NUCLEAR PHYSICS 17

EFFECTIVE FIELD THEORY

FORMULATIONS OF

THE NUCLEAR FORCE

RECAP!

Nuclear forces $\Rightarrow$ Fundamental input for any calculation of nuclear properties

a) Nucleon-nucleon scattering

b) Deuteron properties

c) Triton and ^{3}He properties + triton decays

d) ^{4}He properties

e) Nuclei w/ $A \geq 4$

$\Rightarrow$ [WANTED :]

A derivation of the nuclear forces from first principles

RECAP | Historical overview:

0) Yukawa's idea ⇒ Exchange of a boson (the pion)

1) Before-QCD era:

1.a) Multipion theories ⇒ Failed (lack of knowledge about pion dynamics and renormalization)

1.b) The OBE model ⇐

2) After-QCD era:

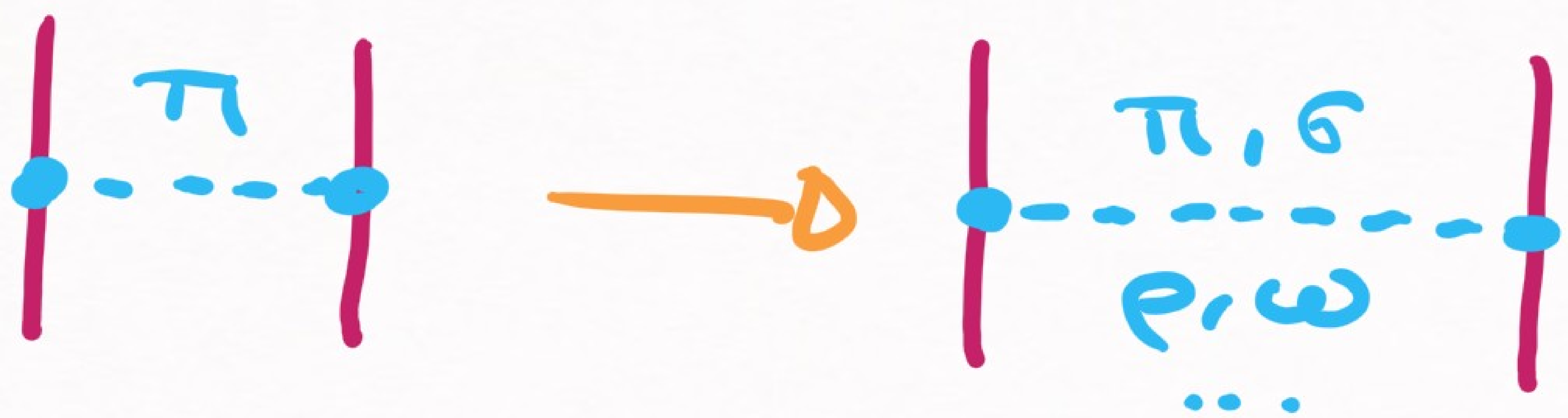
2.a) Asymptotic freedom ⇒ [Impossible to solve QCD directly]

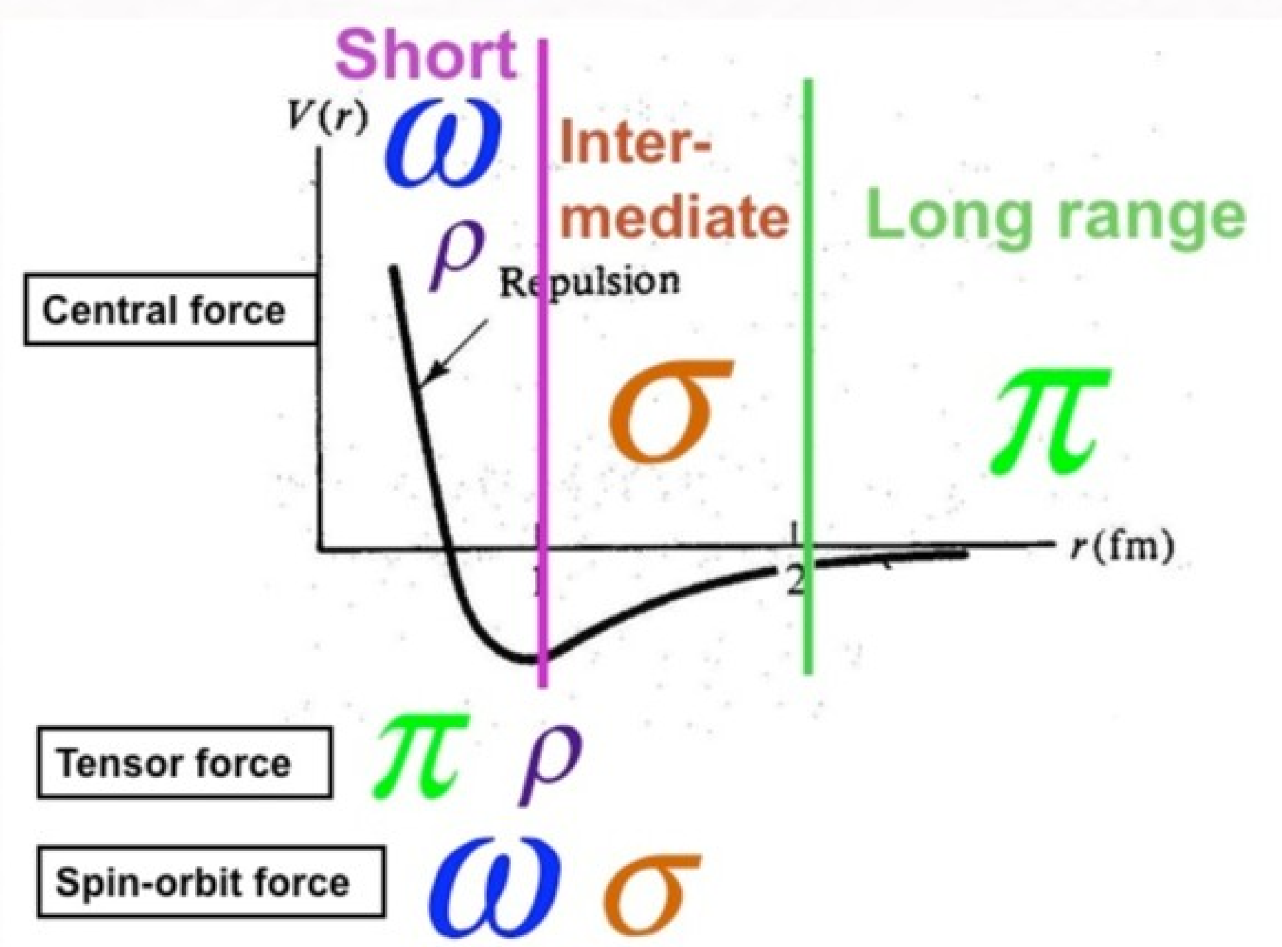
2.b) EFT approaches ⇐

RECAP Past lesson: [THE OBE MODEL]

1) The one boson exchange model:

natural extension of Yukawa's idea





2) Each meson has a job $\Rightarrow$

3) Infinities, singular potentials

$\Rightarrow$ We include form factors (pre-renormalization)

sort of ...

[THE FUNDAMENTAL PROBLEM OF NUCLEAR PHYSICS] 3

[Derivation of nuclear forces from first principles]

Before QCD:

OBE model was as close as we could get to first principles

$\Rightarrow$

After QCD:

not true anymore (because nucleon/mesons were no longer fundamental)

[THE FUNDAMENTAL PROBLEM OF NUCLEAR PHYSICS] ②

⇒ After QCD : OBE model simply became a phenomenological model

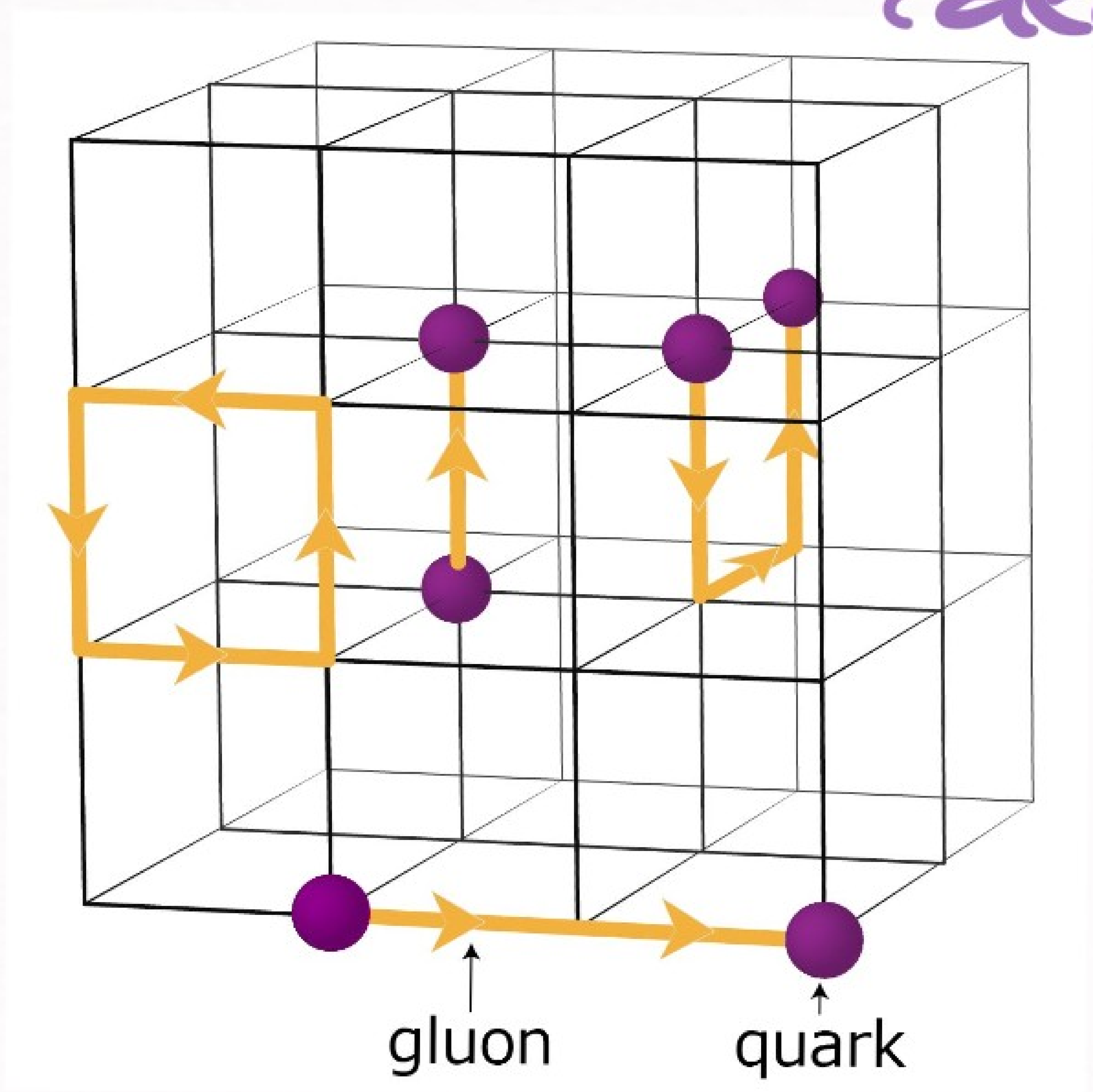
something that describes a given phenomenon (i.e. data) but lacks a solid theoretical foundation

⇒ Post QCD : a new kind of theory was required

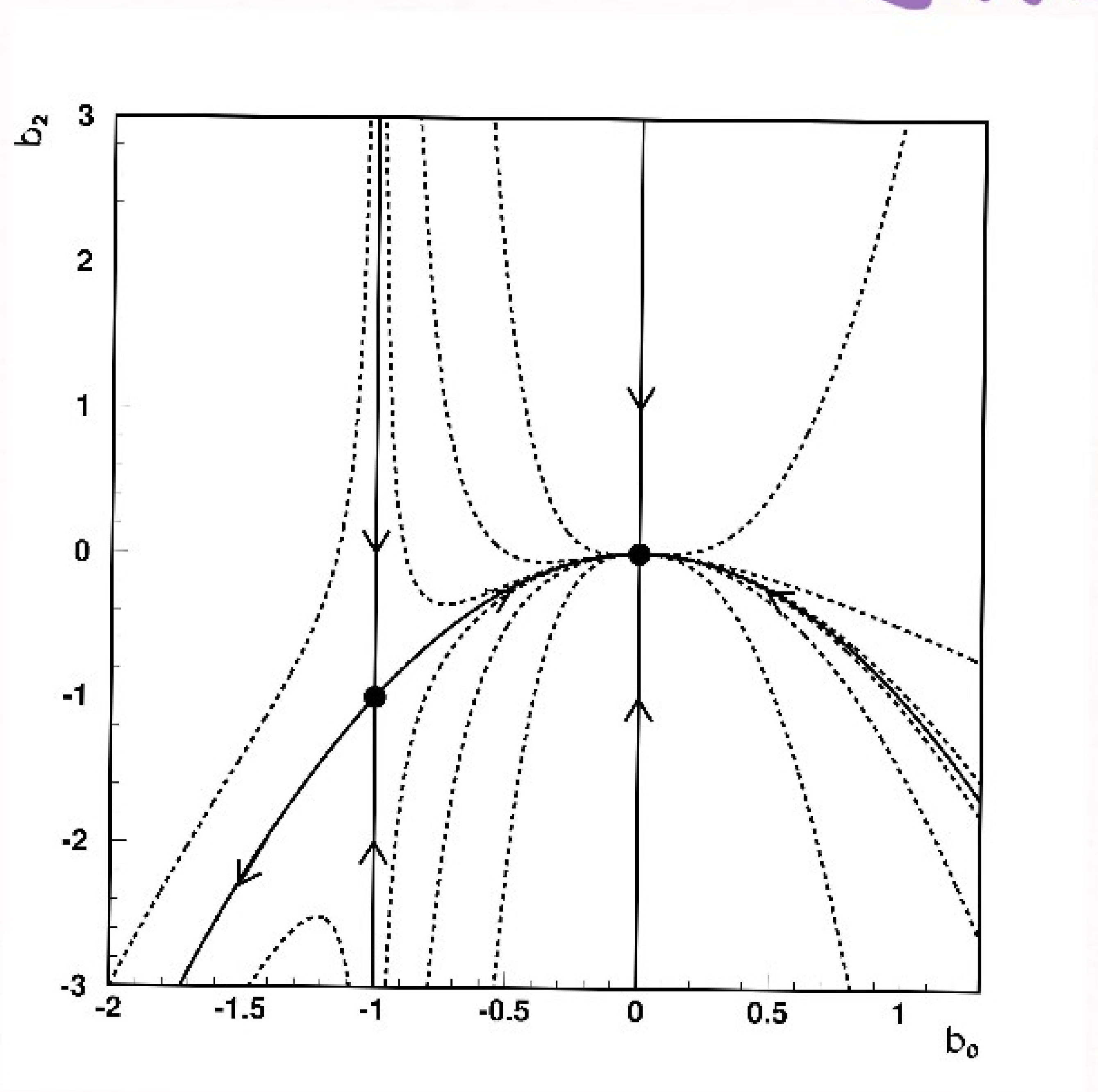
[THE FUNDAMENTAL PROBLEM OF NUCLEAR PHYSICS] 3

