NUCLEAR PHYSICS 6

a) PROPERTIES OF THE NUCLEAR FORCE

(CONTINUATION FROM PAST LESSON)

b) ORIGIN OF THE NUCLEAR FORCES

→ Exchange of a meson ←

RECAP

PROPERTIES OF THE NUCLEAR FORCE

1) Nuclear forces have a short range ✓

2) Nuclear forces are attractive at intermediate ✓
distances (i.e. distances of (1-2) fm)

3) Nuclear forces are repulsive at short distances ✓
(≤ 0.7 fm)

4) Nuclear forces do not distinguish
neutrons & protons (new)

5) Nuclear forces are not central (new)
↳ important for understanding
their nature

So far we have studied 1), 2) & 3):

1) SHORT RANGE $\Rightarrow$ Because saturation

why?

$$\boxed{\underbrace{\frac{B}{A}}_{} \to \text{constant}}$$

(8–9) MeV per nucleon

binding energy

number (#) of nucleons

Infinite range force: $\left(\frac{1}{r}\right)$

$$\boxed{B \propto A^2}$$

Finite range force: $\left(\frac{e^{-mr}}{r}\right)$

$$\boxed{B \propto A}$$

2) ATTRACTIVE AT MEDIUM DISTANCES $\sim (1-2)\,\text{fm}$

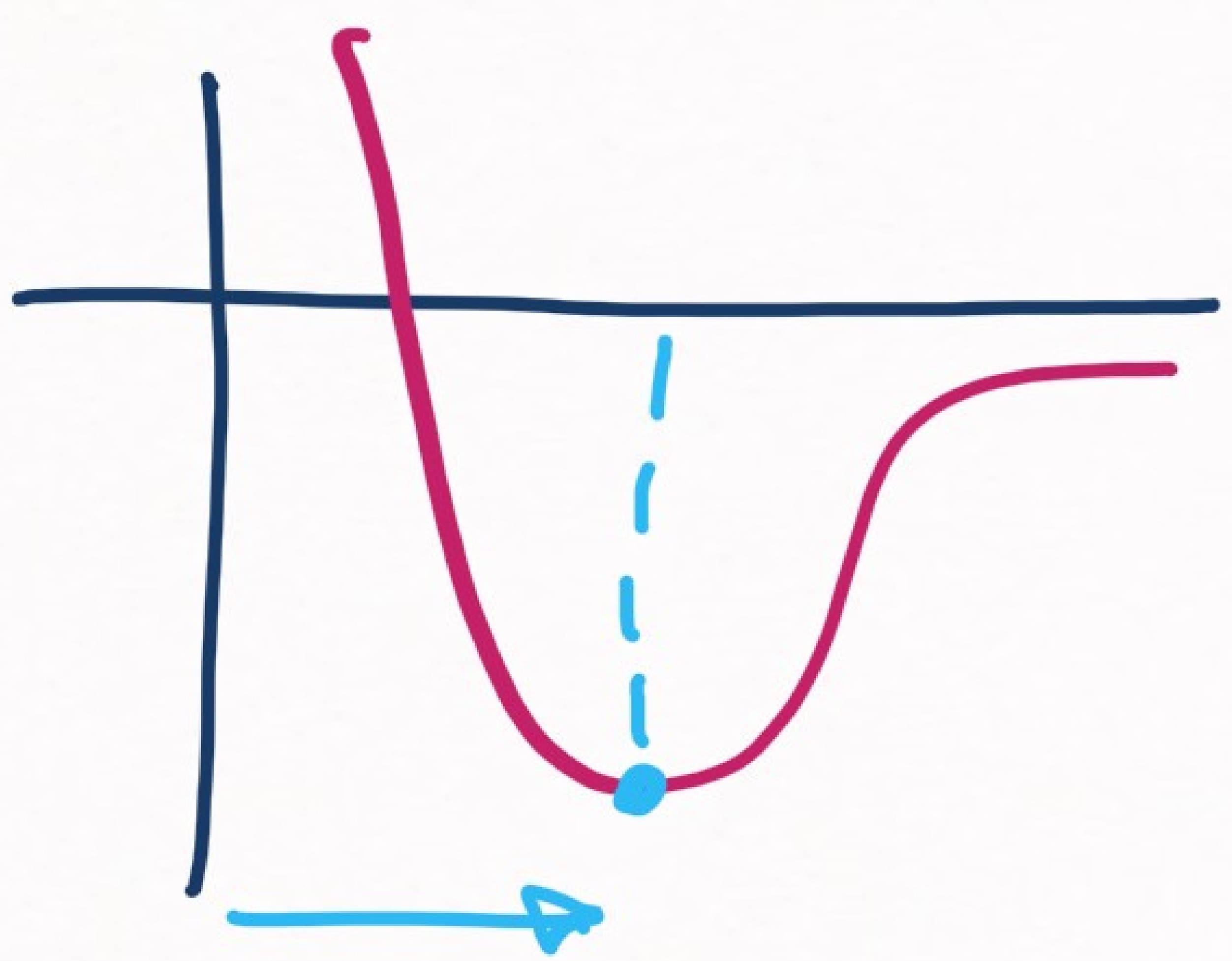
$\Downarrow$

Because of the density of nuclei : ← why?

$\rho \neq \rho(A)$ but $\rho \simeq \rho_0$

Hofstadter in the 50's

$\Downarrow$



R_{min}

$\Leftrightarrow$

$\rho_0 \simeq 0.17$ nucleons / fm^3

$\simeq \left(\frac{1}{R_{min}}\right)^3 \Rightarrow \boxed{R_{min} \simeq 1.8\ \text{fm}}$

(not exactly, but close enough)

3) REPULSIVE AT SHORT DISTANCES

$\Downarrow$

(previous argument)

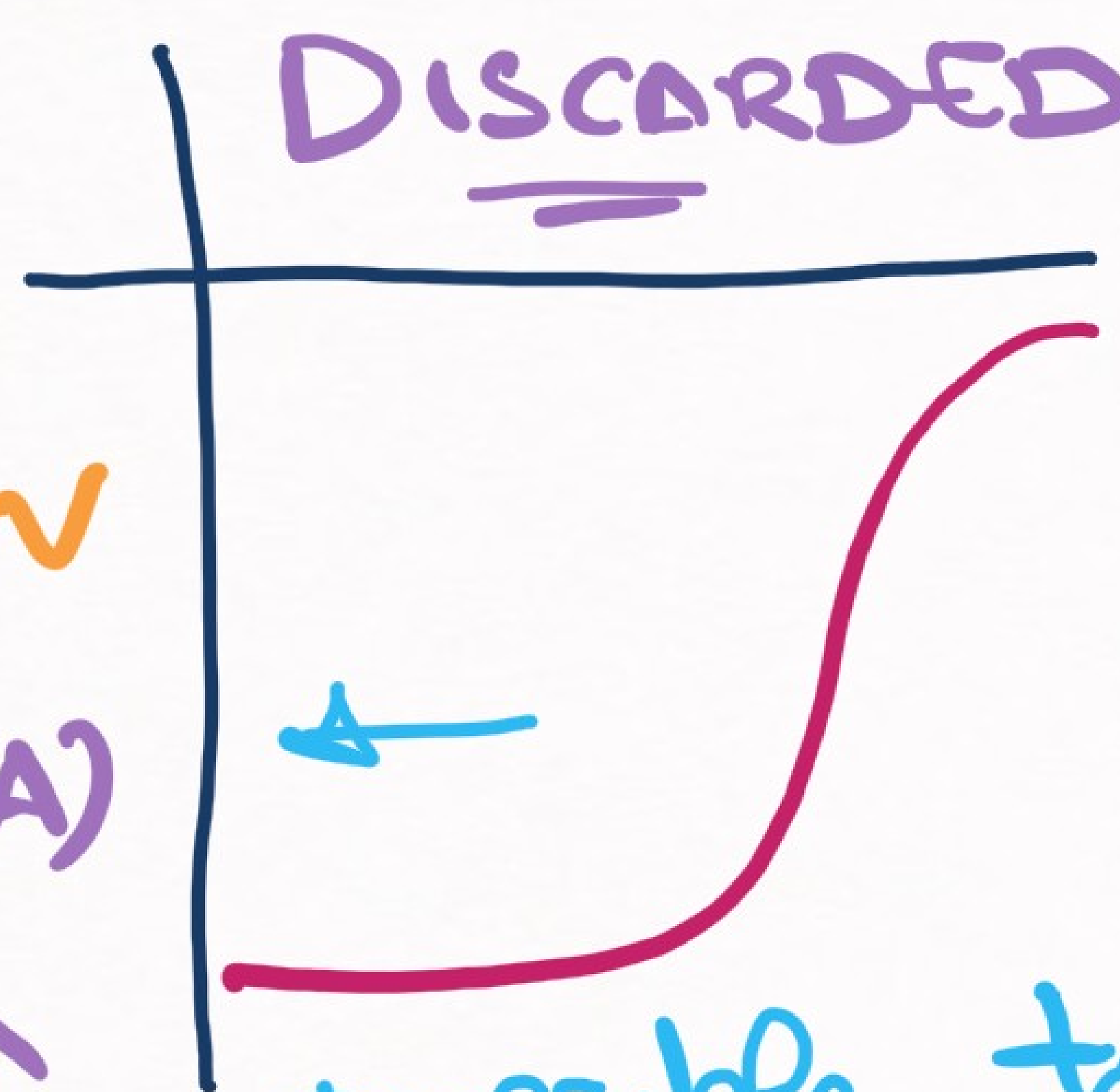
3.a) Again, because of constant density of nuclei:

$$\rho \simeq \rho_0 \quad \text{w/} \quad \rho_0 \simeq \underline{\underline{0.17}} \text{ nucleons/fm}^3$$

$B \propto A$ ✓

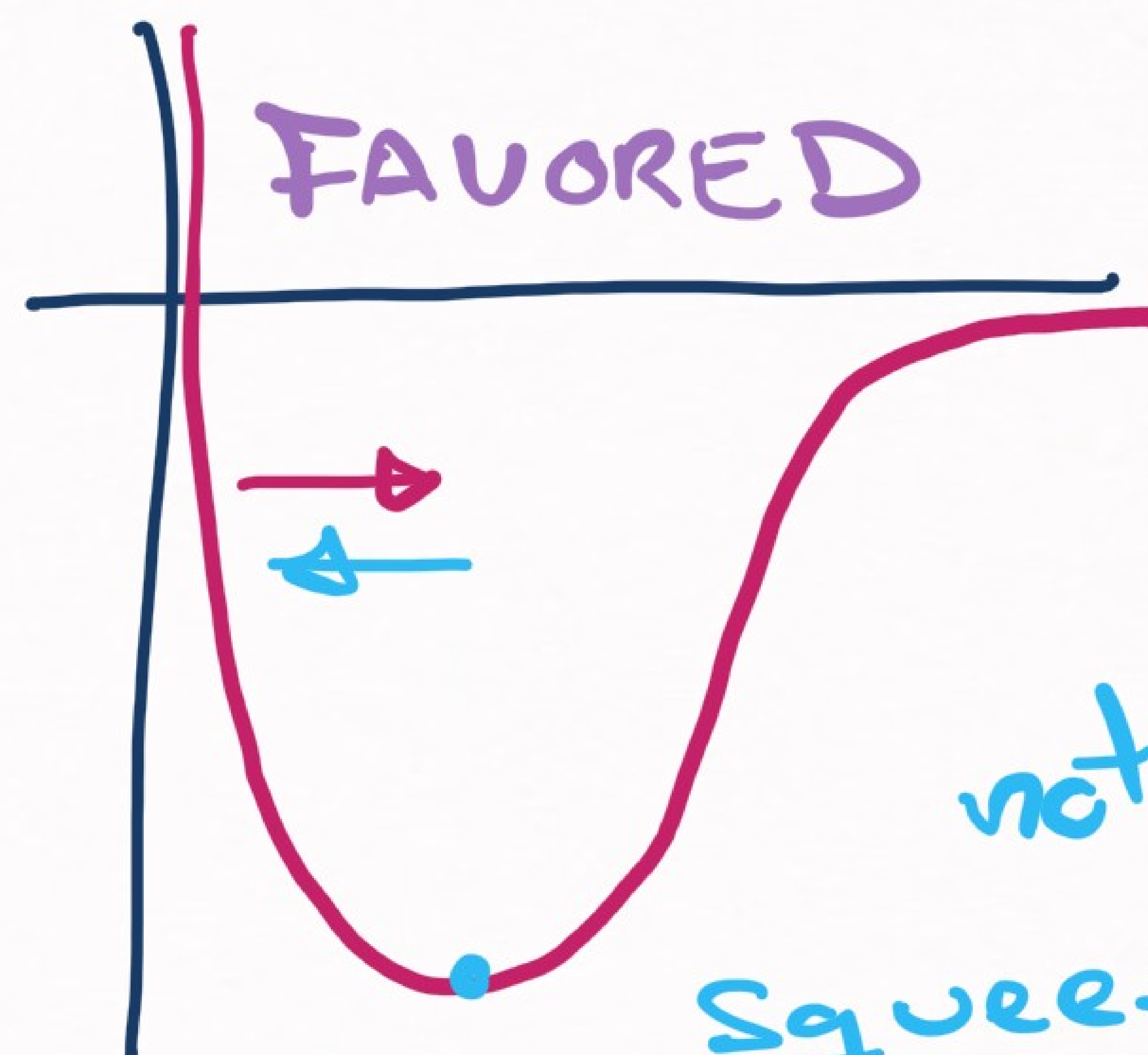
$\rho = \rho(A)$

$A\uparrow \ \rho\uparrow$ ✗



possible to squeeze nucleons

VS



$B \propto A$ ✓

$\rho \simeq \rho_0$ ✓

not possible to squeeze nucleons beyond a certain point

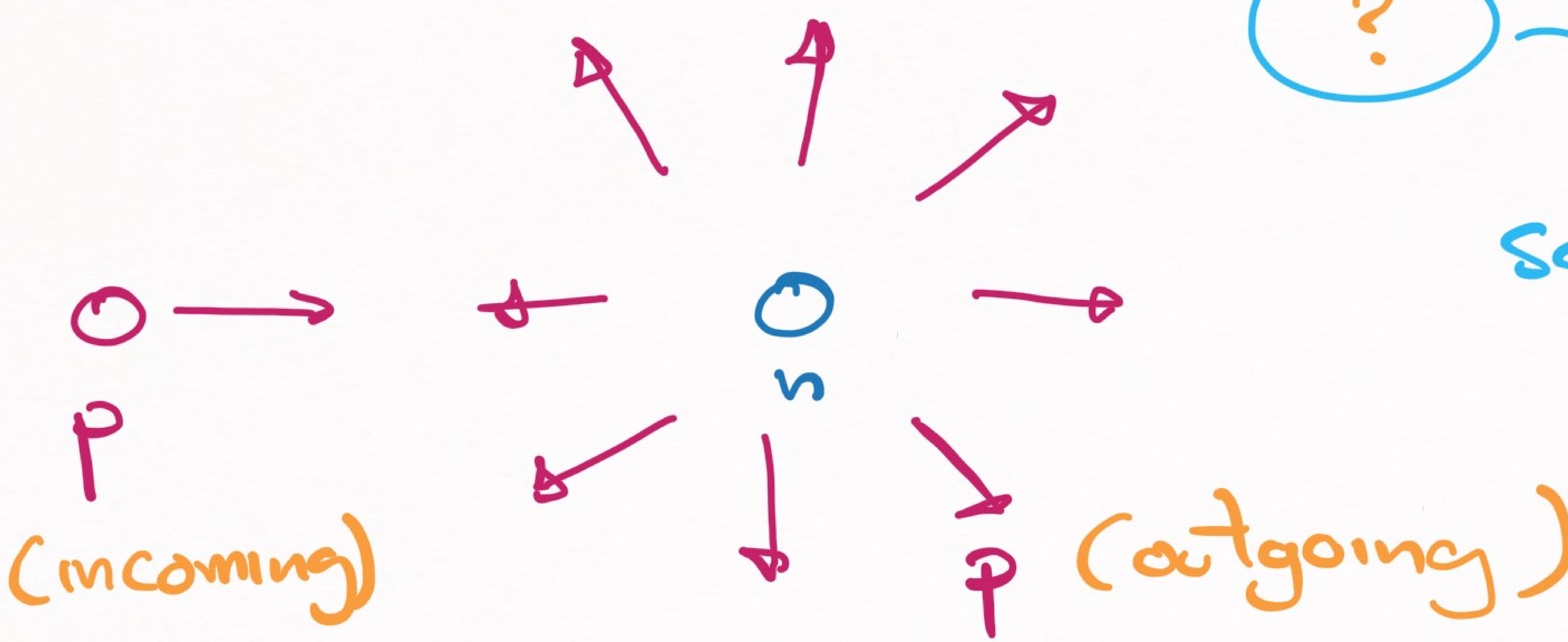
But, there are more reasons

3.b) Because of the phase shifts

?

