NUCLEAR PHYSICS 5

a) NATURAL & UNNATURAL SYSTEMS IN PHYSICS

CHECK A FEW EXAMPLES

b) THE NUCLEAR FORCE & IT'S PROPERTIES

QUALITATIVE PROPERTIES

RECAP

[Physical systems $\longrightarrow$ Scales]

(scales) $\longrightarrow$ $Q = \{ Q_1, Q_2, Q_3, \dots \}$

(momentum)

1) Natural systems : one scale dominates

$\circledast \leftarrow Q_1 \ll Q_2, Q_3, Q_4$ $\qquad \circledast \rightarrow$ this scale is special

$$\langle \psi | \mathcal{O} | \psi \rangle = c_{\mathcal{O}} \, Q_1^{d} \, , \quad c_{\mathcal{O}} \sim \mathcal{O}(1)$$

all observables are natural in terms of the dominant scale

But we also have ...

2) Unnatural systems : two or more scales fight with each other

$\boxed{Q_1 \sim Q_2} \ll Q_3, Q_4, \ldots$

or $\boxed{Q_1 \sim Q_2 \sim Q_3} \ll Q_4, Q_5, \ldots$

or any other variation

let's compute observable

$\langle \psi | \hat{O} | \psi \rangle =$ (it's complicated) $\longrightarrow$ [there might be fine-tuning]

the calculation could be a surprise

FINE-TUNING !

Basically, a cancellation among the effects of several scales

$\Rightarrow$ { (similar to obtaining a small number as the difference of two big numbers)

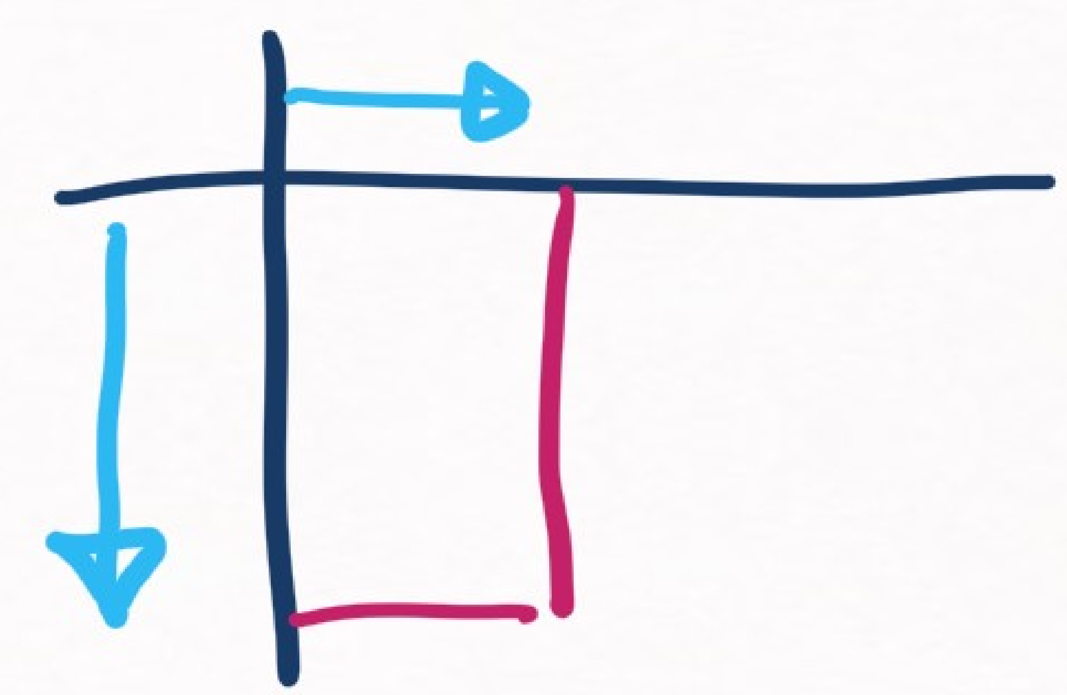
$$A - B = C$$

(but sometimes it can happen)

$$\left.\begin{aligned} A &\sim \mathcal{O}(10^6) \\ B &\sim \mathcal{O}(10^6) \\ C &\sim \mathcal{O}(1) \end{aligned}\right\} \text{usually this will not happen}$$

FINE-TUNING $\longrightarrow$ THE SQUARE WELL

a) Two parameters : range & strength



$\Rightarrow$ Two scales : $\{a, R_s\}$

b) When we have $\boxed{\frac{a}{R_s} = \frac{\pi}{2}}$ there is a zero-energy bound state

c) Condition very specific : $\frac{a}{R_s} = \frac{\pi}{2}(1+x)$

$\Rightarrow$ $x \lesssim 0.2$ for this to happen (fine tuning)

FINE-TUNING → [NUCLEAR FORCES]

a) np systems w/ $S = 0, 1$ (spin) (S-waves or $L = 0$)

b) for $S = 1$, we have the deuteron

$|E_B| \sim 2$ MeV

$E_B = \langle T \rangle + \langle V \rangle$

$|\langle T \rangle|, |\langle V \rangle| \sim 50$ MeV

fine-tuning of 1/25 (2/50)

[c) for $S = 0$, we have the virtual state (failed bound state)

$|E_V| \sim 0.07$ MeV fine tuning of 1/700 !!

[We want to understand why fine-tuning happens]

↳ TWO ANSWERS:

a) FORTUITOUS / BY CHANCE

→ just the way nature happens to be

b) "CONSPIRACY" / $\exists$ A MORE PROFOUND REASON

→ fine-tuning points toward new physics

a) BY CHANCE

FINE-TUNING

→ $\exists$ of eclipses : relative size of the moon and the sun are almost identical on earth (no fundamental reason for this)

→ Solar system & the location of earth

we just happen to be in the habitable zone of th sun

(no reason, only selection bias)

if this not the case, we will not be here

[b) "CONSPIRACY"]

FINE-TUNING

→ classical electron radius :
points towards new physics : QM & QFT

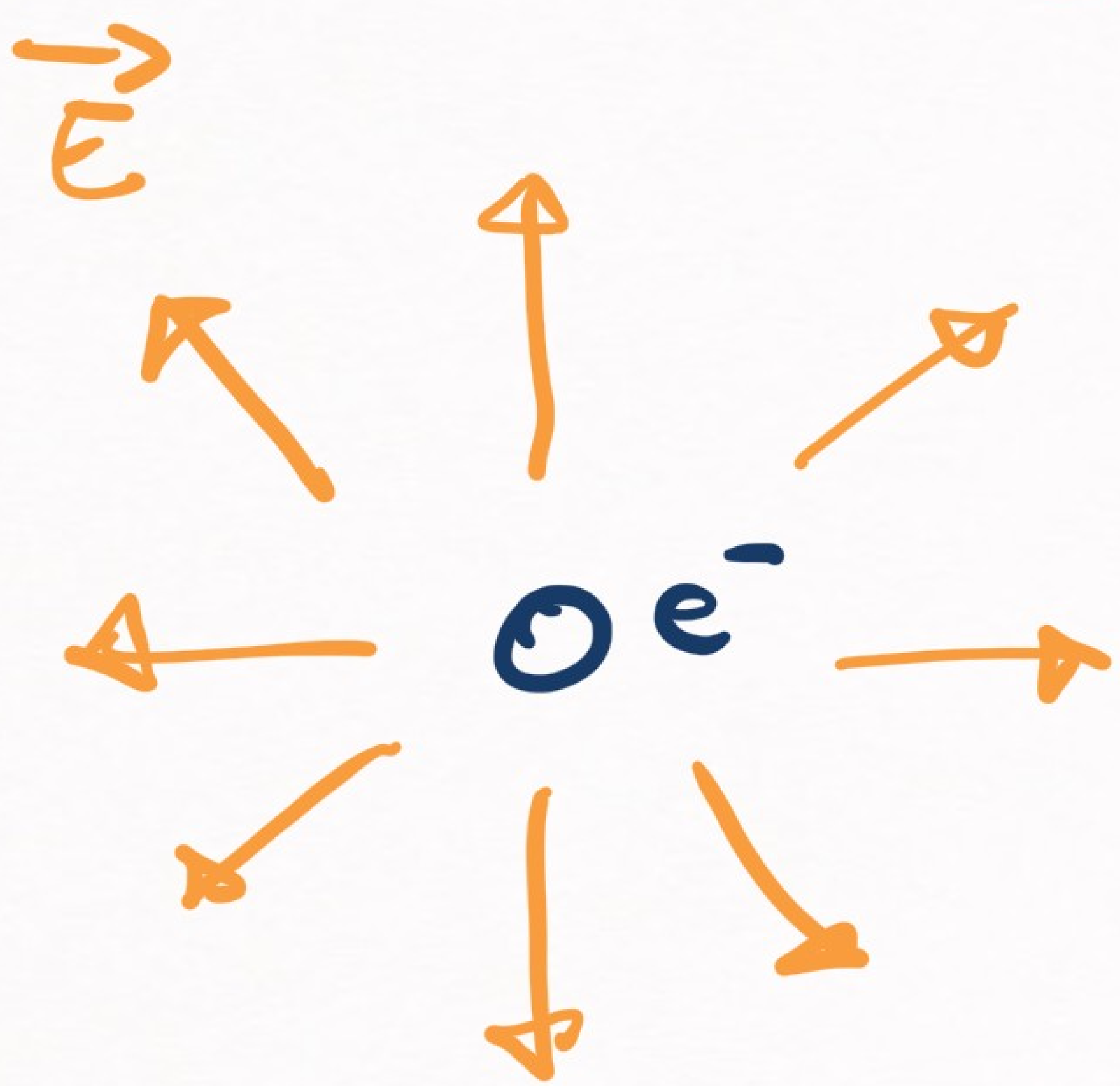
→ pion mass : (pion is the lightest hadron)
points towards the ρ meson & chiral symmetry

