NUCLEAR PHYSICS 11

a) FLAVOR SYMMETRY (PART II)

b) WHY IS THE PION SO LIGHT?

→ THE LINEAR SIGMA MODEL

RECAP | ISOSPIN & FLAVOR SYMMETRIES

a) $\frac{m_u}{\Lambda_{QCD}}, \frac{m_d}{\Lambda_{QCD}} \ll 1 \Rightarrow \begin{pmatrix} u \\ d \end{pmatrix} \rightarrow U_I \begin{pmatrix} u \\ d \end{pmatrix}$

b) $\frac{m_s}{\Lambda_{QCD}} < 1 \Rightarrow \begin{pmatrix} u \\ d \\ s \end{pmatrix} \rightarrow U_F \begin{pmatrix} u \\ d \\ s \end{pmatrix}$

Symmetries

c) $\underbrace{U_I \in SU(2)}_{\text{Isospin symmetry}}, \quad \underbrace{U_F \in SU(3)}_{\text{Flavor symmetry}}$

[REPRESENTATIONS OF $SU(2)_I$ & $SU(3)_F$]

a) $SU(2)_I$ ⟹ Analogous to $SU(2)$-spin ⟹ Same representations

$$|I\, m_I\rangle \xrightarrow[SU(2)_I]{} U^{I}_{m_I m_I'}\, |I\, m_I'\rangle$$

(Exactly the same as we would have done w/ spin, $|S\, m_S\rangle$)

Couple isospin:

$$I_1 \otimes I_2 = |I_1 - I_2| \oplus \cdots\cdots \oplus (I_1 + I_2)$$

$$\underbrace{\tfrac{1}{2} \otimes \tfrac{1}{2}}_{\text{isospin}} = 0 \oplus 1$$

Alternatively, we can use multiplicity:

$$\underline{\underline{2 \otimes 2 = 1 \oplus 3}}$$

[REPRESENTATONS OF $SU(2)_I$ & $SU(3)_F$]

b) $SU(3)_F$ $\Rightarrow$ More complicated (requires two quantum numbers)

b.1) $SU(2)_I \subseteq SU(3)_F$

→ one of the quantum numbers will be spin

b.2) Different possible choices: [$Q = \frac{Y}{2} + m_I$]

→ Strangeness: s-quark has $\boxed{S = -1}$

→ Hypercharge (Y): $Y = B + S$

B → baryon number

Y → strangeness

$\boxed{\text{ANTIPARTICLES}}$ → Represantions in $SU(3)_F$ are denoted by their multiplicity

a) $\begin{pmatrix} u \\ d \\ s \end{pmatrix} \to U_F \begin{pmatrix} u \\ d \\ s \end{pmatrix}$ ⇒ triplet representation 3

b) $\hat{C}\lambda|P\rangle = \lambda^* |\bar{P}\rangle$ (charge conjugation) ←

$\begin{pmatrix} \bar{u} \\ \bar{d} \\ \bar{s} \end{pmatrix} \to U_F^* \begin{pmatrix} \bar{u} \\ \bar{d} \\ \bar{s} \end{pmatrix}$ ⇒ $(\bar{u}\ \bar{d}\ \bar{s}) \to (\bar{u}\ \bar{d}\ \bar{s})(U_F^*)^T$

(transpose) $\to (\bar{u}\ \bar{d}\ \bar{s})\, U_F^\dagger$

⇒ antitriplet representation $\bar{3}$

[TWO LIGHT QUARK & SU(3)F] ③

a) $|q_1 q_2\rangle \Rightarrow \underbrace{\frac{1}{\sqrt{2}}(|q_1 q_2\rangle - |q_2 q_1\rangle)}_{\text{3 states }(3 \text{ or } \bar{3}\,?)}, \underbrace{\frac{1}{\sqrt{2}}(|q_1 q_2\rangle + |q_2 q_1\rangle)}_{\text{6 states }(6)}$

b) $\langle a| = \epsilon_{abc}\langle q_b q_c| \Rightarrow \langle a| \to \langle a'|U^{\dagger}_{a'a}$ $(\bar{3})$

$\epsilon_{abc}\langle q_b q_c| \to \langle q_{b'} q_{c'}|\epsilon_{abc} U_{bb'} U_{cc'}$] ①

$(\epsilon_{abc} U_{aa'} U_{bb'} U_{cc'} = \det U\, \epsilon_{a'b'c'} = \epsilon_{a'b'c'}) \times (U^{\dagger})_{a'\rho}$

$\to (U_{aa'}(U^{\dagger})_{a'\rho} = \delta_{a\rho}) \to \epsilon_{\rho bc} U_{bb'} U_{cc'} = \epsilon_{a'b'c'} U^{\dagger}_{a'\rho}$

Rename $\rho \to a$: ① $= \epsilon_{a'b'c'} U^{\dagger}_{a'a}$

[TWO LIGHT QUARKS & SO(3)F] (2)

$$\Rightarrow \boxed{3 \otimes 3 = \bar{3} \oplus 6}$$

AND NOW WE WANT TO ADD A THIRD QUARK:

$$3 \otimes 3 \otimes 3 = 1 \oplus \underbrace{8_S}_{} \oplus \underbrace{8_A}_{} \oplus 10$$

"symmetric" (only for $q_1 q_2$)

"antisymmetric" (only for $q_1 q_2$)

mixed symmetry representations

[THREE LIGHT QUARKS & SU(3)] 1

$\Rightarrow |q_1 q_2 q_3\rangle \Rightarrow$ Same trick as before: symmetry

a) Completely antisymmetric

$$\varepsilon_{abc} |q_a q_b q_c\rangle \xrightarrow[SU(3)_F]{} (\det U)\, \varepsilon_{abc} |q_a q_b q_c\rangle$$

$\det U = 1$

$\rightarrow$ Good representation $\longrightarrow$ $\boxed{\text{1 state (singlet)}}$

b) Completely symmetric $\quad |uuu\rangle, \; \frac{1}{\sqrt{3}}\left(|uud\rangle + |udu\rangle + |duu\rangle\right),$

+ all the other possibilities

$\boxed{\text{10 states (decuplet)}}$

[THREE LIGHT QUARKS & SU(3)] 2

c) Mixed symmetry $\Rightarrow$ only quarks 1 & 2 have well-defined symmetry

c.1) $|(q_1q_2 - q_2q_1)q_3\rangle$

8_A $\rightarrow$ if you count, you will find 8 states

c.2) $|(q_1q_2 + q_2q_1)q_3\rangle + ($terms to ensure orthogonality with 10$)$

$\rightarrow$ if we count we find 8 states (8_S)

[THREE QUARKS & SU(3)] 3

c) Mixed symmetry (continued)

$$|p_A\rangle = \frac{1}{\sqrt{2}}\left(|ud\rangle - |du\rangle\right)|u\rangle$$

$$|p_S\rangle = \frac{1}{\sqrt{6}}\left[\left(|ud\rangle + |du\rangle\right)|u\rangle - 2|uud\rangle\right]$$

proton states

$$\langle \Delta^+| = \frac{1}{\sqrt{3}}\left(\langle uud| + \langle udu| + \langle duu|\right)$$

$$\langle 10|8_S\rangle = 0$$

(see orthogonality condition in previous slide)

[HOW TO BUILD A BARYON] 1

a) Baryon $\Rightarrow$ 3 quarks (fermions) $\Rightarrow$ Totally antisymmetric wave function

REMINDER: COLOR

$$|\text{Baryon w.f.}\rangle = \underbrace{|\text{color w.f.}\rangle}_{\text{Hadrons are colorless} \rightarrow 1} \times |\text{flavor w.f.}\rangle \times |\text{spin w.f.}\rangle$$

Baryons : $\epsilon_{ijk} |q_i q_j q_k\rangle$, $ijk \rightarrow$ refers to color

$\rightarrow$ always antisymmetric $\leftarrow$ 1

[HOW TO BUILD A BARYON] 2

(spin w.f.) × (flavor w.f.)
$\underbrace{\phantom{(\text{spin w.f.}) \times (\text{flavor w.f.})}}_{\text{always symmetric}}$

b) Singlet :

$$|1\rangle = \underbrace{\epsilon_{abc}\,|q_a q_b q_c\rangle}_{\text{flavor wavefunction}} \Rightarrow \text{antisymmetric}$$

$\Rightarrow$ Require a fully antisymmetric spin w.f.

$\epsilon_{ijk}\,|j_1 m_i;\, j_2 m_j\, j_3 m_k\rangle \Rightarrow$ 3 different values of $\underline{\underline{m_s}}$

$\Rightarrow$ Only possible if at least one quark has $j = 3/2$
(excited states only) $(\underline{\underline{\ell = 1}},\ s = 1/2)$

[HOW TO BUILD A BARYON] 3

c) Decuplet : $|000\rangle$ → totally symmetric

⇒ spin w.f. also totally symmetric

$$\underbrace{|\uparrow\uparrow\uparrow\rangle}_{J=3/2} \quad , \quad \frac{1}{\sqrt{3}}\left(|\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle\right), \dots$$

⇒ Decuplet baryons :

$$J = 3/2 \, , \underbrace{P = +1}_{\text{because } \ell = 0} \rightarrow \left(\tfrac{3}{2}\right)^{+} \text{ states}$$

[HOW TO BUILD A BARYON] (4)

d) Octet : (flavor w.f.) × (spin w.f.)

$$\underbrace{(\text{flavor w.f.})}_{\text{mixed symmetry}} \times \underbrace{(\text{spin w.f.})}_{\text{mixed symmetry}} = \underline{\underline{\text{totally symmetric}}}$$

$\Rightarrow$ Study spin wavefunctions : $\underbrace{\frac{1}{2} \otimes \frac{1}{2}}_{} \otimes \frac{1}{2} = \frac{1}{2} \oplus \frac{1}{2} \oplus \frac{3}{2}$

$(0_A \oplus 1_S) \otimes \frac{1}{2} =$

$\left(\frac{1}{2}\right)_A \oplus \left(\frac{1}{2}\right)_S \oplus \left(\frac{3}{2}\right)_S$

$|\frac{3}{2}(S)\rangle \rightarrow$ previous slide

$|\frac{1}{2}(S)\rangle = \frac{1}{\sqrt{6}}[(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)|\uparrow\rangle]$, ...

$|\frac{1}{2}(A)\rangle = \frac{1}{\sqrt{2}}[(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)|\uparrow\rangle]$, ...

[HOW TO BUILD A BARYON] (5)

c) Octet (continued): → the spin wavefunction ←

$$|\tfrac{3}{2}(S)\rangle = |\uparrow\uparrow\uparrow\rangle \;,\; \frac{1}{\sqrt{3}}\left(|\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle\right), \dots$$

$$|\tfrac{1}{2}(A)\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)|\uparrow\rangle \;, \dots$$

$$|\tfrac{1}{2}(S)\rangle = \frac{1}{\sqrt{6}}\left[\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)|\uparrow\rangle - 2|\uparrow\uparrow\downarrow\rangle\right], \dots$$

[How to build a Baryon] 6

c) Octet (continued):

symmetric configurations : $|8_S\rangle \left|\tfrac{1}{2}(S)\right\rangle$
(1 ↔ 2) $|8_A\rangle \left|\tfrac{1}{2}(A)\right\rangle$ } (1 ↔ 2)