scattering

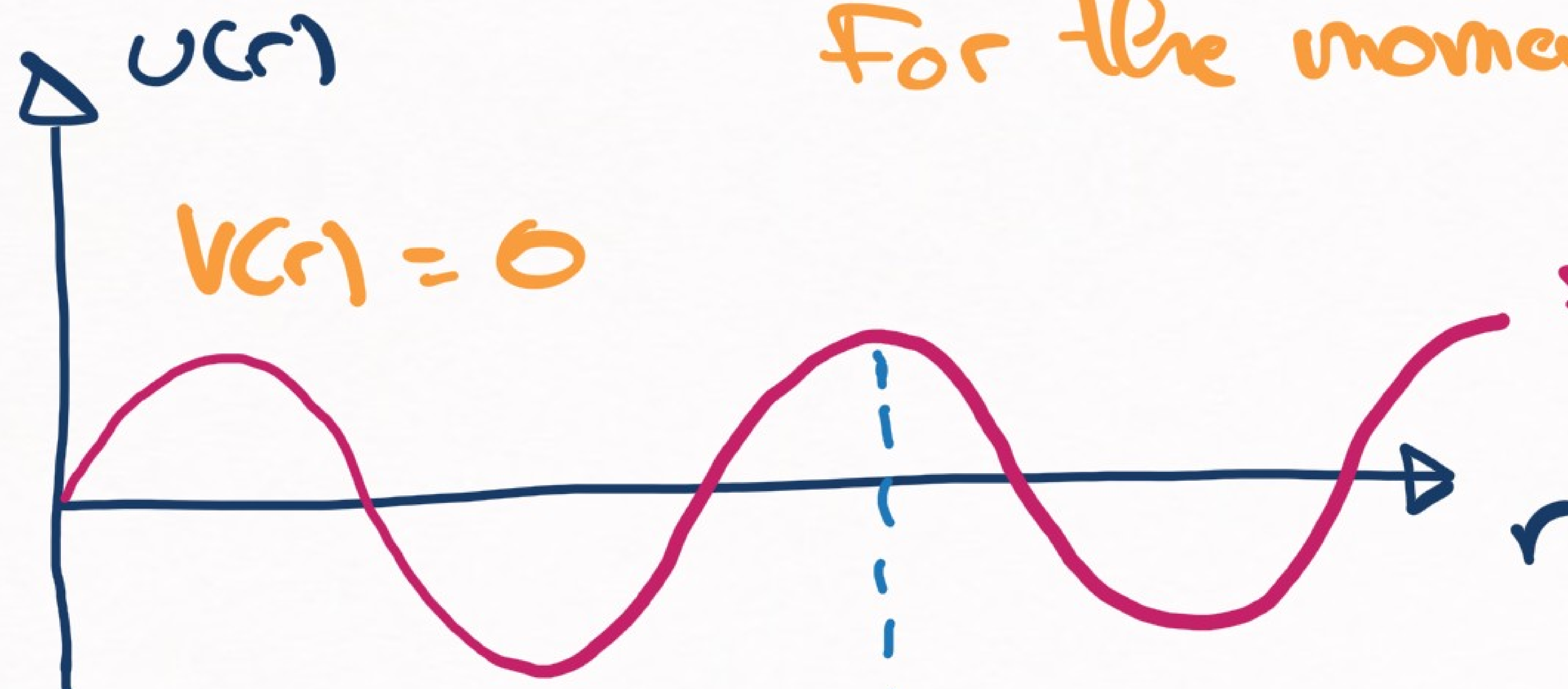
What are the phase shifts?



"σ" (cross section)

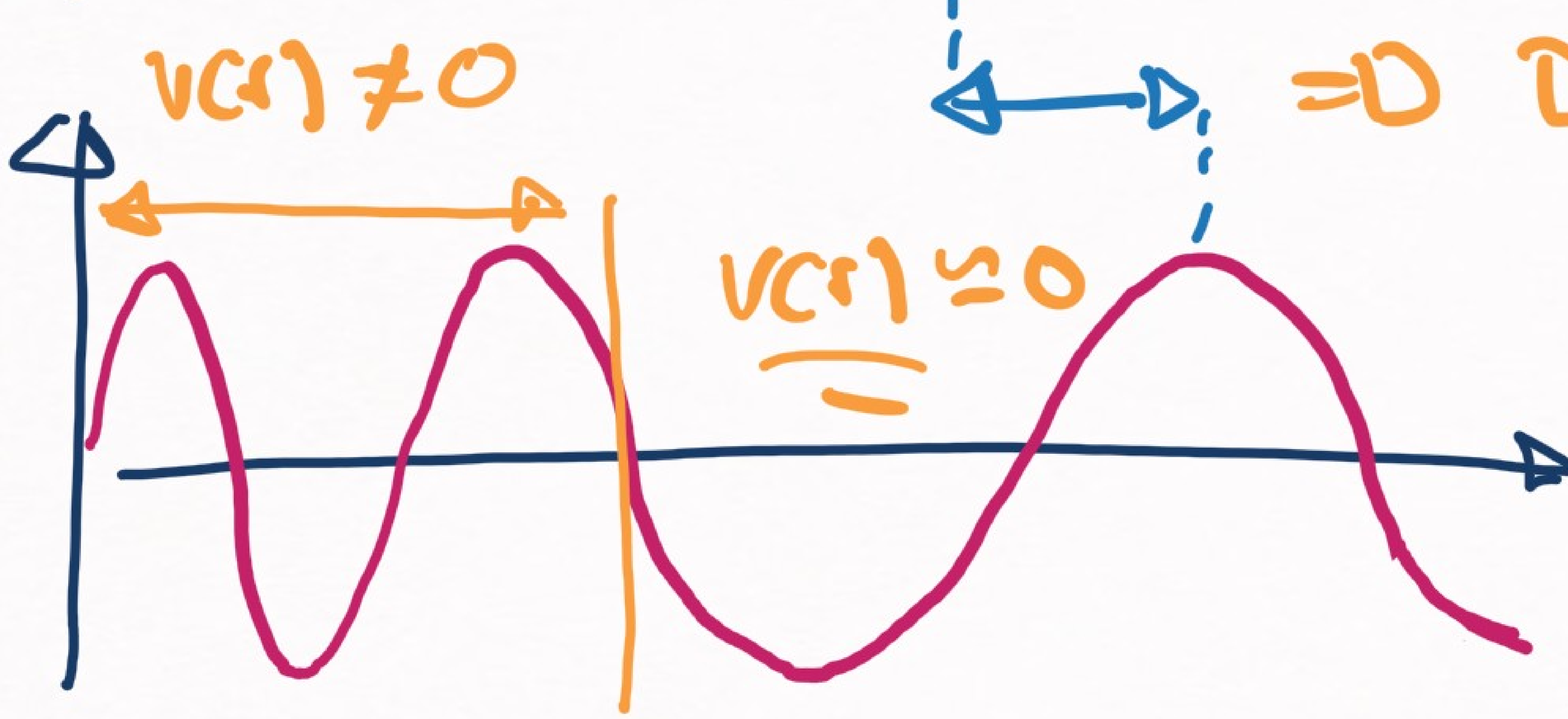
$$\left[\sigma = \frac{4\pi}{k^2}\sum_{\ell} \sin^2\delta_\ell\right] \longrightarrow$$ Phase shifts (we will explain this later)

For the moment just believe it



FREE WAVE FUNCTION (no V)

$u(r) = \sin kr$



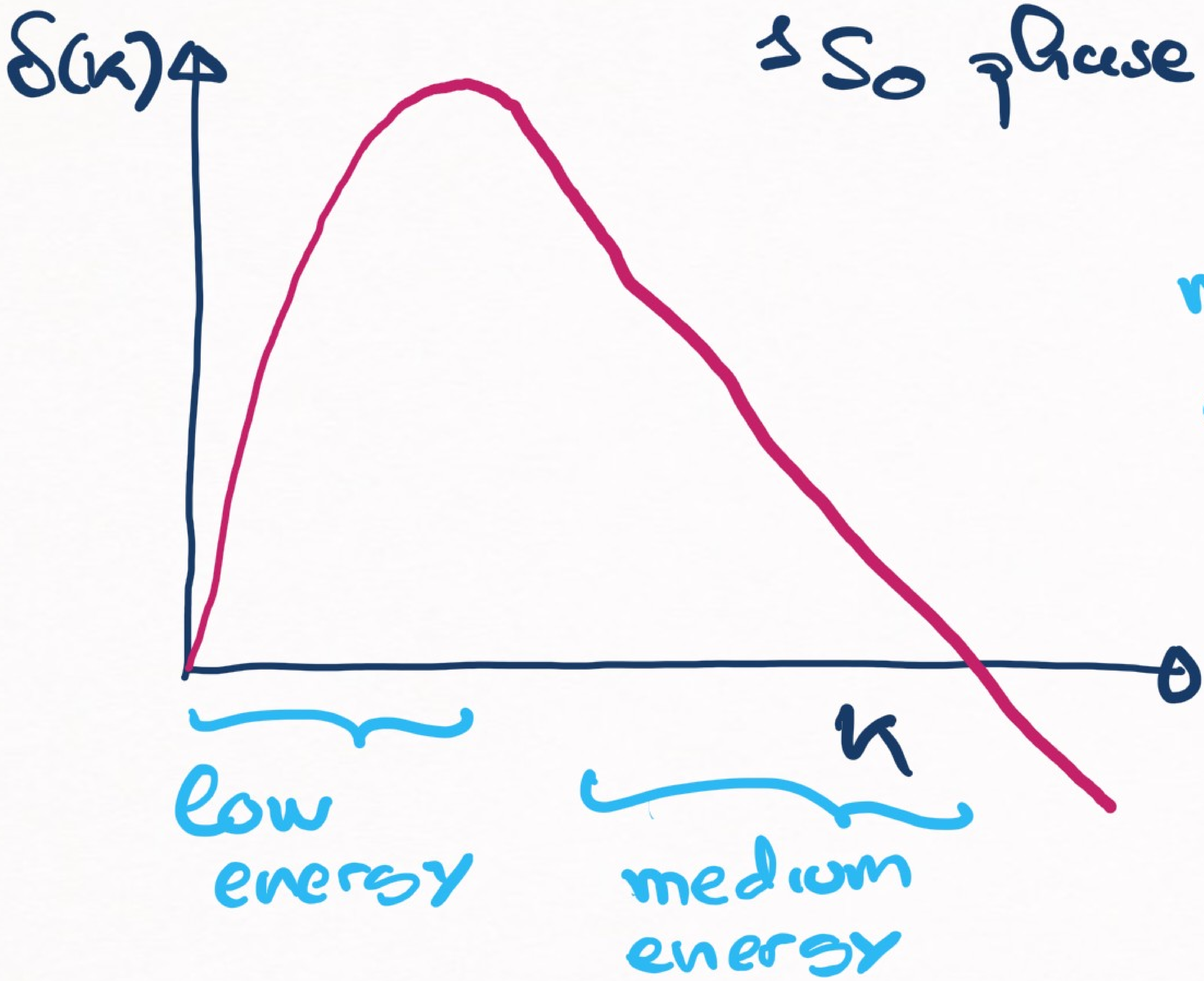
$\Rightarrow$ Difference in the phase

$u(r) = \sin(kr + \delta)$ (V)

WAVE FUNCTION W/ POTENTIAL

[$S=0$ neutron-proton phase shifts:]

($\psi(r) \sim \frac{1}{r}\sin(kr+\delta)$ → δ is the phase shift)



$^1S_0$ phase shift : $^{2S+1}L_J$

$2S+1$ → multiplicity of the spin

L → orbital angular momentum

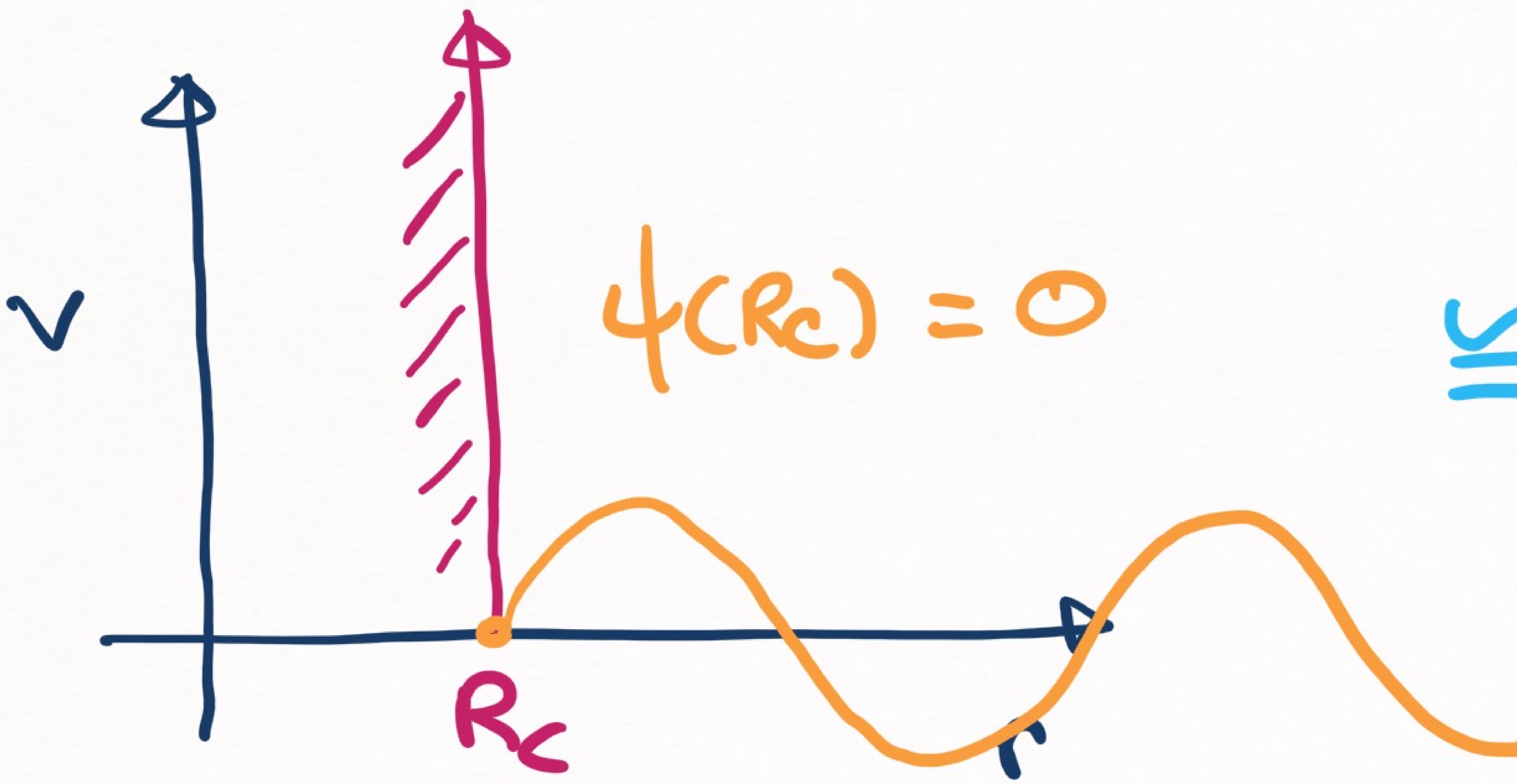
J → total angular momentum

$^1S_0$ → $S=0$ (spin)

