NUCLEAR PHYSICS 4

DEALING WITH MULTISCALE SYSTEMS

→ IDENTIFYING SCALES
IN TWO-BODY SYSTEMS ?

→ FINE TUNING IN A TWO SCALE SYSTEM
(~ two effects cancelling each other out)

RECAP |

PHYSICAL SYSTEMS $\longleftrightarrow$ SCALES] $\leftarrow$

$Q = \{Q_1, Q_2, Q_3, \dots\}$] $\leftarrow$ ALSO POSSIBLE $R = \{R_1, R_2, \dots\}$ (LENGHT)

$\rightarrow$ Ideal situation: $Q_1 \ll Q_2, Q_3, \dots$

$\Downarrow$

small ratios

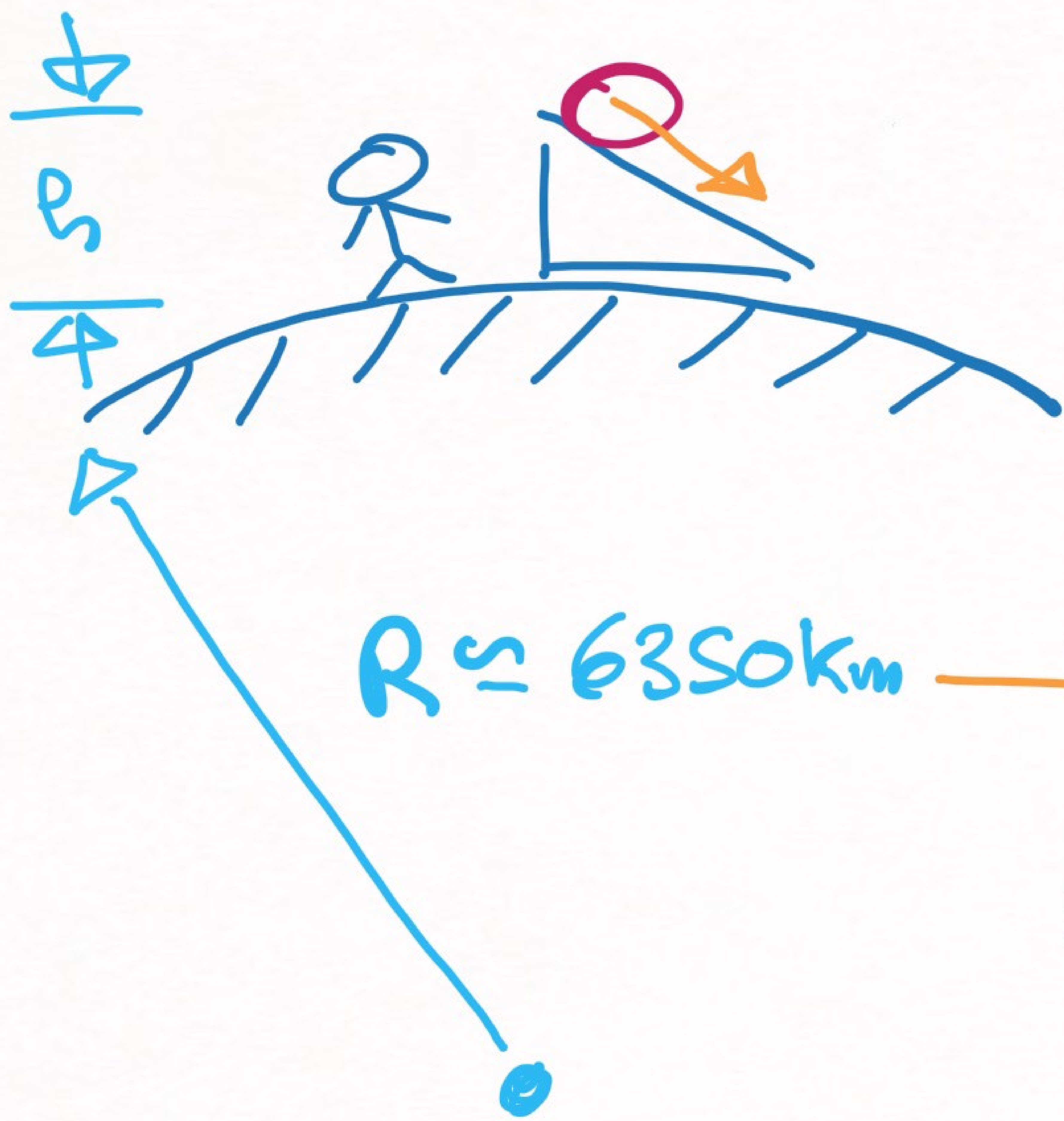
[$\langle \hat{O} \rangle$: observable of the system]

$$\langle \hat{O} \rangle = c_0 Q_1^d \left[1 + c_2^{(1)} \frac{Q_1}{Q_2} + c_3^{(1)} \frac{Q_1}{Q_3} + c_2^{(2)} \left(\frac{Q_1}{Q_2}\right)^2 + c_3^{(2)} \left(\frac{Q_1}{Q_3}\right)^2 + \dots \right]$$

(Taylor expansion)

It doesn't have to be a quantum system

("Galilean" gravity from past lesson)



$$\Delta V = mgh\left[1 - \frac{h}{R} + \frac{h^2}{R^2} - \dots\right]$$

$$g \simeq 9.8\ m/s^2$$

$R \simeq 6350km$ $\longrightarrow$ Two scales: h, R

$h \ll R$ $\Rightarrow$ Taylor expansion on $\frac{h}{R}$ possible

→ Non-ideal situation $Q = \{Q_1, Q_2, Q_3 \dots\}$

$\boxed{Q_1 \sim Q_2} \ll Q_3, Q_4, \dots$

or

$\boxed{Q_1 \sim Q_2 \sim Q_3} \ll Q_4, Q_5, \dots$

or

anything similar

unexpected complications are possible

But before finding an example (*) of this non-ideal situation...