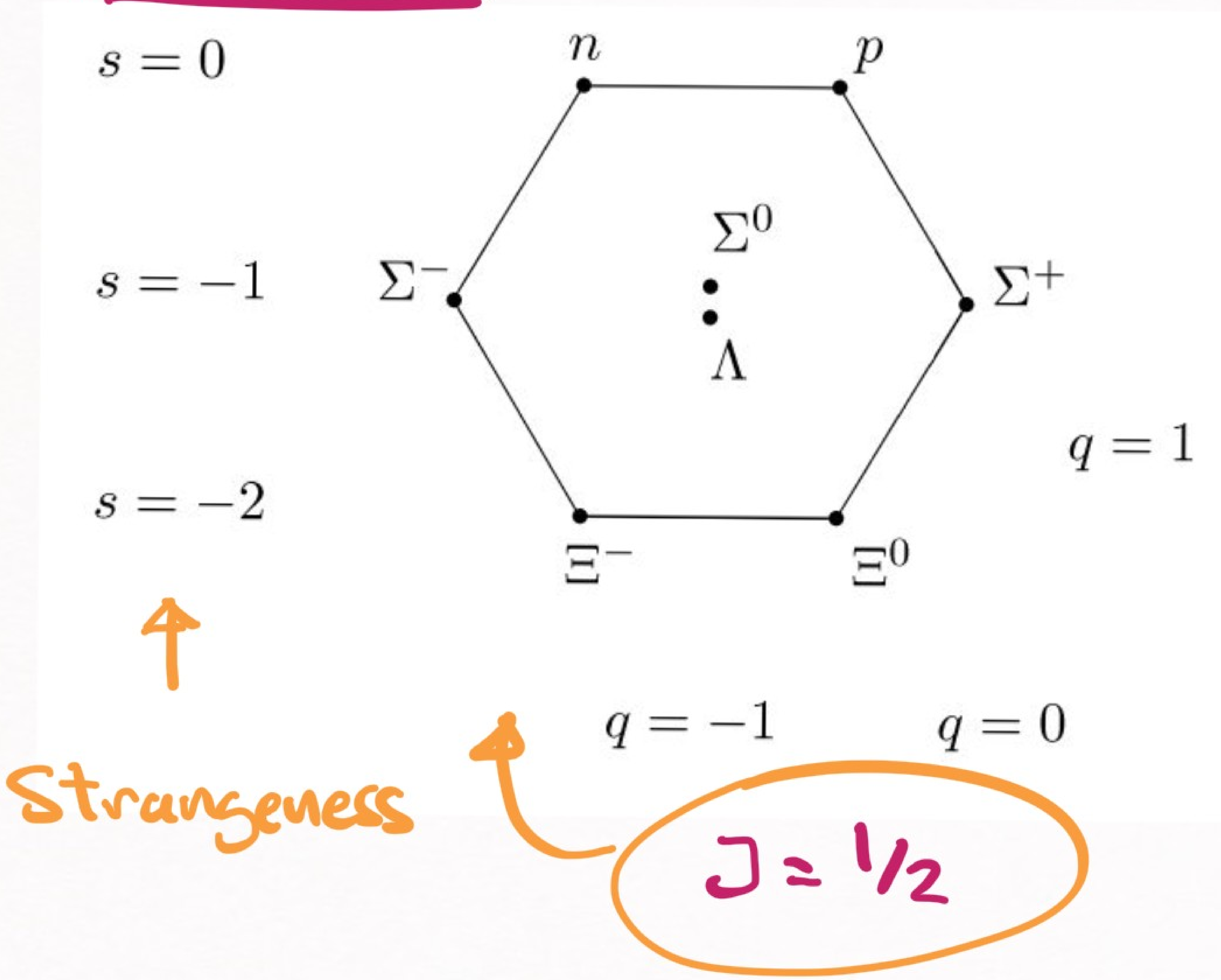
But what we really want are symmetric configurations in 1, 2, 3 (1 ↔ 2, 2 ↔ 3, 3 ↔ 1)

$$\frac{1}{\sqrt{2}}\left(|8_S\rangle \left|\tfrac{1}{2}(S)\right\rangle + |8_A\rangle \left|\tfrac{1}{2}(A)\right\rangle\right) \longrightarrow \text{Octet}$$

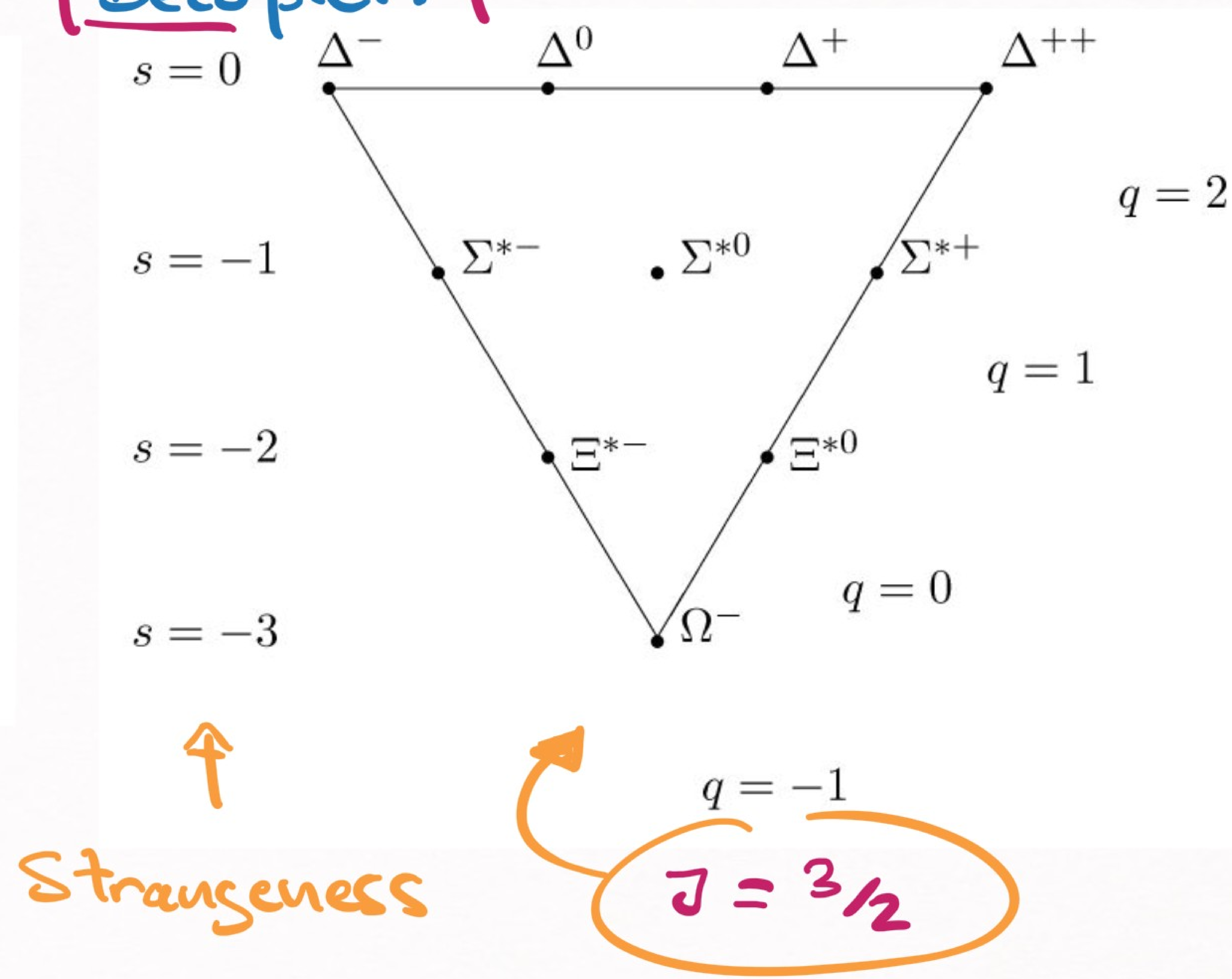
flavor & spin wavefunction

[Ground State Baryons]

Octet:



Decuplet:



[HOW TO BUILD A MESON] $\Rightarrow$ Quark-antiquark systems

$$\Rightarrow \begin{pmatrix} u \\ d \\ s \end{pmatrix} \rightarrow U \begin{pmatrix} u \\ d \\ s \end{pmatrix}, \quad (\bar{u}\ \bar{d}\ \bar{s}) \rightarrow (\bar{u}\ \bar{d}\ \bar{s})\, U^{\dagger}$$

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix} (\bar{u}\ \bar{d}\ \bar{s}) = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix}$$

$SU(3)_F$ transformation for quark-antiquark states $\rightarrow$ (*)

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix} (\bar{u}\ \bar{d}\ \bar{s}) \longrightarrow U \left[\begin{pmatrix} u \\ d \\ s \end{pmatrix} (\bar{u}\ \bar{d}\ \bar{s}) \right] U^{\dagger}$$

(*) $\rightarrow$ change of basis $\rightarrow$ trace will be invariant

[HOW TO BUILD A MESON] 2

a) $$\mathrm{Tr}\left[\begin{pmatrix}u\\d\\s\end{pmatrix}(\bar u\ \bar d\ \bar s)\right] \xrightarrow[SU(3)_F]{} \mathrm{Tr}\left[\begin{pmatrix}u\\d\\s\end{pmatrix}(\bar u\ \bar d\ \bar s)\right]$$

$$\frac{1}{\sqrt3}\left(|u\bar u\rangle + |d\bar d\rangle + |s\bar s\rangle\right) \longrightarrow \frac{1}{\sqrt3}\left(|u\bar u\rangle + |d\bar d\rangle + |s\bar s\rangle\right)$$

$\Rightarrow$ Singlet state or $|1\rangle$

b) $$\begin{pmatrix}u\\d\\s\end{pmatrix}(\bar u\ \bar d\ \bar s) - \frac{1}{3}\mathrm{Tr}\left[\begin{pmatrix}u\\d\\s\end{pmatrix}(\bar u\ \bar d\ \bar s)\right]\mathbb{1}_{3\times3} \xrightarrow[SU(3)_F]{} \text{(into itself)}$$

$\Rightarrow$ 8 states (9 basis states minus the trace)

$\Rightarrow$ octet or $|8\rangle$

[HOW TO BUILD A MESON] (3)

$\Rightarrow$ Check how these w.f.s correspond to particles

$\Rightarrow$ $3 \otimes \bar{3} = 1 \oplus 8$

$$|1\rangle = \frac{1}{\sqrt{3}}\left(|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle\right) \leftarrow \text{singlet w.f.}$$

octet w.f.

$$|8\rangle \rightarrow \begin{pmatrix} |u\bar{u}\rangle - \frac{1}{\sqrt{3}}|1\rangle & |u\bar{d}\rangle & |u\bar{s}\rangle \\ |d\bar{u}\rangle & |d\bar{d}\rangle - \frac{1}{\sqrt{3}}|1\rangle & |d\bar{s}\rangle \\ |s\bar{u}\rangle & |s\bar{d}\rangle & |s\bar{s}\rangle - \frac{1}{\sqrt{3}}|1\rangle \end{pmatrix}$$

Let's simplify the matrix:

a) Reminder: isospin of quark-antiquark systems

$|1\,1\rangle = -|u\bar{d}\rangle \rightarrow -|\pi^+\rangle$ (this could

$|1\,0\rangle = \frac{1}{\sqrt{2}}(|u\bar{u}\rangle - |d\bar{d}\rangle) \rightarrow |\pi^0\rangle$ change depending

$|1\,-1\rangle = |d\bar{u}\rangle \rightarrow |\pi^-\rangle$ on conventions)

b) We can define an isospin zero octet state:

$$|0\,0\rangle = \underbrace{\frac{1}{\sqrt{2}}(|u\bar{u}\rangle + |d\bar{d}\rangle)}_{\text{mixture of } |1\rangle \,\&\, |8\rangle} \neq |\eta\rangle = \frac{1}{\sqrt{6}}(|u\bar{u}\rangle + |d\bar{d}\rangle - 2|s\bar{s}\rangle)$$

(pure octet w/ $I=0$)