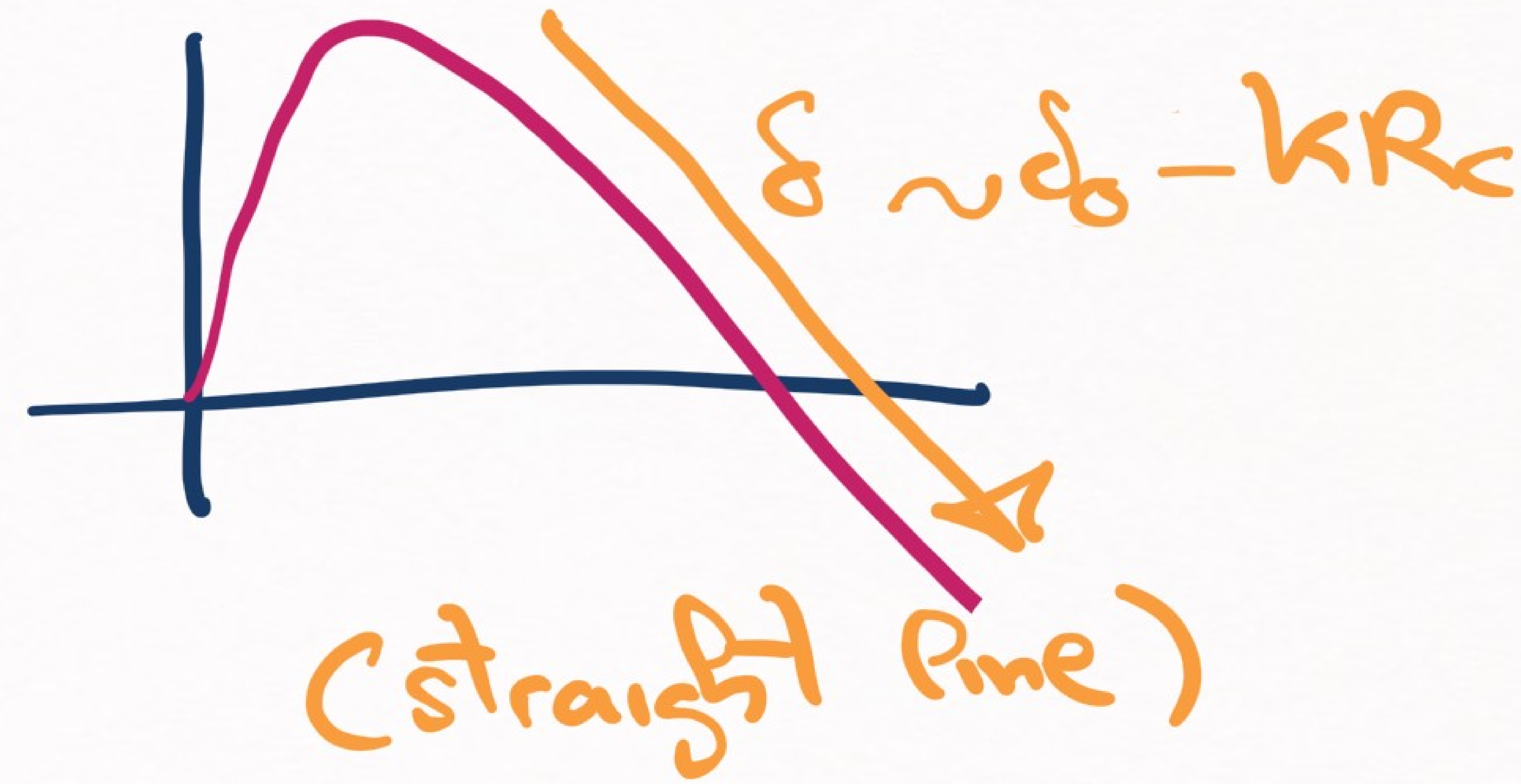
$^1S_0$ → $L=0$

$^1S_0$ → $J=0$

[HARD CORE POTENTIAL] ⇒ Explain the phase shift at medium energies



$\cong$



$$\Rightarrow \boxed{\delta = -kR_c}$$

$V(r) = \infty \quad , r < R_c$

$$\psi(r) = \frac{\sin(kr+\delta)}{r} \Rightarrow \sin(kR_c + \delta) = 0$$

$= 0$ (or $n\pi$)

4) NEUTRONS & PROTONS ARE NOT DISTINGUISHED

$\rightarrow$ ISOSPIN FORMALISM (LATER LESSONS)

4.a) Mirror nuclei

nuclei for which the number of neutrons & protons are swapped

^3He	^3H
ppn	nnp

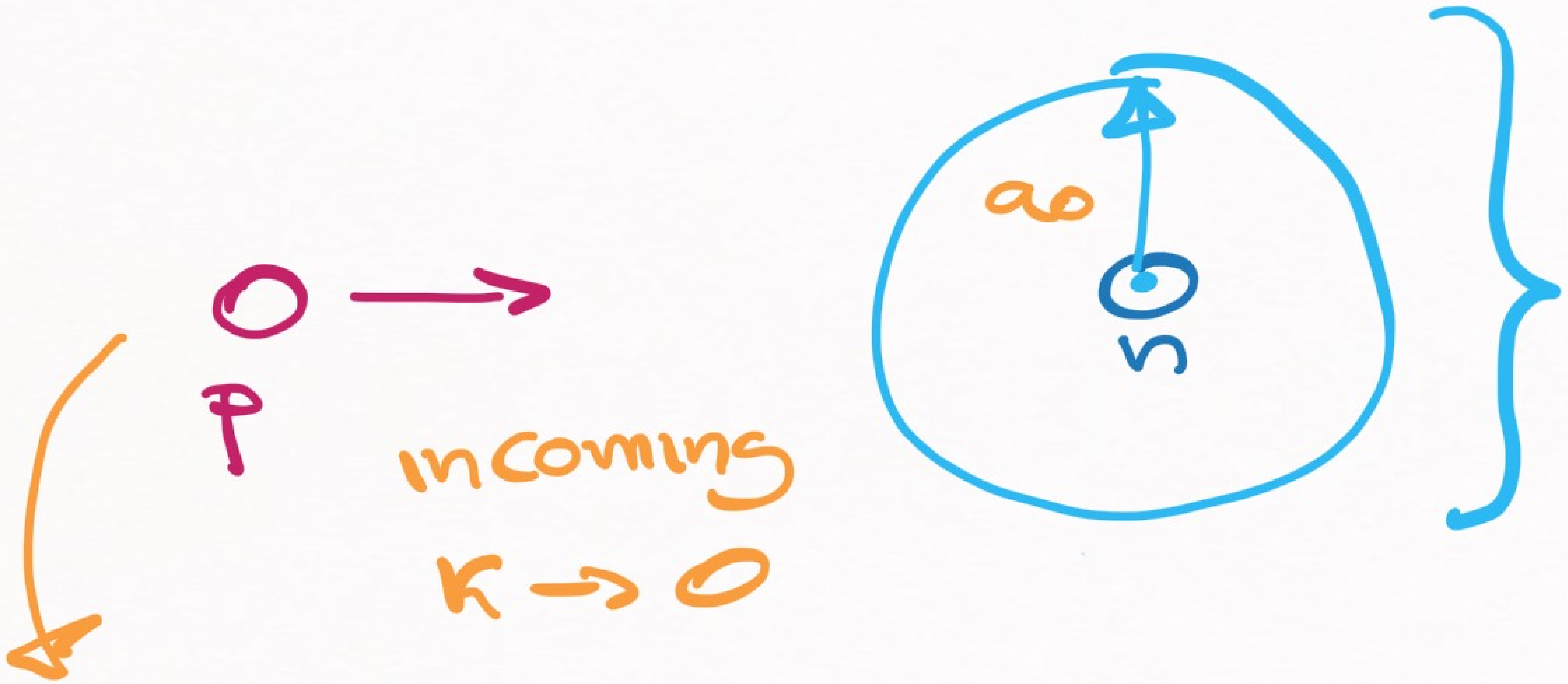
$$B(^3He) = 7.72\ MeV$$
$$B(^3H) = 8.48\ MeV$$

difference comes from Coulomb repulsion in ^3He

$$B(^3He) \simeq B(^3H)$$

4.b) Scattering lengths in $S=0$ nn, np, pp scattering

$$\left[\ \sigma \to 4\pi |a_0|^2\ \right]$$



for $k \to 0$, it behaves as a hard sphere of radius a_0

SCATTERING LENGTH

Classical scattering from a rigid sphere:

$$\sigma = 4\pi |R|^2$$

$$a_0(np) \simeq -24\ \text{fm}$$
$$a_0(nn) \simeq -19\ \text{fm}$$
$$a_0(pp) \simeq -17\ \text{fm}$$

very similar numbers

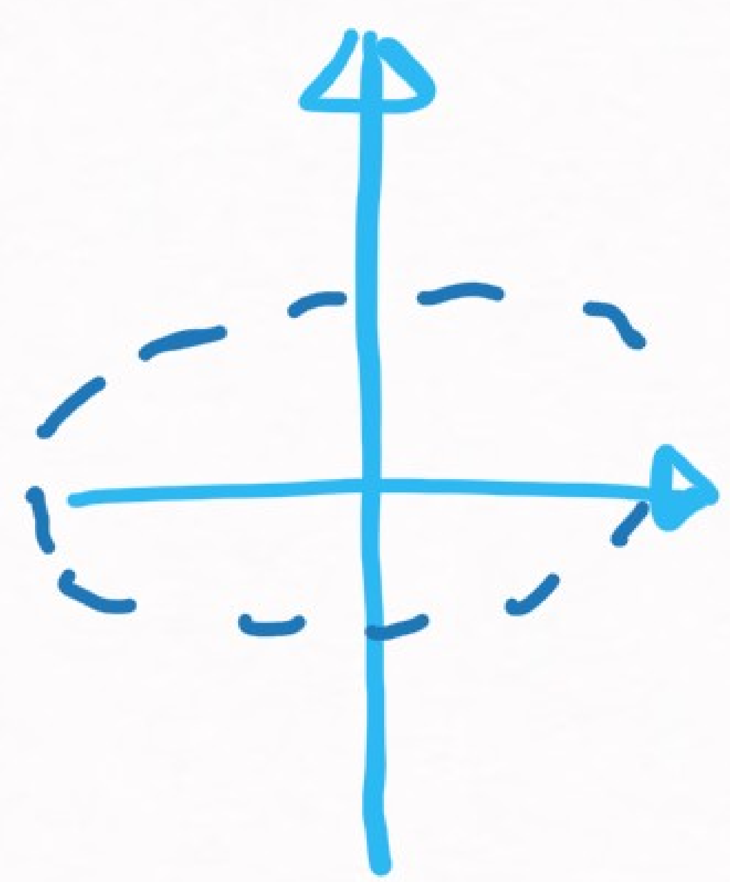
5) THE NUCLEAR FORCE IS NOT CENTRAL

Reminder → $V(\vec{r}) = V(|\vec{r}|)$ for central forces

Example: COULOMB

⇓

Because of the shape of the deuteron



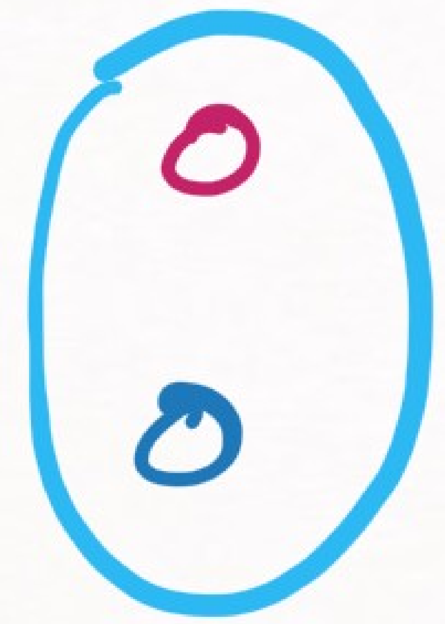
$\rho(\vec{r}) = |\psi(\vec{r})|^2$

Deuteron → spin 1

→ Longer in the direction of its spin

$V(r) \sim -(\vec{S} \cdot \hat{r})^2$

[How do we know the deuteron is not spherical?]



?

DEUTERON'S QUADRUPOLE MOMENT

$Q_d = 0.286\ \mathrm{fm}^2$

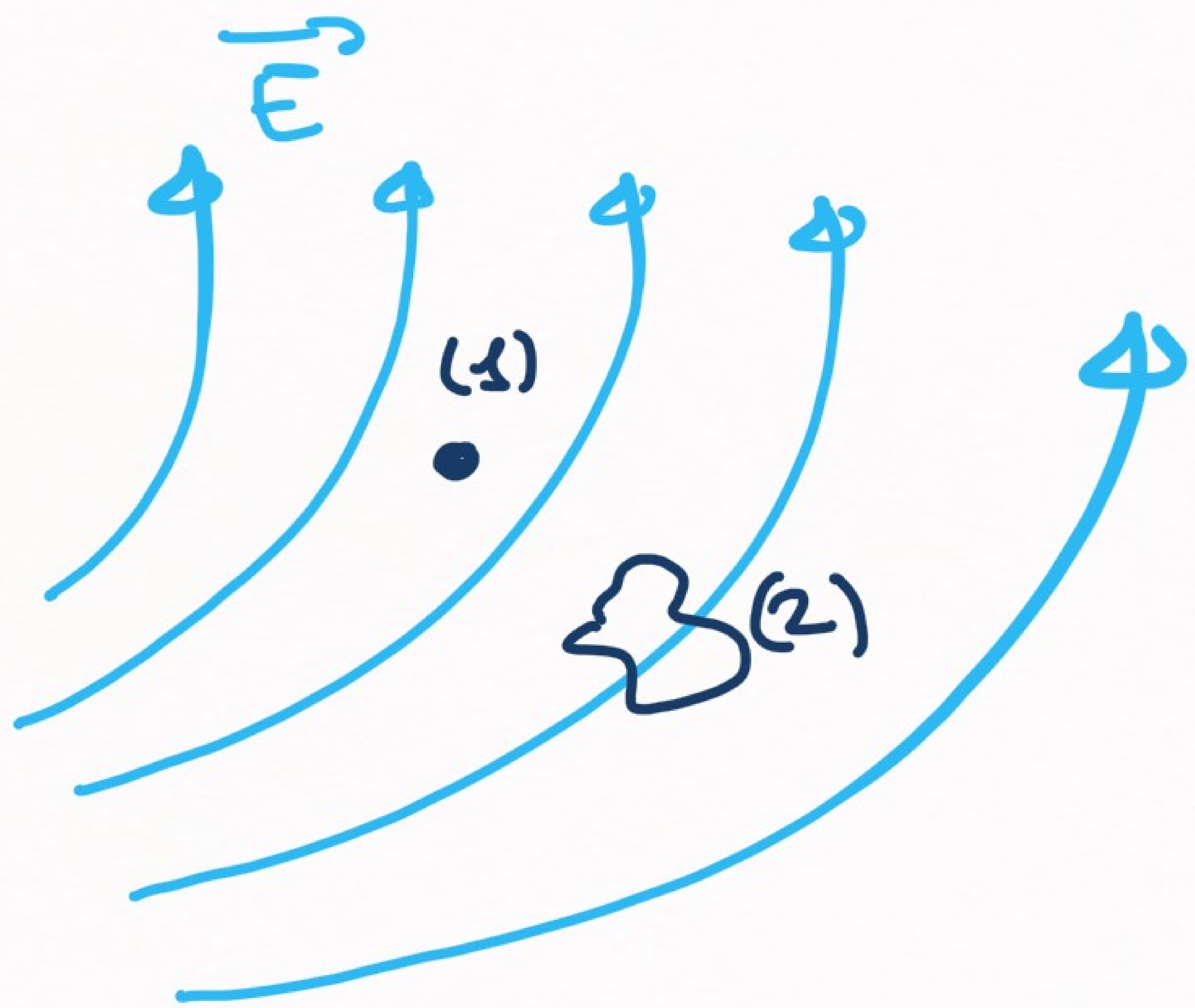
$Q_d \longrightarrow$ electromagnetic property related to the charge distribution of some extended object

