NUCLEAR PHYSICS 8

a) QUANTUM CHROMODYNAMICS (PART II)

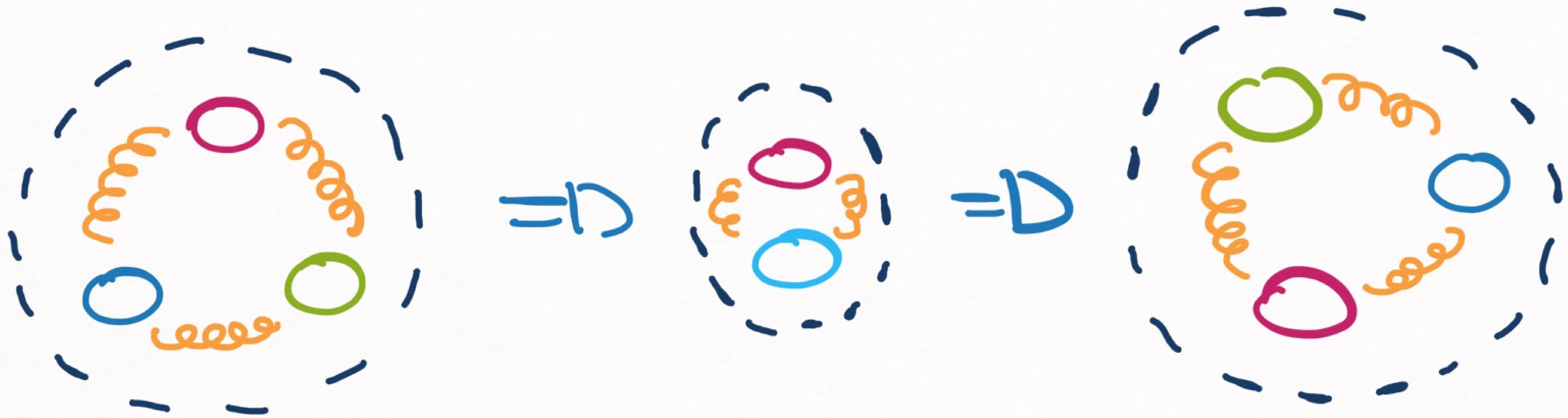
NUCLEONS → QUARKS & GLUONS

b) RENORMALIZATION & EFFECTIVE FIELD THEORY

NUCLEONS ⟷ QUARKS & GLUONS

relate these two explanations

RECAP NUCLEONS NOT FONDAMENTAL



NUCLEON PION NUCLEON

(QUARK & GLUONS)

$\Rightarrow$ When a pion is exchanged between two nucleons then this needs to have an explanation in terms of quarks & gluons

[QUANTUM CHRONODYNAMICS] ⇒ THEORY DESCRIBING QUARKS & GLUONS

⇓

IT'S A GAUGE FIELD THEORY! → RECAP

⇓

JUST LIKE QED

⇓

WE CAN GAIN INSIGHT INTO QCD
BY STUDYING QED } → Previous Lesson

→ QED is a simplified version of QCD

QED → gauge theory → U(1) symmetry
QCD → gauge-theory → SU(3) symmetry

① → main difference between QED & QCD

→ HOW DOES THIS WORKS THEN ?

a) Begin w/ Dirac field

$$\psi(x) \to e^{ie\alpha}\psi(x) \quad \text{symmetry}$$

b) Make the symmetry local

$$\psi(x) \to e^{ie\alpha(x)}\psi(x) \quad \text{now local}$$

c) Modify Dirac's theory to be invariant under the local symmetry

c.1) Include a new field $\rightarrow A_\mu$ (vector field)

$$A_\mu \xrightarrow[U(1)]{} A_\mu + \partial_\mu \alpha$$ (gauge transformation)

c.2) Define a new type of derivative

$$\cancel{\partial_\mu} \rightarrow \quad D_\mu \equiv \partial_\mu - ieA_\mu$$

(covariant derivative)

c.3) Complete the theory with a kinetic term for the new field A_μ

$\rightarrow F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ $F_{\mu\nu} \xrightarrow[U(1)]{} F_{\mu\nu}$

$\rightarrow F_{\mu\nu} F^{\mu\nu} \xrightarrow[U(1)]{} F_{\mu\nu} F^{\mu\nu}$ $\Big\{ \rightarrow$ only contains derivatives of the field A_μ

$\rightarrow \mathcal{L}_{QED} = \bar{\psi}(i\not{D} - m)\psi + \lambda F_{\mu\nu} F^{\mu\nu}$

$\rightarrow$ Determine λ from some condition

$$\boxed{\lambda = -1/4}$$

probably contains a kinetic term

[HOW DO WE GO FROM QED TO QCD?]

1) First, let's see the Dirac Lagrangian:

$\Rightarrow$ Quarks come in different types (flavor)

u, d, s, c, b, t

Now: ($n_F = 6$)

Before

$$\mathcal{L}_{e^-} = \bar{\psi}(i\partial\!\!\!/ - m)\psi$$

(only 1 type of field)

$$\rightarrow \quad \mathcal{L}_{quarks} = \sum_{j=1}^{n_F} \bar{q}_j (i\partial\!\!\!/ - m_j) q_j$$

2) Now we add a global symmetry (e.g. U(1) before in QED)

$\Rightarrow$ Quarks also have an additional property called **COLOR**

How do we know this? $\rightarrow$ Δ isobar (a hadron) $\rightarrow J = 3/2$

$$|\Delta^{++}(J=\tfrac{3}{2}, M=\tfrac{3}{2})\rangle = |\underline{u\uparrow}\ \underline{u\uparrow}\ \underline{u\uparrow}\rangle$$

$e_u = +\frac{2}{3}$

$\Downarrow$

$3e_u = +2$

$M = \sum_j m_j$

$\frac{3}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$

$\rightarrow$ completely **symmetric**

PROBLEM:

Quarks are fermions (antisymmetric w.f.)

How do we solve this problem? $\rightarrow$ include a new quantum number

$\downarrow$

$\boxed{\text{WE CALL IT COLOR}}$

$$|\Delta^{++}, \tfrac{3}{2}\,\tfrac{3}{2}\rangle = \epsilon_{abc}\,|u_a^{\uparrow}\; u_b^{\uparrow}\; u_c^{\uparrow}\rangle$$

$\Rightarrow$ $a, b, c = 1, 2, 3$ or R, G, B

$\Rightarrow$ ϵ_{abc} is the Levi-Civita symbol

$\Rightarrow$ The wave function is antisymmetric now!

$$\epsilon_{abc} = -\epsilon_{bac} = -\epsilon_{acb} = \ldots$$

(completely antisymmetric)

[WHICH GLOBAL SYMMETRY?] 1) $U \to (U_R\ U_G\ U_B)$

$\epsilon_{abc} |q_a q_b q_c\rangle \longrightarrow \epsilon_{abc} |q_a' q_b' q_c'\rangle$ =D ⊗

Group

$(\bar{q}_a\ \bar{q}_b\ \bar{q}_c)\begin{pmatrix} q_a \\ q_b \\ q_c \end{pmatrix} \to (\bar{q}_a'\ \bar{q}_b'\ \bar{q}_c')\begin{pmatrix} q_a' \\ q_b' \\ q_c' \end{pmatrix}$

$$\begin{pmatrix} q_a \\ q_b \\ q_c \end{pmatrix} \to \underline{\underline{U}} \begin{pmatrix} q_a \\ q_b \\ q_c \end{pmatrix}$$

b) =D $\underline{\underline{U^\dagger U}} = \mathbb{1}_{3\times3}$

$\epsilon_{abc} \to \det U\, \epsilon_{abc}$

a) U is a $\underline{\underline{3\times3}}$ matrix

(trivial) → ∃ three "colors"

c) ⊗ =D $\underline{\underline{\det U = 1}}$

[WHICH GLOBAL SYMMETRY?] ②

a) U is a 3×3 matrix

b) $UU^\dagger = \mathbb{1}_{3\times3}$

c) $\det U = 1$

(I might have cheated with this one)

$\underline{\underline{SU(3)}} = \{ U \in GL(3,\mathbb{C}),\ U^\dagger U = \mathbb{1},\ \det U = 1 \}$

$GL(n,\underline{\underline{\mathbb{C}}}) \rightarrow$ general linear group

($\forall$ $n\times n$ complex matrices)

heuristical derivations are not rigorous ($\underline{\underline{\text{caveat}}}$)

[NOW, WE MAKE THIS SYMMETRY LOCAL] (3)

$$\begin{pmatrix} q_a \\ q_b \\ q_c \end{pmatrix} \rightarrow U(x) \begin{pmatrix} q_a \\ q_b \\ q_c \end{pmatrix}$$ → Everything is a matrix now

a) $U(1)$: $U(x) = e^{i\alpha(x)}$ → complex numbers are just the $U(1)$ group

b) $SU(2)$: ⇒ group for the rotations of a spin-½ particle

$$\Rightarrow U(x) = e^{i \vec{\alpha}(x) \cdot \vec{\sigma}}$$

→ Pauli matrices: $[\sigma_i, \sigma_j] = i\epsilon_{ijk}\sigma_k$

$\vec{\alpha} = \alpha \hat{n}$

$\vec{\alpha}(x) = (\alpha_1(x), \alpha_2(x), \alpha_3(x))$

$$U(x) = \mathbb{1}_{2\times 2} \cos(\alpha) + i(\hat{n} \cdot \vec{\sigma}) \sin(\alpha)$$

c) SU(3): How we make the extension to this group?

$$\underbrace{U(x) = e^{i\vec{\alpha}(x)\cdot\vec{\sigma}}}_{2\times 2 \text{ matrix}} \Rightarrow \underbrace{U(x) = e^{i\alpha_a(x)t_a}}_{3\times 3 \text{ matrix}}$$

$a = 1, \dots, 8$

$t_a \to 8$ matrices called the SU(3) "generators"

$$[\sigma_i, \sigma_j] = i\epsilon_{ijk}\sigma_k$$

$$\Downarrow$$

$$[t_b, t_c] = i f_{abc} t_c$$

structure constants

$f_{abc} = -f_{acb}$

If you ever studied Lie groups, then this sounds familiar

[COMMENT ABOUT NOTATION]

Usually we don't use t_a but $\lambda_a = \frac{t_a}{2}$

$$\left[\frac{\lambda_b}{2}, \frac{\lambda_c}{2}\right] = i f_{abc} \frac{\lambda_a}{2}$$

$\Rightarrow$ From now on we use λ_a

We will concentrate on these

[EXTRA NOTE] $\rightarrow$ $U(x) = \exp\left(i \alpha_a(x) \frac{\lambda_a}{2}\right) = \overbrace{1 + \frac{1}{2} i \alpha_a t_a}$

$+$ corrections

[WE KEEP MAKING THIS SYMMETRY LOCAL] 2

$$\begin{pmatrix} q_a \\ q_b \\ q_c \end{pmatrix} \rightarrow \left(1 + i\underbrace{g\,\alpha_a(x)\frac{\lambda_a}{2}}\right)\begin{pmatrix} q_a \\ q_b \\ q_c \end{pmatrix}$$ (infinitesimal transformation)

→ convenient notation

1) Include a new field:

$$A_\mu \underset{U(1)}{\rightarrow} A_\mu + \partial_\mu \alpha \Rightarrow A^a_\mu \xrightarrow[SU(3)]{} A^a_\mu + \partial_\mu \alpha^a + g f_{abc} A^b_\mu A^c_\nu$$