→ maybe the Higgs mass
→ maybe the cosmological constant
} ?

We will review a few examples of the "conspiracy-type" of fine tuning:

1) CLASSICAL ELECTRON RADIUS

(old calculation in classical electrodynamics)



(electric field generated by the electron)

contains energy → mass

→ if we take into account the energy
of the electric field around an electron:

$$m(e^-) = \underbrace{m_{bare}(e^-)}_{\text{"bare" mass}} + \underbrace{m(\vec{E})}_{\text{energy/mass of the } \vec{E} \text{ field}}$$

2

"bare" mass

⇓

mass of the electron if we "switch off"
the electric force

$m(\vec{E}) \rightarrow$ can be calculated

If we assume a point-like electron:

$m(\vec{E}) \longrightarrow \infty$!! ☹ → infinite fine-tuning

$$m(e^-) = \underbrace{m(\vec{E})}_{\infty} + \underbrace{m_{bare}(e^-)}_{-\infty} \quad [\infty + -\infty = \text{finite}]$$

☺

Infinite fine-tuning? → this doesn't make much sense

What are the alternatives?

Condition: $m_{bare}(\bar{e}) \geq 0$

(looks like a sensible additional condition)

$\Rightarrow$

CLASSICAL RADIUS OF THE ELECTRON:

$$m(e^-) = m(\vec{E}) + m_{bare}(e^-) \rightarrow \boxed{m(e^-) = m(\vec{E})}$$

(≥ 0)

$$|\vec{E}| \propto \frac{1}{r^2} \rightarrow$$ this energy will depend on the size of the electron

$\rightarrow$ classical electron radius $\boxed{r_{cl}(e^-) = \frac{\alpha}{m_e}}$

Classical radius of the electron :

$$\left(r_{ce}(e^-) = \frac{\alpha}{m_e} \simeq \underline{\underline{2.8\ fm}} \right.$$

What interpretation for this number ? $\rightarrow$ $\exists$ new physics before we reach r_{ce}

$\boxed{r > r_{ce}}$ $\rightarrow$ something new (not classical) must happen

Indeed we have new physics for $\boxed{r \gtrsim r_{cl}(e^-)}$

1) Quantum mechanics : $r \sim a_B \sim \frac{1}{m_e \alpha} \gg \frac{\alpha}{m_e}$

($\frac{1}{\alpha^2}$ larger than $r_{cl}(e^-)$)

2) Quantum field theory :

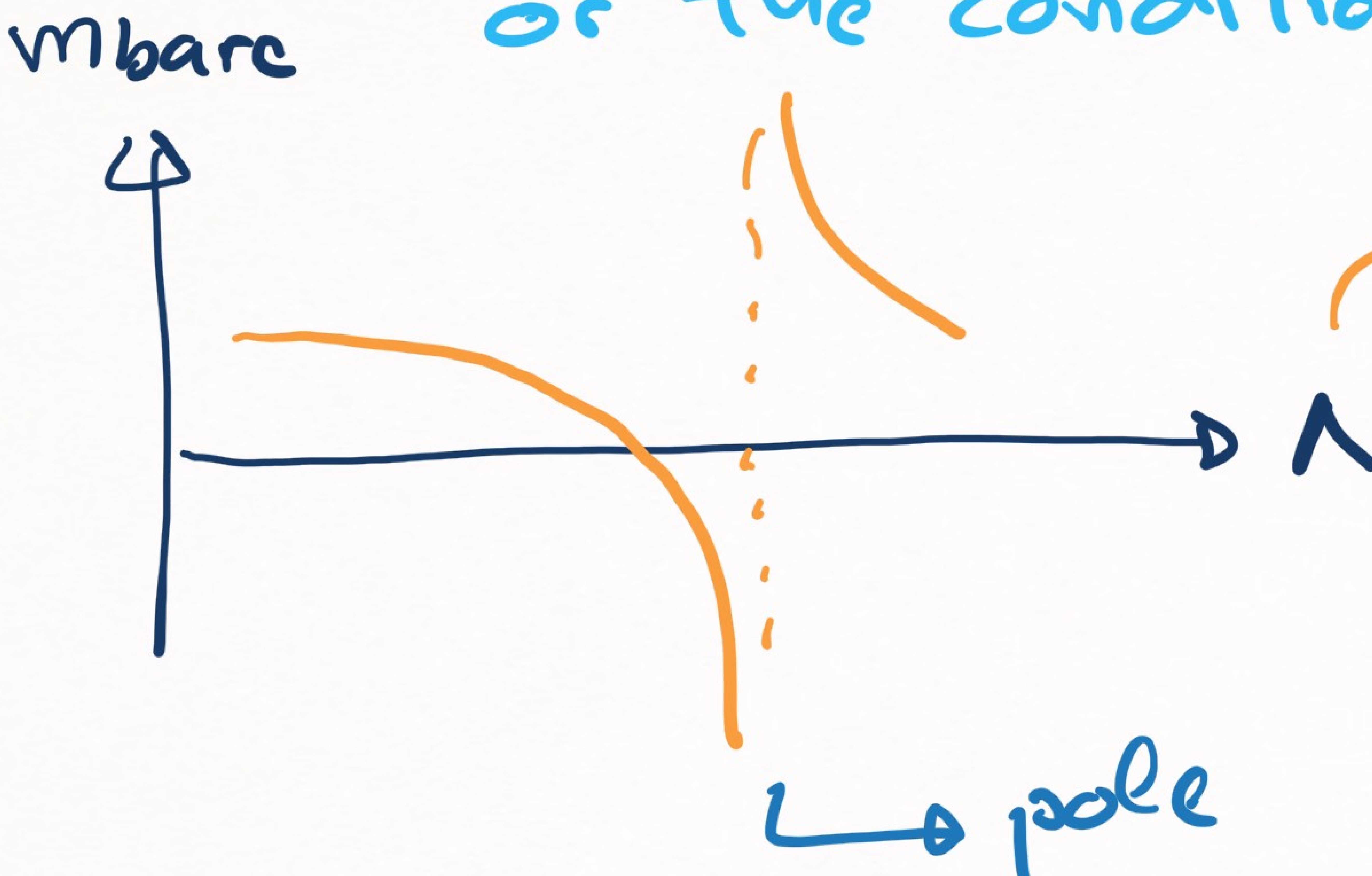
$r \sim \frac{1}{m_e} \gg \frac{\alpha}{m_e}$ → gives us a new bound for new physics

($\frac{1}{\alpha}$ times larger than $r_{cl}(e^-)$)

$m(e^-)$ in quantum field theory

at which energies do we have $m_{bare}(\bar{e}) \geq 0$

or the condition $m_{bare}(\bar{e}) \geq -\infty$



it's called a cutoff

(max. energy/momentum in our calculation)

$\boxed{\text{QFT}}$ → At which energies do we have a problem?