→ [HOW DO WE FIND THE TYPICAL SCALES OF A SYSTEM?]

(*) → Square-well (you can review it, very easy to understand)

A SCALE TUTORIAL | (finding scales)

Schrödinger equation $\longrightarrow$ $\left[-\frac{\nabla^2}{2\mu} + V(\vec{r})\right]\psi(\vec{r}) = E\psi(\vec{r})$

$\Downarrow$ (multiply by 2μ)

Reduced Schrödinger equation $\longrightarrow$ $\left[-\vec{\nabla}^2 + 2\mu V(\vec{r})\right]\psi(\vec{r})$

$$= \underbrace{-\gamma^2\psi(\vec{r})}$$

$E > 0$, $E = \frac{k^2}{2\mu}$ (scattering state)

$E < 0$, $E = -\frac{\gamma^2}{2\mu}$ (bound state)

Bound states are more simple

Reminder $\longrightarrow$ $2\mu V(\vec{r})$ has dimensions of $[L^{-2}]$

We can extract scales from this observation

(anything in $2\mu V(\vec{r})$ that is not "r" will be part of a scale of the system)

Example ① $\longrightarrow$ Coulomb potential $V(\vec{r}) = -\frac{\alpha}{r}$ ε

$$2\mu V(\vec{r}) = -\frac{2}{a_B r}$$

the factor of 2 is convention

a_B : Bohr radius

Example ② $\longrightarrow$ van der Waals forces $V(\vec{r}) = -\frac{C_6}{r^6}$

$$2\mu V(\vec{r}) = -\frac{R_{vdW}^4}{r^6}$$

$\longrightarrow$ R_{vdW} is the length scale associated w/ van der Waals

Example 3 $\longrightarrow$ any inverse power-law potential

$$\left.\begin{aligned} V(r) &= -\frac{C_n}{r^n} \\ 2\mu V(r) &= -\frac{R_n^{\,n-2}}{r^n} \end{aligned}\right\} \longrightarrow \underbrace{R_n = (2\mu C_n)^{1/(n-2)}}$$

(this is only a question of dimensions)

But $\exists$ more tricky examples

Example 4 $\rightarrow$ inverse square-law potential

$$V(\vec{r}) = -\frac{C_2}{r^2}$$

$$2\mu V(\vec{r}) = -\frac{g}{r^2}$$

$$\boxed{g = 2\mu C_2}$$

Inverse square-law potential $\rightarrow$ $\boxed{\text{NO SCALE}}$

$\downarrow$

CAVEAT HERE

$$\underbrace{\left[-\vec{\nabla}^2 - \frac{g}{r^2}\right]}_{\text{Here no scales}} \psi(\vec{r}) = \underbrace{-\gamma^2}_{\gamma \text{ is a scale}} \psi(\vec{r})$$

Here no scales
$\Rightarrow$
(INPUT)

γ is a scale
(but γ is a prediction)
(OUTPUT)

[What is special about it?]

$\boxed{\text{NO SCALE}} \Rightarrow \boxed{\text{SYMMETRY}}$

$$\left[-\vec{\nabla}^2 - \frac{g}{r^2}\right]\psi(\vec{r}) = -\gamma^2\psi(\vec{r})$$

Which symmetry? Let's see...

$\vec{r} \to \lambda \vec{r}\,,\ \lambda \in \mathbb{R}$

$\frac{1}{r^2} \to \frac{1}{\lambda^2}\frac{1}{r^2}$

$\vec{\nabla} \to \frac{1}{\lambda^2}\vec{\nabla}$

$$\left[\vec{\nabla}^2 + \frac{g}{r^2}\right] \to \frac{1}{\lambda^2}\left[\vec{\nabla}^2 + \frac{g}{r^2}\right]$$