2) Include a new type of derivative

$$\cancel{\partial_\mu} \rightarrow D_\mu \equiv \partial_\mu - ie A_\mu \Rightarrow \cancel{\partial_\mu} \rightarrow D_\mu \equiv \partial_\mu - ig\frac{\lambda_a}{2}A^a_\mu$$

3) Define the kinetic term

$$\left.\begin{aligned} &F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \\ &F_{\mu\nu} \xrightarrow[U(1)]{} F_{\mu\nu} \\ &F_{\mu\nu}F^{\mu\nu} \xrightarrow[U(1)]{} F_{\mu\nu}F^{\mu\nu} \end{aligned}\right\} \Rightarrow$$

$$\Downarrow$$

$$\lambda F_{\mu\nu}F^{\mu\nu}$$

$$(\lambda = -1/4)$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f_{abc} A^b_\mu A^c_\nu$$

$$F^a_{\mu\nu} \xrightarrow[SU(3)]{} F^a_{\mu\nu} - \underbrace{f_{abc}}_{\text{antisymmetric}} \alpha_b F^c_{\mu\nu}$$

$$\underline{F^a_{\mu\nu}F^{\mu\nu a}} \xrightarrow[SU(3)]{} \underline{F^a_{\mu\nu}F^{\mu\nu a}}$$

$$\Downarrow$$

$$\lambda F_{\mu\nu a}F^{\mu\nu a} \Rightarrow \lambda = -\frac{1}{4}$$

4) Add the kinetic term:

$$\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi - \underbrace{\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\text{Photons}} \qquad U(1)$$

$$\Downarrow$$

$$\mathcal{L} = \sum_{j=1}^{n_F} \bar{q}_j(i\not{D} - m_j)q_j - \underbrace{\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a}}_{\text{Gluons}} \qquad SU(3)$$

SUMMARY

[QCD is just a complicated version of QED]

However, besides n_f (flavors)

and n_c (colors) there will be

a very important difference

between QCD & QED

$\boxed{\text{QCD vs QED}}$

→ There is a fundamental difference between QCD & QED

→ $\boxed{\text{ASYMPTOTIC FREEDOM}}$

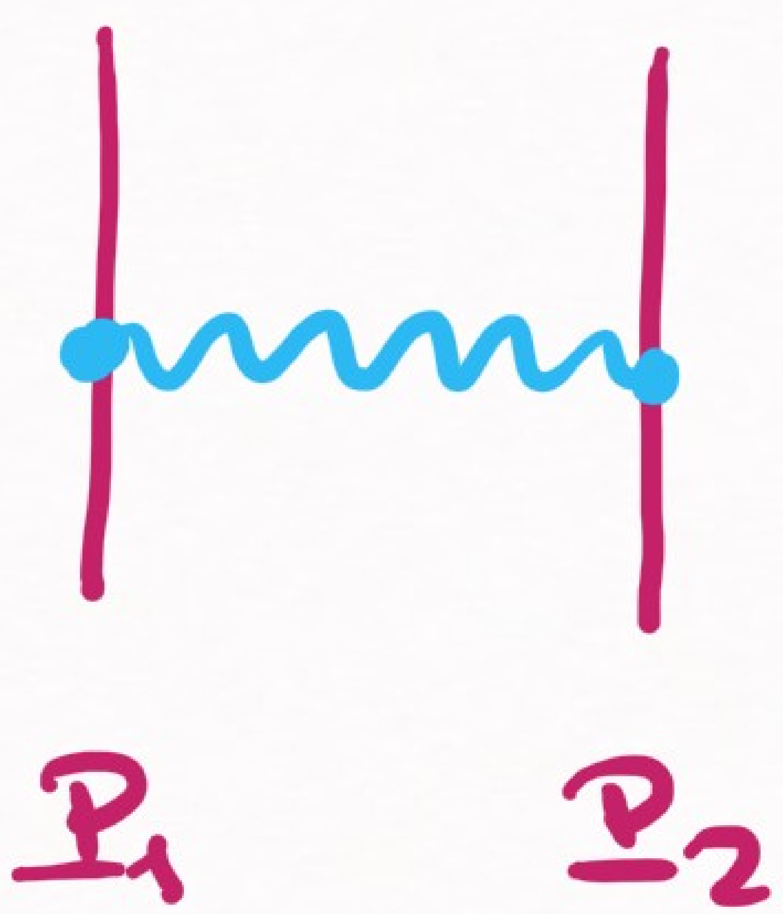
→ Strength of QCD decreases at short distances
(increases at long distances)

(contrary to what happens in QED)

QED & LOOPS → simpler version

How does the strength of QED change with energy?

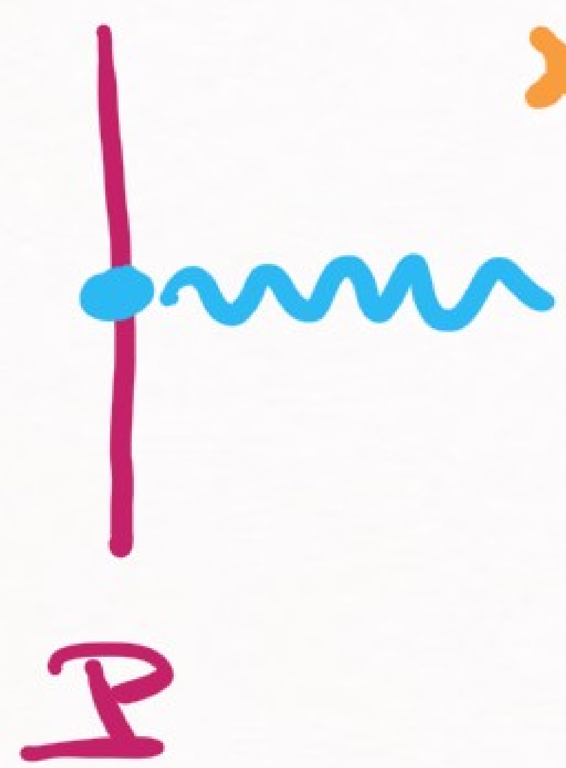
COULOMB



$$= \frac{e_1 e_2}{|\vec{q}\,|^2} \longrightarrow$$

Strength seems to be always $e_1 e_2$

[WE CAN CONSIDER THE VERTEX:]



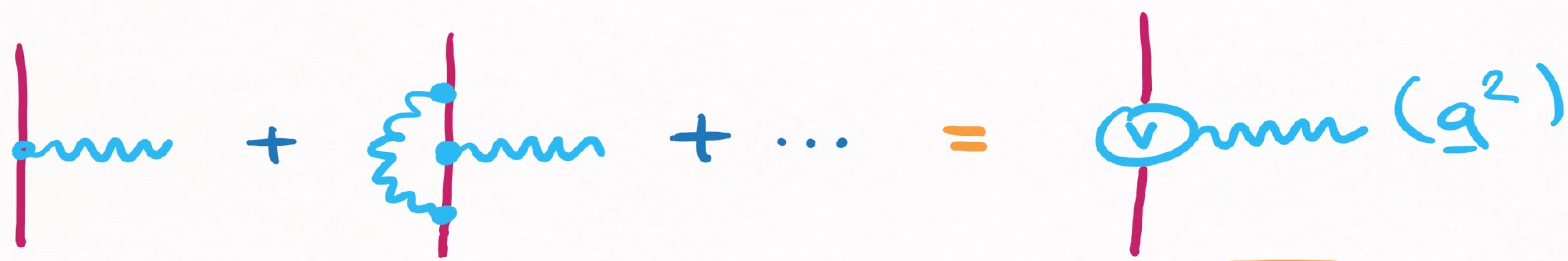
$\propto e$ ← [in QFT]

(more compact notation)

$$\left[\langle p\,\gamma | H | p \rangle = -ie\frac{1}{\sqrt{2\omega}}\, e^{-i\vec{q}\cdot\vec{r}}\, \vec{\epsilon}^{\,*}_{\lambda}(\vec{q}) \quad \text{in QM}\right]$$

(both equivalent ways to understand the vertex function)

[BUT THERE ARE QUANTUM CORRECTIONS TO THE VERTEX...]



[EQUIVALENTLY] $\Rightarrow$ $\boxed{e = e(q^2)} \to \boxed{\alpha = \alpha(q^2)}$

$$\alpha = \frac{e^2}{4\pi}$$

(effective parametrization of the previous sum of amplitudes)

[NOW, LOOK CAREFULLY] → We use a first order calculation of previous sum

$$\alpha(q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi} \log\left(\frac{q^2}{\mu^2}\right)} \Rightarrow \boxed{\begin{array}{l} \alpha(q^2) > \alpha(\mu^2) \\ \text{for } q^2 > \mu^2 \end{array}}$$

That is, the strength of the electromagnetic interaction increases with energy

[KEEP LOOKING CAREFULLY]

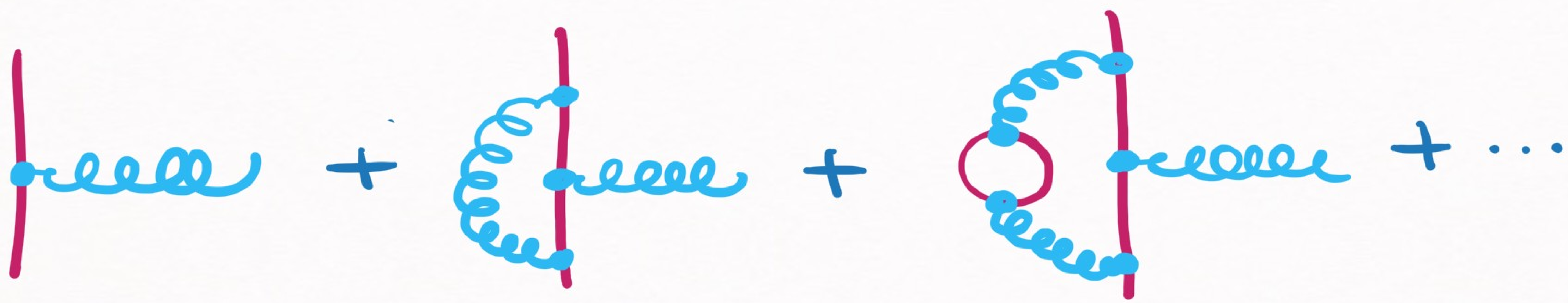
$\alpha(m_e^2) \simeq \frac{1}{137}$ but $\exists\, \Lambda_0 \,/\, \alpha(\Lambda_0^2) \to \infty$

$\Rightarrow \left[\Lambda_0 = m_e \exp\left(\frac{3\pi}{2\alpha}\right) \sim 10^{280}\ \mathrm{MeV} \right]$

At this scale, the electromagnetic force diverges

$\Rightarrow$ $\boxed{\text{LANDAU POLE}}$

QCD $\Rightarrow$ SIMILAR EFFECT



But something different happens...

it will become weaker as the energy grows

(contrary situation as w/ QED)

[QCD : RUNNING OF THE STRONG COUPLING]

$$\alpha_s(q^2) = \frac{\alpha_s(\mu^2)}{1 + \frac{\alpha_s(\mu^2)}{12\pi}(33 - 2n_f)\log\left(\frac{q^2}{\mu^2}\right)} \Rightarrow (*)$$

QCD equivalent of $\alpha(q^2)$ in QED

$$(*) \Rightarrow \boxed{\begin{array}{c} \alpha_s(q^2) < \alpha_s(\mu^2) \\ q^2 > \mu^2 \end{array}} \rightarrow \text{grows weaker w/ energy}$$