⇒ QCD is not solvable at low energies ⇒ some strategy is required

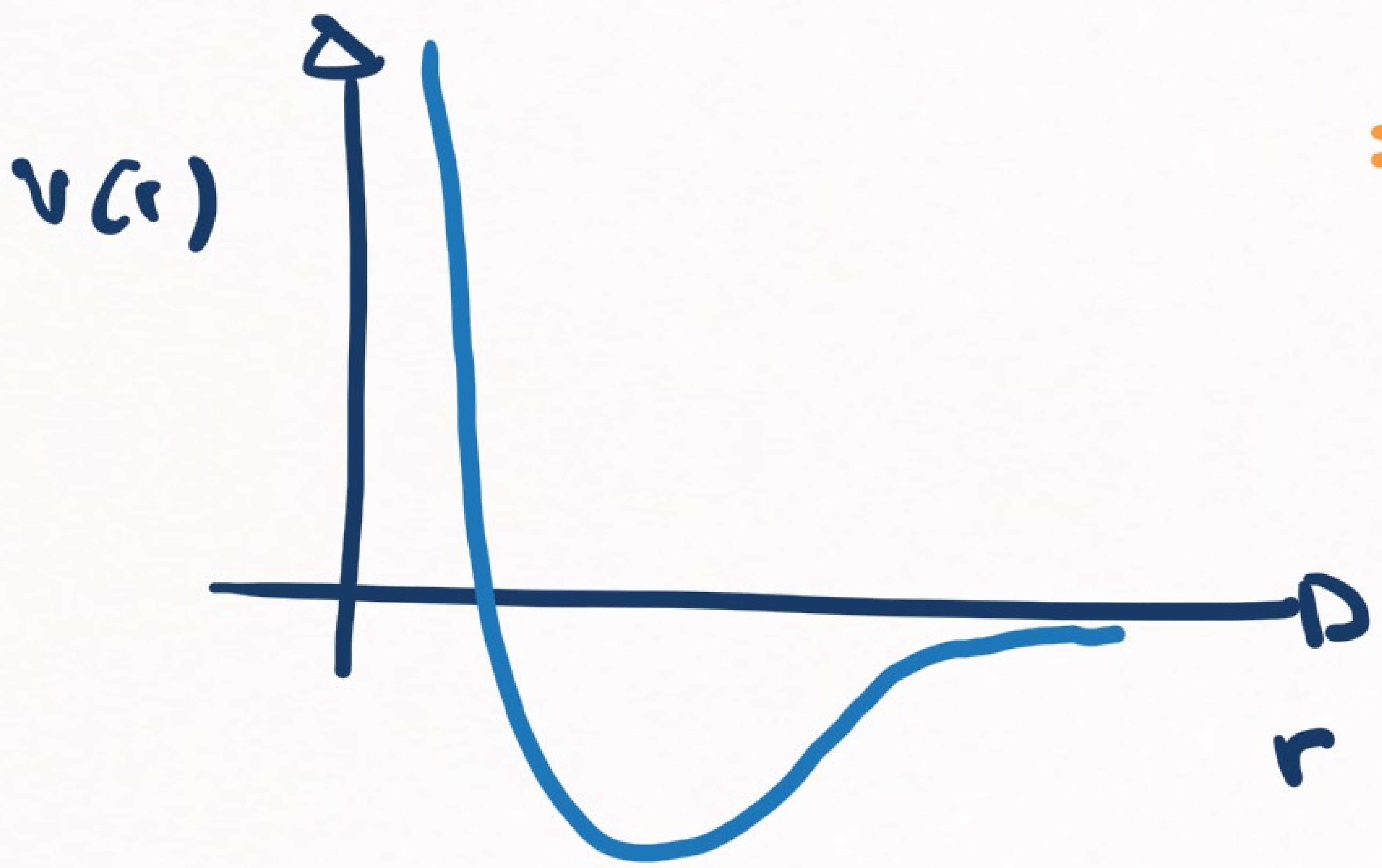
1) Lattice QCD (direct calculation)



2) EFT (indirect strategy)



[EFFECTIVE FIELD THEORIES (EFTs)] 1



⇒ Imagine that this is the true NN potential

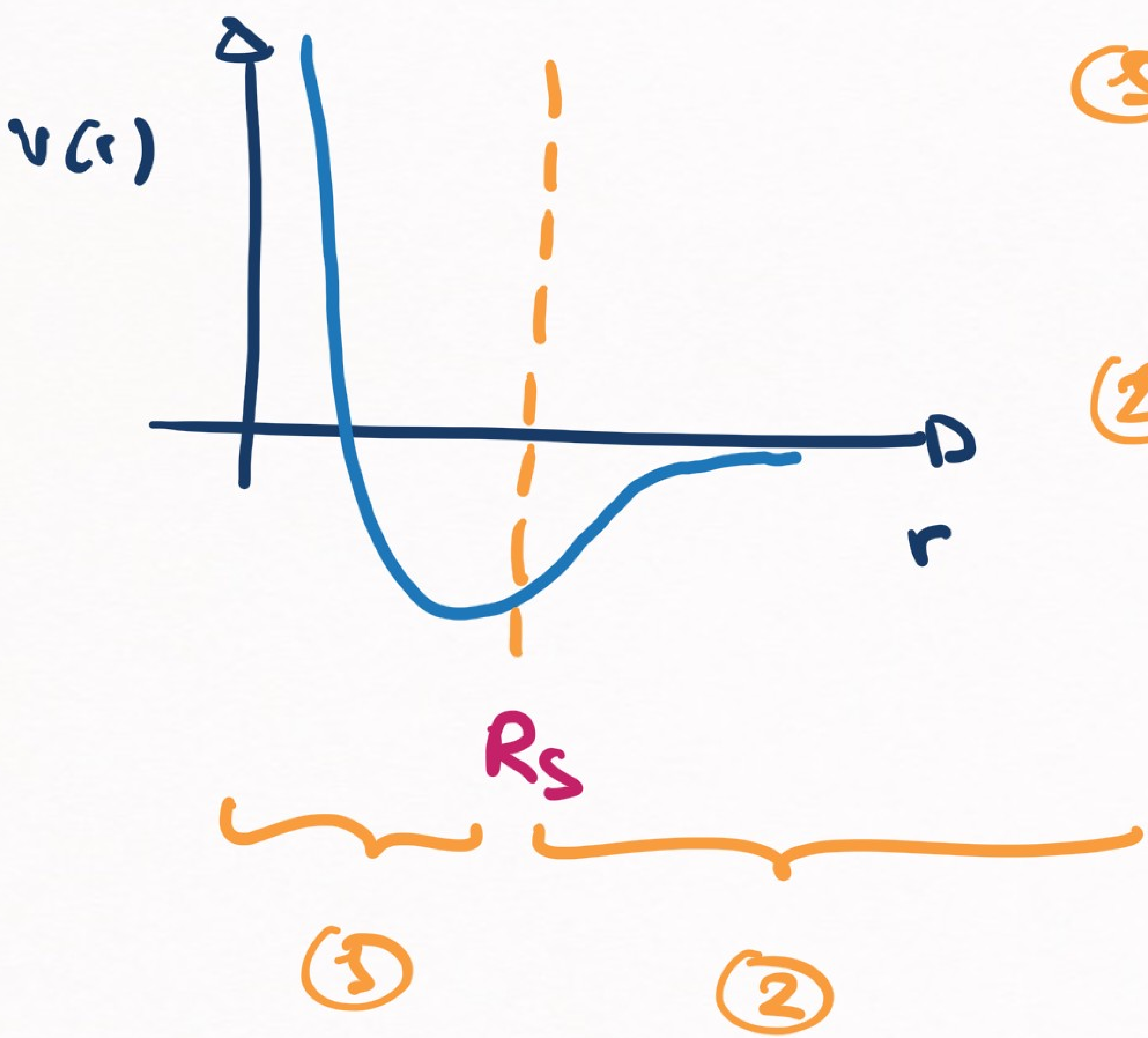
⇓

But then, what happens if we do not know the true potential at all distances (e.g. short distances)

This is why we have EFTs ⇐

⇒ We will begin by acknowledging that we do not know what happens at $R < R_s$

EFTs (2)



1 → Unknown physics
(short-range)

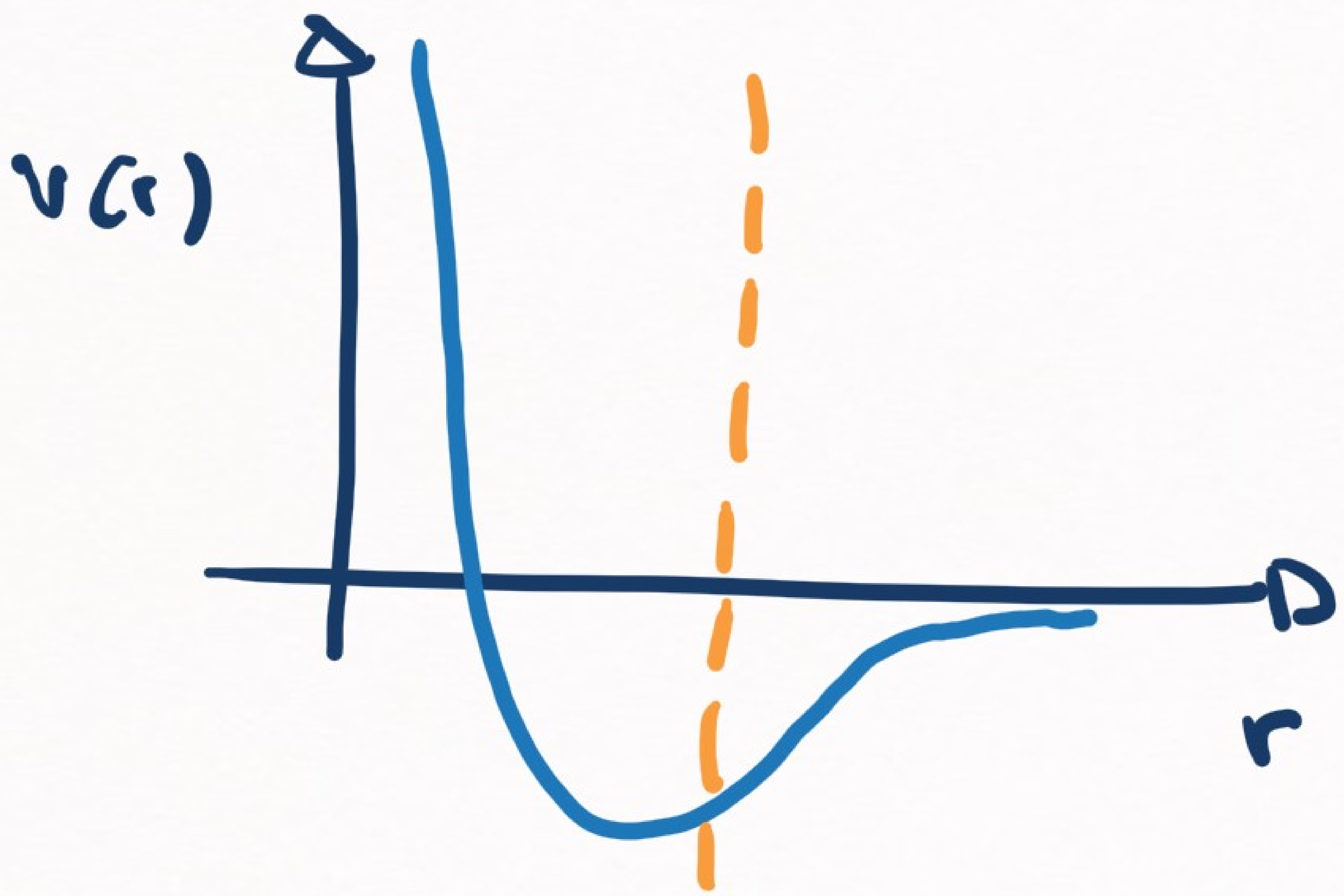
2 → Known physics
(long-range)

$\Downarrow$

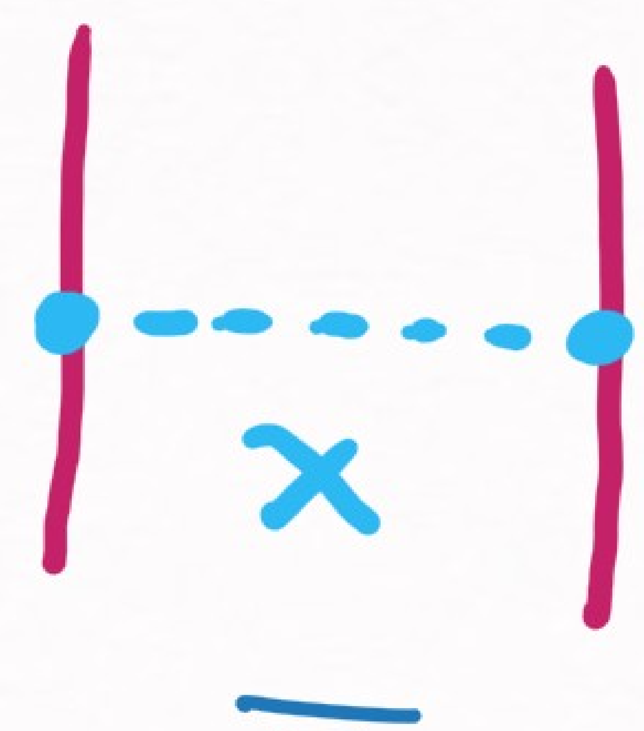
This is why we introduce the idea of R_s

EFTs ③

⇒ [The unknown part:]

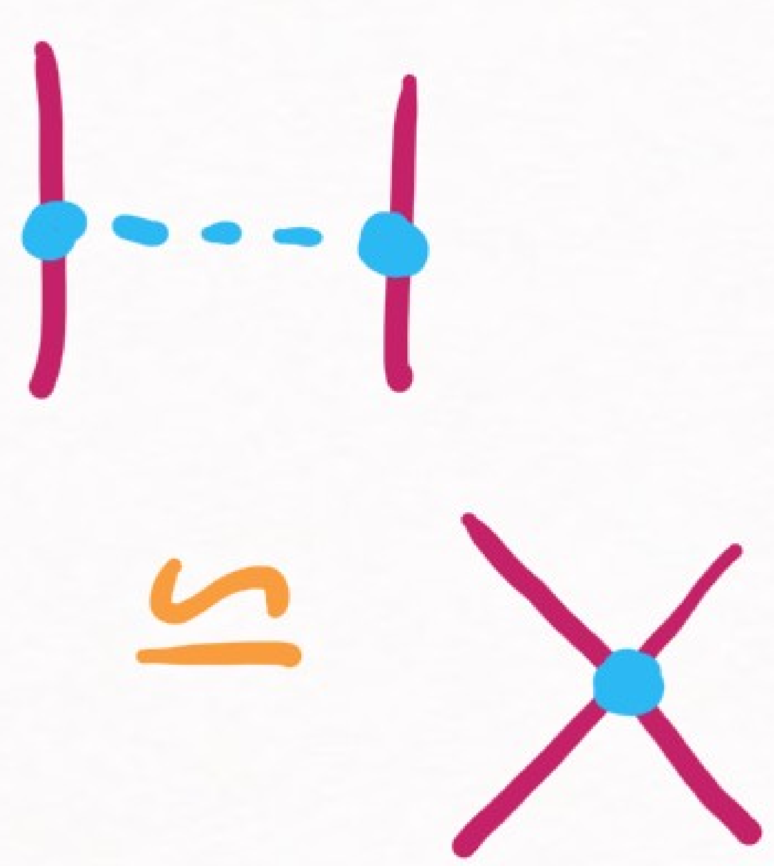


a) Imagine that its origin is the exchange of a massive meson X



⇒ $V_X(r) \sim f(m_X r)\, e^{-m_X r}$

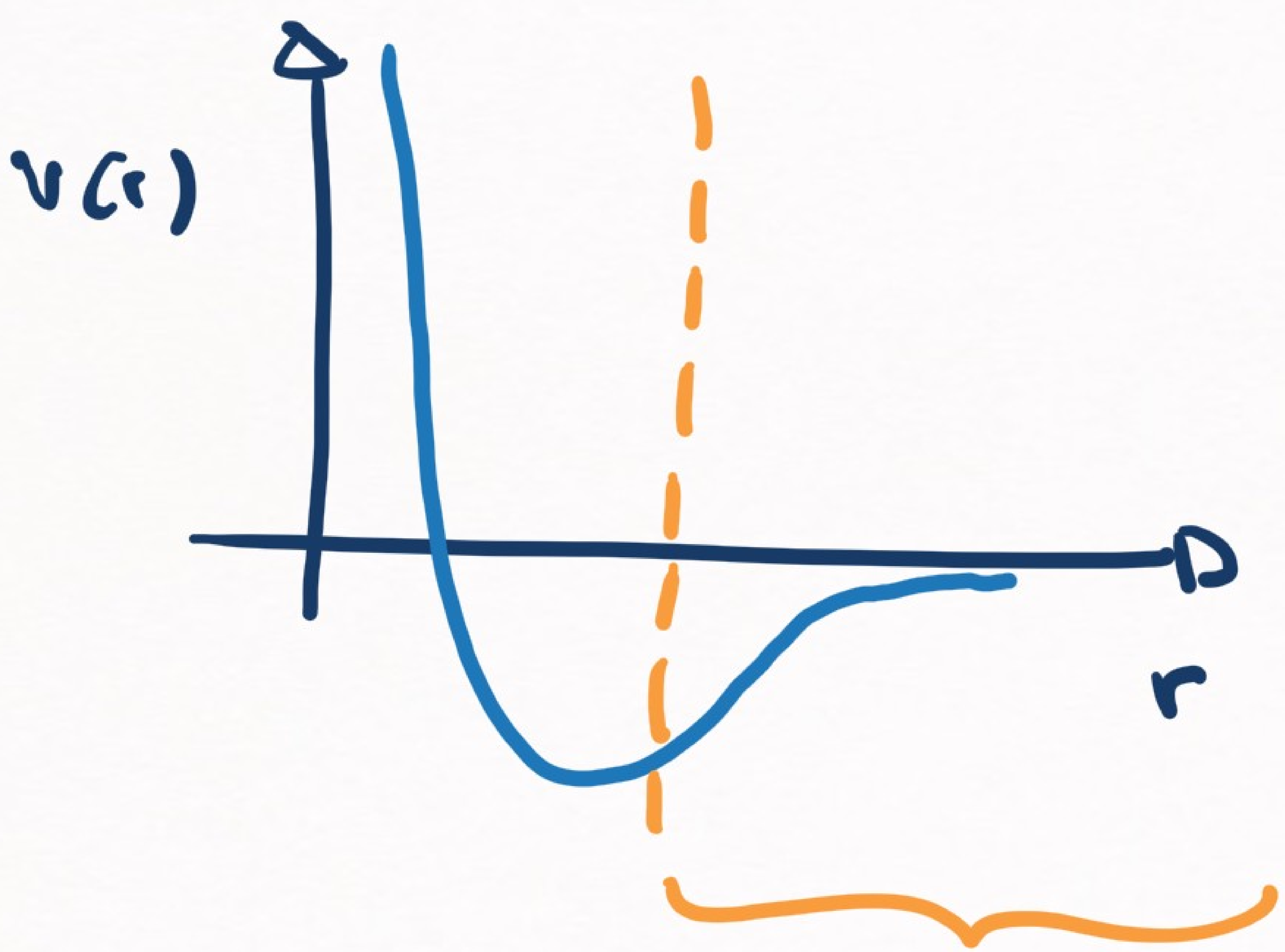
b) for $m_X R_S \gg 1$ ⇒ We can approximate this exchange with a contact-range pot