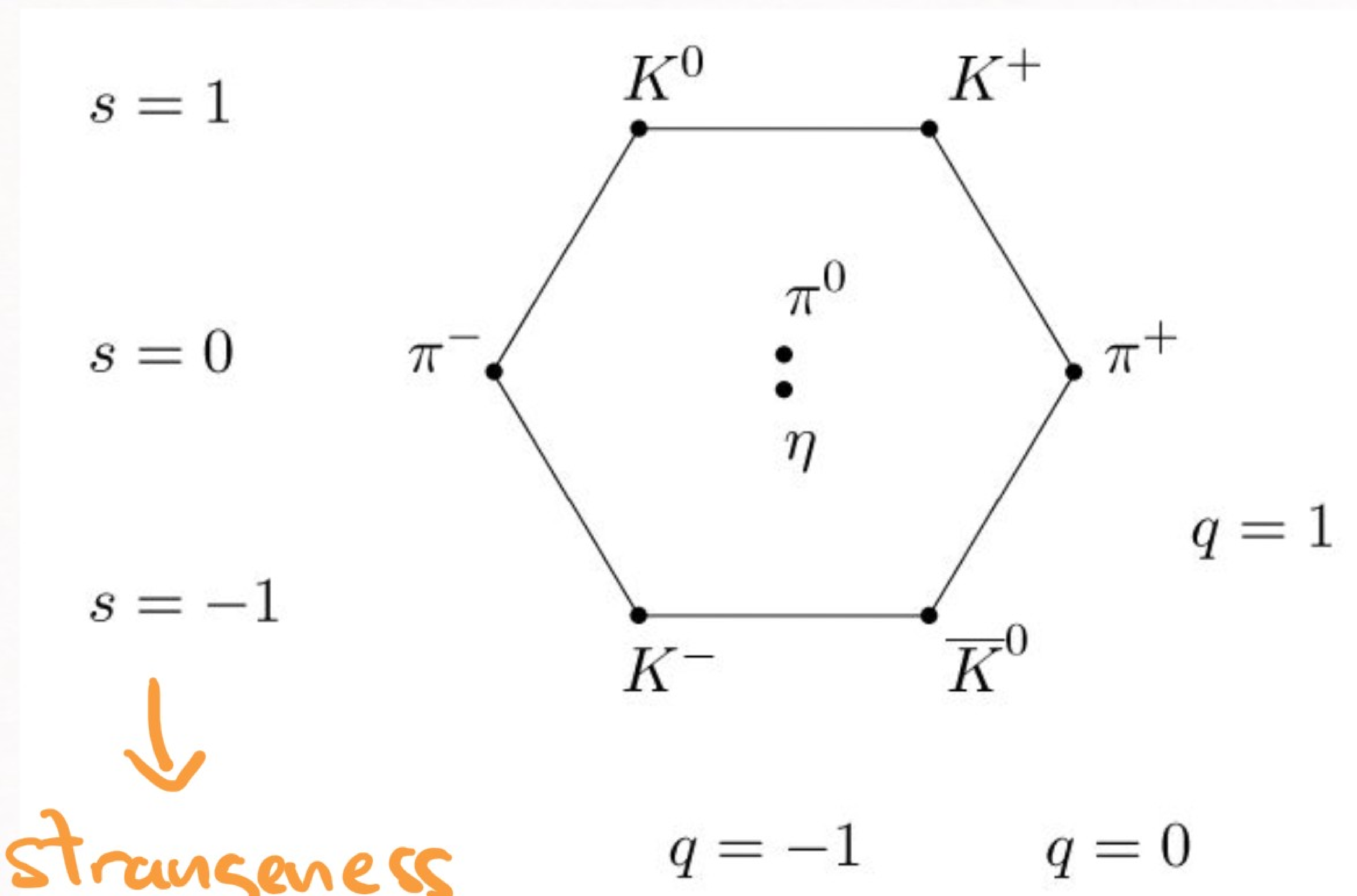
$\boxed{\text{PSEUDOSCALAR OCTET}}$

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix}(\bar u \ \bar d \ \bar s) - \frac{1}{3}\mathrm{Tr}\left[\begin{pmatrix} u \\ d \\ s \end{pmatrix}(\bar u \ \bar d \ \bar s)\right] \Rightarrow \underbrace{\begin{pmatrix} \frac{\pi^0}{\sqrt{2}}+\frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}}+\frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & \sqrt{\frac{2}{3}}\eta \end{pmatrix}}_{\text{Caveat: depends on sign conventions}}$$

Caveat: depends on sign conventions

$\Downarrow$

corresponds w/ the matrix in slide $\underline{\underline{20}}$



$\downarrow$

strangeness

$\underline{s}$

But $SU(3)_F$ is not always a good symmetry:

$(m_s \gg m_u, m_d)$

a) $|1\rangle = \frac{1}{\sqrt{3}}(|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle)$

$|8\rangle = \frac{1}{\sqrt{6}}(|u\bar{u}\rangle + |d\bar{d}\rangle - 2|s\bar{s}\rangle)$

$\Rightarrow$ (invert) $\Rightarrow$ ☺

☺ $\Rightarrow$
$$\begin{pmatrix} |s\bar{s}\rangle \\ \frac{1}{\sqrt{2}}(|u\bar{u}\rangle + |d\bar{d}\rangle \end{pmatrix} = \begin{pmatrix} \sqrt{1/3} & -\sqrt{2/3} \\ -\sqrt{2/3} & \sqrt{1/3} \end{pmatrix} \begin{pmatrix} |1\rangle \\ |8\rangle \end{pmatrix}$$

(check it just in case)

$\boxed{m_s \gg m_u, m_d}$ → Often, physical states will correspond to $\frac{1}{\sqrt{2}}(|u\bar{u}\rangle + |d\bar{d}\rangle)$ and $|s\bar{s}\rangle$ (instead of $|1\rangle$ and $|8\rangle$)

MESON NONETS

(instead of meson octets)

Example → vector mesons ($J^P = 1^-$)

$|\omega\rangle = \frac{1}{\sqrt{2}}(|u\bar{u}\rangle + |d\bar{d}\rangle)$

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix} \begin{pmatrix} \bar{u} & \bar{d} & \bar{s} \end{pmatrix} \longrightarrow \begin{pmatrix} \frac{\rho^0 + \omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0 + \omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}$$

$|\phi\rangle = |s\bar{s}\rangle$

∃ also baryon nonets (excited baryons)

[$SU(2)_I$ & $SU(3)_F$] ⟹ How to use them?

a) Probabilities (cross sections & decays): $\underline{\underline{SU(2)_I}}$

↑

$\underline{\underline{\text{Relative}}}$

$$\underline{\underline{P_1}} = P(\Delta^{++} \to p\pi^+) \propto |\langle \Delta^{++} | p\pi^+ \rangle|^2 = 1$$

$$P_2 = P(\Delta^+ \to p\pi^0) \propto |\langle \Delta^+ | p\pi^0 \rangle|^2 = 2/3$$

$$P_3 = P(\Delta^+ \to n\pi^+) \propto |\langle \Delta^+ | n\pi^+ \rangle|^2 = 1/3$$

$$\left.\begin{aligned} \frac{P_1}{P_2} &= \frac{3}{2} \\ \frac{P_1}{P_3} &= 3 \end{aligned}\right\} \Leftarrow$$

$|\Delta^{++}\rangle = |\tfrac{3}{2}\,\tfrac{3}{2}\rangle$ $\quad |p\rangle = |\tfrac{1}{2}\,\tfrac{1}{2}\rangle$

$|\Delta^{+}\rangle = |\tfrac{3}{2}\,\tfrac{1}{2}\rangle$ $\quad |n\rangle = |\tfrac{1}{2}\,-\tfrac{1}{2}\rangle$

$|\pi^+\rangle = |1\,1\rangle$, $|\pi^0\rangle = |1\,0\rangle$

$$\langle p\pi^+ | \Delta^{++}\rangle = \underbrace{\langle \tfrac{1}{2}\,\tfrac{1}{2}\,1\,1 | \tfrac{3}{2}\,\tfrac{3}{2}\rangle}_{\text{Clebsch-Gordan coefficient}} = \circledast$$

$\circledast = 1$

[$SU(2)_I$ & $SU(3)_F$] Clebsch-Gordan coefficients

b) Probabilities (cross section & decays): $SU(3)_F$

$$\underbrace{\Sigma^* \to \Sigma\pi}, \quad \Sigma^* \to \Lambda\pi \;\{\to \Sigma^{*+} \to \Lambda^0\pi^+, \text{ etc.}$$

$$\Sigma^{*+} \to \frac{1}{\sqrt{2}}(\Sigma^+\pi^0 - \Sigma^0\pi^+)$$

$$\Sigma^{*0} \to \underbrace{\frac{1}{\sqrt{2}}(\Sigma^+\pi^- - \Sigma^-\pi^+\rangle}_{}$$

etc. from $SU(2)_I$ Clebsch Gordan coefficient

$10 \to 8 \otimes 8$ transition

$$\langle \Sigma^* | \Sigma\pi \rangle = \sqrt{\frac{2}{12}}$$

$$\langle \Sigma^* | \Lambda\pi \rangle = -\sqrt{\frac{3}{12}}$$

$$\frac{P(\Sigma^* \to \Sigma\pi)}{P(\Sigma^* \to \Lambda\pi)} = \frac{2}{3}$$

[$SU(2)_I$ & $SU(3)_F$] $\Rightarrow$ We need to use tables of $SU(3)_F$ Clebsch-Gordan coefficients

Example: $\left.\begin{array}{l} \Sigma^* \to \Sigma\pi \\ \Sigma^* \to \Lambda\pi \end{array}\right\}$ $10 \to 8 \otimes 8$

$\Downarrow$ Only thing required

10 → 8 ⊗ 8

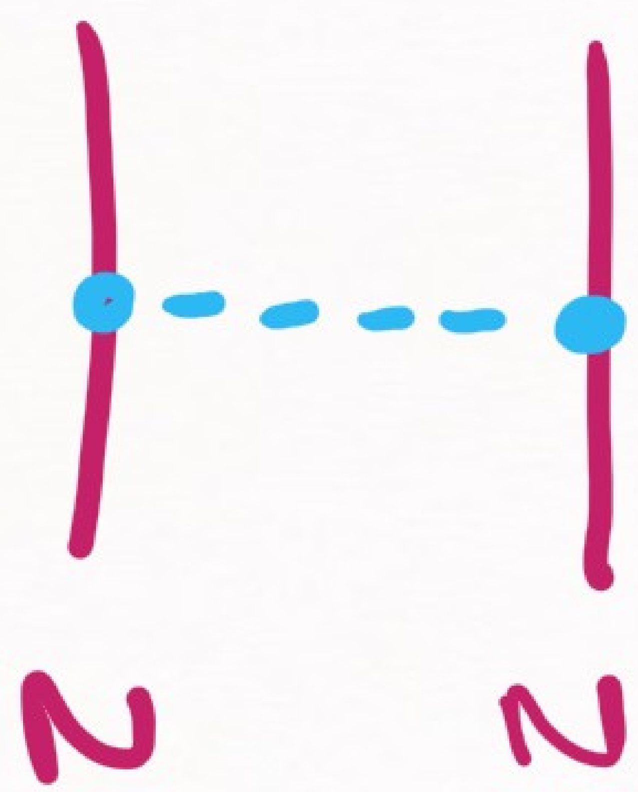
$$\begin{pmatrix} \Delta \\ \boxed{\Sigma} \\ \Xi \\ \Omega \end{pmatrix} \to \begin{pmatrix} & N\pi & \Sigma K & & \\ N\overline{K} & \Sigma\pi & \Lambda\pi & \Sigma\eta & \Xi K \\ & \Sigma\overline{K} & \Lambda\overline{K} & \Xi\pi & \Xi\eta \\ & & \Xi\overline{K} & & \end{pmatrix} = \frac{1}{\sqrt{12}} \begin{pmatrix} & -6 & 6 & & \\ -2 & 2 & -3 & 3 & 2 \\ & 3 & -3 & 3 & 3 \\ & & 12 & & \end{pmatrix}^{1/2}$$

check convention of your $SU(3)$ table

from review of "SU(3) isoscalar form factors and representation matrices" (within Review of Particle Physics)

$[SU(2)_I$ & $SU(3)_F] \Rightarrow$ How to use them?

b) Baryon-baryon potentials: $\leftarrow$ Our example



$$= \left\{ \begin{array}{l} V_1 \text{ for } \underline{\underline{I=1}} \\ \\ V_0 \text{ for } \underline{\underline{I=0}} \end{array} \right] \Rightarrow$$

Can we derive from this the potential in other configurations?

