NUCLEAR PHYSICS 9

RENORMALIZATION
& EFFECTIVE FIELD THEORIES
(Part II)

Low energy expansions
& Power countings

RECAP | In the previous lesson we learnt:

a) QCD is not solvable at low energies (at least using analytical methods)

→ Nuclear physics can't be explained by solving QCD

b) But there are ways around this...

$\boxed{\text{Baryons \& Mesons}} \longleftrightarrow \boxed{\text{Quarks \& Gluons}}$

[RENORMALIZATION]

[WHAT IS RENORMALIZATION?]

Basic idea: [Physics at long distances should not depend on the short distance details]

CONCEPT

(Easier to understand)

RENORMALIZATION IS THE MATHEMATICAL REALIZATION OF THIS CONCEPT

[LET'S CONSIDER A FEW EXAMPLES]

$$\frac{1}{R+h} - \frac{1}{R} = -\left(\frac{h}{R}\right)\frac{1}{R} + \left(\frac{h}{R}\right)^2\frac{1}{R} - \dots$$

→ Galilean & Newtonian gravity:

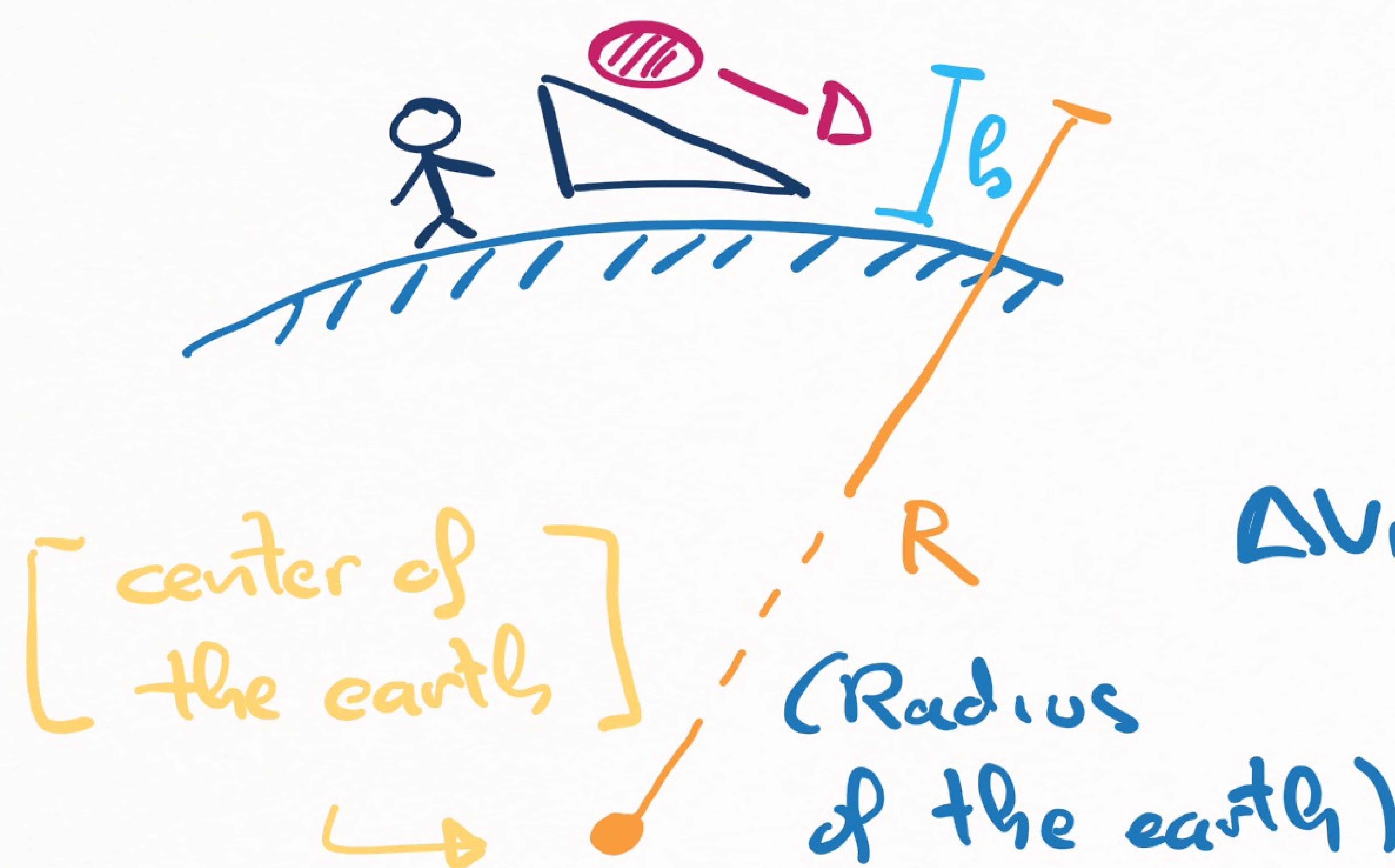
$$\Delta V_G = mgh$$ (high school physics)

$$\Uparrow$$

$$V_N = -\frac{GMm}{r}$$

$$\Delta V_N = -GMm\left[\frac{1}{R+h} - \frac{1}{R}\right]$$

$$= mgh\left[1 + O\left(\frac{h}{R}\right)\right]$$



[IMPORTANT DETAIL:]

$$\Delta V_{\text{Effective}} = \underbrace{mgh}_{\text{"Galilean" gravity}} \underbrace{\left[1 - \left(\frac{h}{R}\right) + \left(\frac{h}{R}\right)^2 - \left(\frac{h}{R}\right)^3 + \cdots\right]}_{\text{Corrections}}$$

Corrections → ⊛

⊛ → This "effective" description is organized as a power series

POWER COUNTING: the rules defining this power series

(above our power counting is $\frac{h}{R} \ll 1$)

[REMEMBER THE CONCEPTUAL DEFINITION OF RENORMALIZATION?]

[Physics at long distances are independent of the short distance details]

Imagine that instead of Newton's gravity we had something <u>different</u>.

$$V_{eff} = - \frac{G' M m}{r^n}$$

What will we see if we do experiments on the surface of earth?

[IT DOESN'T MATTER WHICH IS THE UNDERLYING THEORY OF GRAVITY]

$$V_k = -\frac{GMm}{r^k} \Rightarrow \Delta V_{\text{effective}} = mgh\left[1 - k\left(\frac{h}{R}\right) + \frac{1}{2}k(k+1)\left(\frac{h}{R}\right)^2 - \frac{1}{6}k(k+1)(k+2)\left(\frac{h}{R}\right)^3 + \dots\right]$$

a) Same limit for $\frac{h}{R} \to 0$: $[\Delta V = mgh]$

b) Same power counting: [we count powers of $\frac{h}{R}$]

c) Only difference is the coefficients in front of the powers of $\frac{h}{R}$

[WE CAN BE MORE GENERAL]

Underlying theory $\longrightarrow$ $V = -G \underline{\underline{Mm}} f(R)$ $\begin{cases} f(R) > 0 \\ f'(R) < 0 \end{cases}$

$\Downarrow$

Effective theory $\longrightarrow$ $\Delta V = \underbrace{mgh}_{\text{Galilean gravity}} \Big[1 + \underbrace{c_1 \left(\frac{h}{R}\right) + c_2 \left(\frac{h}{R}\right)^2 + c_3 \left(\frac{h}{R}\right)^3 + \dots}_{\text{corrections}} \Big]$

$$g = -GMf'(R)$$

$$c_1 = \frac{1}{2!} R f''(R) \frac{1}{f'(R)}$$

$\boxed{c_1, c_2, c_3, \dots} \longrightarrow$ low energy constants of the theory

$$c_2 = \frac{1}{3!} R^2 \frac{f'''(R)}{f'(R)} \qquad c_n = \frac{1}{(n+1)!} R^n \frac{f^{(n+1)}(R)}{f'(R)}$$

OBSERVATION

Underlying theory #1

Underlying theory #2

Uderlying theory #3

Effective theory A

TECHNICAL TERM FOR THIS:

[fixed point of the renormalization group]

Example:

Different theories of gravity all lead to same approximation in the surface of earth

A SECOND EXAMPLE

General Relativity $\rightarrow$ Classical Newtonian gravity

$\rightarrow$ Expansion in $\frac{v}{c}$ (this defines our power counting)

$\rightarrow$ $\exists$ could be different relativistic theories of gravity all leading to Newtonian gravity

(Nordstrom gravity, Brans-Dicke theory, etc)

a) But QCD is different... (previous examples: analytically solvable)

(QCD) [Quarks & Gluons] $\longrightarrow$ [Baryons & Mesons] $\neq$

We do not know how to solve QCD in the first place $\rightarrow$ this problem can be overcome

b) Then let's consider an example with similar characteristics

A second example where we can't easily solve the fundamental theory

[THE THEORY OF TEACUPS & TEAPOTS]

(not a joke)

teapot

teacup

Our objective:

Short-distance description (that can't be solve)

⇓

Long-distance description (simple & solvable)

WHICH ONE COOLS FASTER?

THE PROBLEM

[Where does tea cools faster ?]

1 the teapot ?

2 the teacup ?

1

But ...

how do you know the answer without solving any equation?

2

If your answer was 2 : CONGRATULATIONS! You were right

BUT WHY?

1 TEAPOT :

a) Ceramic surface → large

b) Surface exposed to air → small

2 TEACUP :

a) Ceramic surface → smaller

b) Exposed surface → bigger

HEAT TRANSFER

"surface exposed to air"

Intuitively, convection is faster than conduction

"ceramic surface"

$\boxed{\text{ERGO}}$ (Therefore in Latin)

The cup cools faster than the teapot

(we didn't solve any equation)

[SHORT-DISTANCE DESCRIPTION
(UNDERLYING THEORY)] → (Equivalent of QCD for teacups & teapots)

a) Fourier law of heat conduction

$$q = -k \nabla T$$

b) Convection-diffusion equation

$$\frac{\partial c}{\partial t} = \nabla \cdot (D \nabla c) - \nabla (vc) + R$$

It looks really difficult to solve these equations
(partial differential equations) ≤ QCD