$\boxed{\text{LANDAU POLE}}$

$\Lambda \sim 10^{280}$ MeV

(renormalization of electron charge goes to infinity)

$$\left(r \sim \frac{\hbar c}{\Lambda} \sim \frac{200}{10^{280}} \text{ fm} \sim 2 \cdot 10^{-278} \text{ fm} \right)$$

$\boxed{\text{LANDAU POLE}}$

$$\left.\begin{array}{l} r \sim 2 \cdot 10^{-278} \text{ fm} \\ \Lambda \sim 10^{280} \text{ MeV} \end{array}\right\} \to \oplus$$

$\oplus$ → Again, we should expect new physics for $r > 2 \cdot 10^{-278}$ fm

(kind of obvious → $\Lambda_{\text{Landau}} \gg M_{\text{Planck}}$)

[Quantum gravity]

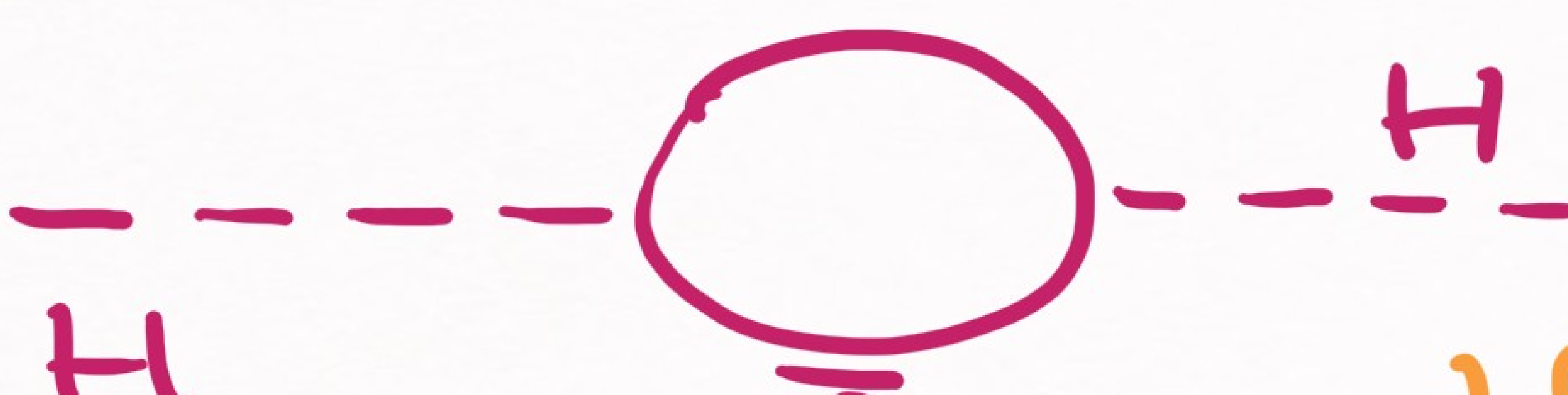
2) MASS OF THE HIGGS

(ANALOGOUS TO THE PION MASS PROBLEM)

$$m_H^2 = m_{bare}^2 + \underbrace{\Delta m_H^2}_{\text{in QFT this comes from loop corrections}}$$

→ like before w/ the electron

f → fermion



$\bar{f}$ → antifermion

$$\Delta m_H^2 \propto -\frac{|\lambda_f|^2}{8\pi^2}\left[\Lambda^2 + \dots\right]$$

The point is that corrections to the mass of the Higgs goes as Λ (the cutoff of the theory)

characteristic of spin-0 fields

(Higgs, pion, etc.)

In contrast, fermions receive mass corrections that go as $\log\left(\frac{\Lambda}{Q}\right) \rightarrow$ Landau pole

Fermions $\rightarrow$ $\Delta m \propto \log\left(\frac{\Lambda}{Q}\right)$ $\rightarrow$ EXAMPLE: Landau pole at incredibly high energies

Spin-0 particles $\rightarrow$ $\Delta m \propto \Lambda$

$\downarrow$ super sensitive to short-range physics

(grows really fast $\downarrow$ problems will appear before)

What is the amount of fine-tuning for the Higgs?

→ $\frac{m_H}{\Lambda}$ (ratio of the Higgs mass over the ultraviolet scale of the standard model)

→ ultraviolet → happen for Λ large (short-distances)

→ infrared → happen for Λ small (long-distances)

How large is Λ ?

$\frac{m_H}{\Lambda}$ a) A possibility is that the standard model is valid up to the Planck scale (distances at which we can see quantum gravity)

$G \sim \frac{1}{M_{Planck}^2} \longrightarrow M_{Planck} \simeq 10^{18}\ \text{GeV}$

Newton's constant (gravity)

$$\boxed{\frac{m_H}{\Lambda} \sim 10^{-16}}$$

b) Second possibility $\rightarrow$ standard model stops to be valid sooner

$$\frac{m_H}{\Lambda} \sim \frac{1}{10} \text{ or } \frac{1}{100} \text{ or } \frac{1}{1000} \rightarrow \underline{\text{depends on } \Lambda}$$

EXAMPLE $\rightarrow$ SUSY (supersymmetry) $\underline{\Lambda \sim M_{SUSY}}$

Usual estimation : $M_{SUSY} \sim (1-10)$ TeV

$$\frac{m_H}{\Lambda} \sim \left(\frac{1}{10} - \frac{1}{100}\right) \rightarrow \underline{\text{less fine-tuned}}$$

$\boxed{\text{PROBLEM}}$ → We don't know what new physics lies beyond the standard model

$\boxed{\frac{m_H}{\Lambda} \gg 10^{-16}}$

→ ∃ infinite reasons why this is not so fine-tuning

BOTTOM-LINE $\rightarrow$ $\frac{m_H}{\Lambda}$ probably indicates new physics

$$\left\{ \left(\Lambda \sim 10\, m_H,\ 100\, m_H,\ 1000\, m_H \text{ etc.} \right) \right.$$

any of this options look better than $10^{16}\, m_H$

3) COSMOLOGICAL CONSTANT (super fine-tuning example)

Reminder :

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \underbrace{\Lambda_c g_{\mu\nu}}_{\text{cosmological term}} = 8\pi G T_{\mu\nu}$$

$[\Lambda_c] = [E^4] \longrightarrow$ What is its expected natural size ?

$\Lambda_c = Q^4$ with Q a natural scale in gravity

one answer:

$G \sim \frac{1}{M_{Planck}{}^2} \rightarrow \underline{\underline{Q \sim M_{Planck}}}$

$\Lambda_c \sim (10^{18}\,GeV)^4 = (10^{27}\,eV)^4$

$= 10^{108}\,eV^4$

a) natural estimation of Λ_c

$$\Lambda_c \sim 10^{108} \text{ eV}^4$$

b) but we can measure Λ_c from the acceleration of the expansion of the universe

$$\boxed{\Lambda_c \sim 10^{-12} \text{ eV}}$$

HOLY MOLLY !!

