NUCLEAR PHYSICS 7

a) BEYOND THE PION ← PROPOSED ORIGIN OF NUCLEAR FORCES

WHAT DO NUCLEONS EXCHANGE (BESIDE PIONS)?

b) QUANTUM CHROMODYNAMICS (PART I)

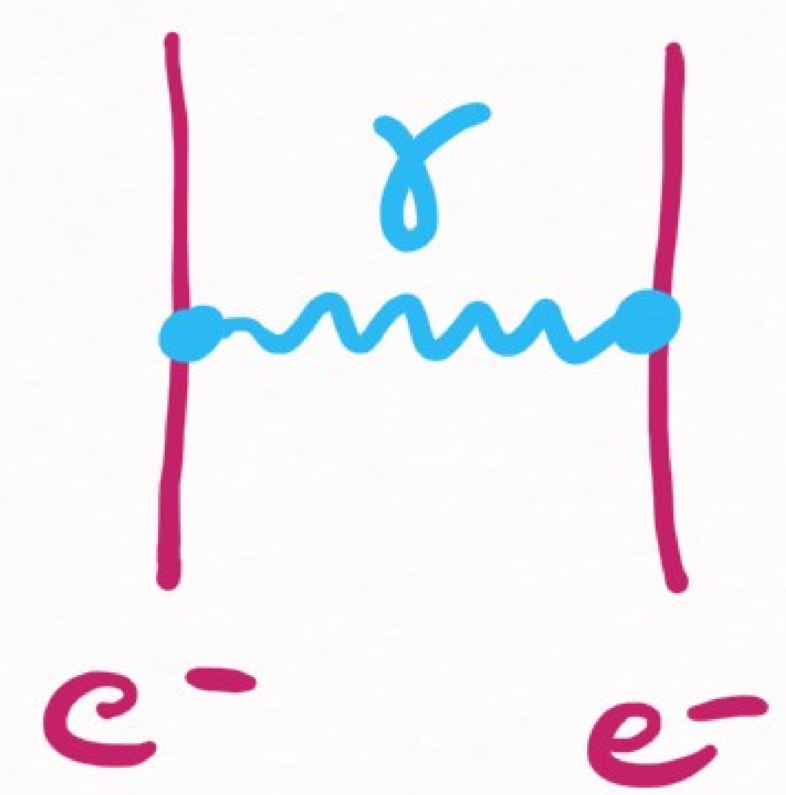
NUCLEONS → $\underbrace{\text{QUARKS \& GLUONS}}_{\text{QCD}}$

(Qualitative explanation of QCD only)

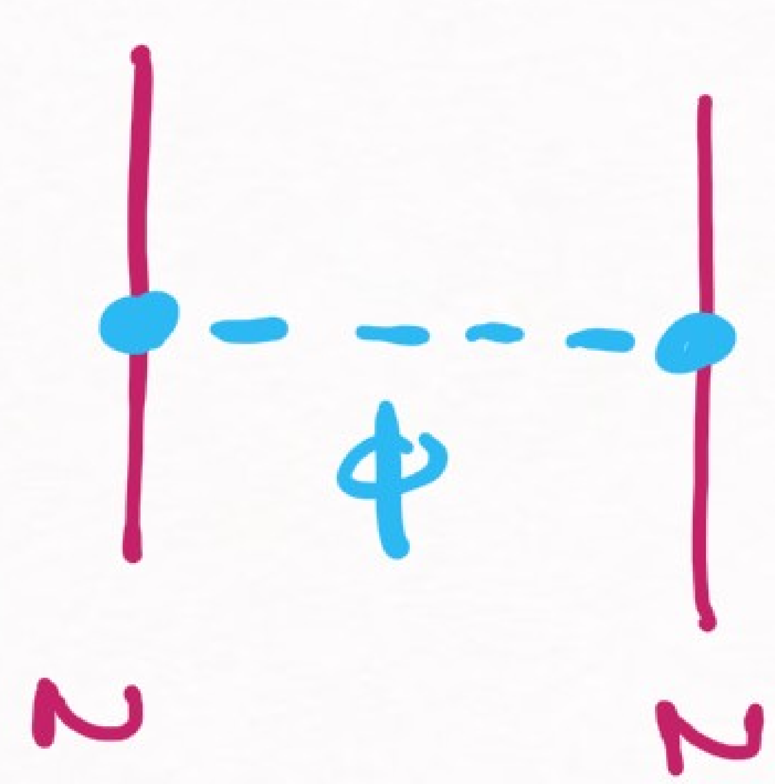
RECAP

HOW DID WE DETERMINE THE ORIGIN OF THE NUCLEAR FORCES?

1) QFT: forces are the result of the exchange of a boson

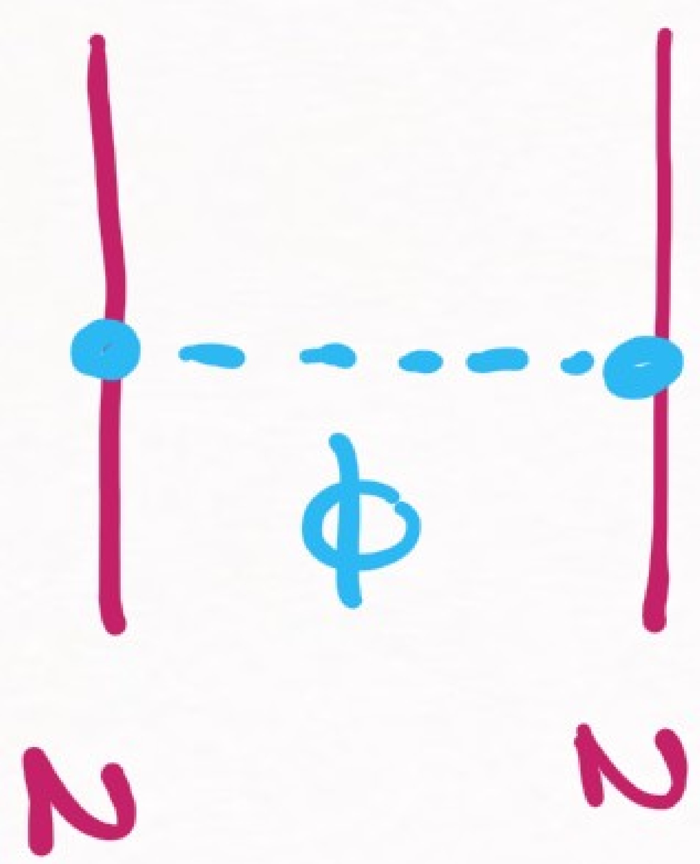


(Exchange of a photon) $\Rightarrow$ [COULOMB] $\quad V(r) = \dfrac{e_1 e_2}{4\pi r^2}$



$\Rightarrow$ A DIFFERENT POTENTIAL (depends on the exchanged particle)

2) Yukawa : the nuclear force comes from the exchange of a massive boson



$\Rightarrow$ Easiest assumption : a scalar field $\rightarrow$ (*)

(*) **Reminder** | $\phi = \phi(\vec{r})$

$$\left[V(\vec{r}) = -g_Y^2 \, \frac{e^{-mr}}{4\pi r} \right]$$ YUKAWA POTENTIAL

$$\left.\begin{array}{l} \vec{r} \rightarrow -\vec{r} \\ \phi(\vec{r}) \rightarrow \phi(-\vec{r}) \end{array}\right\}$$ this is a scalar field

3) There is a constraint though:

THE DEUTERON QUADRUPOLE MOMENT

$\Rightarrow$ (non-central potential)

$$V(\vec{r}) = \underbrace{-g_r^2 \frac{e^{-mr}}{4\pi r}}_{\text{~~Exchange of a scalar~~}} \Rightarrow \text{central potential}$$

$\Rightarrow$ OUT

$\Downarrow$

$$\boxed{Q_d = 0}$$

But $Q_{d,exp} > 0$ ($Q_{d,exp} = 0.286\ \text{fm}^2$)

(Need to try a different hypothesis)

4) SOLUTION: the meson we are looking for is actually a pseudoscalar ⊛

↳ REMINDER $\phi = \phi(r)$

$$\vec{r} \rightarrow -\vec{r}$$

$$\phi(\vec{r}) \rightarrow -\phi(-\vec{r})$$

this is a pseudoscalar field ↳ ⊛

[The tensor force]

$$V(\vec{r}) = \vec{\tau}_1 \cdot \vec{\tau}_2 \left[\vec{\sigma}_1 \cdot \vec{\sigma}_2 \, W_C(r) + \underbrace{(3\,\vec{\sigma}_1 \cdot \hat{r} \; \sigma_2 \cdot \hat{r} - \vec{\sigma}_1 \cdot \vec{\sigma}_2)}_{[S_{12}(\hat{r})]} W_T(r) \right]$$

$W_C(r), W_T(r) \geq 0$

Deuteron case:

$$\boxed{\vec{\tau}_1 \cdot \vec{\tau}_2 = -3} \longrightarrow \text{ISOSPIN FACTOR}$$

$$+$$

$$\boxed{Q_d > 0} \Rightarrow \boxed{\text{In agreement with experiment}}$$

NOTE: We also tried a vector meson, but this predicted $Q_d < 0$

(and we rejected this hypothesis)

$\boxed{\text{SUMMARY :}}$ the nuclear force is generated by the exchange of a pseudoscalar meson

→ We call this meson the pion

$\boxed{\text{BUT} \ldots}$ → the pion does not explain all the properties of the nuclear force

1) Short-range → $V(r) \sim \frac{e^{-mr}}{r} \times$ (a lot of things)

OK ✓

$\boxed{mr \lesssim 1}$ → short-ranged ✓

2) Attractive at medium distances → ← [It depends]

MAYBE [Ignore $S_{12}(\vec{r})$] $V(r) \sim \underbrace{\vec{\tau}_1 \cdot \vec{\tau}_2}_{\pm} \, \vec{\sigma}_1 \cdot \vec{\sigma}_2 \, \frac{e^{-mr}}{r}$

S-waves : $\left[\begin{array}{l} S=0, \ \vec{\tau}_1\cdot\vec{\tau}_2 = 1, \ \vec{\sigma}_1\cdot\vec{\sigma}_2 = -3 \\ S=1, \ \vec{\tau}_1\cdot\vec{\tau}_2 = -3, \ \vec{\sigma}_1\cdot\vec{\sigma}_2 = +1 \end{array}\right)$ → correct in S-waves

~ 3) Repulsive at short-distances → See previous slide

(MAYBE) S-waves ⇒ Incorrect

✓ 4) Non-central → $\boxed{\text{Yes !!}}$ → this is how we chose a pseudo scalar meson

(CHECK!)