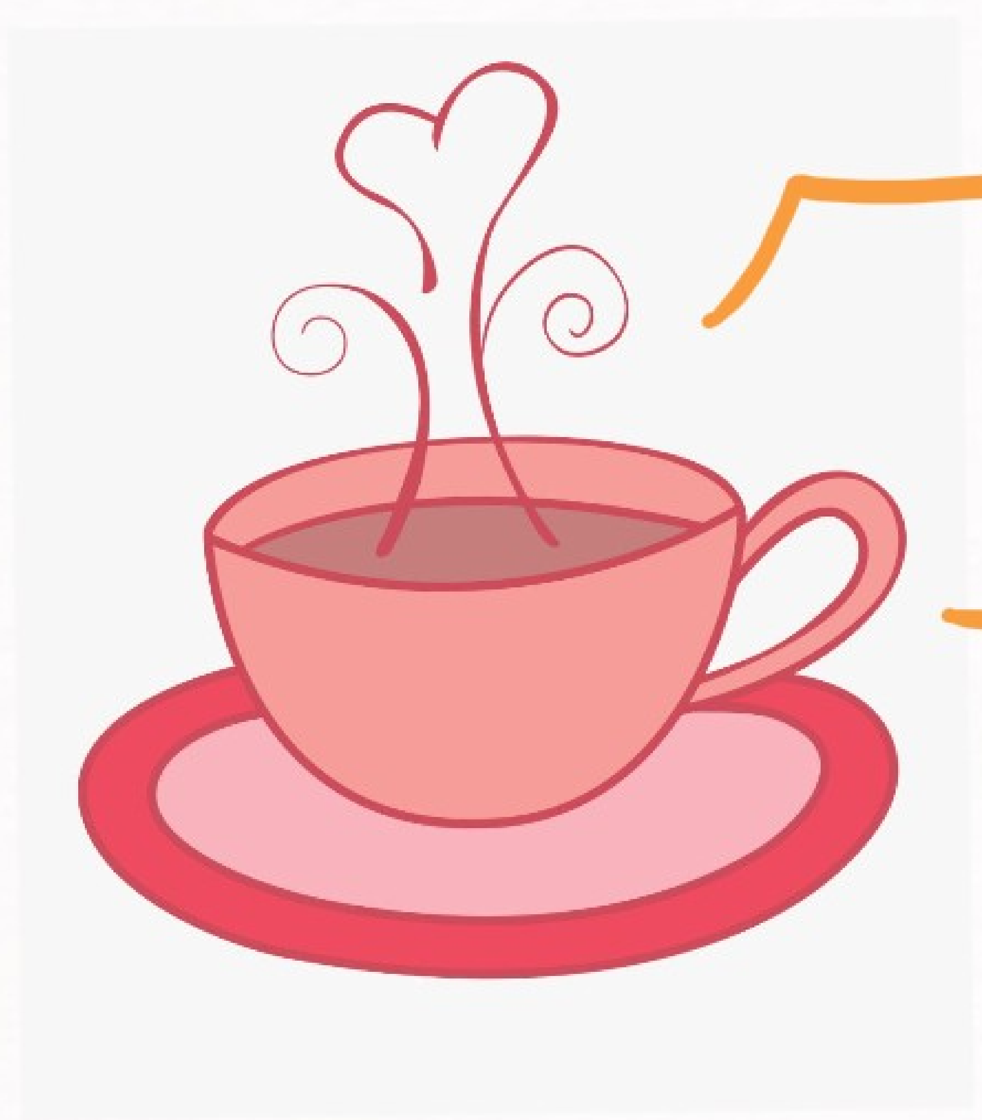
But I'm lazy ... $\longrightarrow$ $\boxed{\text{EFFECTIVE THEORY}}$

1) Newton's law of cooling : $\leftarrow$ include some dynamics

$$T = T_0\, e^{-\lambda(t-t_0)} \quad \begin{cases} T, t \to \text{temperature \& time} \\ T_0, t_0 \to \text{initial temp \& time} \end{cases}$$

$\lambda \to$ constant

2) Find the relevant degrees of freedom :



S : surface exposed to air

Σ : surface made of a ceramic material (or other insulator)

3) Propose a power counting: → what is our expansion parameter?

S is more important than Σ

$$x = \frac{\rho \Sigma}{S} \rightarrow \rho < 1$$ (correction factor to take into account that ceramic conducts heat worse)

4) Write down the theory: [x < 1 in cup S]

$$T = T_0 e^{-\lambda(t - t_0)}$$ → We want to calculate $\underline{\underline{\lambda}}$

Expansion: $\lambda = S\left[\underline{\underline{c_0}} + \underline{\underline{c_1}}x + \underline{\underline{c_2}}x^2 + \dots\right]$

$c_0, c_1, c_2, \dots$ → Low energy constants (LECs) of the theory

Power series → Infinite terms → Infinite parameters

↓

We have to put a limit in the number of parameters

5) Choose the accuracy we want & truncate the series

Accuracy $\mathcal{O}(x)$ $(x<1)$: $\lambda = S \times [c_0 + \mathcal{O}(x)]$ → Leading order (LO)

Accuracy $\mathcal{O}(x^2)$: $\lambda = S[c_0 + c_1 x + \mathcal{O}(x^2)]$

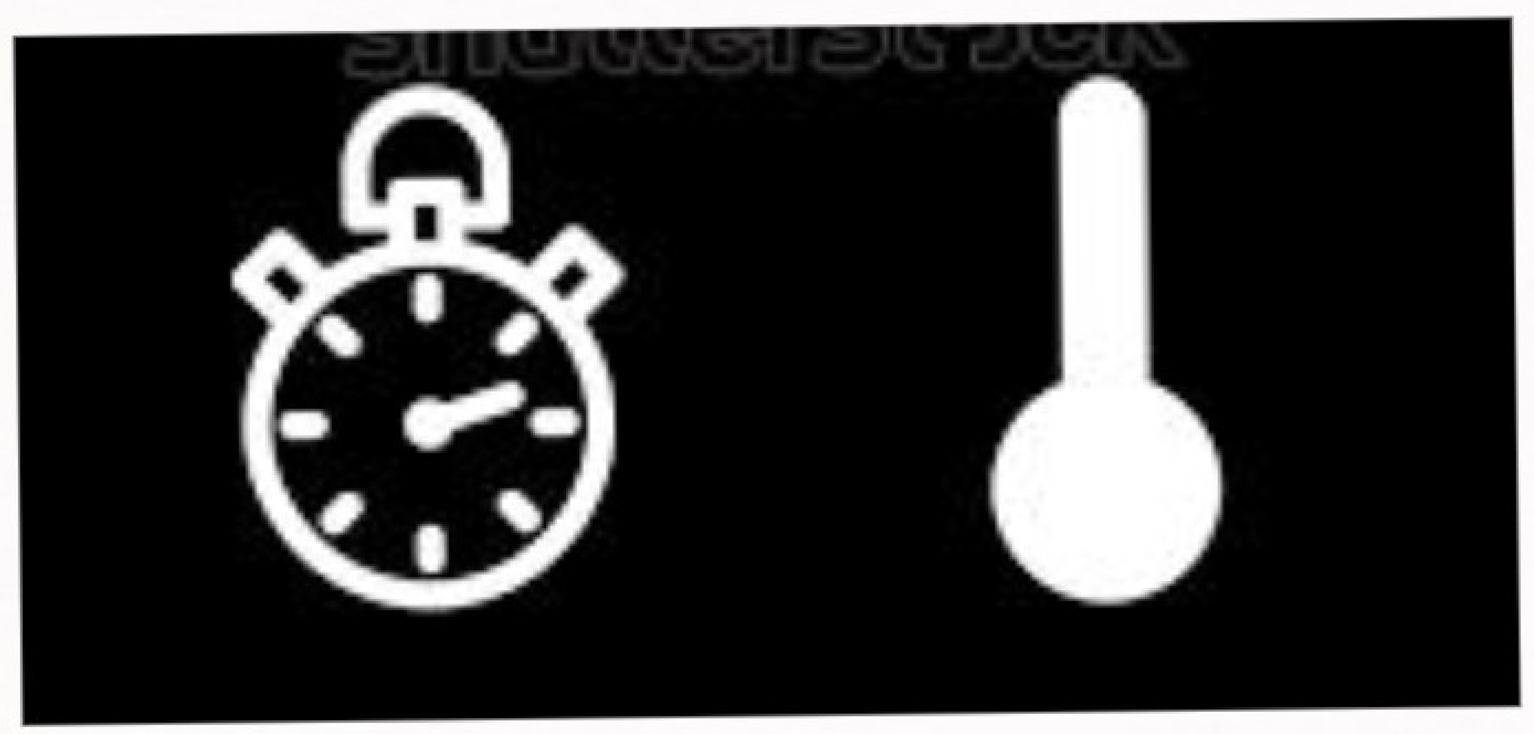
Next-to-leading order (NLO)

Accuracy $\mathcal{O}(x^3)$: $\lambda = S[c_0 + c_1 x + c_2 x^2 + \mathcal{O}(x^3)]$

Next-to-next-to-leading order (N^2LO)

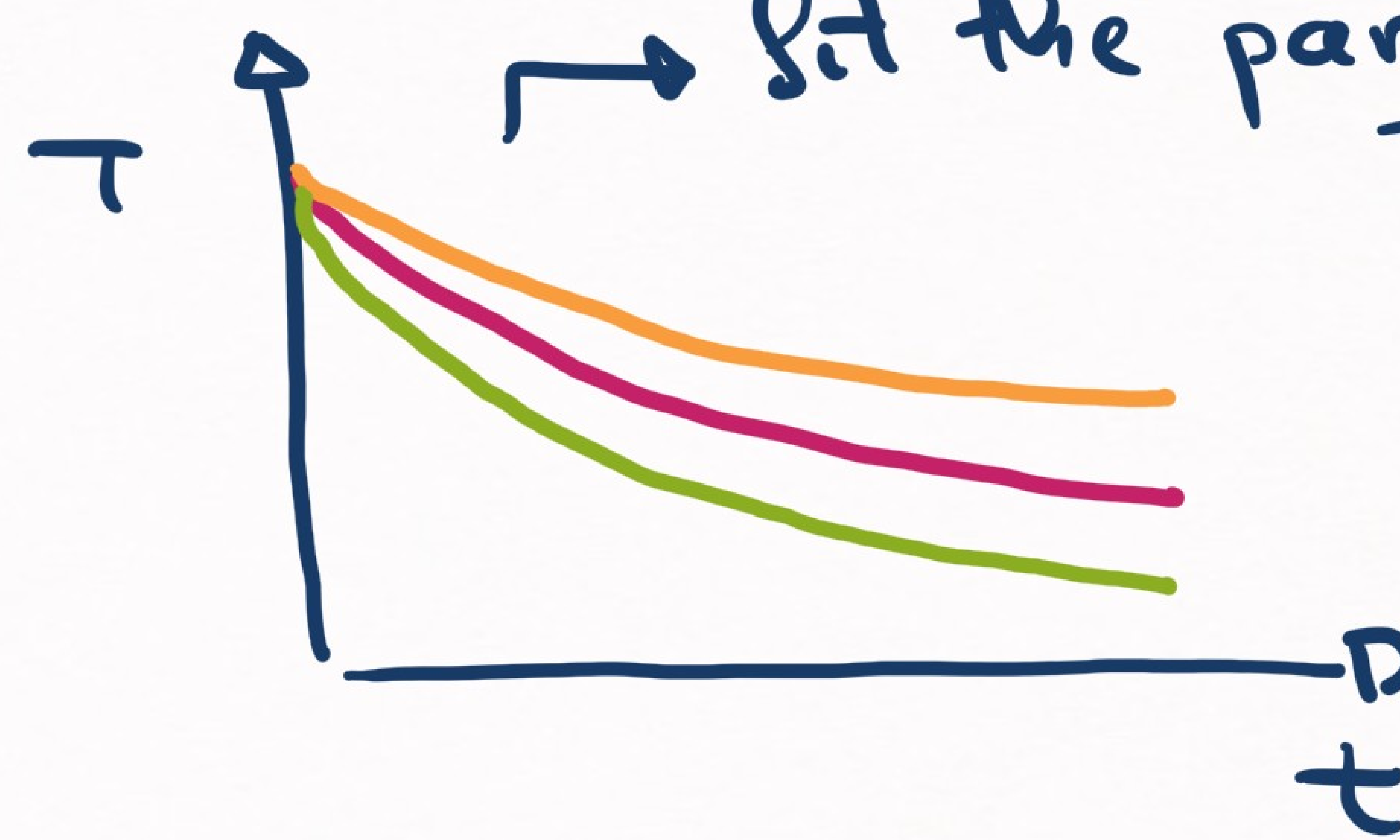
[More terms → More accuracy]

6) Fit the LECs to experimental data



→ Our instruments

S_1, Σ_1 S_2, Σ_2 S_3, Σ_3



→ fit the parameters

RECAP | How to build an effective theory:

1) Some dynamics → cooling's law / QM / QFT / classical mechanics / etc.
2) Choose the degrees of freedom
   → surface type / types of particles / potential energy / etc.
3) Power counting → small parameter
4) Write down the theory → LECs
5) Choose the accuracy → LO, NLO, N^2LO, ...
6) Fit to experimental data → get the value of the LECs

[BUT THERE IS MORE...] $\Longrightarrow$ many types of power counting

$\leftarrow$ **TEAPOT THEORY**

Expansion parameter : $x = \frac{\rho \Sigma}{S}$, $\rho < 1$

$\Sigma \gg S$, $\rho < 1$

$\Downarrow$

$x = \frac{\rho \Sigma}{S} > 1$ $\Rightarrow$ This can't be our small expansion parameter

$x > 1$, WHAT SHOULD WE DO?