(reminder)

REMINDER

The electromagnetic moments

ELECTRIC



$$\vec{E} = -\vec{\nabla}\underbrace{\Phi}$$

Scalar potential

(1) Charge (point-like):

$$V(\vec{r}) = q\,\Phi(r)$$

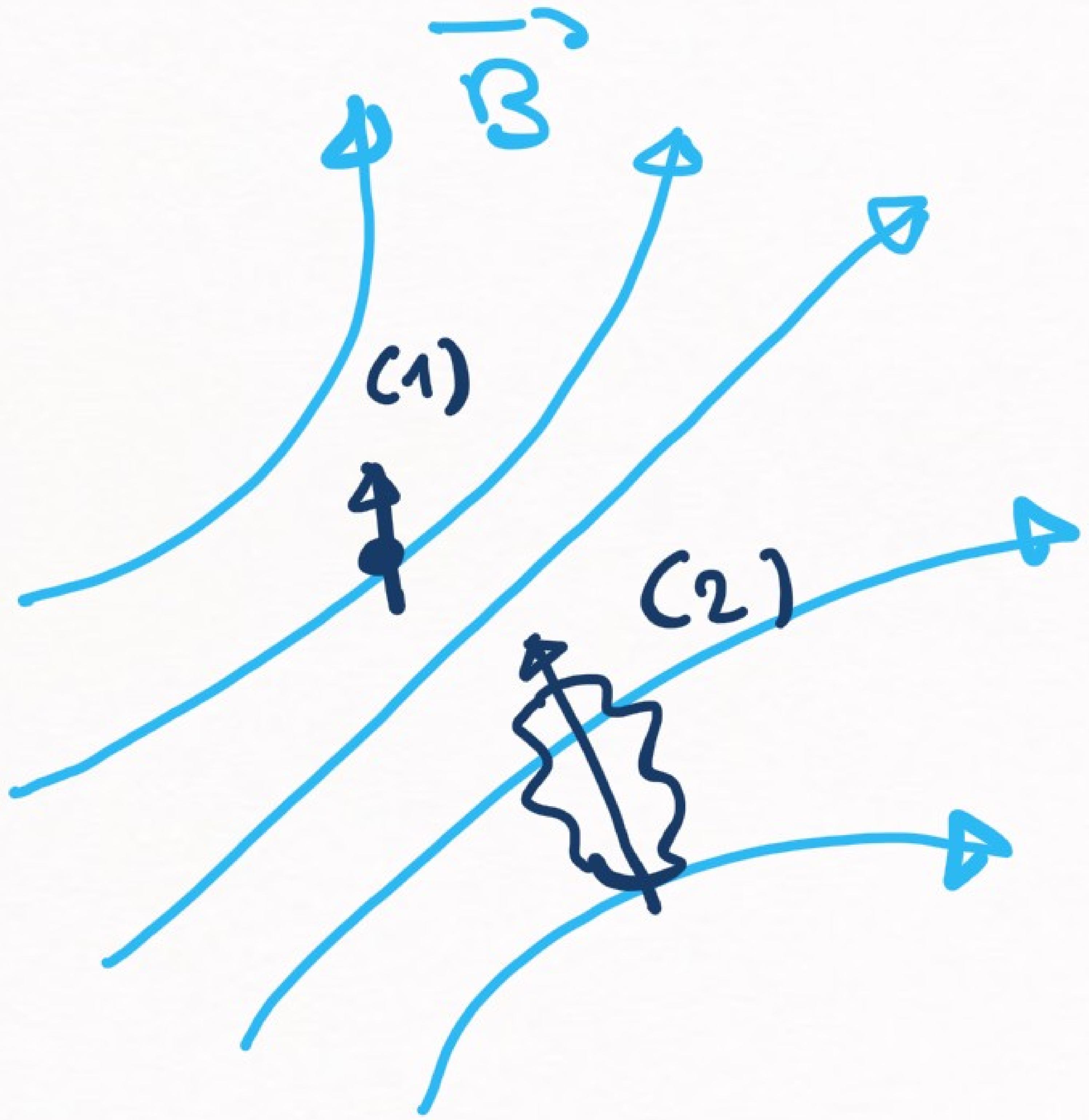
(2) Charge distribution:

$$V = \int d^3\vec{r}\,\rho(\vec{r})\,\Phi(\vec{r})$$

$$\int d^3\vec{r}\,\rho(\vec{r}) = q$$

We also have magnetic moments:

| MAGNETIC |



(1) point-like magnet

$$V(\vec{r}) = -\vec{\mu}\cdot\vec{B}(\vec{r})$$

(2) extended magnet

$$V = -\int d^3\vec{r}\,\vec{\mu}(\vec{r})\cdot\vec{B}(\vec{r})$$

$$\vec{B} = \underbrace{\vec{\nabla}\times\vec{A}}_{\text{vector potential}}$$

$$\int d^3\vec{r}\,\vec{\mu}(\vec{r}) = \vec{\mu}$$

($\vec{\mu} \to$ magnetic moment)

DEUTERON $\Rightarrow$ CHARGE DISTRIBUTION (wave function of the proton)

$$V(\vec{r}) = \int d^3\vec{r}\, \rho(\vec{r})\, \Phi(\vec{r})$$

$\neq$ (neutron is neutral)

MULTIPOLAR EXPANSION:

If $\Phi(\vec{r})$ changes at distance scales larger than the charge distribution:

$$\Phi(\vec{r}) = \underbrace{\Phi(\vec{0})} + \vec{r}\cdot\vec{\nabla}\Phi(\vec{r})\Big|_{\vec{r}=0} + \frac{1}{2} r_i r_j \partial_i \partial_j \Phi(\vec{r})\Big|_{\vec{r}=\vec{0}} + \dots$$

$\vec{r} = \vec{0}$ could be the center of the charge distribution

MULTIPOLAR EXPANSION $\to$ MULTIPOLAR MOMENTS

$$V = \int d^3\vec{r}\, \rho(\vec{r})\, \Phi(\vec{r})$$

$$= \Phi(0) \int d^3\vec{r}\, \rho(\vec{r}) + \vec{\nabla}\Phi \cdot \int d^3\vec{r}\, \vec{r}\, \rho(\vec{r})$$

$$+ \frac{1}{2} \partial_i \partial_j \Phi \cdot \int d^3\vec{r}\, r_i r_j\, \rho(\vec{r}) + \dots$$

$$= q\Phi + \vec{d} \cdot \vec{\nabla}\Phi + \frac{1}{6} Q_{ij} \partial_i \partial_j \Phi + \dots$$

$$\frac{1}{6} \langle r^2 \rangle \vec{\nabla}^2 \Phi \qquad \qquad \frac{1}{6} Q_{ij} \left(\partial_i \partial_j - \frac{1}{3} \vec{\nabla}^2 \right) \Phi$$

THE MOMENTS:

$$q = \int d^3\vec{r}\, \rho(\vec{r})$$ charge (we already knew this one)

$$\vec{d} = \int d^3\vec{r}\, \vec{r}\, \rho(\vec{r})$$ electric dipole moment

$$Q_{ij} = \int d^3\vec{r}\, (3r_i r_j - \underline{r^2\delta_{ij}})\, \rho(\vec{r})$$ electric quadrupole moment

$\left(Q_{ij} = Q_{ji},\ \sum_i Q_{ii} = 0\right) \rightarrow 5$ moments

THE QUADRUPOLAR MOMENT:

$Q = Q_{33}$ → Usual definition

It can be measured by the behavior of a nucleus in a non-uniform electric field

$Q_d = 0.286 \, fm^2 \longrightarrow Q_d = \int d^3\vec{r}\,(3z^2 - r^2)\rho(\vec{r})$

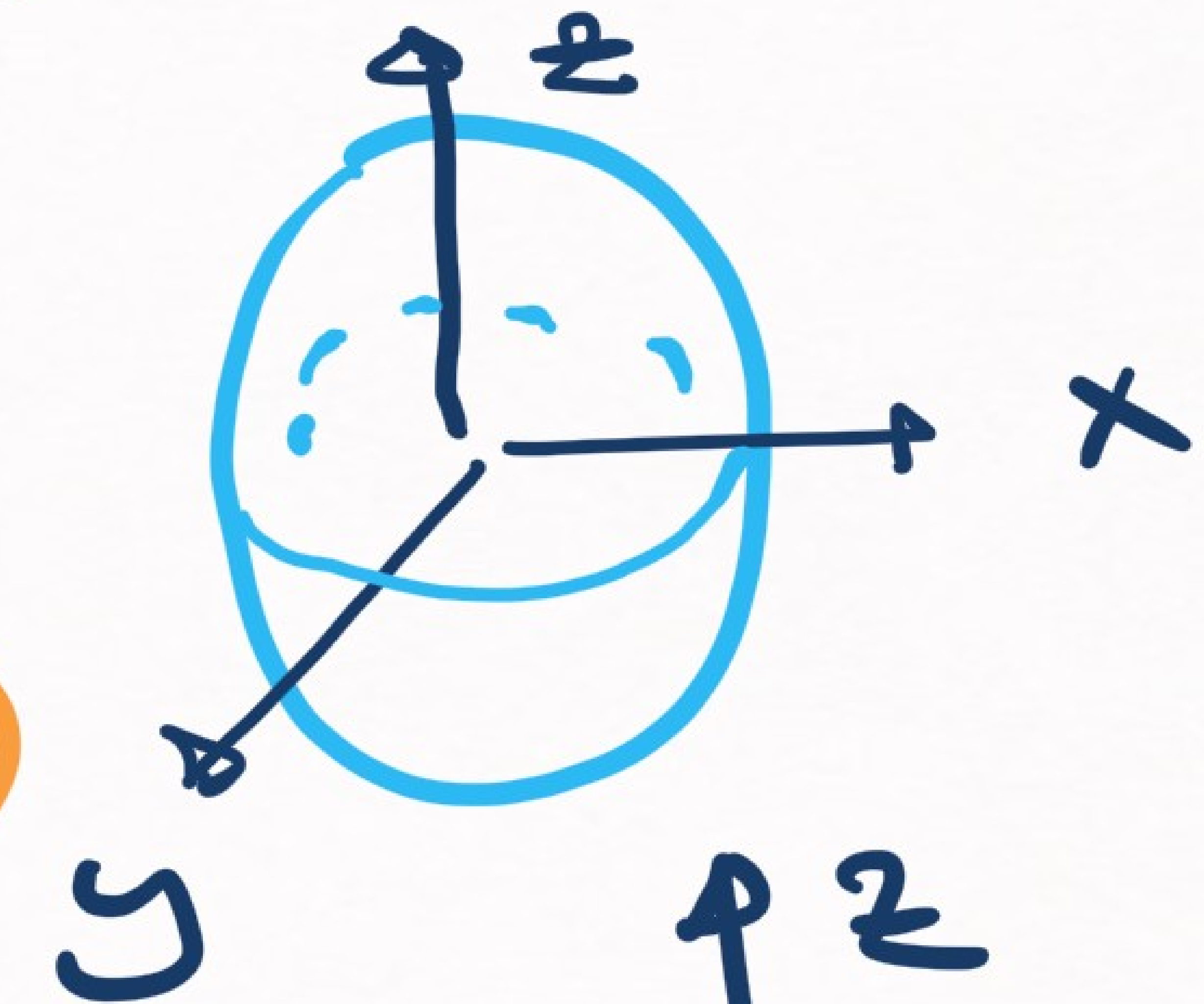
geometry of the distribution

$$\propto 3\langle z^2\rangle - \langle r^2\rangle$$

SIGN OF THE QUADRUPOLAR MOMENT & SHAPE OF THE CHARGE DISTRIBUTION

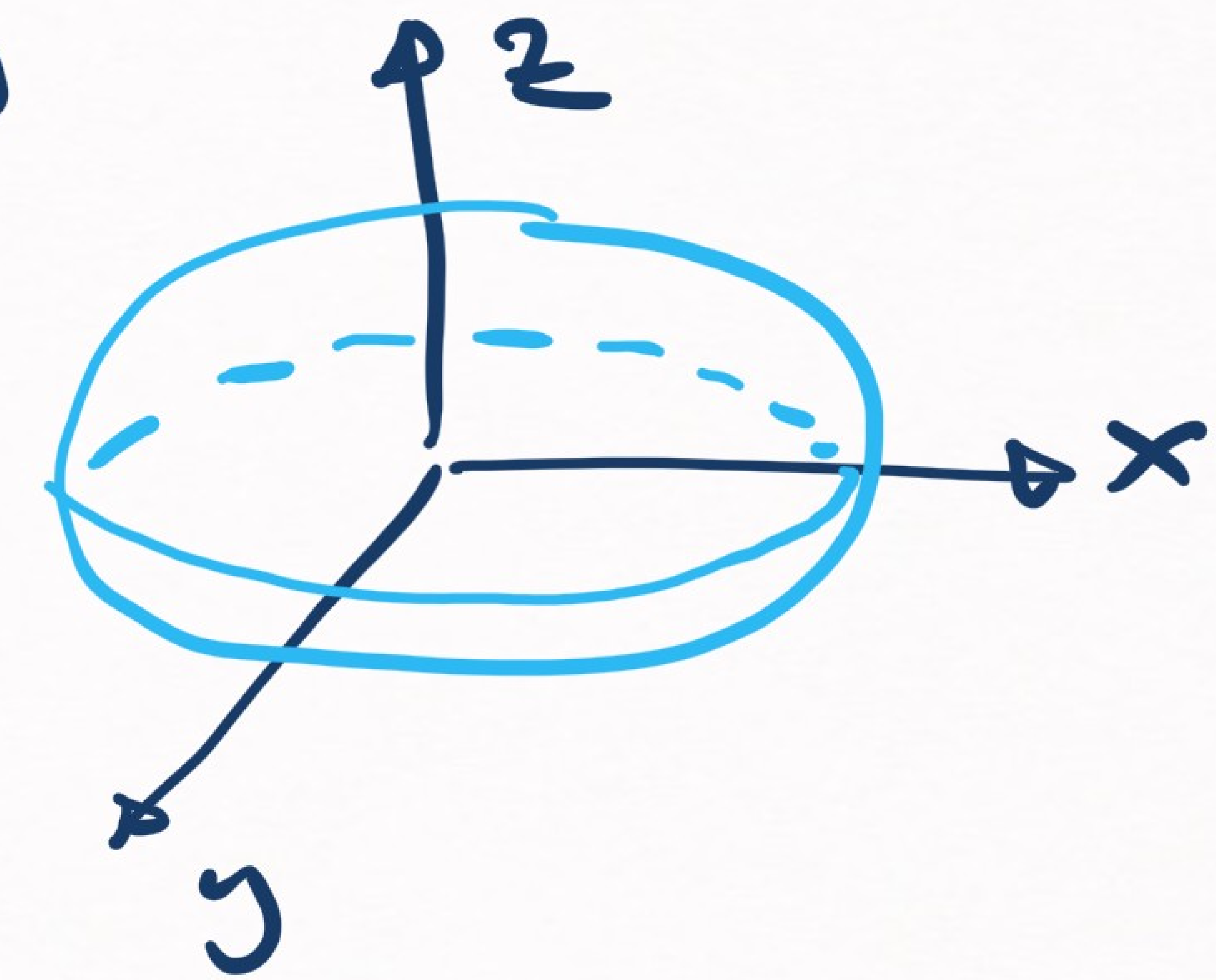
$Q > 0 \longrightarrow 3\langle z^2 \rangle \geq \langle r^2 \rangle \longrightarrow$

$\langle z^2 \rangle \geq \frac{1}{2}\left(\langle x^2 \rangle + \langle y^2 \rangle\right)$



$Q < 0 \longrightarrow 3\langle z^2 \rangle \leq \langle r^2 \rangle \longrightarrow$

$\langle z^2 \rangle \leq \frac{1}{2}\left(\langle x^2 \rangle + \langle y^2 \rangle\right)$



$Q = 0 \longrightarrow$ Spherical distribution

AND NOW WE WILL CONTINUE WITH ...