✓ 5) Does not distinguish neutrons & protons

(CHECK!) → Isospin formalism ($\vec{\tau}_1 \cdot \vec{\tau}_2$ factor)

$\boxed{\text{REMINDER :}}$ ISOSPIN

$|p\rangle, |n\rangle \longrightarrow$ Equivalent : $\begin{pmatrix} |p\rangle \\ |n\rangle \end{pmatrix} \rightarrow U \begin{pmatrix} |p\rangle \\ |n\rangle \end{pmatrix}$

(What does it mean?)

$\underbrace{\text{with } UU^\dagger = \mathbb{1}}_{\text{this must be a symmetry of the system}}$

[U 2x2 matrix such that $UU^\dagger = \mathbb{1}$] $\longleftarrow$

$\Downarrow$

Definition of the SU(2) group

Isospin symmetry $\longleftrightarrow$ SU(2) symmetry

→ [SU(2) is also the symmetry group for spinors:]

$$\text{spin-}\tfrac{1}{2}\text{ particle} \rightarrow \left.\begin{aligned} |+\rangle &= |\overset{s}{\tfrac{1}{2}}\ \overset{m_s}{\tfrac{1}{2}}\rangle \\ |-\rangle &= |\tfrac{1}{2}\ -\tfrac{1}{2}\rangle \end{aligned}\right\}\ \text{spin up \& down}$$

$$\begin{pmatrix} |+\rangle \\ |-\rangle \end{pmatrix} \longrightarrow U \begin{pmatrix} |+\rangle \\ |-\rangle \end{pmatrix} \Big\{ \text{SU(2)-spin symmetry}$$

$$UU^\dagger = \mathbb{1}$$

$\Rightarrow$ ISOSPIN $\cong$ SPIN

(structurally identical)

$\boxed{\text{ISOSPIN} \cong \text{SPIN}}$ → (COPY THE SPIN FORMALISM TO ISOSPIN)

$|p\rangle = |\tfrac{1}{2}\ \tfrac{1}{2}\rangle_I \cong \underline{|+\rangle_S}$

$|n\rangle = |\tfrac{1}{2}\ {-\tfrac{1}{2}}\rangle_I \cong \underline{|-\rangle_S}$

$\quad\quad\quad\ \ I\ \ M_I$

[SPIN OPERATORS :] ⟹ ISOSPIN OPERATORS:

$\vec{\sigma} = \{\sigma_1, \sigma_2, \sigma_3\}$ ⟶ $\vec{\tau} = \{\tau_1, \tau_2, \tau_3\}$

$\sigma_3|+\rangle = |+\rangle,\ \sigma_3|-\rangle = -|-\rangle$ → $\tau_3|p\rangle = +|p\rangle$

$\tau_3|n\rangle = -|n\rangle$

$\vec{\sigma}_1 \cdot \vec{\sigma}_2 = +1$ or -3

$(S=1)\quad (S=0)$

$\vec{\tau}_1 \cdot \vec{\tau}_2 = +1$ or -3

$(I=1)\quad (I=0)$

BACK TO THE PROBLEM AT HAND...

3) Nuclear forces are repulsive at short distances

(the pion alone doesn't explain this)

=D We need something beyond pions to explain this property

What can we do? =D

POSSIBLE IMPROVEMENTS TO THE PION THEORY:

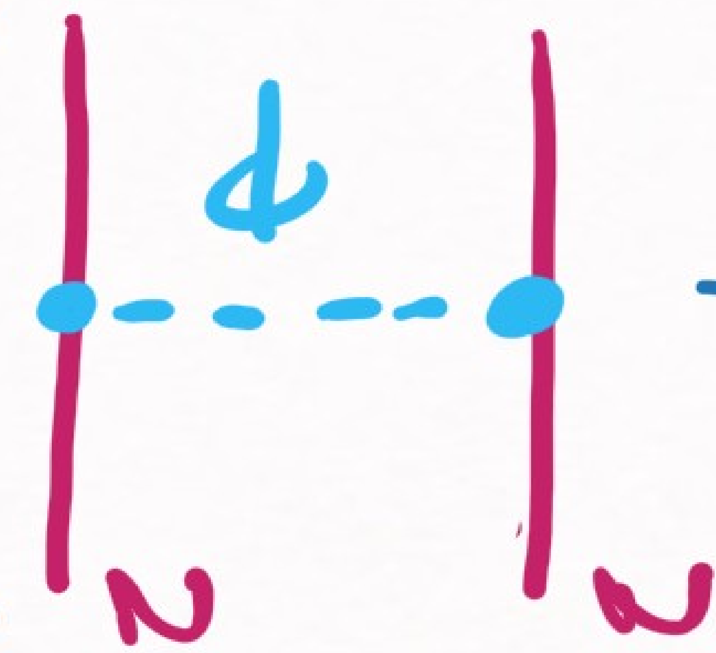
a) Include more pions $\leftarrow$

b) Include more types of mesons $\leftarrow$

$\Rightarrow$ Very much related to the history of nuclear physics

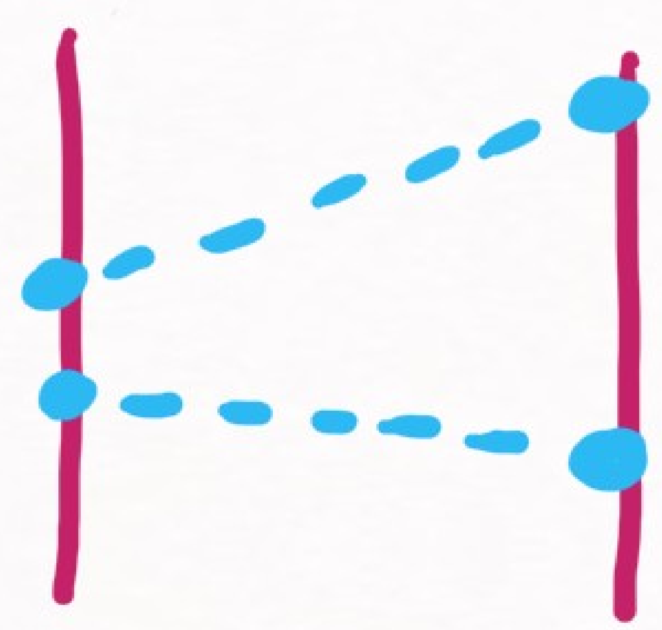
a) MORE PIONS

REMINDER: HOW WE CALCULATE V(r)

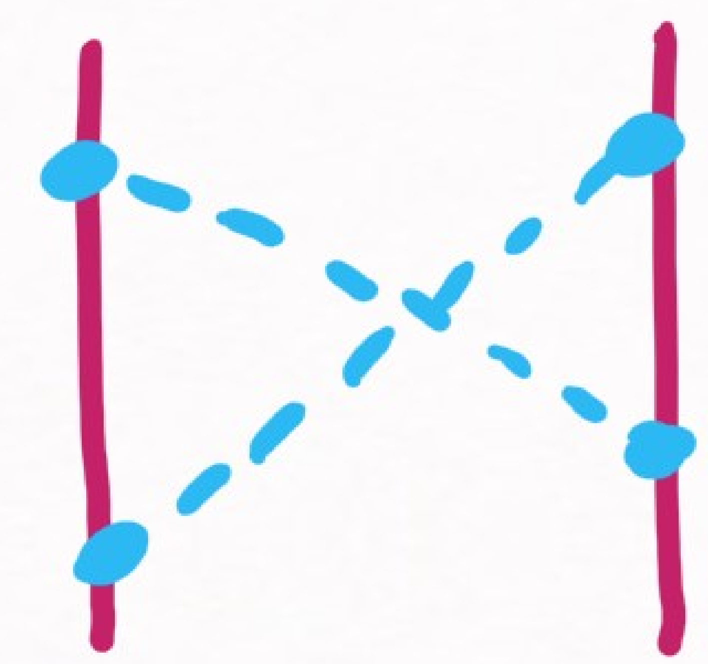
$$V(r) = E\Big(\ $$  $$\ + \cdots\Big) - E\Big(\ $$  $$\ \Big)$$