POWER COUNTINGS

→ We can have several PCs

a) $x = \frac{f\Sigma}{S} < 1$, $\lambda = S(c_0 + c_1 x + c_2 x^2 + \dots)$

b) $x = \frac{f\Sigma}{S} > 1$,

$$\lambda = \Sigma\left(d_0 + \frac{d_1}{x} + \frac{d_2}{x^2} + \dots\right)$$

(use $\frac{1}{x}$ as the expansion parameter)

[TAKE-HOME MESSAGE] $\Rightarrow$ [POWER COUNTING IS NOT UNIQUE]

Underlying theory #1

Effective theory A

Effective theory B

Infrared fixed points

or

Universality classes

In renormalization group (RG) language we call these:

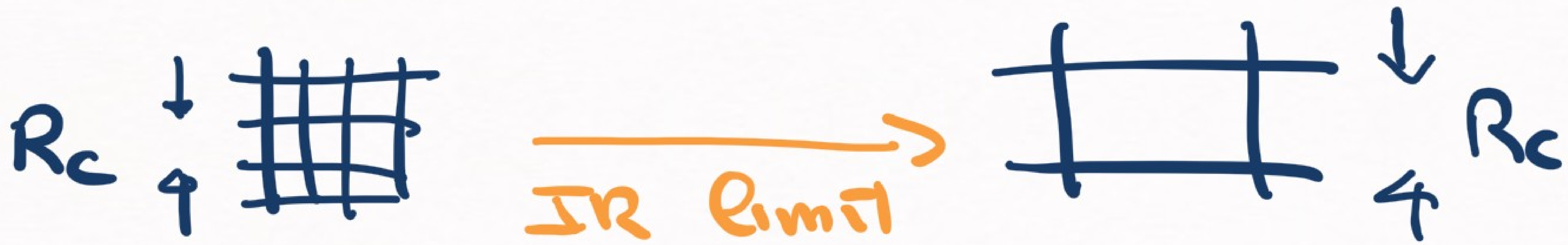
Underlying theory #1

Effective theory A

Effective theory B $\rightarrow$ IR fixed points / Universality classes

[NOTE ABOUT RG-LANGUAGE]

a) Infrared : very low energies / long distances



$R_c \to \infty$
$(\Lambda \to 0)$

b) Ultraviolet : very high energies / short distances

$R_c \to 0$
$(\Lambda \to \infty)$

[EFFECTIVE THEORIES IN QM]

a) We already know the basic ideas

→ Galilean gravity, cooling of teacups & teapots

→ (several fundamental theories) → (same effective theory)

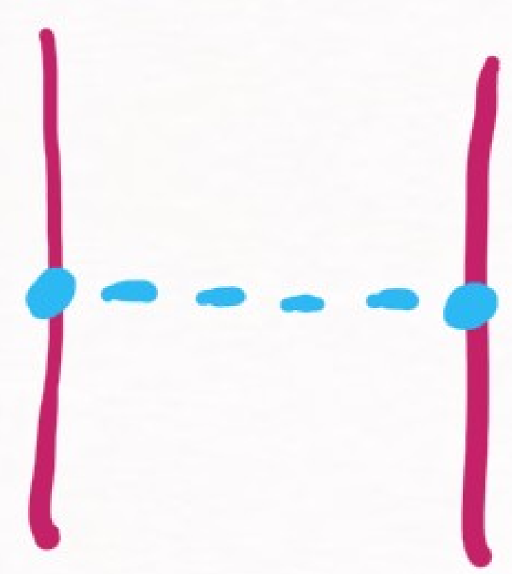
→ (unique fundamental theory) → (several effective theories)

b) We will try now an example from QM

→ two-body QM problem

TWO-BODY BOUND STATE

a) Consider a Yukawa potential



$$V(\vec{q}) = -\frac{g_Y^2}{\vec{q}^{\,2} + m^2} \;, \quad V(\vec{r}) = -\frac{g_Y^2}{4\pi}\frac{e^{-mr}}{r}$$

a.1) two-particles of mass M $(M_1 = M_2 = M)$

a.2) they will exchange a scalar $\rightarrow$ V = Yukawa

a.3) described by $\left[-\frac{\nabla^2}{M} + V(r)\right]\psi(\vec{r}) = E\psi(\vec{r})$

1) Schrödinger equation : $\left[-\frac{\vec{\nabla}^2}{M} - \frac{g_\gamma^2}{4\pi}\frac{e^{-mr}}{r}\right]\psi(\vec{r}) = E\psi(\vec{r})$

2) When does this system have bound states?

$E < 0$, $E = -\frac{\gamma^2}{M}$

$\longrightarrow \quad \lambda = \frac{M}{m}\frac{g_\gamma^2}{4\pi}$, $\lambda_c = 1.68$

(we define this quantity)

$M = 1\,\text{GeV}$

$m = 0.5\,\text{GeV}$

a) $\lambda < \lambda_c \longrightarrow$ no binding

b) $\lambda = \lambda_c \longrightarrow$ zero energy bound state

c) $\lambda > \lambda_c \longrightarrow$ binding

REDUCED UNITS

$$E_B = -\frac{\gamma^2}{M} \rightarrow \gamma : \text{wave number}$$

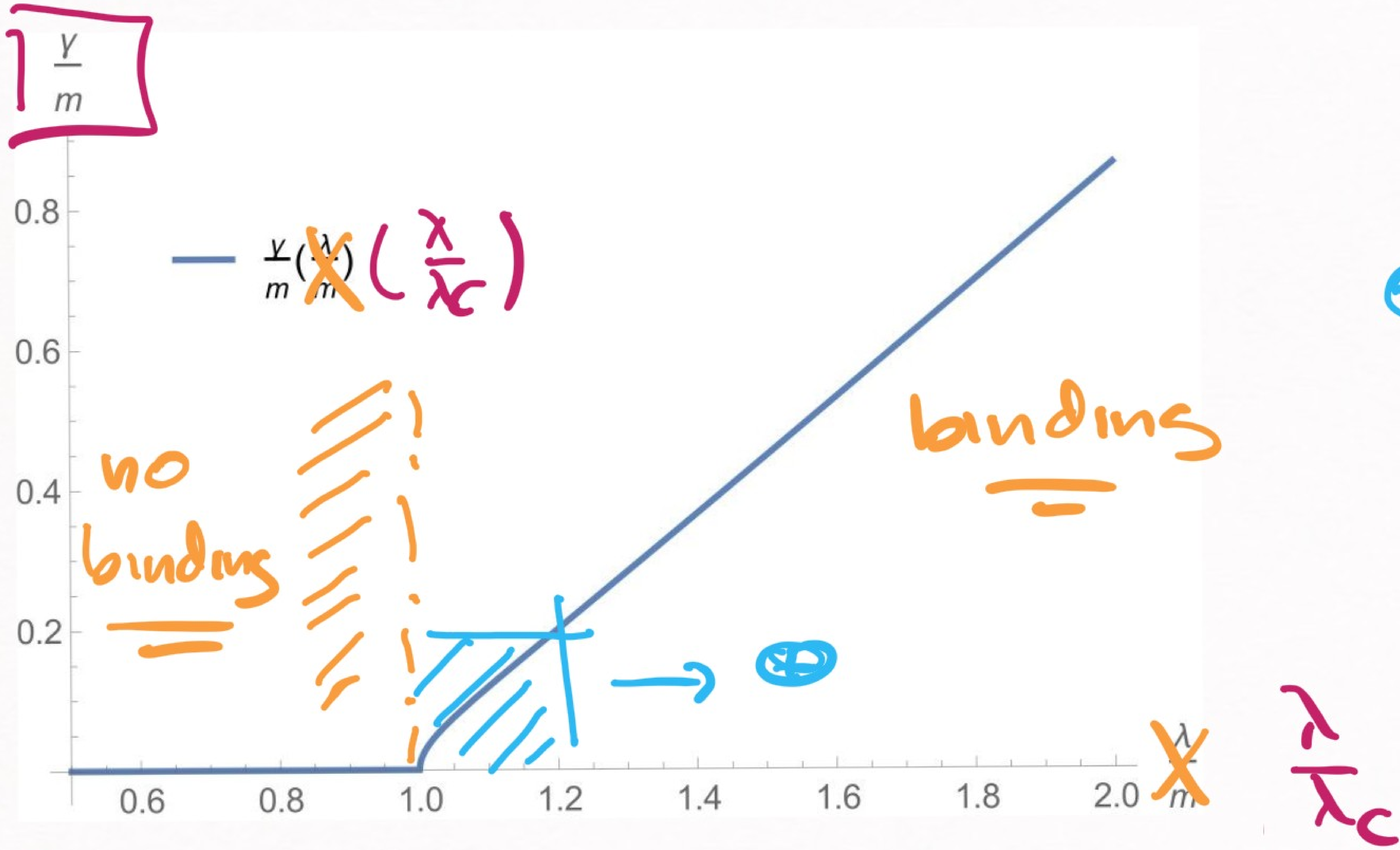
→ We will try to write everything in units of m and λ_c (actually dimensionless)

$$\hat{\gamma} = \frac{\gamma}{m}, \quad \hat{\lambda} = \frac{\lambda}{\lambda_c} \quad (\hat{\lambda} > 1 \Rightarrow \text{binding})$$

(the idea is to better identify scale separation)

$\hat{\gamma} = \hat{\gamma}(\hat{\lambda})$ or $\frac{\gamma}{m} = \frac{\gamma}{m}\left(\frac{\lambda}{\lambda_c}\right)$:

REMEMBER NATURALNESS



⊗ → bound states are unnatural here ($\gamma \ll m$)

↑ Solve Schrödinger and obtain the values of γ

[SOMETIMES WE HAVE AN UNNATURAL SYSTEM]

For $\frac{\lambda}{\lambda_c} \to 1$ $\Rightarrow$ Spatial extent of the wave function (size of the w.p.) is going to be much larger than the range of the potential

$\boxed{\text{WHY?}}$

$$\psi(\vec{r}) \xrightarrow[mr \gg 1]{} \frac{A_s}{\sqrt{4\pi}} \frac{e^{-\gamma r}}{r}$$

($A_s \to$ number,
$\gamma \to$ wave number)

Explanation $\rightarrow$ $\left[-\frac{\nabla^2}{\mu} - \frac{g_Y^2}{4\pi}\frac{e^{-mr}}{r}\right]\psi(\vec{r}) = -\frac{\gamma^2}{\mu}\psi(\vec{r})$