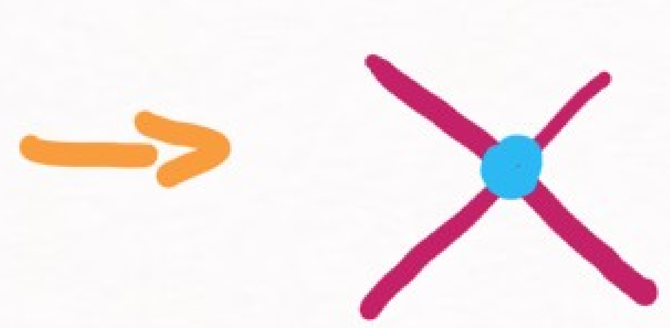


$$V_X(\vec{r}) \to C_X\, \delta^{(3)}(\vec{r}) + D_X\, \vec{\nabla}^2 \delta^{(3)}(\vec{r}) + \dots$$

(for $|\vec{q}| \ll m_X$)

$\boxed{\text{EFTs}}$ ④





We only keep explicitly $R > R_c$

Different choices of R_S

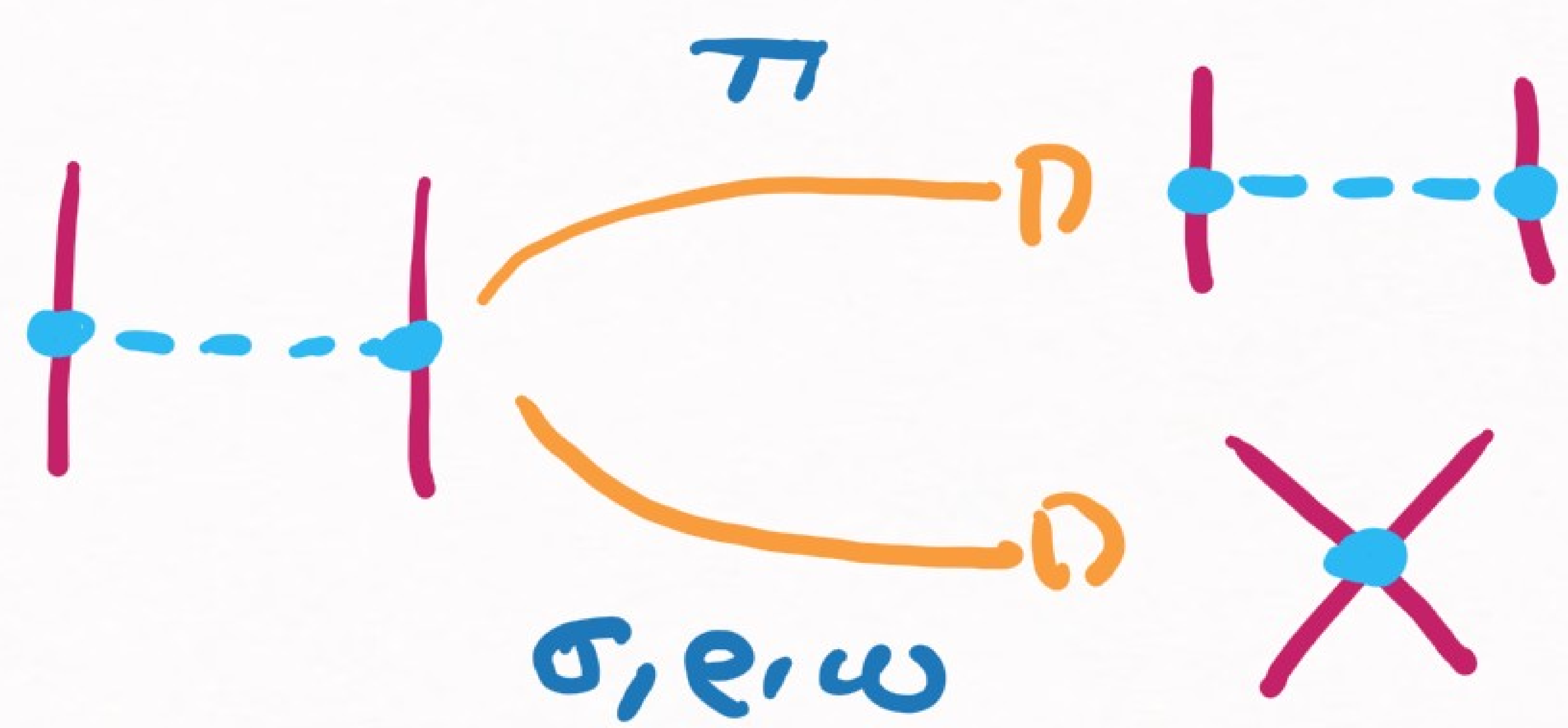
⇒ Different EFTs :

1) $m_\pi R_S \sim 1$ (most extreme case)

⇒

pionless EFT

2) $m_\rho R_S \sim 1$ ⇒ pionful EFT



$\boxed{\text{PIONLESS EFT}}$ 1

1) Most simple example of an EFT

   $\Rightarrow$ If interested in EFTs, it is worth your effort to study this EFT

2) Only contains contact-range interactions (i.e. Dirac-deltas)

3) Only valid for $k < m_\pi$ (otherwise, if $k \gtrsim m_\pi$ we will be able to resolve (or see) the pion physics)

PIONLESS EFT ②

⇒ For S-waves the effective potential will be:

$$\langle p'|V_c|p\rangle = C_0 + C_2(p'^2+p^2) + C_4(p'^4+p^4) + C_4' p'^2 p^2 + \dots$$

a) Each new term is smaller than the previous one at low momenta

b) We will cut the expansion where we find it convenient (i.e. according to the desired accuracy)

PIONLESS EFT 3

$\Rightarrow$ The most important EFT trick:

$\Rightarrow$ [All regularizations lead to the same results]

$\Rightarrow$ We might simply choose the easiest one possible

Delta-shell: $V_C(r;R_c) = \dfrac{C_K(R_c)}{4\pi R_c^2}\,\delta(r-R_c)$

with $C_K(R_c) = C_0 + k^2 C_2 + k^4 C_4 + \dots$

$\boxed{\text{PIONLESS EFT}}$ 4

a) $\underline{\underline{V_c(r;R_c)}} = \dfrac{C_k(R_c)}{4\pi R_c^2}\,\underline{\underline{\delta(r-R_c)}}$ with $C_k = C_0 + C_2 k^2 + \underline{\underline{C_4 k^4}} + \ldots$

b) Explicit solution:

$$k\cot(kR_c + \underline{\underline{\delta}}) - k\cot kR_c = 2\mu \frac{C_k(R_c)}{4\pi R_c^2}$$

c) Interpretation of R_c:

→ Auxiliary parameter to make calculations easier

PIONLESS EFT

5

⇒ About the cutoff R_c : → really important part of EFT

a) R_c is not R_s :

a.1) R_c is introduced to regularize the Dirac-deltas

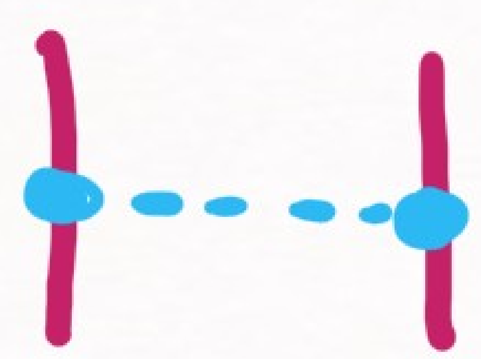
a.2) R_s is our choice of a physical separation scale → our boundary between known & unknown physics

⇒ Controls convergence

Pionless EFT 6

$$\frac{1}{q^2+m^2} = \frac{1}{m^2} - \frac{|q|^2}{m^4} + \frac{|q|^4}{m^6} - \dots$$

⇒ About the breakdown scale R_s:

1)  $\xrightarrow[kR_s \ll 1]{}$ 

$$C_0 + C_2 k^2 + C_4 k^4 + \dots$$

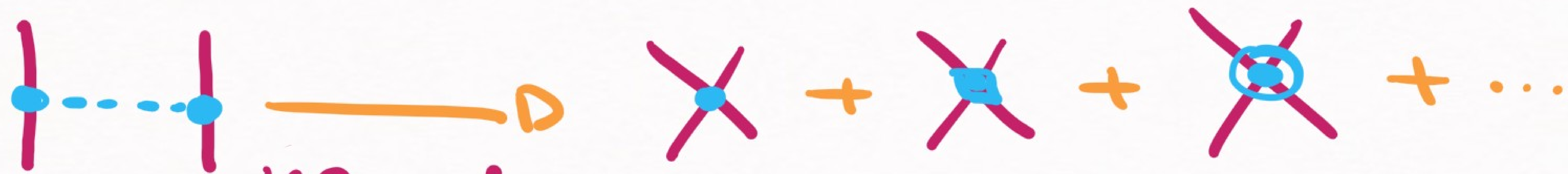
Namely $C_{2n} \sim \frac{2\pi}{\mu} R_s^{2n+1}$ ⇒ Each term is smaller than the previous one (for $kR_s < 1$)

$$C_0 \sim \frac{2\pi}{\mu} R_s \qquad C_{2n} k^{2n} \sim \left(\frac{2\pi}{\mu} R_s\right) \times (k R_s)^{2n} \sim [C_0] \times (kR_s)^{2n}$$

$$\frac{C_{2n} k^{2n}}{C_0} \sim (kR_s)^{2n}$$

Pionless EFT ⑦

⇒ About the breakdown scale $\underline{R_s}$:

1) 

$kR_s \ll 1$

$$C_0 + C_2 k^2 + C_4 k^4 + \dots$$

Actually, if a_0 is large (e.g. if we have a bound/virtual state near threshold) the scaling will be given by:

$$\left[C_0 \sim \frac{2\pi}{\mu} a_0 \,,\; C_{2n} \sim \frac{2\pi}{\mu} a_0^2 \underline{R_s^{2n-1}} \right] \to$$ still, series is converging for $kR_s < 1$

PIONLESS EFT ⑧

R_s determines the convergence radius of the theory

⇒ About the cutoff R_c:

3) Not a real parameter of the EFT ← Cutoff: to make calculations easier

⇒ Calculations should be independent of R_c

→ $k\cot(kR_c+\delta)-k\cot\delta = 2\mu\frac{C_n(R_c)}{4\pi R_c^2} = 2\mu\frac{C_0(R_c)}{4\pi R_c^2} + 2\mu\frac{C_2(R_c)}{4\pi R_c^2}k^2 + \ldots$

Expand everything in k^2 & match coefficients $\Big\}$ ⇒ $\frac{1}{C_0(R_c)} = \frac{\mu}{2\pi}\left(\frac{1}{a_0}-\frac{1}{R_c}\right)$

+ similar eqs for C_{2n}

$\boxed{\text{PIONLESS EFT}}$ (9)

⇒ About the cutoff R_c:

1) Not a real parameter of the EFT

$$\frac{1}{C_0(R_c)} = \frac{\mu}{2\pi}\left(\frac{1}{a_0} - \frac{1}{R_c}\right), \quad \underline{C_{2n}(R_c) = 0} \Rightarrow$$ LO calculation

$$k \cot\delta = -\frac{1}{a_0} + \frac{2}{3} R_c\left(1 - \frac{1}{2}\frac{R_c}{a_0}\right) k^2 + O(k^4)$$

(it will have some R_c dependence because of the truncation)

↓

$C_{2n}(R_c) = 0$

$n > 0$

[Is this a problem?]

PIONLESS EFT (10)

⇒ About the cutoff R_c:

3) Not a real parameter of the EFT ⇒ $\left[\frac{d}{dR_c}\langle\hat{O}\rangle = 0\right]$

$\frac{d}{dR_c}[k\cot\delta_{EFT}] = 0$ (modulo the EFT truncation error)

$= \mathcal{O}(R_s k^2)$ at LO

⇒ $k\cot\delta_{LO}(R_{c1}) - k\cot\delta_{LO}(R_{c2}) = \frac{2}{3}(R_{c1}-R_{c2})k^2 + \dots = \mathcal{O}(R_s k^2)$

⇒ $\boxed{R_c < R_s}$ ⇒ $\frac{d}{dR_c}[k\cot\delta_{LO}] = 0$ mod expansion

PIONLESS EFT

(11)

⇒ About the cutoff R_c:

3) Not a real parameter of the EFT

$$\frac{d}{dR_c}\left[k\cot\delta_{N^\nu LO}\right] = \mathcal{O}(R_s^\nu k^{\nu+2})$$

$= 0$ (from the EFT viewpoint)