The natural estimation fails by a factor of:

$$\left[\frac{\Lambda_c}{M_{Planck}^4} \sim \frac{1}{10^{120}} \right] \to \text{fine-tuning of 1 part in } 10^{120} \text{ !!!}$$

Looks improbable

→ Requires an explanation

Explanations:

1) Observation bias: we will only be alive in universes w/ certain values of Λ_c

2) Multiverse: more than 10^{120} universes

it's likely that there will be fine-tuned universes like ours

3) New physics: QFT understanding of vacuum energy

→ We don't know the answer &

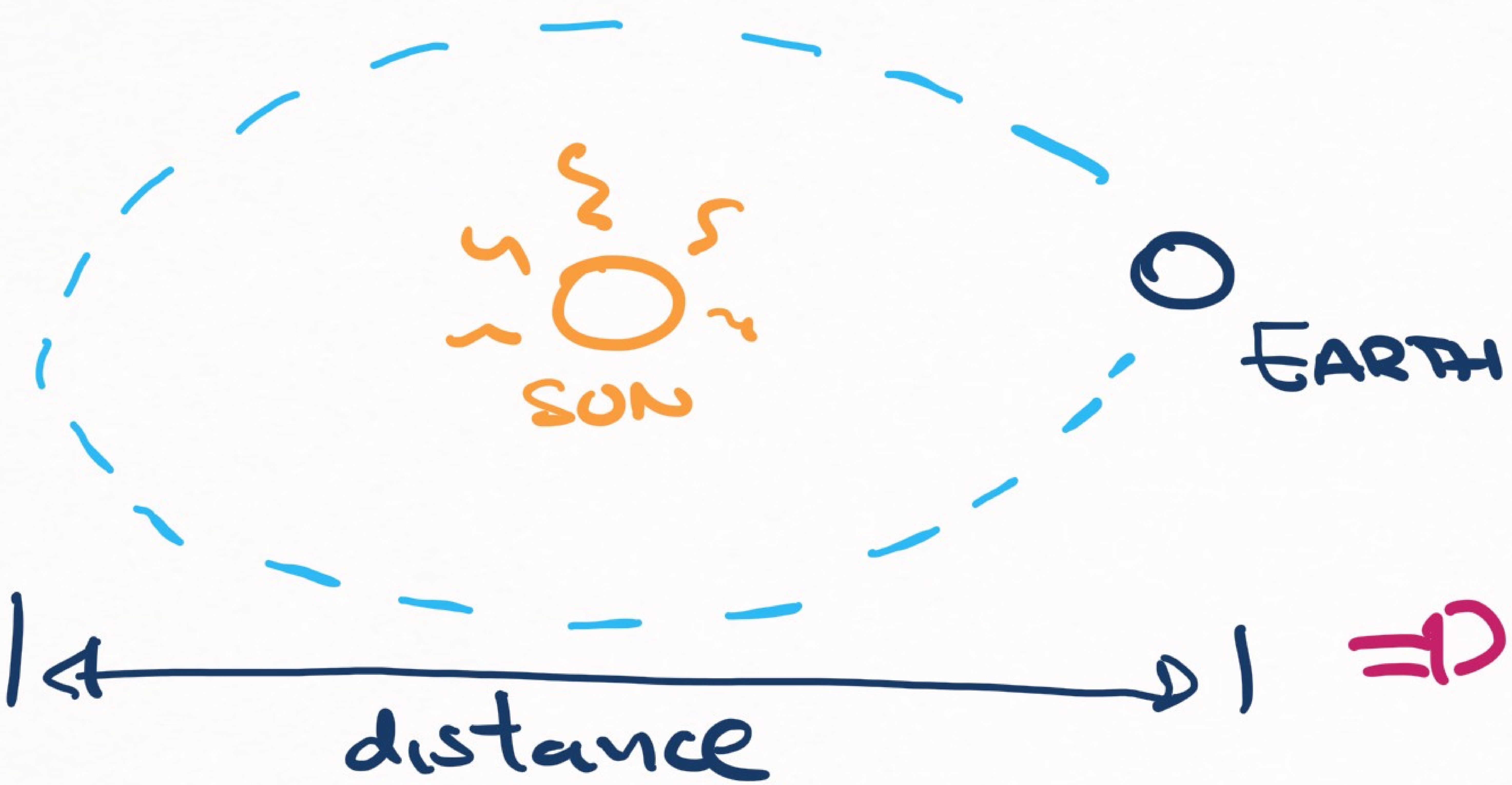
Sometimes assuming quantities must be natural leads us to wrong answers