$\Downarrow$ $\underline{\underline{mr \gg 1}} \Rightarrow e^{-mr} \ll 1$ $(\to 0)$

$$-\frac{\nabla^2}{\mu}\psi(\vec{r}) = -\frac{\gamma^2}{\mu}\psi(\vec{r})$$

$\underbrace{\psi(\vec{r}) \propto \frac{1}{r}e^{-\gamma r}}$ $\Rightarrow$ $\vec{\nabla}^2\psi(\vec{r}) = -\gamma^2\psi(\vec{r})$

$\hookrightarrow$ for $mr \gg 1$, this is a solution of the system

[A FEW EXAMPLES:]

a) $\lambda = \underline{1.05\,\lambda_c}$, $\gamma \simeq \underline{0.075\, m}$

$$\boxed{\psi(r) = \frac{1}{\sqrt{4\pi}}\frac{u(r)}{r}}$$

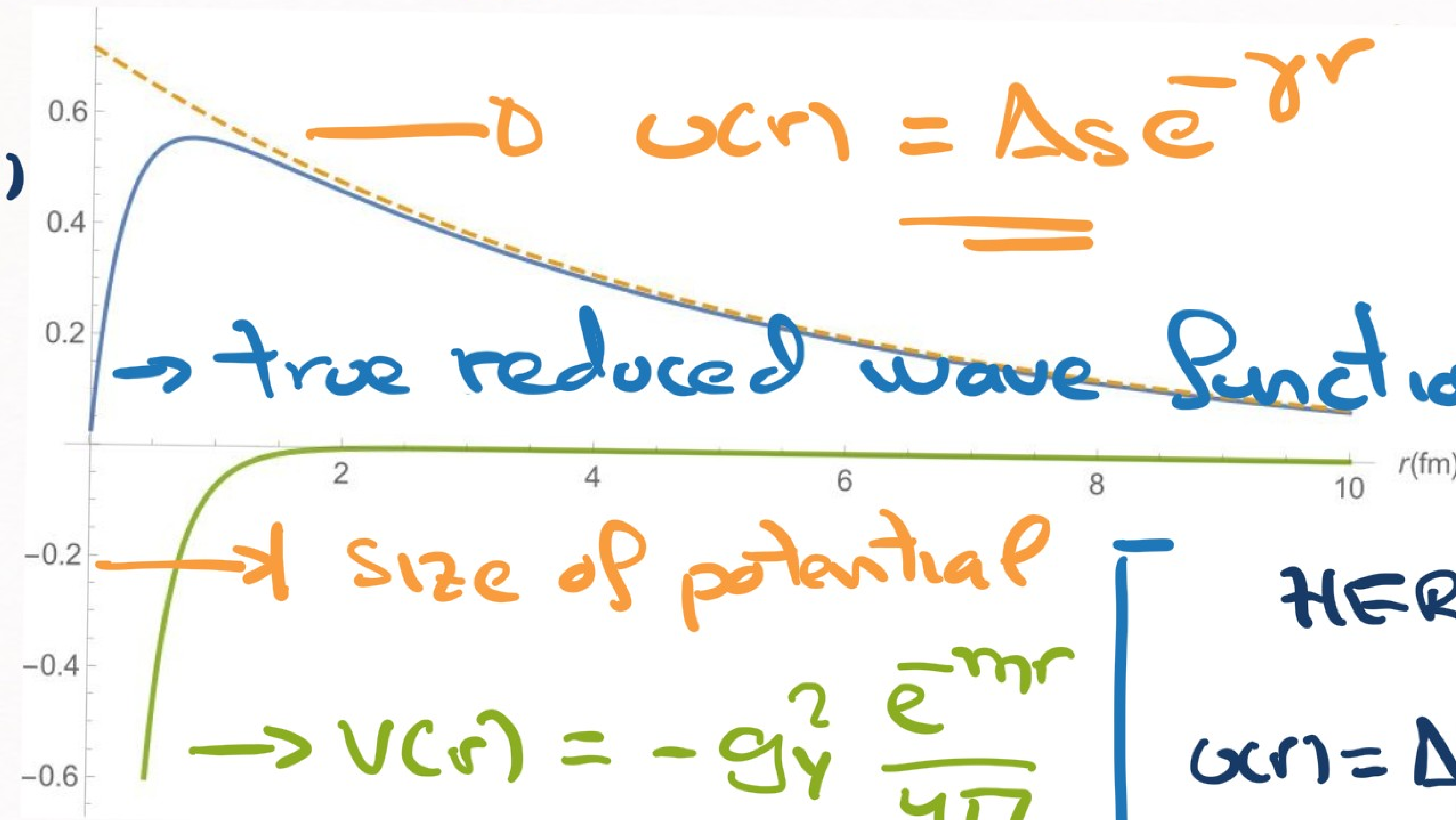
$\downarrow$

$u(r)$

$\downarrow$

reduced wave function (defined because it's really convenient)

$\underline{u(r)}$



$\longrightarrow$ $\underline{u(r) = A_s e^{-\gamma r}}$

$\rightarrow$ true reduced wave function

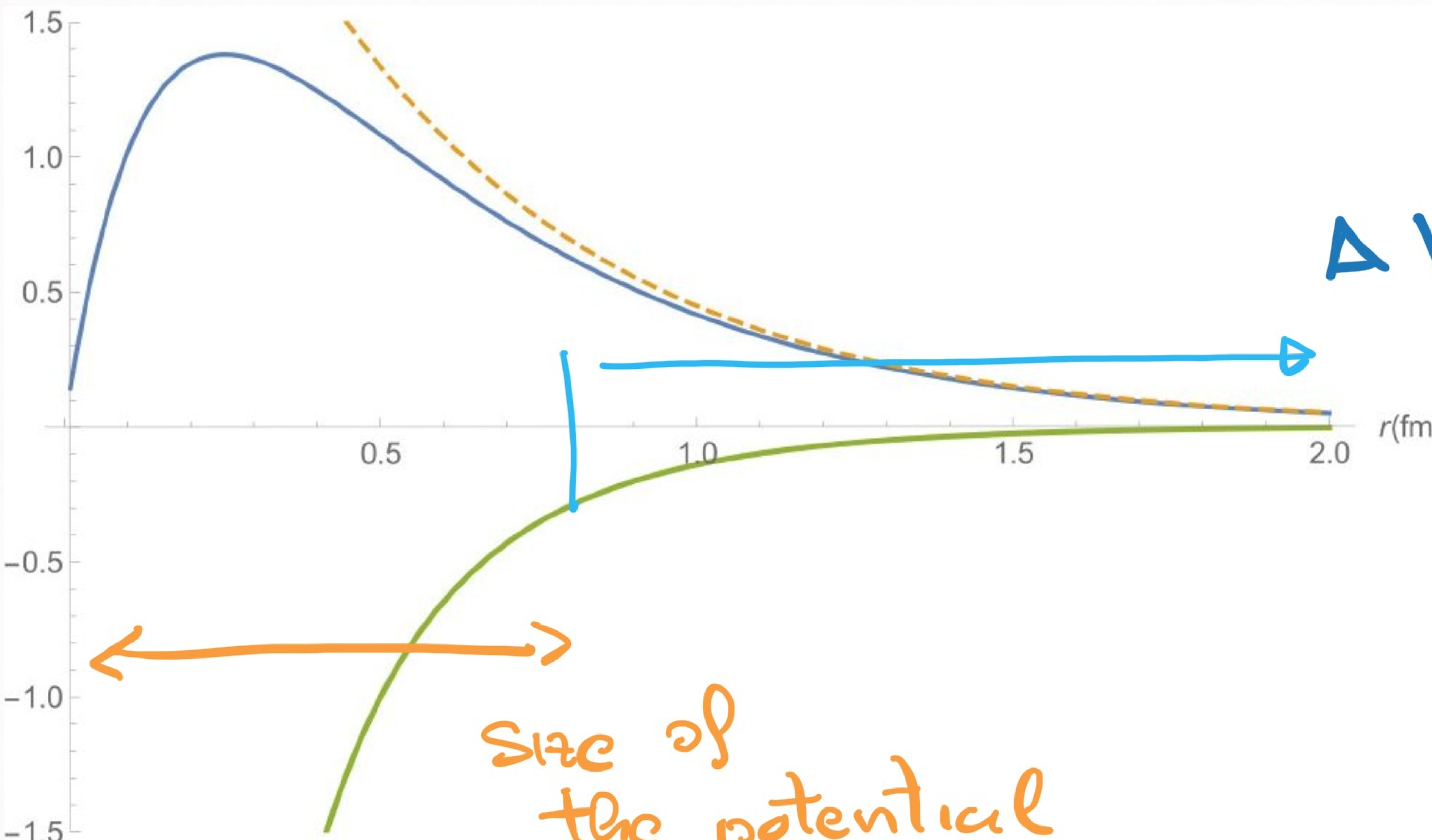
$\rightarrow$ size of potential

$\rightarrow V(r) = -g_Y^2 \frac{e^{-mr}}{4\pi}$

(arbitrary unit)

[HERE: $u(r) = A_s e^{-\gamma r}$ GOOD APPROX.]

b) $\lambda = 2\lambda_c$, $\gamma \simeq 0.86\,m$) $\rightarrow$ this solution is more bound



Size of the potential

A big chunk of the w.f. lives outside the range of the potential

[WHAT DOES THIS MEAN?]

a) First case → Unnatural (check previous lessons & naturalness)

b) Second case → Natural

a) Wave function will be insensitive to the form of the potential $V(r)$ [a bit like how $\Delta V = mgh$ was independent of our theory of gravity]

b) Wave function will be sensitive to the form of the potential $V(r)$

THAT IS...