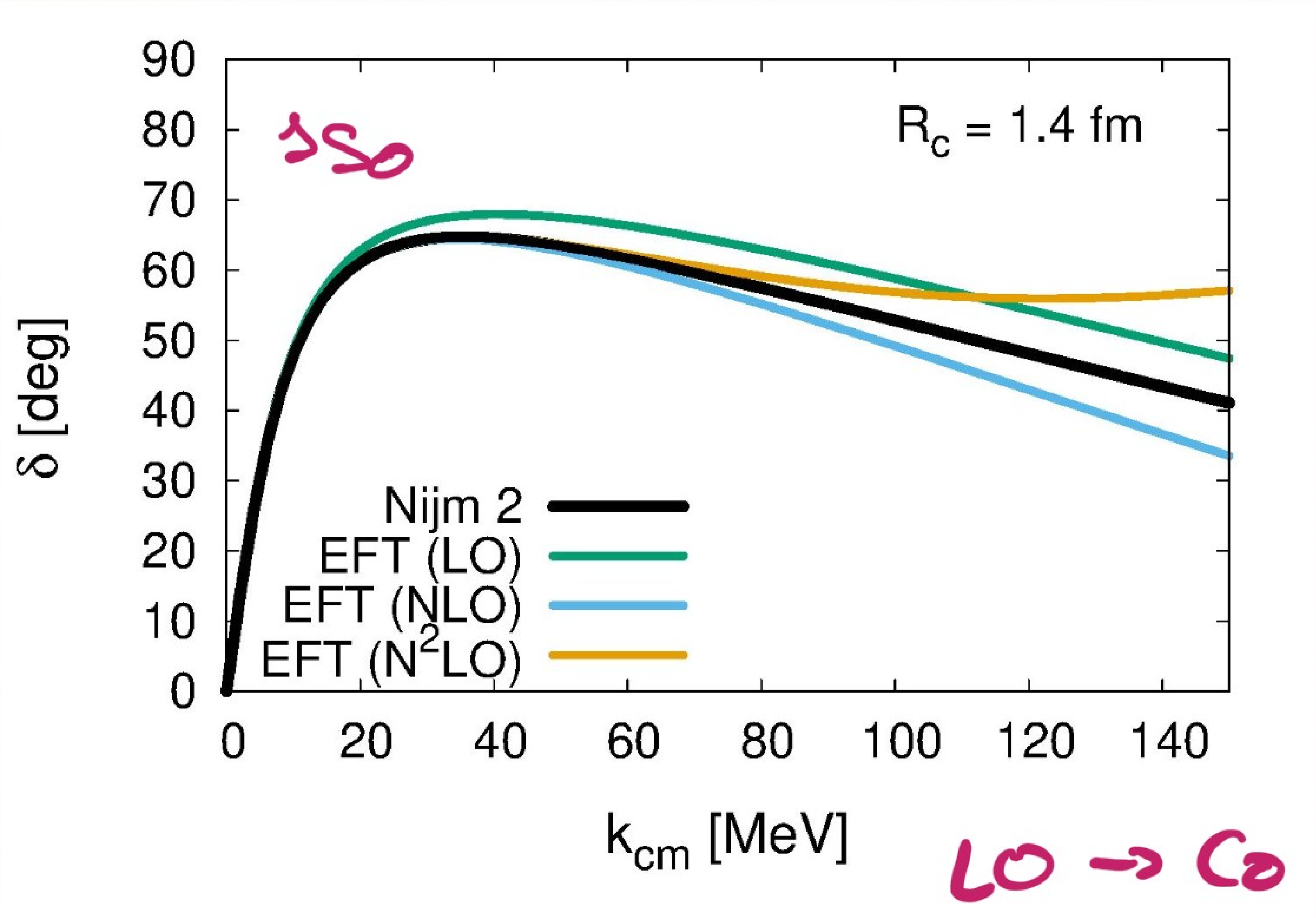
→ This requires $R_c < R_s$

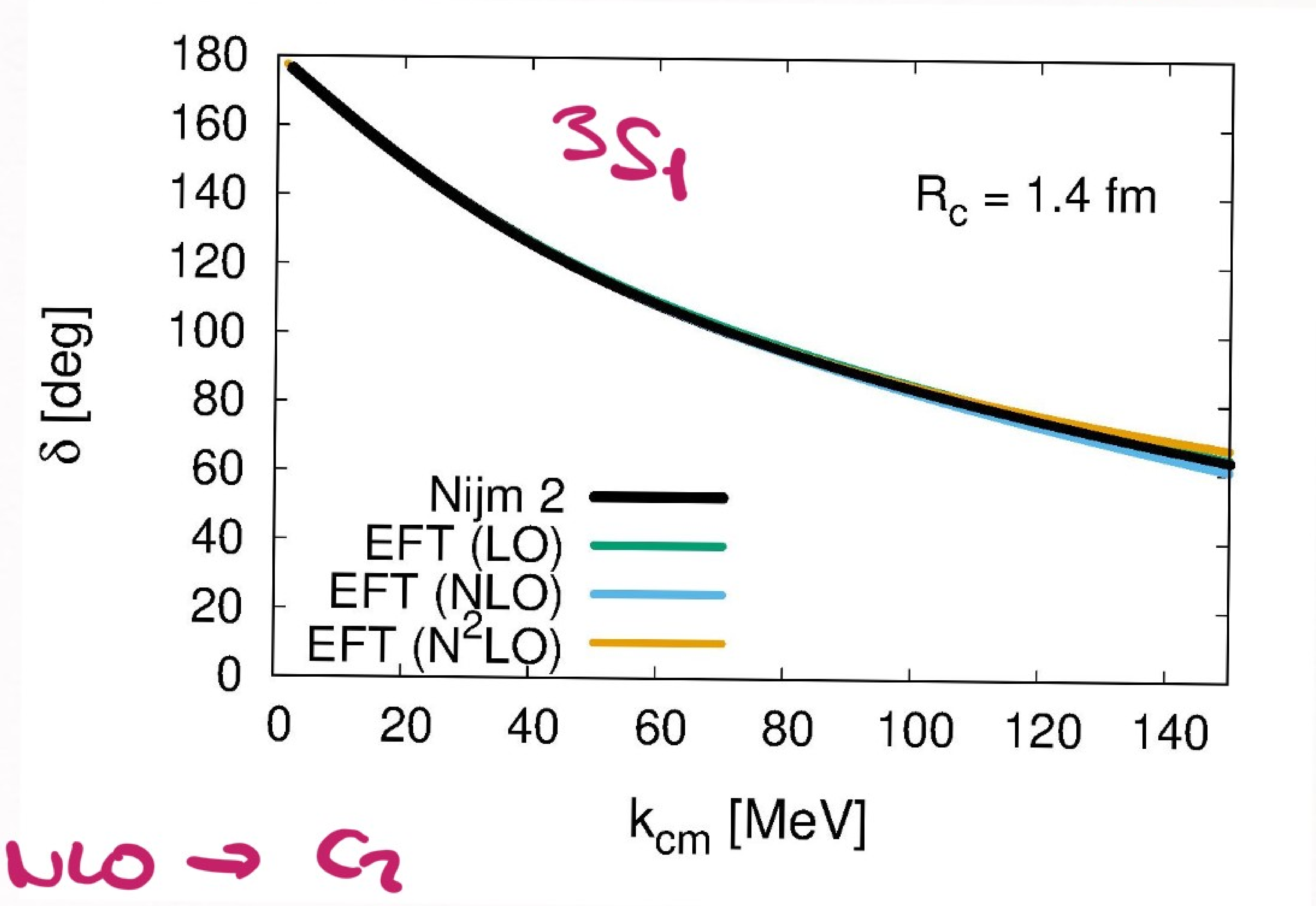
→ Cutoff dependence is not strictly zero, but just compatible with zero

Pionless EFT (12)

$m_\pi R_c \lesssim 1$

⇒ Results up to N^2LO for the $^1S_0$ & $^3S_1$ partial waves ($R_c = 1.4$ fm)



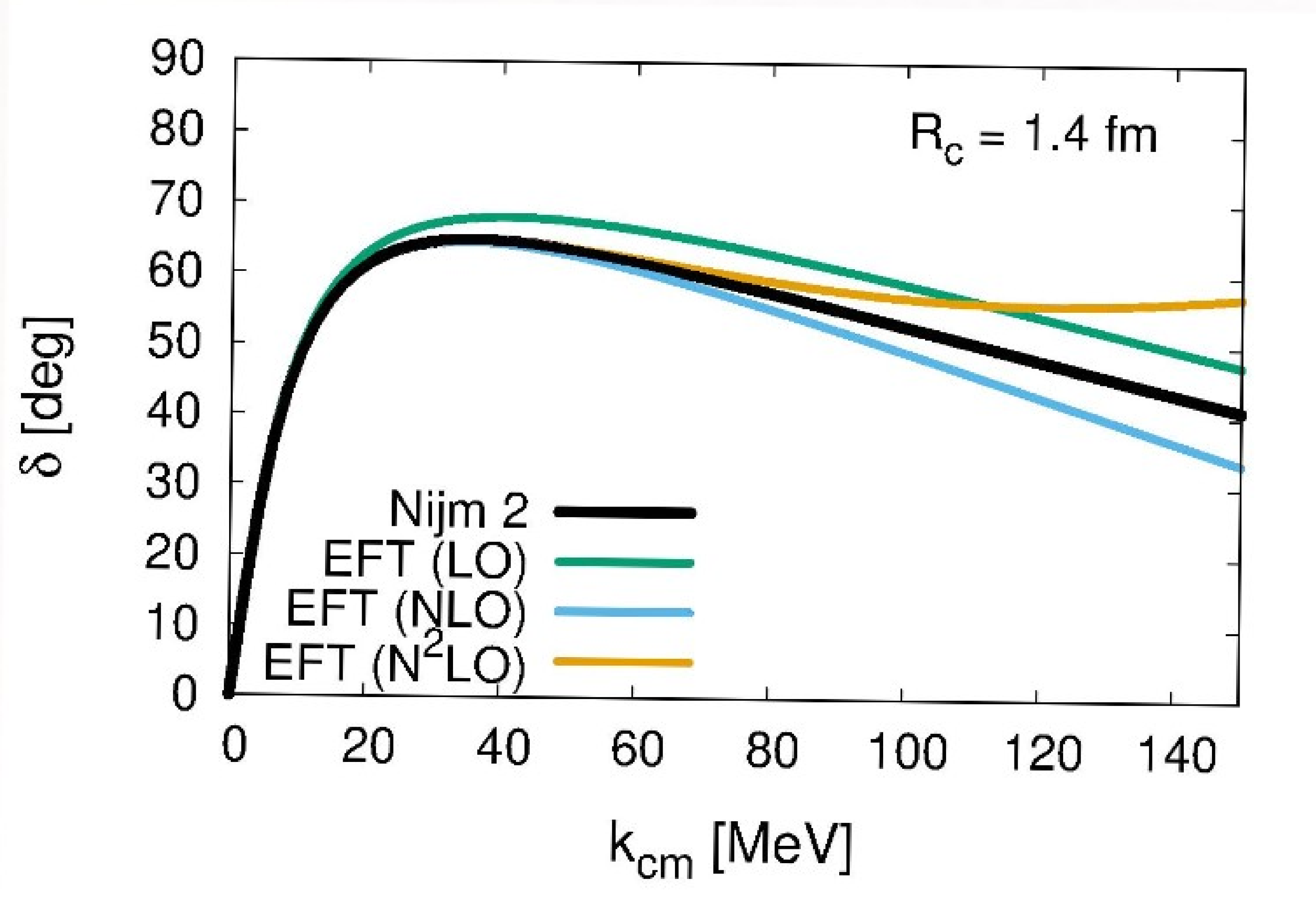


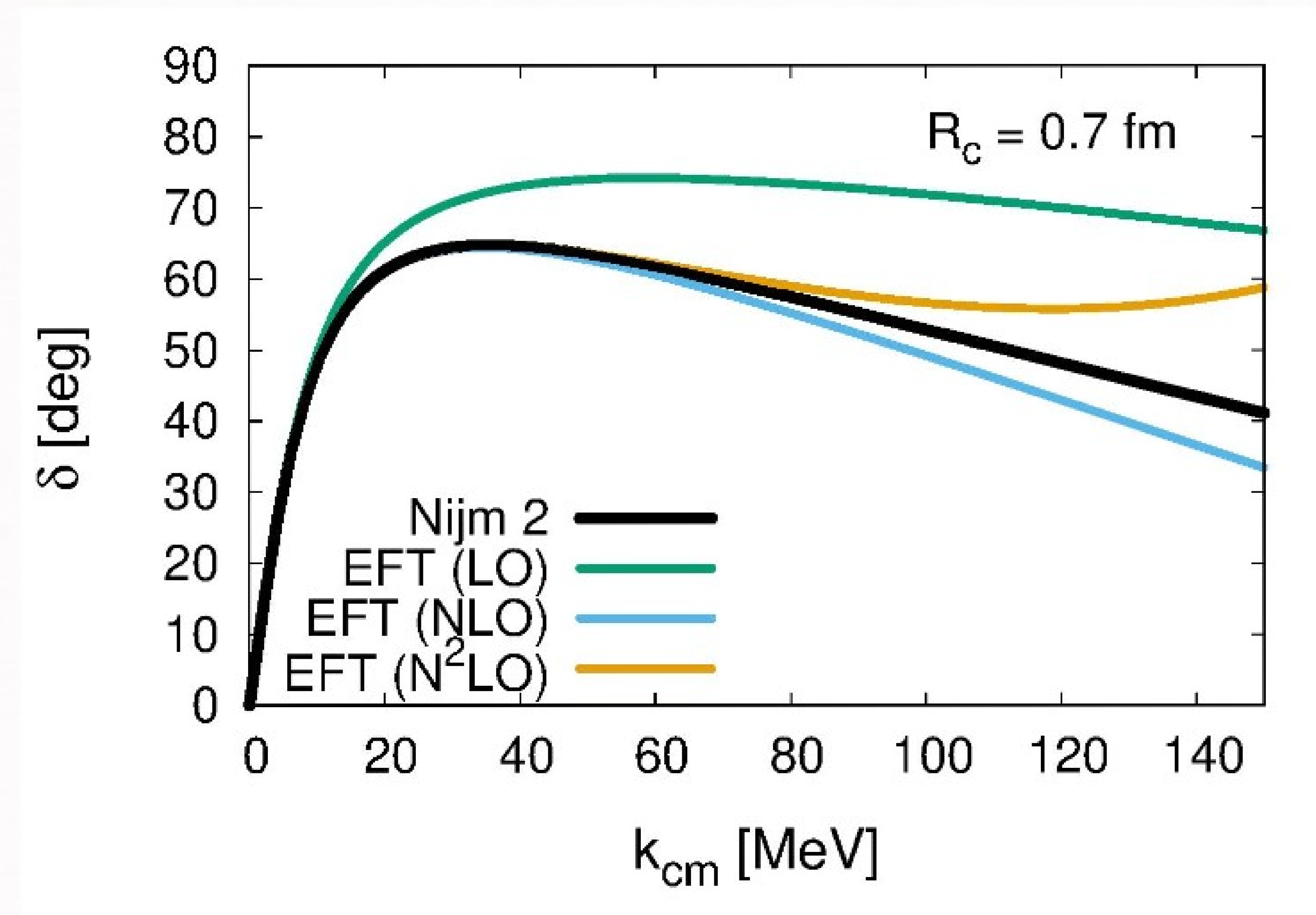
LO → C_0 , NLO → C_2

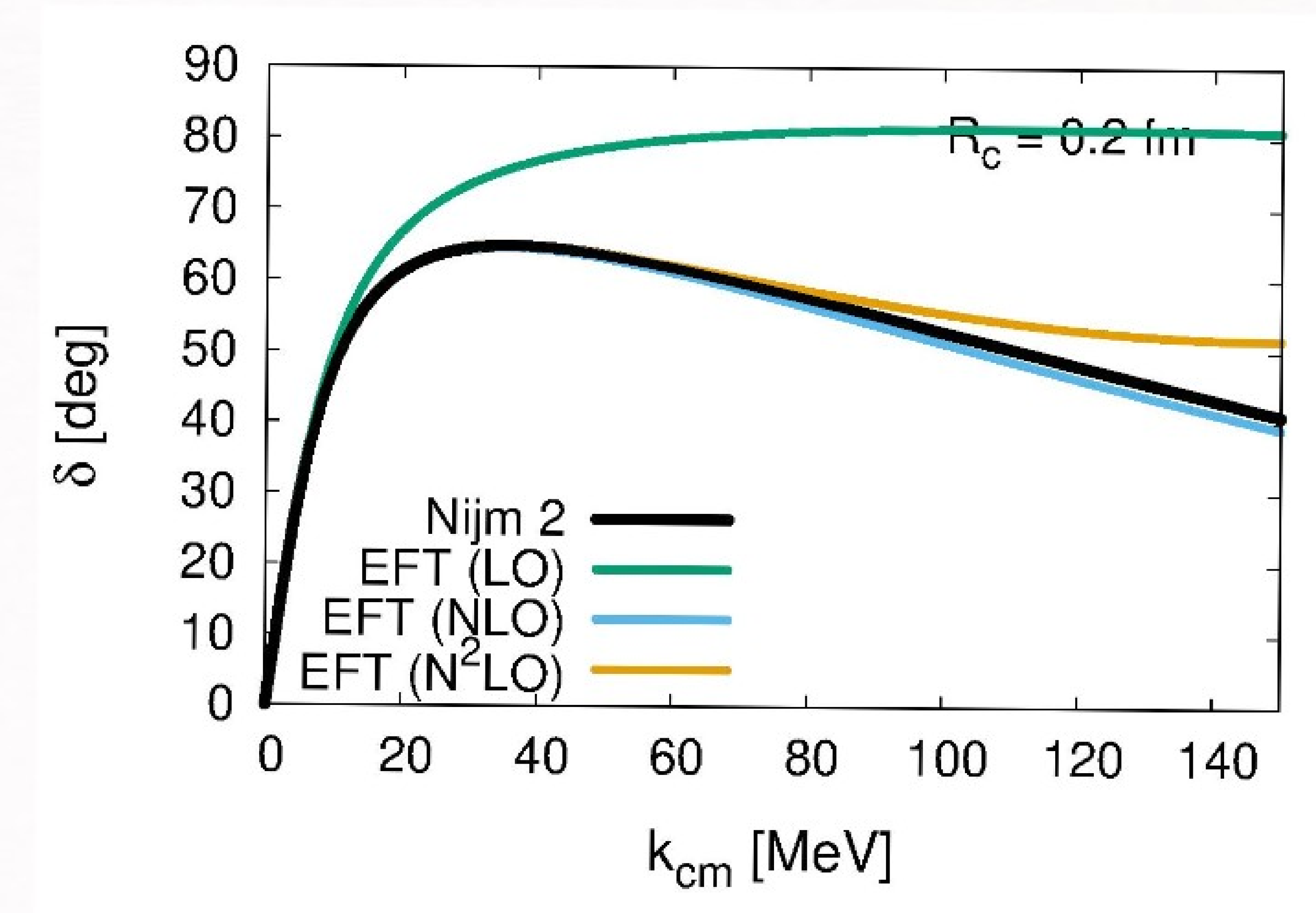
N^2LO → C_4

Pionless EFT (13)

⇒ We can check the cutoff dependence: ($^1S_0$ as an example)