⇓

EXAMPLE : TYCHO BRAHE'S MODEL OF THE SOLAR SYSTEM

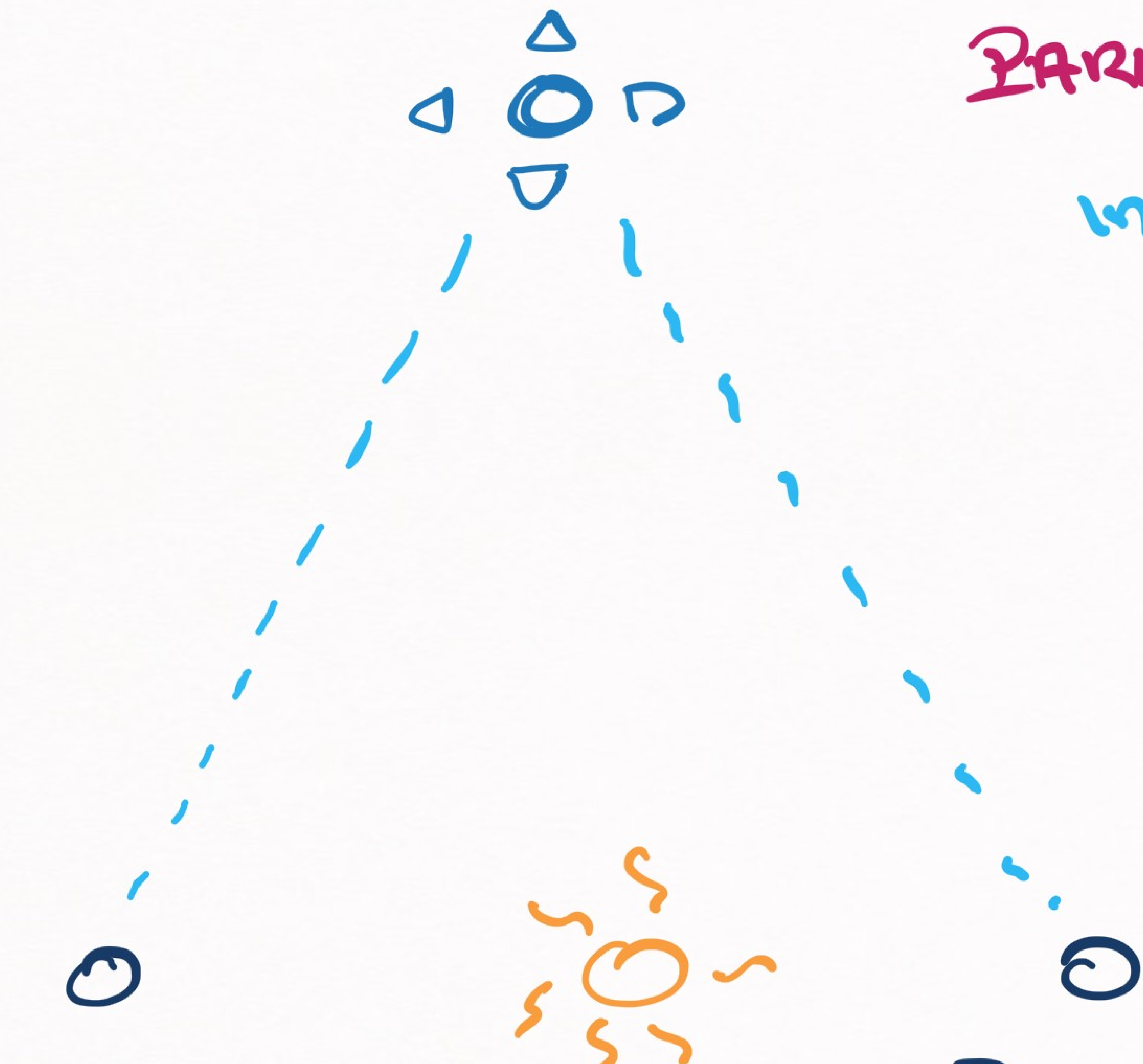
KEPLER → HELIOCENTRIC MODEL

BRAHE → HAD THE GOOD OBSERVATIONS



⇒ PARALAX

PARALAX: small change in the relative position of this star



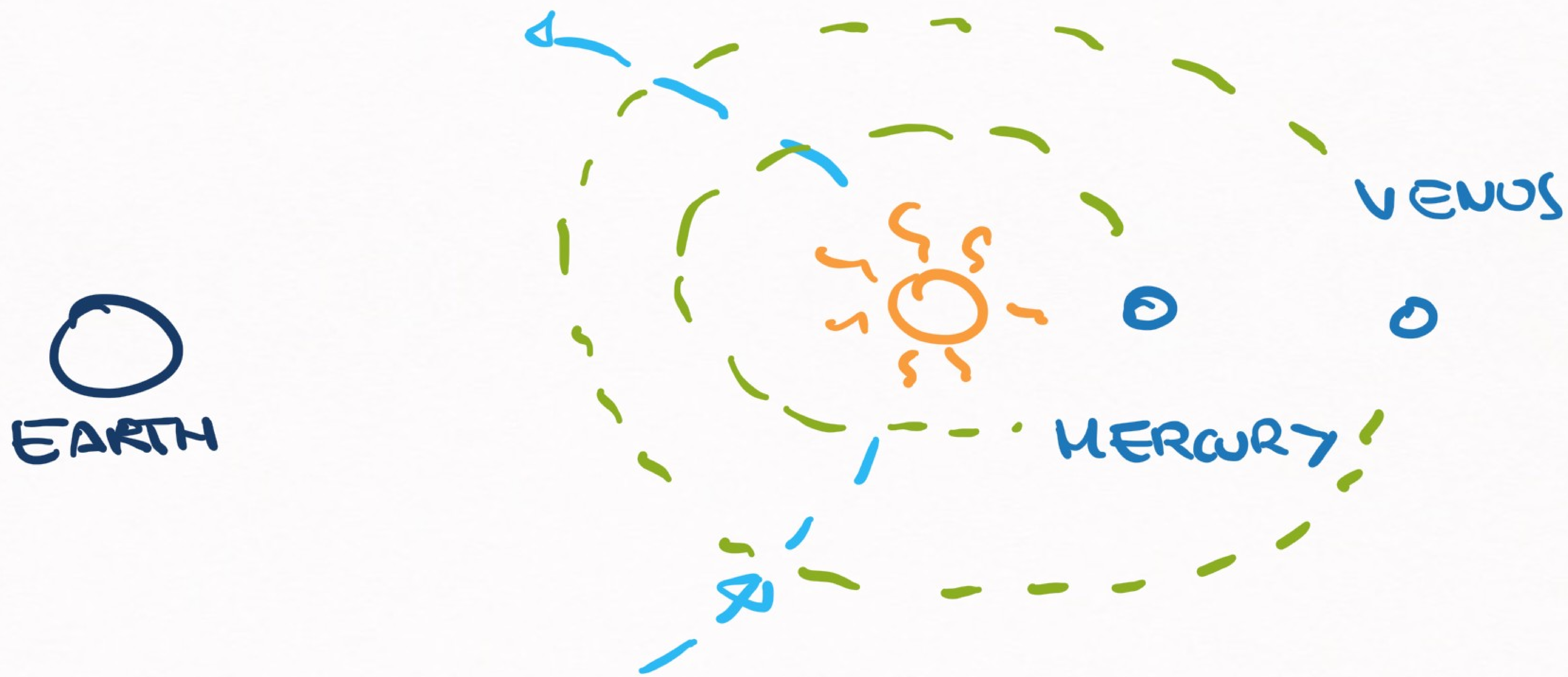
BRAHE → DIDN'T OBSERVE ANY PARALAX

a) Stars really far away

b) Earth is at the center of the solar system $\rightarrow \odot$

$\odot$ → choose for this one because otherwise the distances involved would be too unnatural

BRAHE MODEL:



WEIRD MODEL → BASIS WAS NATURALNESS

[TAKE HOME MESSAGE]

→ FINE-TUNING, WHY?

1) Sometimes this requires an explanation

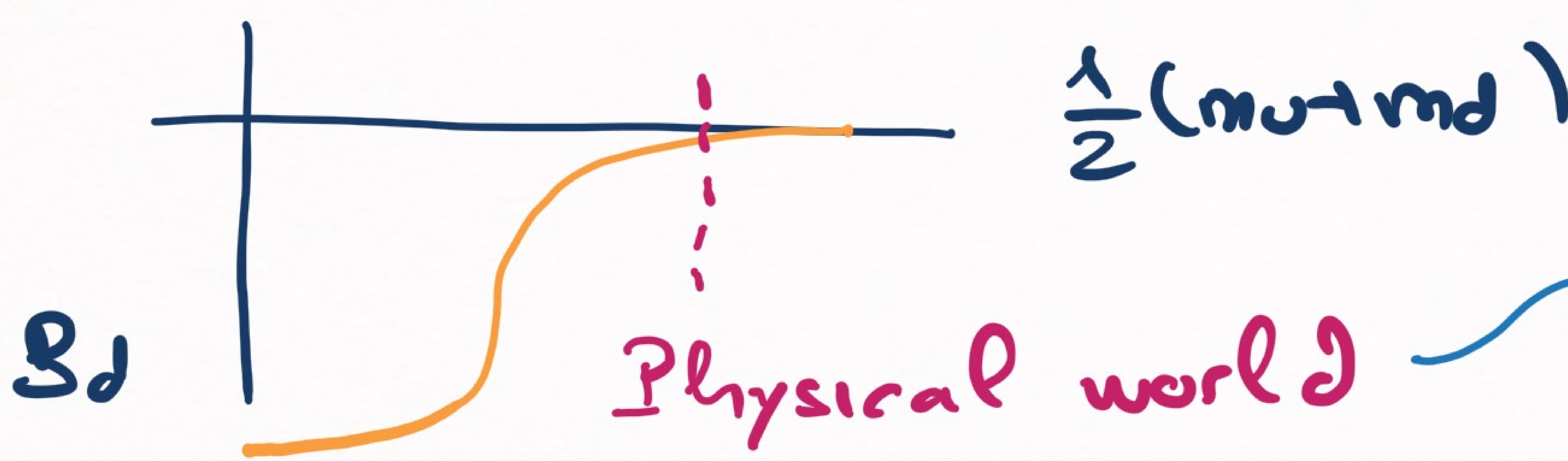
2) Sometimes this is just chance & there is no further explanation ?

FINE-TUNING IN NUCLEAR PHYSICS:

a) binding energy of deuteron → too small

→ by chance (probably)

→ m_u, m_d quark masses → couplings w/ the Higgs



→ here by chance

$m_u = m_d = 0$, $B_d \sim 15$ MeV (not fine tuned)

b) mass of the pion

$m_u, m_d \ll m_s, m_c, m_b, m_t$

(these two quark masses are small)

$m_u = m_d = 0 \longrightarrow \boxed{m_\pi = 0}$

$\Downarrow$

$\exists$ symmetry for this

$\Rightarrow$ (not chance)

→ enough for now about fine-tuning

$$\boxed{\text{PROPERTIES OF THE NUCLEAR FORCES}}$$

Nuclear forces → forces among neutrons & protons

nucleon → neutron, proton

PROPERTIES

1) Nuclear forces have a short range

2) At "intermediate" distances, nuclear forces are attractive (1-2) fm

3) At short distances the nuclear force is repulsive $\leq 0.7\ fm$

4) Nuclear forces do not distinguish neutrons & protons ($\rightarrow$ nucleons)

5) Nuclear forces are not central

$$V(\vec{r}) \neq V(|\vec{r}|)$$

(dependence on direction)

PROPERTIES (IN MORE DETAIL)