$\boxed{\text{POLE IN } \alpha_s(q^2)}$ ← "Equivalent" of Landau pole

$$\underline{\underline{\alpha_s(\Lambda^2_{QCD})}} \longrightarrow \infty \quad \text{with} \quad \underline{\underline{\Lambda_{QCD}}} = \mu \exp\left[-\frac{12\pi}{(33-2n_f)\,\alpha_s(\mu^2)}\right]$$

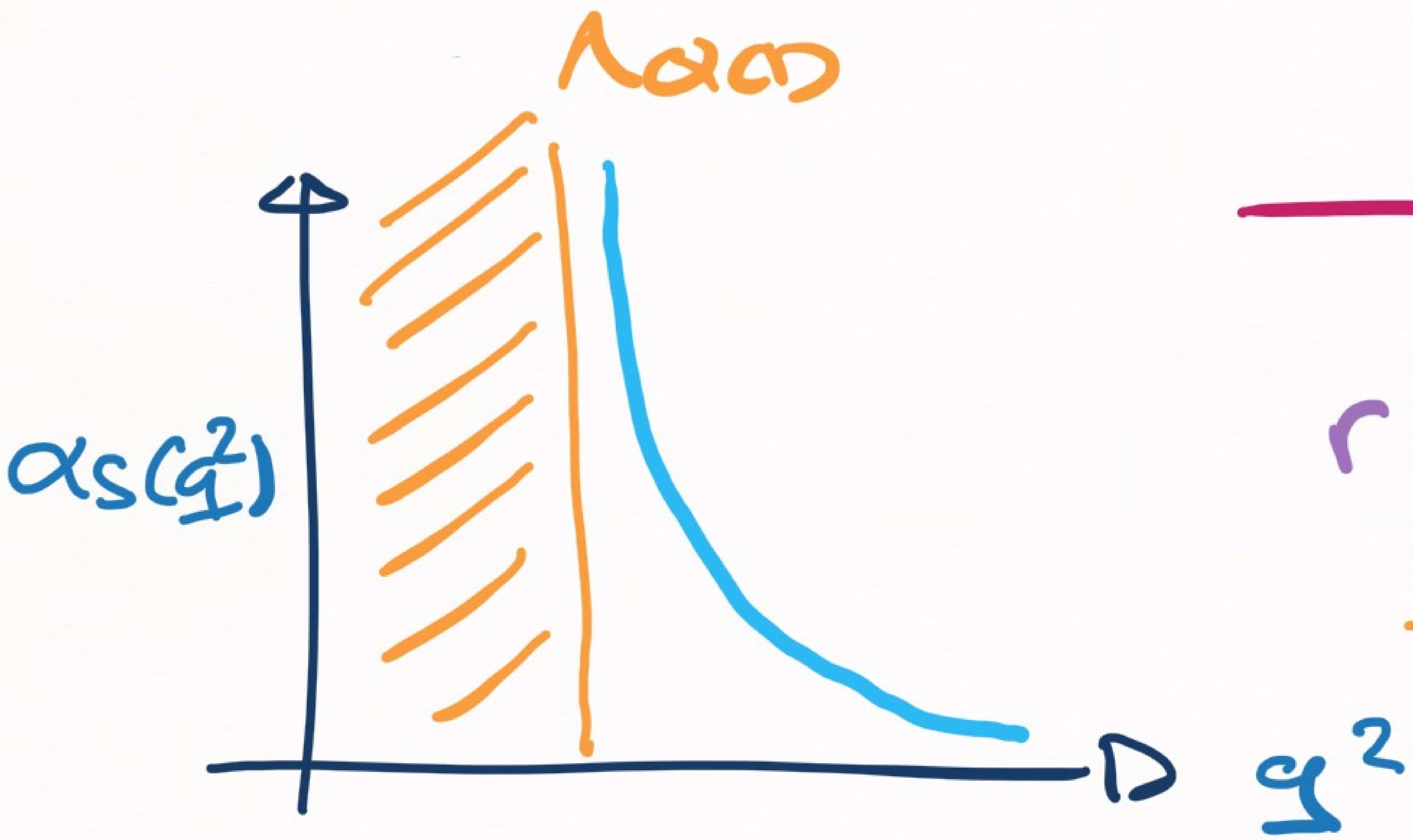
We can rewrite $\alpha_s(q^2)$ as $\Big\} \Rightarrow \left[\alpha_s(q^2) = \dfrac{12\pi}{(33-2n_f)\log\left(\frac{q^2}{\mu^2}\right)}\right]$

$$\boxed{\Lambda_{QCD} \sim (200 - 350)\,\text{MeV}}$$

[INTERPRETATION OF Λ_{QCD}]

PROBLEM

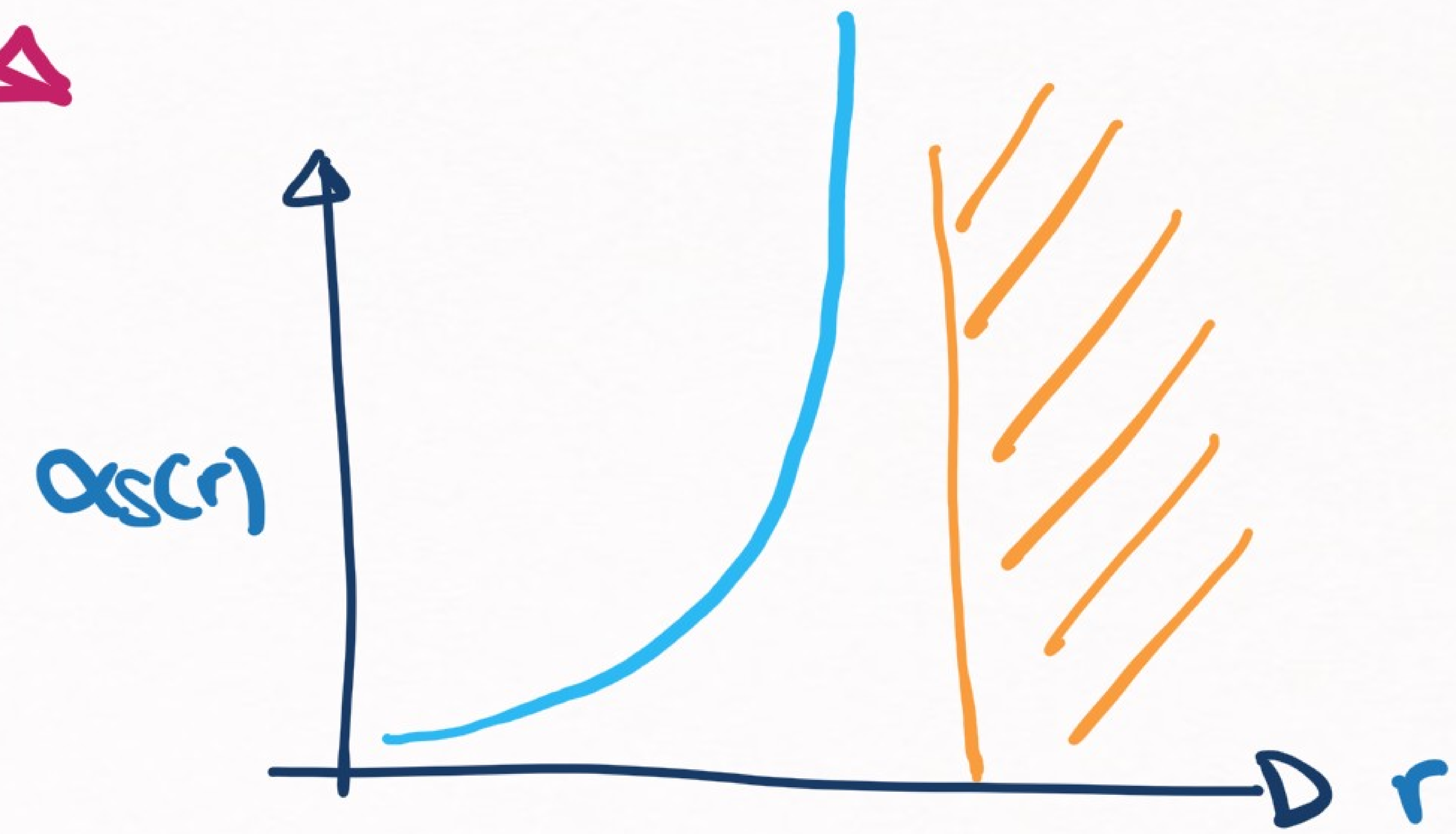
$$\boxed{\frac{1}{\Lambda_{QCD}} \sim 0.5\,fm}$$



not solvable

QCD is solvable w/ perturbation theory

$$r \propto \frac{1}{|q|}$$



QCD solvable

not solvable

[AND... THIS IS REALLY PROBLEMATIC]

NUCLEON SIZE → 0.85 fm → Lies in the region where QCD is not solvable

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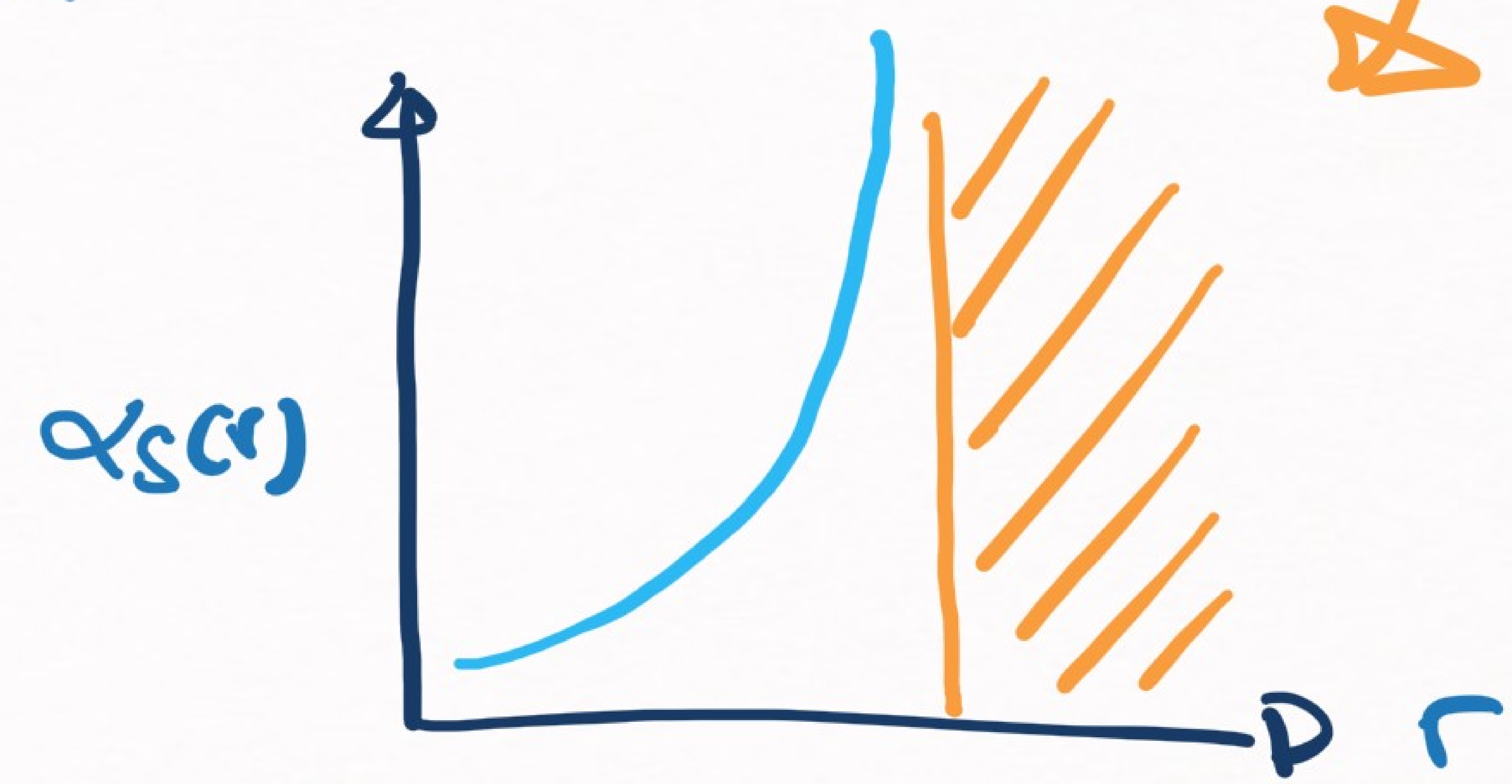
It will be difficult to explain nuclear physics from QCD