→ by calculating this difference (in second order pertorbation theory) ← QM explanation
(or in QFT)

BUT WE CAN GO FURTHER:



(planar box)

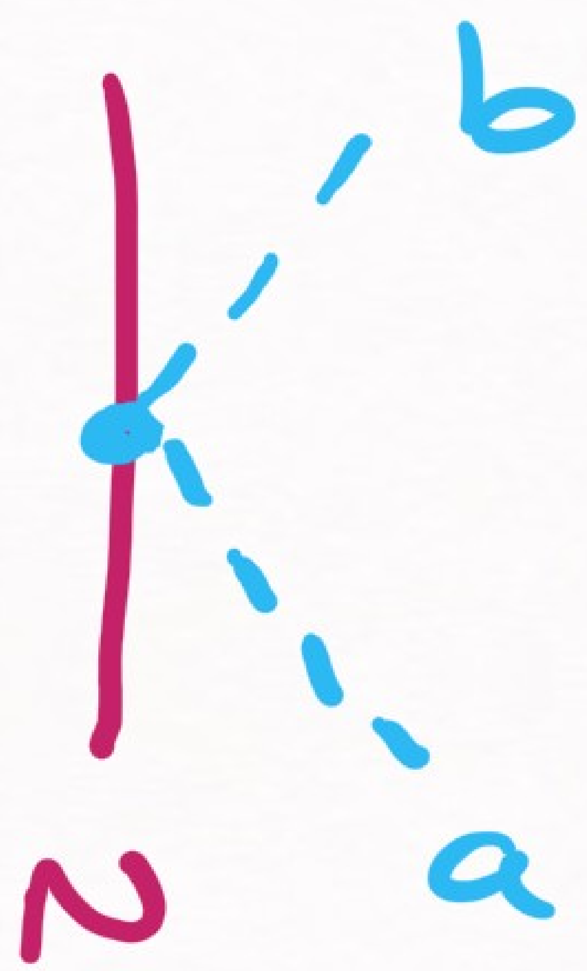


(crossed box)

(EXCHANGING TWO PIONS)

V(r) CAN BE CALCULATED → FOURTH ORDER PERTURBATION THEORY

OR CONSIDER NEW INTERACTIONS:



→ Weinberg-Tomozawa term

$$\langle N(\underbrace{2\pi}_{\pi_a \pi_b})|\phi|N\rangle \propto \epsilon_{abc}\tau_c$$

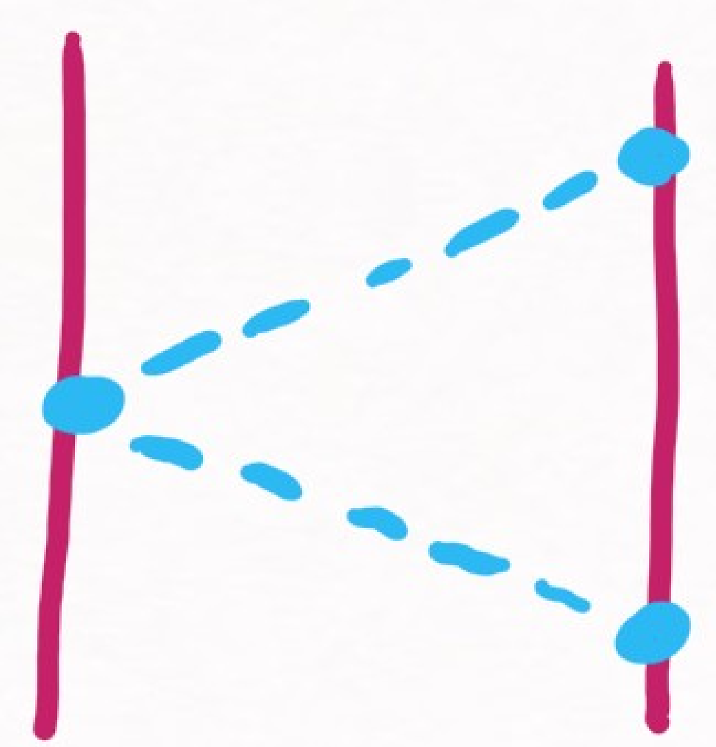
Levi-Civita symbol

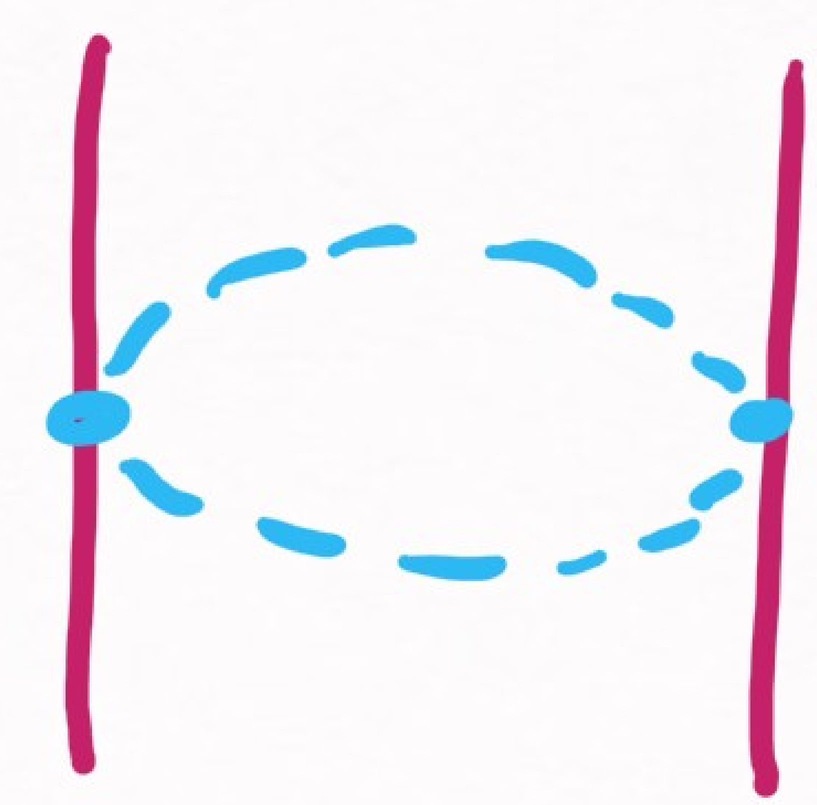
isospin operator

(this for the creation or annihilation of two pions at the same time)

→ (THE DETAILS ARE NOT IMPORTANT)

WEINBERG-TOMOZAWA → NEW TWO-PION EXCHANGE CONTRIBUTIONS





→ USE PERTURBATION THEORY TO CALCULATE THESE TWO CONTRIBUTION

(OR USE QFT IF YOU ALREADY KNOW IT)

TO THE POINT: these diagrams were calculated in the 50's and....

WHY? ← ... failed!

a) They were using the wrong pion dynamics

b) Back then, renormalization was unknown (previous diagrams contain divergences)

[WRONG PION DYNAMICS?] WHAT!?

→ [INVERSE SCATTERING PROBLEM]

→ Different potentials lead to the same predictions

$V(r), V'(r), V''(r) \Rightarrow$ SAME OBSERVABLES

[OFF-SHELL BEHAVIOR] ⇐ (MOMENTUM SPACE REPRESENTATION OF THE POTENTIAL)

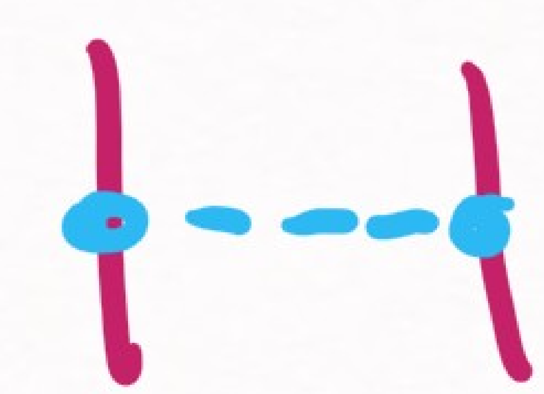
[THIS PROBLEM ALSO EXISTS IN QFT:] (happens in all quantum theories)

1) Pion dynamics (1950's version)

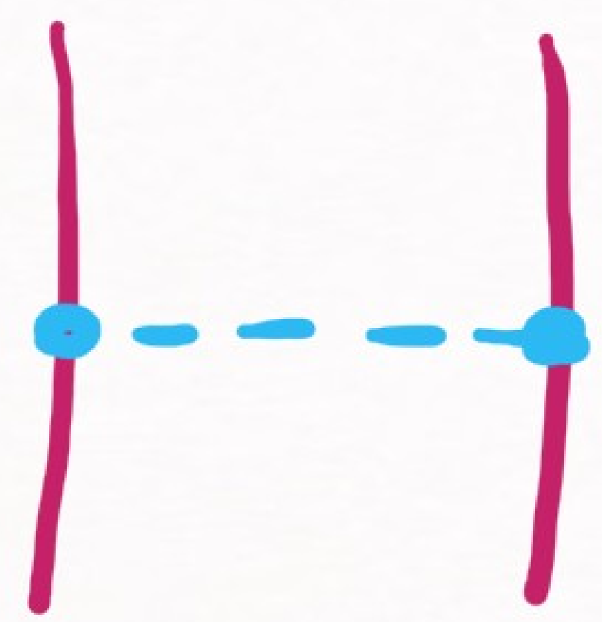
$$\mathcal{L}_{int} = i g_\pi \bar{\psi}_N \gamma^5 \vec{\tau} \cdot \vec{\pi} \, \psi_N$$