1) SHORT-RANGED ($\neq$ SHORT RANGE)

$\downarrow$ I will explain the difference

two types of potential (check previous lessons)

1.a) LONG-RANGED: Coulomb, van der Waals

$V(r) \sim 1/r^n \longrightarrow$ extends to infinity

1.b) SHORT-RANGED: Yukawa, square well

$$V(r) \sim \frac{e^{-mr}}{r^n}, \quad V(r) \sim \frac{\Theta(a-r)}{r^n}$$

SHORT-RANGED FORCES $\longrightarrow$ Easier to handle

$\Rightarrow$

Difference between short-ranged & short range

long-ranged & long range

SCALES:

short & long range

(pedantic, do not assume that other people will differentiate them)

$\Rightarrow$

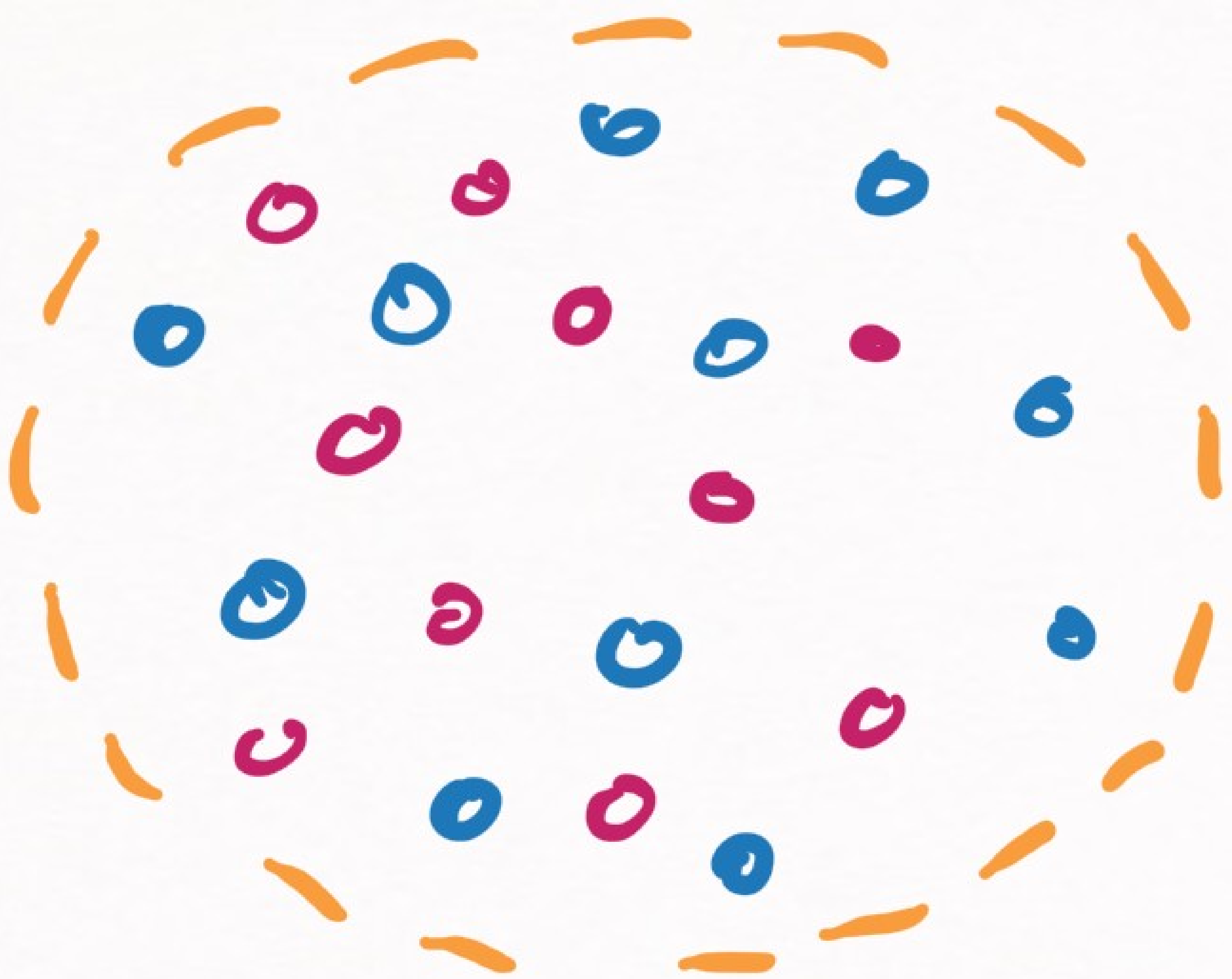
$$V(r) = \underbrace{V_S(r)}_{R_S} + \underbrace{V_L(r)}_{R_L}$$

$$R_S < R_L$$

(only language, not physics)

[HOW DO WE KNOW THAT NUCLEAR FORCES ARE SHORT-RANGED?]

→ Binding energy of nuclei

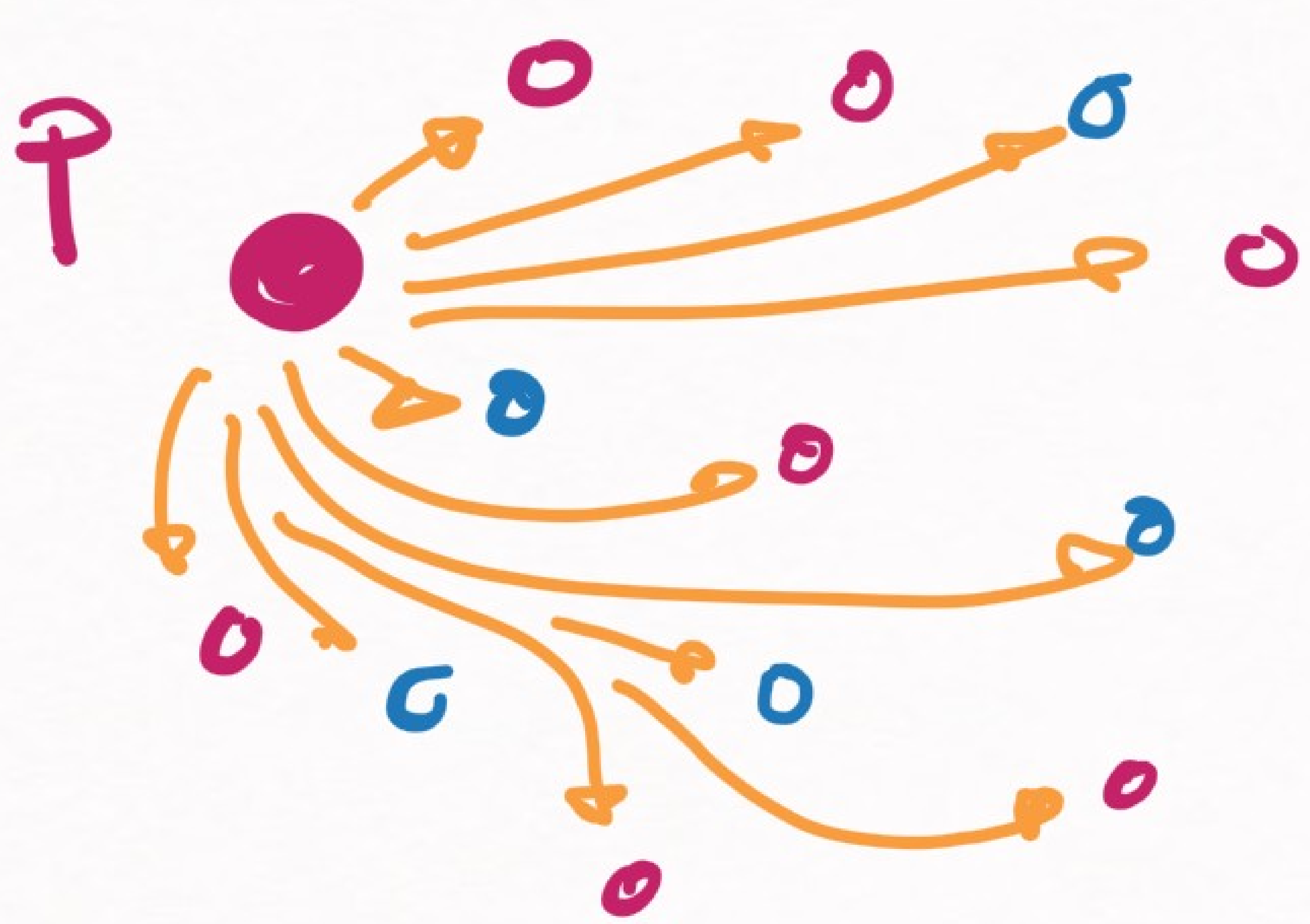


$B = B(\underbrace{A}_{\#\text{ of nucleons}})$ → depends on range of forces

$B = B(A)$ A is number of nucleons

a) if you have a force w/ infinite range (e.g. gravity)