BUT ON THE BRIGHT SIDE

1) Λ_{QCD} is special $\Rightarrow$ Natural scale of QCD

(We can explain lots of things from Λ_{QCD})

2) Though, not the only scale : u, d, s, c, b, t

$$m_u, m_d, m_s < \Lambda_{QCD} < m_c, m_b, m_t$$

Λ_{QCD} & SYMMETRIES

this quark does not hadronize (decays too fast)

HIERARCHY :

$$\underbrace{m_u, m_d, m_s}_{\text{New symmetry:}} < \Lambda_{QCD} < \underbrace{m_c, m_b}_{\text{Another new symmetry:}} (, m_t)$$

New symmetry: CHIRAL SYMMETRY

Another new symmetry: HEAVY QUARK SYMMETRY

BOTH ARE REALLY USEFUL

LIGHT SECTOR & Λ_{QCD}

$m_u \sim 2\,\text{MeV}$
$m_d \sim 5\,\text{MeV}$
$\Lambda_{QCD} \sim 300\,\text{MeV}$

$\Rightarrow m_u, m_d \ll \Lambda_{QCD}$

1) mass of the ρ meson ($u\bar{d} \to \rho^+$) $m_\rho \sim 770\,\text{MeV}$

$m_\rho \neq m_u + m_d \longrightarrow m_\rho \simeq 2\Lambda_{QCD} \simeq 600\,\text{MeV}$

(acceptable)

2) mass of the nucleon ($uud \to p$) $m_N \sim 940\,\text{MeV}$

$m_p \neq 2m_u + m_d \longrightarrow m_p \simeq 3\Lambda_{QCD} \simeq 900\,\text{MeV}$

(acceptable)

3) mass of the D-meson $(c\bar{u})$

$m(D) \simeq 1.8\,\text{GeV}$

$m_c \simeq 1.2\,\text{GeV}$

$m(D) \cancel{=} m_u + m_c$

$\simeq 1.2\,\text{GeV}$

$\Rightarrow \quad m(D) = m_c + m_u + 2\Lambda_{QCD}$

$\simeq 1.8\,\text{GeV}$

(acceptable)

$\boxed{\text{MORAL:}}$

Λ_{QCD} actually provides non-trivial information about QCD dynamics

[EXCEPTION : THE PION] $\longrightarrow$ very interesting exception

4) mass of the pion ($u\bar{d} \to \pi^+$) $m_\pi \simeq 140$ MeV

$m_\pi \cancel{\simeq} m_u + m_d$ $\Rightarrow$ $m_\pi \cancel{\simeq} 2\Lambda_{QCD} \simeq 600$ MeV

doesn't work doesn't work either

$$\frac{m_\pi}{2\Lambda_{QCD}} \sim \frac{1}{4} \quad , \quad \frac{m_\pi}{m_e} \sim \frac{1}{6} \Rightarrow \text{fine-tunings}$$

[FINE-TUNING OF THE PION MASS]

$\Rightarrow$ pion mass much smaller than expected

QUESTION: a) Is this a coincidence?

[b) Or is this a conspiracy?]

It is a symmetry

(CHIRAL SYMMETRY)

$$\Leftarrow \boxed{\frac{m_u}{\Lambda_{QCD}}, \frac{m_d}{\Lambda_{QCD}} \ll 1}$$

RECAP

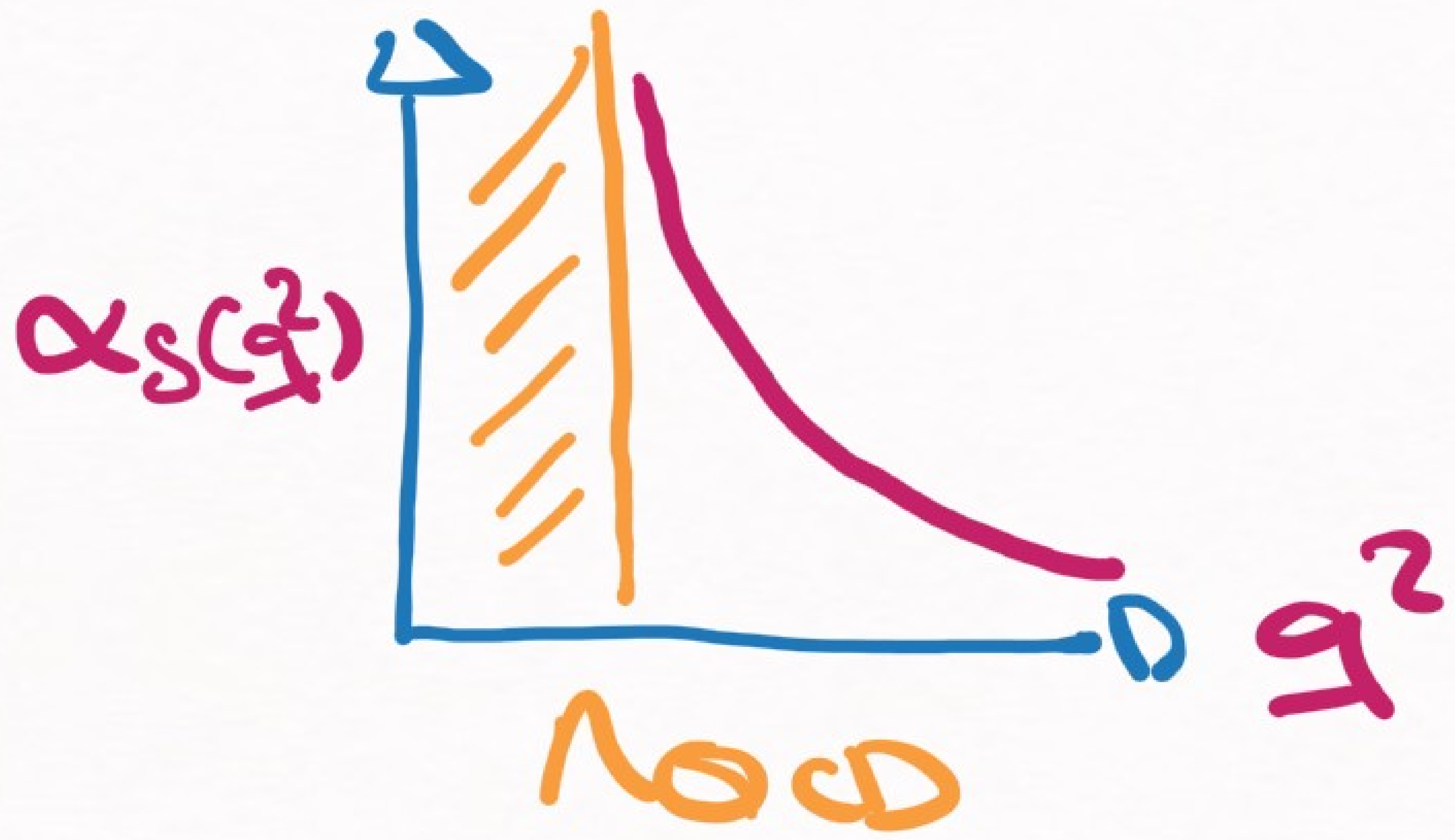
1) Hadron → Quarks & Gluons

This type of explanation would be wonderful

2) Quarks & Gluons → Explained by QCD

We can try to explain hadrons in terms of QCD

3) QCD → Asymptotic freedom



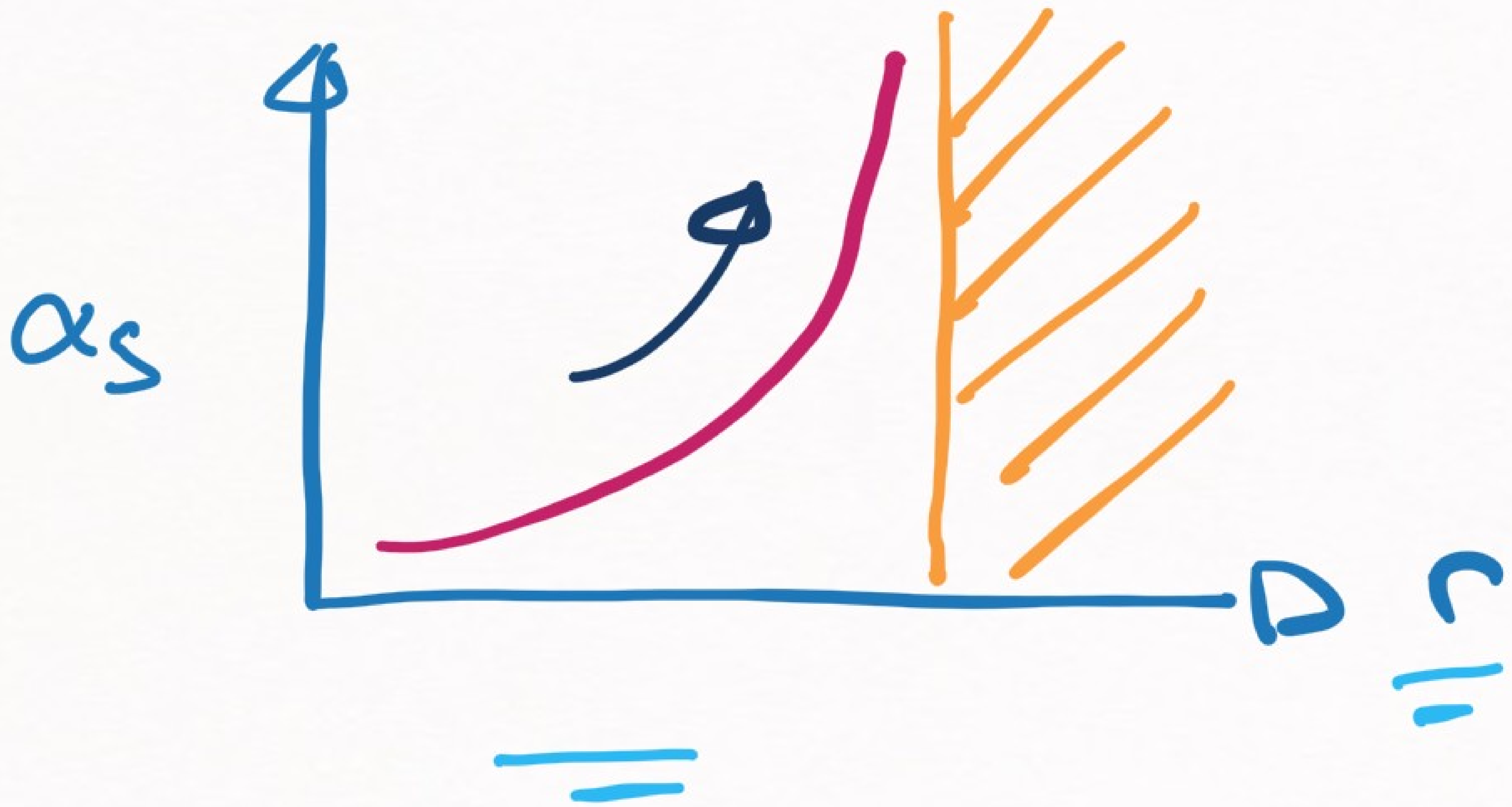
⟹ We can't use techniques such as perturbation theory to explain hadrons in terms of quarks & gluons

