NUCLEAR PHYSICS 20

THE COLLECTIVE MODEL

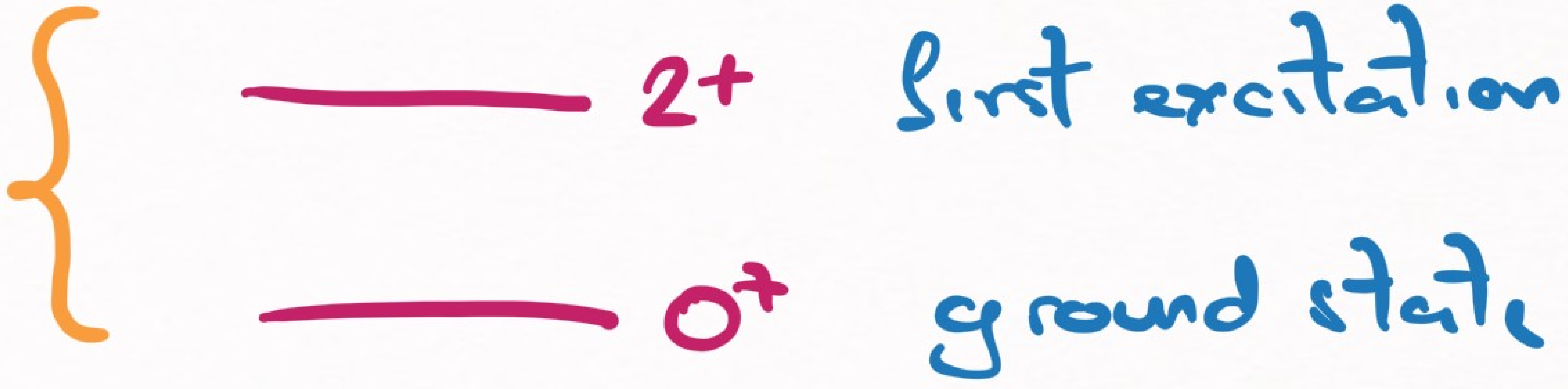
RECAP!

SHELL MODEL {
a) Assume an average potential (ideally containing $\vec{\ell}\cdot\vec{s}$) ✓
b) Then we fill shells ✓

⇓

A few examples:

[$^{17}O/^{17}F$, $^{15}O/^{15}N$, ^{41}Ca , $\underbrace{^{38}Ar}_{}$]



This pattern ($0^+/2^+$) happens in almost all even-even nuclei ⇐

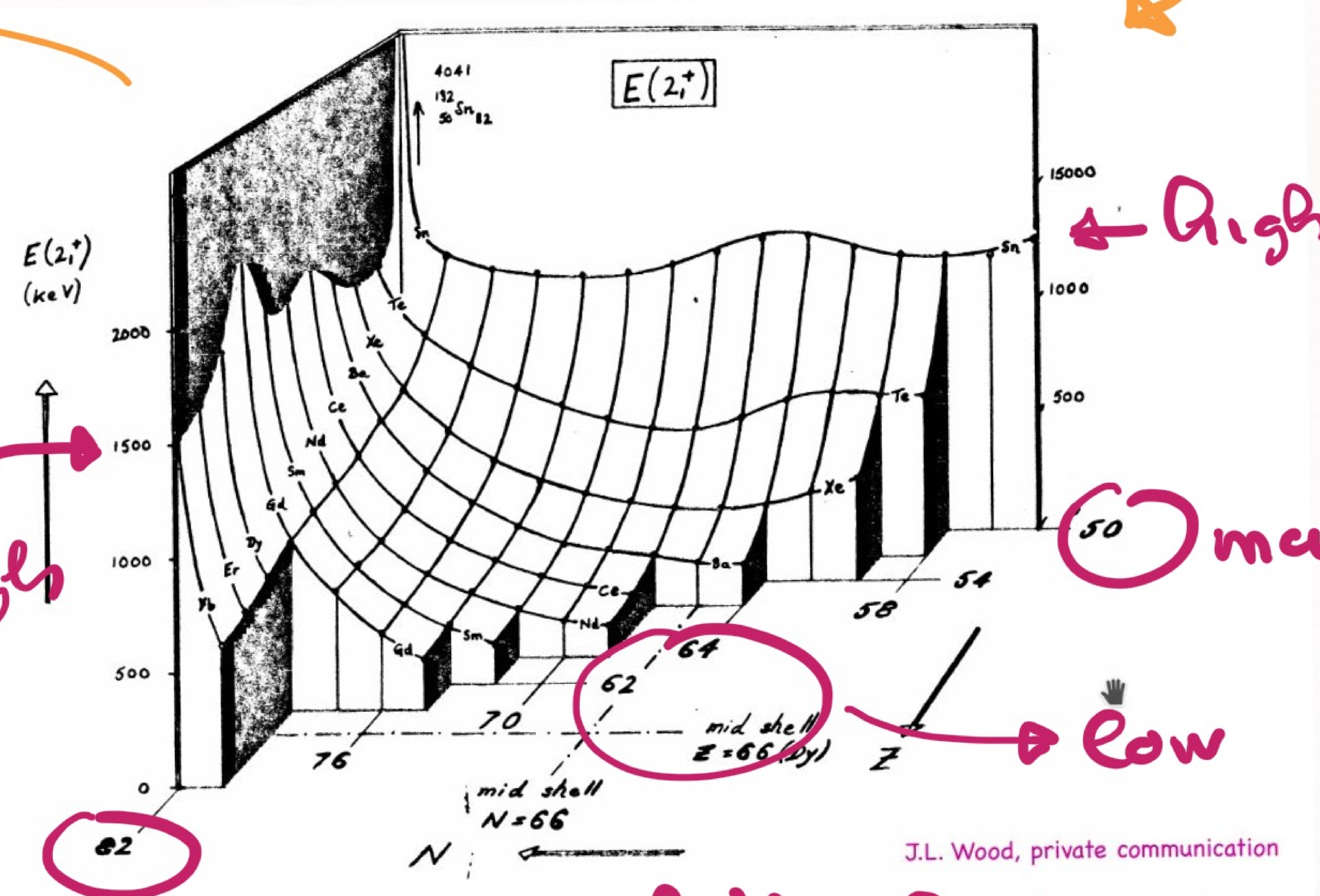
↳ We would like to have an explanation

$\Rightarrow$ [This can indeed be explained in the shell model, but...]

Far away from the shell closures (the magic numbers) the energy of 2^+ states is sort of small

⇓

Remember the scale arguments from our first few lessons



J.L. Wood, private communication

high

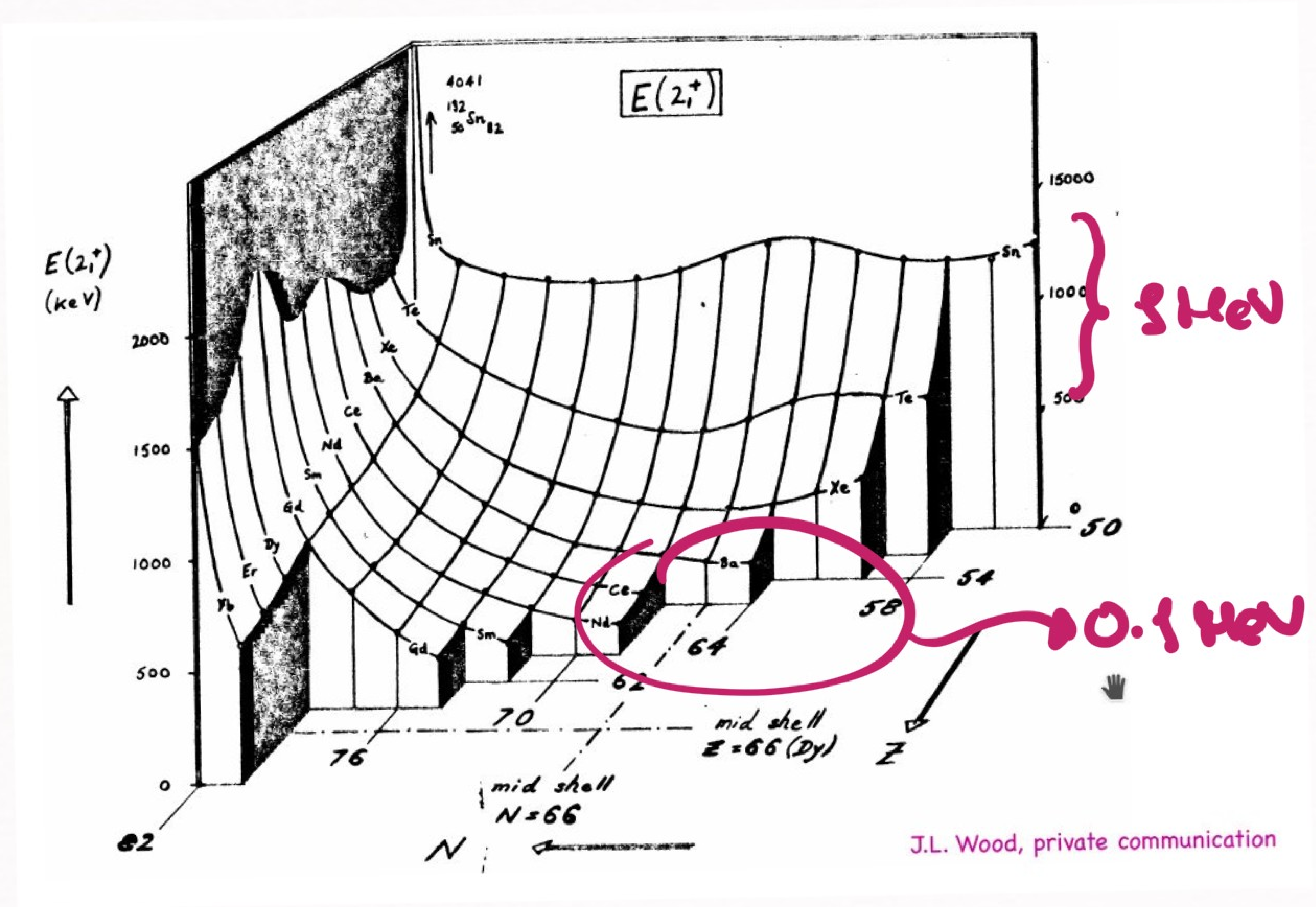
magic

low

high

magic

energy of the first 2^+ excited state (2_1^+)



J.L. Wood, private communication

(If scales are very different, this usually means that the physics are different)

=> In the shell model:

$E(2^+) \sim (1-2)\,\text{MeV}$

(at least close to N, Z magic)

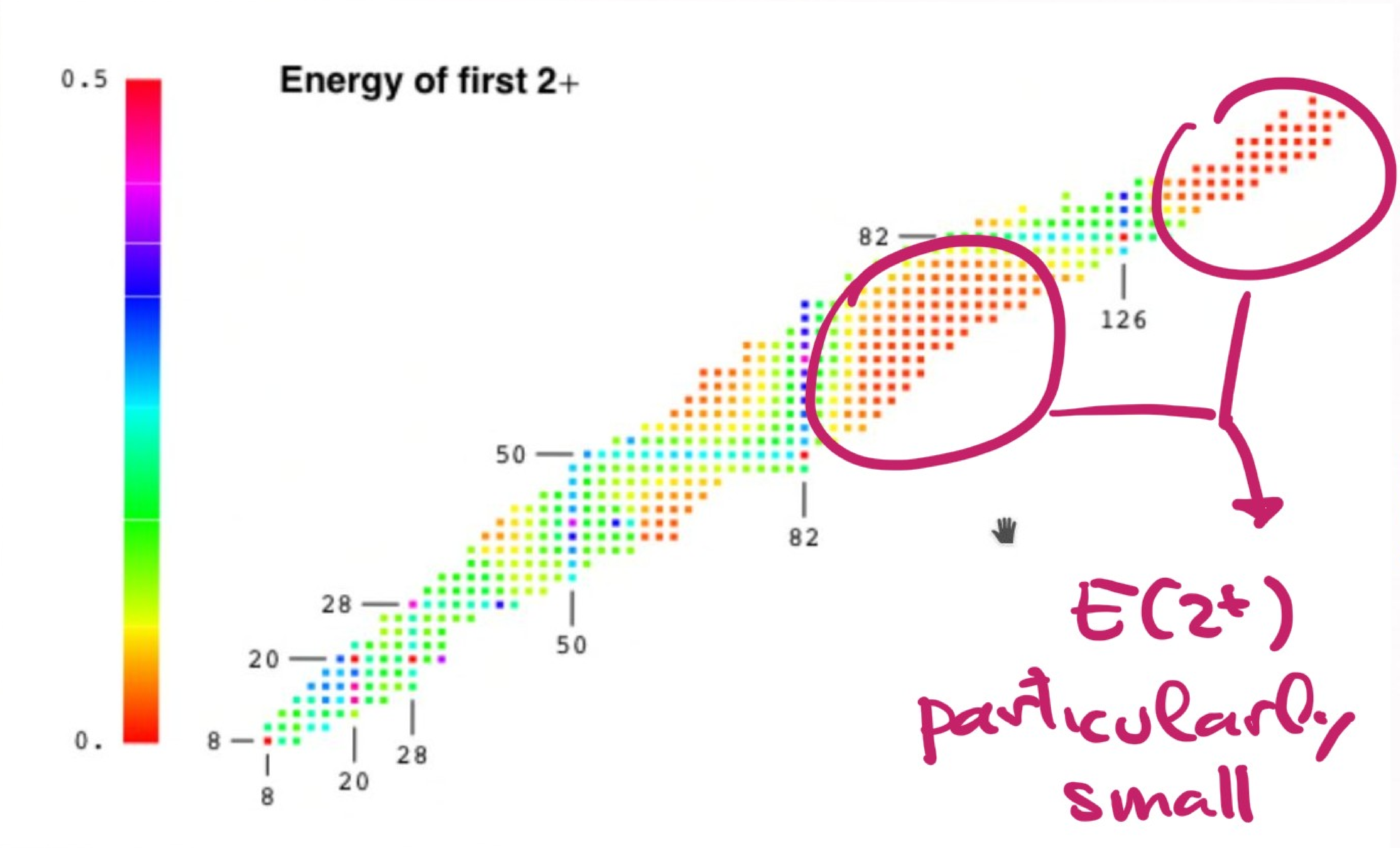
$$\hbar\omega = \frac{40}{A^{1/3}}\,\text{MeV} \sim 8\,\text{MeV}$$

$$\hbar^2\zeta = \underbrace{\frac{20}{A^{2/3}}}\,\text{MeV} \sim \underline{\underline{1\,\text{MeV}}}$$