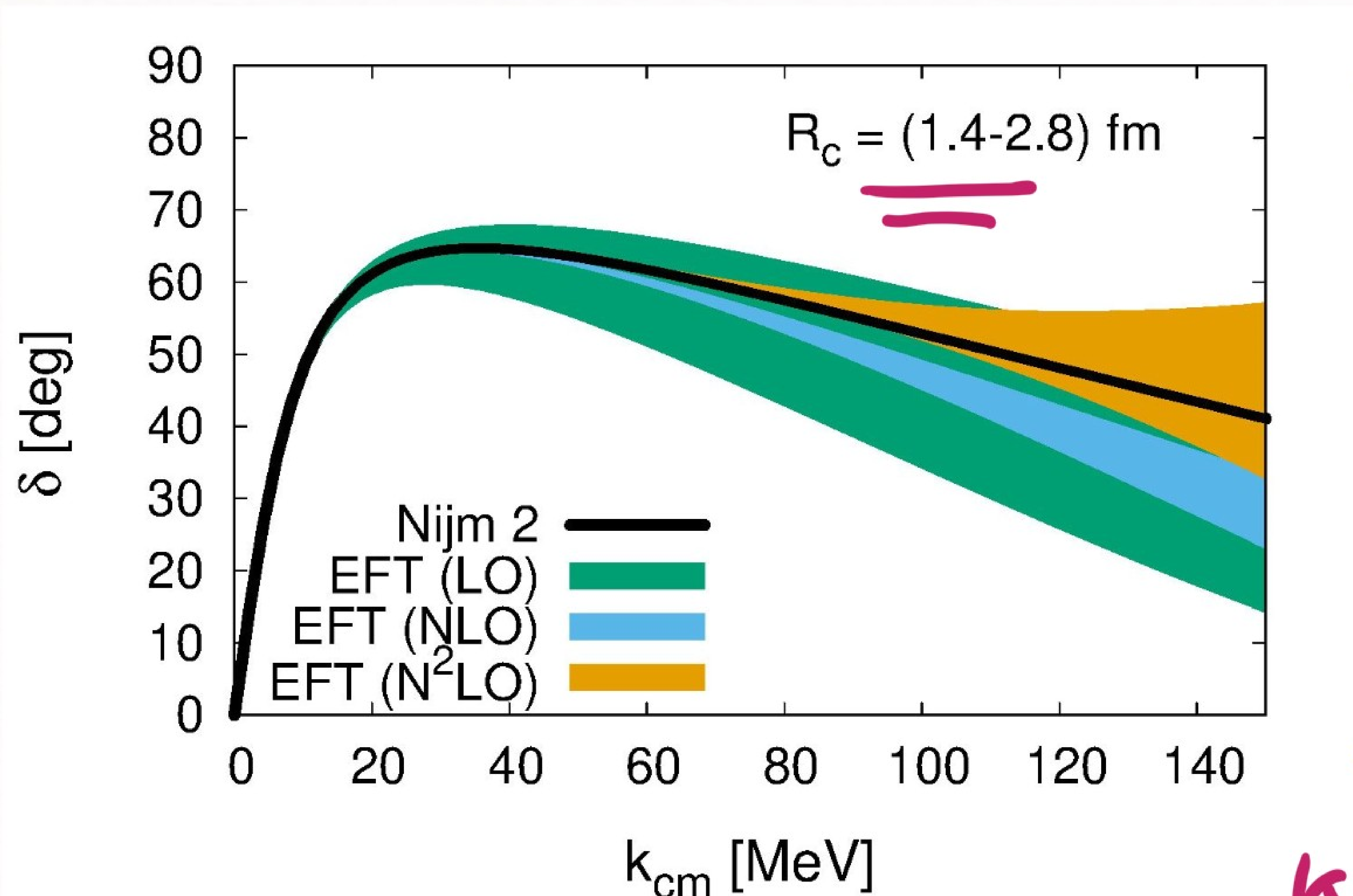


→ There is convergence for $R_c \to 0$

(correct renormalization)

Pionless EFT (14)

=> Alternatively, we can use R_c dependence to calculate uncertainties



a) $k\cot\delta_{EFT}$ only accurate up to $\mathcal{O}(R_s^\nu k^{\nu+1})$ (naively)

b) $k\cot\delta(R_c)$ only accurate up to $\mathcal{O}(R_c^\nu k^{\nu+1})$

=> Difference of $k\cot\delta(R_{c1}) - k\cot\delta(R_{c2})$ as a proxy for the EFT error

imperfect method though (but really useful)

$\boxed{\text{PIONLESS EFT}}$ 1S

$\Rightarrow$ What happens if $R_c \to 0$?

Renormalization condition ($R_c \to 0$) $\Rightarrow$ $k\cot\delta - \frac{1}{R_c} = \frac{2\mu C_K}{4\pi R_c^2} + \dots$

a) LO : $k\cot\delta \xrightarrow[R_c \to 0]{} -\frac{1}{a_0}$

b) NLO : $k\cot\delta \xrightarrow[R_c \to 0]{} -\frac{1}{a_0} + \frac{1}{2} r_0 k^2$

c) N^2LO : $k\cot\delta \xrightarrow[R_c \to 0]{} -\frac{1}{a_0} + \frac{1}{2} r_0 k^2 + v_2 k^4$

$\Rightarrow$ Convergence into the ERE

Pionless EFT

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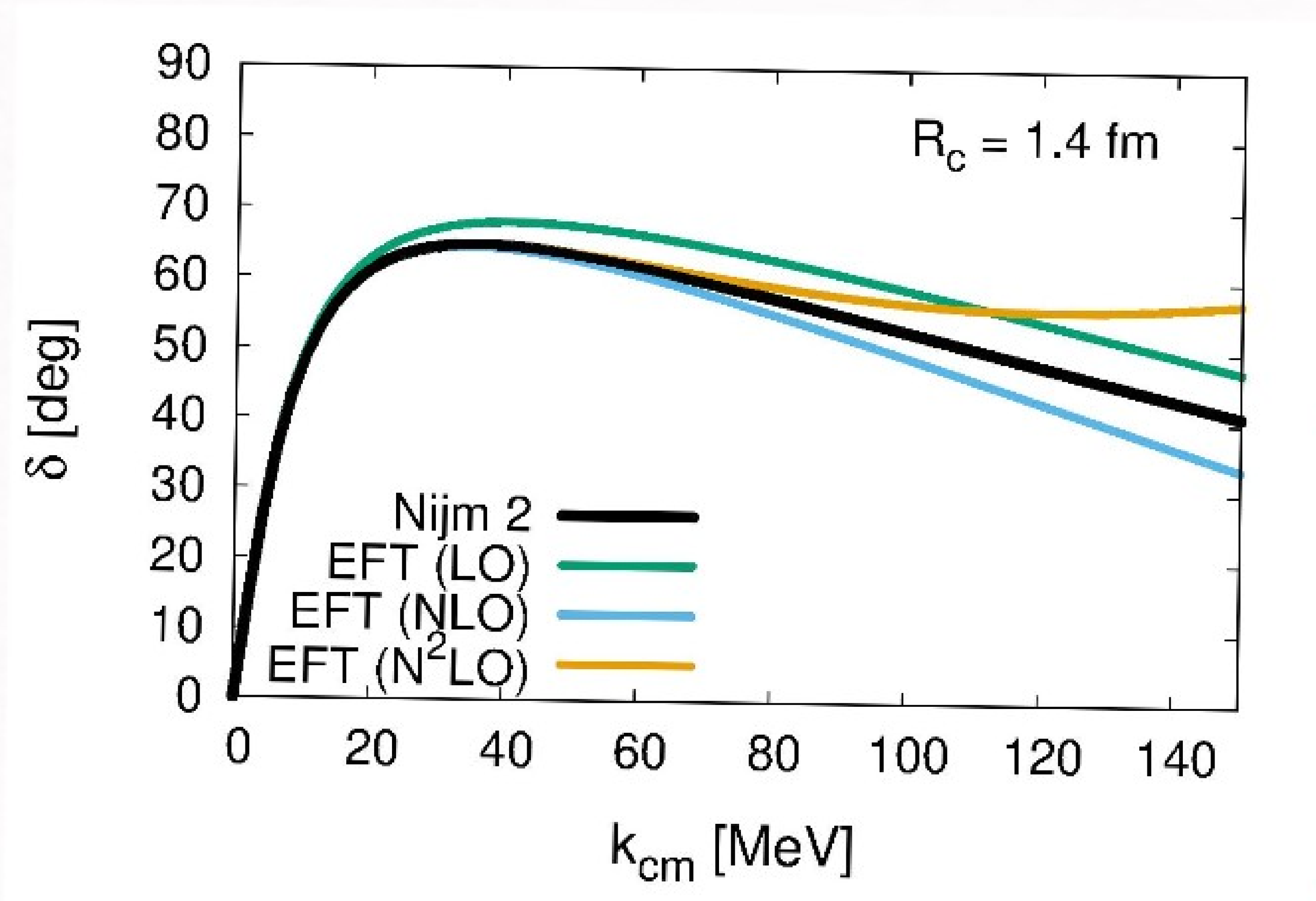
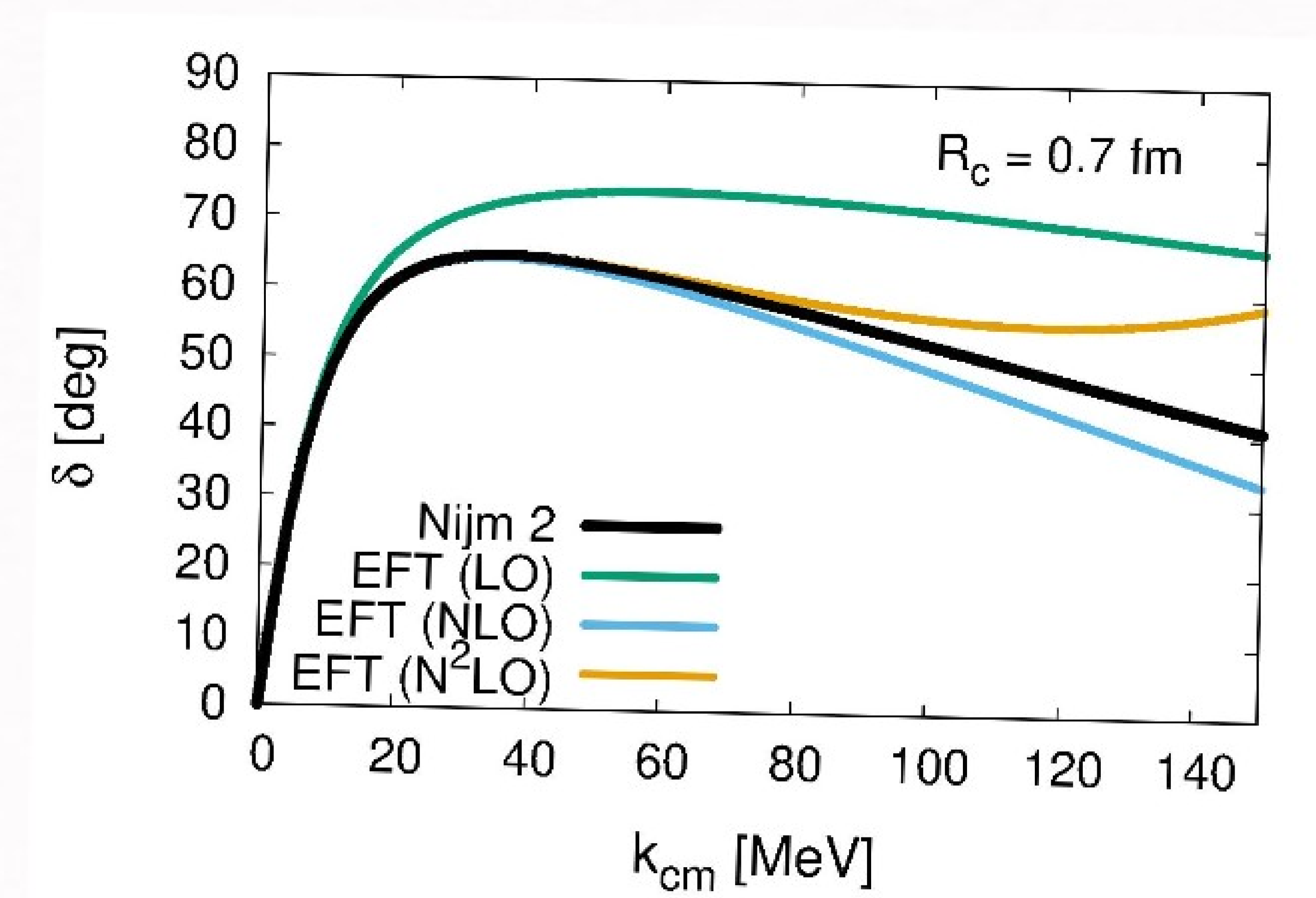
⇒ What happens if $R_c \to 0$? We converge to the ERE



⇒



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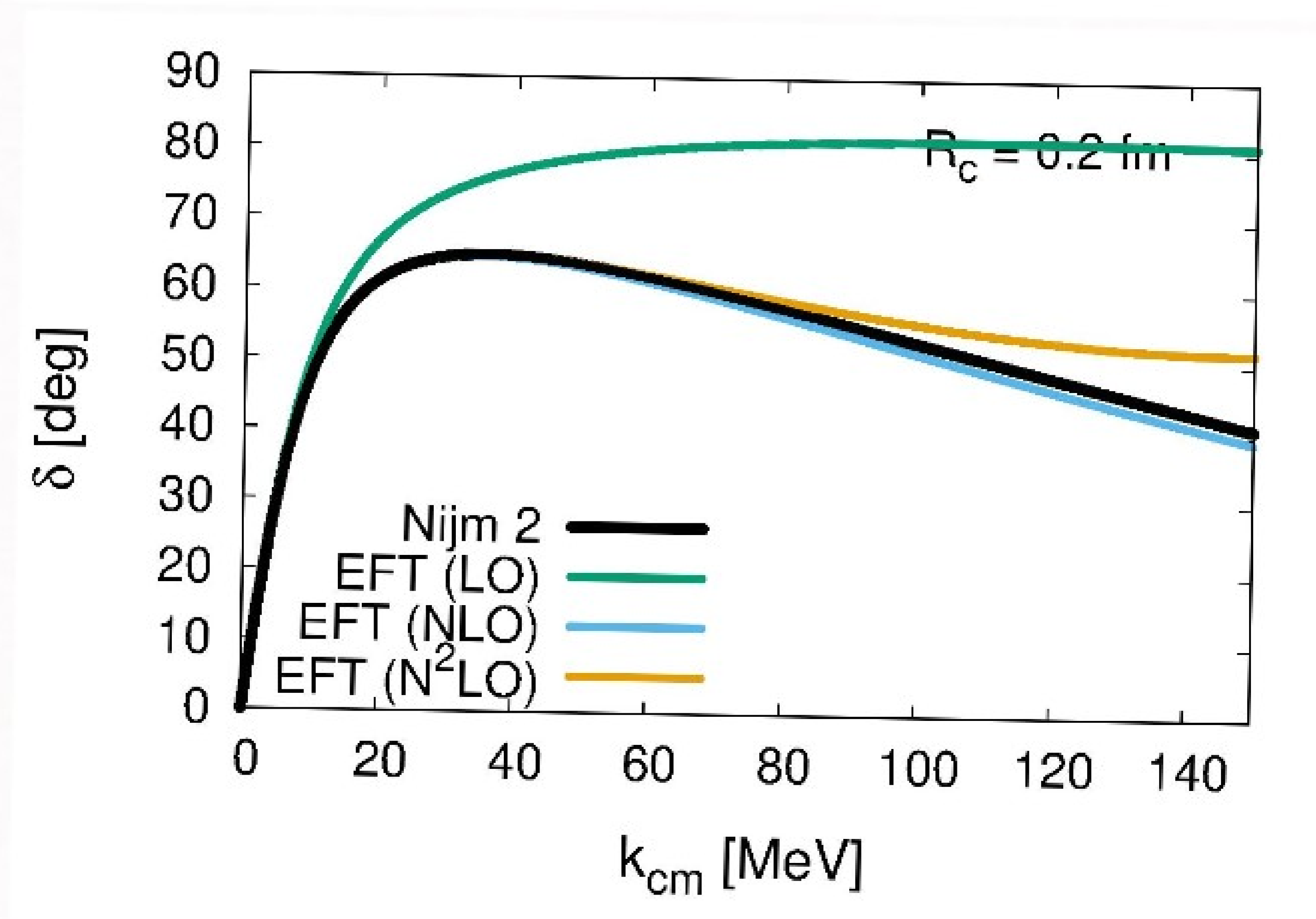