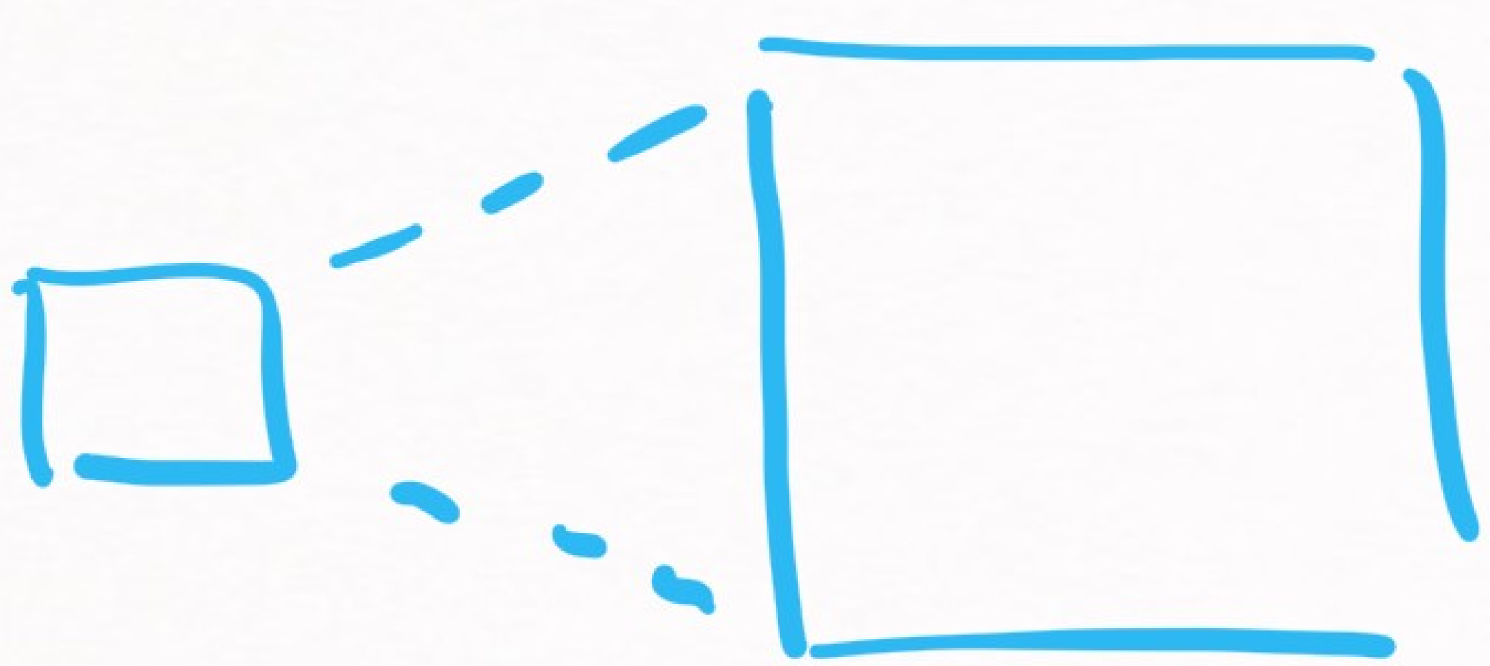
But if we have ≠ potential:

$$\left[\vec{\nabla}^2 + \frac{2}{a_B r}\right] \to \left[\frac{1}{\lambda^2}\vec{\nabla}^2 + \frac{1}{\lambda}\frac{2}{a_B r}\right]$$ → can't factor out the λ

$$\left.\begin{array}{l}\vec{r} \to \lambda \vec{r} \\ \vec{p} \to \frac{1}{\lambda} \vec{p}\end{array}\right\} \text{dilation symmetry}$$

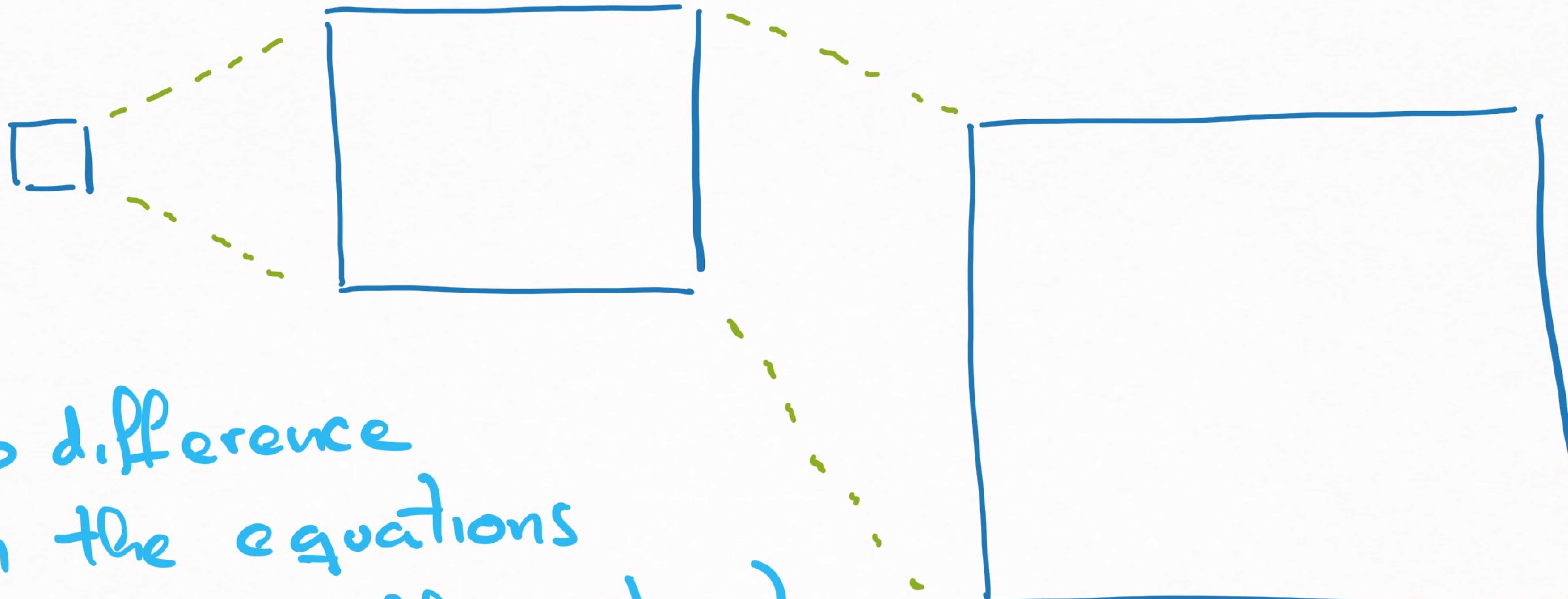
$(\forall \lambda / \lambda \in \mathbb{R})$