$\rightarrow$ Yes, but it will be troublesome

$$8 \otimes 8 = 1 \oplus 8_S \oplus 8_A \oplus 10 \oplus 10^* \oplus 27$$

(that is really daunting)

b) Baryon-Baryon potential → most simple case:

Reminder: $\underline{\underline{Y = B + S}}$ ⇒ $Y = 1$ for proton ⇒ $\underline{\underline{}}$

pp we have $Y = 2$ ⇐
($I = 1$)

$$\boxed{\begin{array}{c} NN \text{ with } I = 1 \\ \left(pp, \frac{1}{\sqrt{2}}(np+pn), nn\right) \end{array}}$$

TABLE II. Isoscalar factors for $\{8\} \otimes \{8\}$. Given are the isoscalar factors

$$\begin{pmatrix} 8 & 8 & \mu_\gamma \\ I_1 Y_1 & I_2 Y_2 & I\ Y \end{pmatrix}$$ → this table uses $\underline{\underline{I}}$ ☺

for the CG series $\{8\} \otimes \{8\} = \{27\} \oplus \{10\} \oplus \{10^*\} \oplus \{8\}_1 \oplus \{8\}_2 \oplus \{1\}$.

$Y = 2 \quad I = 1$

$I_1,\ Y_1;\ I_2,\ Y_2$	27	μ_γ
$\frac{1}{2},\ 1;\ \frac{1}{2},\ 1$	1	

$Y = 2 \quad I = 0$

$I_1,\ Y_1;\ I_2,\ Y_2$	10^*	μ_γ
$\frac{1}{2},\ 1;\ \frac{1}{2},\ 1$	-1	

de Swart, RMP 35, number 4, 916   ☺ → $\underline{\underline{|pp\rangle = |27\rangle}}$

b) Baryon-baryon: the most simple case!

$|NN(I=1)\rangle = |27\rangle \Rightarrow$ Look for more $|27\rangle$'s (because they will have the same V)

$|\Sigma\Sigma(I=2)\rangle = |27\rangle$

$|\Xi\Xi(I=1)\rangle = |27\rangle$

$\frac{1}{\sqrt{2}}|N\Sigma+\Sigma N(I=3/2)\rangle = |27\rangle$

$\frac{1}{\sqrt{2}}|\Xi\Sigma+\Sigma\Xi(I=3/2)\rangle = |27\rangle$

$Y = 1 \quad I = \frac{3}{2}$

$I_1, Y_1; I_2, Y_2$	27	10	μ_γ
$\frac{1}{2}, 1; 1, 0$	$\sqrt{2}/2$	$-\sqrt{2}/2$	
$1, 0; \frac{1}{2}, 1$	$\sqrt{2}/2$	$\sqrt{2}/2$	

$Y = 1 \quad I = \frac{1}{2}$

$I_1, Y_1; I_2, Y_2$	27	8_1	8_2	10^*	μ_γ
$\frac{1}{2}, 1; 1, 0$	$\sqrt{5}/10$	$3\sqrt{5}/10$	$1/2$	$-1/2$	
$1, 0; \frac{1}{2}, 1$	$-\sqrt{5}/10$	$-3\sqrt{5}/10$	$1/2$	$-1/2$	
$\frac{1}{2}, 1; 0, 0$	$3\sqrt{5}/10$	$-\sqrt{5}/10$	$1/2$	$1/2$	
$0, 0; \frac{1}{2}, 1$	$3\sqrt{5}/10$	$-\sqrt{5}/10$	$-1/2$	$-1/2$	

$Y = 0 \quad I = 2$

$I_1, Y_1; I_2, Y_2$	27	μ_γ
$1, 0; 1, 0$	1	

$Y = 0 \quad I = 1$

$I_1, Y_1; I_2, Y_2$	27	8_1	8_2	10	10^*	μ_γ
$\frac{1}{2}, 1; \frac{1}{2}, -1$	$\sqrt{5}/5$	$-\sqrt{30}/10$	$\sqrt{6}/6$	$-\sqrt{6}/6$	$\sqrt{6}/6$	
$\frac{1}{2}, -1; \frac{1}{2}, 1$	$\sqrt{5}/5$	$-\sqrt{30}/10$	$-\sqrt{6}/6$	$\sqrt{6}/6$	$-\sqrt{6}/6$	
$1, 0; 1, 0$	0	0	$\sqrt{6}/3$	$\sqrt{6}/6$	$-\sqrt{6}/6$	
$1, 0; 0, 0$	$\sqrt{30}/10$	$\sqrt{5}/5$	0	$1/2$	$1/2$	
$0, 0; 1, 0$	$\sqrt{30}/10$	$\sqrt{5}/5$	0	$-1/2$	$-1/2$	

$Y = 0 \quad I = 0$

$I_1, Y_1; I_2, Y_2$	27	8_1	1	8_2	μ_γ
$\frac{1}{2}, 1; \frac{1}{2}, -1$	$\sqrt{15}/10$	$\sqrt{10}/10$	$1/2$	$\sqrt{2}/2$	
$\frac{1}{2}, -1; \frac{1}{2}, 1$	$-\sqrt{15}/10$	$-\sqrt{10}/10$	$-1/2$	$\sqrt{2}/2$	
$1, 0; 1, 0$	$-\sqrt{10}/20$	$-\sqrt{15}/5$	$\sqrt{6}/4$	0	
$0, 0; 0, 0$	$3\sqrt{30}/20$	$-\sqrt{5}/5$	$-\sqrt{2}/4$	0	

$Y = -1 \quad I = \frac{3}{2}$

$I_1, Y_1; I_2, Y_2$	27	10^*	μ_γ
$\frac{1}{2}, -1; 1, 0$	$\sqrt{2}/2$	$-\sqrt{2}/2$	
$1, 0; \frac{1}{2}, -1$	$\sqrt{2}/2$	$\sqrt{2}/2$	

$Y = -1 \quad I = \frac{1}{2}$

$I_1, Y_1; I_2, Y_2$	27	8_1	8_2	10	μ_γ
$\frac{1}{2}, -1; 1, 0$	$-\sqrt{5}/10$	$-3\sqrt{5}/10$	$1/2$	$1/2$	
$1, 0; \frac{1}{2}, -1$	$\sqrt{5}/10$	$3\sqrt{5}/10$	$1/2$	$1/2$	
$\frac{1}{2}, -1; 0, 0$	$3\sqrt{5}/10$	$-\sqrt{5}/10$	$-1/2$	$1/2$	
$0, 0; \frac{1}{2}, -1$	$3\sqrt{5}/10$	$-\sqrt{5}/10$	$1/2$	$-1/2$	

$Y = -2 \quad I = 1$

$I_1, Y_1; I_2, Y_2$	27	μ_γ
$\frac{1}{2}, -1; \frac{1}{2}, -1$	1	

$Y = -2 \quad I = 0$

$I_1, Y_1; I_2, Y_2$	10	μ_γ
$\frac{1}{2}, -1; \frac{1}{2}, -1$	1	

b) Baryon-baryon potential: $\underline{\underline{|27\rangle}}$

$|NN(I=1)\rangle \longrightarrow (S=0) \longrightarrow$ because total w.f. is antisymmetric

$|\Sigma\Sigma(I=2)\rangle \longrightarrow (S=0)$

$|\Xi\Xi(I=1)\rangle \longrightarrow (S=0)$

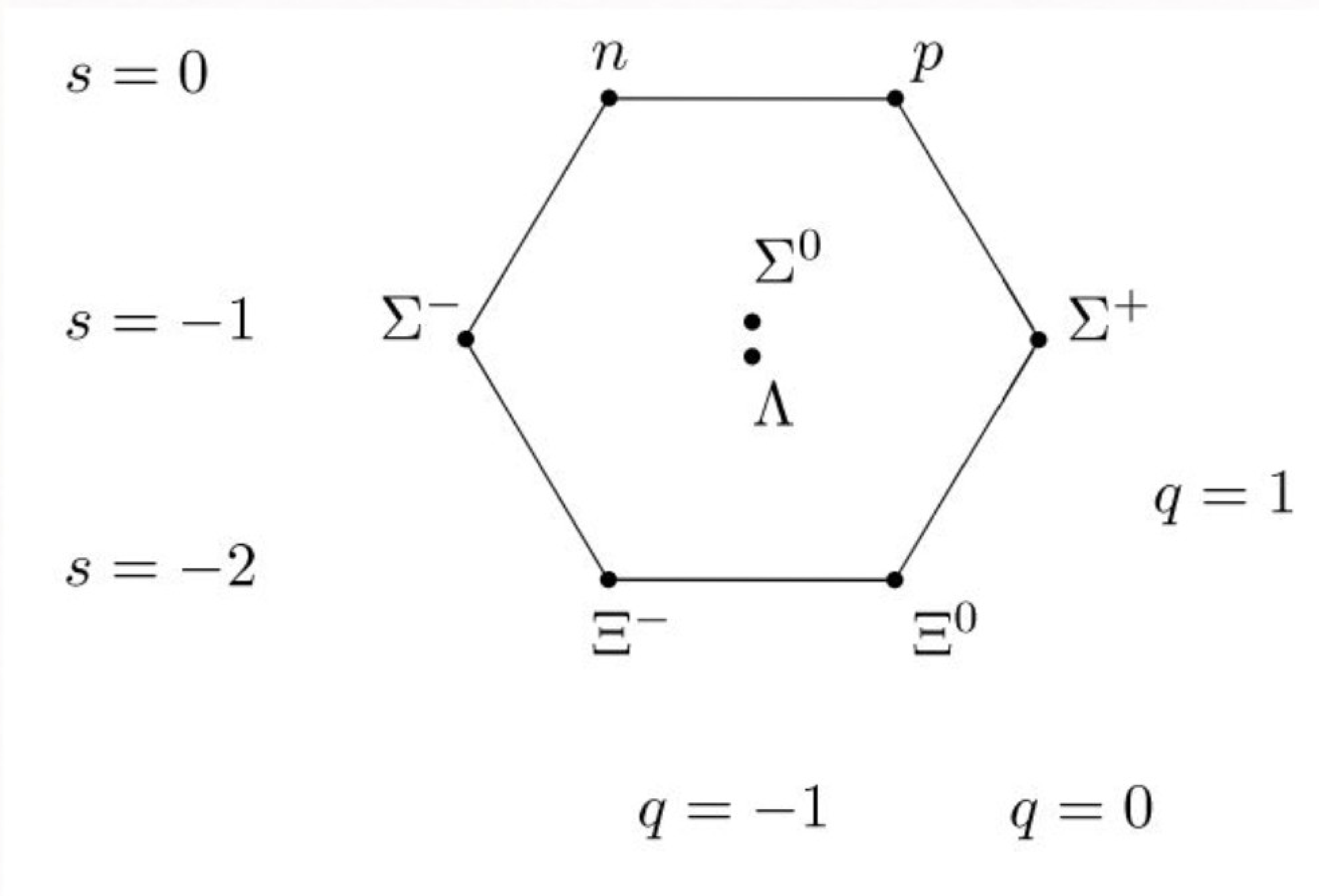
$\frac{1}{\sqrt{2}}|\Sigma N(3/2) + N\Sigma(3/2)\rangle \longrightarrow (S=0)$

$\frac{1}{\sqrt{2}}(|\Xi\Sigma(3/2) + \Sigma\Xi(3/2)\rangle \longrightarrow (S=0)$

$\Rightarrow$ All these systems will have the same potential (except for symmetry breaking effects)
(e.g. $m_s \gg m_u, m_d$)

$\boxed{SU(3)_F}$: HOW WELL DOES IT WORK?