→ ORIGIN OF THE NUCLEAR FORCES !

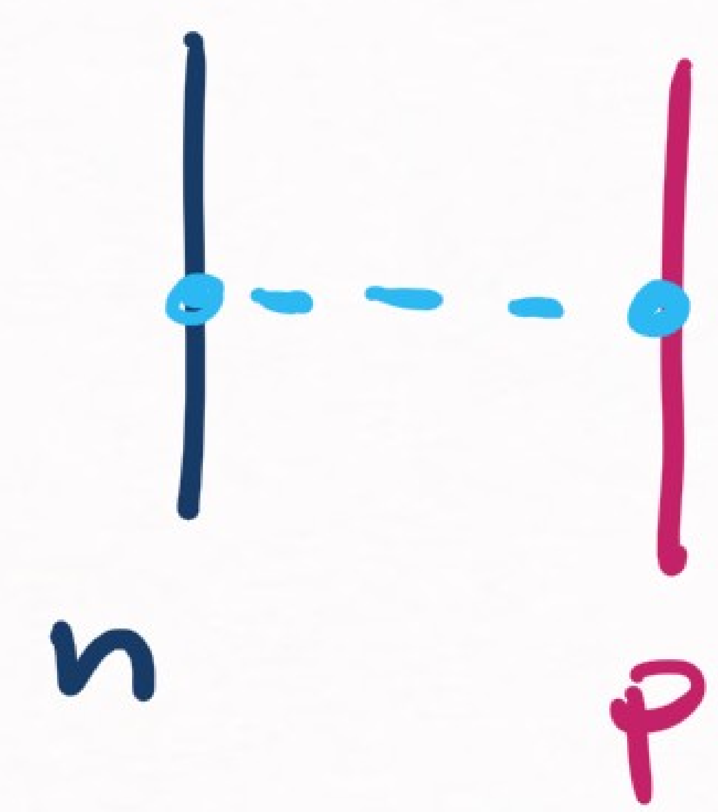
ORIGIN OF THE NUCLEAR FORCES

Where does $V_{NN}(\vec{r})$ comes from?

QFT → [Forces come from the exchange of particles]

Quantum Field Theory

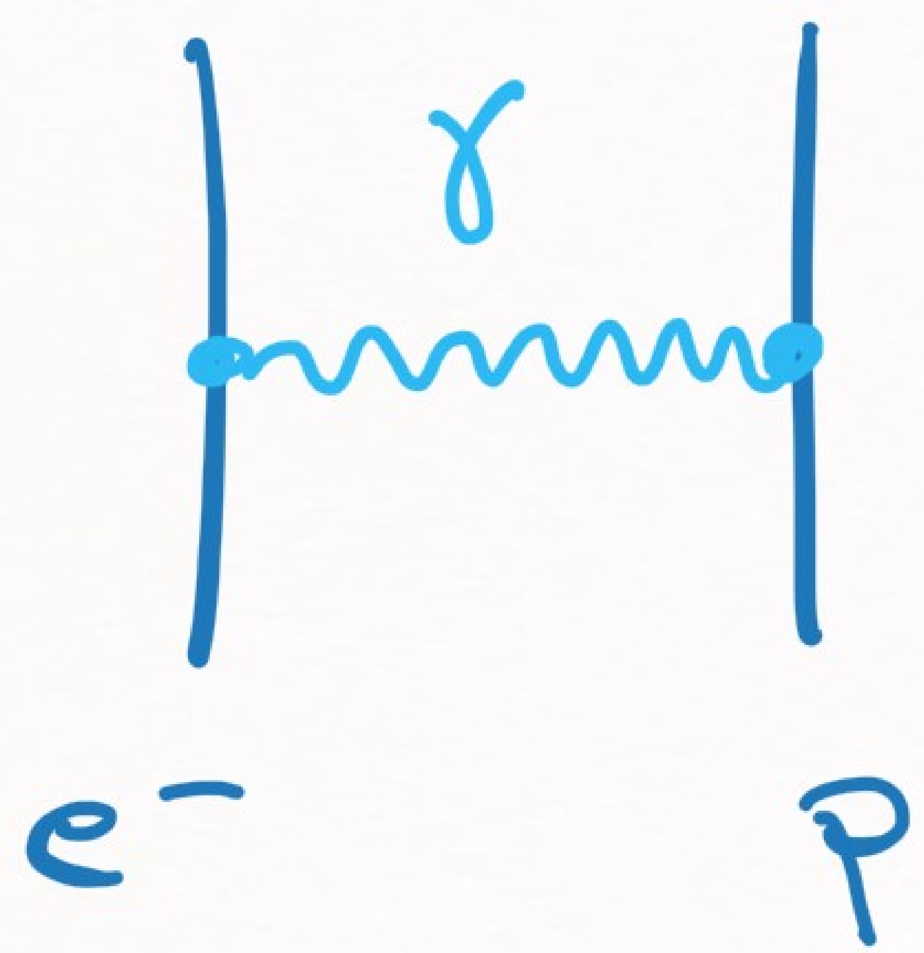
also true for Coulomb



→ some particle will be exchanged

$\boxed{\text{Coulomb}}$ → Exchange of a virtual photon

→ Example of how this mechanism works



→ (Feynman rules) → $V_c(\vec{q}) = -\frac{e^2}{|\vec{q}|^2}$

$\underbrace{\text{(Feynman rules)}}_{2}$

3

↓ (Fourier transform)

$$V_c(\vec{r}) = \int \frac{d^3\vec{q}}{(2\pi)^3} V_c(\vec{q})\, e^{-i\vec{q}\cdot\vec{r}} = -\frac{e^2}{4\pi r}$$

4

$\underbrace{\text{Feynman diagram}}_{1}$

Detailed calculation in Peskin & Schröder chapter 4.7 (Yukawa), chapter 4.8 (Coulomb)

$\boxed{\text{COULOMB}}$ → Can also be explained in QM ← ⊛

(second quantization formalism
for the electromagnetic field)

Usually studied
for explaining the decays of the excited levels
of the hydrogen atom

(usually appears in undergraduate courses)

⊛ → Useful as a derivation for those of you
who still have not studied QFT.

Decay of 2p state into the 1s state + photon:

(Reminder)

$$d\Gamma(2p \to 1s+\gamma) = |\langle 1s+\gamma|\hat{t}|2p\rangle|^2 \left(\frac{\omega}{2\pi}\right)^2 d^2\hat{q}$$ Fermi's Golden Rule

$$\hat{t} = -ie\left[\underbrace{\vec{A}(\vec{q})}\cdot\frac{\vec{p}}{2m_e} + \frac{\vec{p}}{2m_e}\cdot\vec{A}(\vec{q})\right]$$

in second quantization $\vec{A}(\vec{q})$ is an operator (creates/destroys photons)

$$\langle 1_\gamma(\vec{q},\lambda)|\vec{A}(\vec{q})|0_\gamma\rangle = \frac{-ie}{\sqrt{2\omega}}\, e^{i\vec{q}\cdot\vec{r}}\, \vec{\epsilon}_\lambda(\vec{q})$$

polarization vector

For the decay of $2p \to 1s + \gamma$, what we did was:

1) Dipolar approximation $e^{i\vec{q}\cdot\vec{r}} \simeq 1$ (check!)

2) $\langle 1s+\gamma | \hat{t} | 2p \rangle = -ie \frac{1}{\sqrt{2\omega}} \vec{\epsilon}_\lambda^*(\vec{q}) \cdot \langle 1s | \frac{\vec{P}}{m_e} | 2s \rangle$

$\Rightarrow$ $[H, \vec{r}] = \frac{\vec{P}}{m_e}$ $\Rightarrow$ $-ie\omega \frac{1}{\sqrt{2\omega}} \vec{\epsilon}_\lambda^*(\vec{q}) \langle 1s | \vec{r} | 2p \rangle$

(check!)

3) We put all the pieces together and .... voilá!

$$\Gamma(2p \to 1s) = \frac{4\alpha}{3} \omega^3 |\langle 1s | \vec{r} | 2p \rangle|^2$$

(check!)

→ $\boxed{\text{END OF THE REMINDER}}$ ←

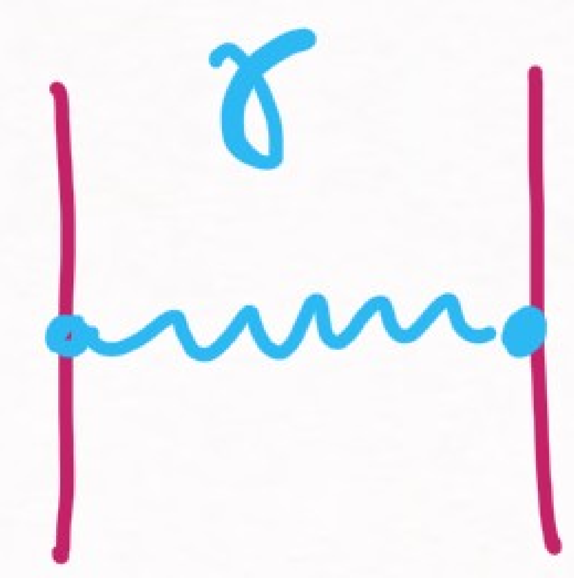
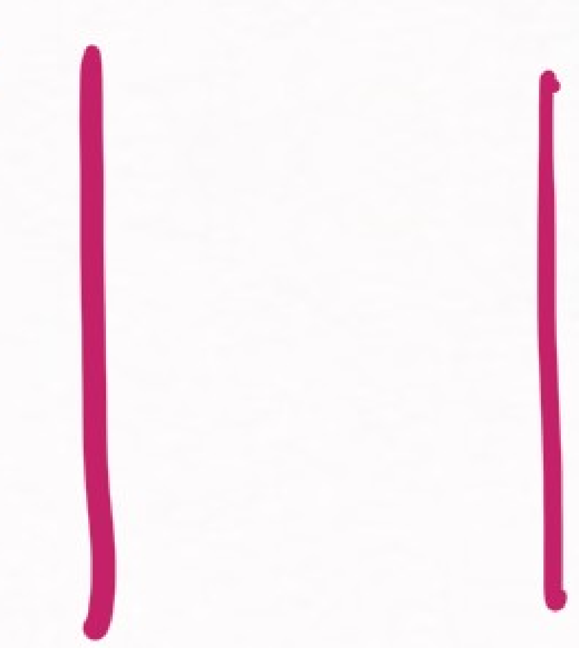
You probably studied this in the past for electromagnetic decays

→ So... HOW TO USE THIS TO DERIVE THE COULOMB POTENTIAL?

[HOW DO WE CALCULATE THE COULOMB POTENTIAL?]

→ photon

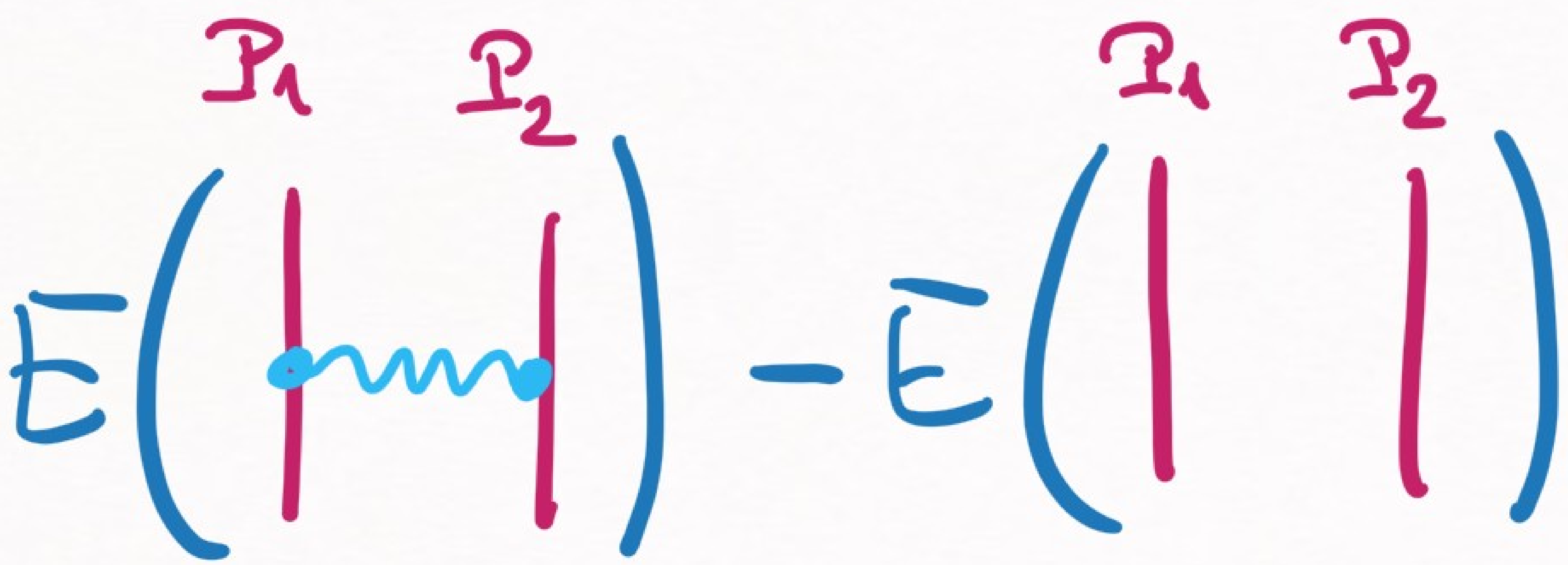
→ no photon

$V(\vec{r}) = E\Big($  $\Big) - E\Big($  $\Big)$

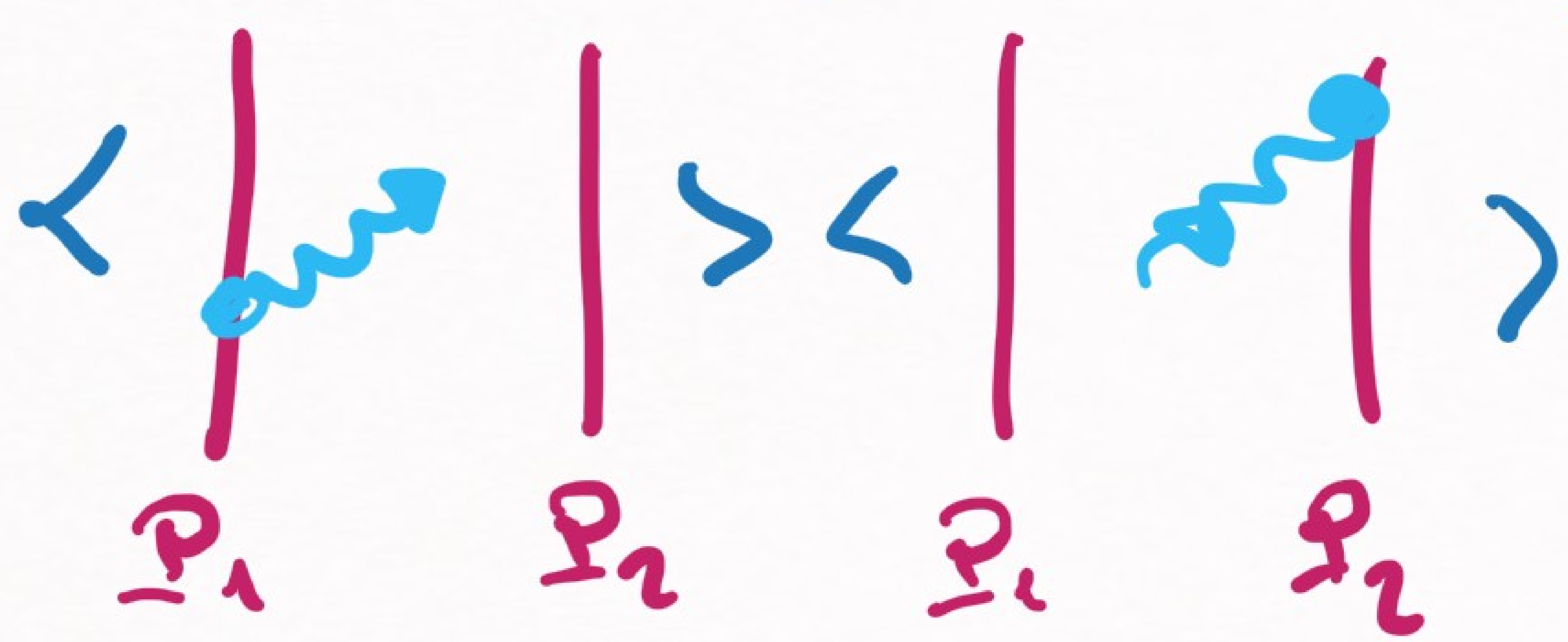
$\underbrace{P_1, P_2}_{\text{two particles}}$

