW TO CALCULATE $\langle \Delta V \rangle$] (3)

We will follow this scheme :

0) Begin with a mean-field

e.g. harmonic oscillator + $\vec{\ell}\cdot\vec{s}$ term $\rightarrow V^{MF} \rightarrow h^{MF}$

1) Find the single-particle levels :

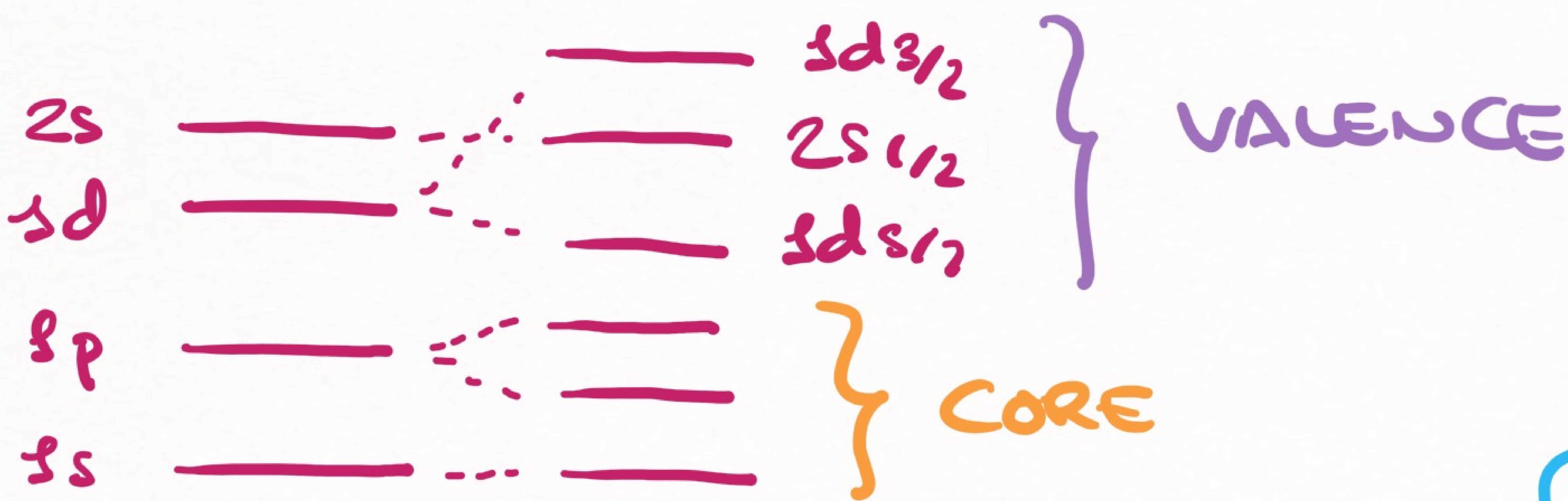
$$h^{MF}\phi_a = \epsilon_a \phi_a \quad \text{for} \quad a = \underbrace{1d_{5/2},\ 1s_{1/2},\ 1d_{3/2}}_{^{18}O \text{ example}}$$

$\Downarrow$

The energies can actually be obtained by comparing the energy of different nuclei

[SHELL MODEL : HOW TO CALCULATE $\langle \Delta V \rangle$] (2)

1) Find the single particle energy levels: $\underline{\underline{^{18}O}}$



1.a) $\underbrace{\epsilon(1d_{5/2})}_{^{17}O(\frac{5}{2}^{+})} = E_B(^{17}O) - E_B(^{16}O)$

$^{17}O = {}^{16}O$ core + neutron in $1d_{5/2}$

this helps us improve our shell model calculation

[SHELL MODEL : HOW TO CALCULATE $\langle \Delta V \rangle$] ③

1) Find the single particle energy levels: $\underline{\underline{^{18}O}}$

1.b) $\underbrace{\epsilon(2s_{1/2})} = \epsilon(1d_{5/2}) + [E_B^*(^{17}O, \tfrac{1}{2}^+) - E_B(^{17}O)]$

$$= E_B^*(^{17}O, \tfrac{1}{2}^+) - E_B(^{16}O)$$

$^{17}O^*(\tfrac{1}{2}^+)$ excited state of ^{17}O

[SHELL MODEL: HOW TO CALCULATE $\langle \Delta V \rangle$] (4)

1) Find the single particle energy levels: ^{18}O

1.c) $\underbrace{\epsilon(1d_{3/2})}_{^{17}O^{*}(\frac{3}{2}^{+})} - \epsilon(1d_{5/2}) = E_B^{*}(^{17}O, \frac{3}{2}^{+}) - E_B(^{17}O, \frac{5}{2}^{+})$