(symmetry under dilations of the system)



(a.k.a. conformal symmetry)

$\frac{1}{r^2}$ potential $\longrightarrow$ System invariant under dilations



(no difference in the equations describing the system)

$\longrightarrow$ (problematic) why? $\rightarrow$

$\frac{1}{r^2}$ potential $\longrightarrow$ Most simple example of an anomaly (*)

(*) a classical symmetry that is lost in the quantization process

anomaly

[classical symmetry $\longrightarrow$ (quantize) $\longrightarrow$ stops being a symmetry]

[Why is there an anomaly?] ← qualitative argument

$$\left[-\vec{\nabla}^2 - \frac{g^2}{r^2}\right]\psi(\vec{r}) = -\gamma^2 \psi(\vec{r})$$

$$\Big\downarrow \text{ dilation } (\vec{r} \to \lambda\vec{r})$$

$$\left[-\frac{\vec{\nabla}^2}{\lambda^2} - \frac{g^2}{\lambda^2 r^2}\right]\psi(\lambda\vec{r}) = -\gamma^2 \psi(\lambda\vec{r})$$

or : $$\left[-\vec{\nabla}^2 - \frac{g^2}{r^2}\right]\psi(\lambda\vec{r}) = -\underline{\underline{\lambda^2}}\gamma^2 \psi(\lambda\vec{r})$$

[Physical interpretation :]

$\left[-\vec{\nabla}^2 - \frac{g}{r^2}\right]\psi(\vec{r}) = -\gamma^2\psi(\vec{r})$ solution

$\Rightarrow \left[-\nabla^2 - \frac{g}{r^2}\right]\psi(\lambda\vec{r}) = -\lambda^2\gamma^2\psi(\lambda\vec{r})$ is also a solution

If $\exists$ a bound state w/ $E_B = -\frac{\gamma^2}{2\mu}$

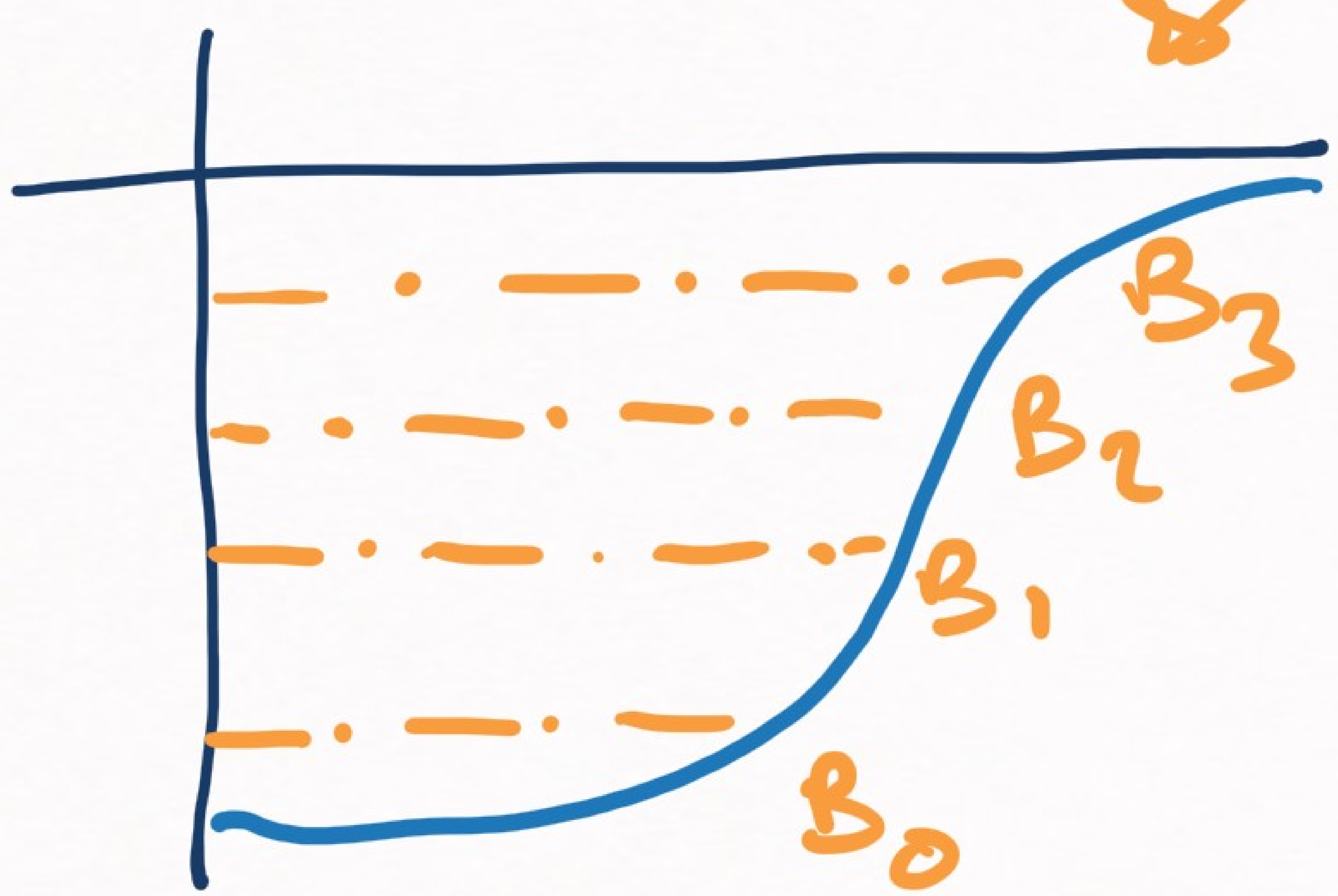
then $\exists$ another bound state w/ $E_B = -\lambda^2\frac{\gamma^2}{2\mu}$