$E(2^+)$

this comes from fitting the $V^{MF} = \frac{1}{2}m\omega^2 r^2 - \zeta\,\vec{\ell}\cdot\vec{s}$

⇒ But we observe that far away from N/Z magic $E(2_1^+)$ is much smaller :



$E(2^+)$ particularly small

These energies will be difficult to explain in the shell model (at least with the parameters in the previous slide)

⇒ Thus, we need a different explanation:

→ **COLLECTIVE MODEL** ←

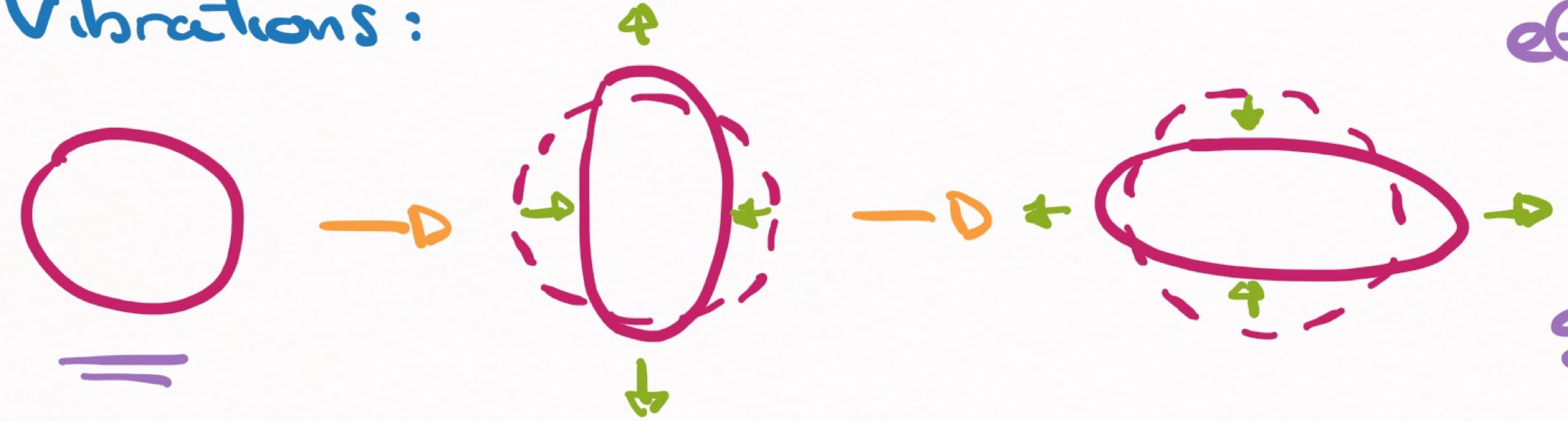
↓

It treats the nucleus as a "whole"

1) Liquid drop model → nucleus is basically a liquid ~ collective
2) Shell model → individual nucleons in a mean field
3) Collective model → we sort of go back to the analogy of the nucleus as some sort of fluid (or the nucleus as a solid)

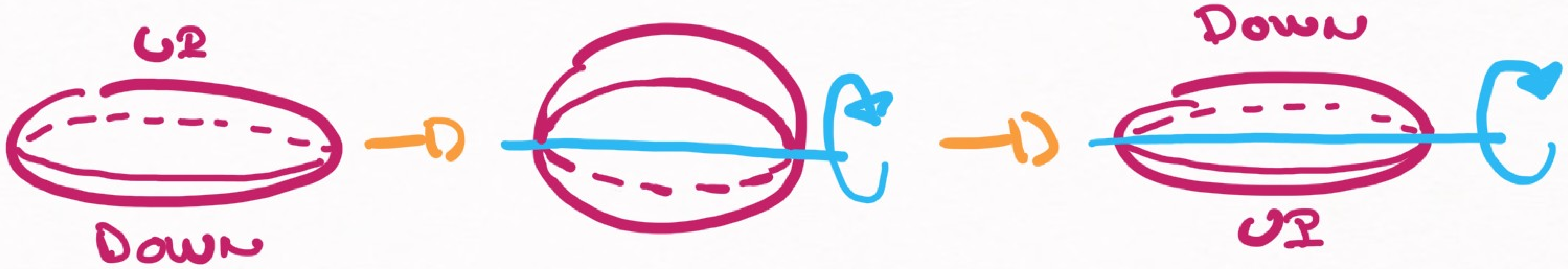
COLLECTIVE MODEL

1) Vibrations:



nucleus is some sort of elastic liquid that vibrates around its ground spherical form

2) Rotations:

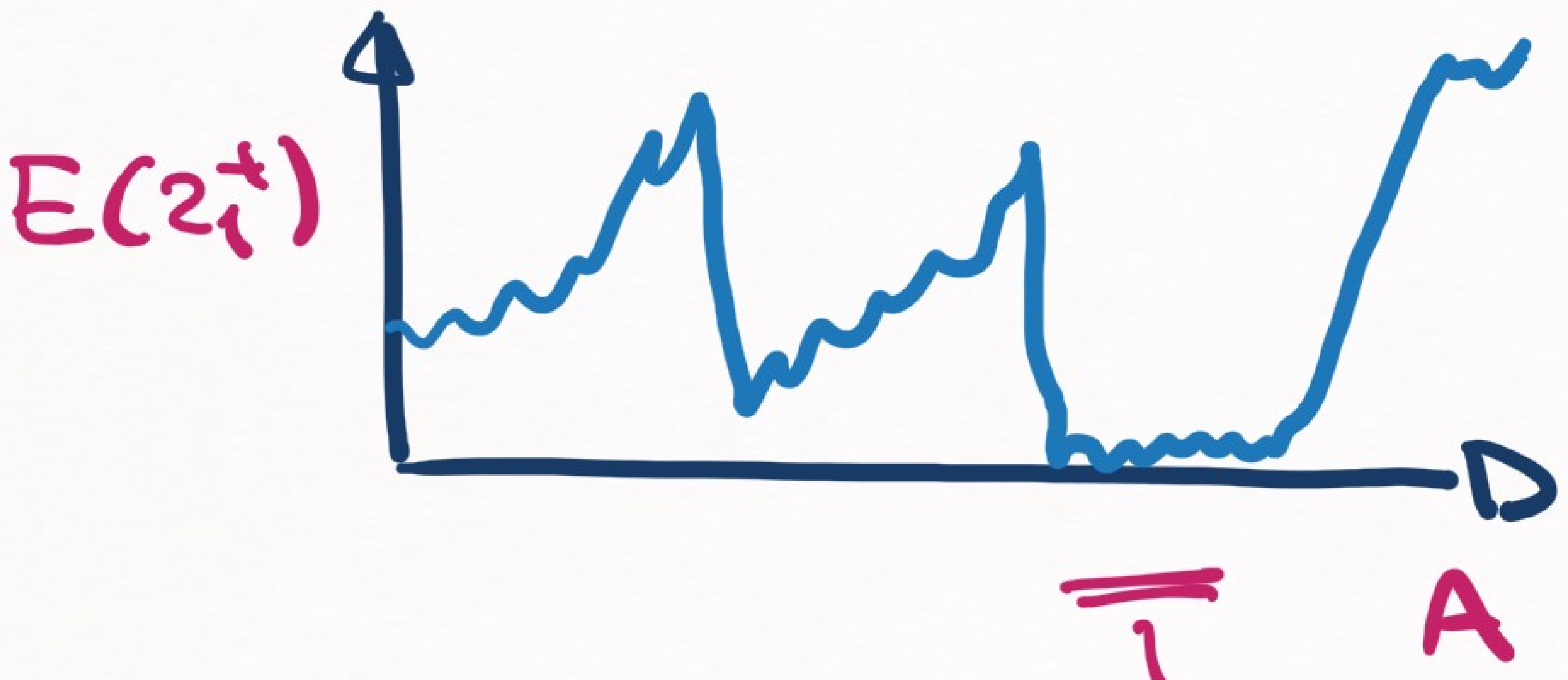


a rigid nucleus (non-spherical in its ground state) rotating around one of its axes

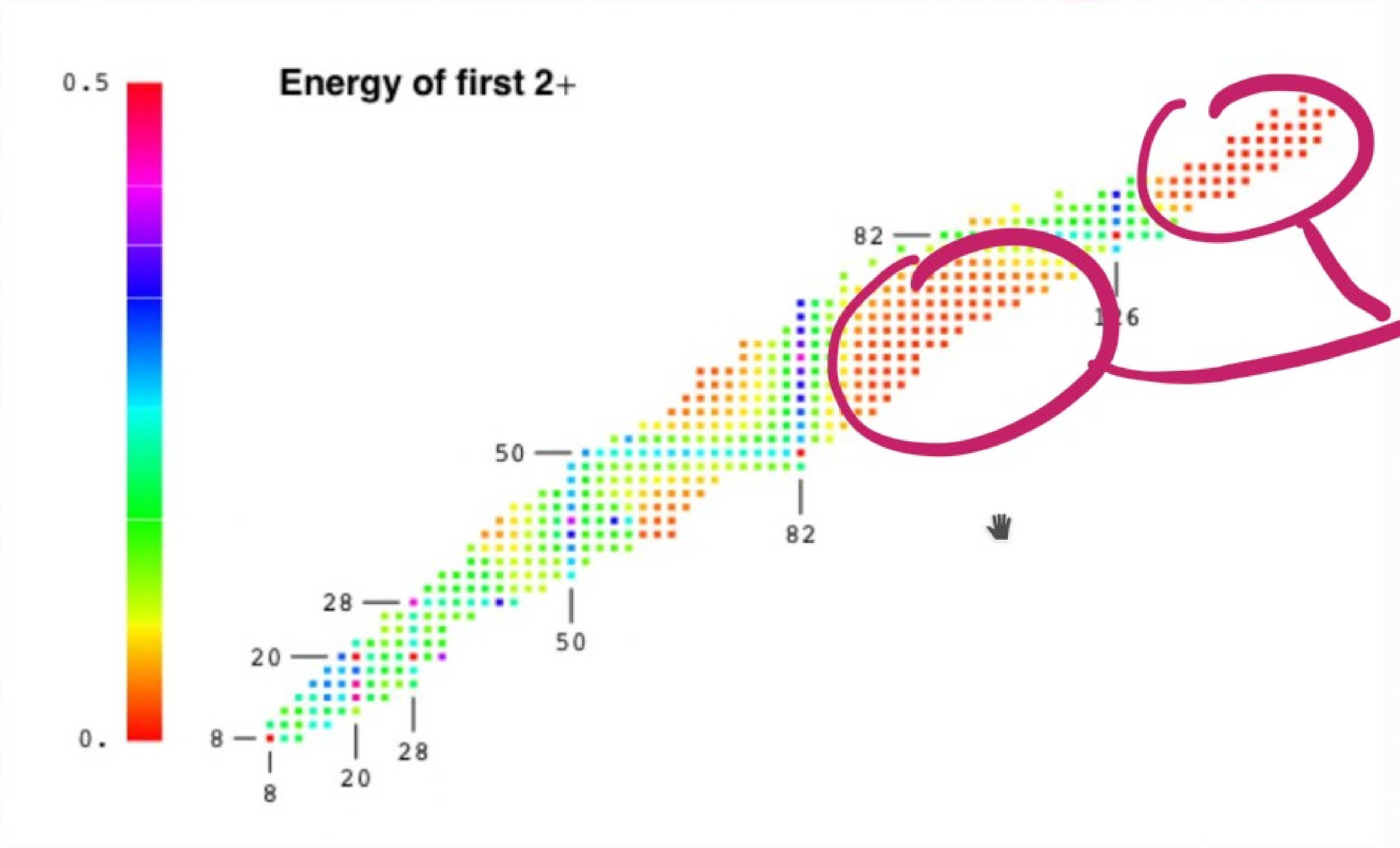
ROTATIONS ①

⇒ They are the easier to understand

(these are not going to be our standard classical rotations)



$A \sim (150-190) \Rightarrow E(2_1^+)$ are really small

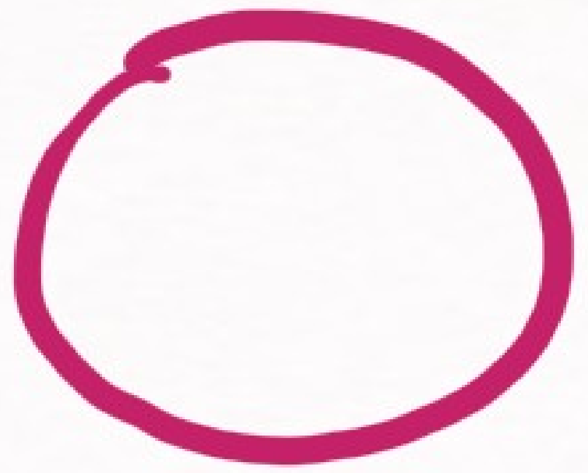


$E(2_1^+) \sim 0.1$ MeV or less

ROTATIONS ②

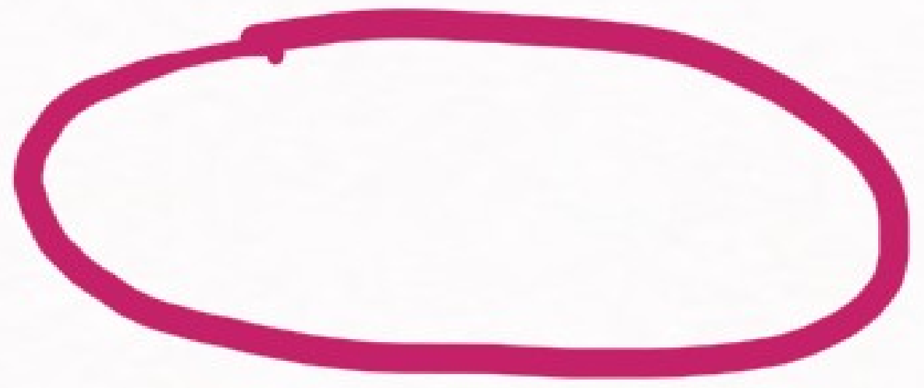
⇒ $A \sim (150-190)$ these nuclei are really deformed

1) Vibrations?