It's just a game of comparisons

[SHELL MODEL : HOW TO CALCULATE $\langle \Delta V \rangle$] 5

1) Find the single particle energy levels: $^{18}\underline{\underline{O}}$

$$\left.\begin{aligned} \epsilon(1d_{5/2}) &= -4.1\ \mathrm{MeV} \\ \epsilon(2s_{1/2}) &= -3.3\ \mathrm{MeV} \\ \epsilon(1d_{3/2}) &= +0.9\ \mathrm{MeV} \end{aligned}\right\}$$

just from looking at different energy levels

2) Write down the hamiltonian (^{18}O, $J^P = 0^+$)

$$H_{\alpha\beta} = \underline{\underline{2\delta_{\alpha\beta}\epsilon_\alpha}} + \underline{\underline{\Delta V_{\alpha\beta}}}$$

} slides 36 & 37

→ matrix elements for ΔV (we will not explain this)

[SHELL MODEL : HOW TO CALCULATE $\langle \Delta V \rangle$] ⑥

3) Calculate $\Delta V_{\alpha\beta}$ in the valence basis :

$$\Delta V = \begin{pmatrix} -2.8 & -1.3 & -3.3 \\ -1.3 & -2.1 & -1.1 \\ -3.3 & -1.1 & -2.2 \end{pmatrix}$$

Imagine this is what you obtain (with some potential)

[SHELL MODEL : HOW TO CALCULATE $\langle \Delta V \rangle$] (7)

4) Calculate the eigenvalues of $H_{\alpha\beta} = \underline{\underline{2\delta_{\alpha\beta}\epsilon_\beta}} + (\Delta V)_{\alpha\beta}$

→ We just have a 3x3 matrix

Eigenvalues: $\underbrace{-12.6}_{\text{Ground state}}\ ,\ \underbrace{-8.1\ ,\ +0.6}_{\text{Excited states (two)}}$

However, we will usually use relative energies (the energy difference with respect to the ground state)

[SHELL MODEL : HOW TO CALCULATE $\langle \Delta V \rangle$] 8

4) Calculate the eigenvalues of $H_{\alpha\beta} = 2\delta_{\alpha\beta}\epsilon_\beta + (\Delta V)_{\alpha\beta}$

Eigenvalues (0^+) → $-12.6,\ -3.1,\ +0.6$
$0.0,\ +4.5,\ +13.2$ } (MeV)

$\Delta E(^{18}O^*, 0_2^+) = 4.5$ MeV
$\Delta E(^{18}O^*, 0_3^+) = 13.2$ MeV } my excitation energies!

Of course, the difficult part here is calculating the oscillator matrix elements of $\underline{\underline{\Delta V_{\alpha\beta}}}$

[SHELL MODEL : HOW TO CALCULATE $\langle \Delta V \rangle$] (9)

5) For calculating the full spectrum we simply ...

... just repeat the process for all J^P

$^{18}O \rightarrow J^P = 0^+, 1^+, 2^+, 3^+, 4^+$

(5^+ not possible because it was symmetric)

[SHELL MODEL : HOW TO CALCULATE $\langle \Delta V \rangle$] (10)

RECAP!

0) A choice of mean-field

^{18}O case an **example**

1) Finding the single particle levels
2) Writing down the Hamiltonian (in the basis given by the mean field) $h_{\alpha\beta} = 2\delta_{\alpha\beta}\epsilon_\beta + \Delta V_{\alpha\beta}$
3) Calculate $\Delta V_{\alpha\beta}$
4) Diagonalize $h_{\alpha\beta}$ to obtain the energy levels

[SHELL MODEL : HOW TO CALCULATE $\langle \Delta V \rangle$] 11

$\Rightarrow$ For an alternative example we might consider:

[^{18}O with $J^P = 3^+$]

We begin with the problem of finding the antisymmetric combinations with $J^P = 3^+$

$1d_{3/2}$ ———

$1s_{1/2}$ ———

$1d_{5/2}$ ———

$\frac{3}{2} \otimes \frac{3}{2} = 0 \oplus 1 \oplus 2 \oplus \cancel{3}$

$\frac{5}{2} \otimes \frac{5}{2} \rightarrow$ same problem

Only options possible $\Rightarrow \begin{cases} \frac{1}{2} \otimes \frac{5}{2} \Big|_3 \\ \frac{3}{2} \otimes \frac{5}{2} \Big|_3 \end{cases}$ $\Rightarrow$ [$\mathrm{Dim}(3^+, n=3) = 2$]

[SHELL MODEL : HOW TO CALCULATE $\langle \Delta V \rangle$] (12)

$\Rightarrow$ For an alternative example we might consider:

[^{18}O with $J^P = 3^+$]