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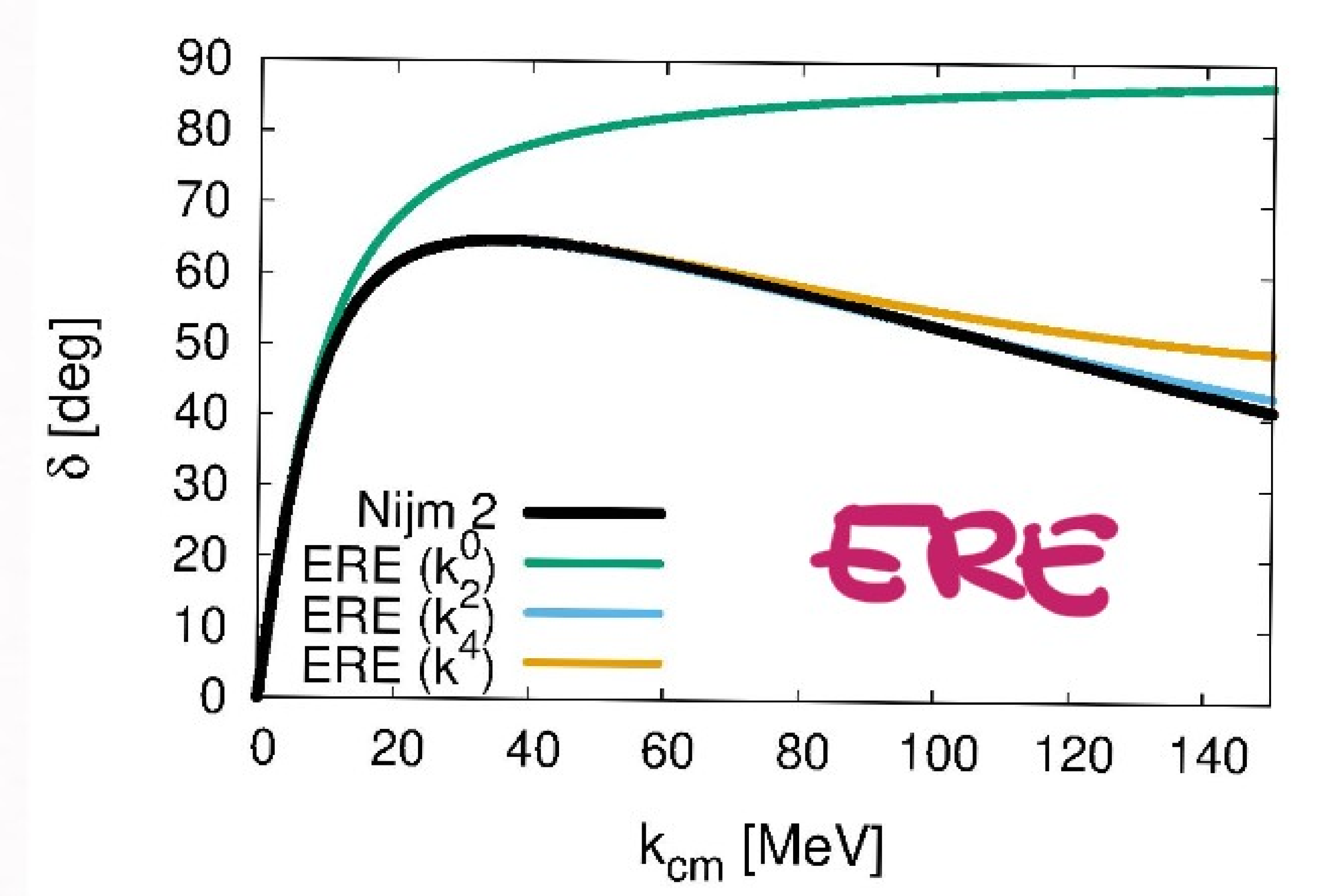


ERE

PIONLESS EFT (17)

⇒ Pionless EFT is equivalent to the ERE
(in the two-body sector)

⇓

EFT contains a few extra things that are not in ERE
for the general case (e.g. reactions:
$np \to \gamma d$)

Pionless EFT (18)

So far we have used delta-shell

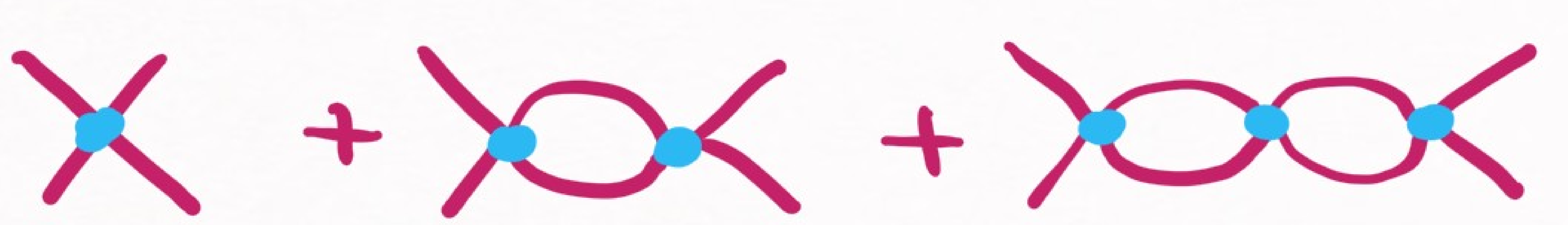
$\Rightarrow$ Of course, you can try any regularization you like:

3) Power divergence subtraction

$\rightarrow$ very easy regulator for calculations

$$\int \frac{d^3\vec{q}}{(2\pi)^3}\,\frac{q^{2n}}{E \pm i\epsilon - \frac{q^2}{2\mu}} = -\frac{\mu}{2\pi}\,k^{2n}\left(\Lambda_{PDS} \pm ik\right)$$

$$k = \sqrt{2\mu E}$$



$+ \ldots \Rightarrow$ it's analytically solvable

(you might try to play with this regulator)

PIONLESS EFT (19)

2) Gaussian regulator (non-local):

$$\langle p' | V_c | p \rangle = \Big[C_0(\Lambda) + C_2(\Lambda)(p^2 + p'^2) + C_4(\Lambda)(p^4 + p'^4)$$
$$+ C_4'(\Lambda) p^2 p'^2 + \ldots \Big] \times f\left(\frac{p'}{\Lambda}\right) f\left(\frac{p}{\Lambda}\right)$$

$f(x) = e^{-x^{2n}}$ $\Rightarrow$ now calculations are mostly numeric

(not a big problem if you are using a computer algebra system)

$\boxed{\text{PIONLESS EFT}}$ (20)

3) Gaussian regulator (local):

$$V_C(\vec{q}) = \left[C_0 + C_2 \vec{q}^{\,2} + C_4 \vec{q}^{\,4} + \cdots \right] \times f\left(\frac{|\vec{q}|}{\Lambda} \right)$$

$$f(x) = e^{-x^{2n}}$$

RECAP!

1) EFTs are a way to manage our unknowns about physics at short distances

2) Different choices of R_s (separation scale)

$\Rightarrow$ Different EFTs

2.a) $m_\pi R_s \ll 1$, $m_e R_s \sim 1$ $\Rightarrow$ pionful EFT

2.b) $m_\pi R_s \sim 1$ $\Rightarrow$ pionless EFT

the most simple EFT possible for nuclear physics

RECAP

3) Pionless EFT : best test case for learning about EFTs

3.a) Analytic for particular choices of regulator $\Rightarrow$ delta-shell, PDS

3.b) For two-body problems, it is equivalent to the ERE

3.c) Most of its applications can be studied as an usual problem in quantum mechanics

Pionful EFT ①

Derivation of nuclear forces

But QCD not solvable at low energies

EFT approach

$= Q \sim k, 1/a_0$
$M \sim 1/R_s$

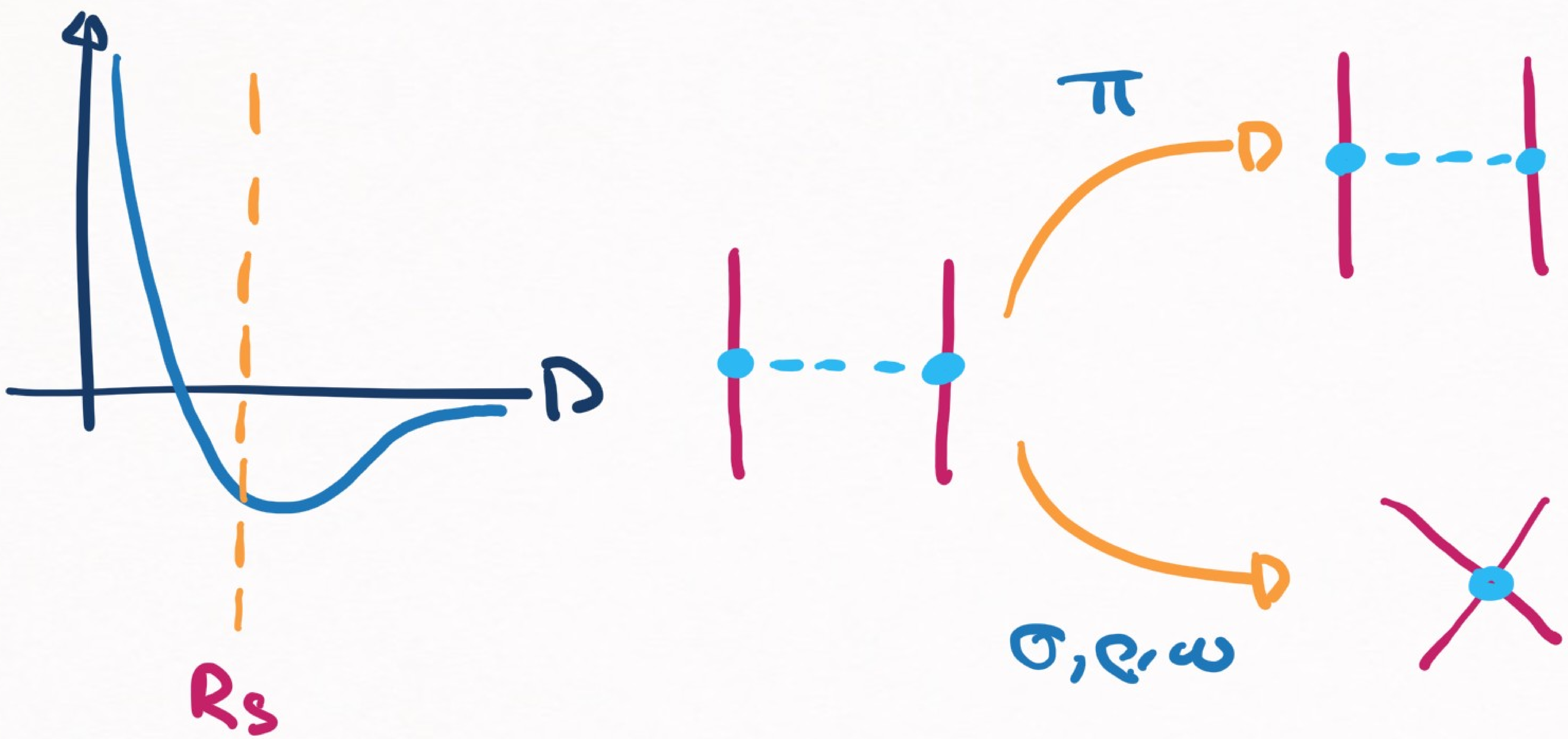
Expansion in Q/M
(depends on choices of Q & M)

Pionless:
$Q \sim 1/a_0$
$M \sim m_\pi$

Pionful
$Q \sim m_\pi$
$M \sim m_\rho$

PIONFUL EFT (2)

⇒ If we choose $m_\pi R_s \ll 1$ ($m_\rho R_s \sim 1$ or $M \sim m_\rho$):



[keep the pions explicitly]

include the other mesons implicitly via contacts

PIONFUL EFT

③

$\Rightarrow$ EFT description now contains two parts:

$$V_{EFT}(\vec{q}) = \underline{\underline{V_F}}(\vec{q}) + \underline{\underline{V_C}}(\vec{q})$$

a) $V_C(\vec{q}) \rightarrow$ contact-range piece (just as before in pionless)

b) $\underline{\underline{V_F}}(\vec{q}) \rightarrow$ finite-range piece (pion exchanges)

Pionful EFT (4)

⇒ There is an expansion: $V_{EFT}(\vec{q}) = \sum_{\nu=\nu_0}^{\infty} V_{EFT}^{(\nu)}(\vec{q})$

a) Contact-range part:

$$V_C = C_0 + C_2(p^2 + p'^2) + C_4(p^4 + p'^4) + C_4' p^2 p'^2 + \dots$$

b) Finite-range part:

$$V_F = \underbrace{V_F^{(0)}}_{\text{one pion exchange}} + \underbrace{\cancel{V_F^{(1)}}}_{=0} + \underbrace{V_F^{(2)} + V_F^{(3)}}_{\text{two pion exchange}} + \dots$$

PIONFUL EFT ⑤

⇒ How do we determine the order of a contribution?

0) Counting rules: $|\vec{q}| \sim Q$, $m_\pi \sim Q$, $|\vec{r}| \sim 1/Q$

(everything else) $\sim M$

1) Contacts:

$$\underline{C_{2n}} \boxed{\vec{\nabla}^{2n} \delta(\vec{r})} \rightarrow Q^{2n} \times [\delta^{(3)}(\vec{r})] \sim Q^{2n+3}$$

$$\delta^{(3)}(\lambda\vec{r}) = \frac{1}{\lambda^3}\delta^{(3)}(\vec{r})$$

⇒ Q^3 operator

⇒ $C_{2n} \sim \frac{1}{(\text{mass})^{2n+2}} \sim \frac{1}{M^{2n+2}}$

2) Finite range: $\frac{1}{r^n} \sim Q^n$

$\boxed{\text{PIONFUL EFT}}$ 6

3) Behavior of the lowest order part of the potential:

$\boxed{\text{LO}}$ $\rightarrow$ $V_C^{(0)} = C_0 \delta^{(3)}(\vec{r}) \sim Q^3$

$V_F^{(0)} \sim Q^3 \rightarrow V_F^{(0)}(r) \sim \dfrac{1}{r^3}$

$\Rightarrow$ If the expansions of V_C & V_F begin at the same order : $V_F^{(0)}(r) \sim \dfrac{1}{r^3}$

This is the behavior of the tensor part of the OPE potential

PIONFUL EFT ⑦

⇒ Counting depends on the quantity/representation

⇒ $V(\vec{r})$ and $V(\vec{q})$ are counted differently:

(because of diff in dimension)

⇒ $V(\vec{r}) = \int \frac{d^3\vec{q}}{(2\pi)^3} V(\vec{q}) e^{-i\vec{q}\cdot\vec{r}}$ implies that:

⇒ Q^3 ⟶ $V(\vec{r})$ is $Q^3 \times [V(\vec{q})]$

⇒ Convention:

counting is usually defined with respect to the p-space representation of the potential

Pionful EFT (3)

⇒ In p-space:

1) Contacts: $V_c = \underbrace{C_0}_{\frac{1}{\Lambda^2}} \times \underbrace{1}_{Q^0} + \underbrace{C_2}_{1/\Lambda^4} \times \underbrace{(p^2 + p'^2)}_{Q^2} + \underbrace{C_4}_{1/\Lambda^6} \times \underbrace{(p^4 + p'^4)}_{Q^4} + \ldots$

2) OPE (one pion exchange):

$$V_{OPE}(\vec{q}) = -\frac{g^2}{4f_\pi^2} \frac{\overbrace{\vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q}}^{\sim Q^2}}{\underbrace{\vec{q}^2}_{\sim Q^2} + \underbrace{m_\pi^2}_{\sim Q^2}} \vec{\tau}_1 \cdot \vec{\tau}_2 \sim \frac{Q^2}{Q^2} \sim Q^0$$

In principle this is easy

PIONFUL EFT (9)

3) How do we count the pion diagrams?