$$B \propto \frac{A(A-1)}{2} \qquad (B \propto A^2)$$

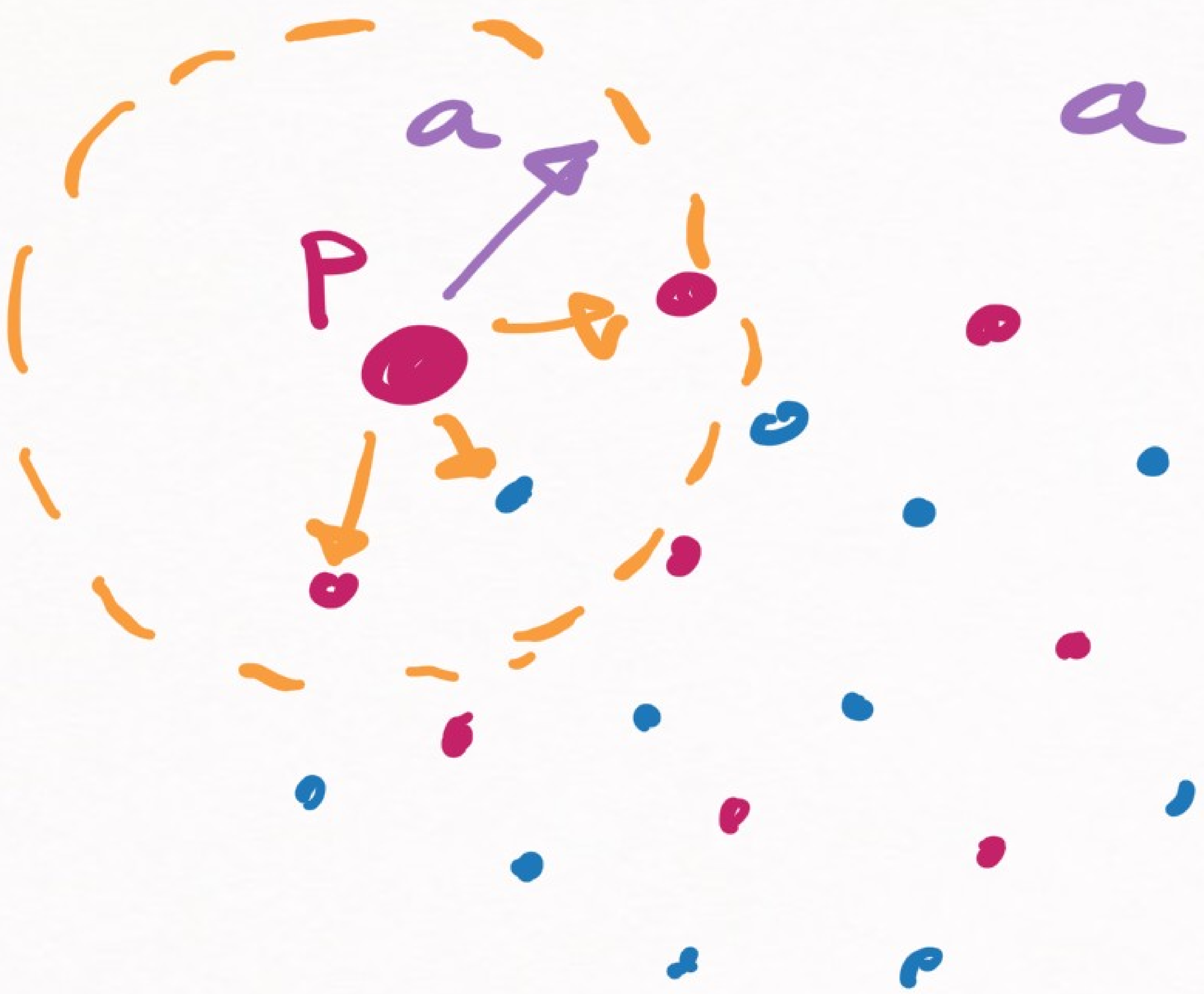


Every particle interacts with all others

$B = B(A)$ A is number of nucleons

b) if we have a short-range force:

$B \propto A$ → why?



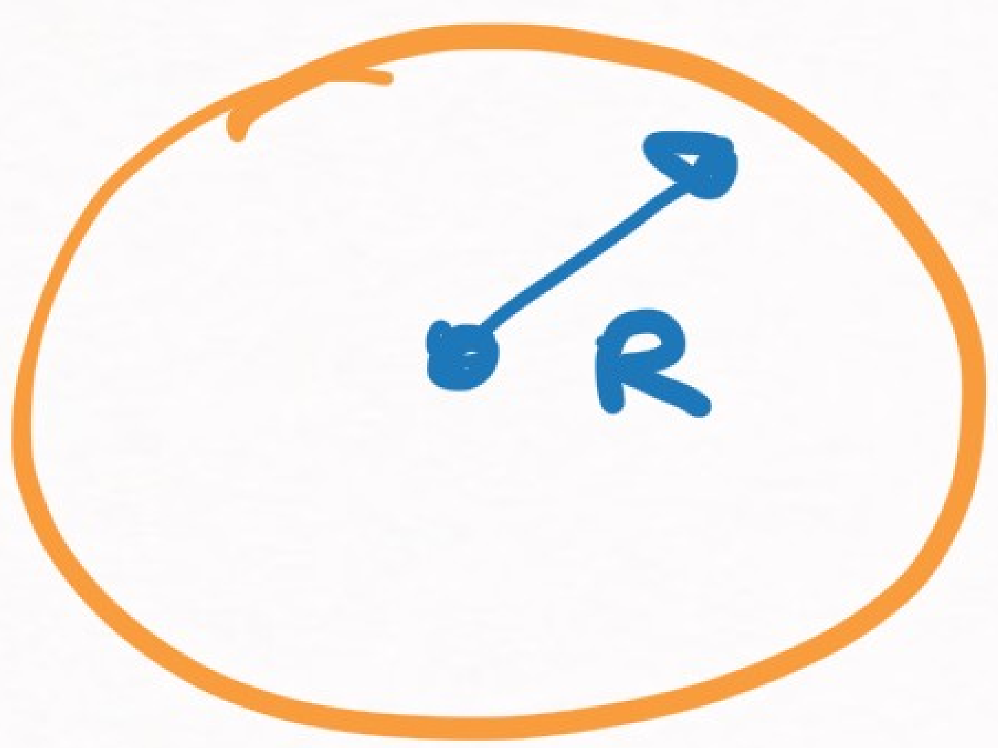
a → range of the force

(outside the range, the interaction is practically zero)

Each particle only interacts with the closest neighbors $\underline{\underline{B \propto A}}$

Exercise to understand this idea:

TO CALCULATE THE GRAVITATIONAL SELF ENERGY OF A SPHERICAL MASS DISTRIBUTION



constant density

$$\underline{\underline{U \propto M^2}}$$

HOWEVER, IF WE MODIFY GRAVITY WITH A RANGE:

$$V \sim \frac{e^{-\lambda r}}{r} \Rightarrow \underline{\underline{U \propto M}}$$

like nuclear forces

HOW DO WE KNOW THAT NUCLEAR FORCES HAVE A SHORT RANGE?