CASE 1 $\Rightarrow$ Octet baryons



$m_n \simeq m_p \simeq 940$ MeV

$m_\Lambda \simeq 1115$ MeV (+15%)

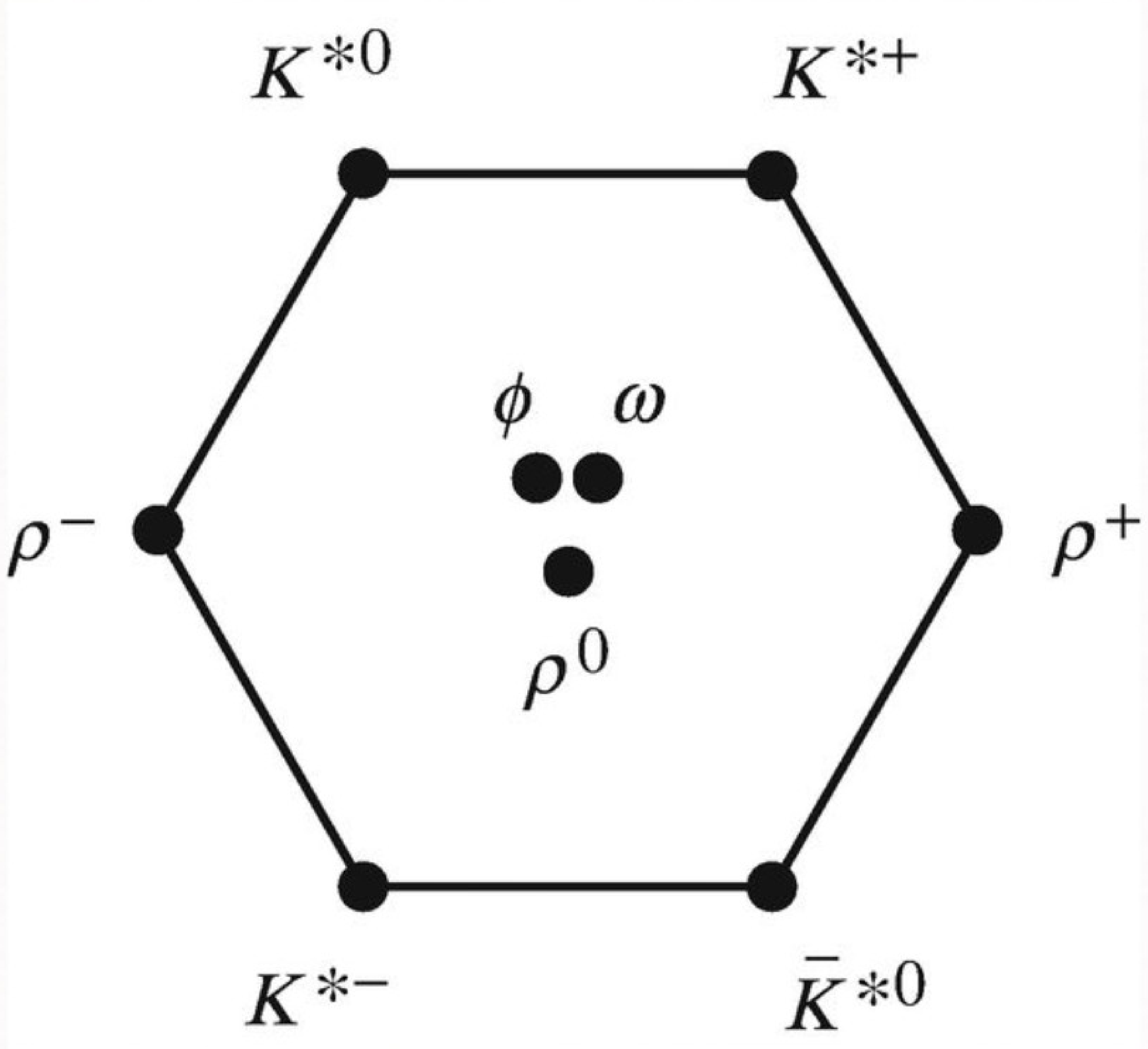
$m_\Sigma \simeq 1193$ MeV (+20%)

$m_\Xi \simeq 1318$ MeV (+30%)

$\rightarrow$ not terribly bad $\leftarrow$

$\boxed{SU(3)_F}$: HOW WELL IT WORKS?

CASE 2 $\Rightarrow$ vector meson nonet



$m_\rho \simeq 770\,\text{MeV}$

$m_\omega \simeq 780\,\text{MeV}$

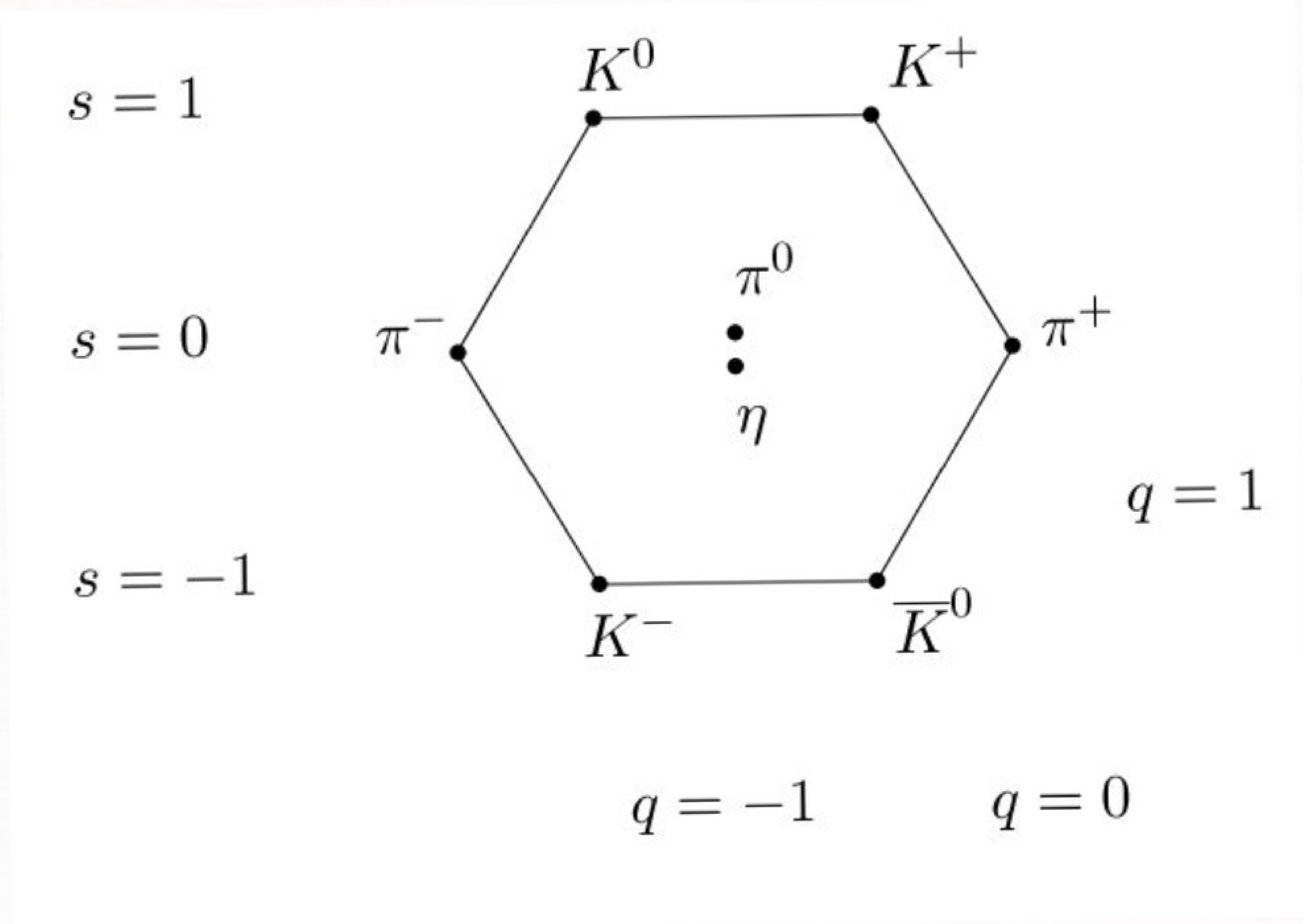
$m_{K^*} \simeq 890\,\text{MeV}$

$m_\phi \simeq 1020\,\text{MeV}$

$\rightarrow$ not terribly bad $\leftarrow$

$\boxed{SU(3)_F}$: HOW WELL IT WORKS?

CASE 3 $\Rightarrow$ Pseudoscalar meson octet



$m_\pi \simeq 138$ MeV

$m_K \simeq 495$ MeV

$m_\eta \simeq 548$ MeV

$\rightarrow$ there is a problem $\leftarrow$

$\rightarrow$ Needs an explanation $\leftarrow$

THE QUESTION HERE IS ... [WHY IS THIS HAPPENING?]

→ [CHIRAL SYMMETRY]

Getting here ↗ will require a few steps

1) the linear sigma model

2) Goldstone theorem

RECAP

a) SU(3)-flavor works (more or less) well in most cases

→ Baryon masses, vector meson masses ✓

b) But the pseudoscalar mesons are an exception ←

→ Pion is unnaturally light

c) Path to Chiral symmetry (the explanation for the small pion mass)

c.1) the linear σ model

c.2) Nambu-Goldstone theorem

Let's reconsider this problem
from the point of view of NATURALNESS

$\Rightarrow$ NATURAL : $\frac{m_\rho}{m_N} \sim 0.8 \sim \mathcal{O}(1)$ (from previous lessons)

$m_{meson} \sim 2\Lambda_{QCD} \sim 0.7\,GeV + (\text{mass of s-quark})$
$m_{baryon} \sim 3\Lambda_{QCD} \sim 0.9\,GeV + (\text{mass of s-quarks})$

$\Rightarrow$ BUT THE PION : $\frac{m_\pi}{m_\rho} \sim \frac{1}{7} \ll \mathcal{O}(1) \rightarrow$ fine-tuning

The pion is different:

$$\boxed{\frac{m_\pi}{m_e} \sim \frac{1}{7}} \Rightarrow \left\{ \begin{array}{l} \text{a) } \cancel{\text{Chance}} \\[2ex] \text{b) Symmetry} \rightarrow \text{Chiral symmetry} \end{array} \right.$$

$\Downarrow$

this fine-tuning comes from a symmetry

NOTE THIS $\Rightarrow$ $\frac{m_\pi}{m_e} \sim \frac{1}{7}$ also indicates a good separation of scales $\Rightarrow$ EFT

(EFT for pion $\rightarrow$ Chiral perturbation theory)

PROBLEM: $\boxed{\text{explain } \frac{m_n}{m_e} \sim \frac{1}{7}}$

SOLUTION: (long & tortuous) → this is just how physics <u>works</u>

⇓

Series of steps

For preparing yourself for chiral symmetry:

a) Gell-Mann's & Levi's linear σ model

b) Goldstone theorem

[THE TWO INTERMEDIATE STEPS]

a) Linear σ-model (Gell-Mann & Levi, 70's)

a theory in which we begin with a massless nucleon and a massive pion and end up with a massive nucleon and massless pion

b) Goldstone theorem

general explanation for what is happening in the LσM & a lot of other models

LINEAR SIGMA MODEL

1) We begin with this basic QFT:

$$\mathcal{L}_{L\sigma M} = \underbrace{i\bar{N}\not{\partial}N}_{(a)} + \overbrace{g\bar{N}(\phi_0 + i\gamma_5\vec{\tau}\cdot\vec{\phi})N}^{(b)} + \underbrace{\sum_{i=0}^{3}\partial_\mu\phi_i\partial^\mu\phi_i}_{(c)} - \underbrace{V(\phi)}_{(d)}$$

(a) massless Dirac field (nucleon)

(b) ϕ-N interaction

(c) kinetic term of the ϕ-field

(d) mass + interaction terms of ϕ-field

2) Let's see $V(\phi)$ in more detail:

$$V(\phi) = \underbrace{\frac{1}{2}\mu^2 \sum_{i=0}^{3} \phi_i^2}_{\text{mass term}} + \underbrace{\frac{\lambda}{4}\left(\sum_{i=0}^{3} \phi_i^2\right)^2}_{\text{kinetic term}} \longrightarrow \phi^4 \text{ theory}$$

3) Details to pay attention to:

3.a) N is a massless field

3.b) $(\phi_0, \vec{\phi})$ are massive $(m = \mu)$

[THE MEXICAN HAT POTENTIAL] 1

=> But this is not what we want! (N massless, ϕ massive)

What we want is the following:

[N massive, ϕ massless]

=> Luckily there is a magic trick to make this happen:

=> The mexican hat potential

[THE MEXICAN HAT POTENTIAL] ②

a) What does this trick achieves?