$\Rightarrow$ We check the Feynman rules:

$$\mathcal{L}_{\pi NN} = \frac{g_A}{2f_\pi} N^\dagger \vec{\sigma}\cdot\vec{\nabla}(\tau_a \pi_a) N \longrightarrow$$

$\sim Q$

$$\left(\sim \frac{g}{2f_\pi}\vec{\sigma}\cdot\vec{q}\,\tau_a\right)$$

$$\mathcal{L}_{\pi\pi NN} = -\frac{1}{4f_\pi^2} N^\dagger (\epsilon_{abc}\tau_a \pi_b \partial_0 \pi_c) N$$

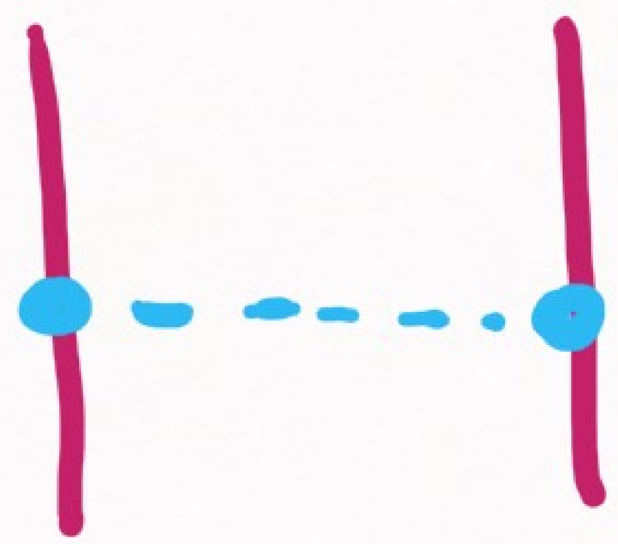
Weinberg-Tomozawa interaction $\longrightarrow$

$\sim Q$

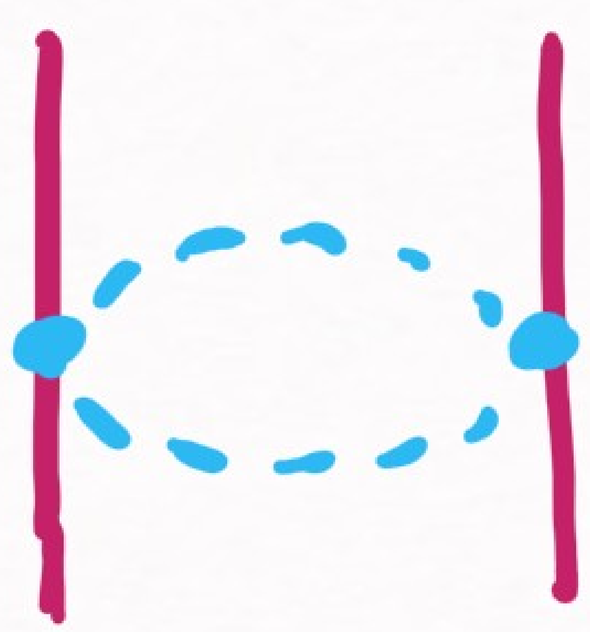
$$\left(\sim \frac{1}{2f_\pi^2}(\omega+\omega'),\ \omega = \sqrt{m_\pi^2+q^2}\right)$$

PIONFUL EFT (10)

3) How do we count pion diagrams (cont'ed)?



$$\sim Q\,\Big[\underbrace{\text{pion propagator}}_{1/(q^2+m_\pi^2)\,\sim\, Q^{-2}}\Big]\,Q \sim Q\,\frac{1}{Q^2}\,Q \sim Q^0$$



WT WT

$$\sim Q\left[\int \frac{d^4 l}{(2\pi)^4} \times (\text{pion prop})^2\right] Q \sim Q^2$$

$$Q^4 \times 1/Q^4 \sim Q^0$$

PIONFUL EFT

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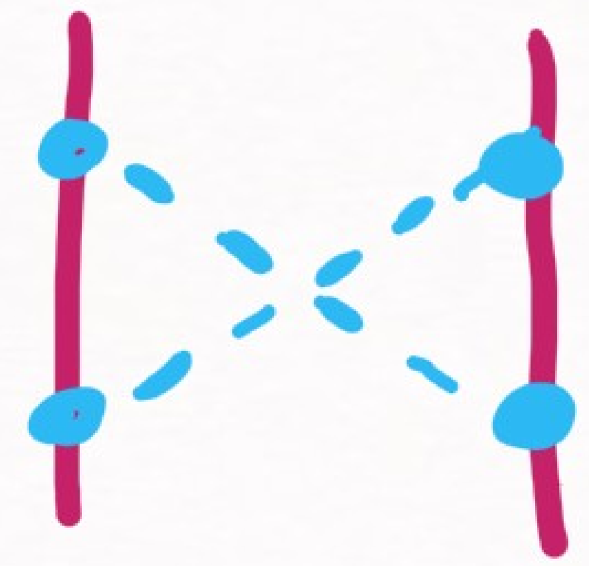
3) How do we count pion diagrams (contd)?



$$\sim Q^2 \Big[\underbrace{(\text{loop})}_{Q^4} \times \underbrace{\frac{1}{\text{pion}}}_{Q^{-2}} \times \underbrace{\frac{1}{\text{pion}}}_{Q^{-2}} \times \underbrace{\frac{1}{\text{nucleon}}}_{Q^{-1}} \times \underbrace{\frac{1}{\text{nucleon}}}_{Q^{-1}} \Big] Q^2$$

nucleon propagators

$$\sim \frac{Q^8}{Q^4 Q^2} \sim Q^2$$

$$\frac{1}{\not{p} - m_N} \sim \frac{1}{Q}$$

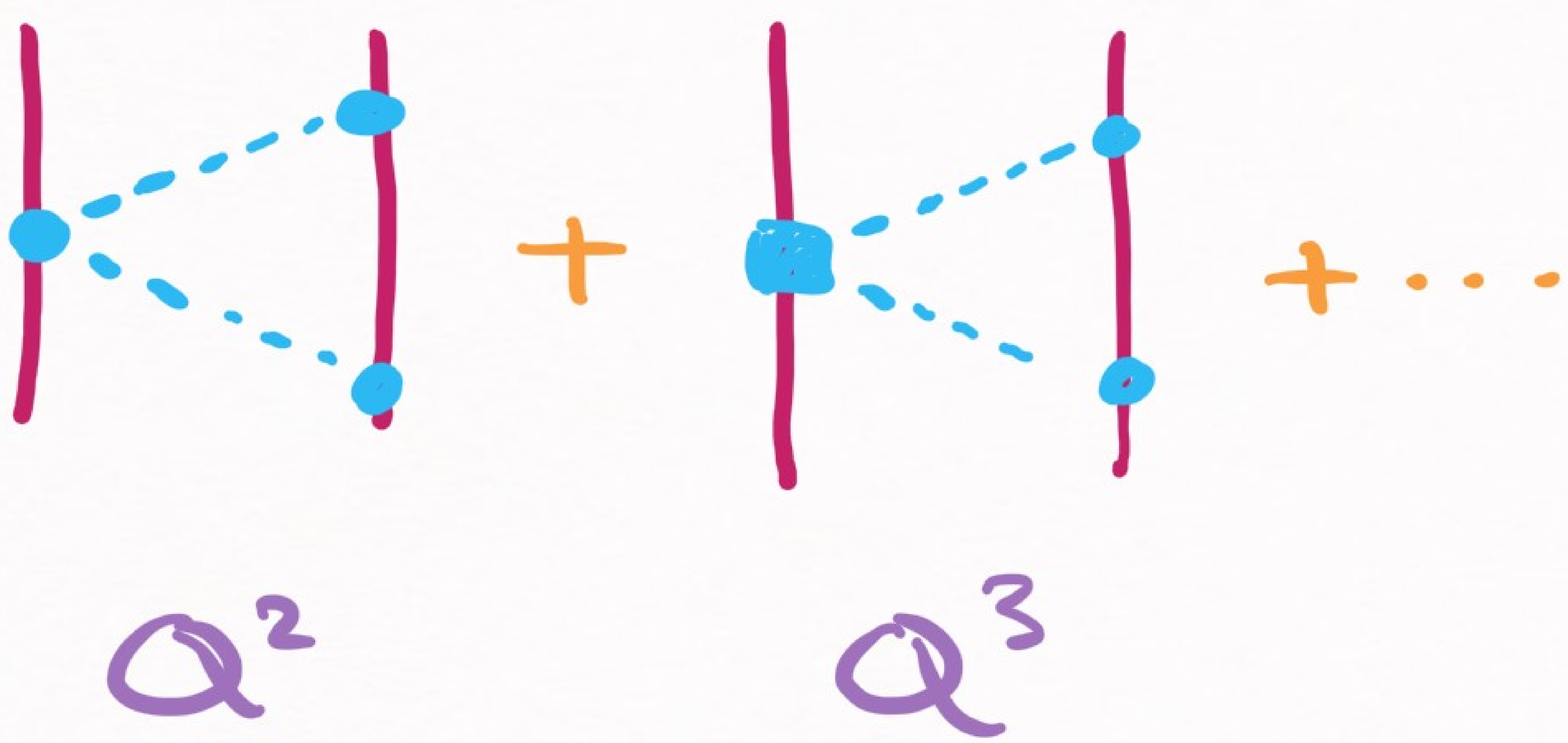
(fermion propagator)

PIONFUL EFT (12)

3) How do we count pion diagrams (cont'd)?

$\Rightarrow$ all vertices can in principle accept an expansion

(further possible expansion, much like $C_0 + C_2(p^2+p'^2)+\dots$)



Pionful EFT

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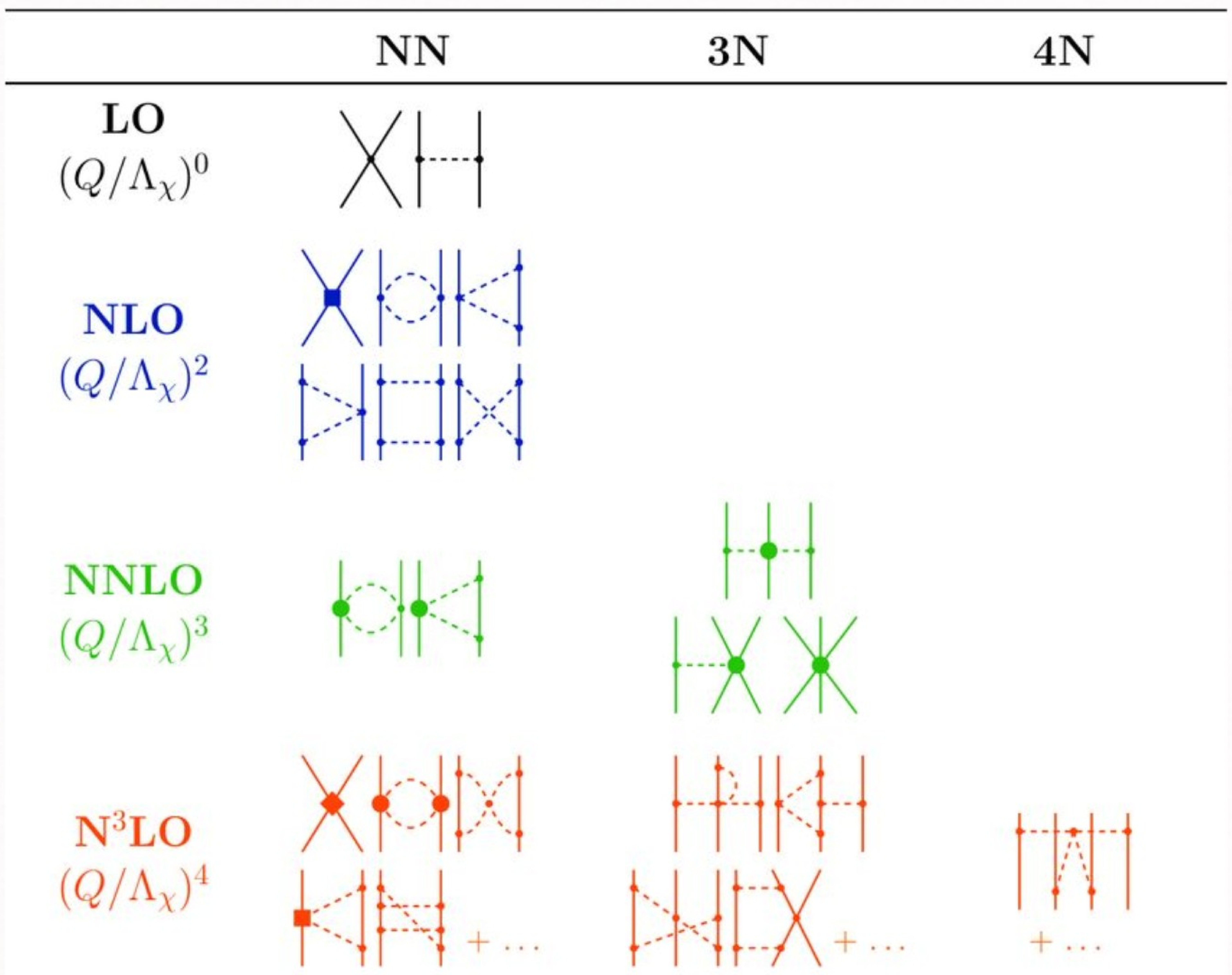

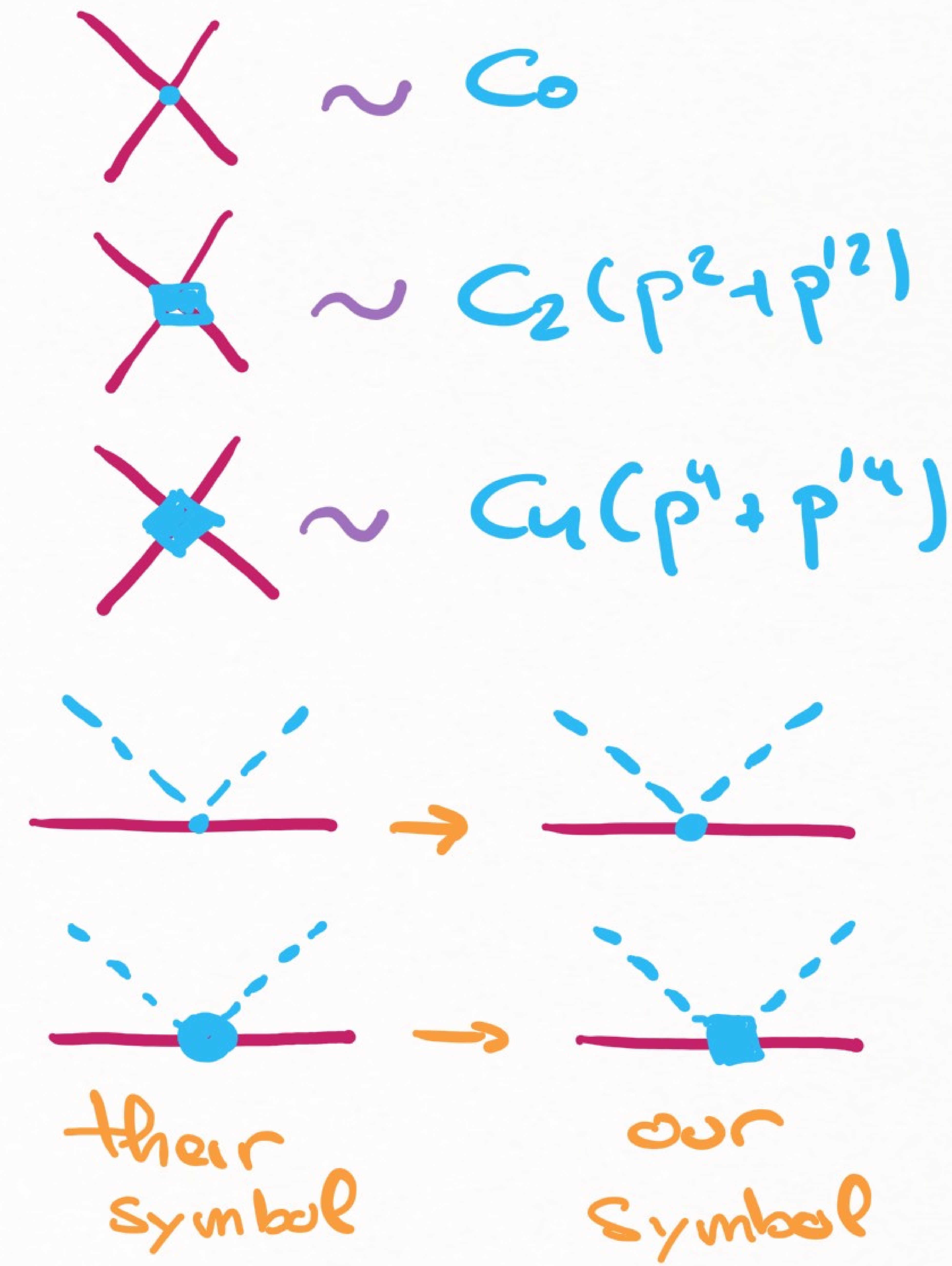

PIONFUL EFT

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4) But, naive power counting does not necessarily coincide with actual power counting

Example: Pionless

naive $\rightarrow$ $C_{2n} \sim \frac{2\pi}{\mu} R_s^{2n+1}$ (only valid for perturbative systems)

reality $\rightarrow$ $C_0 \sim \frac{2\pi}{\mu} a_0$, $C_{2n} \sim \frac{2\pi}{\mu} a_0^2 R_s^{2n+1}$

(if we have a_0 large: virtual / bound state close to $E_{cm} = 0$)

PIONFUL EFT

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4) But, naive power counting does not necessarily coincide with actual power counting

Pionful $\Rightarrow$ Same issues as in pionless one, that they are much more complex

(no consensus on the solution: open research problem)

PIONFUL EFT (16)

⇒ How do we do calculations in pionful?