(spherical)

{ → NO! → this is not the lowest energy type of excitations for these nuclei

2) Rotation?


(deformed)

{ ⇒ they are indeed deformed ⇒ this could be a good explanation

[QUANTUM ROTATOR] ①

These rotations will be quantum-mechanical

⇒ Let's begin with the solid rigid and the moment of inertia

0) Classical: $\underline{\underline{H = \frac{1}{2} I \vec{\omega}^2}}$

I : moment of inertia

$\vec{\omega}$: rotation frequency (with $\vec{\omega} \parallel \vec{L}$)

at first sigh, it is not clear how to quantize it

1) Quantum:

1.a) $\vec{L} = I \vec{\omega}$

1.b) $H = \frac{1}{2} I \vec{\omega}^2 = \underline{\underline{\frac{1}{2I} \vec{L}^2}}$ ⟶ but now it is clear how to quantize it

[QUANTUM ROTATOR] ②

⇒ Naively, the energy levels are: $\vec{L}^2 = L(L+1)$ with $L = 0, 1, 2, 3, \ldots$

$$H = \frac{1}{2I}\vec{L}^2 = \frac{1}{2I} L(L+1)$$

with $L = 0, 1, 2, 3, \ldots$

⇒ But... there are complications (as usual)

⇒ The rotation that we do most do something to the system

(we are not rotating particles, but an object)

[QUANTUM ROTATOR] ③

⇒ Imagine you try to rotate this:

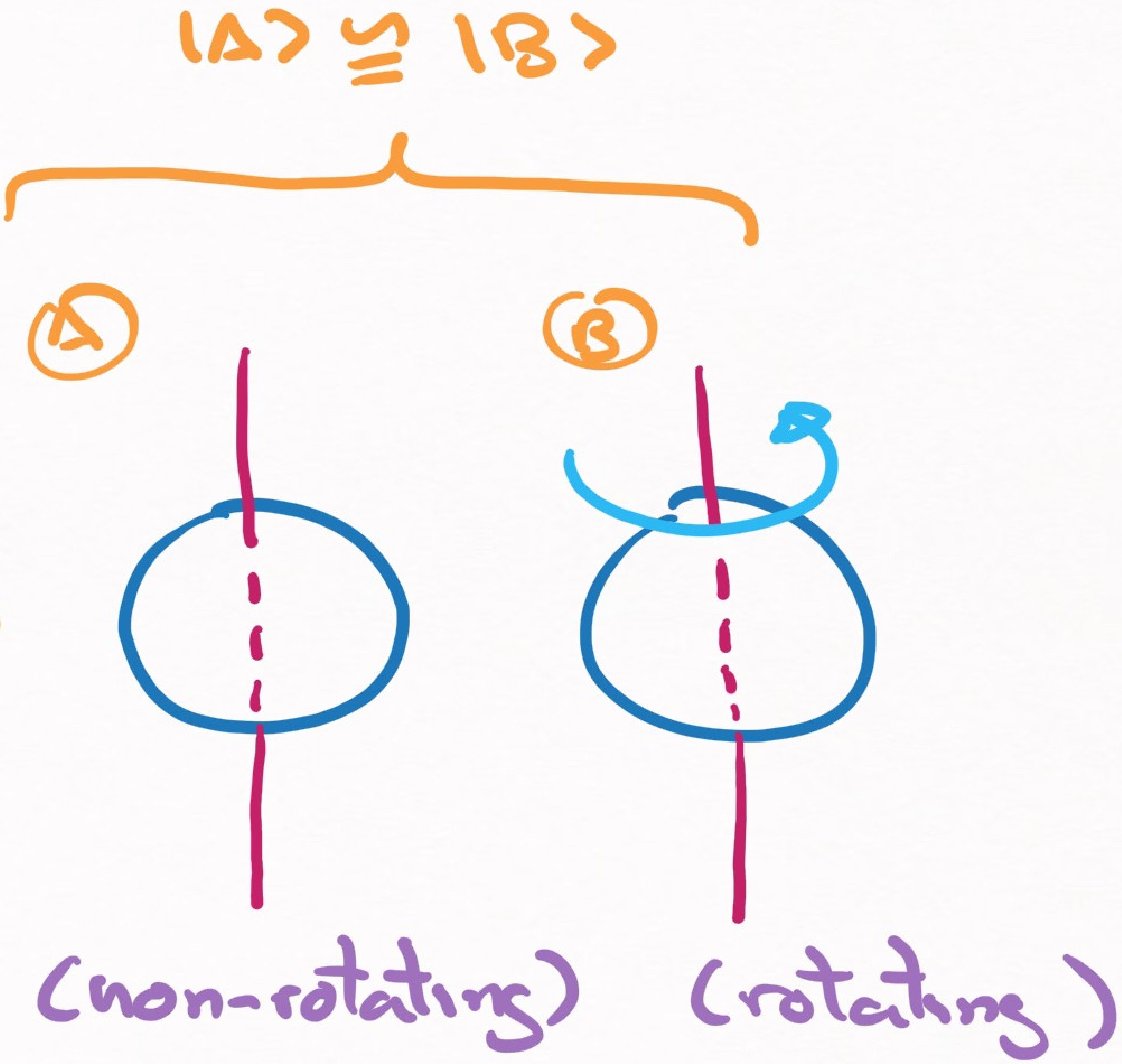
A rotation will not change the quantum state of a sphere

↑

⇒ You quickly notice that there is no difference between a sphere and a rotated sphere

[QUANTUM ROTATOR] ④

⇒



⇒ Since $|A\rangle \cong |B\rangle$, there is no difference in their internal state, and hence no difference in energy

⇒ no rotational levels

[QUANTUM ROTATOR] ⑤

Spherical nucleus $\hat{=}$ Rotated spherical nucleus

(equivalent to)

$\Rightarrow$ No difference in their internal states

$\Rightarrow$ No difference in their energy levels

$\Rightarrow$ [No rotational level]

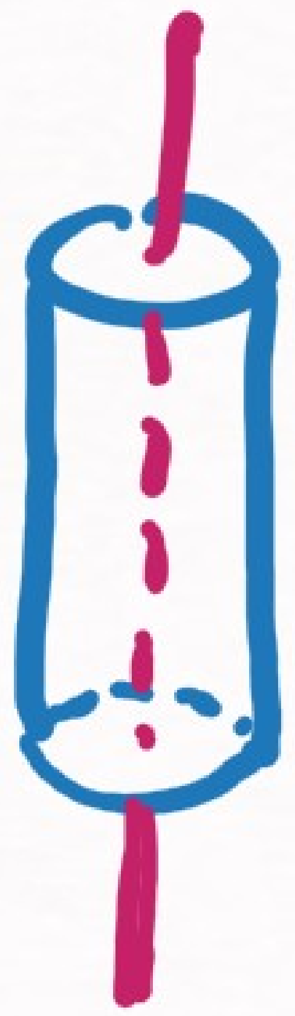
What about a non-spherical nucleus?

[QUANTUM ROTATOR] ⑥

Exaggerated case

⇒ Non-spherical case : a (quantum) rod

A1

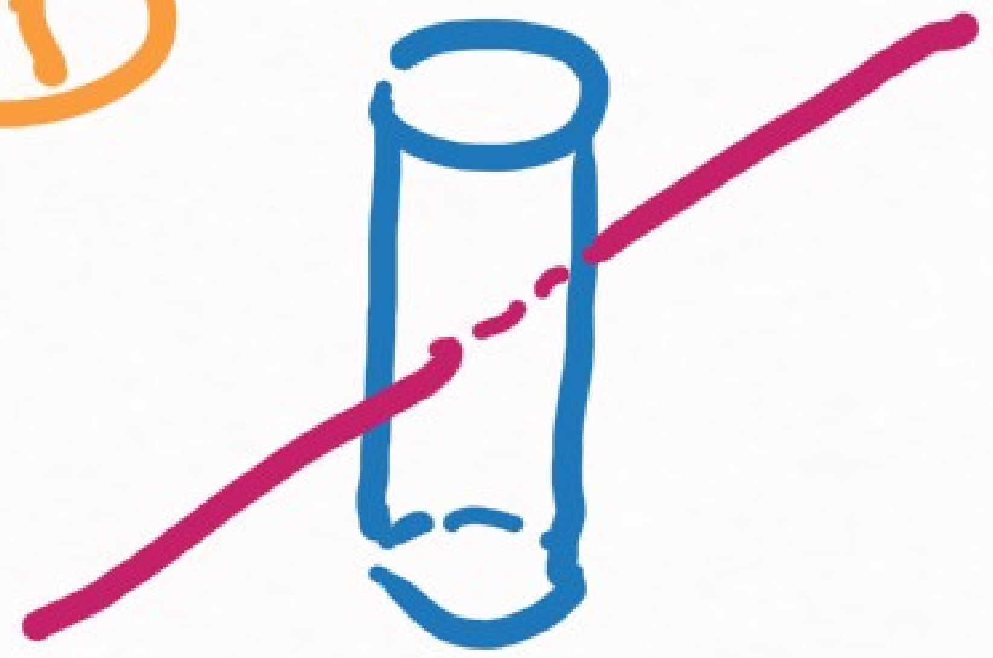


A2

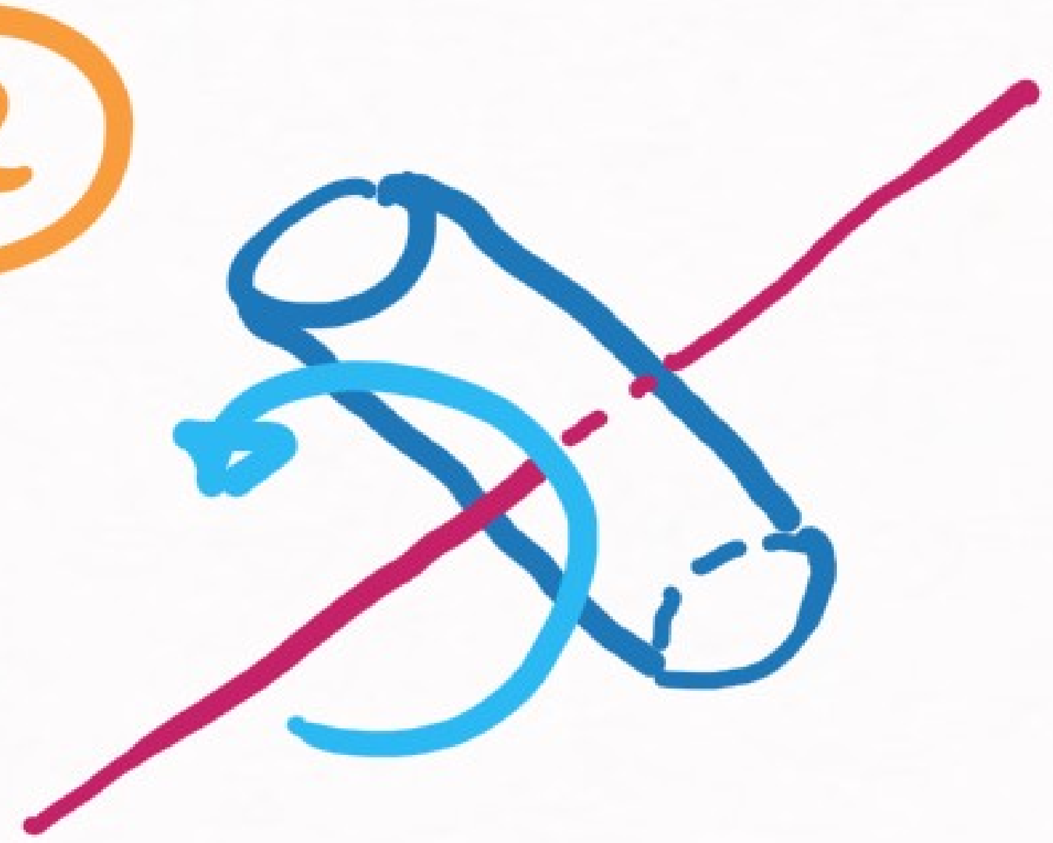
it depends on the type of rotation

⇒ $|A_1\rangle \cong |A_2\rangle$ ⇒ this rotation is trivial (does not count)