4) The pion is special ⟹ REASON: CHIRAL SYMMETRY

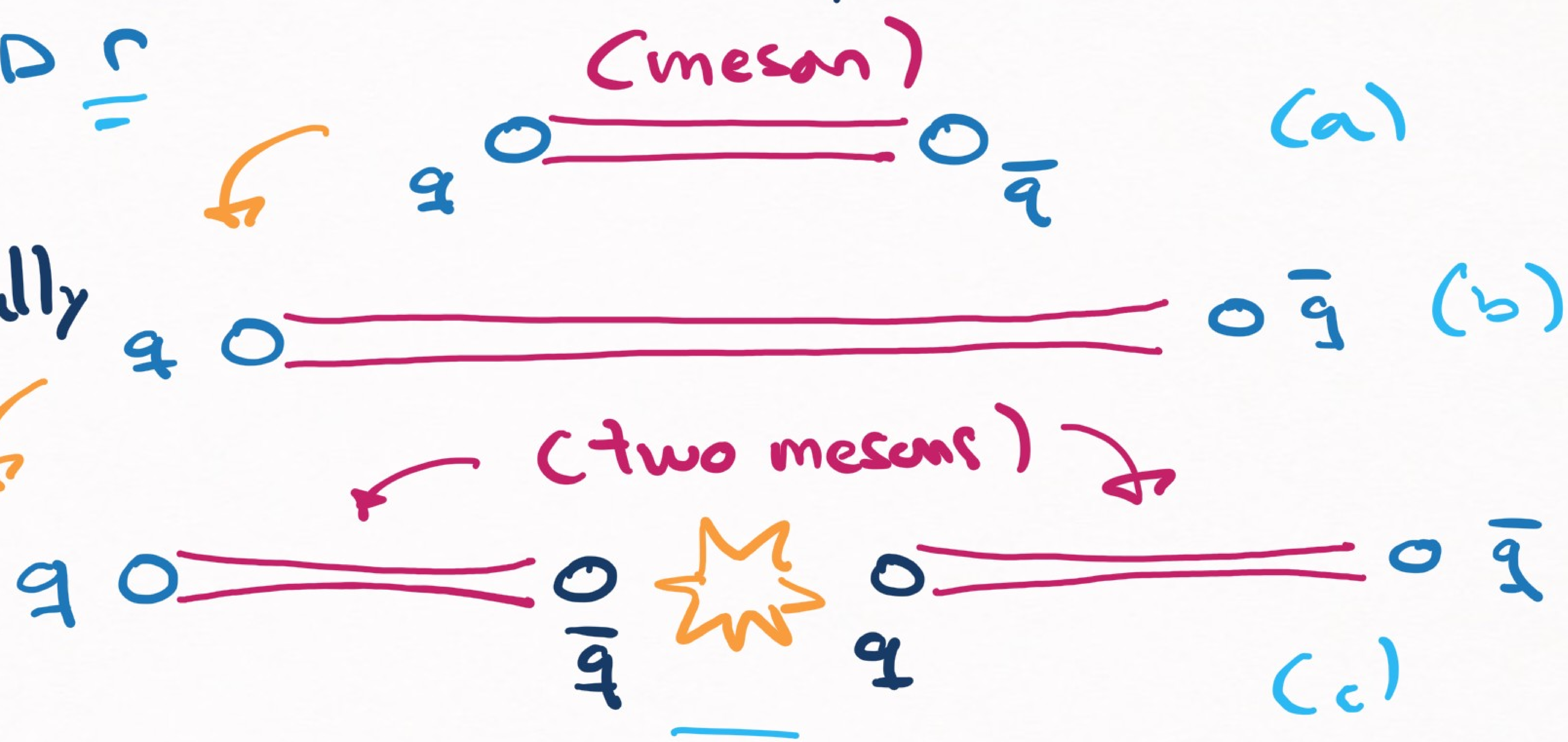
$$\frac{m_u}{\Lambda_{QCD}}, \frac{m_d}{\Lambda_{QCD}} \ll 1$$

CONFINEMENT

$\Rightarrow$ We never see isolated quarks



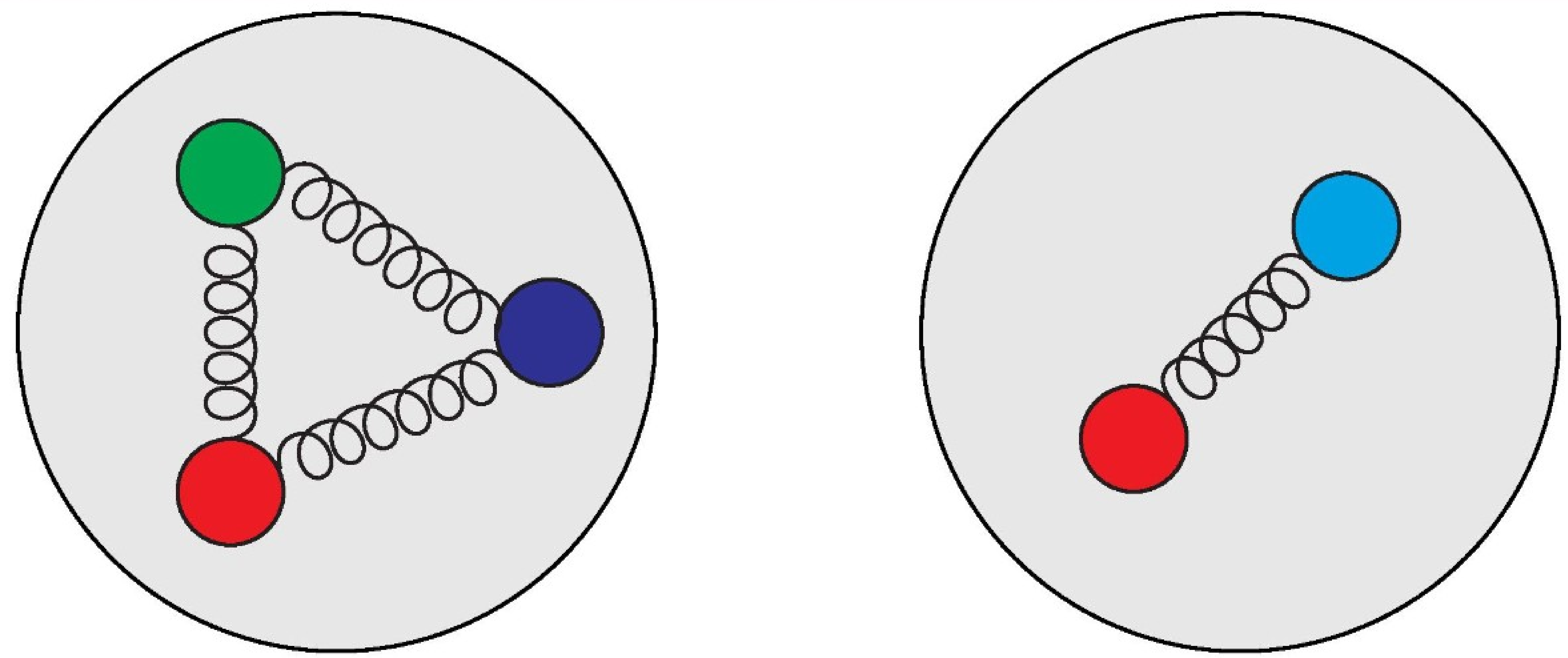
$\Rightarrow$ What happens if we try to separate two quarks?



(c) is energetically favored over just separating the two quarks

[THERE IS A PROBLEM IN EXPLAINING NUCLEAR FORCES FROM QCD]

→ How to solve it?



BARYON

⇓

PROTON

(r ∼ 0.85 fm)

MESON

⇓

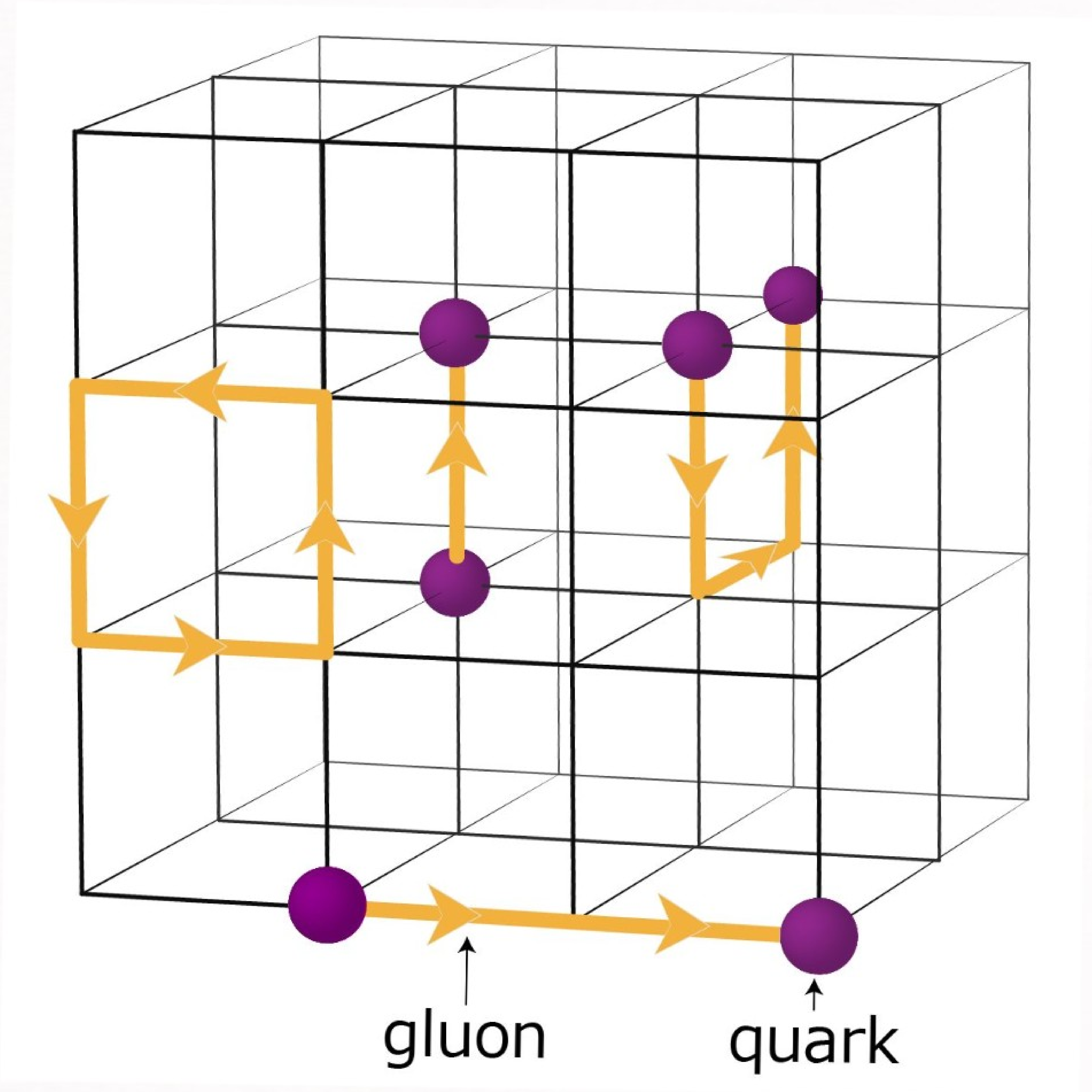
PION

(r ∼ 0.65 fm)