N massless, ϕ massive

$\Downarrow$

N massive, ϕ massless

b) How this is achieved?

$$V(\phi) = \frac{1}{2}\mu^2\left(\sum_{i=0}^{3}\phi_i^2\right) + \frac{\lambda}{4}\left(\sum_{i=0}^{3}\phi_i^2\right)^2 \Rightarrow \text{We take } \mu^2 < 0$$

maths allow this

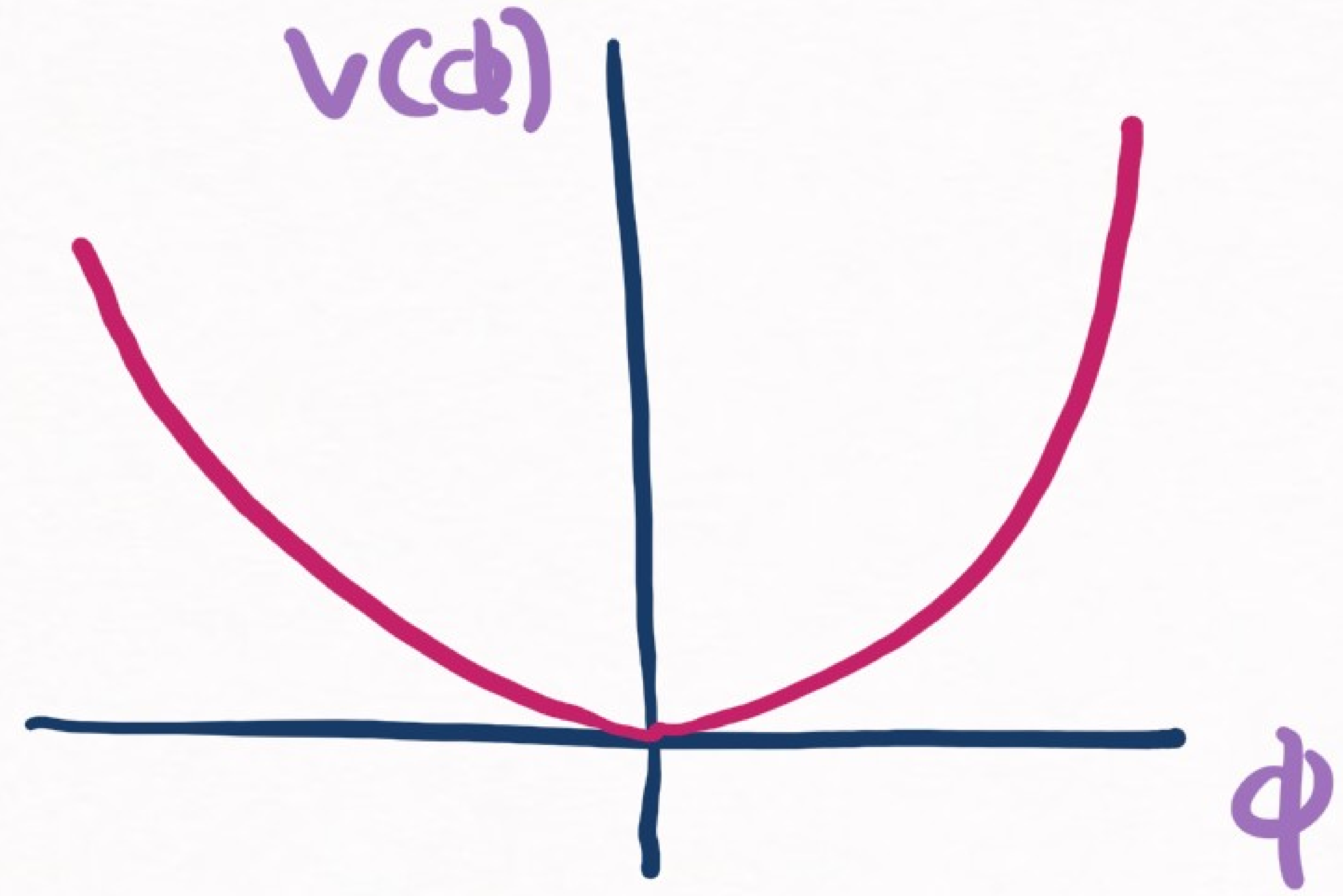
[THE MEXICAN HAT POTENTIAL] (3)

SM → 4 ϕ fields

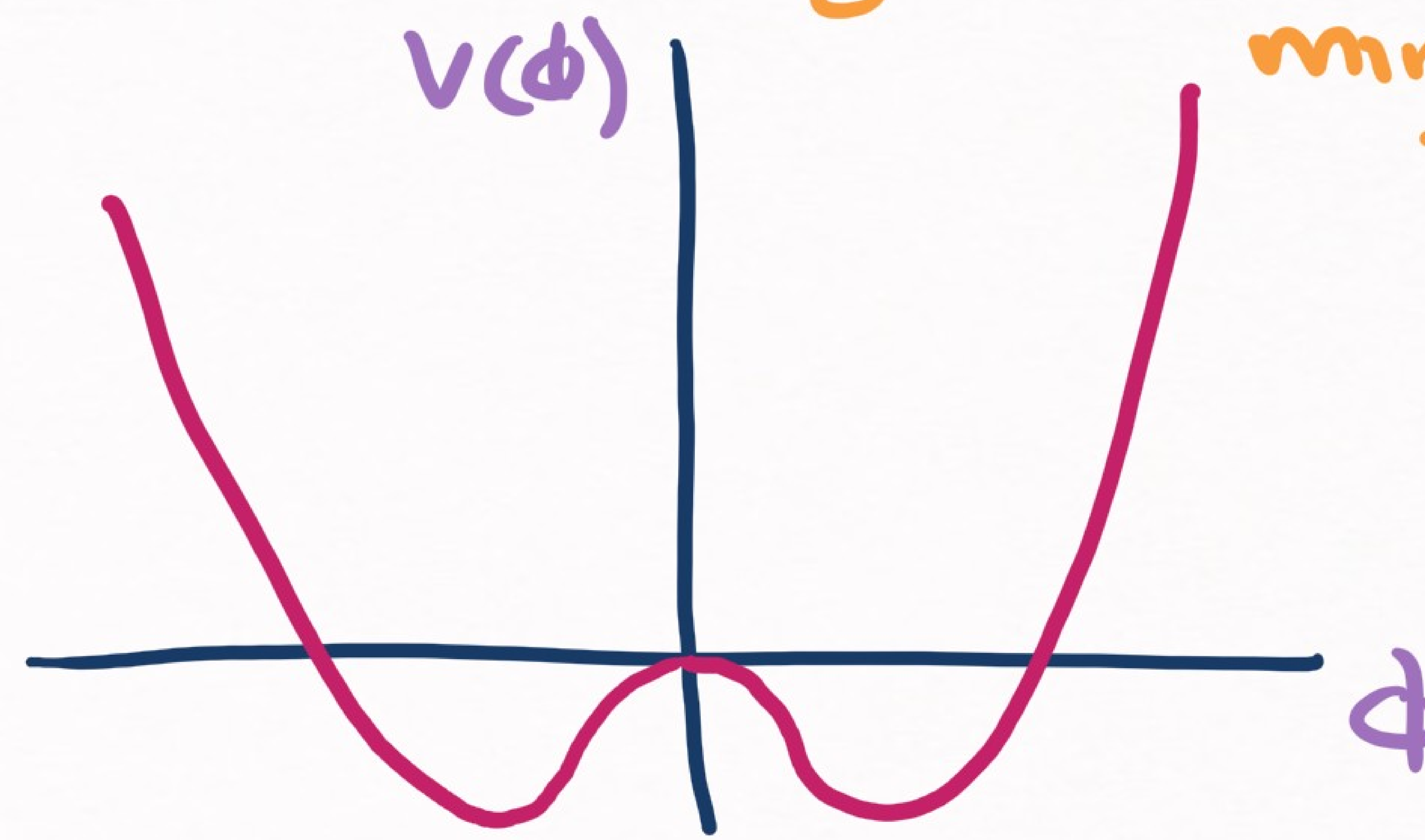
1) Imagine we only have one ϕ field:

$$V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4}\phi^4$$

→ this choice generates two minima



$\mu^2 > 0 \, , \lambda > 0$



$\mu^2 < 0 \, , \lambda > 0$

$\boxed{\text{TWO SCENARIOS}}$

1.a) Usual quartic potential: $\mu^2 > 0$, $\lambda > 0$

[μ is the mass of the ϕ field]

1.b) Mexican hat potential: $\rightarrow$ try to interpret this

$\mu^2 < 0$, $\lambda > 0$

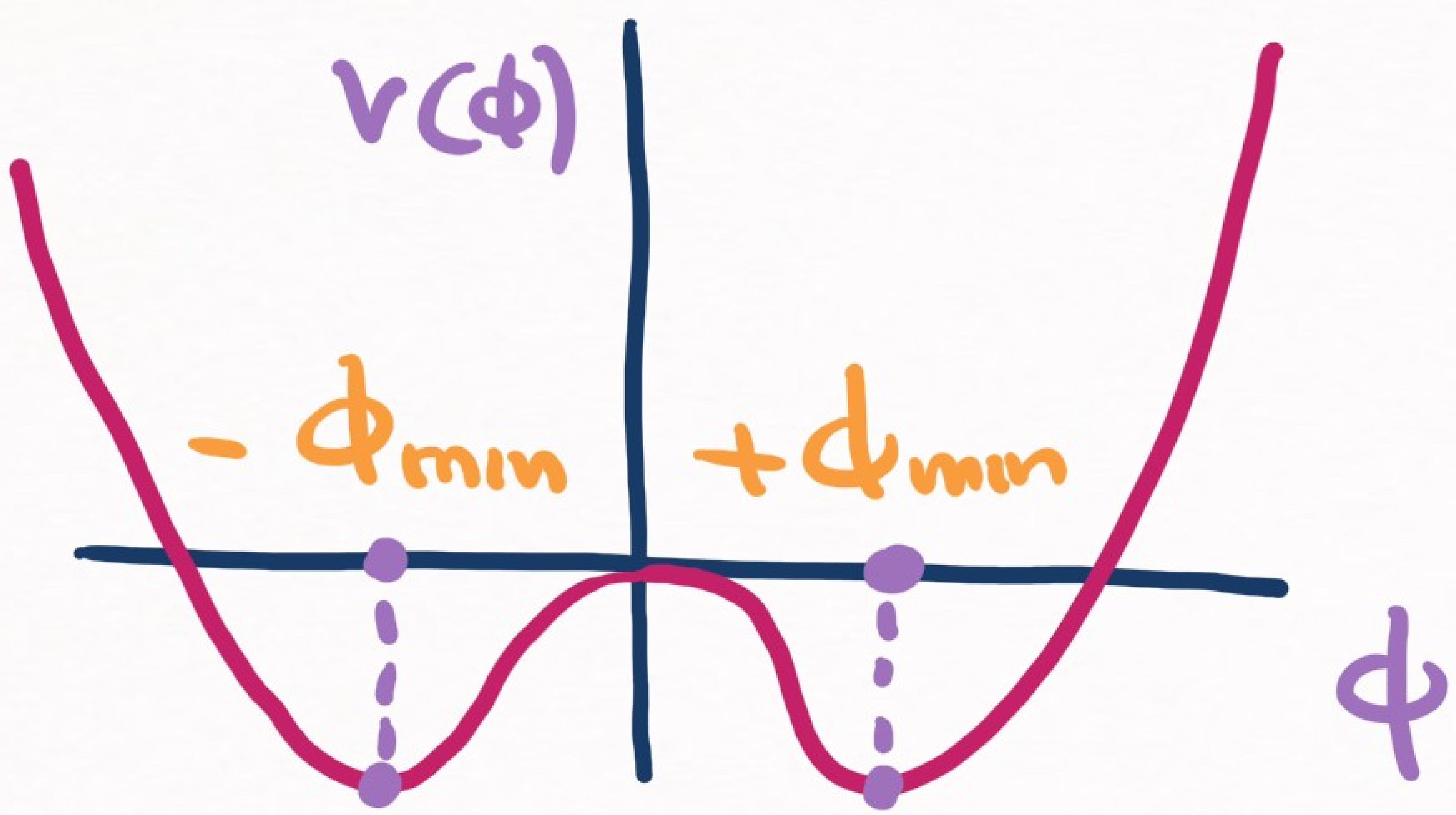
$\rightarrow$ allowed by mathematics, but μ cannot be interpret as the mass of the ϕ field

[HOW TO INTERPRET THIS?] 1

a) If $\mu^2 < 0$, μ can't be a mass ← ground state of our QFT

b) Nature likes minimal energy states :

$$\langle \phi \rangle \neq 0 \quad \text{but} \quad \langle \phi \rangle = \pm \phi_{min}$$



$$V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4}\phi^4 \, , \quad \left.\frac{dV}{d\phi}\right|_{\phi=\phi_{min}} = 0$$