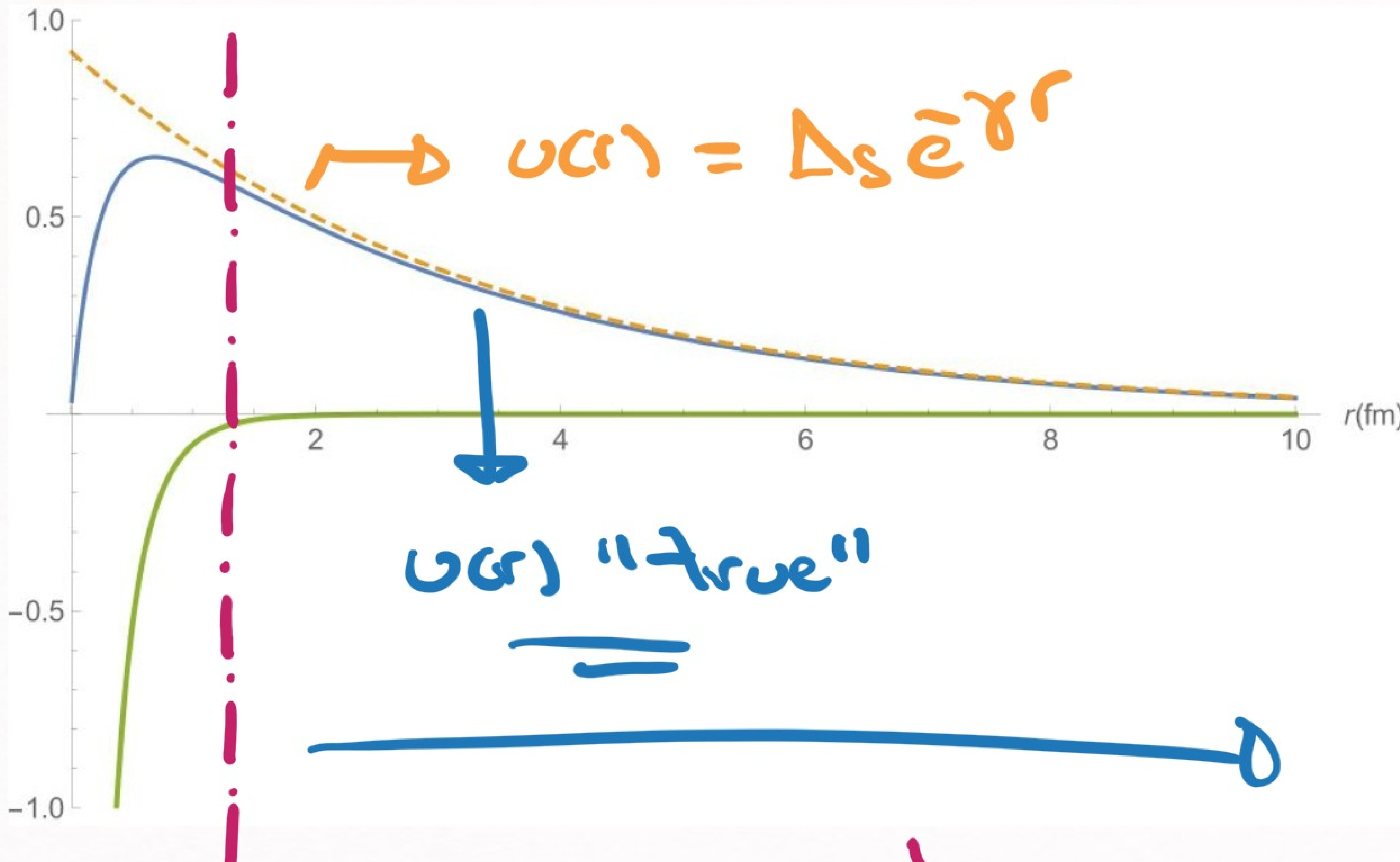
$\Rightarrow$ For $\frac{\lambda}{\lambda_c} \to 1$, it will be possible to build an effective description of this system

$(\gamma \ll m)$

[NOW WE SHOW HOW THIS IS DONE]

IF WE TAKE A SECOND LOOK AT a)

[$\chi = 1.05\,\lambda_c$, $\gamma \sim 0.075\,m$]

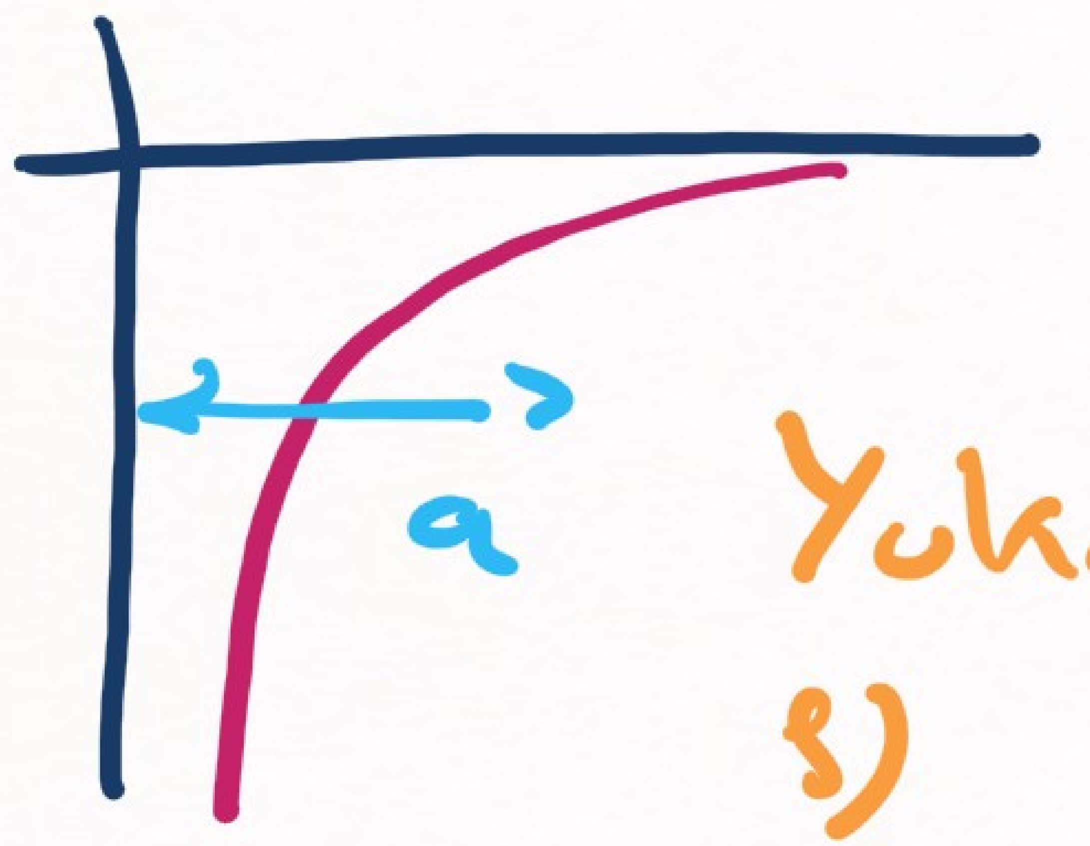


$R_c \sim (1-2)\,\text{fm}$

For $r > R_c$ there is almost no difference between $u(r) = A_s e^{-\gamma r}$ (approx. for $mr \gg 1$) and the "true" wave function

$\Rightarrow$ Most of the wave function is outside R_c

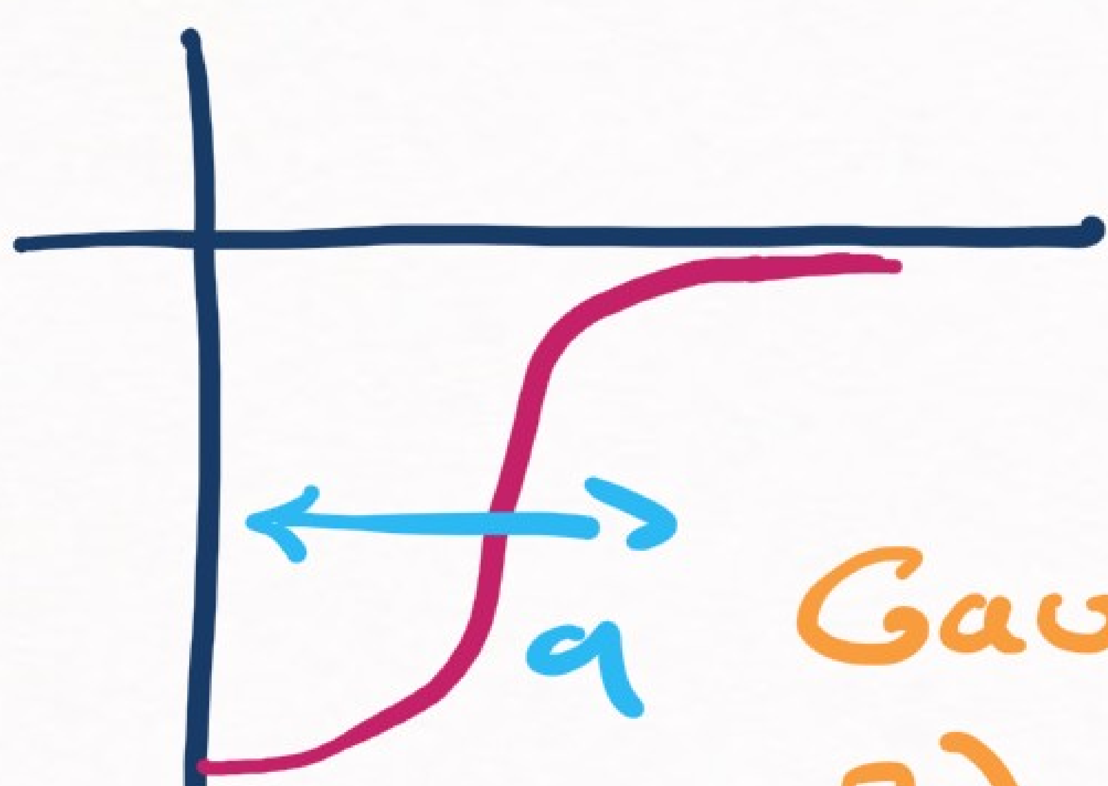
This means that: (Connatural system)



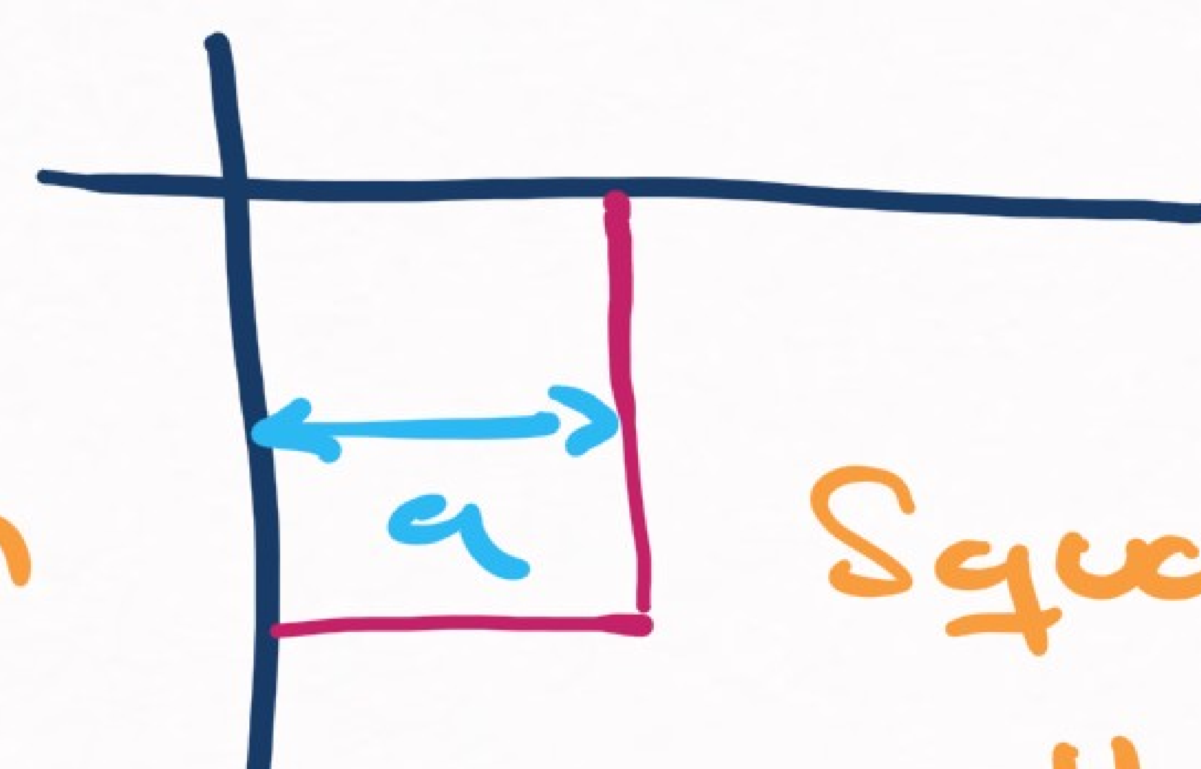
Yukawa
1)

if a) is the case ($\gamma \ll m$, $\frac{1}{\gamma} \gg a$)
it doesn't matter which potential we have