B1



B2

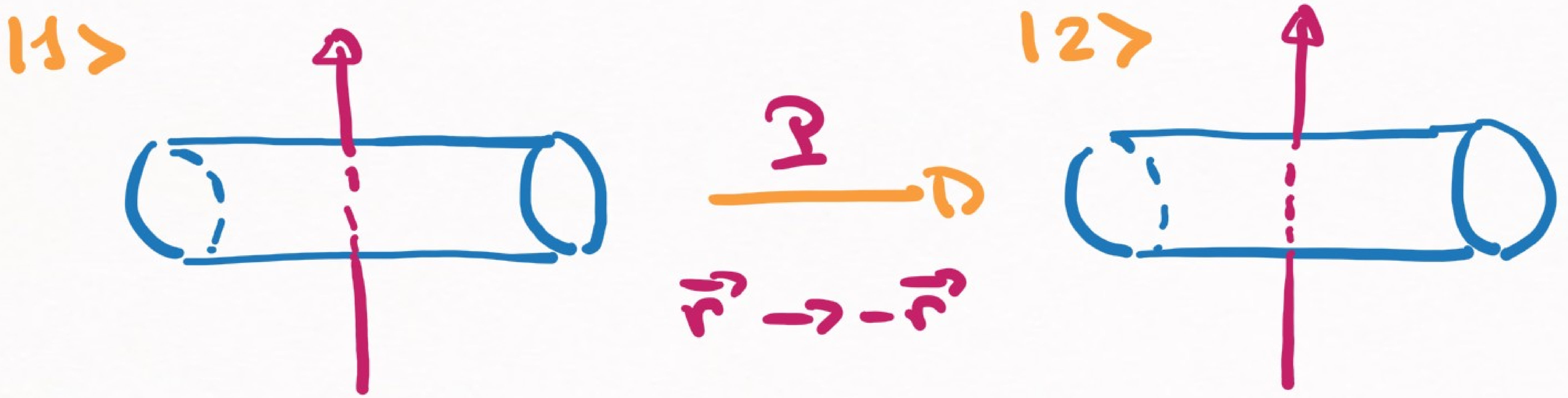


⇒ $|B_1\rangle \not\cong |B_2\rangle$

⇒ this rotation is not trivial (counts) ⇒ energy levels

[QUANTUM ROTATOR] ⑦

$\Rightarrow$ Besides, the rotation must have even L



$$\underbrace{\hat{P}|1\rangle = |2\rangle \qquad |1\rangle \cong |2\rangle}_{\text{Parity transformations do not change the nucleus}}$$

Why? ← [Only L = 0, 2, 4, ... will be allowed] ←

(next slide)

Reminder: for nuclei $\rho(\vec{r}) = \rho(-\vec{r})$ → shape of nuclei should be symmetric under parity

[QUANTUM ROTATOR] ⑧

⇒ Notice that $\underbrace{\vec{r} \to -\vec{r}}_{\text{parity transformation}}$ is identical to a $\theta = \pi$ rotation for $\underline{\underline{L = 1, 3, 5, \ldots}}$

$$R(\theta) = e^{i\theta L}$$ (change of phase from rotations)

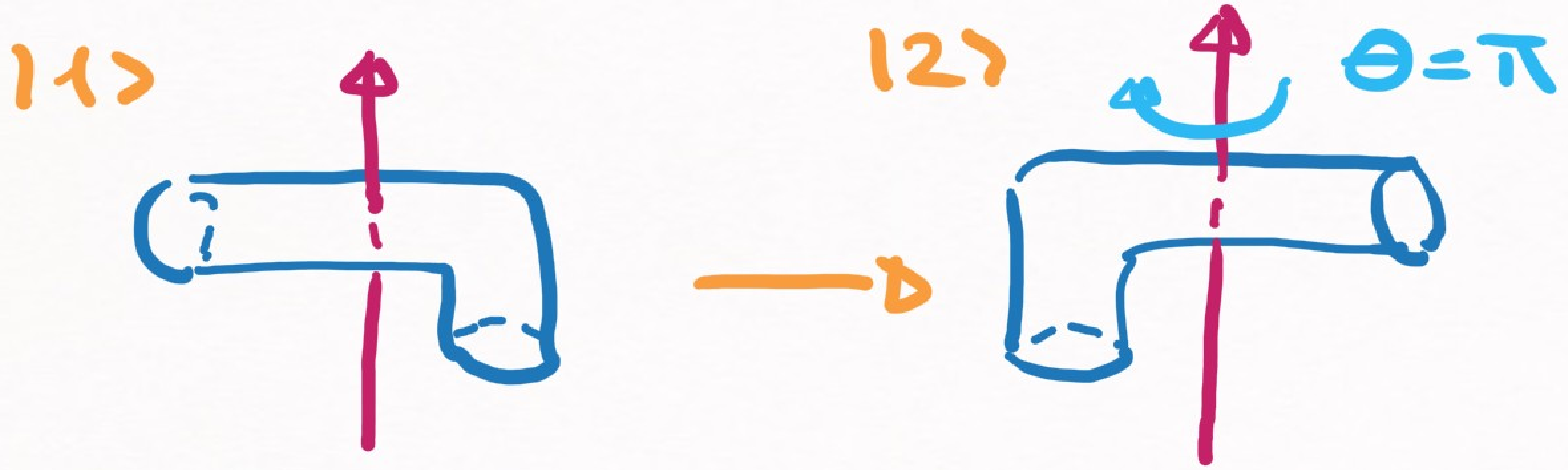
⇒ $\theta = \pi$ ⇒ $R(\pi) = (-1)^L$

$R(\pi)|1\rangle = (-1)^L|1\rangle = |1\rangle$ ⇒ $(-1)^L = +1$

⇒ $L = 0, 2, 4, \ldots$
(even)

[QUANTUM ROTATOR] ⑨

⇒ $L = 1, 3, 5, \dots$ is possible though in certain cases:



$$|1\rangle \neq |2\rangle$$

$$P|1\rangle = |2\rangle$$

$$P^2|1\rangle = |1\rangle$$

⇒ In this case we could have rotations with arbitrary L

⇒ However, these are system with chirality ($P|1\rangle \neq \pm|1\rangle$)

⇒ Nuclei are always parity invariants ($L = 0, 2, 4, \dots$)

[ROTATIONAL STATES] ①

Rotational levels in nuclei:

1) non-spherical nucleus

2) $L = 0, 2, 4, \ldots$ (even)

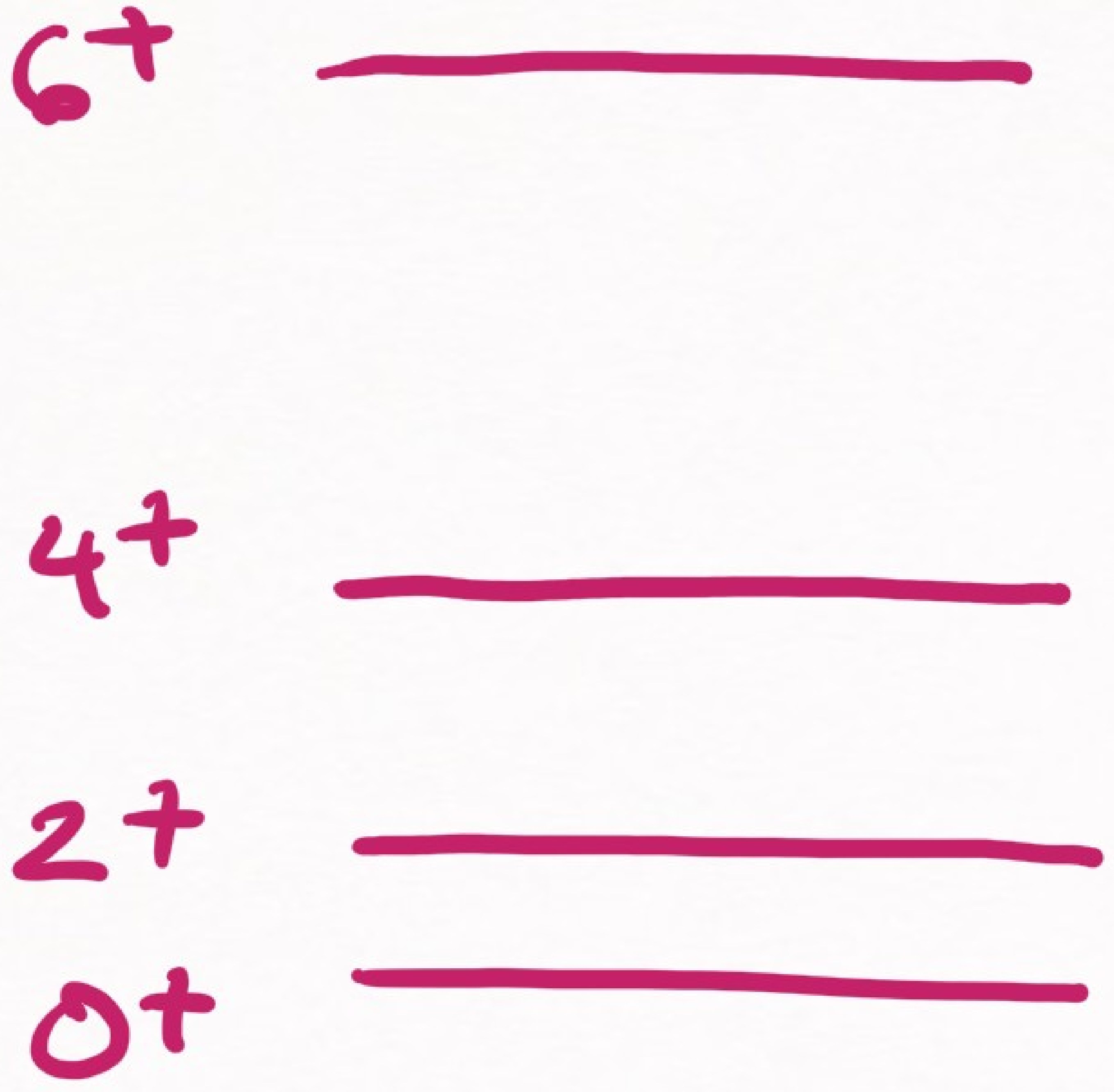
⇒ The energy levels must then be:

$$\left[E(L) = \frac{1}{2J} L(L+1) \right]$$

$$J^P \longrightarrow J = L \, , \; P = (-1)^L = +$$

[ROTATIONAL STATES] ②

Rotational levels in nuclei: $\underline{\underline{E(L) = \frac{1}{2\mathcal{I}} L(L+1)}}$



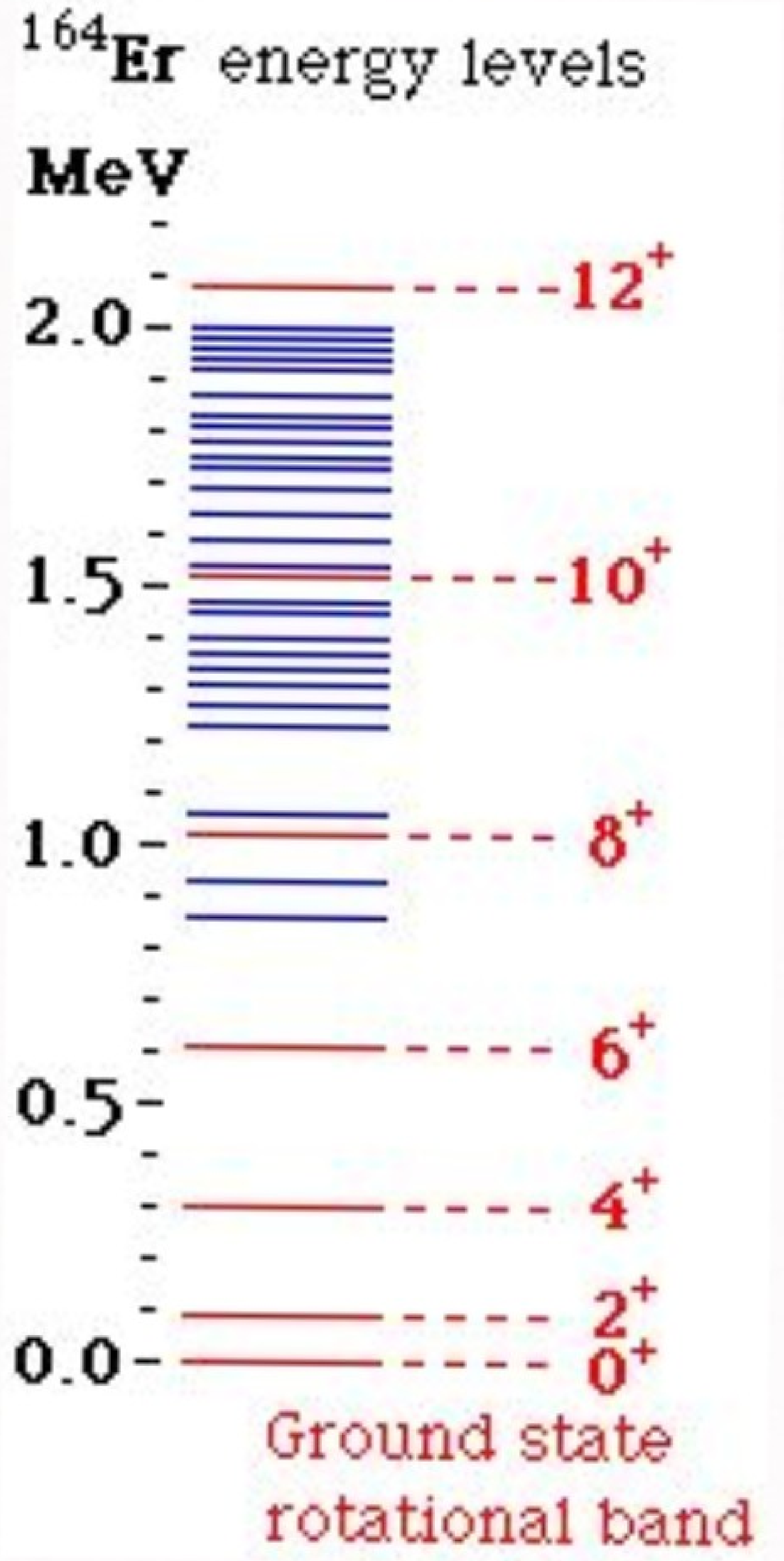
$$\frac{E(4^+)}{E(2^+)} \simeq \frac{4\cdot 5}{2\cdot 3} \simeq 3.33$$