→ TOO LARGE TO BE DESCRIBED BY QCD

WE NEED ALTERNATIVE EXPLANATIONS

TWO POSSIBILITIES : THE FIRST ONE IS LATTICE QCD

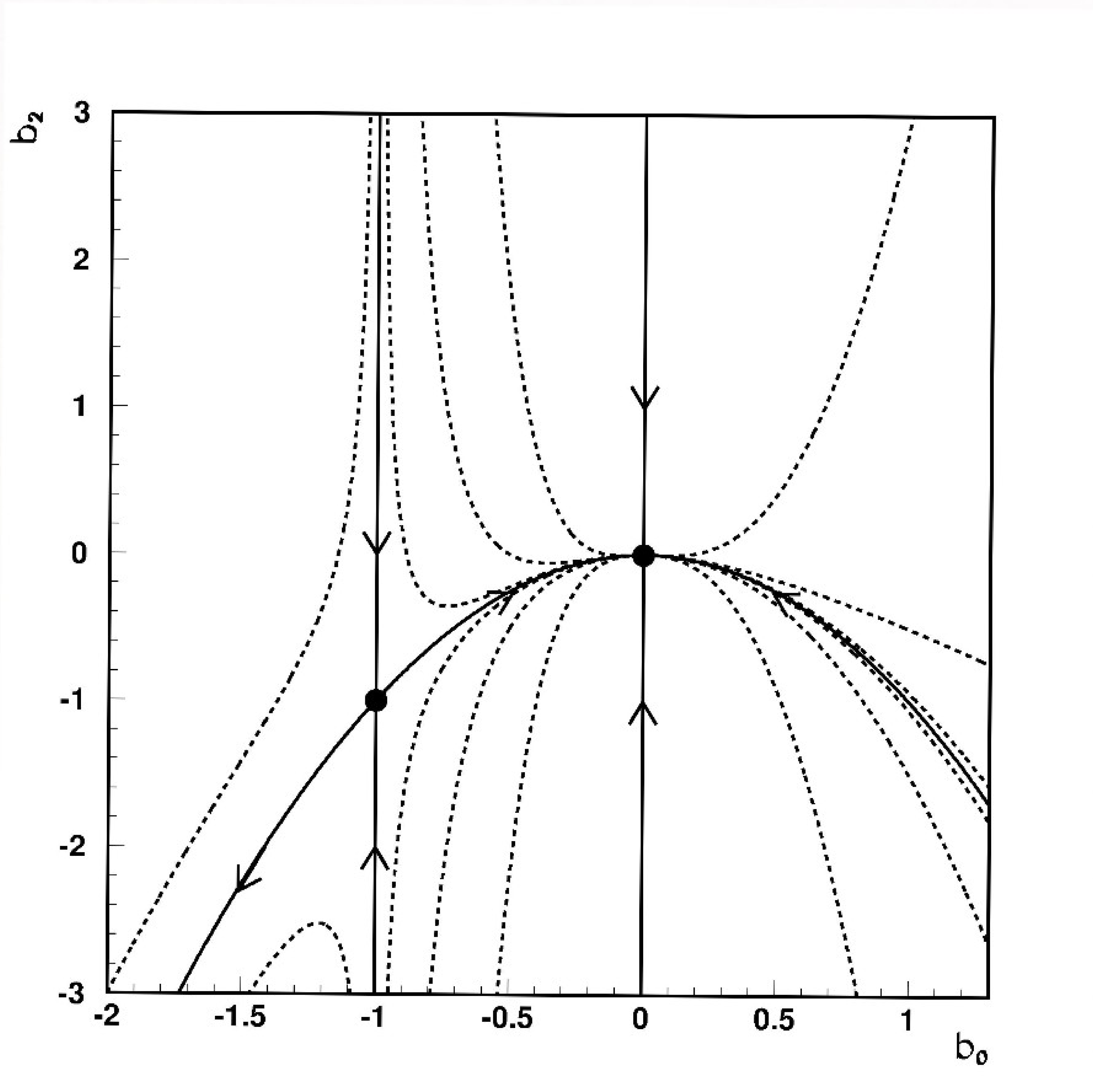


I will use a supercomputer to solve QCD numerically

$\Downarrow$

I don't have a supercomputer

THE SECOND ONE IS [EFFECTIVE FIELD THEORY]



I will use a series of techniques (called renormalization group analysis) to try to solve QCD indirectly at low energies

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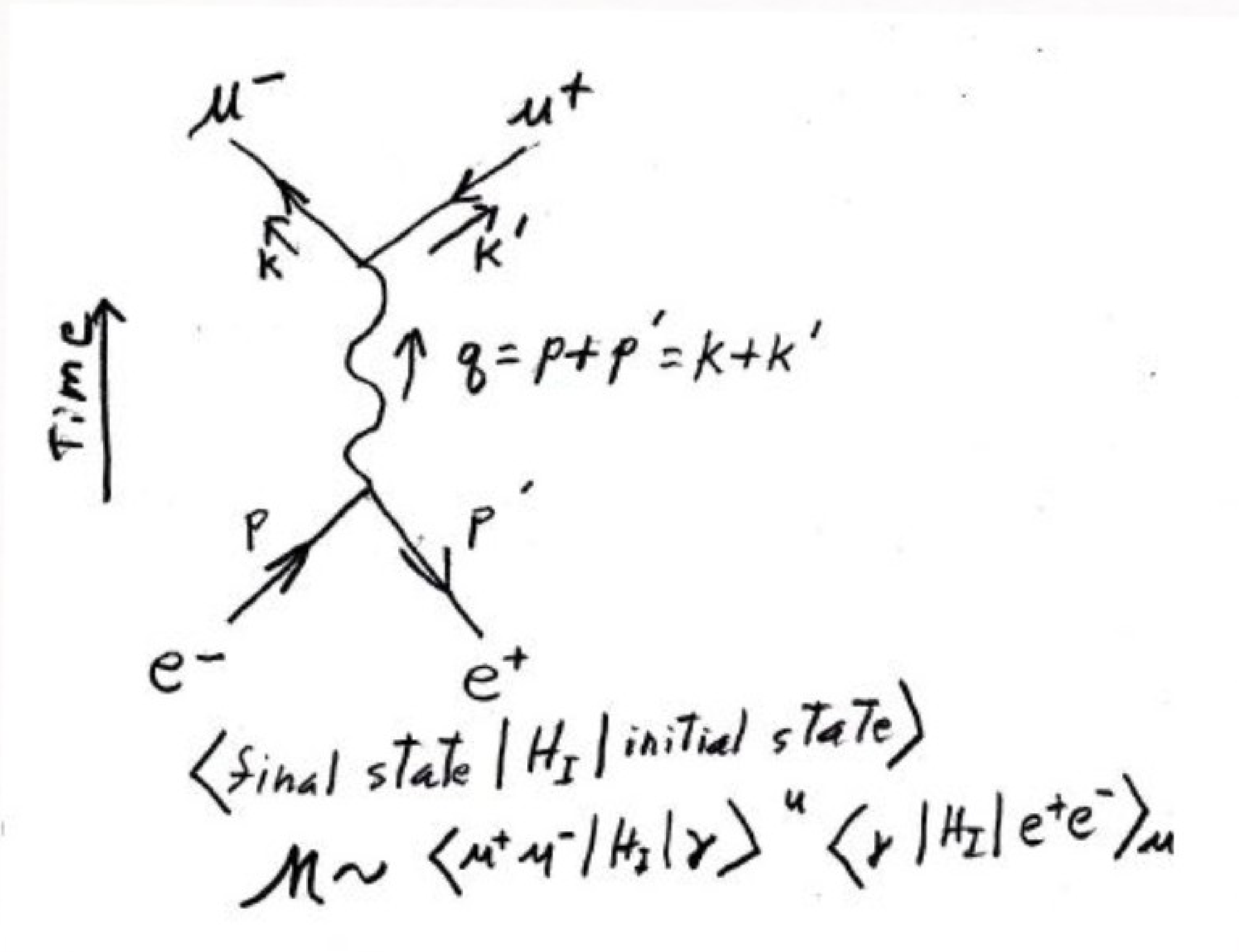
What I have is pen & paper (laptop, ChatGPT, mathematica)

[WHAT ARE EFFECTIVE FIELD THEORIES?]

→ Let's begin with RENORMALIZATION : ←

1) Once upon a time ... → conference in a town called Pocono in 1948



Feynman & Schwinger presented very weird methods to solve the infinities of QED

ORIGINALLY, RENORMALIZATION WAS...

→ a set of arcane methods to remove infinities

1) Example: the harmonic series $H(n) = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$

Diverges: $H(n) \sim \log n$ for large n

But we can use tricks: $\zeta(s) = 1 + \frac{1}{2^s} + \frac{1}{3^s} + \dots$

$$= \sum_{n=1}^{\infty} \frac{1}{n^s}$$ (Riemann zeta function)

ζ-regularization:

$$\lim_{n\to\infty} H(n) = \zeta(1) = -\frac{1}{12} \Rightarrow \underline{1 + \frac{1}{2} + \frac{1}{3} + \dots} = -\frac{1}{12}$$

[FEYNMAN DESCRIBED IT AS:]

我们为求出n和j所玩的壳层游戏，在专业上叫做“重正化”（renormalization）。但是，不管这个词听来多聪明，我却说这个过程是蠢笨的！求助于这类戏法妨碍了我们去证明量子电动力学在数学上的自洽性（self-consistent）。令人不解的是，尽管人们用了各种办法，这个理论至今仍未被证实是自洽的；我猜想，重正化在数学上是不合法的。我们还没有一种好的数学方法描述量子电动力学，这是肯定的——像这样描述n、j同m、e之间关系的语言不是好的数学。[23]

=D from "QED: the strange theory of light & matter"

You can read it latter

=D

- So it appears that the only things that depend on the small distances between coupling points are the values for n and j-theoretical numbers that are not directly obseroable any- way; everything else, which can be observed, seems not to be affected. [The shell game that we play to find n and j is technically called "renormalization." But no matter how clever the word, it is what I would call a] dippy process! Having to resort to such hocus-pocus has prevented us from proving that the theory of quantum electrodynamics is mathematically self-consistent. It's surprising that the theory still hasn't been proved self-consistent one way or the other by now; I suspect that renormalization is not mathematically legitimate. What is certain is that we do not have a good mathematical way to describe the theory of quantum electrodynamics: such a bunch of words to describe the connection between n and j and m and e is not good mathematics.
    - Richard Feynman, *QED: The Strange Theory of Light and Matter* (1985), Chap. 4. Loose Ends