$\Rightarrow$



Gaussian
2)



Square well
3)

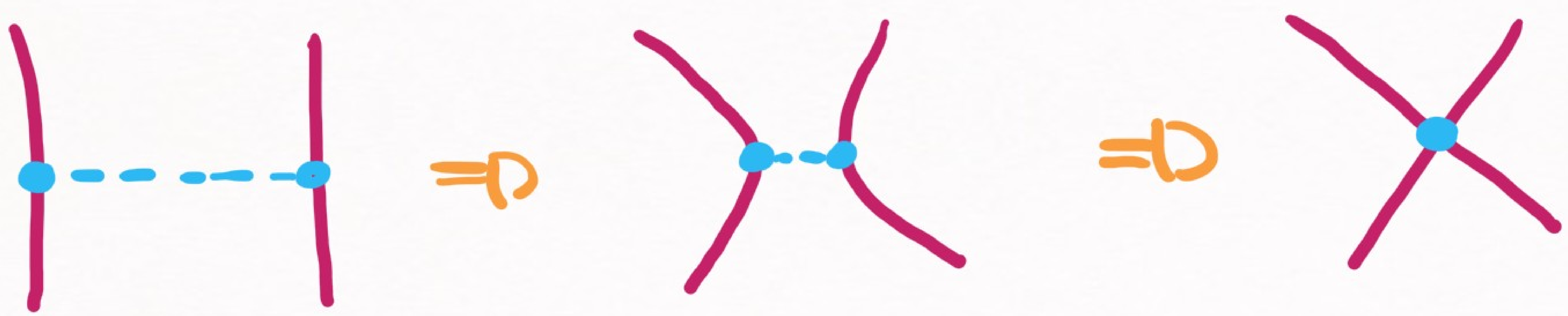
1), 2), 3) $\rightarrow$ All of them will give me

$$u(r) \simeq A_s e^{-\gamma r}$$

for $\frac{r}{a} \gg 1$

$\left(\frac{1}{\gamma} \gg a\right)$

IF $\frac{\ell}{r} \gg a$, WE DON'T NEED TO KNOW THE FORM OF THE POTENTIAL

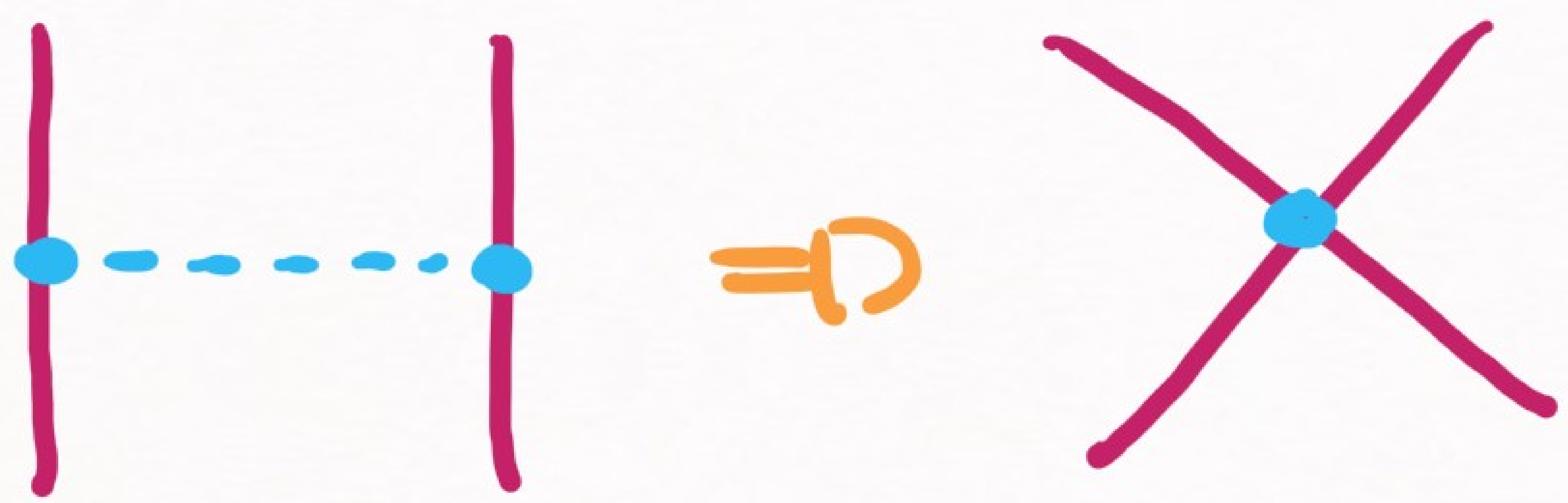


When I reduce the resolution, I won't be able to see which particle is being exchange

$$V(\vec{q}) = -\frac{g_Y^2}{\vec{q}^2+m^2} = -\frac{g_Y^2}{m^2}\left[1 - \frac{\vec{q}^2}{m^2} + \frac{\vec{q}^4}{m^4} - \frac{\vec{q}^6}{m^6} + \frac{\vec{q}^8}{m^8} + \cdots\right]$$

$$(|\vec{q}|^2 < m^2)$$

AT LONG DISTANCES, WE DON'T KNOW
IF TWO PARTICLES EXCHANGE A BOSON (OR NOT)



$$V(\vec{q}^{\,2}) = -\frac{g_Y^2}{\vec{q}^{\,2} + m^2} \Rightarrow V(\vec{q}^{\,2}) = \overbrace{\underline{C_0 + C_2\vec{q}^{\,2}} + C_4\vec{q}^{\,4}}^{\text{the universal limit at low momenta}} + \ldots$$

ANALOGOUS TO $\rightarrow$ $V(r) = -GMm\,f(r)$

$$\Rightarrow \Delta V = mgh\left[1 + c_1\left(\frac{h}{R}\right) + c_2\left(\frac{h}{R}\right)^2 + \ldots\right]$$

$\boxed{\text{POWER COUNTING}}$

$\Rightarrow$ $V_{\text{Effective}}(q) = C_0 + C_2\,\vec{q}^{\,2} + C_4\,\vec{q}^{\,4} + \dots$

For $|\vec{q}| \ll m$, we expect to have:

$$|C_0| > |C_2\,\vec{q}^{\,2}| > |C_4\,\vec{q}^{\,4}| > \dots$$

$\Rightarrow$ At LO this simplifies to:

$$\boxed{V_{LO}(q) = C_0} \rightarrow \text{Universal low energy approx.}$$

[A PROBLEMATIC POTENTIAL] 1

$V_{LO}(\vec{q}) = C_0$ → r-space version? → Fourier transform the potential

$$V_{LO}(\vec{r}) = C_0 \int \frac{d^3\vec{q}}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} = C_0 \underbrace{\delta^{(3)}(\vec{r})}_{\text{Dirac-delta}}$$

Reminder

$$\delta^{(3)}(\vec{r}) = \begin{cases} \infty, & \vec{r}=0 \\ 0, & \vec{r}\neq 0 \end{cases}$$

$$+ \int d^3\vec{r}\, \delta^{(3)}(\vec{r}) = 1$$ (3-dimensional Dirac-delta)

[A PROBLEMATIC POTENTIAL] 2

Schrödinger equation: $\left[-\frac{\vec{\nabla}^2}{\mu} + C_0\,\delta^{(3)}(\vec{r})\right]\psi(\vec{r}) = -\frac{\gamma^2}{\mu}\psi(\vec{r})$

If $\vec{r} \neq 0 \Rightarrow -\frac{\vec{\nabla}^2}{\mu}\psi(\vec{r}) = -\frac{\gamma^2}{\mu}\psi(\vec{r}) \Rightarrow \psi(\vec{r}) = \frac{A_s}{\sqrt{4\pi}}\frac{e^{-\gamma r}}{r}$

↳ Dirac-delta recovers our universal w.f.

(compatible with our previous observations about the long-distance w.f.)

[A PROBLEMATIC POTENTIAL] 3

But there is something wrong with the Dirac-delta ...

→ The variational principle

$$\left.\begin{array}{l}|\psi_t\rangle \text{ a trial w.f.} \\ (\langle\psi_t|\psi_t\rangle = 1)\end{array}\right\} \Rightarrow \boxed{E_B \leq \langle\psi_t|H|\psi_t\rangle}$$

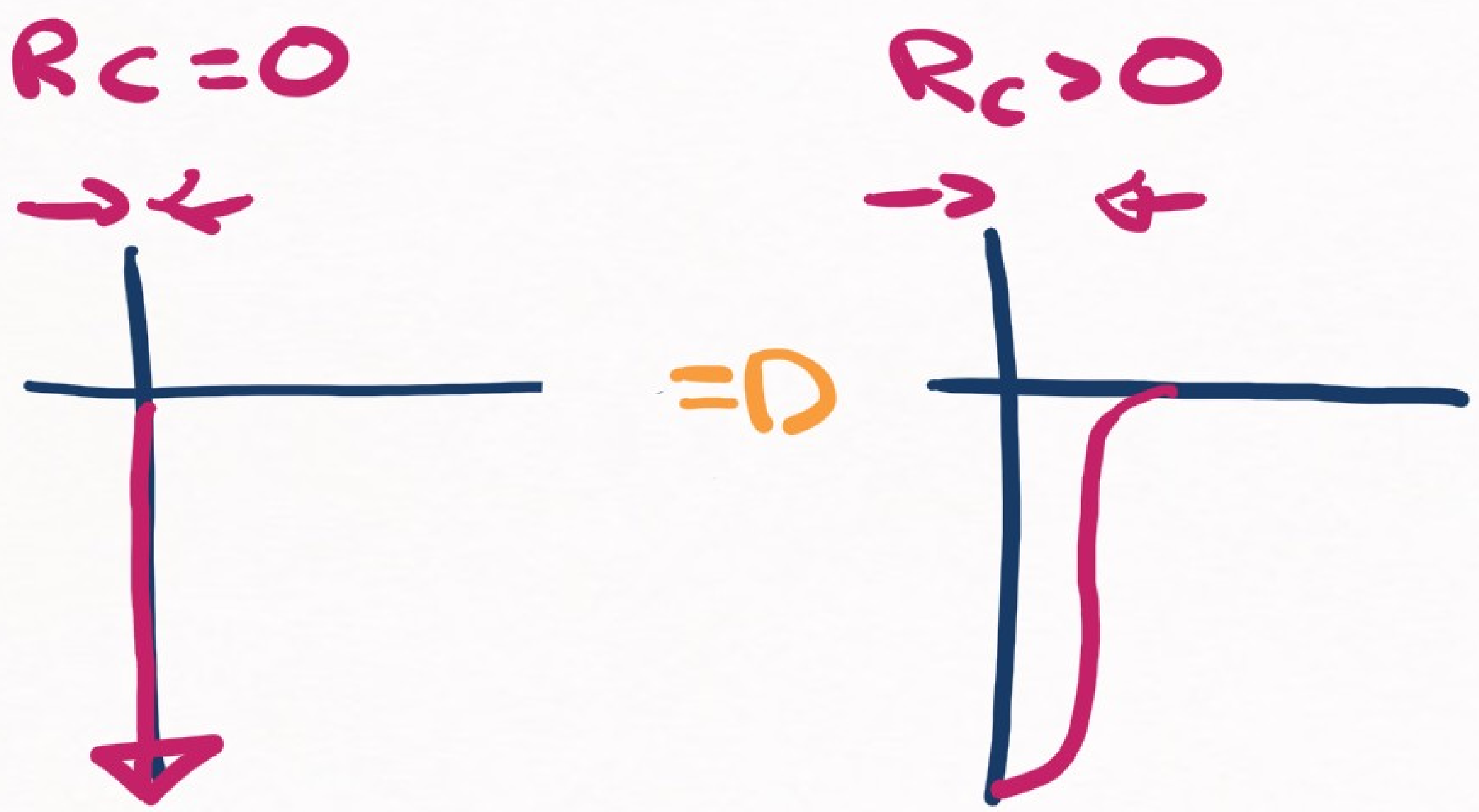
For $V(\vec{r}) = C_0\,\delta^{(3)}(\vec{r})$, $\langle\psi_t|H|\psi_t\rangle \to \underline{\underline{-\infty}}$

$$\Rightarrow \boxed{E_B < -\infty} \Rightarrow$$ Describes a two-body system that collapses