[Reminder → a potential is just a difference in energy]

QUANTUM MECHANICS $\longrightarrow$ Second order perturbation theory



$$= \sum_{p_1 p_2 \gamma} \frac{\langle p_1 p_2 | H | p_1 p_2 \gamma \rangle \langle p_1 p_2 \gamma | H | p_1 p_2 \rangle}{E^{(0)}_{p_1 p_2} - E^{(0)}_{p_1 p_2 \gamma}}$$



Check your QM textbook

We put a few of the pieces together:

$$E^{(0)}_{P_1P_2} = m_1 + m_2 + \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2}$$

$$E^{(0)}_{P_1P_2\gamma} = E^{(0)}_{P_1P_2} + \omega \longrightarrow \text{energy of the photon}$$

$(\omega = |\vec{q}\,|)$

$$V_c(\vec{r}\,) = -\sum_{P_1P_2\gamma} \frac{\langle P_1P_2|H|P_1P_2\gamma\rangle\langle P_1P_2\gamma|H|P_1P_2\rangle}{\omega}$$

$$\sum_{P_1P_2\gamma} \longrightarrow \sum_{(\dots)} \int \frac{d^3\vec{q}}{(2\pi)^3}$$

$(\dots) \to$ we check later

But we still need the matrix element:

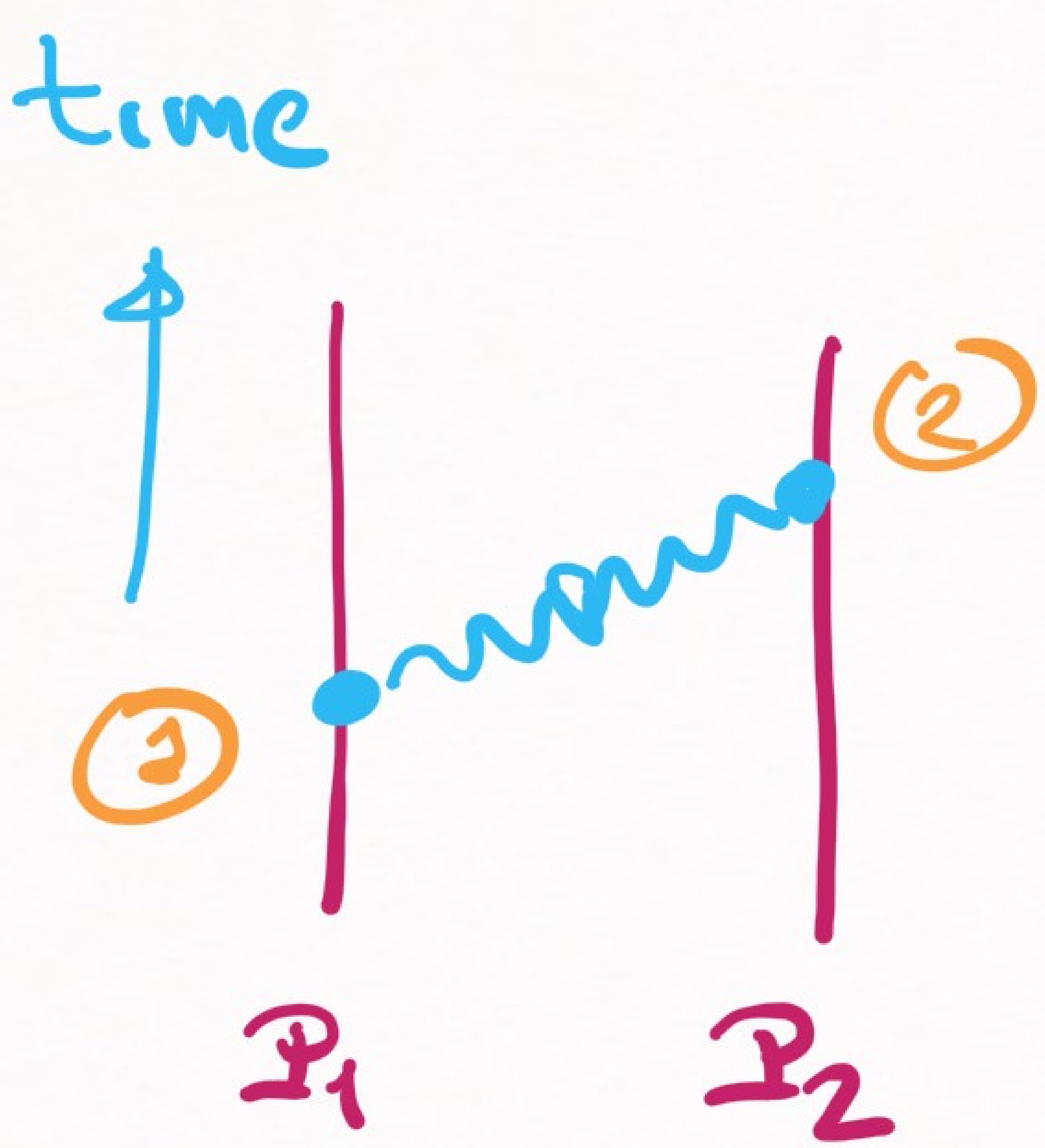
$$\langle p_1 p_2 | H | p_1 p_2 \gamma \rangle = \underbrace{-ie \frac{1}{\sqrt{2\omega}} e^{-i\vec{q}\cdot\vec{r}} \, \vec{\epsilon}^{\,*}_{\lambda}(\vec{q})}_{\text{(from before) should be similar}} \; ?$$

NO, WE HAVE TO ADAPT IT

(there are important differences)

TWO PROBLEMS W/ THE PREVIOUS MATRIX ELEMENT

1) We have to indicate which of the particles is emitting the photon :



$$= \langle P_1 P_2 | H | \underbrace{\overbrace{(P_1 \gamma)}^{\text{particle 1}}}_{\text{emit a photon}} P_2 \rangle \langle P_1 \underbrace{\overbrace{(P_2 \gamma)}^{\text{particle 2}}}_{\text{absorb the photon}} | H | P_1 P_2 \rangle$$

2) The Coulomb potential is related to the scalar potential!

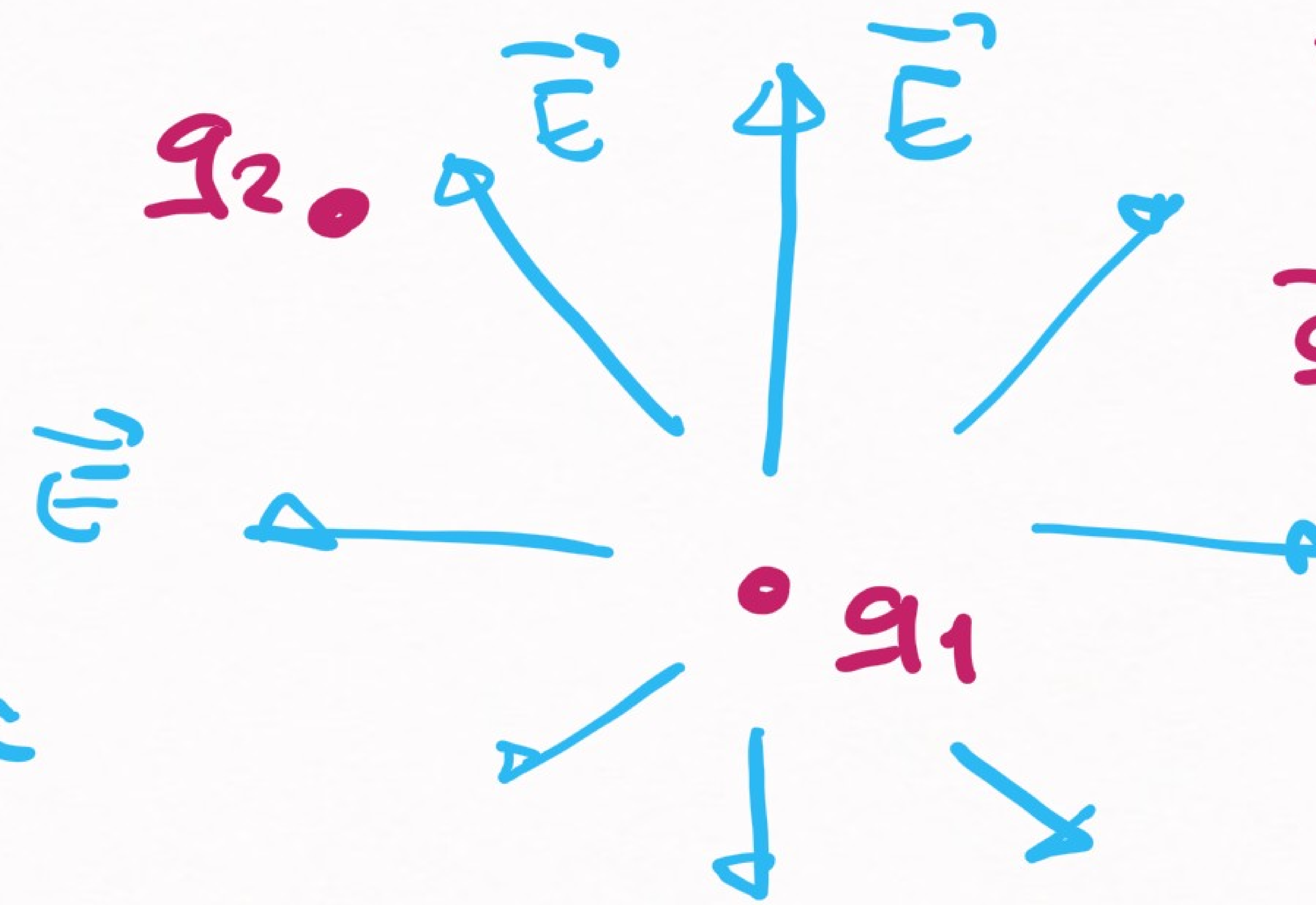
$$\boxed{\vec{E} = -\vec{\nabla}\Phi}$$

$$\vec{B} = \vec{\nabla}\wedge\underline{\underline{\vec{A}}}$$

(static sources)

w/ time dependence

Coulomb is related to $\Phi(\vec{r})$



$$V = q_2\Phi_1(\vec{r})$$

$$\Phi_1(\vec{r}_1) = \frac{q_1}{4\pi r}$$

[But before we saw the creation/annihilation rule for $\vec{A}$ (not for Φ) !]

$$\langle \underline{p}_j \gamma | \underline{\underline{\vec{A}}} | \underline{p}_j \rangle = -ie_j \frac{1}{\sqrt{2\omega}} e^{-i\vec{q}\cdot\vec{r}_j} \underbrace{\vec{\epsilon}_\lambda(\vec{q})}_{\text{polarization}}$$

vector potential

$$\left[\underline{\underline{\vec{\epsilon}_\lambda(\vec{q})\cdot\vec{q} = 0}} \right]$$

two physical polarization for photons

HOW TO HANDLE THIS?

$$\boxed{\Phi, \vec{A}} \rightarrow (\Phi, \vec{A}) \text{ or } (\underline{\underline{A_0}}, \vec{A})$$

We put them together within a 4-vector

$$\underline{\underline{\Phi = A_0}}$$

AND WE NOW MODIFY THE RULE :

$(\epsilon_0, \vec{e})$

$$\langle p_j \gamma | A_\mu | p_j \rangle = -ie_j \frac{1}{\sqrt{2\omega}} e^{-i\vec{q}\cdot\vec{r}} \epsilon^*_{\lambda\mu}(\vec{q})$$

$(A_0, \vec{A})$

$\vec{\epsilon}_\lambda \cdot \vec{q} = 0$, $\sum_\lambda \epsilon^*_{\lambda i} \epsilon_{\lambda j} = \delta_{ij} - \hat{q}_i \hat{q}_j$

① $\vec{A} \rightarrow (A_0, \vec{A})\ (A_\mu)$

② $\rightarrow$ now we have more polarizations

$\epsilon_{\lambda\mu} q^\mu = 0$, $\sum_\lambda \epsilon^*_{\lambda\mu} \epsilon_{\lambda\nu} = g_{\mu\nu} - \cancel{\hat{q}_\mu \hat{q}_\nu}$

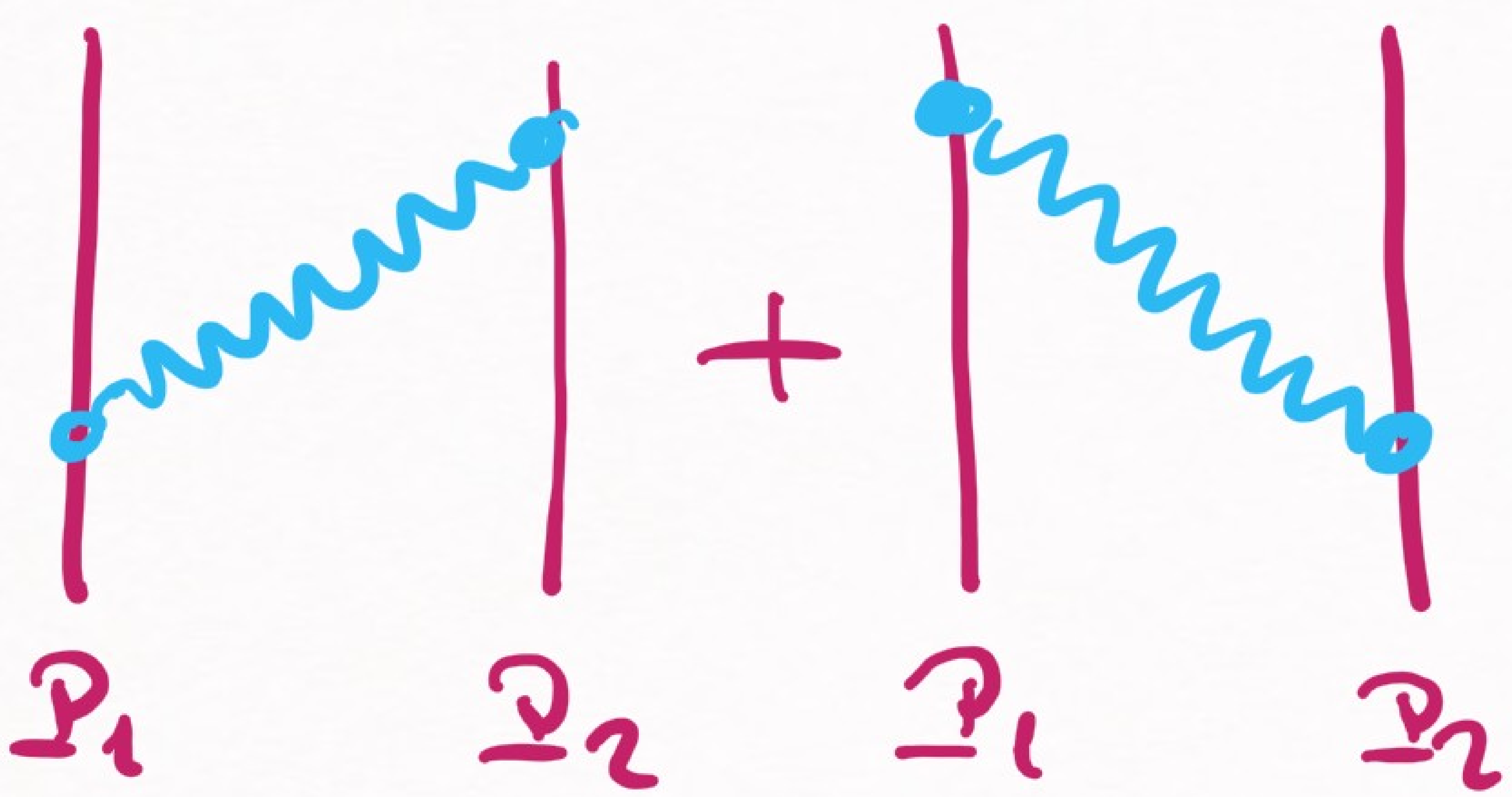
important!