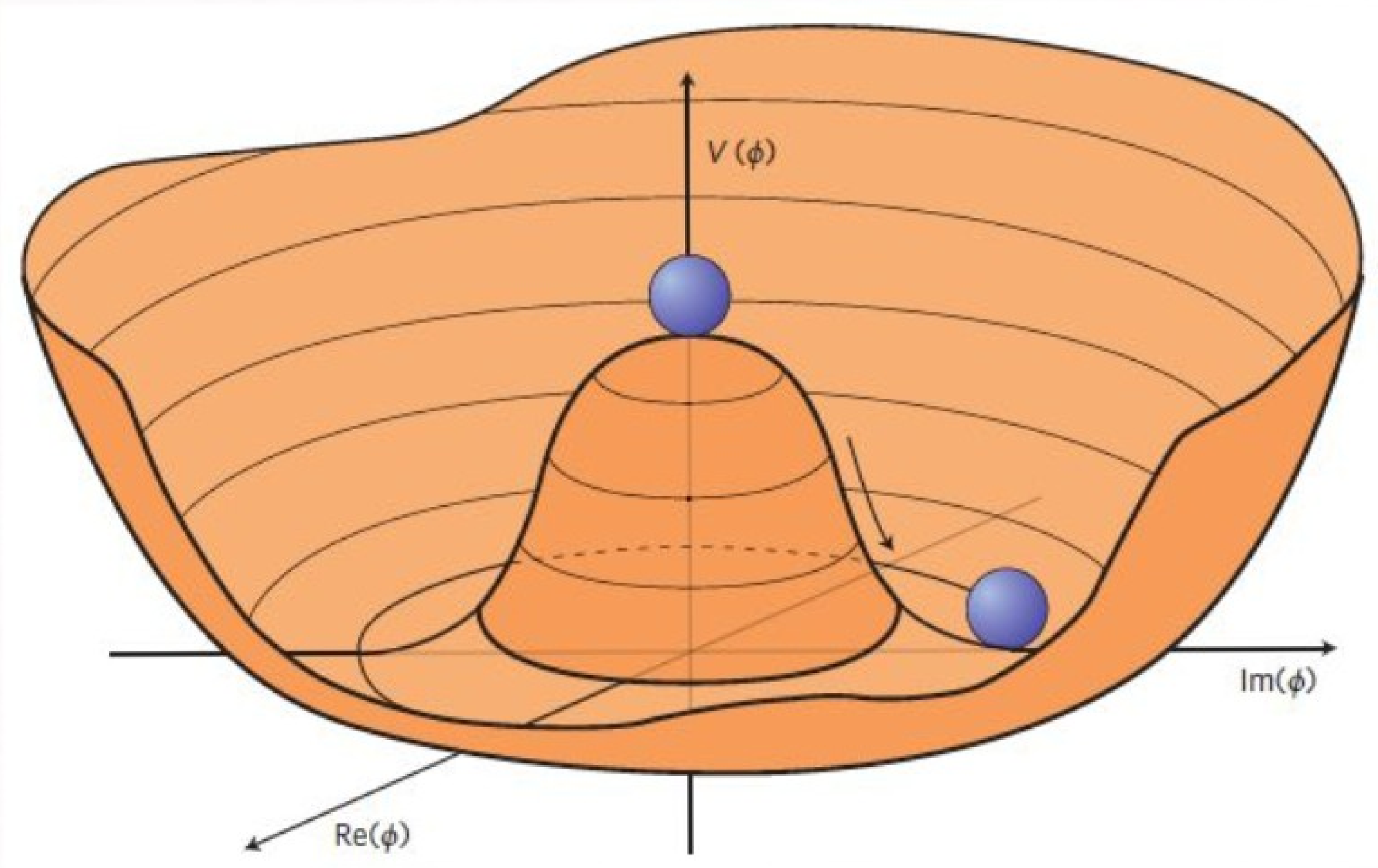
$$\Rightarrow (\mu^2 + \lambda\phi^2)\phi = 0$$

$$\Rightarrow \phi_{min}^2 = -\frac{\mu^2}{\lambda} \, , \quad \phi_{min} = \pm\sqrt{-\frac{\mu^2}{\lambda}}$$

$$= \pm \underline{\underline{v}}$$

[HOW TO INTERPRET THIS?] (2)

=> Graphically, we have : → classically, the ball (the field) will fall to the lowest energy state



=> This example shows a complex ϕ-field

$$V(\phi,\phi^*) = \frac{1}{2}\mu^2|\phi|^2 + \frac{\lambda}{4}|\phi|^4 \quad , \quad \phi_{min} = \sqrt{\frac{-\mu^2}{\lambda}}\, e^{i\theta_0}$$

[HOW TO INTERPRET THIS?] (3)

a) Observation → All QFT we know how to solve at the end reduce to a bunch of harmonic oscillators

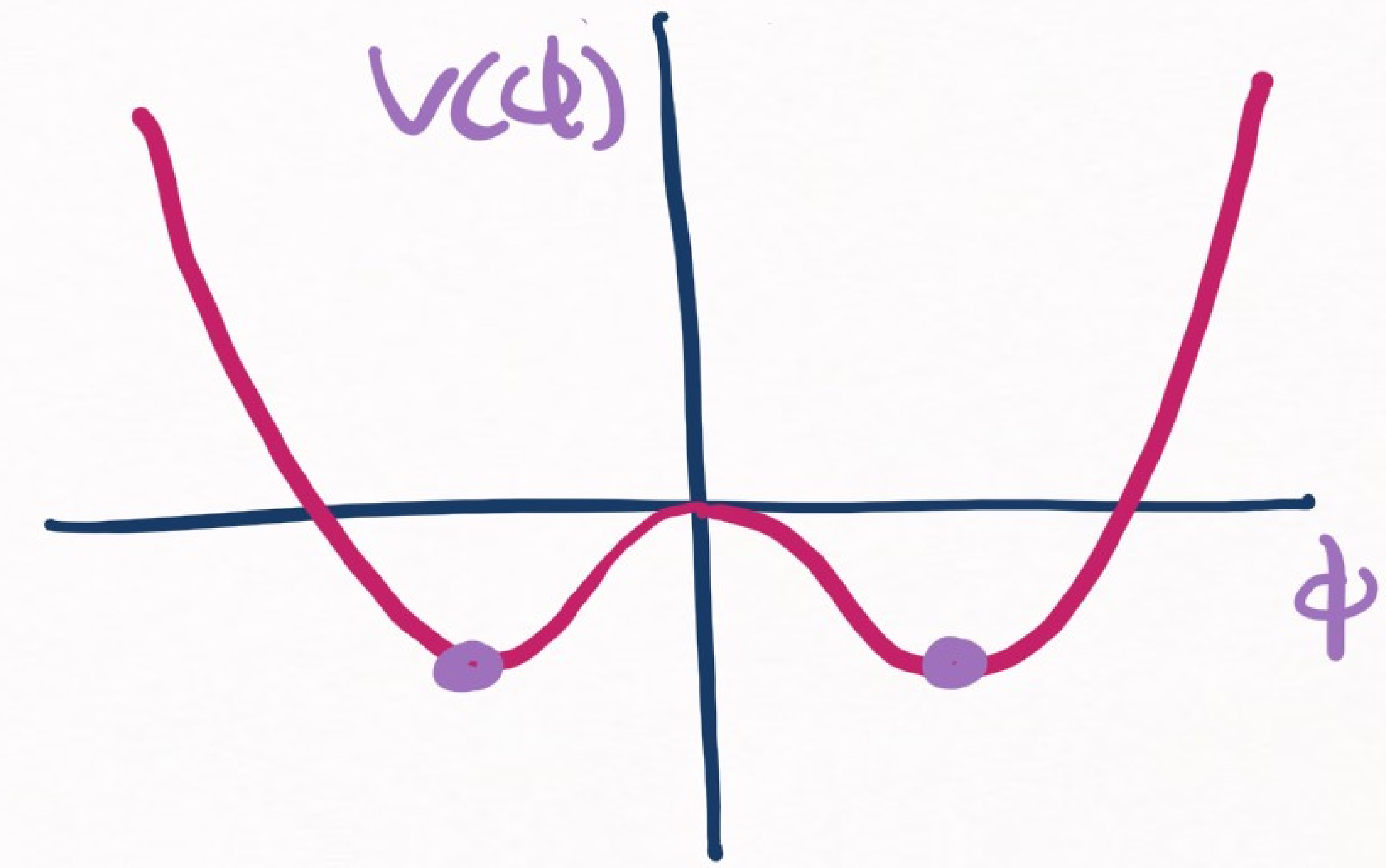
$$[\phi(x), \dot{\phi}(y)] = i\delta^{(3)}(\vec{x}-\vec{y})$$ (why? → canonical quantization)

$$\phi(x) = \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{1}{\sqrt{2\omega}} [a(\vec{p})\, e^{i\vec{p}\cdot\vec{x}} + a^{\dagger}(\vec{p})\, e^{-i\vec{p}\cdot\vec{x}}]$$

$$[a(\vec{p}), a^{\dagger}(\vec{p}\,')] = (2\pi)^3 \delta^{(3)}(\vec{p}-\vec{p}\,')$$

↳ totally analogous to the creation/annihilation operators for the harmonic oscillator

⇒ This QFT observation means we know how to quantize the minima of $V(\phi)$



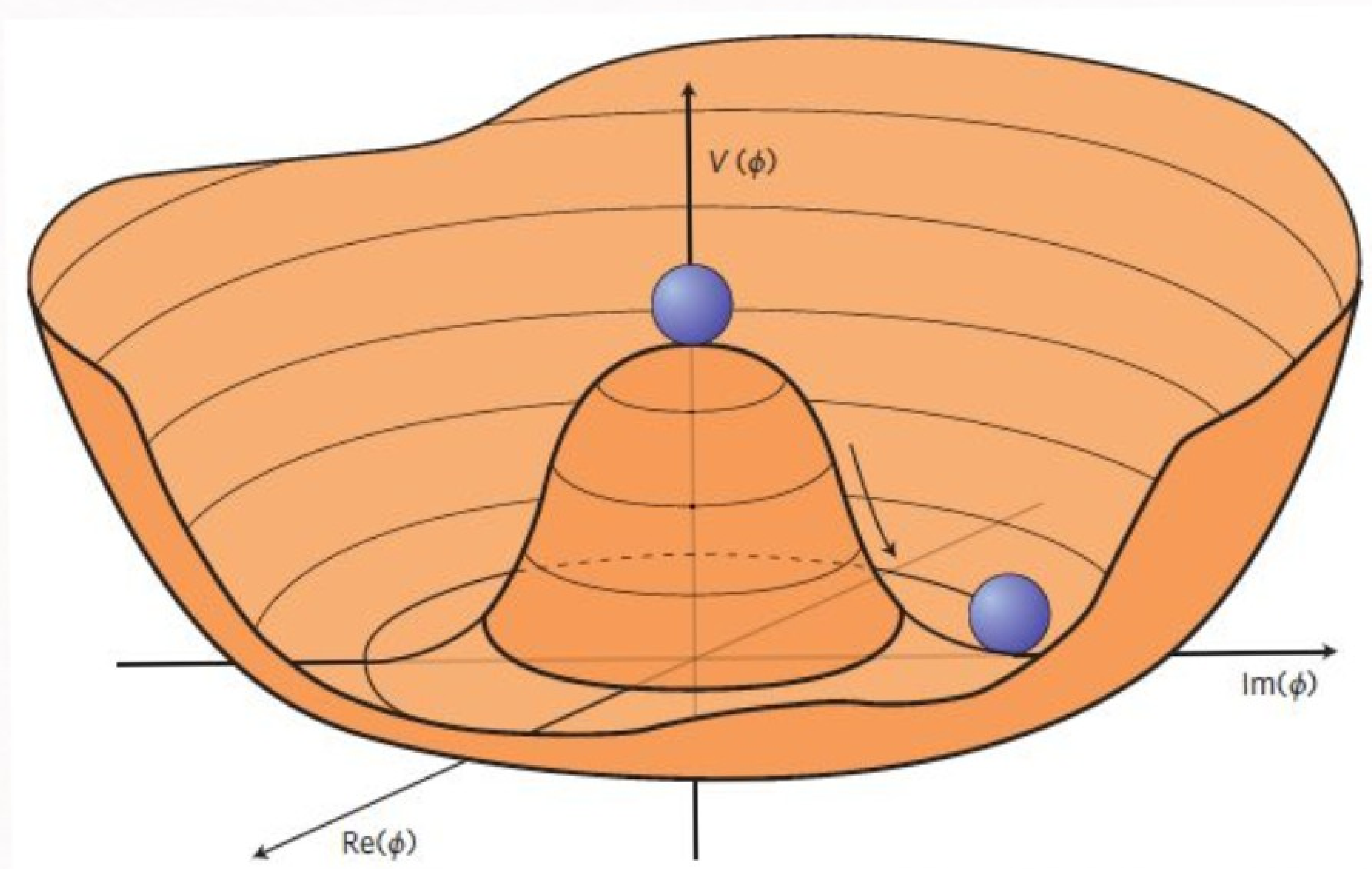
Why?. ⇒

$$V(\phi_{min} + \delta\phi) = V(\phi_{min}) + \delta\phi \cancel{\frac{dV}{d\phi}\Big|_{\phi_{min}}} + \frac{1}{2}(\delta\phi)^2 \frac{d^2V}{d\phi^2}\Big|_{\phi_{min}} + \dots$$

0 (because the minimum)

⇒ Around $\phi = \phi_{min}$, $V(\phi)$ will look like a quadratic potential (like a h.o.)