HOW TO SOLVE THIS ? $\longrightarrow$ NEW CONCEPT :

REGULARIZATION

$\Downarrow$



$$V_{LO}(\vec{q}) = C_0$$

$$\Rightarrow V_{LO}(\vec{q};\Lambda) = C_0 \, f\left(\frac{|\vec{q}|}{\Lambda}\right)$$

a) $f(x \to 0) \to 1$

b) $f(x \to \infty) \to 0$

Make this potential a bit more broad

$\Downarrow$

↳ We will "smear" the potential

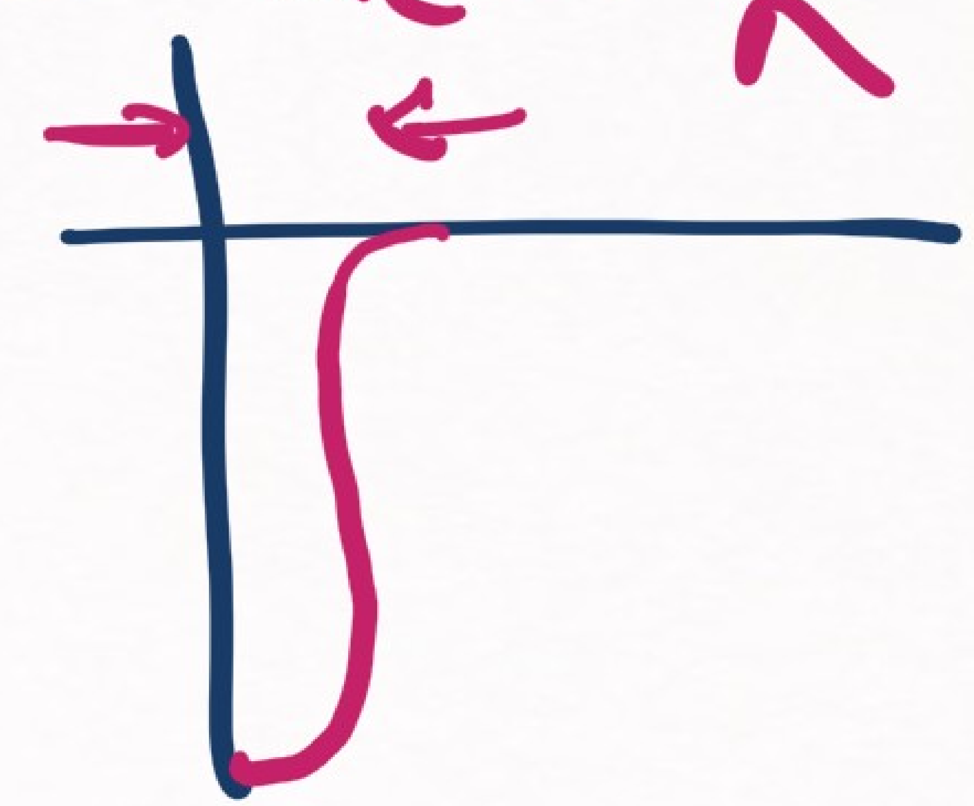
[REGULARIZATION] $\longrightarrow$ EXAMPLE : $f\left(\frac{|\vec{q}|}{\Lambda}\right) = e^{-|\vec{q}|^2/\Lambda^2}$

$$V_{LO}(\vec{q};\Lambda) = C_0 e^{-\left(\frac{q}{\Lambda}\right)^2} \longrightarrow V_{LO}(\vec{r}) = C_0 \frac{\Lambda^3}{8\pi^{3/2}} e^{-\frac{1}{4}(\Lambda^2 r^2)}$$

(Gaussian regulator)

Fourier transform

$= R_c \sim \frac{2}{\Lambda}$



$\rightarrow$ If $\Lambda \gg \gamma$, the bound state will not notice the difference betttween the original & regularized potentials

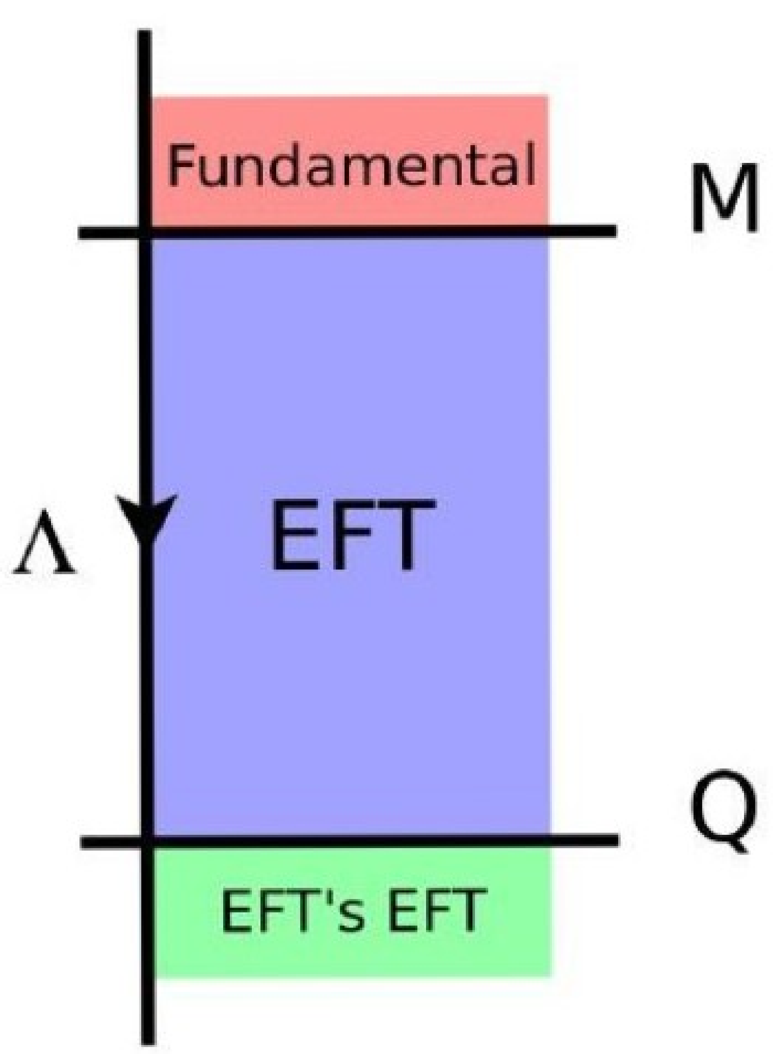
$\boxed{\text{PROBLEM}} \rightarrow$ [Λ is a new parameter]

WHAT WE WANT IS ... **RENORMALIZATION** ← Aim

a) Regularization: just include Λ + reg. function to make everything finite

b) But what we want is ... ?

hint!



Physics is unique, but choice of theory depends on resolution Λ:

- $\Lambda \geq M$: Fundamental
- $M \geq \Lambda \geq Q$: EFT

For equivalent descriptions:

$$\boxed{\frac{d}{d\Lambda}\langle\Psi|\mathcal{O}|\Psi\rangle = 0}$$

← this condition

Renormalization group invariance

What we want to achieve is independence with respect to the cutoff Λ

[HOW DO WE RENORMALIZE?]

a) $V_{LO}(r;\Lambda) = \underbrace{C_0(\Lambda)}_{\Downarrow} \frac{\Lambda^3}{8\pi^{3/2}} e^{-\frac{1}{2}\Lambda^2 r^2}$

Potential is not an observable quantity (in EFT)

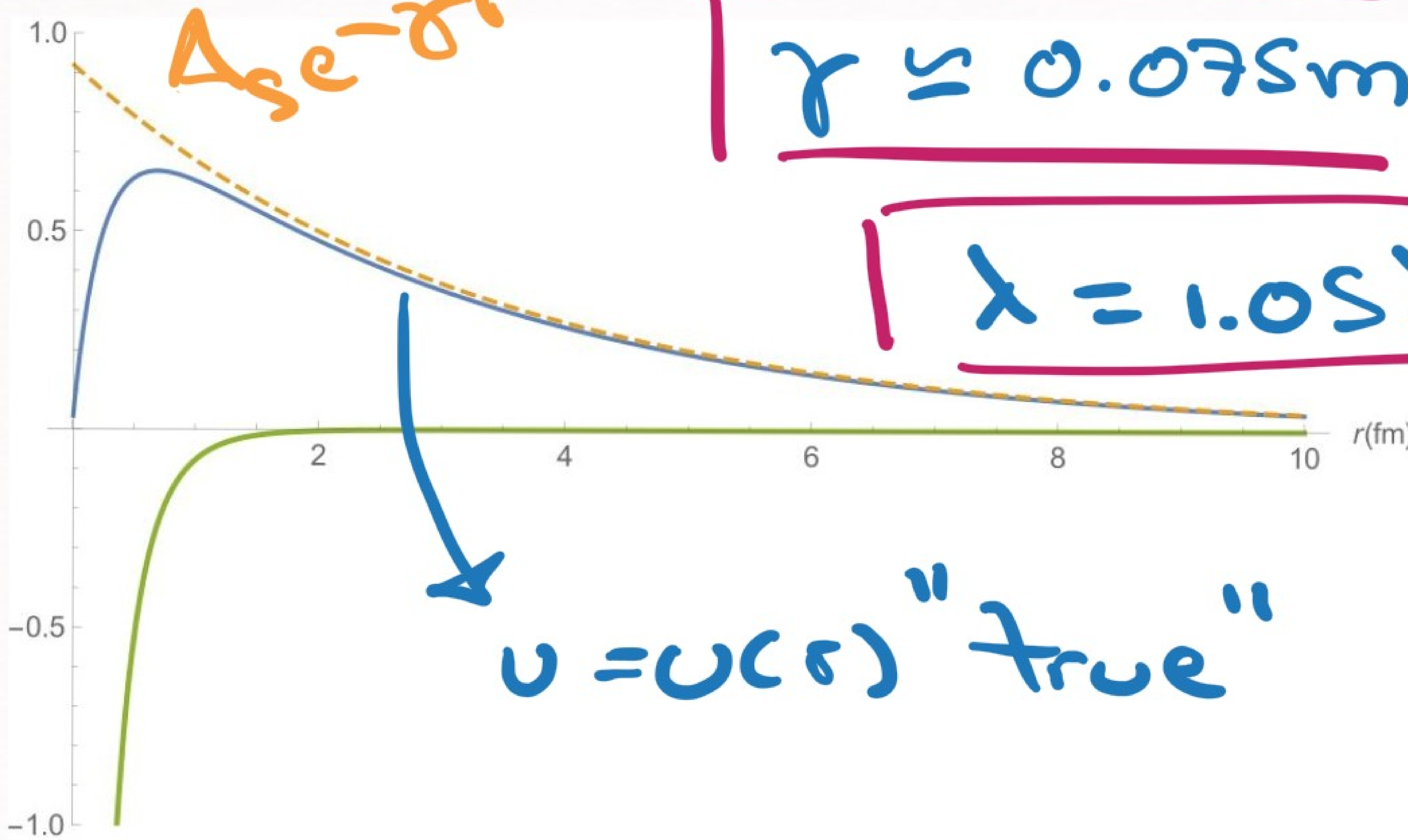
[Couplings now dependent on Λ]

b) We determine $C_0 = C_0(\Lambda)$ from...