$$\frac{E(6^+)}{E(2^+)} \simeq 7$$

these are characteristic ratios of the rotational states

[ROTATIONAL STATES] ③

⇒ Example: ^{164}Er $A \sim (150-190)$

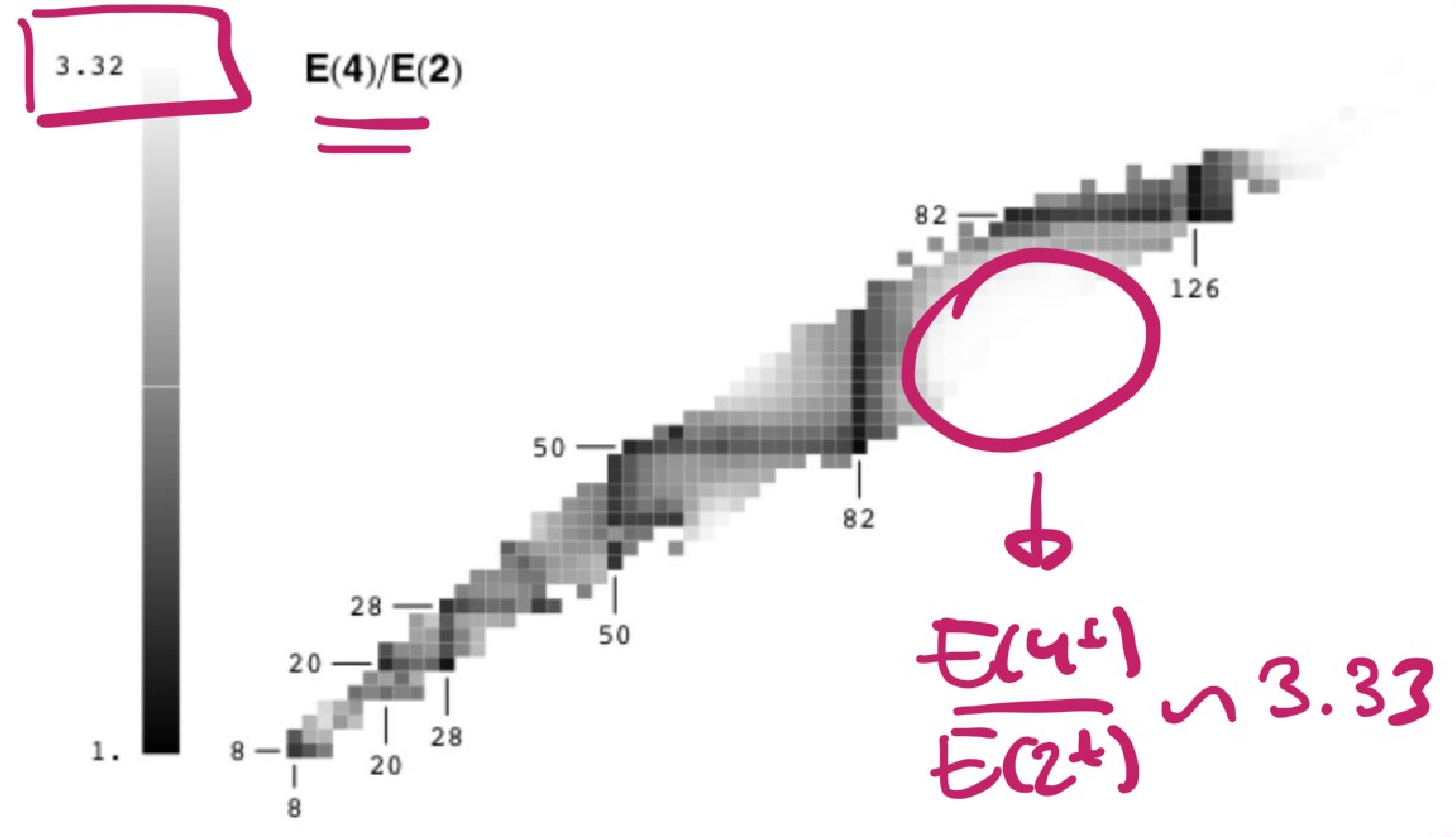


⇒ Notice the multitude of energy levels

⇒ First few excited states are very rotational

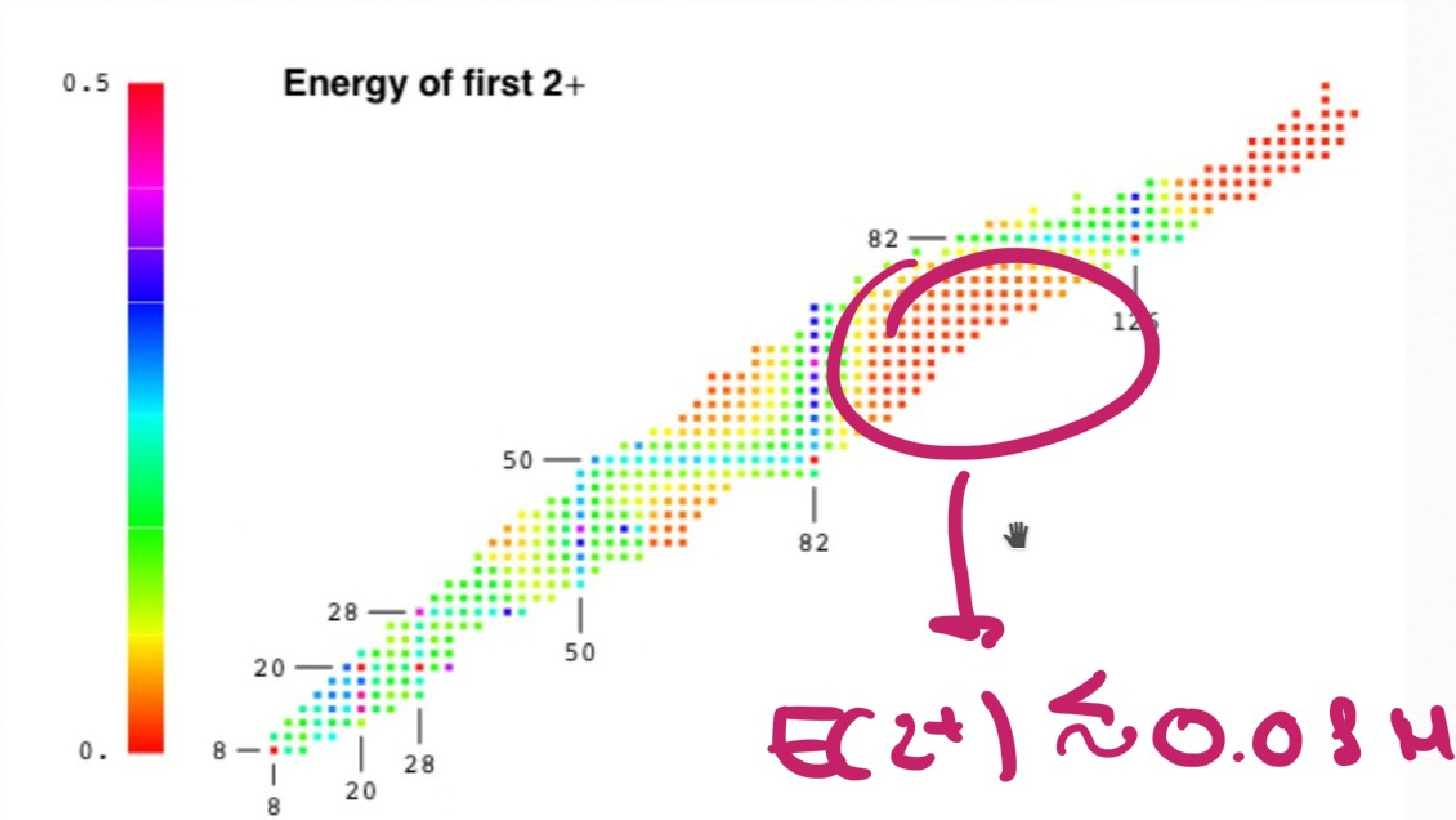
[ROTATIONAL STATES] ④

=D We can also plot $\frac{E(4^+)}{E(2^+)}$



$\frac{E(4^+)}{E(2^+)} \sim 3.33$

(rotational)



$E(2^+) \approx 0.08$ MeV

[ROTATIONS : RECAP] → Physics is about scales

1) Shell-model : $\sim \underline{\underline{(1-2)}}$ MeV

2) Rotational levels : $\sim \underline{\underline{0.1}}$ MeV (or even less)

$$\left[E(4^+)/E(2^+) \simeq 3.33 \right]$$

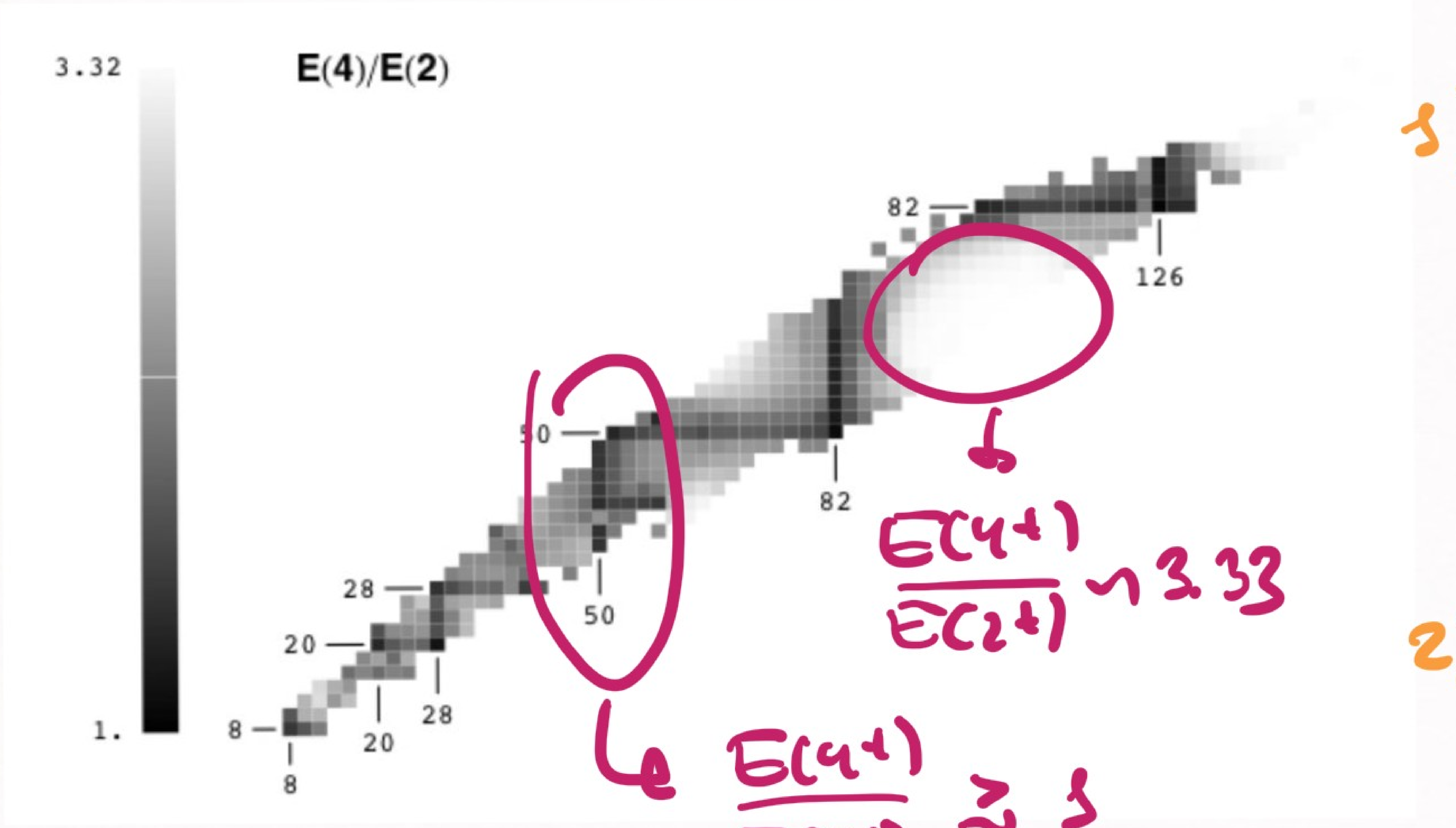
3) Vibrational levels : ~ 0.5 MeV

$$\left[E(4^+)/E(2^+) \simeq \underline{\underline{2}} \right]$$

⇓

Next part of the lesson

$\Rightarrow$ Let's take a second look at $E(4^+)/E(2^+)$



$$\frac{E(4^+)}{E(2^+)} \sim 3.33$$

$$\frac{E(4^+)}{E(2^+)} \gtrsim 1$$

1) Shell-model of $j > 2$

It's perfectly possible to build $2^+, 4^+$ states with similar energies

(modulo ΔV)