2) Pion dynamics (modern version)

$$\mathcal{L}_{int} = \frac{g}{2 f_\pi} \bar{\psi}_N \gamma^5 \gamma^\mu \vec{\tau} \cdot \partial_\mu \vec{\pi} \, \psi_N$$

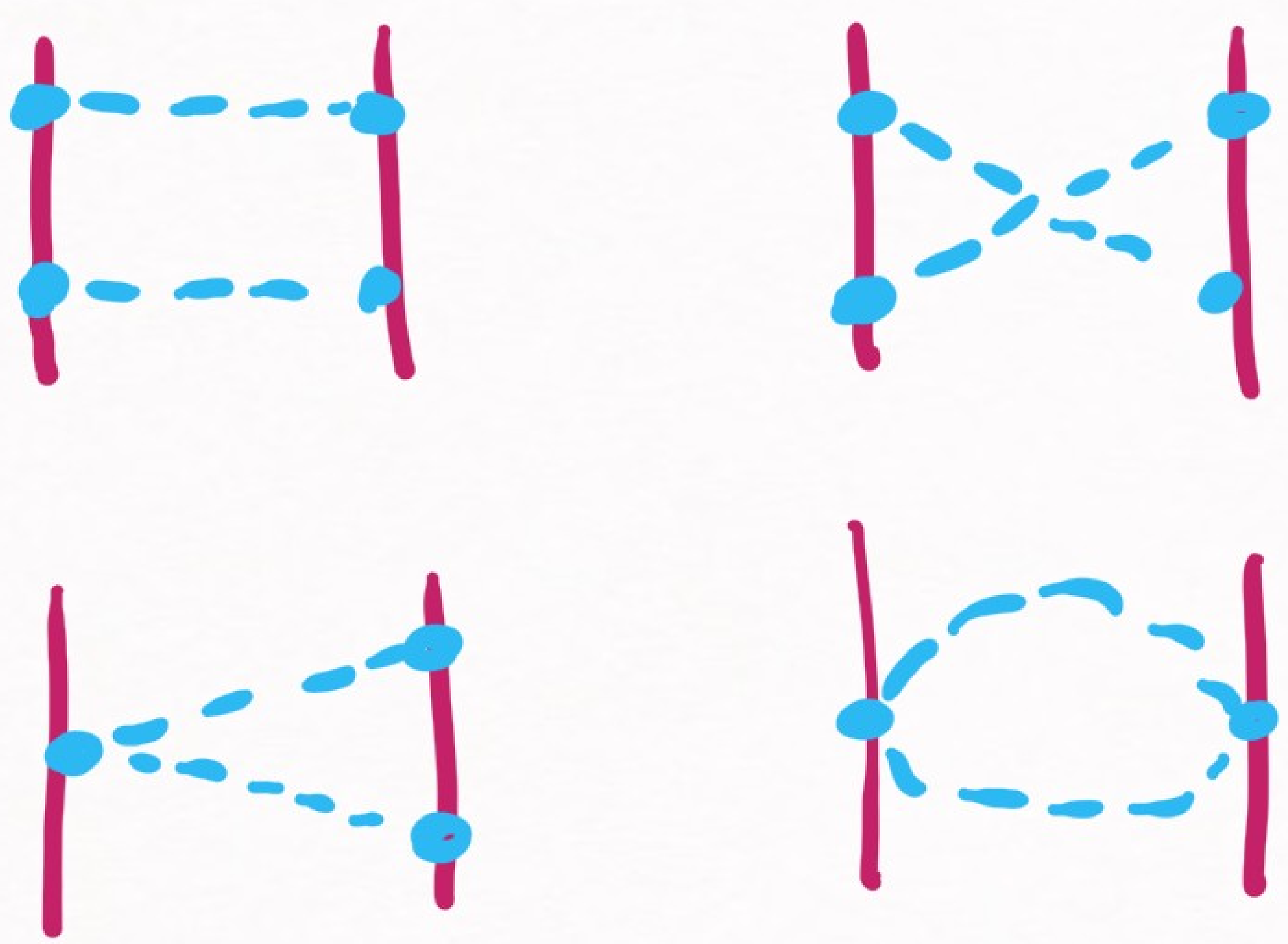
1), 2) generate the same potential for 

[ONE PION EXCHANGE]



1) & 2) $\Rightarrow$ IDENTICAL RESULTS

[TWO PION EXCHANGE]



1) & 2) $\Rightarrow$ DIFFERENT RESULTS

IT SO HAPPENS THAT 1) LEADS TO WRONG RESULTS

Only type of pion dynamics known back then (1950)

There is a problem w/ multipion exchanges! CHANGE STRATEGY!!

Thus, at that time the only solution was...

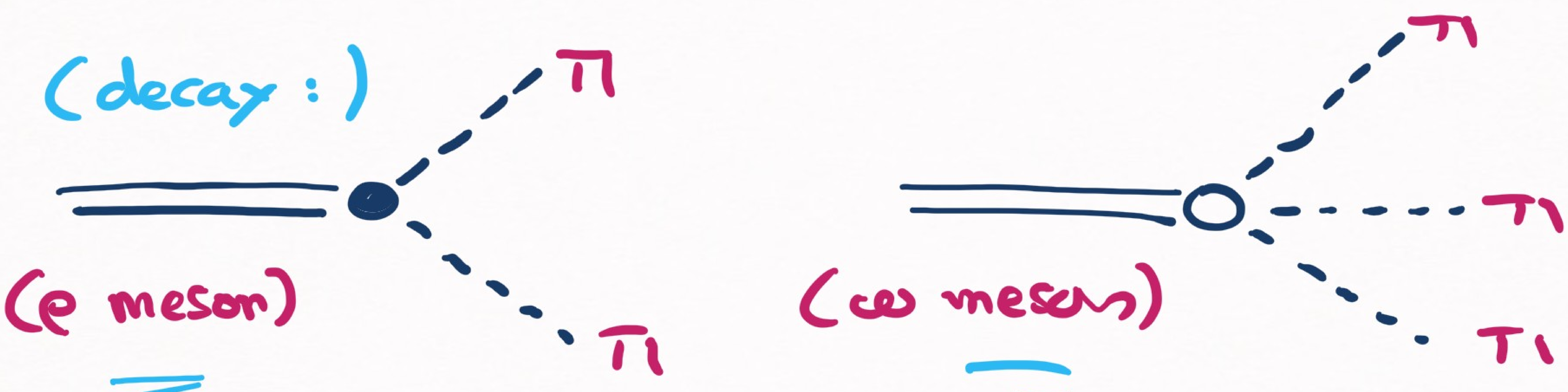
b) TO INCLUDE MORE TYPES OF MESONS

[MESON THEORY OF NUCLEAR FORCES]

We are in the 1950's and we notice:

1) pions don't work → they gave up to this idea

2) pions are related to other particles

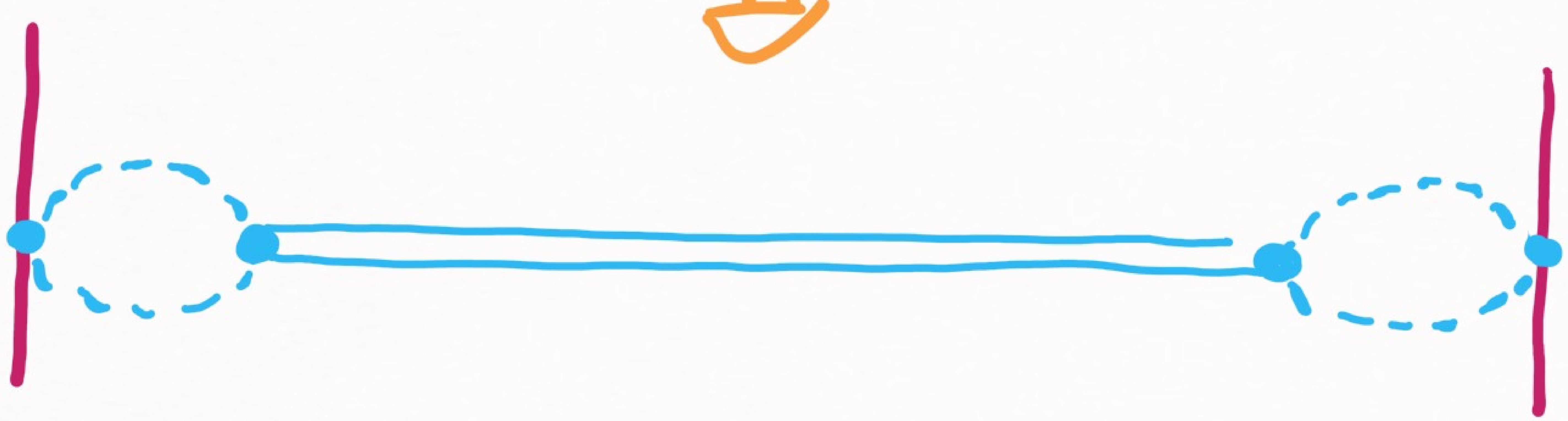


So we are still in the 1950's and we think:

We might try this (maybe this solves our pion problem)

(a lot of bubbles)

$\Downarrow$



Or we might simplify the previous situation into this

Thus, we invent a distinction: (1950's)

⇒ a) Correlated two-pion exchange (because: )

IMPORTANT → It can be substituted by other mesons

→ Most important contribution to multipion exchanges

⇒ b) Uncorrelated two-pion exchange

UNIMPORTANT → Cannot be substituted by other mesons

→ Small contribution to the nuclear force

And we propose a new type of potential:

$$\Rightarrow V_{NN}(\vec{r}) = \underline{\underline{V_\pi(\vec{r})}} + \underbrace{\sum}_{\text{(other mesons)}} V_{meson}(\vec{r}) + \cancel{\text{corrections}}$$

(2π, 3π, ...)