ANSWER: SATURATION

$$\underbrace{\frac{B}{A} \sim (8-9)\,\text{MeV}}_{\text{only possible if finite range of interaction}}$$

} Very good approx. to binding energy

PROPERTIES |

2) NUCLEAR FORCES ATTRACTIVE AT MEDIUM DISTANCES (1 – 2) fm more or less

how do we know this?

→ Density of nuclei is more or less constant

$$\rho \neq \rho(A) \qquad \rho \simeq \rho_0$$

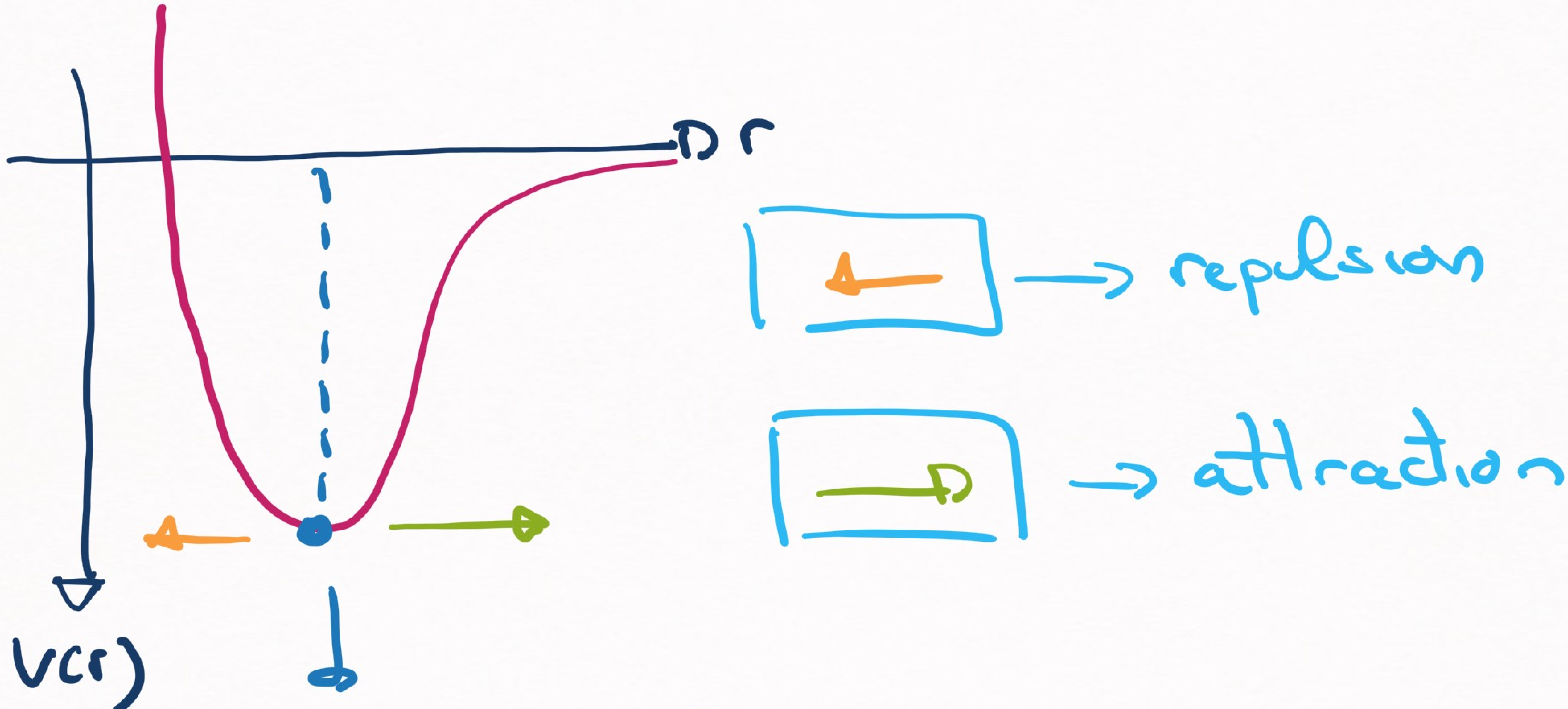
PROPERTIES

3) NUCLEAR FORCES ARE REPULSIVE AT SHORT DISTANCES

Again, an important reason is density

$$\rho \simeq \rho_0 \,, \quad \rho \neq \rho(A)$$

→ Let's explain 2) & 3) together

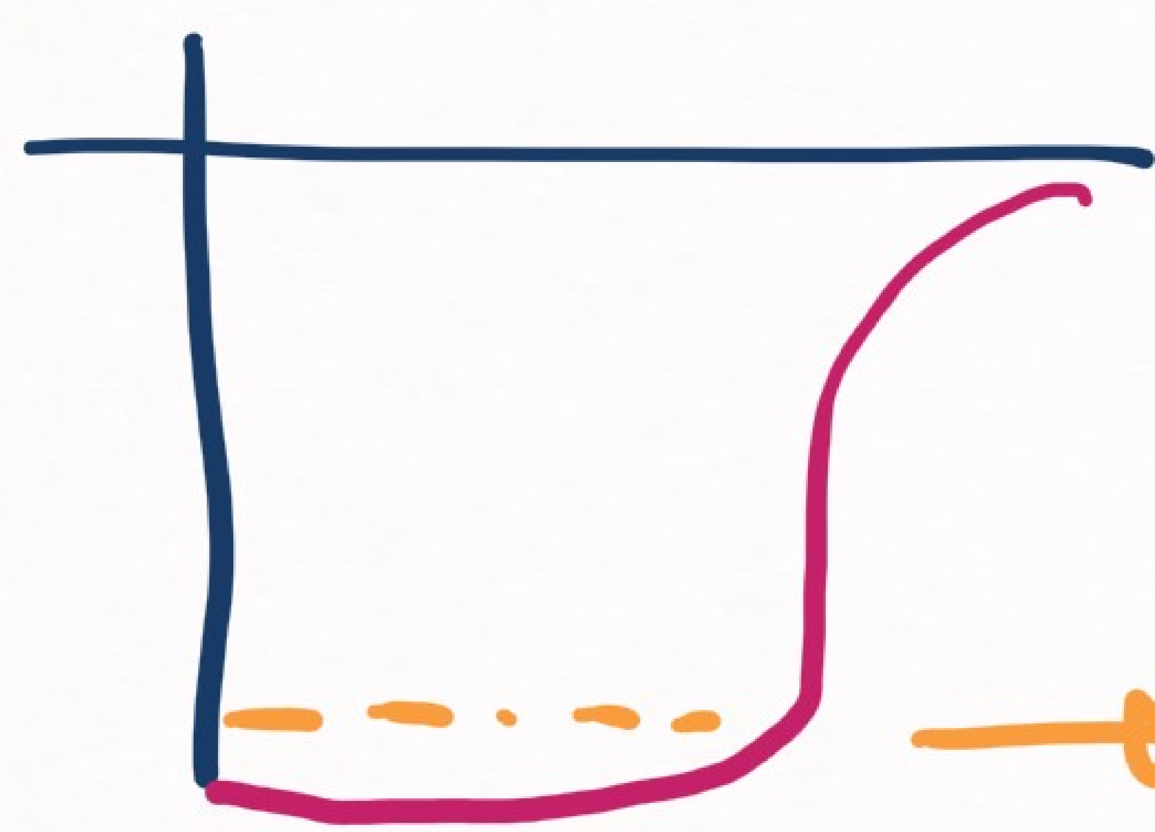


preferred relative distance between two nucleons

You need a minimum for $\rho \simeq \rho_0$

(because you want nucleons to be equally spaced)

NOTICE THAT $\frac{B}{A} \sim$ constant DOES NOT IMPLY A MINIMUM



minimum energy independent of distance

→ We will extend on this
on next lesson

FRIDAY AT 15:50

See you then!