$\boxed{g_{00} = -1}$, $g_{ii} = +1$

$(g_{0i} = 0)$ $(g_{ij} = +\delta_{ij})$

AND NOW WE CAN PUT THE PIECES TOGETHER:

$$V_C(\vec{r}) = -\sum_{\lambda}\int\frac{d^3\vec{q}}{(2\pi)^3}\left[\frac{\langle p_1 p_2|H|(p_1\gamma)\,p_2\rangle\langle p_1(p_2\gamma)|H|p_1 p_2\rangle}{\omega} + \frac{\langle p_1 p_2|H|p_1(p_2\gamma)\rangle\langle(p_1\gamma)\,p_2|H|p_1 p_2\rangle}{\omega}\right]$$



We just go to the previous slides and begin to substitute things

NEXT → We ignore the parts that depend on $\vec{\epsilon}$ (or $\vec{A}$)

$\Downarrow$

$$\langle P_1 P_2 | H | (P_1 \gamma) P_2 \rangle = -i e_s \sqrt{\frac{\ell}{2\omega}}\, e^{-i\vec{q}\cdot\vec{r}_1}\, \epsilon^{*}_{\lambda 0} \quad (1)$$

H now means Φ, the scalar potential

(1) → Because Coulomb is related to Φ only (not to $\vec{A}$)

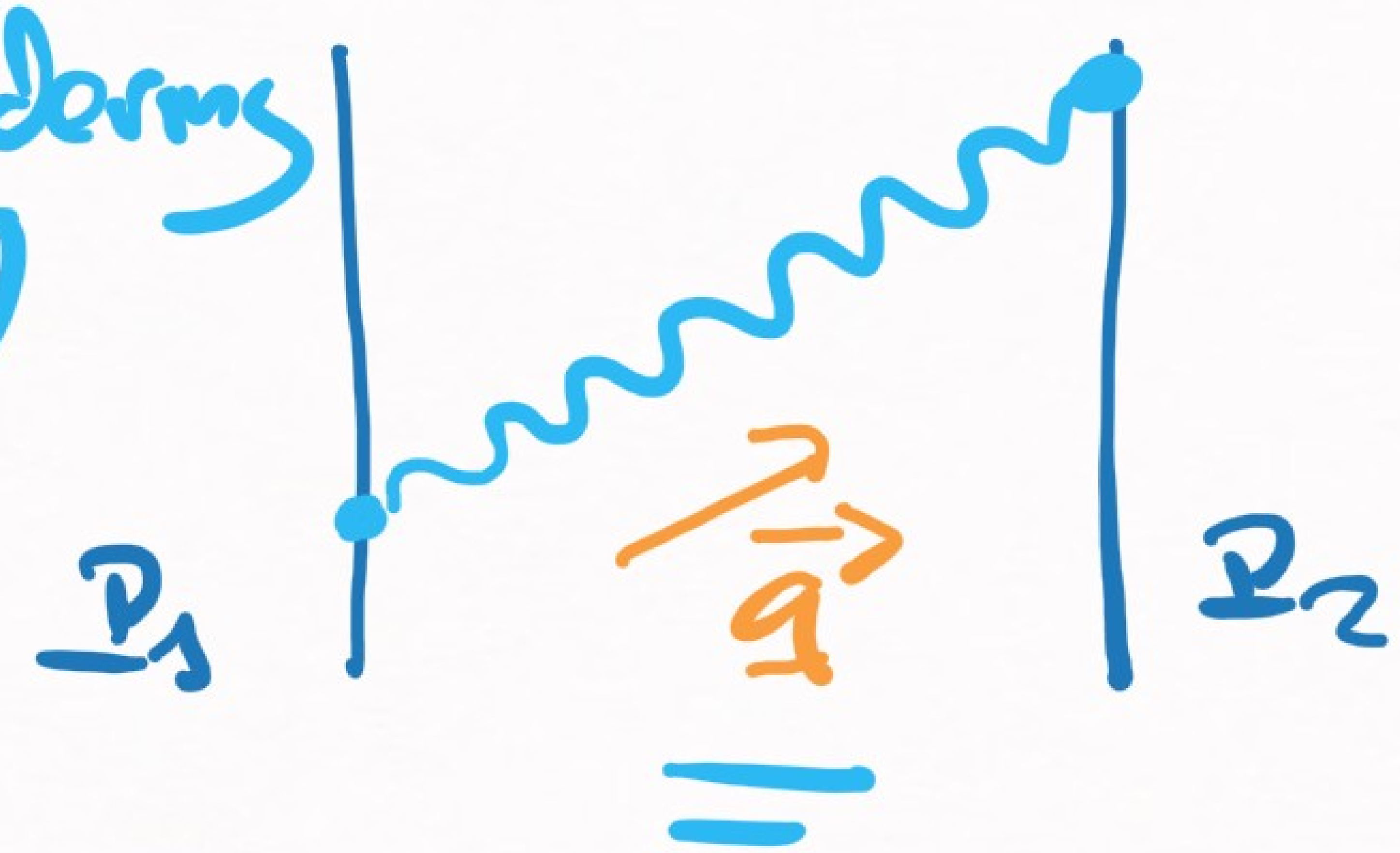
And we calculate now:

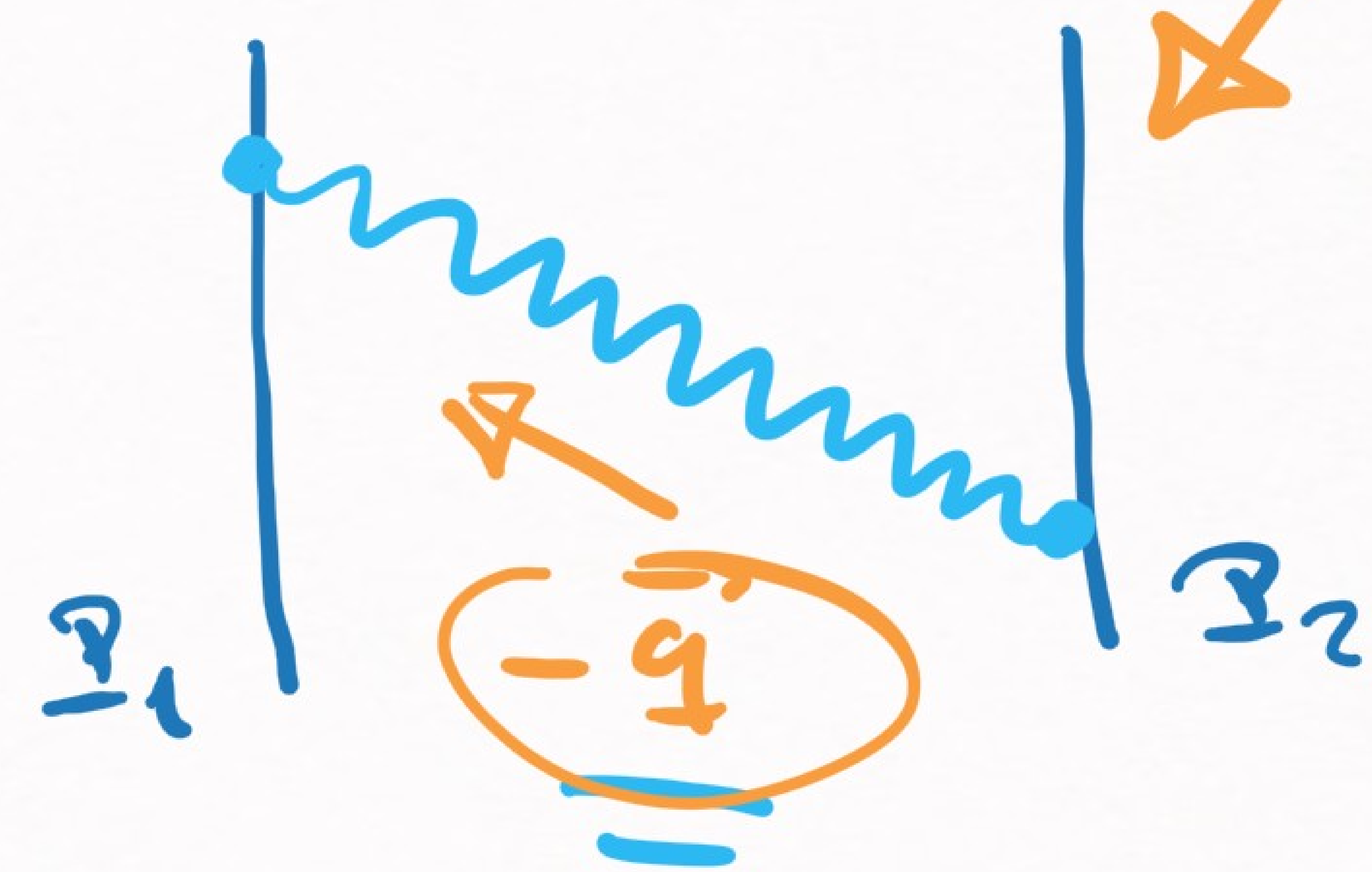
surprise factor

$$V(\vec{r}) = -\sum_{\lambda} \int \frac{d^3\vec{q}}{(2\pi)^3} \underbrace{(-ie_1)(ie_2)}_{e_1 e_2} \frac{1}{\omega} \boxed{\epsilon_{\lambda 0}\epsilon^*_{\lambda 0}} \times \circledast$$

$$\circledast \times \left[\frac{e^{-i\vec{q}(\vec{r}_1-\vec{r}_2)}}{2\omega} + \frac{e^{+i\vec{q}(\vec{r}_2-\vec{r}_1)}}{2\omega} \right]$$

(here, time-orderings is important)



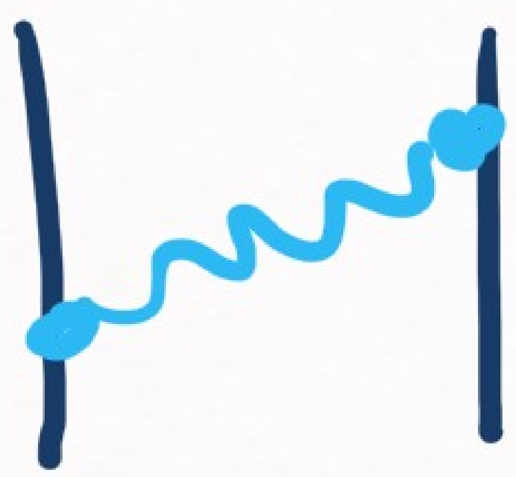


AND THERE IS SOMETHING WEIRD HERE :

$$\sum_{\lambda} \epsilon_{\lambda 0} \epsilon^{*}_{\lambda 0} = g_{00} = -1$$

We will have the opposite sign that we would have expected from second order perturbation theory

SECOND ORDER PERTURBATION THEORY



$$= \sum \frac{\langle p_1 p_2 | H | p_1 p_2 \phi \rangle \langle p_1 p_2 \phi | H | p_1 p_2 \rangle}{E_{p_1 p_2} - E_{p_1 p_2 \phi}} = \circledast$$

$E_{p_1 p_2 \phi} > E_{p_1 p_2}$ → because of the additional particle

$$\circledast = -\sum \frac{|\langle p_1 p_2 | H | p_1 p_2 \phi \rangle|^2}{|E_{p_1 p_2} - E_{p_1 p_2 \phi}|}$$

→ negative & therefore attractive

Why does this expectation fails w/ photons?

→ Technical reason: relativistic extension of the sum over polarizations

$$\sum_\lambda \epsilon_{\lambda i}\epsilon^*_{\lambda j} = \delta_{ij} - \hat{q}_i\hat{q}_j \rightarrow \sum_\lambda \epsilon_{\lambda\mu}\epsilon_{\lambda\nu} = g_{\mu\nu}$$

$(g_{ij} = \delta_{ij})$

→ If we exchange a "vector boson" → $(\phi, \vec{A})$

→ Sign different than expected

→ On the contrary, for a "scalar boson" we will obtain universal attraction

AND FINALLY WE ARRIVE TO COULOMB:

$$V(\vec{r}) = e_1 e_2 \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{e^{-i\vec{q}\cdot(\vec{r}_1-\vec{r}_2)}}{|\vec{q}|^2}$$

$(\omega = |\vec{q}|)$

$$= \frac{e_1 e_2}{4\pi r} \longrightarrow \text{Coulomb}$$

electron-proton $\left.\begin{array}{l} e_1 = -e \\ e_2 = +e \end{array}\right\} \rightarrow V(\vec{r}) = -\frac{e^2}{4\pi r} = -\frac{\alpha}{r}$

$\boxed{\text{CAVEAT}}$ → I HAVE CHEATED A BIT WITH THE PREVIOUS CALCULATION

(I did not explain why a few things do really come out)

This is what we call a heuristical derivation

(Time-ordered perturbation theory)

→ END OF ORIGIN OF COULOMB ←

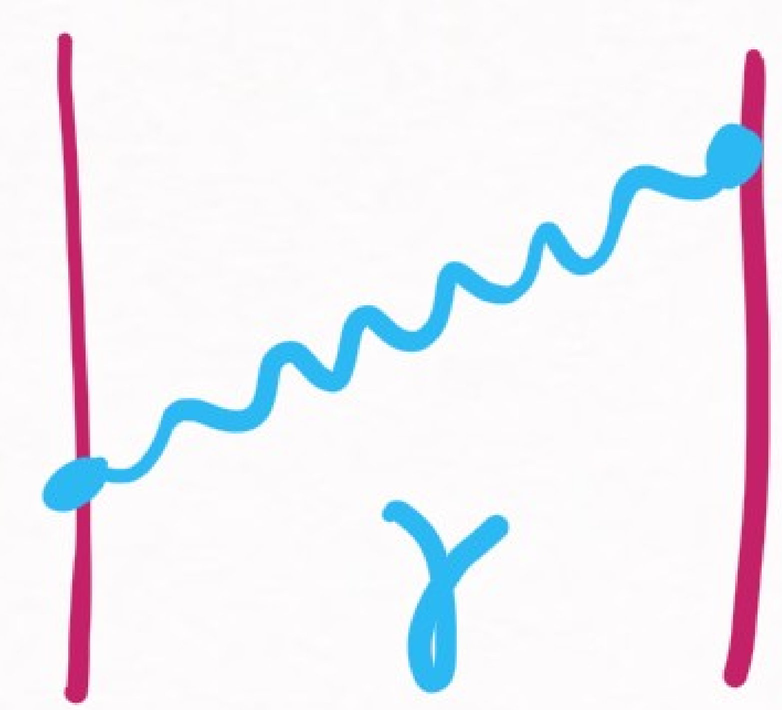
(this is not crucial,
focus on the qualitative ideas)

RECAP

In QFT forces are the result of the exchange of a particle (*)

$$A_\mu = (\underbrace{A_0}_{\text{scalar}}, \underbrace{\vec{A}}_{\text{vector}})$$

COULOMB →



Photon → quantization of the electromagnetic field (*)

PARITY TRASFORMATION

$$\boxed{\begin{aligned} A_0 &\to A_0 \\ \vec{A} &\to -\vec{A} \end{aligned}}$$

[PHOTON IS A $J^P = 1^-$ PARTICLE/FIELD]

J: spin of the photon

P: parity of the photon

Photon $\rightarrow (A^0, \vec{A})$

Rotations: $\vec{A} \rightarrow R\vec{A} \Rightarrow$ spin-1 quantity

Parity $(\vec{r} \rightarrow -\vec{r})$: $\underbrace{\vec{A} \rightarrow -\vec{A}}_{\Rightarrow\ P=-1 \text{ quantity}}$ $(A_0 \rightarrow A_0)$