UNIMPORTANT

(other mesons) = $\boxed{\sigma}$, $\boxed{\rho}$, $\boxed{\omega}$ + others

σ: $I = 0$, $J^P = 0^+$ (SCALAR)

ρ: $I = 1$, $J^P = 1^-$

ω: $I = 0$, $J^P = 1^-$

(VECTORS: ~ HEAVY PHOTONS)

"LIKE"

Each of these mesons has a job:

$$V_{NN}(\vec{r}) = \underbrace{V_\pi(\vec{r})}_{\underline{Q_d > 0}} + \underbrace{V_\sigma(\vec{r})}_{\text{INTERMEDIATE RANGE ATTRACTION}} + \overbrace{\underbrace{V_\rho(\vec{r})}_{\Delta Q_d < 0}}^{\text{BALANCE}} + \underbrace{V_\omega(\vec{r})}_{\text{SHORT RANGE REPULSION}}$$

(Q_d FROM PIONS IS USUALLY TOO LARGE)

$$Q_d = \underbrace{Q_d^\pi}_{(>0)} + \underbrace{Q_d^\rho}_{(<0)} > 0$$

SCALAR MESON → POTENTIAL ALWAYS ATTRACTIVE

However, there were problems: 1), 2)

(this always happens)

1) σ meson was not found experimentally back then

(nowadays we know it exists)

2) necessary repulsion from the ω meson incompatible with known symmetry relations

SU(3)-flavor $\rightarrow$ $\boxed{g_\omega = 3 g_\rho}$

But what we have:

$$g_\omega \gg 3 g_\rho$$

FORM FACTORS

3) Singular potentials: What are they?

tensor force

$V_\pi(\vec{r})$ contains $\boxed{S_{12}(\hat{r})\, W_T(r)}$

$W_T(r) \xrightarrow[mr \ll 1]{} \frac{1}{r^3}$

problematic

$V_\rho(\vec{r})$ also contains it

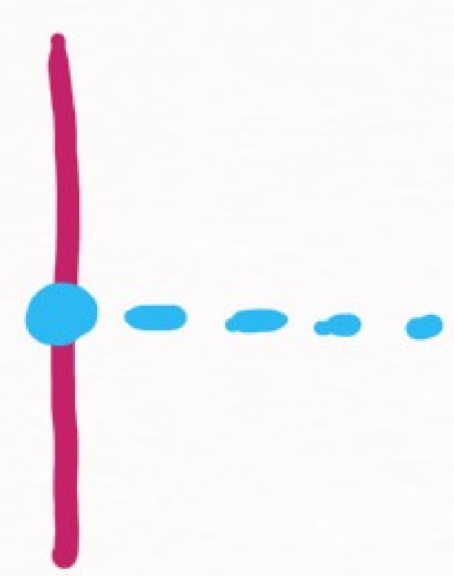
PREDICTION $\Rightarrow$ $\boxed{B_{deuteron} \to \infty}$

NOT COMPATIBLE WITH EXPERIMENT

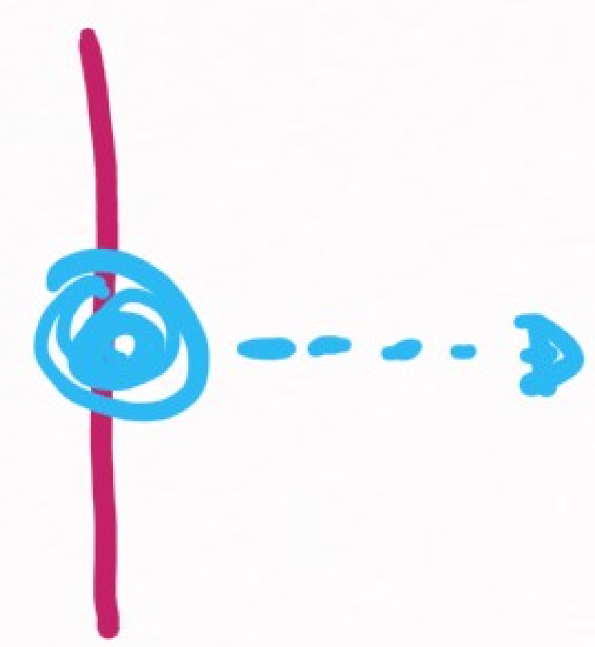
$B_d \simeq 2.2$ MeV

FORM FACTORS (2)

SOLUTION → Include form factors → Takes into account that hadrons have a finite size



$\Rightarrow$



$q = |\vec{q}|$ $\quad q \to 0$

$$\langle N\phi | H | \phi \rangle \propto 1$$

$$\langle N\phi(\vec{q}) | H | \phi \rangle \propto f\left(\frac{q}{\Lambda}\right) \to 1 \;(q \to 0), \quad \to 0 \;(q \to \infty)$$

JUSTIFICATION → HADRONS HAVE A FINITE SIZE

FORM FACTORS ③

[POTENTIALS ARE NOW FINITE]

DEUTERON NOW HAS A FINITE BINDING ENERGY

$$V_Y(\vec{r}) = -g_Y^2 \frac{e^{-mr}}{4\pi r} \Rightarrow V_Y(\vec{r}) = -g_Y^2 \frac{e^{-mr}}{4\pi r} g\left(\frac{r}{R_c}\right)$$

$$W_T(r) \sim \frac{1}{r^3} \Rightarrow W_T(r) \sim \frac{1}{r^3} g\left(\frac{r}{R_c}\right) \xrightarrow[r \to 0]{} \underline{\text{constant}}$$

(R_c related to size of nucleons)

In hindsight, the justification of this model was incorrect

1) Using the wrong pion dynamics (their problem w/ multipions was not real)

2) They didn't know about renormalization

3) They didn't know about quarks

→ Usually, this is how science progress (a never ending succession of approximations)

[THE HISTORY SO FAR]

OLD PION THEORIES (INFINITIES)

⇓

ONE BOSON EXCHANGE MODEL OR MESON THEORY OF NUCLEAR FORCES

(BUT HADRONS ARE NOT FUNDAMENTAL)

⇓

MODERN PION THEORIES

(CONTAIN INFORMATION ABOUT LOW ENERGY QCD → SYMMETRIES)

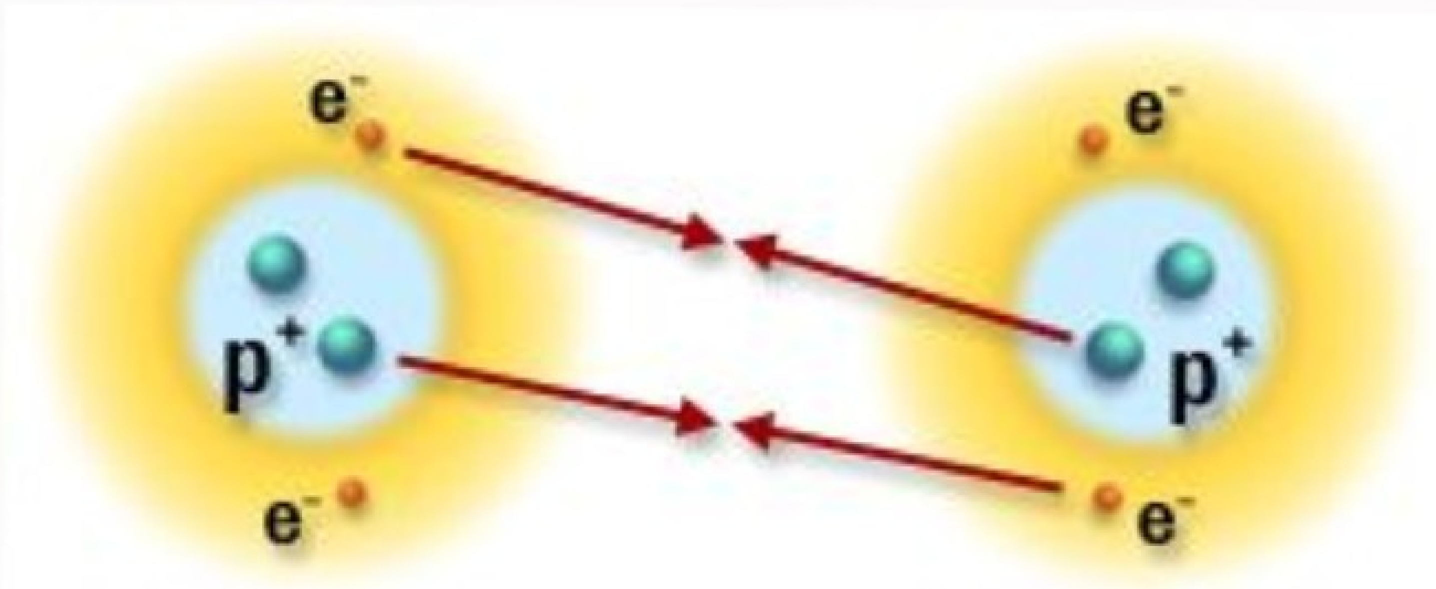
THE MODERN VIEW

$\boxed{\text{nucleon}} \Rightarrow$ [not fundamental, but composed of quarks]

$\Downarrow$

[IDEA: NUCLEAR FORCES ARE $\underbrace{\text{RESIDUAL INTERACTIONS}}_{\text{what is this?}}$]

RESIDUAL INTERACTIONS



RESIDUAL E-M FORCE IN ACTION: THE ATOMS ARE ELECTRICALLY NEUTRAL, BUT THE ELECTRONS IN ONE ARE ATTRACTED TO THE PROTONS IN ANOTHER, AND VICE VERSA!