(true for $\forall\lambda$, $\lambda \in \mathbb{R}$)

PROBLEM !!

But if this is true, we arrive to a $\boxed{\text{contradiction}}$: (3) & (2)

REMINDER $\longrightarrow$ The bound state spectrum is discrete (1)



BUT IF $\vec{r} \to \lambda \vec{r}$ IS A QUANTUM SYMMETRY:

bound state spectrum will be continuous (2)

$\boxed{\text{SOLUTION :}}$ (caveat: not a detailed explanation, only the general idea)

THIS CLASSICAL SYMMETRY MUST BREAK DOWN WHEN QUANTIZING THE SYSTEM

$$\psi(r) \sim \frac{1}{r}\sqrt{r}\,\sin\left(\nu \log\left(\frac{r}{R_0}\right)+\varphi\right)$$

(using WKB approximation)

$\vec{r} \to \lambda \vec{r}$ not a symmetry, $\nu = \nu(g)$

for every $\lambda \in \mathbb{R}$ (but for some λ_0 it is a symmetry)

What is the meaning of this?

Continuous scale invariance

$\vec{r} \to \lambda_0 \vec{r}$

λ_0 a specific value

$\vec{r} \to \lambda \vec{r}, \forall \lambda \in \mathbb{R}$

Discrete scale invariance

Scale invariance $\equiv$ dilation symmetry

(a) continuous (boring)
(b) discrete (interesting)

no structure whatsoever

there is a pattern that repeats itself

Reminder

$$2\mu V(\vec{r}) = -\frac{g^2}{r^2} \quad \vec{r} \to \lambda \vec{r}$$

continuous scale invariance

Two possibilities:

(a) $g^2 < 1/4$ little attraction $\to$ no bound state

(b) $g^2 > 1/4$ more attraction $\to$ infinite number of bound states

discrete scale invariance

$\frac{1}{r^2}$ potential interesting because:

→ If $E_0 = -\frac{\gamma^2}{2M}$ is a bound state energy,

the energy of all other bound states

is $E_n = (E_0)\,\lambda_0^n$

(geometric spectrum)