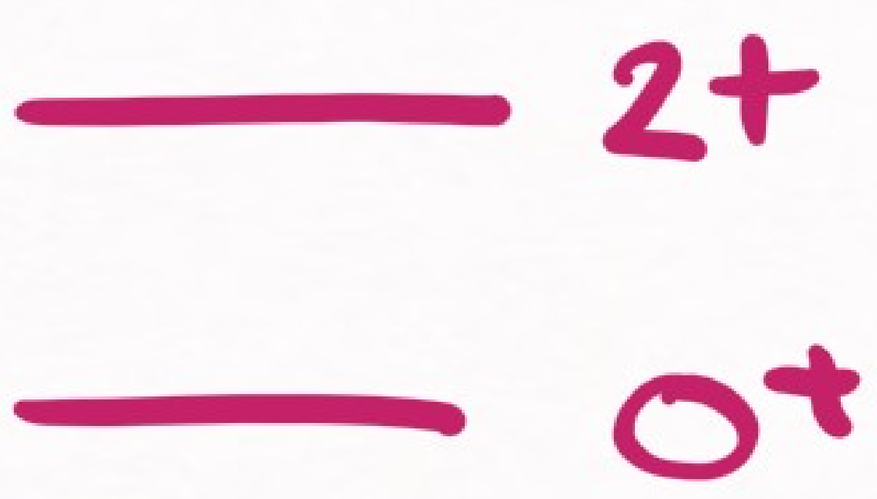
2) Rotational models:

$$E(4^+)/E(2^+) \sim 3.33$$

3) Vibrational models

$$E(4^+)/E(2^+) \sim 2$$

⇒ A few pieces of the nuclear structure puzzle:

1) 

} almost universal pattern in all nuclei

2) $E(2^+)$ { gives an indication of the scale
⇒ tells us which mode to use

3) $\dfrac{E(4^+)}{E(2^+)}$ } this ratio requires an explanation

⇒ let's see how this is explained in the collective model

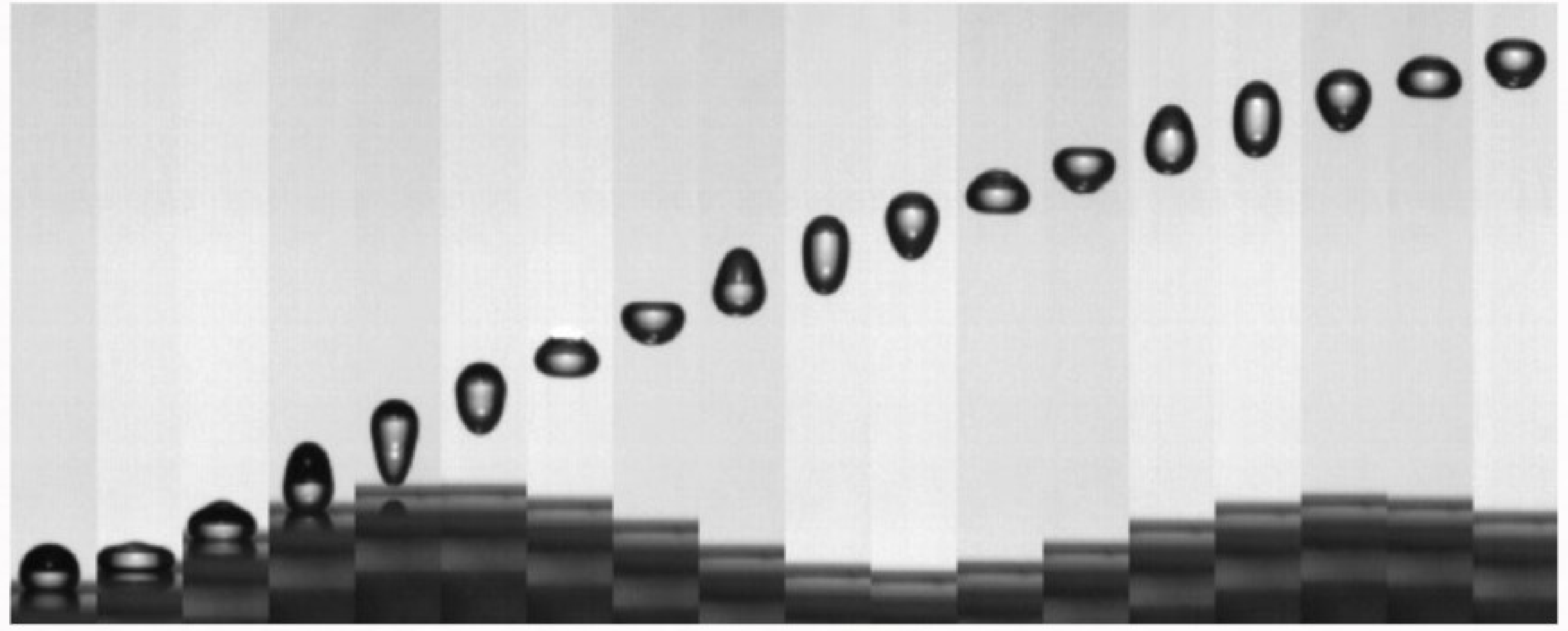
[COLLECTIVE MODEL]

1) Rotational models :

→ non-spherical nucleus
→ rigid body

2) Vibrational models:

→ spherical nucleus
→ liquid

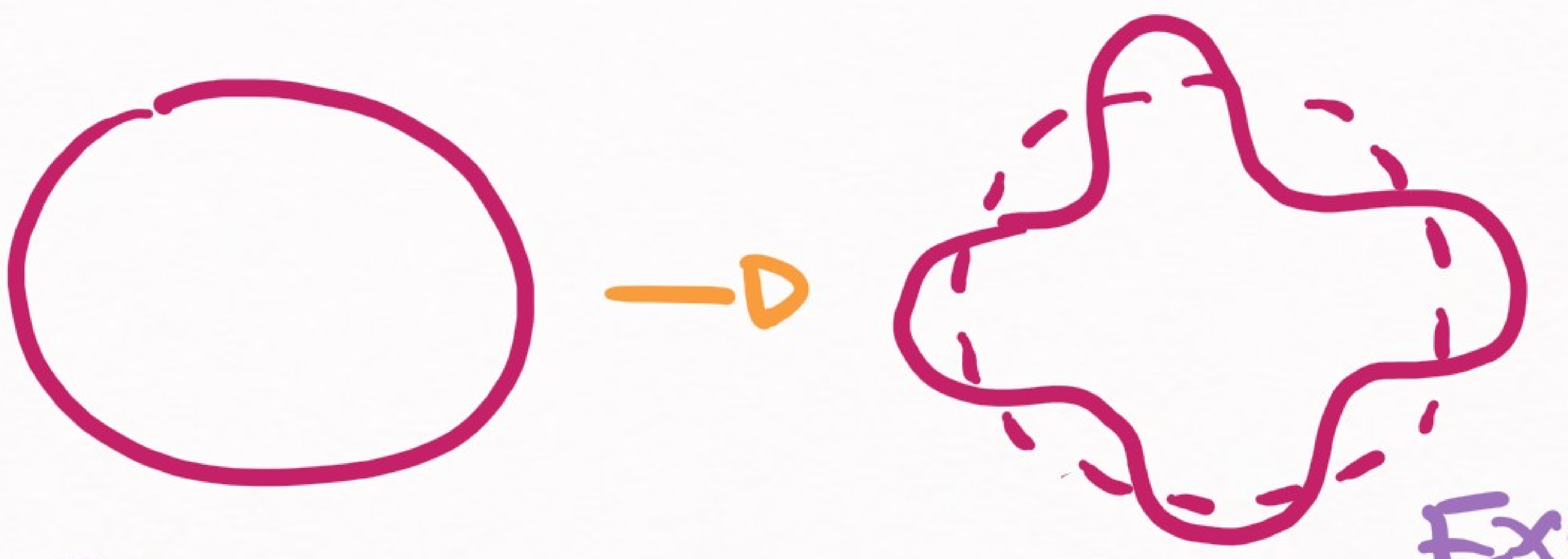


[We want the quantum mechanical version of this]

We will not give a full derivation though

VIBRATIONS ①

⇒ Basic idea:



$L = 4$

↓

We will see why later

⇒ What we need here are quantized vibrations

↳ They behave like phonons (a quasi-particle that is a boson)

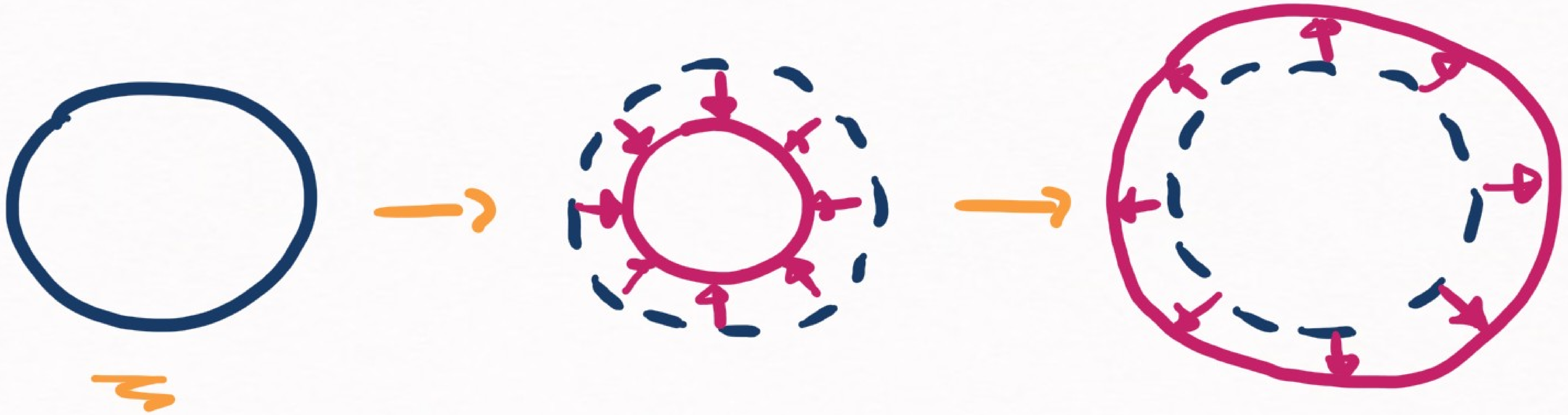
VIBRATIONS ②

⇒ Let's see the different type of vibrations :

0) $\underline{\underline{L = 0 \ (0^+)}}$

⇓

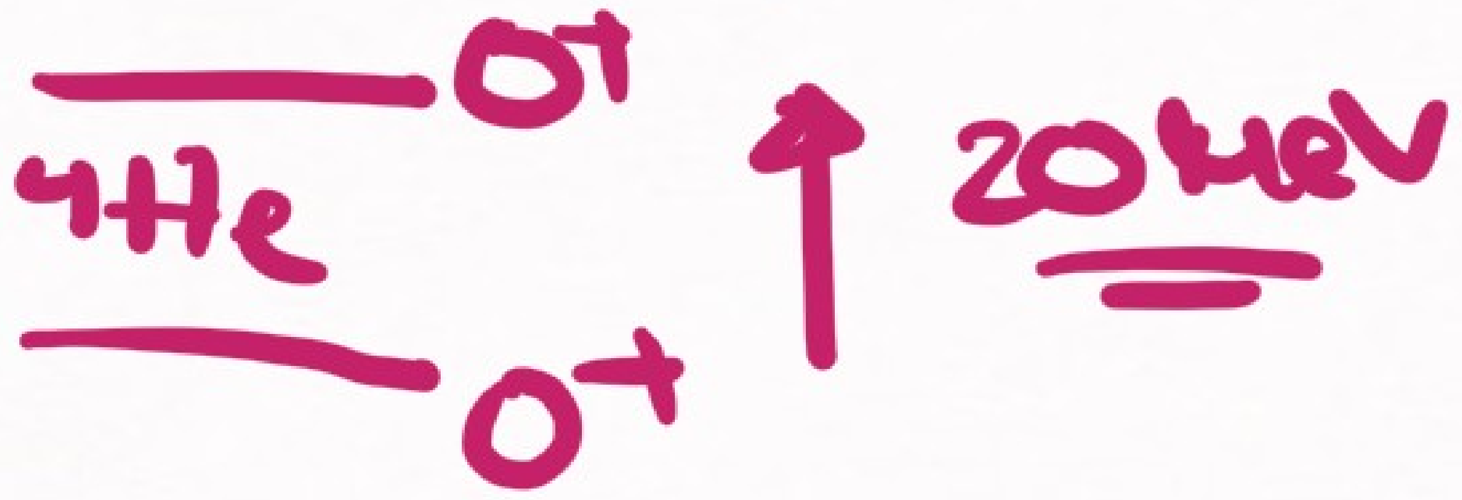
Breathing mode



⇒ Related with the compressibility of nuclear matter

(for liquids in general you need of energy to compress it)

These are very high energy excitations ⇐

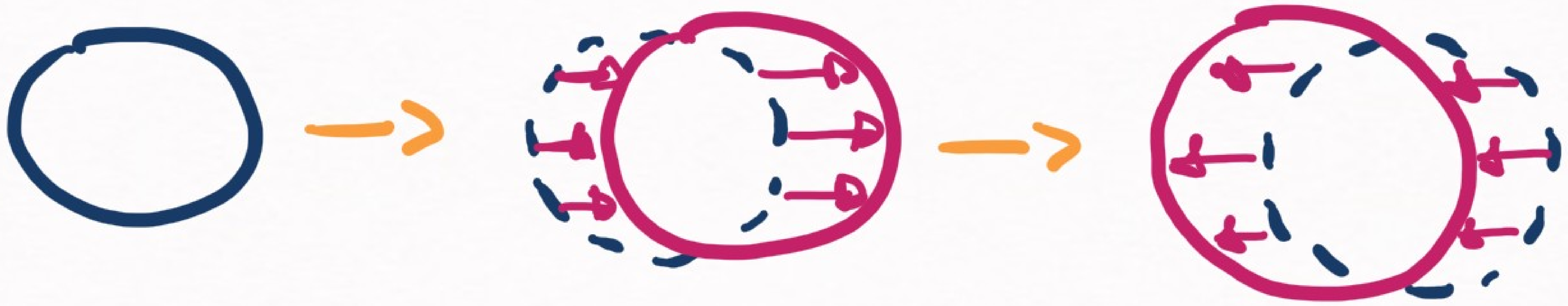


VIBRATIONS ③

⇒ Let's see the different type of vibrations :