But after 70 years our understanding of renormalization has vastly improved

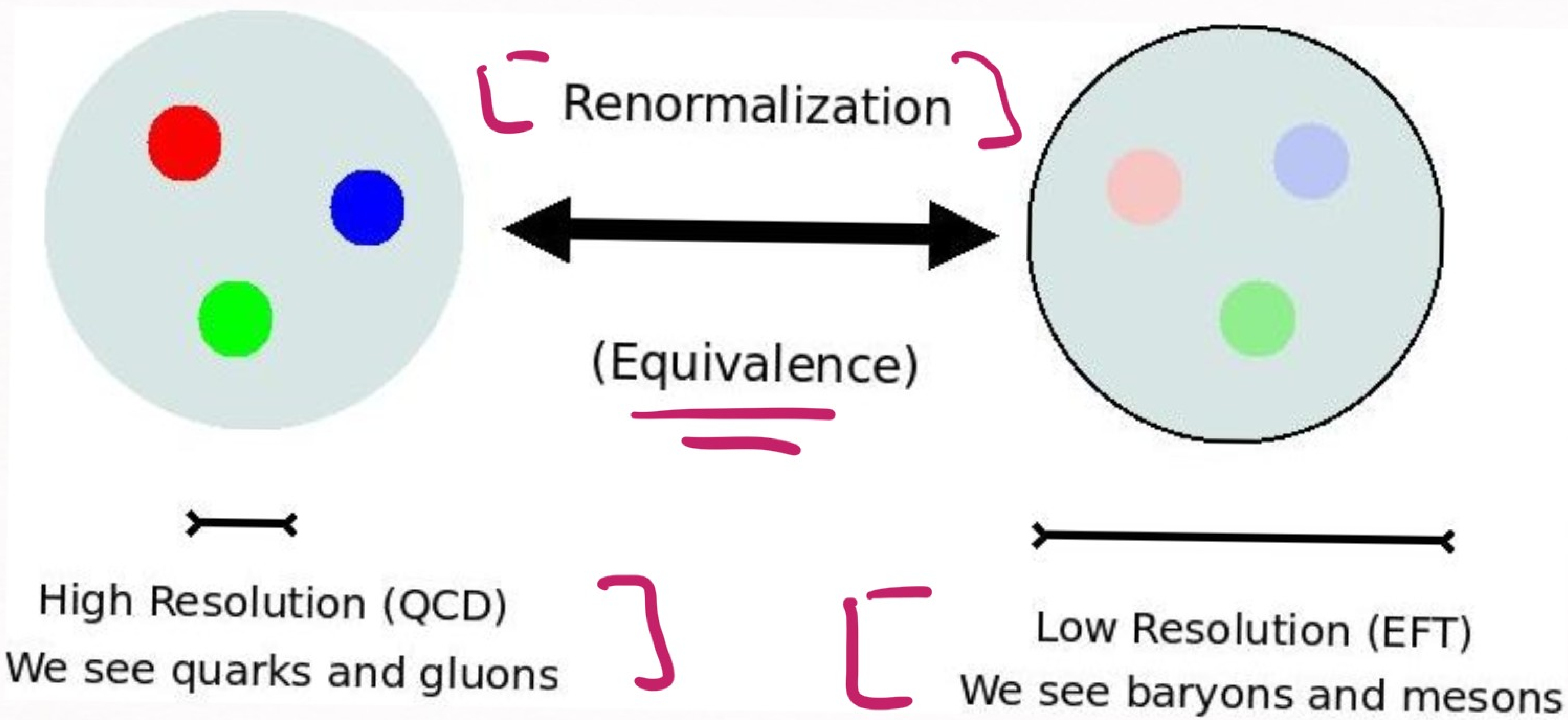
→ (particularly after K.G. Wilson) ←

— ⊗ —

BASIC IDEA : [Physics at long-distances does not depend on short-distance details]

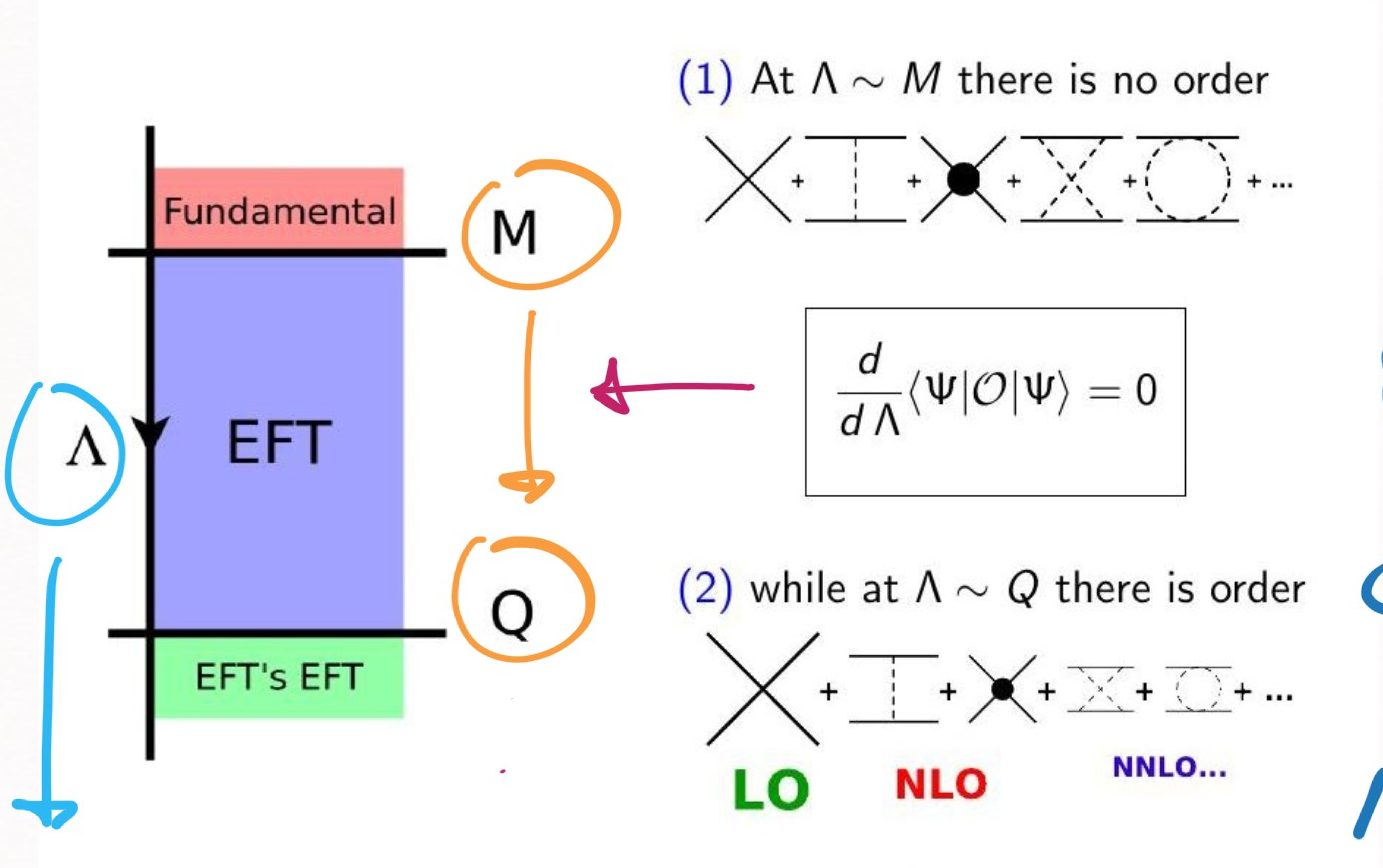
Renormalization is the mathematical implementation of this idea

BASICALLY, FOR NUCLEAR PHYSICS IT LOOKS LIKE THIS



RENORMALIZATION → HOW TO CONNECT IN A RIGOROUS WAY THESE TWO DIFFERENT VIEWS (LOW RESOLUTIONS VS HIGH RESOLUTION)

A CLOSELY RELATED CONCEPT IS: $\boxed{\text{EFFECTIVE FIELD THEORY}}$



Our RESOLUTION

M → scale of my fundamental theory

Q → low energy scale (things I am interested in)

Λ → energy scale we are looking at (RESOLUTION)

(Concentrate on the concepts only)

For QCD :

a) $M \sim 1$ GeV → Coincides w/ the mass of most hadrons

b) $Q \sim m_\pi \sim 140$ MeV → Coincides w/ the separation of nucleons within nuclei : $\frac{1}{Q} \sim \frac{\hbar c}{m_\pi}$

~ 1.4 fm

High & low energy scales in nuclear physics

separation → 1.7 fm

OUR VIEW WILL DEPEND ON Λ: $\rightarrow$ $\frac{1}{\Lambda}$ is the resolution

1) $\Lambda > M$ $\rightarrow$ I will only see quark & gluons

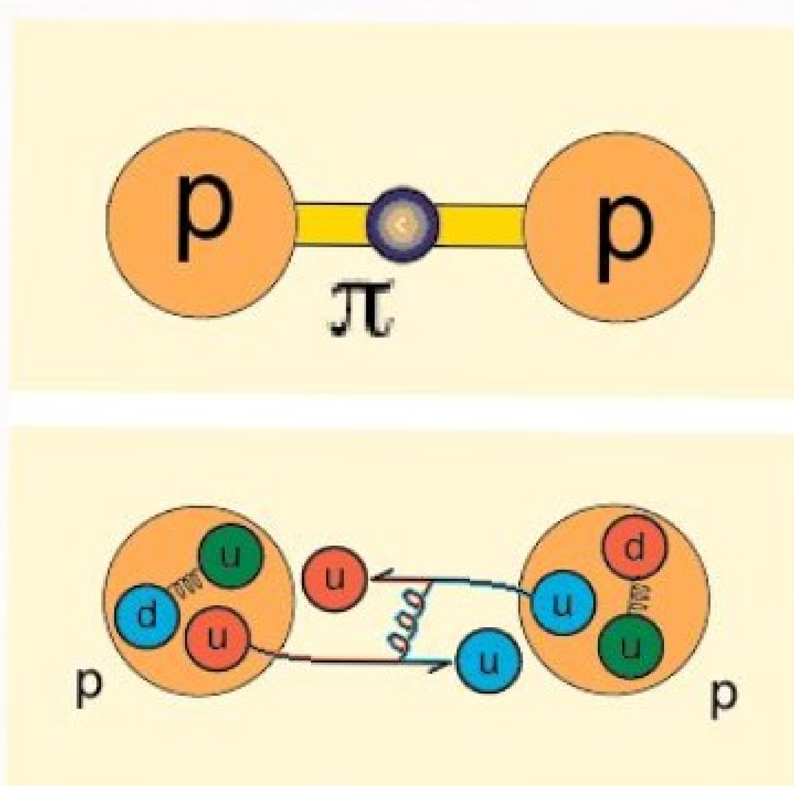
2) $\Lambda < M$ $\rightarrow$ I will only see baryons & meson

3) $\Lambda \sim M$ $\rightarrow$ Both descriptions will be valid

With this, the basics of building an EFT are:

1) $\Lambda \sim M$ $\rightarrow$ two equivalent descriptions

$\frac{d}{d\Lambda}(\,)=0$



$\boxed{\text{baryons \& mesons}} \longleftrightarrow \boxed{\text{quarks \& gluons}}$