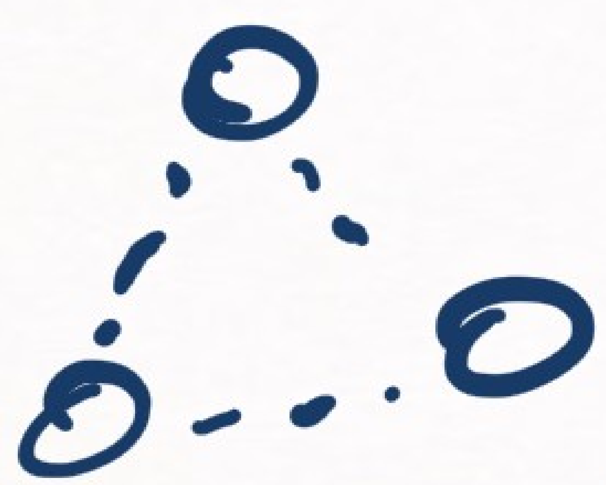
→ Effective potential in 3-body systems

Potential coming from the exchange of a non-relativistic particle

$\sim 1/r^2$

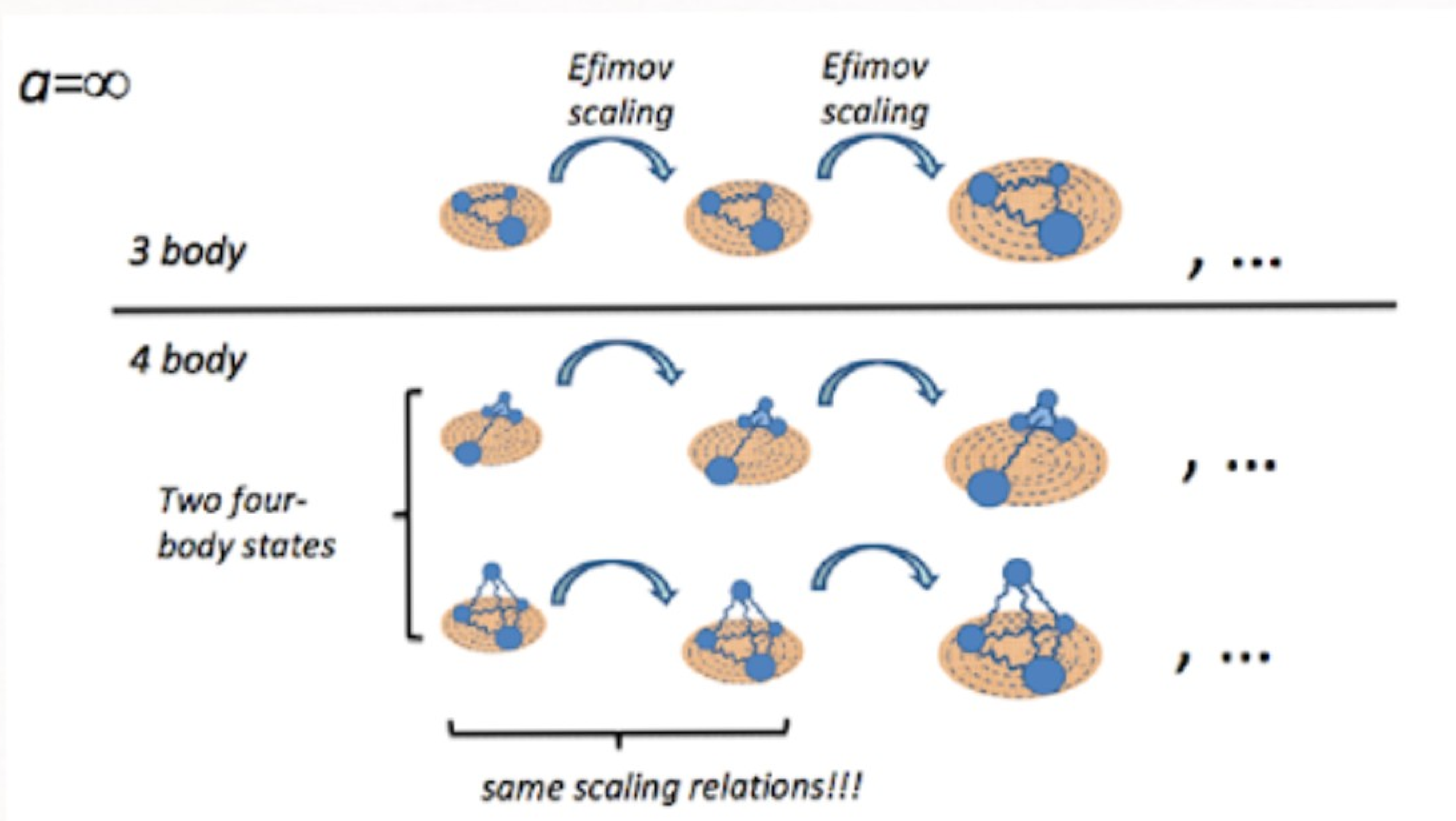
A second example of anomaly in QM:

$$\boxed{\text{THE EFIMOV EFFECT}}$$



3-BOSON SYSTEM +

bound state at $\underline{\underline{E=0}}$



E_0

$$\frac{E_0}{(22.7)^2}$$

$\frac{1}{r^2}$ potential is very interesting

(but also not necessarily related to our course)

We now go to the two-scale systems

Example 5 → Potentials with a finite range

(range is a scale)

→ Coulomb $V(\vec{r}) = -\frac{\alpha}{r}$ ①

(a_B)