Example: atom-atom interactions

atom → neutral

(but composed of charged components)

1) Coulomb → $V_{AA}(\vec{r}) = 0$ → because neutral

2) Coulomb interactions of the electrons & protons inside the atom → $V_{AA}(\vec{r}) \neq 0$ → because components not neutral

[ATOM-ATOM RESIDUAL INTERACTION]

$$V(\vec{r}) = -\frac{C_6}{r^6} - \frac{C_8}{r^8} - \frac{C_{10}}{r^{10}} - \dots$$

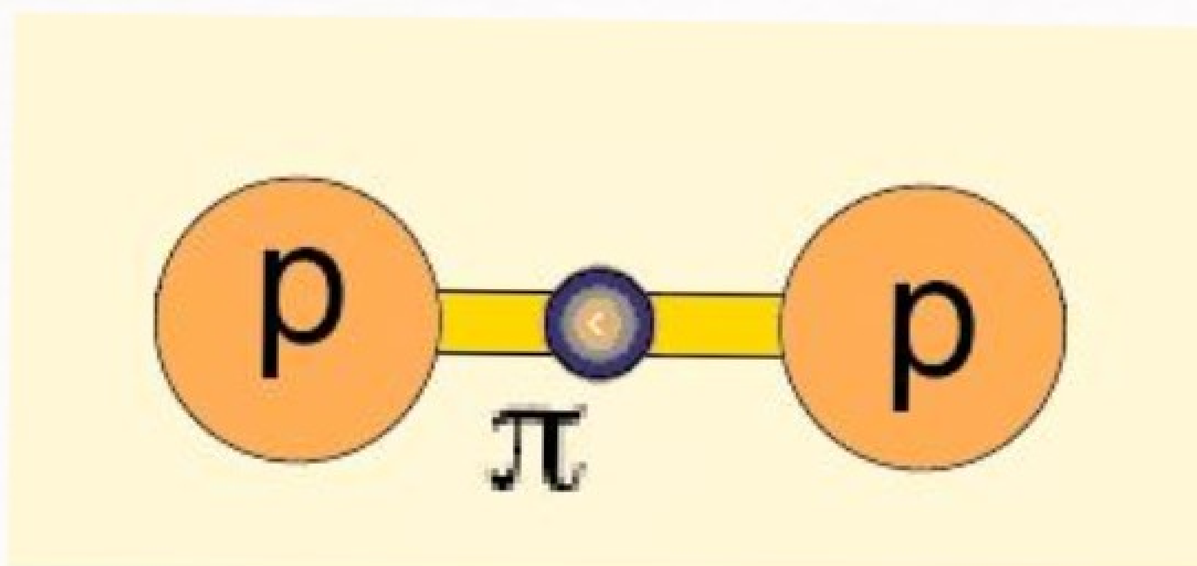
van der Waals potential (residual)

$\Downarrow$

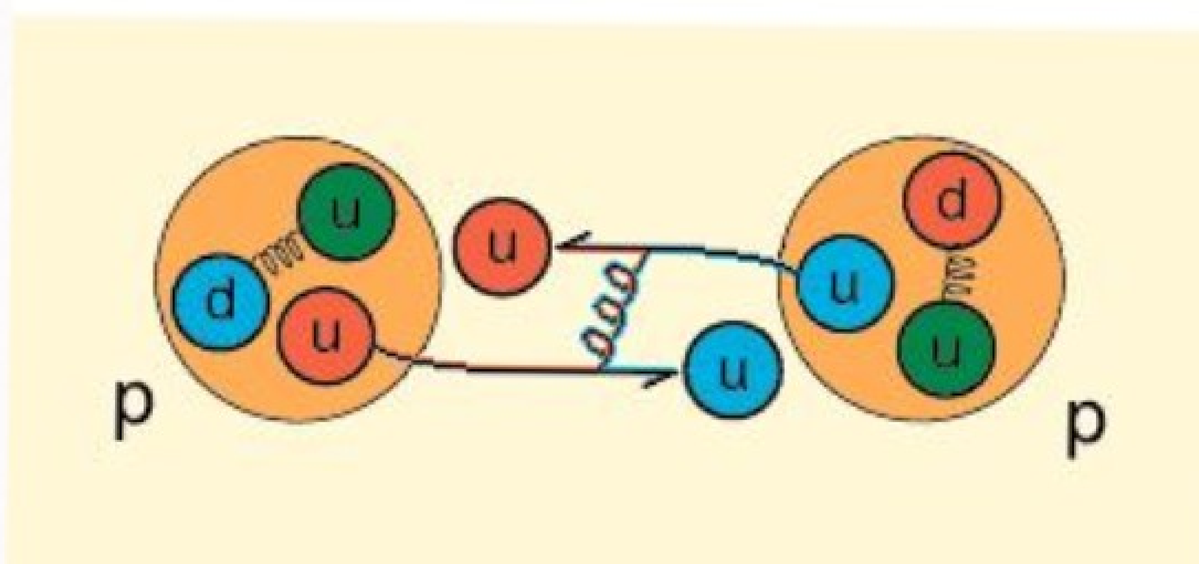
but it comes from a fundamental force (Coulomb)

[NUCLEON - NUCLEON POTENTIAL]

→ Same idea



→ Nucleons & Pions



→ Quark & Gluons

Exchange a quark-antiquark pair

[NUCLEAR FORCES
& QUANTUM CHROMODYNAMICS]

Nucleons → Quarks & Gluons

$\underbrace{\text{Nucleons}}_{\text{Residual force}}$ $\underbrace{\text{Quarks \& Gluons}}_{\text{Fundamental force}}$

Residual force

Fundamental force
(QCD: Quantum chromodynamics)

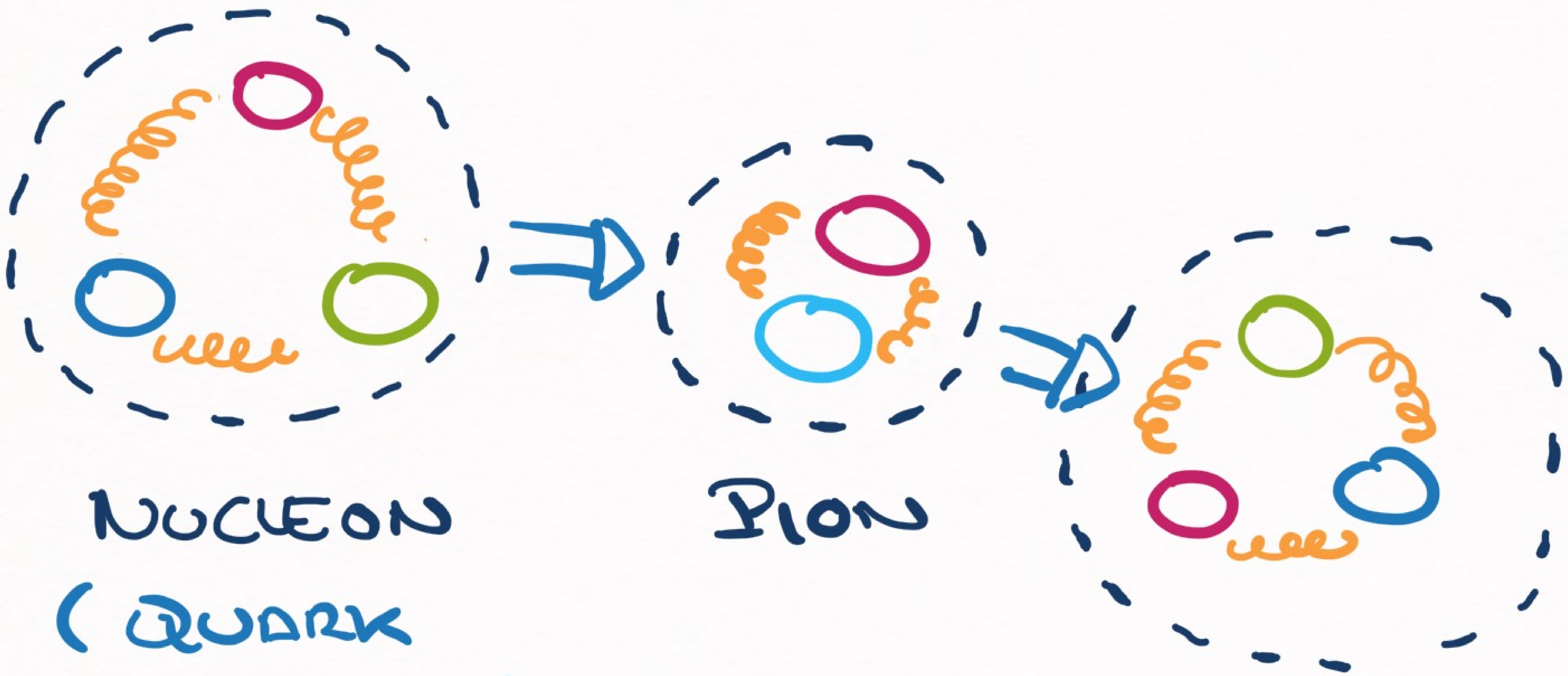
QUANTUM CHROMODYNAMICS

(A QUALITATIVE INTRODUCTION)