1) $L = 1$

↓



dipolar vibration → It is a trivial vibration, because it can be reduced to a translation

↙

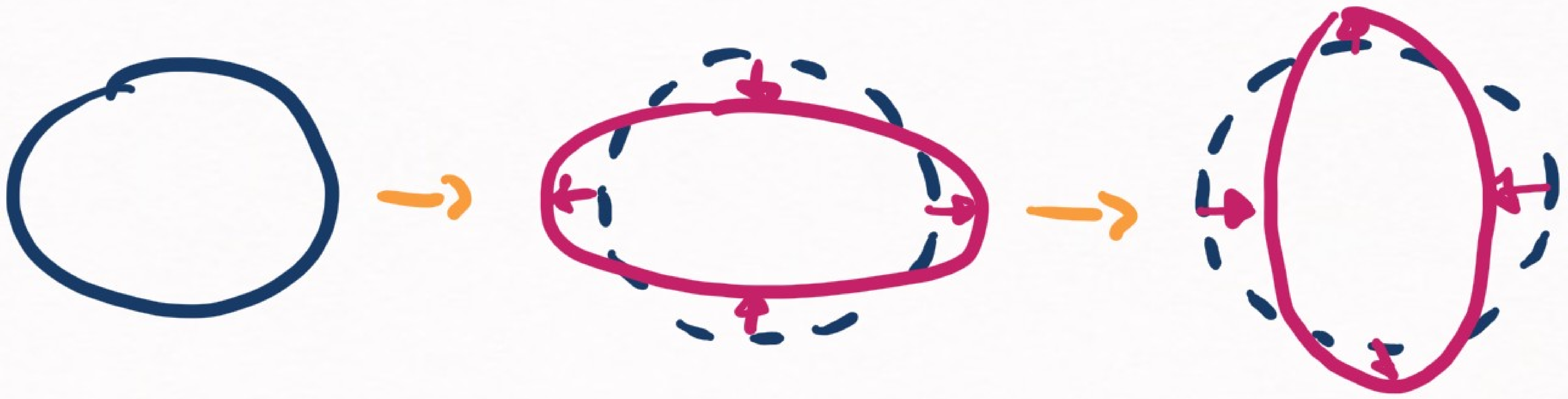
They can be ignored (because they do not do anything)

VIBRATIONS ④

⇒ Let's see the different type of vibrations :

2) L = 2 (2^+)

↓



Quadrupolar vibration → non-trivial, lowest energy vibration available

(most important type of vibration)

⇒ Explains why

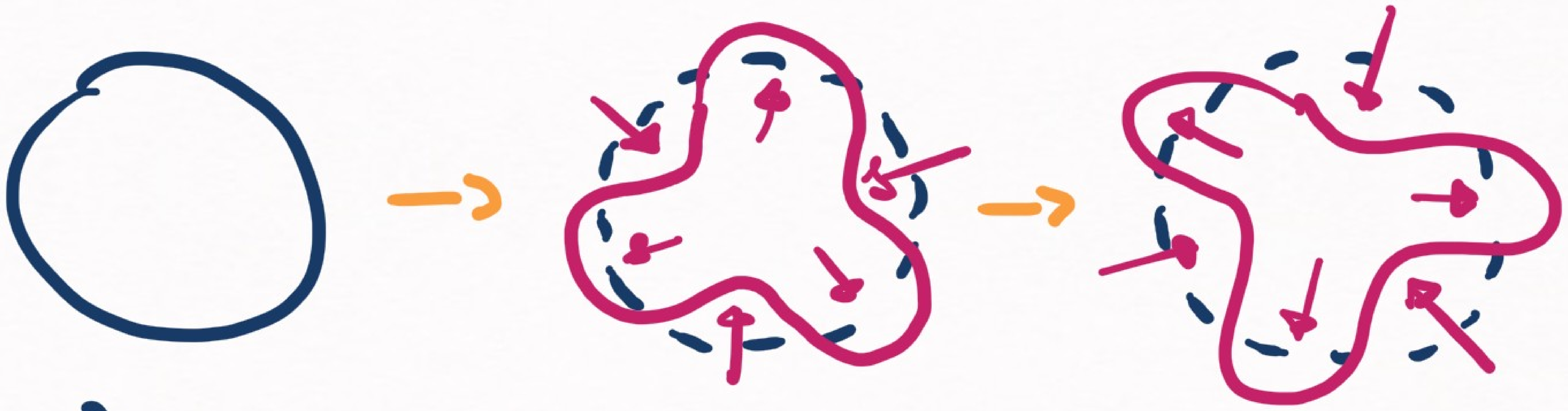
is so common

VIBRATIONS ⑤

=> Let's see the different type of vibrations :

3) $L = 3\ (3^-)$

Octopolar vibration



$$\omega_3 \simeq (2-3)\,\omega_2$$

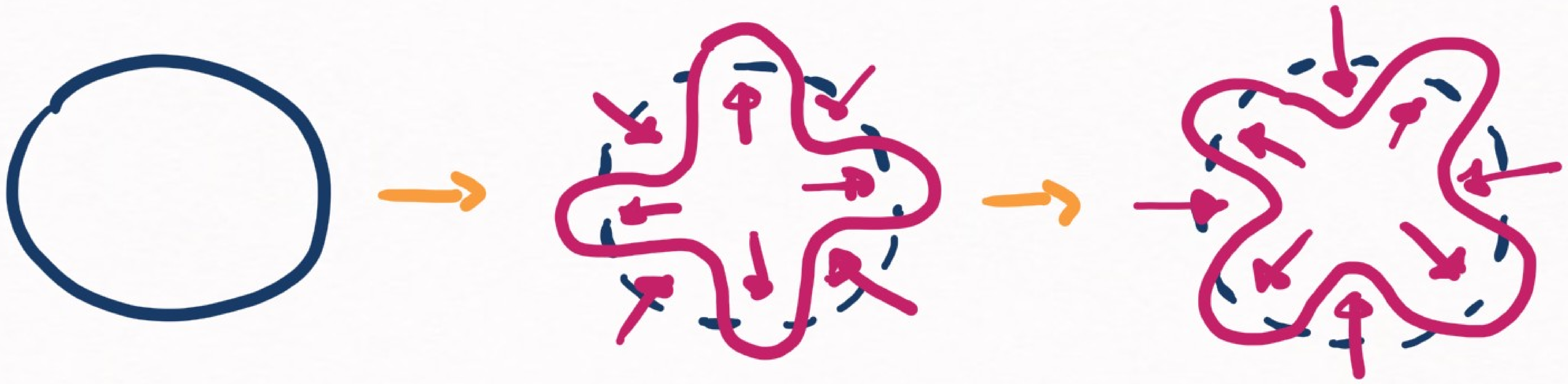
(higher energy,
a bit less important
than the quadrupolar
ones)

VIBRATIONS ⑥

⇒ Let's see the different type of vibrations :

4) L = 4 (4+)

↓



hexadecupolar

(higher energy, usually we won't need it)

[VIBRATIONS TYPES]

$$R(\theta,\varphi) = R_0\left[1 + \sum_{\lambda\mu} \alpha^*_{\lambda\mu} Y_{\lambda\mu}(\theta,\varphi)\right]$$

0) $\lambda=0$ → Compression (high energy)

1) $\lambda=1$ → (translation) (trivial: we ignore it)

2) $\lambda=2$ → Quadrupole deformation (most common)

3) $\lambda=3$ → Octupole deformation ($\omega_3 \simeq (2-3)\omega_2$)

4) $\lambda=4$ → Hexadecapole (usally we will not need this one)

[QUANTIZED VIBRATIONS] ① → How to model them?

⇒ We use the second quantization formalism:

vibrational state → $|n_0, n_2, n_3, \dots\rangle$

n_i: # of vibrations with i

$$\Rightarrow \left[H|n_0, n_2, n_3, \dots\rangle = \hbar(n_0\omega_0 + n_2\omega_2 + n_3\omega_3 + \dots) |n_0, n_2, n_3, \dots\rangle \right]$$

Like harmonic oscillator: $H|n\rangle = \hbar(n\omega + \underline{\underline{\tfrac{1}{2}}})|n\rangle$

[QUANTIZED VIBRATIONS] ②

⇒ $H_0|n_0, n_2, n_3, \ldots\rangle = \hbar(n_0\omega_0 + n_2\omega_2 + n_3\omega_3 + \ldots)$
$\times |n_0, n_2, n_3, \ldots\rangle$

⇒ The vibrations behave like boson (phonons)

Angular momentum + parity:

monopole: 0^+ quadrupole: 2^+ octupole: 3^-

n_0 monopoles ⇒ $0^+ \otimes 0^+ \otimes \ldots \otimes 0^+ = 0^+$

$n_2 = 2$ quadrupoles ⇒ $2^+ \otimes 2^+ = 0^+ \oplus 2^+ \oplus 4^+$

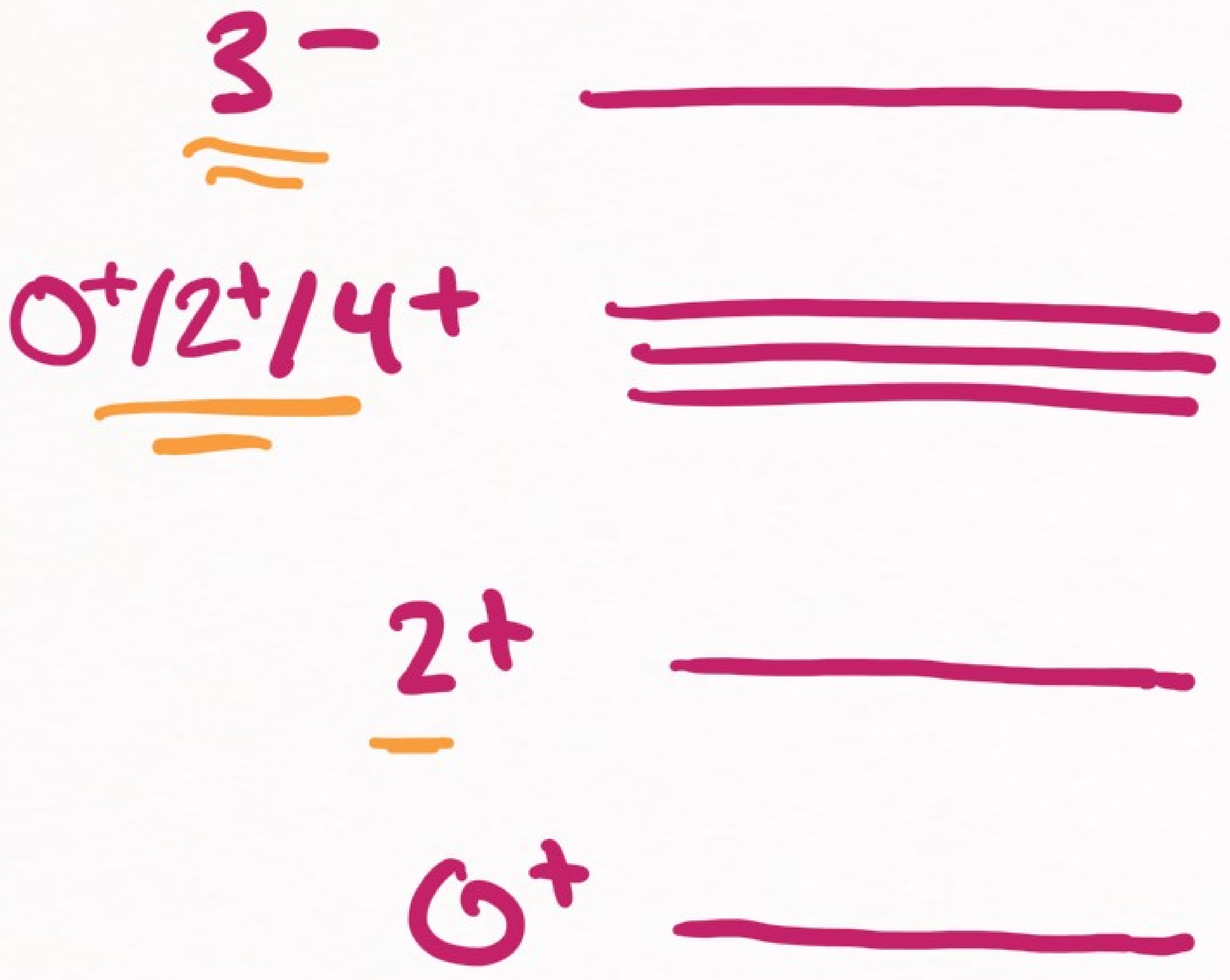
($1^+, 3^+$ not allowed ⟶ ⟶ because antisymmetric)

[VIBRATIONAL SPECTRUM] ③

⇒ What type of spectrum will we obtain?