$\boxed{\vec{r}, \vec{p}} \rightarrow J^P = 1^-$

$\boxed{\vec{L} = \vec{r} \wedge \vec{p}} \rightarrow J^P = 1^+$ (parity: $(-\vec{r}) \wedge (-\vec{p}) = \vec{r} \wedge \vec{p}$)

But besides photons we can exchange other particles

$\Rightarrow$ $\boxed{\text{YUKAWA}}$ $\rightarrow$ Exchange of a massive boson

(this explanation of nuclear forces)

But ... $\exists$ many types of boson

SCALAR BOSON $\rightarrow$ $\underline{\underline{J^P = 0^+}}$ $\rightarrow$ It behaves like a scalar quantity

(Potential $\rightarrow$ just like Coulomb, but without all the weird polarizations)

$\boxed{\text{BOSONS \& FORCES}}$

1) COULOMB $\longrightarrow$ PHOTON $\longrightarrow$ VECTOR BOSON

$$(A^0, \vec{A}) \xrightarrow{\text{rotations}} (A^0, R\vec{A})$$

$$\xrightarrow{\text{parity}} (A^0, -\vec{A})$$

$$V(\vec{r}) = + \frac{e_1 e_2}{4\pi r} \qquad \text{" + "} \longrightarrow g_{\mu\nu} \text{ for } \mu\nu = 0$$

$$(g_{00} = +1)$$

2) SCALAR BOSON $\longrightarrow$ YUKAWA'S PROPOSAL

$$\left.\begin{array}{l} \phi \xrightarrow[\text{rotation}]{} \phi \\ \phi \xrightarrow[\text{parity}]{} \phi \end{array}\right\}$$ potential will be just like Coulomb but without the polarization factors

$(g_{00} = -1) \longrightarrow (1)$

VECTOR BOSON SCALAR BOSON

2) POTENTIAL FOR SCALAR BOSON :

$$\langle P_1 P_2 | H | (P_1 \gamma) P_2 \rangle = -\frac{i g_Y}{\sqrt{2\omega}} e^{-i\vec{q}\cdot\vec{r}_1}$$ ← modify this rule

$(g_{Y1} = g_{Y2} = g_Y)$ (remove ϵ_λ)

$$\frac{1}{\omega^2} = \frac{1}{m^2 + \vec{q}^{\,2}} \Rightarrow V(\vec{r}) = -g_Y^2 \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{1}{\vec{q}^{\,2} + m^2}$$

Boson with $m \neq 0$

$$= -\frac{g_Y^2}{4\pi} \frac{e^{-mr}}{r} \Bigg\} \text{Yukawa potential}$$

[WHY DOES THIS POTENTIAL FAILS?]

$$V(\vec{r}) = -\frac{g_Y^2}{4\pi}\frac{e^{-mr}}{r} \Rightarrow V(\vec{r}) = V(|\vec{r}|)$$

Prediction: QUADRUPOLAR MOMENT OF THE DEUTERON $= 0$

$\boxed{Q_d = 0}$ → not correct → not the origin of the nuclear force

SOLUTION $\rightarrow$ WE NEED A BOSON THAT GENERATES A NON-CENTRAL FORCE

MOST OBVIOUS CANDIDATE :

VECTOR BOSON $(V_0, \vec{V}) \simeq (A_0, \vec{A})$

(like in COULOMB, but this time we will include the effects of $\vec{V}$)

quantum effects
$\rightarrow$ interacts w/spin

WE INCLUDE THE VECTOR POTENTIAL TOO:

$$\langle P_1 P_2 | H | (P_1 V) P_2 \rangle = -\frac{i}{\sqrt{2\omega}} e^{-i\vec{q}\cdot\vec{r}_1} \times \left[g_V \epsilon^*_{\lambda 0} \xrightarrow{} E0 \right.$$

$M_1 \to$ mass of particle 1

$$\left. \underbrace{+ i \frac{f_V}{2M_1} \vec{\epsilon}^* \cdot (\vec{\sigma}_1 \times \vec{q})}_{M1} \right]$$

$\vec{\sigma}_1 \to$ [spin operator of P_1]

E0 $\to$ electric-type part of the interaction

M1 $\to$ magnetic-type part of the interaction $\Big) \to$ we previously ignored it for coulomb

ELECTRIC & MAGNETIC PIECES (3)

$$V_E(\vec{q}) = \frac{g_V^2}{m_V^2 + \vec{q}^2} \longrightarrow \text{electric-like piece}$$

$$V_M(\vec{q}) = \left(\frac{f_V}{2M}\right)^2 \frac{(\vec{\sigma}_1 \times \vec{q}) \cdot (\vec{\sigma}_2 \times \vec{q})}{m_V^2 + \vec{q}^2} \longrightarrow \text{magnetic-like piece}$$

$$\rightarrow \left[\; V(\vec{q}) = V_E(\vec{q}) + V_M(\vec{q}) \;\right]$$

ELECTRIC & MAGNETIC PIECES ②

FOURIER-TRANSFORM INTO R-SPACE

$$V_E(\vec{r}) = \frac{g_v^2}{4\pi}\frac{e^{-mr}}{r}$$ → just Coulomb with a mass

$$V_M(\vec{r}) = \left(\frac{g_v}{2M}\right)^2\Big[\frac{2}{3}\bar{\sigma}_1\cdot\bar{\sigma}_2\,\frac{e^{-mr}}{4\pi r}$$

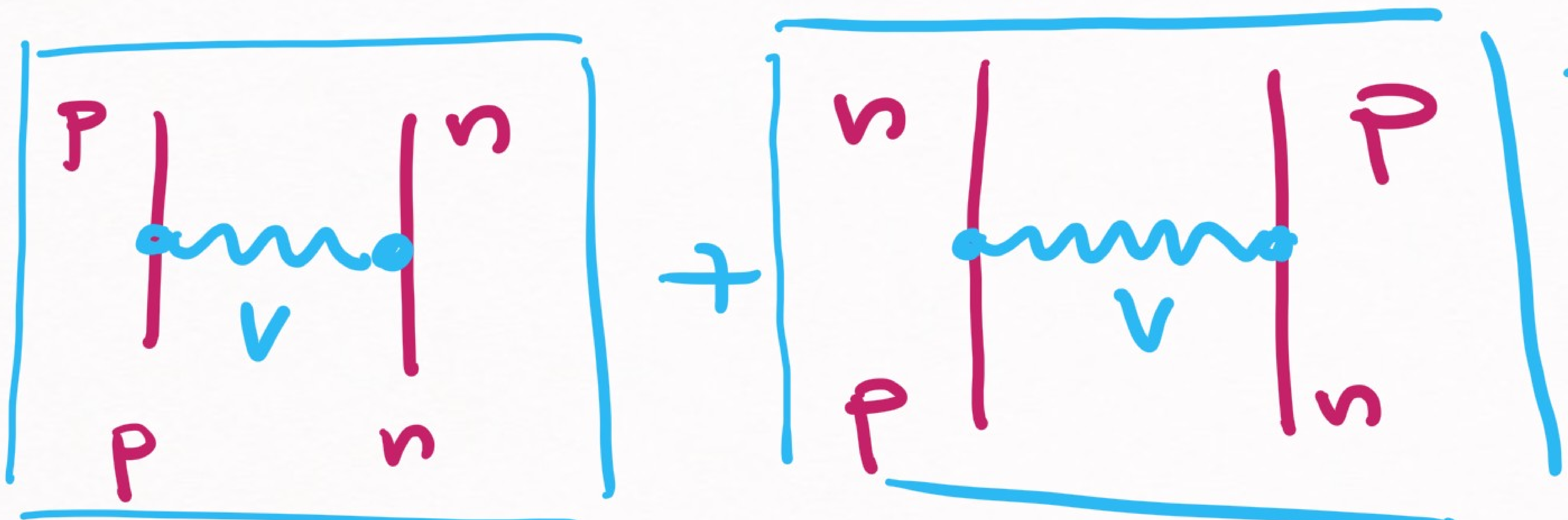
$$-\frac{1}{3}\left(3\bar{\sigma}_1\cdot\hat{r}\,\sigma_2\cdot\hat{r}-\bar{\sigma}_1\cdot\bar{\sigma}_2\right)\frac{e^{-mr}}{4\pi r}\left(1+\frac{3}{mr}+\frac{3}{(mr)^2}\right)\Big]$$

new piece → $\bar{\sigma}_1\cdot\hat{r}\,\bar{\sigma}_2\cdot\hat{r}$ → Quadrupole moment

NON - SPHERICAL $\Rightarrow$ BUT DOES IT SOLVE THE PROBLEM?

1) FIRST CAVEAT $\longrightarrow$ Previous potential is for particles that are distinguished by the force

nuclear forces : $n \longleftrightarrow p$



previous page $V(r)$

$\longrightarrow$ we need to include new terms in $V(r)$

Deuteron : page 56

$$V_d(\vec{r}) = -3\,\boxed{V(\vec{r})}$$

2) SECOND CAVEAT [SIGN OF THE QUADRUPOLAR MOMENT]

$$V(r) = -g^2 \left[a\, \vec{\sigma}_1 \cdot \vec{\sigma}_2\, W_c(r) \pm b \left(3\, \vec{\sigma}_1 \cdot \hat{r}\, \vec{\sigma}_2 \cdot \hat{r} - \vec{\sigma}_1 \cdot \vec{\sigma}_2 \right) W_T(r) \right]$$

with $\underline{\underline{a, b > 0}}$

$\longrightarrow$ same sign

$\Rightarrow \quad Q = \pm |Q|$

We can compare w/ previous potential

$\boxed{\text{VECTOR MESON}} \Rightarrow$ WRONG SIGN OF DEUTERON'S QUADRUPOLAR MOMENT

REJECT THIS HYPOTHESIS

$$V(\vec{r}) = \underbrace{-3}_{\text{Deuteron missing factor}} \left[V_E(\vec{r}) + V_M(\vec{r}) \right] = \circledast$$

$$\circledast = -3 g_V^2 \left[W_C(\vec{r}) + \vec{\sigma}_1 \cdot \vec{\sigma}_2 \, W_C'(\vec{r}) - \left(3 \vec{\sigma}_1 \cdot \hat{r} \, \sigma_2 \cdot \hat{r} - \vec{\sigma}_1 \cdot \vec{\sigma}_2 \right) W_T(\vec{r}) \right]$$

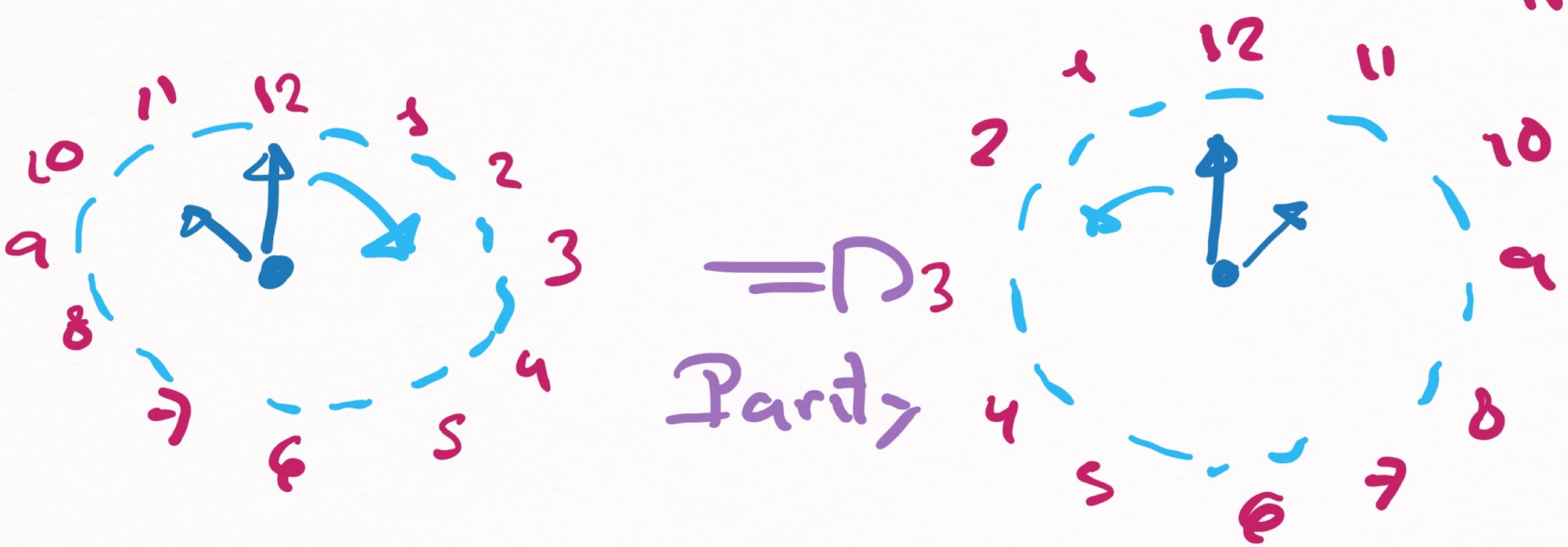
$$Q_d = -|Q_d|$$

NEGATIVE QUADRUPOLE MOMENT

4) PSEUDOSCALAR BOSON $J^P = 0^-$

Difficult to interpret → scalar quantity that changes sign with a parity transformation

Direction of clock's hands



EXAMPLE OF A PSEUDOSCALAR QUANTITY

A SECOND EXAMPLE OF A PSEUDOSCALAR:

$\vec{a}$ vector, $\vec{L} = \vec{r} \times \vec{p}$ pseudovector

$\vec{a} \cdot \vec{L}$ pseudoscalar:

$$\vec{a} \cdot \vec{L} \xrightarrow[\text{parity}]{} (-\vec{a}) \cdot ((-\vec{r}) \times (-\vec{p}))$$

$$= -(\vec{a}) \cdot \vec{L}$$

WHICH MATRIX ELEMENT TO USE?

$$\Rightarrow \langle P_1 P_2 | H | (P_1 \varphi) P_2 \rangle = - \frac{i g_{PS}}{\sqrt{2\omega}} e^{-i\vec{q}\cdot\vec{r}_1} \left[i \frac{\vec{\sigma}_1 \cdot \vec{q}}{2M} \right]$$

This new rule can only be calculated in QFT

$$V(\vec{r}) = - \frac{g_{PS}^2}{4M^2} \int \frac{d^3\vec{q}}{(2\pi)^3} (\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q}) \frac{e^{-i\vec{q}\cdot(\vec{r}_1 - \vec{r}_2)}}{m^2 + \vec{q}^2}$$

We find the potential

NOW WE FOURIER TRANSFORM:

$$V(\vec{r}) = \left(\frac{g_{\pi}}{2M}\right)^2 \left[\vec{\sigma}_1 \cdot \vec{\sigma}_2 \, \frac{e^{-mr}}{4\pi r} + \left(3\vec{\sigma}_1\cdot\hat{r}\,\vec{\sigma}_2\cdot\hat{r} - \vec{\sigma}_1\cdot\vec{\sigma}_2\right)\frac{e^{-mr}}{4\pi r}\left(1+\frac{3}{mr}+\frac{3}{(mr)^2}\right)\right]$$

... and we take a look at pages 57 (missing factor)
& 58 (sign of Q_{33})

THIS POTENTIAL PREDICTS $\boxed{Q_d > 0}$

$\Rightarrow$ and finally we got it right!!

Now we know that the boson generating the nuclear forces must be a pseudoscalar

$$J^P = 0^+$$

$\Rightarrow$ $\boxed{\text{THE PION}}$

$$m \simeq 140\ \text{MeV}$$

Recap

a) Yukawa proposed the exchange of a massive particle

(originally $J^P = 0^+$, scalar)

b) But this give us $\boxed{Q_d = 0}$

(contradicts experiment) ✗

c) If we try a vector meson $Q_d < 0$ ✗

b) Finally, if we try a pseudoscalar $\boxed{Q_d > 0}$

BINGO! ✓

SEE YOU ON TUESDAY

15:50