→ van der Waals $V(\vec{r}) = -\frac{C_6}{r^6}$ ②

$(a_B, \frac{1}{m})$ But we can also have: 1' → Yukawa potential

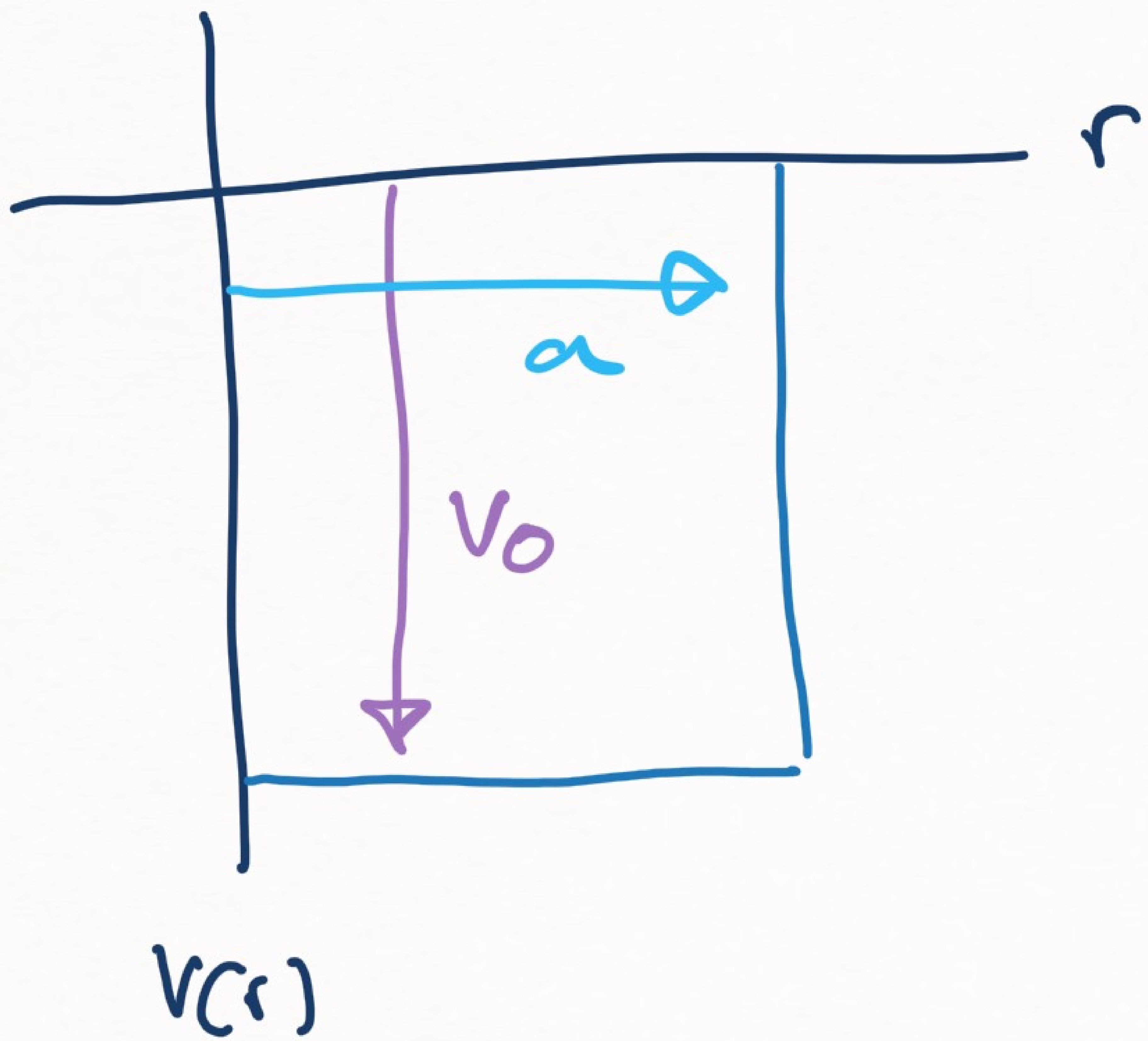
1' → $V(\vec{r}) = -\frac{\alpha}{r} e^{-mr}$

2' → $V(\vec{r}) = -\frac{C_6}{r^6} e^{-2mr}$ ⊛

finite range

⊛ → usually result of the exchange of two bosons w/ mass m

Example 6 $\longrightarrow$ The square-well potential



$$\left[\, V(r) = -\underbrace{V_0}_{\text{stregth}}\, \theta(\overbrace{a}^{\text{range}} - r)\,\right]$$

(we use the convention that $V_0 > 0$ for an attractive well)

SQUARE WELL

We have two scales:

1) Range : a

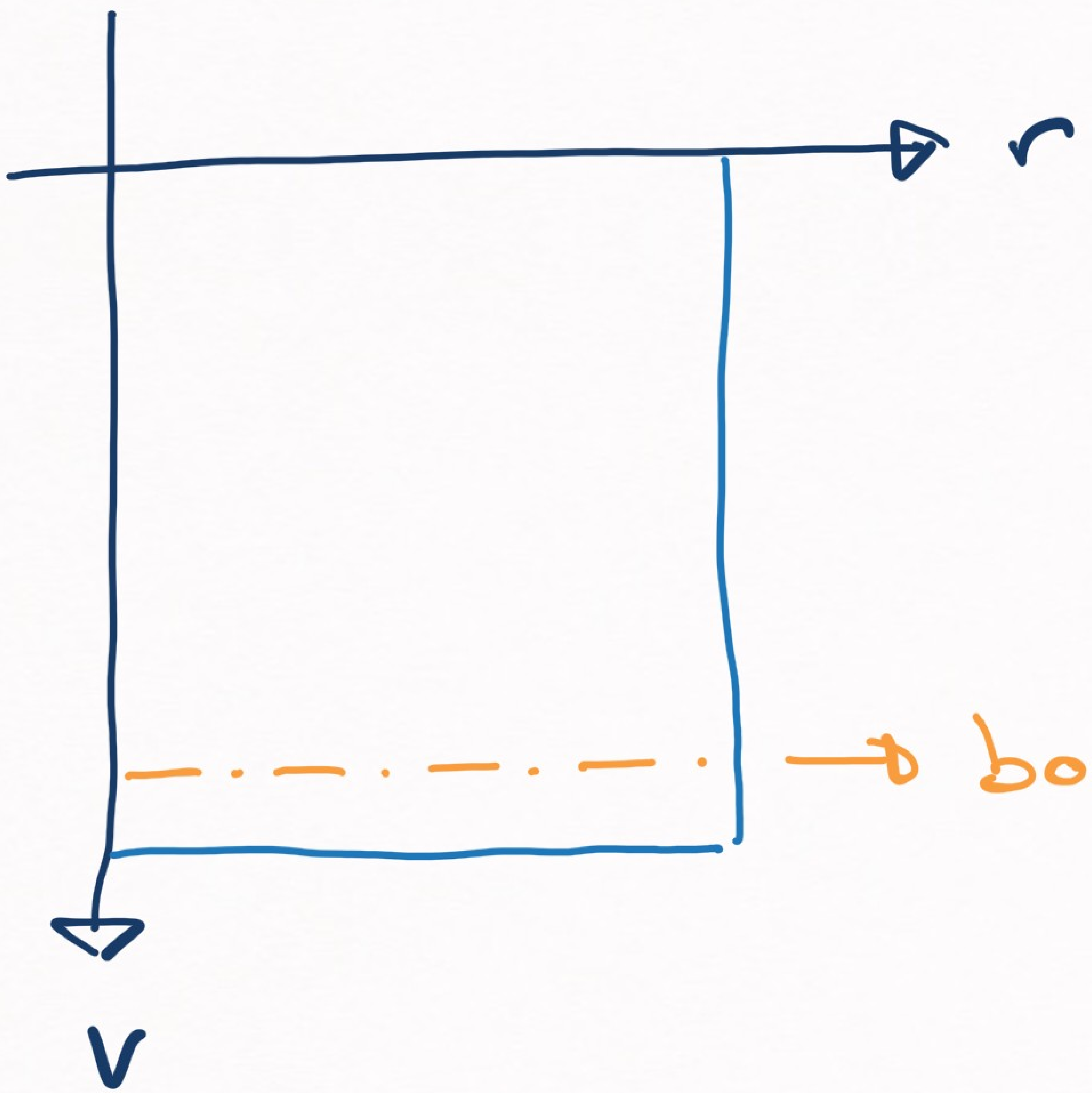
2) Strength : $2\mu V(\vec{r}) = -2\mu V_0 \, \Theta(a-r)$