$C(\Lambda)\,/\,\gamma(\Lambda) = \gamma_{\text{physical}}$

$$\Rightarrow \frac{d}{d\Lambda}\gamma = 0 \iff \boxed{\frac{d}{d\Lambda}\overbrace{\langle\Psi|H|\Psi\rangle}^{E_B = -\frac{\gamma^2}{M}} = 0}$$

EXAMPLE → SOLUTION a) OF OUR YUKAWA POTENTIAL



$A_s e^{-\gamma r}$

$\boxed{\gamma \simeq 0.075 m}$

$\boxed{\lambda = 1.05 \lambda_c}$

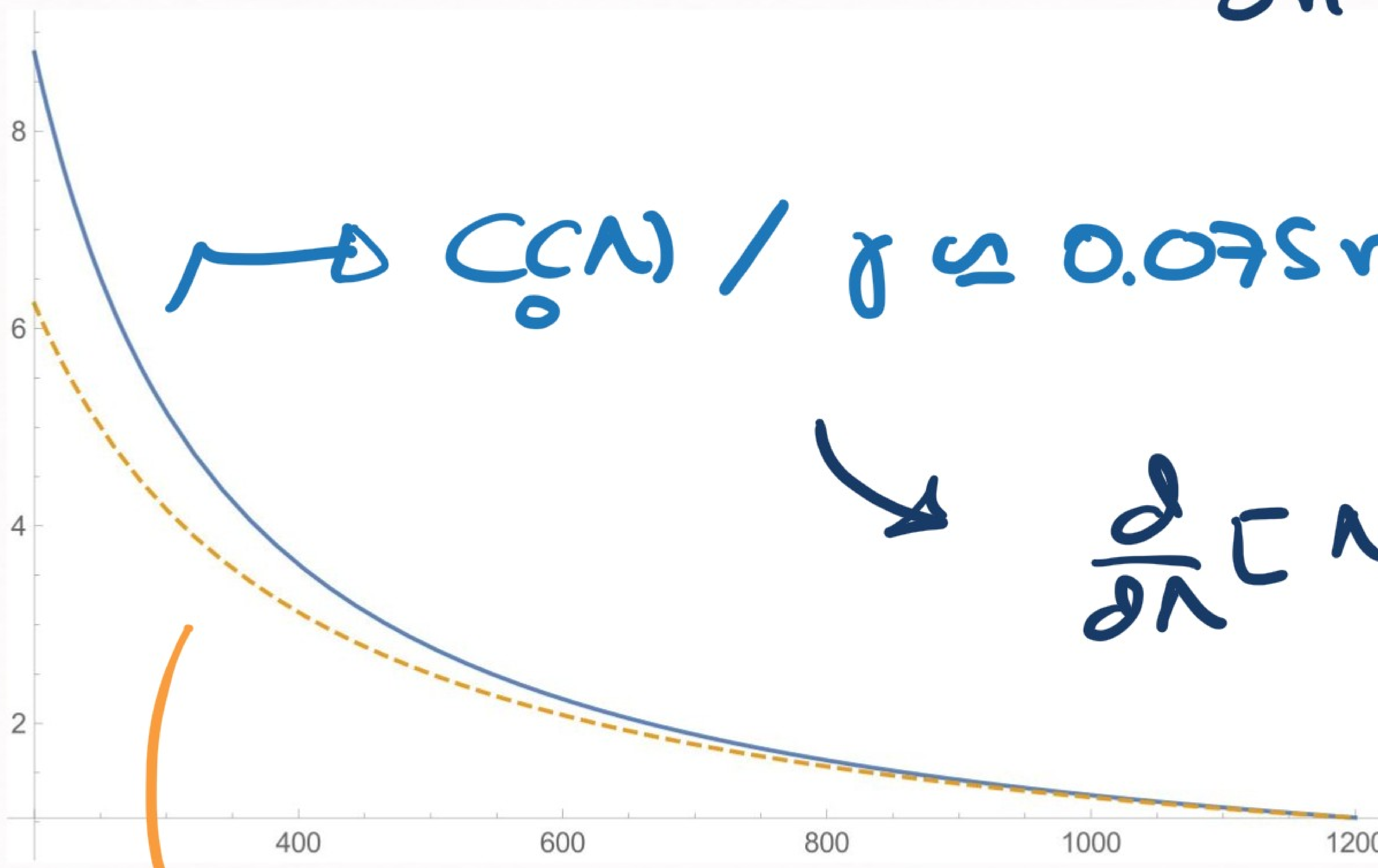
$u = u(r)$ "true"

We will try different Λ

We will reproduce this bound state with $V_{LO}(r;\Lambda)$

$$V_{LO}(r) = C_0(\Lambda) \frac{\Lambda^3}{8\pi^{3/2}} e^{-\frac{1}{2}\Lambda^2 r^2}$$

$\boxed{\text{Now } c_0 = c_0(\Lambda)}$ $\rightarrow$ Dependance of the coupling c_0 on the cutoff



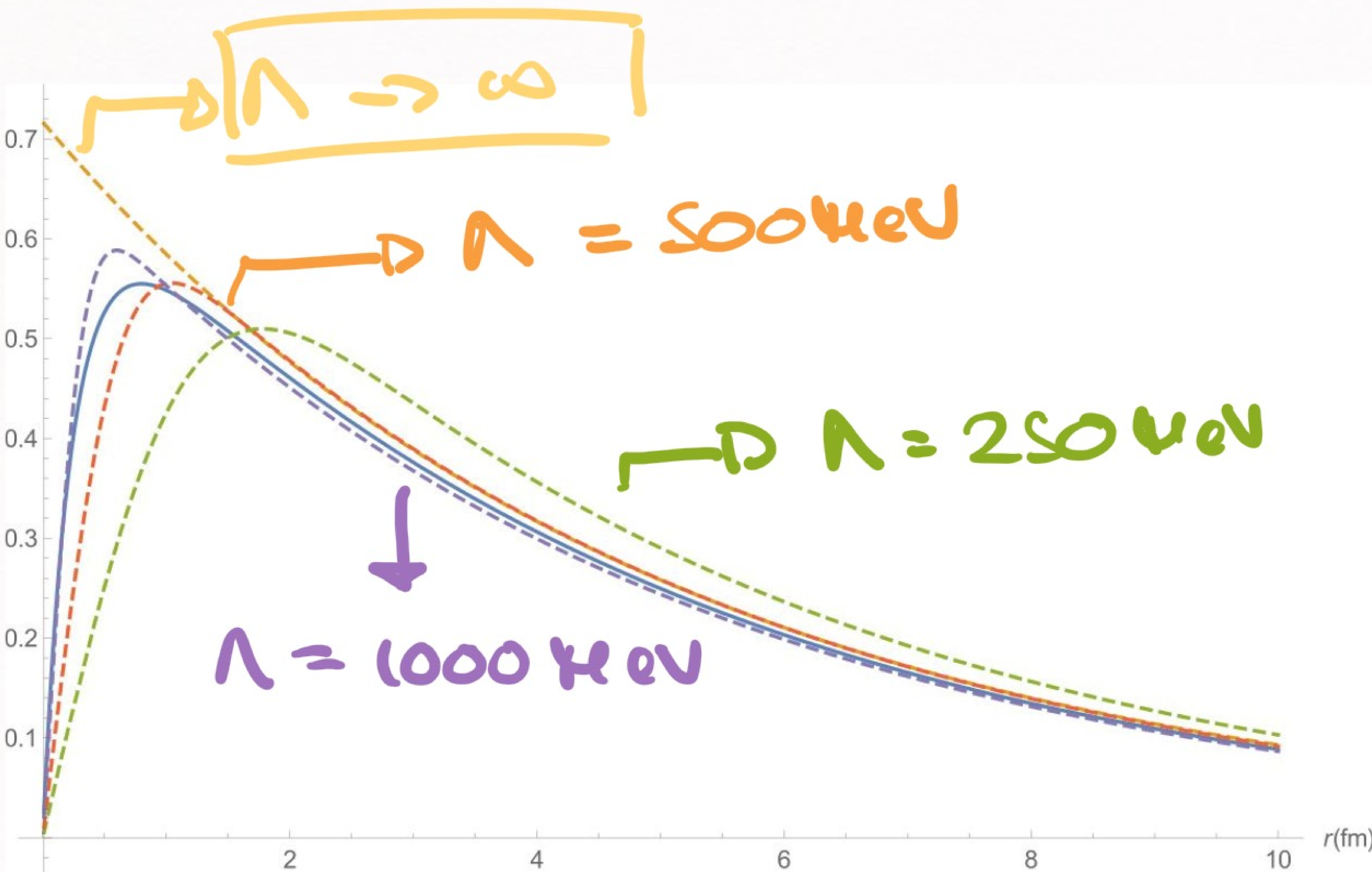
$\rightarrow c_0(\Lambda) \;/\; \gamma \simeq 0.075\, m$

$$\frac{d}{d\Lambda}\left[\Lambda c_0(\Lambda)\right] \simeq 0$$

(RG Equation)

$$c_0(\Lambda) \sim \frac{\text{constant}}{\Lambda}$$

[And we can compare the effective wave functions to the real wave functions]



INDEPENDENTLY OF THE CUTOFF, WE GET ABOUT THE SAME WAVE FUNCTION

— "real" wave function (Yukawa, $\lambda = 1.05\,\lambda_c$)

We see that:

a) the effective wave functions are similar to the real wave function

→ we don't need to know the true potential

b) $\Lambda \gtrsim m$ gives best results

→ the cutoff (its value) is not terribly important

c) for $\Lambda \to \infty$, we recover → $u(r) = A_s e^{-\gamma r}$

(expected w.f. for $mr \gg 1$)

and this is how effective theory works!

SUMMARY

FOR TWO-BODY SYSTEMS SUCH THAT $\gamma \ll m$:

a) No need to know the true potential

→ we only need an effective potential

$$\boxed{V_{eff} = C_0}$$

b) We have to regularize V_{eff}:

$$\left[V_{eff}(\vec{q};\Lambda) = C_0 \, e^{-(q^2/\Lambda^2)} \right]$$ (or any other choice)

c) We renormalize the regularized potential

$C_0 \rightarrow C_0(\Lambda)$ ⟶ make C_0 depend on Λ

$+$

$\frac{d}{d\Lambda}\langle\psi|\vec{O}|\psi\rangle = 0$ $\left(\text{e.g. } \frac{d}{d\Lambda}\gamma = 0\right)$

d) And now we can do the calculations & describe the system relatively well

(without having to know anything about the true potential)