1) Basis: $\{\alpha\} = \{ (1d_{5/2})(2s_{1/2})|_3 \, , (1d_{5/2})(1d_{3/2})|_3 \}$

$= \{1, 2\}$ (ignoring M quantum number)

2) Hamiltonian:

$$h_{11} = \epsilon(1d_{5/2}) + \epsilon(2s_{1/2}) + \Delta V_{11}$$

$$h_{22} = \epsilon(1d_{5/2}) + \epsilon(1d_{3/2}) + \Delta V_{22}$$

$$h_{12} = h_{21} = \Delta V_{12}$$

[SHELL MODEL: HOW TO CALCULATE $\langle \Delta V \rangle$] (13)

$\Rightarrow$ Basically, repeat this procedure with any nucleus of interest and we are done

3) $\Delta V = \begin{pmatrix} 0.8 & 0.7 \\ 0.7 & 0.6 \end{pmatrix}$ MeV

4) Eigenvalues: -6.6, -2.5 MeV

$\Rightarrow$ Just the same story as $J^P = 0^+$ but with $J^P = 3^+$

RECAP | [ΔV IN THE SHELL MODEL] 3

1) Key idea: $h_i \phi_i = \epsilon_i \phi_i \rightarrow \Phi_\alpha = \mathcal{A}[\prod_i \phi_i]$

$\Rightarrow \{|\Phi_\alpha\rangle\}$ form a basis of the A-body Hilbert space

2) The Hamiltonian can be rewritten in this basis $\{|\Phi_\alpha\rangle\}$

$\Rightarrow H|\Psi\rangle = E\Psi$ becomes: $H_{\alpha\beta}\Phi_\beta = E\Phi_\alpha$ } matrix (dim $= \infty$)

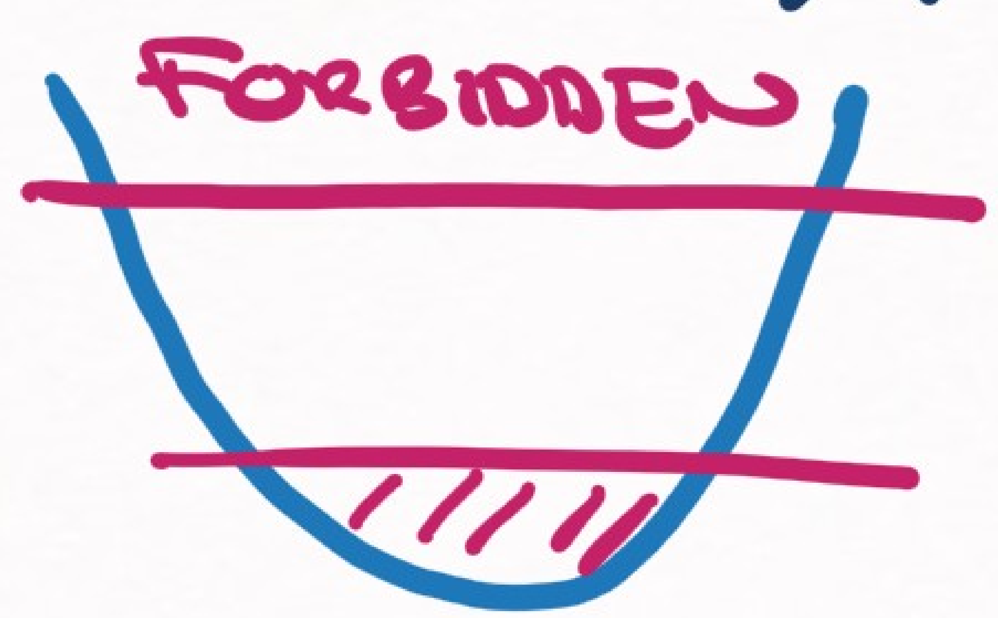
We only have to diagonalize $H_{\alpha\beta}$

(but this is an infinite-dimensional problem)

RECAP | [ΔV IN THE SHELL MODEL] 2

3) We truncate the Hilbert space in this new basis to make calculations possible

$\Rightarrow$ Most typical example:



3.a) External

3.b) Valence] $\rightarrow$ only valence matters

3.c) Core

4) We simplify the truncated Hilbert space further by concentrating on specific values of J^P ||

(and ignoring M in $|JM\rangle$)

RECAP | [ΔV IN THE SHELL MODEL] (3)

5) Then, we write down $\underline{\underline{H}}_{\alpha\beta}$ in the truncated basis and calculate $\underline{\underline{\Delta V}}_{\alpha\beta}$

6) Finally, we diagonalize $H_{\alpha\beta}$ and find the energies of the nucleus we were studying

And that's the general idea!

NEXT LESSON:

HOW TO CALCULATE THE MEAN FIELD

Tuesday 15:50