Example:

$$V^{LO}_{EFT} = V^{LO}_{C} + V^{LO}_{F}$$

$$V^{LO}_{C}(\vec{q}) = C_0 + C_1 \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

$$V^{LO}_{F}(\vec{q}) = -\frac{g_A^2}{4f_\pi^2} \frac{\vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^2 + m_\pi^2} \vec{\tau}_1 \cdot \vec{\tau}_2$$

[We will finish this part of the lesson on Tuesday]

PIONFUL EFT

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⇒ How do we do calculations in pionful?

Example: the LO (leading order) calculation

1)

2)

3)

PIONFUL EFT

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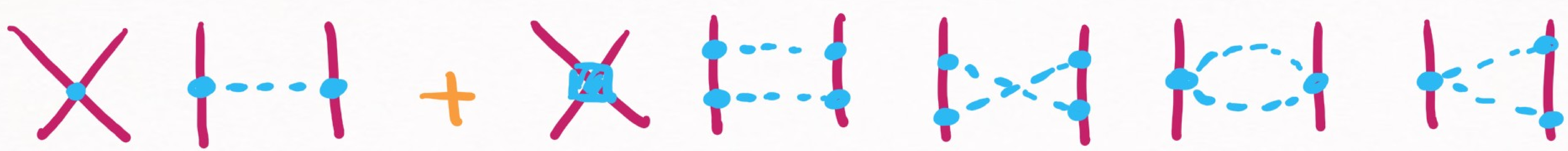
⇒ The LO phase shifts in pionful:

Pionful EFT (18)

⇒ How do we do calculations in pionful?

Now:



$$V_F^{NLO}(\vec{q}) = W_C(\vec{q})\,\vec{\tau}_1\cdot\vec{\tau}_2 + V_S(\vec{q})\,\vec{\sigma}_1\cdot\vec{\sigma}_2 + V_T(\vec{q})\,\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}$$

PIONFUL EFT (19)

⇒ How do we do calculations in pionful?

Now: subleading orders

$$
\begin{aligned}
V_{\rm ct}^{(2)}(\vec{p}^{\,\prime},\vec{p}) &= C_1\, q^2 + C_2\, k^2 \\
&+ \left(C_3\, q^2 + C_4\, k^2\right) \vec{\sigma}_1 \cdot \vec{\sigma}_2 \\
&+ C_5 \left(-i\vec{S}\cdot(\vec{q}\times\vec{k})\right) \\
&+ C_6\, (\vec{\sigma}_1\cdot\vec{q})\,(\vec{\sigma}_2\cdot\vec{q}) \\
&+ C_7\, (\vec{\sigma}_1\cdot\vec{k})\,(\vec{\sigma}_2\cdot\vec{k})\,.
\end{aligned}
$$

V_C^{NLO}

$$
\begin{aligned}
W_C &= -\frac{L(q)}{384\pi^2 f_\pi^4}\left[4m_\pi^2(5g_A^4 - 4g_A^2 - 1) + q^2(23g_A^4 - 10g_A^2 - 1) + \frac{48 g_A^4 m_\pi^4}{w^2}\right], \\
V_T &= -\frac{1}{q^2} V_S = -\frac{3 g_A^4 L(q)}{64\pi^2 f_\pi^4},
\end{aligned}
$$

$$
L(q) \equiv \frac{w}{q}\ln\frac{w+q}{2m_\pi}
$$

$$
w \equiv \sqrt{4m_\pi^2 + q^2}\,.
$$

$$
V_F^{NLO} = W_C\, \vec{\tau}_1\cdot\vec{\tau}_2 + V_S\, \vec{\sigma}_1\cdot\vec{\sigma}_2 + V_T\, \vec{\sigma}_1\cdot\vec{q}\; \vec{\sigma}_2\cdot\vec{q}
$$

Pionful EFT

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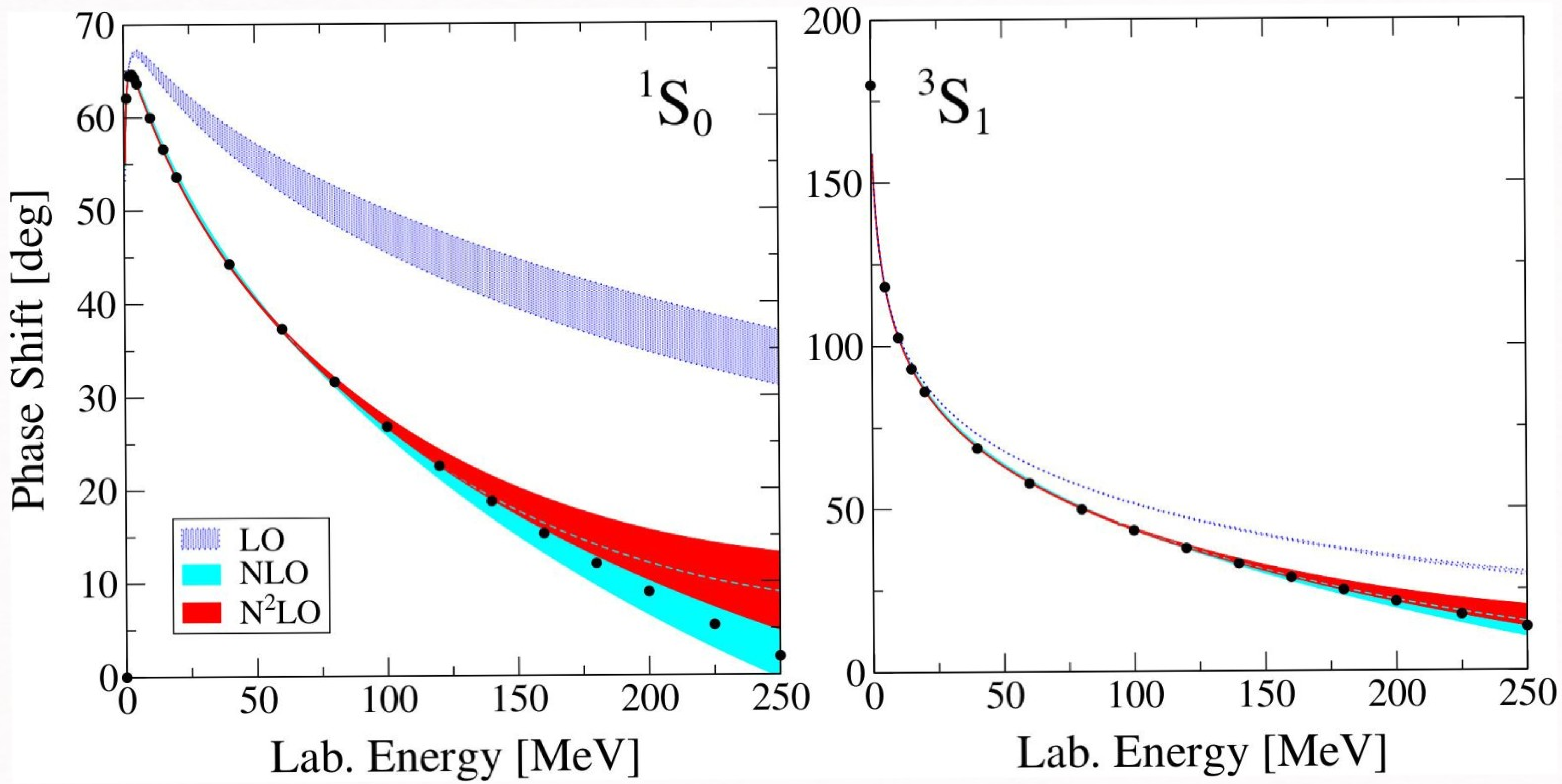
⇒ How do we do calculations in pionful?

Example:



[RENORMALIZATION IN PIONFUL EFT]

$\Rightarrow$ Remember the comment about pionful power counting?

(slides 47 & 48)

$\Rightarrow$

Example: pionful at LO

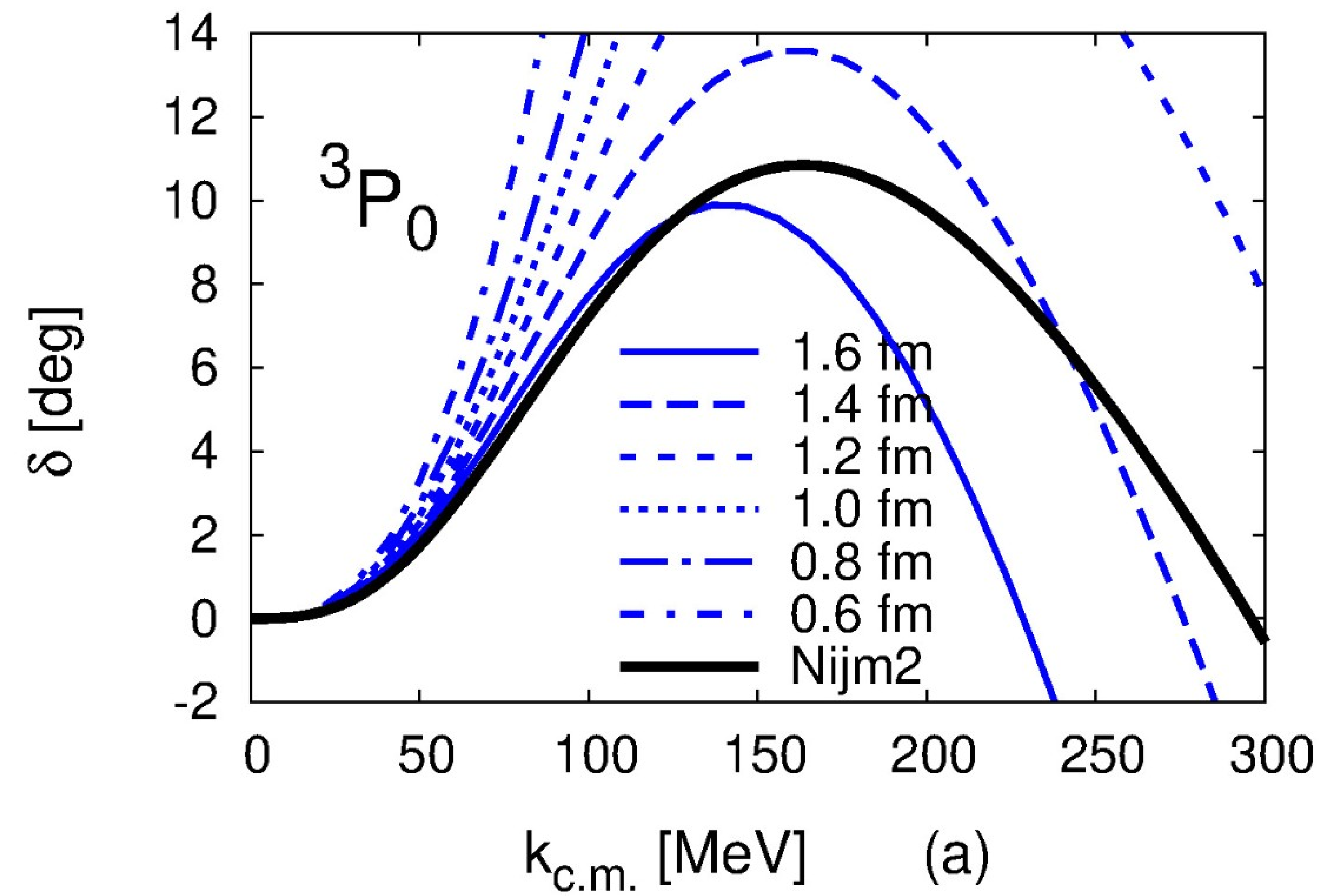
$$V_c^{LO} = C_0 + C_1\,\vec{\sigma}_1\cdot\vec{\sigma}_2 \quad \Rightarrow$$

[RENORMALIZATION IN PIONFUL EFT]

⇒ $^3P_0$ partial wave: a)

b)

$$\left[V_{^3P_0} \propto -\frac{1}{r^3} \right]$$

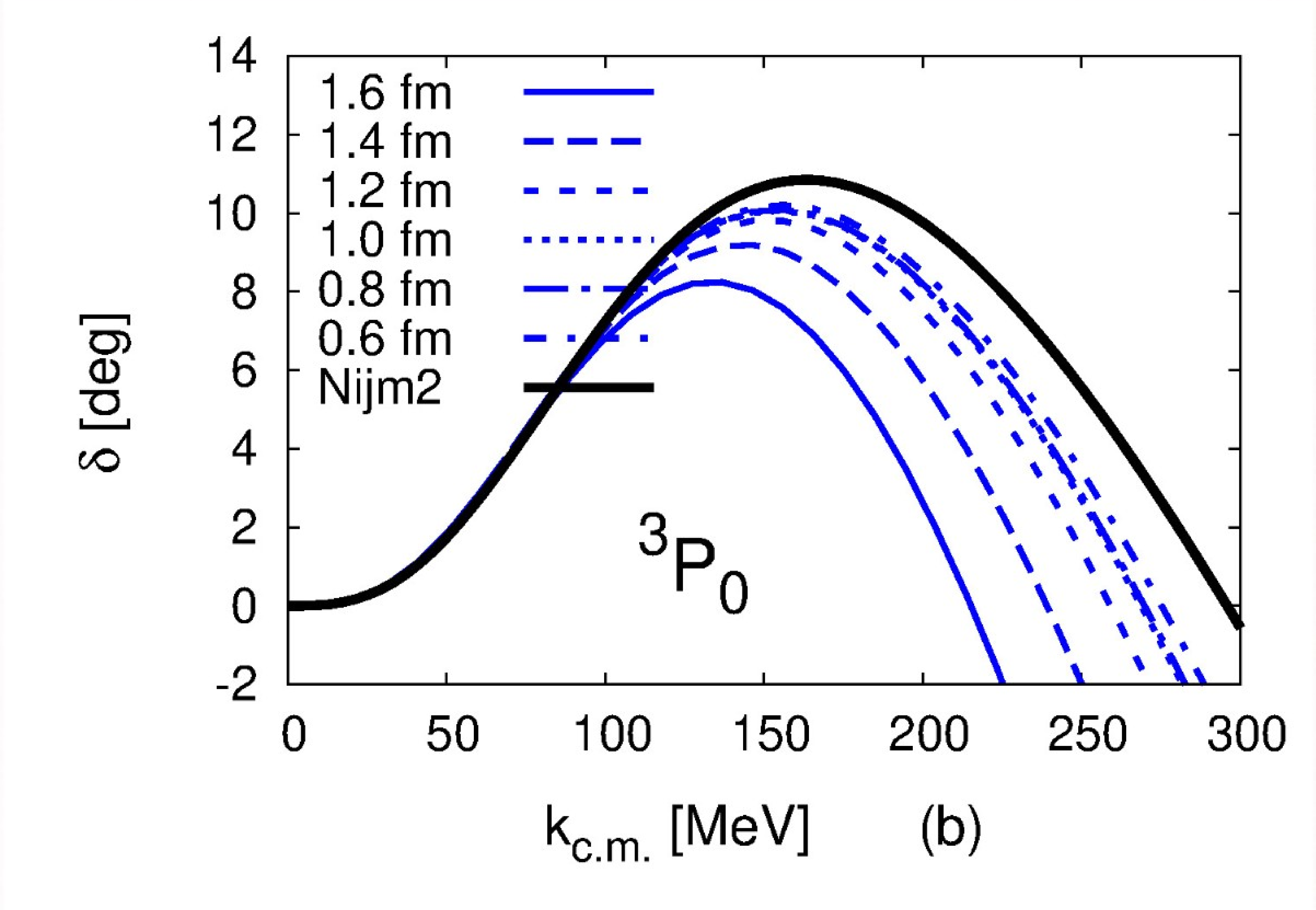


[RENORMALIZATION IN PIONFUL EFT]

$\Rightarrow$ $^3P_0$ partial wave: a) no contact-range interaction

b) tensor force attractive

$$\left[V_{^3P_0} \propto -\frac{1}{r^3} \right]$$



[RENORMALIZATION IN BIONFUL EFT]

$\Rightarrow$ Take-home message:

RECAP !

1) Pionful EFT : contacts + pions

1.a) Contacts are expanded as in pionless EFT (except for the counting, which is more complex)

1.b) Pions are also expanded as a power series

2) Range of validity : 2.a) Pionless: $k < m_\pi$
2.b) Pionful: $k < m_\rho$

3) Renormalization and power counting are still open problems in pionful

See you on Tuesday

15:50