$\underbrace{\text{QUALITATIVE}}_{\text{mostly}}$

(don't worry if there are technical things here & there)

REMINDER: NUCLEAR FORCES ARE RESIDUAL

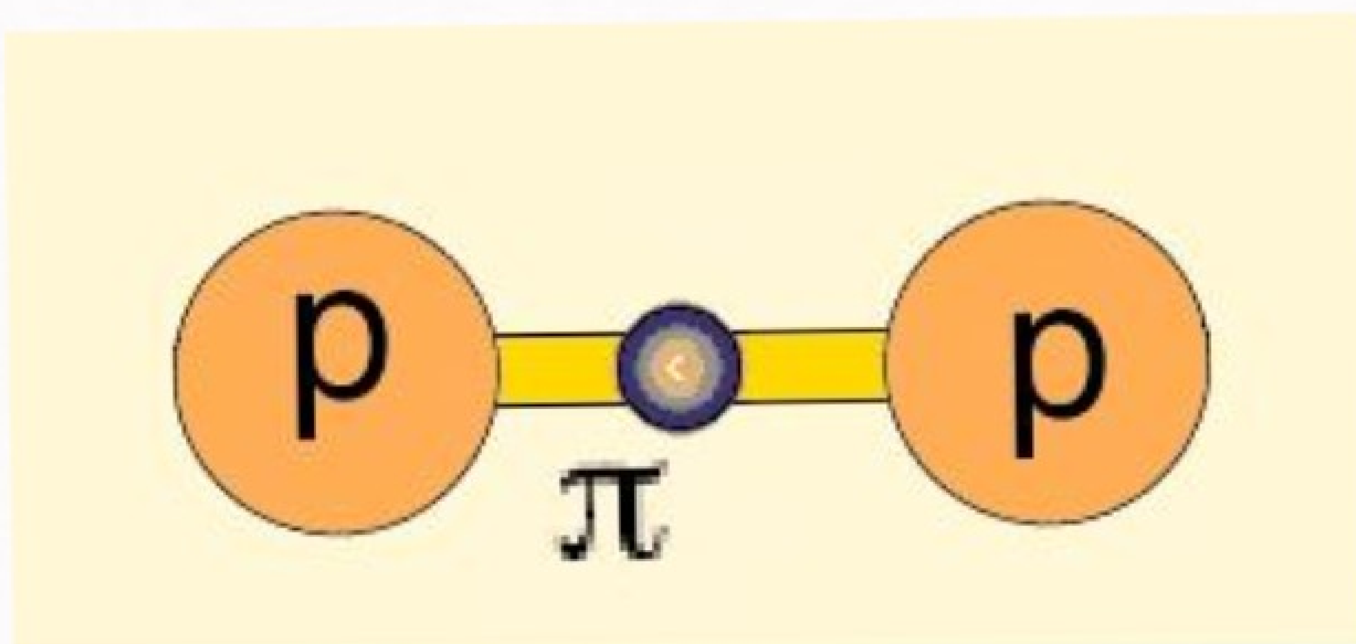


NUCLEON (QUARK & GLUONS)

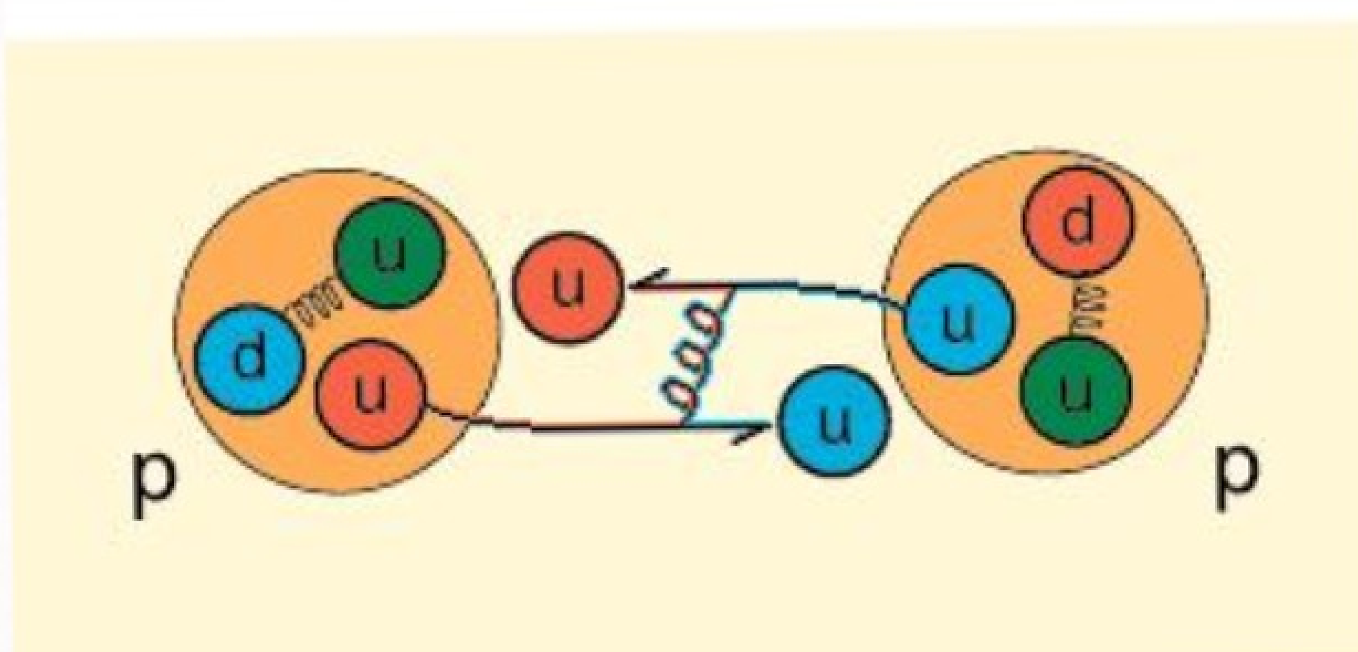
PION

NUCLEONS & PIONS) ⇒ (CAN BE EXPLAINED IN TERMS OF) ⇒ (QUARK & GLUONS

PROBLEM: HOW DO WE EXPLAIN 1 IN TERMS OF 2 ?



$\Rightarrow$ 1



$\Rightarrow$ 2

VERY INTERESTING PROBLEM

↳ FOR THE MOMENT WE WILL EXPLAIN QUARKS & GLUONS

QCD (Quantum Chromodynamics) → QFT

↳ FUNDAMENTAL THEORY OF QUARKS & GLUONS

WE WILL BEGIN WITH QED (Quantum electrodynamics)

(it's an easy version of QCD)

[THE COMPARISON]

1) QED is a well-known theory

→ A lot of info in QFT course

2) QED is a simplified version of QCD

QED → gauge theory → $U(1)$ symmetry

QCD → gauge theory → $SU(3)$ symmetry

QED

Theory of electrons & photons

(Take a look in a QFT textbook)

↳ QED is explained in terms of a Lagrangian

$$\left[\mathcal{L}_{QED} = \bar{\psi}(i\cancel{D} - m)\psi - \frac{1}{4}\underbrace{F_{\mu\nu}F^{\mu\nu}}_{\text{Electromagnetic field}}\right]$$

$\psi \rightarrow$ a Dirac field

$\cancel{D} = \gamma_\mu D^\mu$, D^μ covariant derivative

$m \rightarrow$ mass of the Dirac field

$\boxed{\text{QED}}$ ⇒ The specific details here do not matter

[IMPORTANT CONCEPT] ⇒ [QED is a gauge theory]

Begin with the free Lagrangian

(Dirac Lagrangian) → describes a single fermion (e.g. electron)

[SYMMETRIES OF THE FREE DIRAC LAGRANGIAN]

$$\mathcal{L}_{Dirac} = \bar{\psi}(i\not{\partial} - m)\psi$$

(PESKIN & SCHRÖDER)

$\psi \to$ Dirac field

$\not{\partial} = \underbrace{\gamma^\mu}_{\text{Dirac matrices}} \underbrace{\partial_\mu}_{\text{derivative}}$

$\bar{\psi} = \psi^\dagger \gamma^0$

SYMMETRY

$$\boxed{\begin{aligned} \psi(x) &\to e^{ie\alpha}\psi(x) \\ \bar{\psi}(x) &\to e^{-ie\alpha}\bar{\psi}(x) \end{aligned}}$$

(very silly symmetry)