2) $\Lambda \rightarrow Q$

[both descriptions will be equivalent if and only if observables are cutoff independent]

$$\frac{d}{d\Lambda}[\dots] = 0$$

☺ → We will move smoothly from one description to the other

Mathematically, condition 2) can be written as:

$$\left[\frac{d}{d\lambda} \langle \Psi | \hat{O} | \Psi \rangle = 0 \right] \rightarrow$$ If this doesn't change, the resolution s/λ doesn't matter

$$\langle \hat{O} \rangle = \langle \Psi | \hat{O} | \Psi \rangle$$

Expected value of some observable (described by the operator $\hat{O}$)

It will be complicated:

$|\psi\rangle = |\psi(\lambda)\rangle \quad \hat{O} = \hat{O}_\lambda$

$$\frac{d}{d\lambda} \langle \psi(\lambda) | \hat{O}_\lambda | \psi(\lambda) \rangle = 0$$

(BUT THE IDEA IS EASY)

[EQUIVALENCE :] OBSERVABLE QUANTITIES SHOULD BE THE SAME INDEPENDENTY OF THE RESOLUTION

This cannot change

$$\overline{\left|\langle\hat{O}\rangle\right|} = \langle\psi|\hat{O}|\psi\rangle \Rightarrow |\psi\rangle \text{ is } \underline{\text{not}} \text{ an observable}$$

this is the only thing we can observed

$|\psi(\Lambda > M)\rangle \rightarrow$ Quark & Gluons

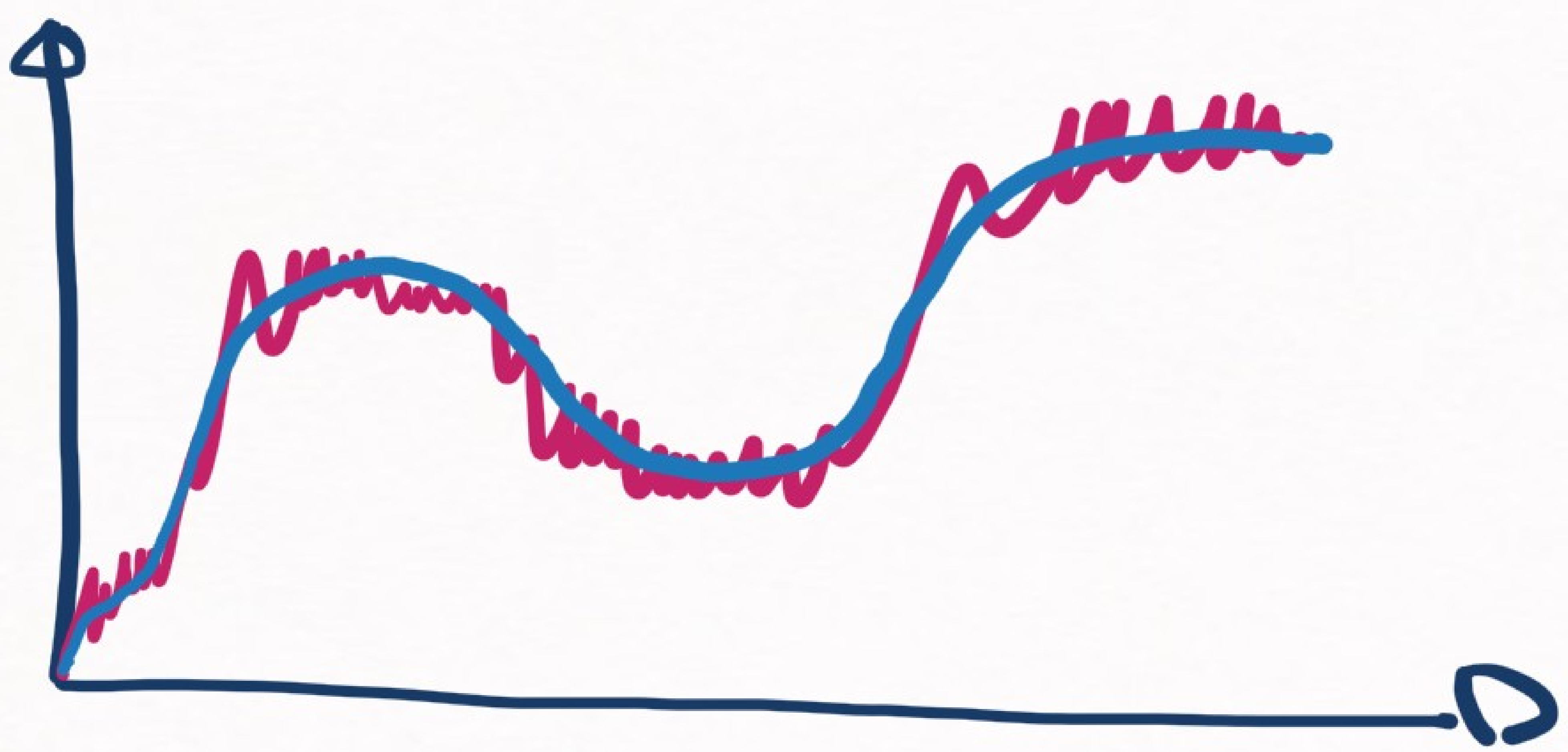
$\hat{O}_{\Lambda > M}$

$|\psi(\Lambda < M)\rangle \rightarrow$ Baryons & mesons

$\hat{O}_{\Lambda < M}$

EXAMPLE :

[LOW RESOLUTION ψ] VS [HIGH RESOLUTION ψ]



$$\boxed{\langle\psi|\hat{O}_L|\psi\rangle \simeq \langle\psi|\hat{O}_R|\psi\rangle}$$

It is not important which $|\psi\rangle$ do I use

[At low energies I can't see these short-distance kinks of $|\psi\rangle$]

→ This is very abstract, but we will understand it better once we see a few examples in the next lesson

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FOR THE MOMENT CONCENTRATE ON THE GENERAL IDEA

RECAP !

a) QCD can't be solved at distances above 0.5 fm

→ Apparently, we can't explain nuclear physics

b) But ∃ techniques to overcome this limitation

Baryons & Mesons (low resolution picture) ⟷ Quarks & Gluons (high resolution picture)

[RENORMALIZATION]

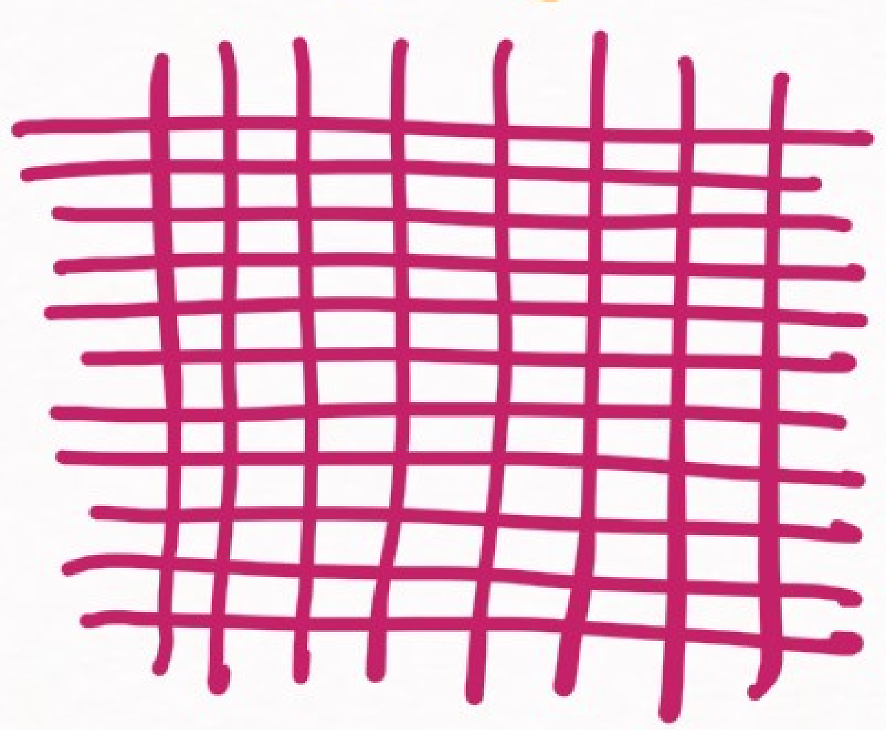
⇑

OUR TOOL

c) Basic idea: [Physics at long distances should not depend on the short distance details]

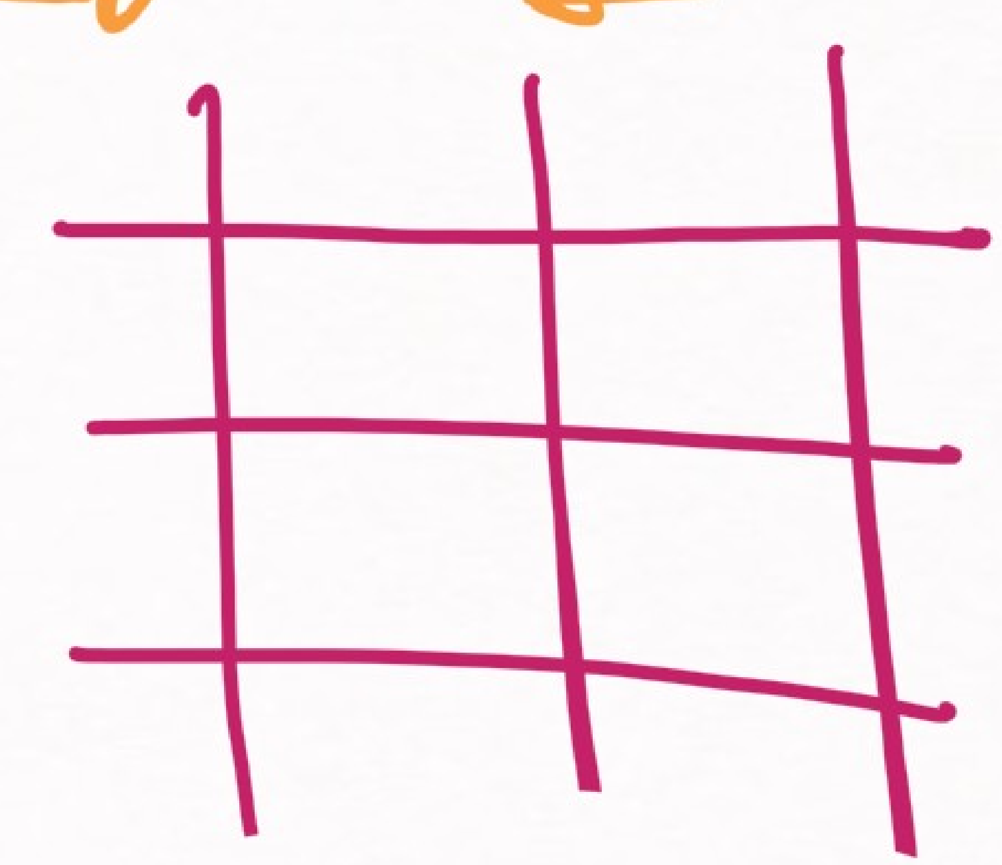
[d) In abstract terms, its mathematical implementation is] $R_c \sim \frac{1}{\Lambda}$

[R_c]



$$\Rightarrow \boxed{\frac{d}{dR_c}\langle\psi|\hat{O}|\psi\rangle = 0} \Rightarrow$$

[R_c]



(high res) implies equivalence (low res)

SEE YOU ON TUESDAY

15:50