Different from rotational states

⇑



$\omega_3 \rightarrow n_3 = 1$ phonons

$2\omega_2 \rightarrow n_2 = 2$ phonons

$\omega_2 \rightarrow n_2 = 1$ phonons

$0 \rightarrow 0$ phonons

$$\underline{\underline{\frac{E(4^+)}{E(2^+)} \simeq 2}}$$

[VIBRATIONAL SPECTRUM] ②

$\Rightarrow$ Examples: ^{64}Zn & ^{122}Te

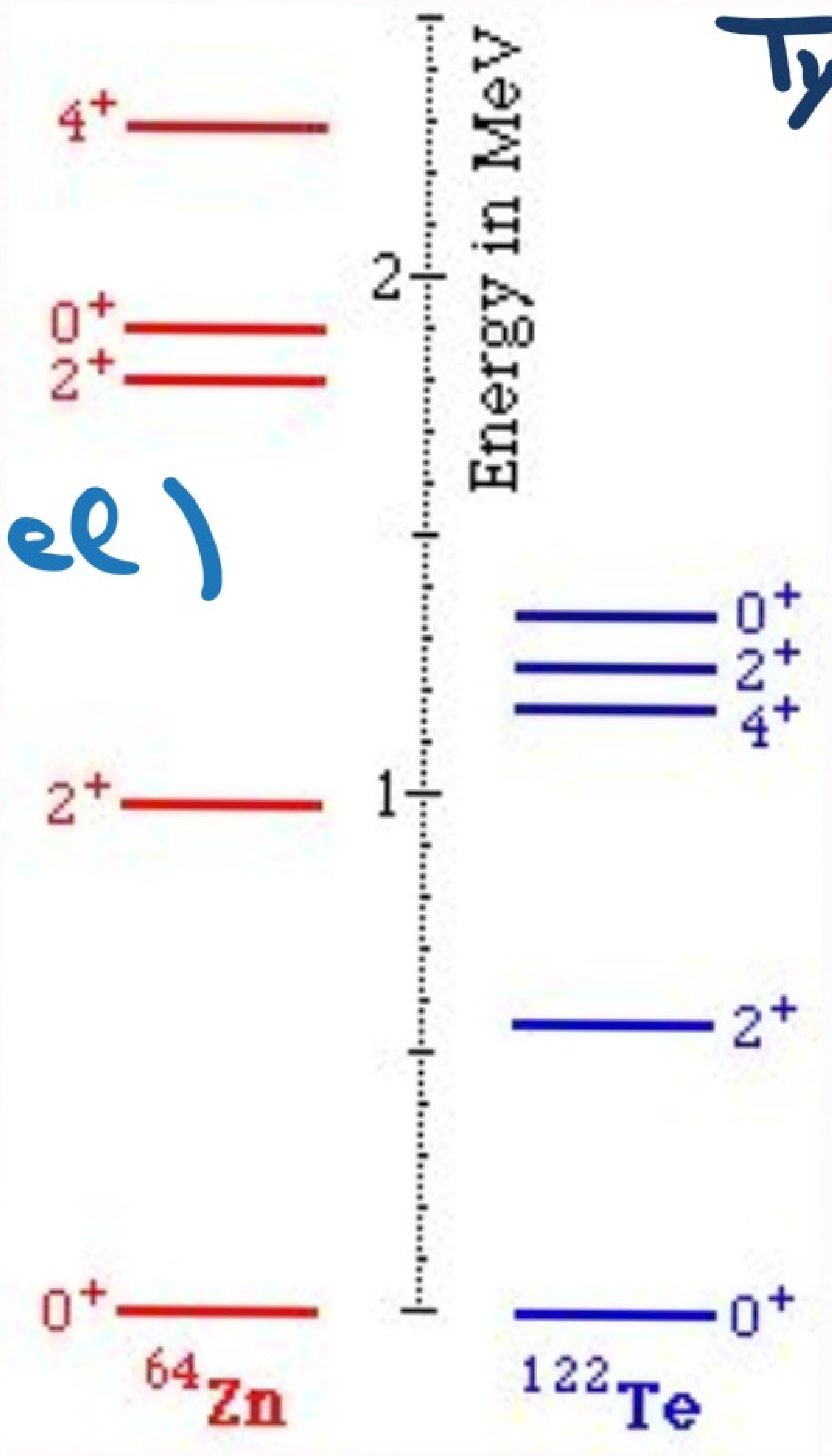
Transitional spectrum

(a bit of vibrational and a bit of shell model)

$E(2^+) \sim 1$ MeV

$\frac{E(4^+)}{E(2^+)} \approx 2.3$

still, pretty close to vibrational



Typical vibrational spectrum

(1) $0^+, 2^+, 4^+$ states are almost together

(2) $E(2^+) \sim 0.5$ MeV

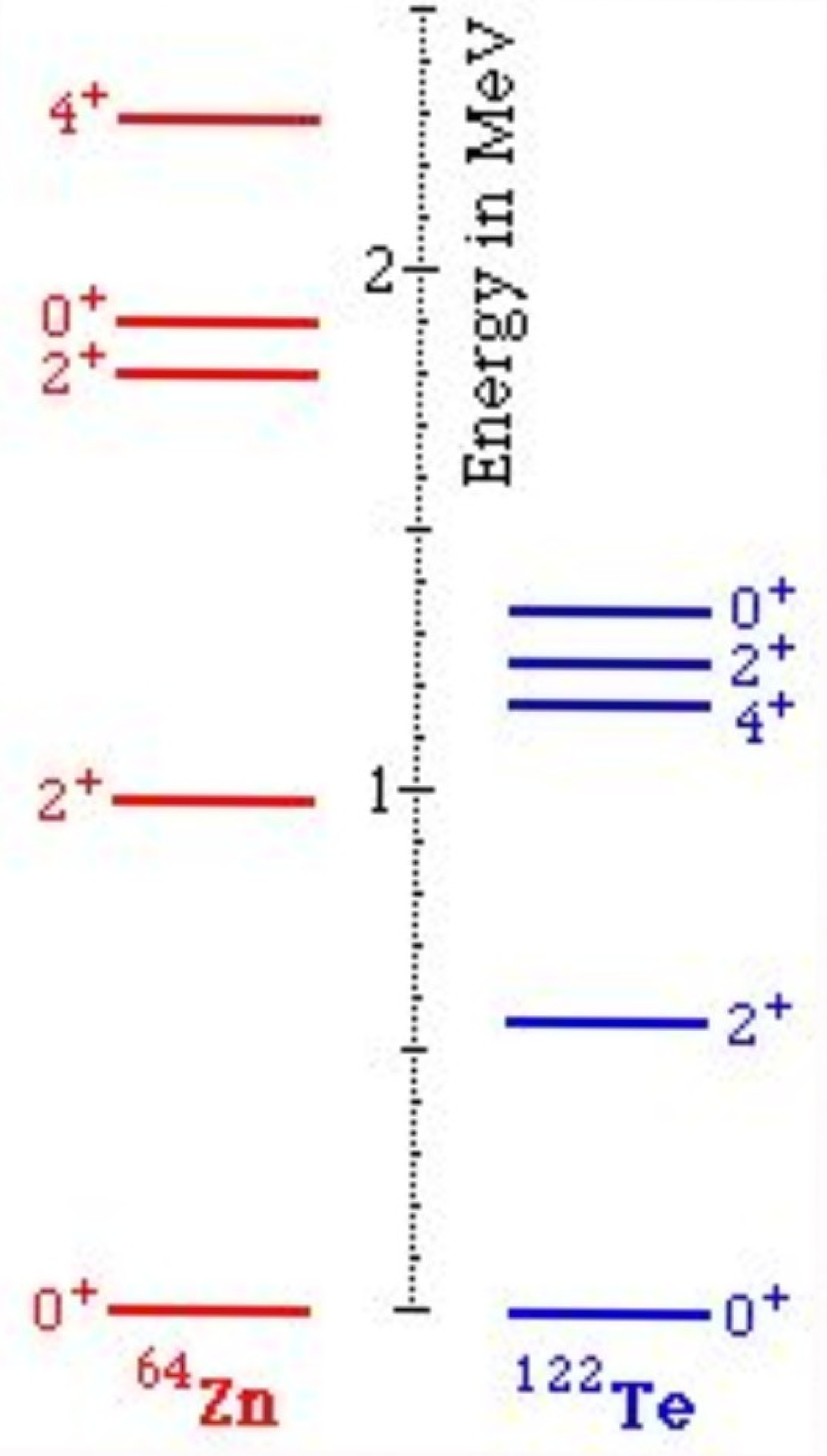
(3) $E(4_1^+)/E(2_1^+) \approx 2$

[VIBRATIONAL SPECTRUM] ③

⇒ Examples : ^{64}Zn & ^{122}Te

$E(2_1^+) \sim 1\,MeV$

$$\frac{E(4_1^+)}{E(2_1^+)} \simeq 2.3$$



$E(2_1^+) \sim 0.5\,MeV$

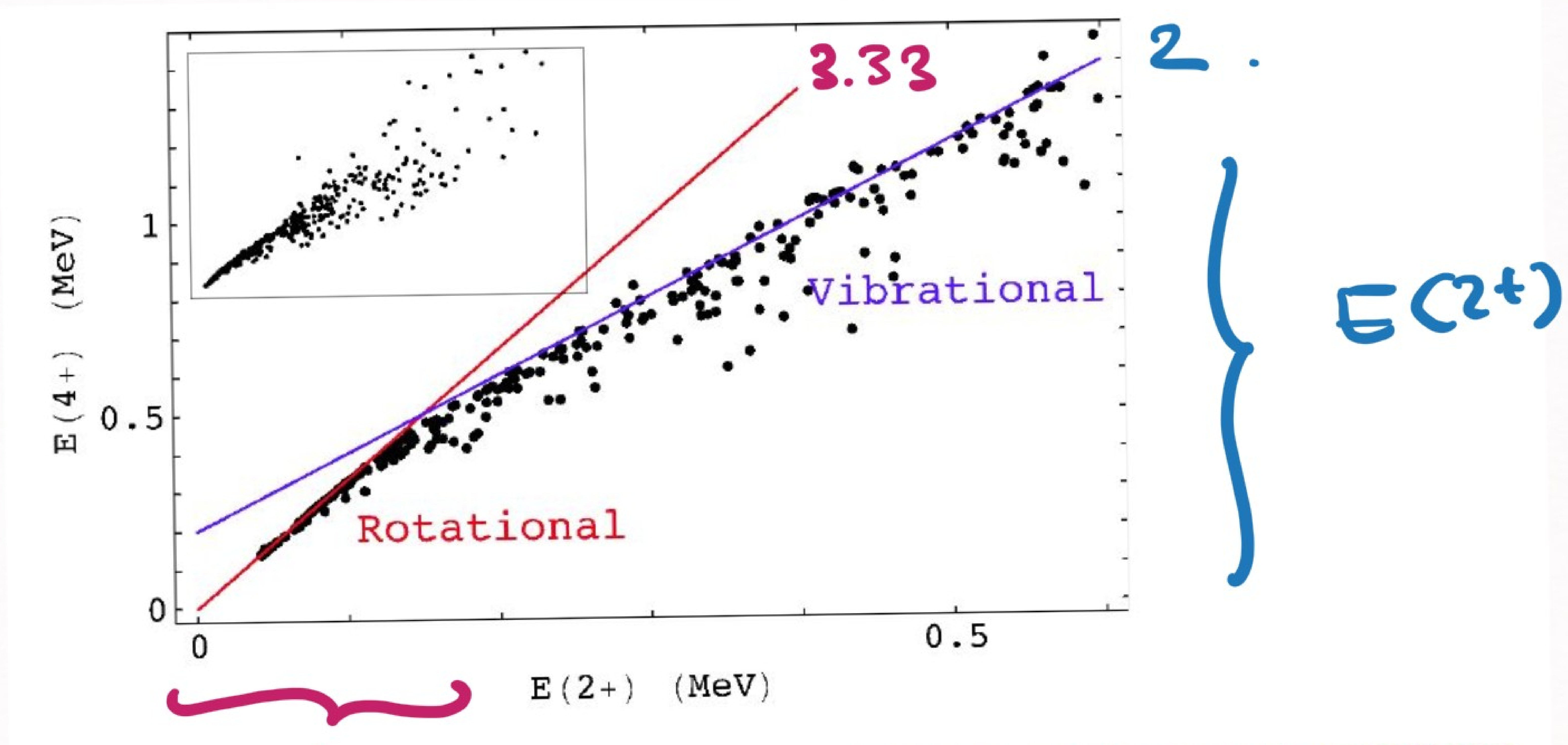
$$\frac{E(4_1^+)}{E(2_1^+)} \simeq 2$$

[VIBRATIONAL vs ROTATIONAL] ①

$$\frac{E(4^+)}{E(2^+)} \sim 3.33$$

$$\frac{E(4^+)}{E(2^+)} \sim 2$$

[VIBRATIONAL vs ROTATIONAL] ②



N.V. Zamfir *et al.*, Phys. Rev. Lett. 72 (1994) 3480

$E(2^+) \gtrsim 0.15$ MeV

Rotational spectra:

$E(2^+) \lesssim 0.15$ MeV

RECAP!

1) Typical excitation scale of each nuclear model

   1.a) Shell-model: $\sim (1-2)\,\mathrm{MeV}$ ✓

   1.b) Vibrational model: $\sim 0.5\,\mathrm{MeV}$ ✓

   1.c) Rotational model: $\sim 0.1\,\mathrm{MeV}$ ✓

2) Typical ratio of $E(4^+)/E(2^+)$:

   2.a) Shell-model: $E(4^+)/E(2^+) < 2$ (but <u>varies</u>)

   2.b) Vibrational model: $E(4^+)/E(2^+) \sim 2$ ✓

   2.c) Rotational model: $E(4^+)/E(2^+) \sim 3.33$ ✓

See you on Friday

15:50

(will be a bit longer than today)