$\to \psi(x) \to e^{iea}\psi(x)$

$\to \mathcal{L}_{Dirac} \to \mathcal{L}_{Dirac}$

} Symmetry of $\mathcal{L}_{Dirac}$

In particular, it is a global symmetry

$\to$ what does this mean?

$$\psi(\underline{x}) \to \underbrace{e^{ie\alpha}}_{\text{does not depend on } x} \psi(\underline{x})$$

} this change of phase happens everywhere (& it's the same everywhere)

REMINDER!

$$\psi(x) \to \boxed{e^{i e \alpha}}\,\psi(x) \qquad U(1) \text{ symmetry}$$

(name)

$$U(N) = \{ N\times N \text{ complex matrices} \,/\, U^\dagger U = \mathbb{1}_{N\times N} \}$$

$$U(1) \to z^\dagger z = 1$$

$$z^\dagger z = z^* z$$

$$z \in \mathbb{C}$$

$$|z|^2 = 1$$

$$\boxed{z = e^{i\varphi}}$$

$$z^\dagger = z^* = e^{-i\varphi}$$

$\to$ [the same]

WE MIGHT WANT TO EXTEND THIS SYMMETRY :

1) Global $U(1)$ symmetry

$$\psi(x) \to e^{ie\alpha} \psi(x)$$

2) Local $U(1)$ symmetry

local → depends on every point

$$\psi(x) \to \underbrace{e^{ie\alpha(x)}}_{\alpha = \alpha(x)} \psi(x)$$

IF WE DO A LOCAL U(1) TRANSFORMATION:

$$\mathcal{L}_{Dirac} = \bar{\psi}(x)(i\cancel{\partial} - m)\psi(x) \left\{ \begin{array}{ll} m\bar{\psi}(x)\psi(x) & (1) \\ i\bar{\psi}(x)\cancel{\partial}\psi(x) & (2) \end{array} \right.$$

(1) $\bar{\psi}(x)\psi(x) \to \bar{\psi}(x)\, e^{-ie\alpha(x)}\, e^{ie\alpha(x)}\psi(x) = \bar{\psi}(x)\psi(x)$

→ OK ✓

(2) $\bar{\psi}(x)\partial_\mu\psi(x) \to \bar{\psi}(x)e^{-ie\alpha(x)}\partial_\mu(e^{+ie\alpha(x)}\psi(x)) = \circledast$

$\circledast = \bar{\psi}(x)\partial_\mu\psi(x) + \boxed{ie\,\partial_\mu\alpha(x)\,\bar{\psi}(x)\psi(x)}$ new term

We find that:

$$\bar{\psi}(x)\,\partial_\mu\psi(x) \longrightarrow \bar{\psi}(x)\,\partial_\mu\psi(x) + \underbrace{ie\,\partial_\mu\alpha(x)\,\bar{\psi}(x)\psi(x)}_{\text{Breaks the local symmetry}}$$

WE DO NOT HAVE LOCAL U(1) SYMMETRY

$$\rightarrow \underset{\text{U(1)-local}}{\mathcal{L}_{Dirac}} \longrightarrow \mathcal{L}_{Dirac} \underbrace{- e\,\partial_\mu\alpha\,\bar{\psi}(x)\psi(x)}_{\text{unexpected new term}}$$

[HOWEVER, WE CAN FIX $\mathcal{L}_{Dirac}$

SO IT WILL BECOME LOCAL U(1) SYMMETRIC]

If we fix this problem,

electromagnetism will appear naturally

$\boxed{\text{HOW TO DO IT:}}$

1) Introduce a new field

$$A_\mu \xrightarrow[U(1)]{} A_\mu + \partial_\mu \alpha$$

2) Define a new type of derivative

$$\cancel{\partial_\mu} \longrightarrow D_\mu \equiv \partial_\mu - ie A_\mu$$

→ contains the new field

→ AND WE CHECK NOW WHAT HAPPENS WITH U(1) SYMMETRY

S

WHAT HAPPENS NOW?

$\circledast \rightarrow$ page 53

$$\bar{\psi}(x) D_\mu \psi(x) \rightarrow \bar{\psi}(x)(\partial_\mu - ieA_\mu - ie\partial_\mu\alpha)\psi(x) + ie\partial_\mu\alpha\,\bar{\psi}(x)\psi(x)^{\circledast} = \circledast$$

$$\circledast = \bar{\psi}(x)(\partial_\mu - ieA_\mu)\psi(x) = \bar{\psi}(x) D_\mu \psi(x)$$

$$\Rightarrow \bar{\psi}(x) D_\mu \psi(x) \xrightarrow[U(1)]{} \bar{\psi}(x) D_\mu \psi(x)$$

SYMMETRIC!

[BUT ∃ STILL A MISSING PIECE]

$$\mathcal{L}_{QED} = \underbrace{\bar{\psi}(i\cancel{\partial} - m)\psi}_{\text{local } U(1) \text{ symmetry}} + \underbrace{(\ldots\ldots)}_{\text{term that describes } A_\mu}$$

NEW FIELD, NEW TERM

1) $A_\mu \xrightarrow[U(1)]{} A_\mu + \partial_\mu \alpha$ (original transformation)

2) $\partial_\mu A_\nu - \partial_\nu A_\mu \xrightarrow[U(1)]{} \partial_\mu A_\nu - \partial_\nu A_\mu + \partial_\mu \partial_\nu \alpha - \partial_\nu \partial_\mu \alpha = \partial_\mu A_\nu - \partial_\nu A_\mu$

[MORE ABOUT THE MISSING PIECE]

$$\mathcal{L}_{QED} = \bar{\psi}(i\not{D} - m)\psi + \lambda \underbrace{F_{\mu\nu}F^{\mu\nu}}_{F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu}$$

$$\boxed{\begin{aligned} F_{\mu\nu} &\to F_{\mu\nu} \\ F_{\mu\nu}F^{\mu\nu} &\to F_{\mu\nu}F^{\mu\nu} \end{aligned}}$$

How do we determine this new constant λ?

→ TRICK : to fix λ from what we expect for a kinetic term

$\boxed{\text{KINETIC TERM}}$

$$\mathcal{L}_{QED} = \bar{\psi}(x)(i\not{\partial} - m)\psi(x) + \underbrace{\lambda}_{\downarrow} F_{\mu\nu}F^{\mu\nu}$$

STRATEGY: COMPARISON WITH A KNOWN KINETIC TERM

SPIN-0 FIELD (KLEIN GORDON)

$$\Rightarrow \mathcal{L}_{KG} = \underbrace{\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi}_{\text{KINETIC}} - \underbrace{\frac{1}{2}m^2\phi^2}_{\text{MASS}}$$

OPEN THE QFT BOOK

[KINETIC TERM OF THE KLEIN-GORDON FIELD] → Rewrite

$\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi \rightarrow \boxed{-\frac{1}{2}\phi\,\Box^2\phi}$ } Why?

1) $\mathcal{L}$ is not a fundamental quantity in QFT
→ the action is what is fundamental

2) $S = \int \mathcal{L}\, d^4x$

$$\int d^4x\left[\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi\right] = \begin{pmatrix}\text{integration}\\ \text{by parts}\end{pmatrix} = \int d^4x\left(-\frac{1}{2}\phi\,\Box^2\phi\right)$$

[ADAPTING THIS IDEA TO SPIN-1]

1) Spin-0 : $\mathcal{L}_{kinetic} = -\frac{1}{2}\phi\,\Box^2\phi$ $\leftarrow$

2) Spin-1 : $\mathcal{L}_{kinetic} = \pm\frac{1}{2}A_\mu\,\Box^2 A^\mu$ $\rightarrow$ (+)

$A_\mu = (\underline{\underline{A_0}}, \underline{\underline{\vec{A}}})$ $\Rightarrow$ How a physical A_μ looks like?

$A_\mu \rightarrow A_\mu + \partial_\mu \alpha$

$$A_\mu = \underbrace{\epsilon_\mu}_{\text{polarization vector}} \underbrace{e^{-iq\cdot x}}_{\text{plane wave}} \Rightarrow \epsilon_\mu \underline{\underline{q^\mu}} = 0$$

Physical photon

$q = (q, -\vec{q})$

$\Downarrow$

$$\boxed{\epsilon_0 = 0}$$

[ADAPTING TO SPIN-1]

$$g_{\mu\nu} = \begin{pmatrix} 1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix} \rightarrow$$ assume we use this convention.

(Caveat: I used the opposite convention in previous lesson)

$$\Downarrow$$

$$A_\mu \Box^2 A^\mu = A_0 \Box^2 A_0 - \vec{A}\,\Box^2 \vec{A}$$

$$= (\text{physical } A_\mu) = \boxed{-\vec{A}\,\Box^2 \vec{A}}$$

$$\mathcal{L}_{KG} = \boxed{-\tfrac{1}{2}\phi\,\Box^2\phi} - \tfrac{1}{2} m^2 \phi^2$$

$\rightarrow$ same sign ✓

[KINETIC TERM]

$$\mathcal{L}_{kinetic} = \frac{1}{2} A_\mu D^2 A^\mu \Rightarrow \boxed{\lambda = -\frac{1}{4}}$$

Expand $F_{\mu\nu} F^{\mu\nu}$

$$\Rightarrow \boxed{\mathcal{L}_{QED} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}}$$

U(1) - local gauge symmetry

[NEXT LESSON: HOW THIS APPLIES TO QCD]

TRICK: extend this argument to a different local gauge symmetry group

FRIDAY 15:50