⟹ Even more clear in the complex ϕ case:



→ classically, the movement of the ball around the minimum is a harmonic oscillator (but only in the direction away from the origin)

second direction: just free movement

[IDEAL FOR QUANTIZATION] ⇐

[THE MEXICAN HAT POTENTIAL]

a) We do a change of variable → this is our trick for quantizing

a.1) One dimensional case:

$$\phi = \phi_{min} + \underline{\underline{\sigma}} = \pm\sqrt{-\frac{\mu^2}{\lambda}} + \underline{\underline{\sigma}}$$

$$\Rightarrow V(\sigma) = -\frac{\mu^4}{4\lambda} \boxed{-\mu^2\sigma^2} \pm \lambda\sqrt{-\frac{\mu^2}{\lambda}}\,\sigma^3 + \frac{\lambda}{4}\sigma^4$$

for simplicity

a.2) Complex field ϕ case : $\phi_{min} = \sqrt{-\frac{\mu^2}{\lambda}}\, e^{i\theta_0} + \theta_0 = 0$

$$\Rightarrow \phi = \phi_{min}\, e^{i\frac{\pi}{\phi_{min}}} + \sigma \simeq \phi_{min} + i\pi + \sigma$$

$$\Rightarrow V(\sigma,\pi) = -\frac{\mu^4}{4\lambda} \boxed{-\mu^2\sigma^2} + \lambda\sqrt{-\frac{\mu^2}{\lambda}}\,\sigma(\sigma^2+\pi^2) + \frac{\lambda}{4}(\sigma^2+\pi^2)^2$$

[THE MEXICAN HAT POTENTIAL]

b) Now we add the kinetic term:

b.1) Real ϕ $\Rightarrow$ $\mathcal{L} = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - V(\phi)$

$= \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \underline{\underline{\mu^2\sigma^2}} + \left(\begin{matrix}\text{interaction}\\ \text{terms}\end{matrix}\right)$

Reminder $\Rightarrow$ $\mathcal{L}_{KG} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2$

(Klein-Gordon Lagrangian)

$\Downarrow$

$m^2(\sigma) = -2\mu^2 + \mu^2 < 0$

We end up w/ a massive field (with positive mass)

$\Rightarrow$ $\boxed{m^2(\sigma) > 0}$

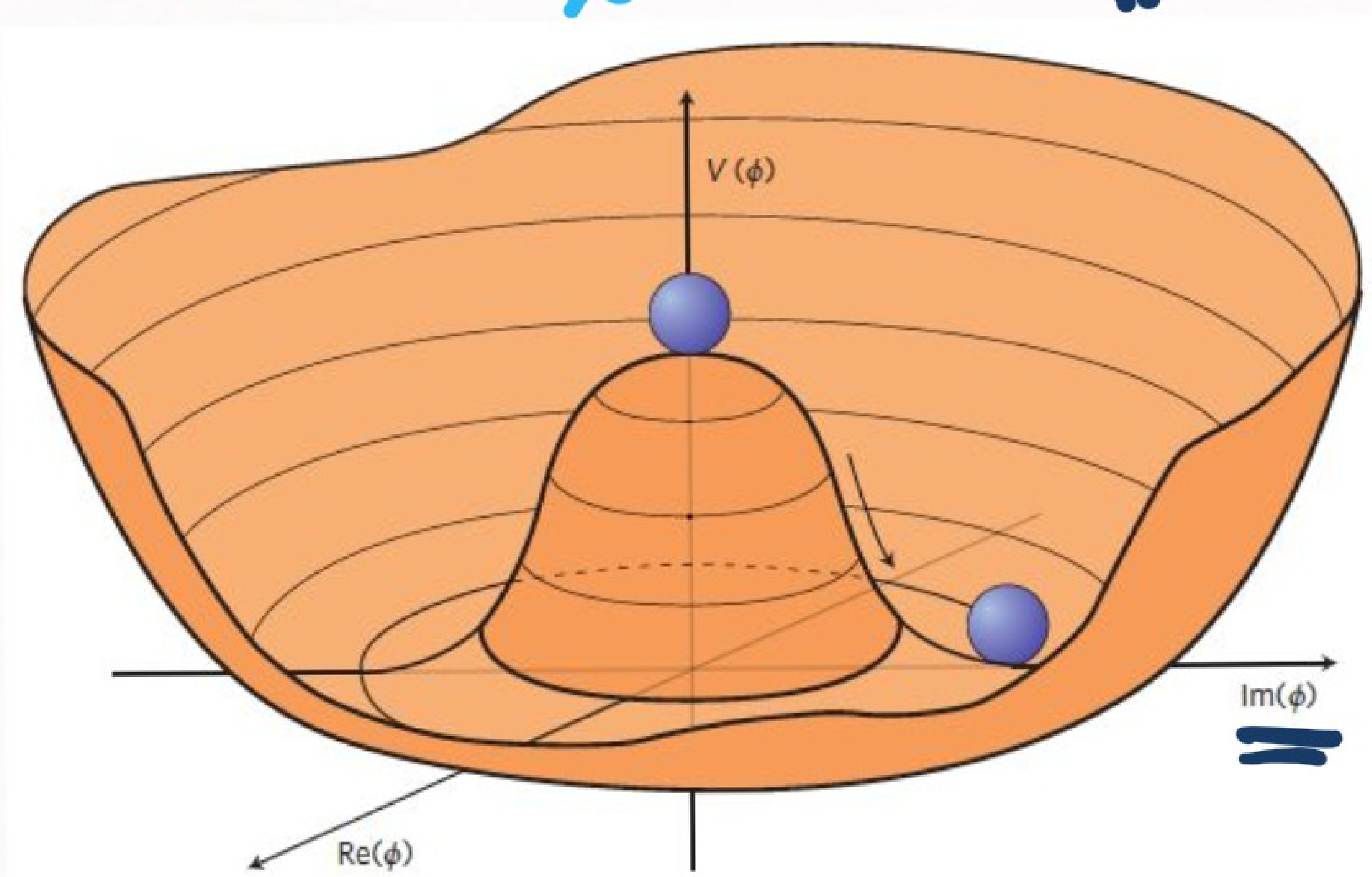
b. 2) Complex ϕ : $\phi = \phi_1 + i\phi_2$ $\quad \mathcal{L}_{kin} = \frac{1}{2}(\partial_\mu \phi_1 \partial^\mu \phi_1 + \partial_\mu \phi_2 \partial^\mu \phi_2)$

$$= \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2}\partial_\mu \pi \partial^\mu \pi$$

$$\mathcal{L} = \mathcal{L}_{kin} + V(\sigma, \pi)$$

$$= \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma - \mu^2 \sigma^2 + \frac{1}{2}\partial_\mu \pi \partial^\mu \pi + \boxed{0 \times \pi^2} + \left(\begin{array}{c}\text{interaction}\\ \text{terms}\end{array}\right)$$

$$\phi_{min} = \sqrt{-\frac{\mu^2}{\lambda}}\, e^{i\pi}$$



σ $\Rightarrow$ just as before (massive, positive mass)

π $\Rightarrow$ a massless field

$\Rightarrow$ a) movement in one direction is harmonic $\rightarrow$ $\boxed{\sigma}$

b) movement in the other direction is free $\rightarrow$ $\boxed{\pi}$

LINEAR SIGMA MODEL

$\to$ complicated version of previous examples

a) We began with: $\underline{\mathcal{L}} = i\bar{N}\cancel{\partial}N + g\bar{N}(\phi_0 + i\gamma^5 \vec{\tau}\cdot\vec{\phi})N + \frac{1}{2}\sum_{i=0}^{3}\partial_\mu\phi_i\partial^\mu\phi_i - V(\phi)$

b) Again, minimum at $\phi_0 = \sqrt{-\frac{\mu^2}{\lambda}}$ $\Rightarrow$ Change of variables

$$\underline{\phi_0 = v + \sigma}\ ,\quad \underline{\vec{\phi} = \vec{\pi}}\qquad \left(v = \sqrt{-\frac{\mu^2}{\lambda}}\right)$$

$$\Rightarrow \mathcal{L} = i\bar{N}(\cancel{\partial} - \underline{gv})N + g\bar{N}(\sigma + i\gamma^5\vec{\tau}\cdot\vec{\pi})N + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \underline{\lambda v^2}\sigma^2 + \frac{1}{2}\partial_\mu\vec{\pi}\partial^\mu\vec{\pi} + \left(\text{interaction terms}\right) + \underline{0\times\vec{\pi}^2}$$

[LINEAR SIGMA MODEL] → Meaning of the terms

$$\Rightarrow \mathcal{L} = \underbrace{\bar{N}(i\partial\!\!\!/ - gv)N}_{1} + \underbrace{g\bar{N}(\sigma + i\gamma^5 \vec{\tau}\cdot\vec{\pi})N}_{2}$$

$$+ \underbrace{\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \lambda v^2\sigma^2}_{3} + \underbrace{\frac{1}{2}\partial_\mu\vec{\pi}\partial^\mu\vec{\pi}}_{4} + \dots$$

1 → nucleons are now massive ($m_N = gv$)

2 → pion-nucleon interaction ($g_{\pi NN} = g_{\sigma NN} = g$)

3 → massive scalar field σ ($m_\sigma = 2\lambda v^2$)

4 → pions are massless (hooray!!)

[A FEW COMMENTS] 1

a) This explains why the pion is so light

→ possible to add a small mass to the pion by adding this term: $\Delta V = -\epsilon v^3 \phi_0$

b) Prediction for $g_{\sigma NN}$ and $g_{\pi NN}$: $(v = f_\pi,\ g v = M_N)$

$$g_{\pi NN} = \underbrace{(g_A)}_{} \frac{M_N}{f_\pi} \quad , \quad g_{\sigma NN} = \frac{M_N}{f_\pi}$$

only difference is $g_A \simeq 1.26$ → 26% discrepancy wrt LSM ($g_A = 1$)

[A FEW COMMENTS] ②

c) Predicts the existence of a scalar meson

→ for many years the σ was not found experimentally (only happened much later → $f_0(500)$ in the PDG)

→ Gell-Mann & Levi proposed an alternative model called non-linear σ Model

RECAP & OUTLOOK

a) Pion is unnaturally light ⟹ Requires an explanation

b) There is a trick : mexican hat potential

c) LσM uses this trick to generate a light pion

d) Next lesson ⟹ this is just a particular case of a much more general result (the Nambu-Goldstone theorem)

Our server

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The lecture notes of this year are under development (ac bit every year), and you should come here from time to t date of the last update):

- General introduction
- The two-nucleon system
- Nuclear Structure

Additional materials: SU(3) Clebsch-Gordan coefficients.

In case you need them