$$= \underbrace{-\frac{1}{R_s^2}}_{\text{because dimensions}} \Theta(a-r)$$

$$R_s = \frac{1}{\sqrt{2\mu V_0}}$$

($R_s \to 0$, strength $\to \infty$)

$\boxed{\text{SQUARE WELL}}$ → What is a natural square well?



Dynamical constraint:

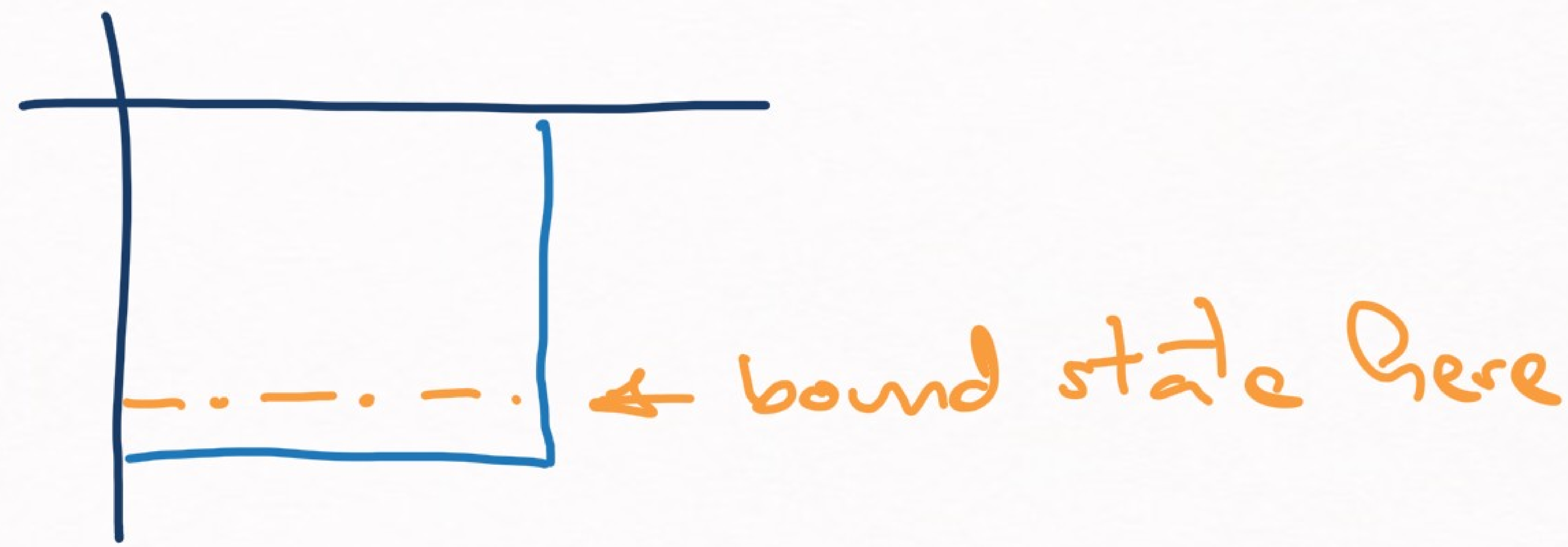
→ $\boxed{B \leq V_0}$

→ bound state $(E_B = -B)$

What is a natural square well?

a) $B \sim O(V_0)$

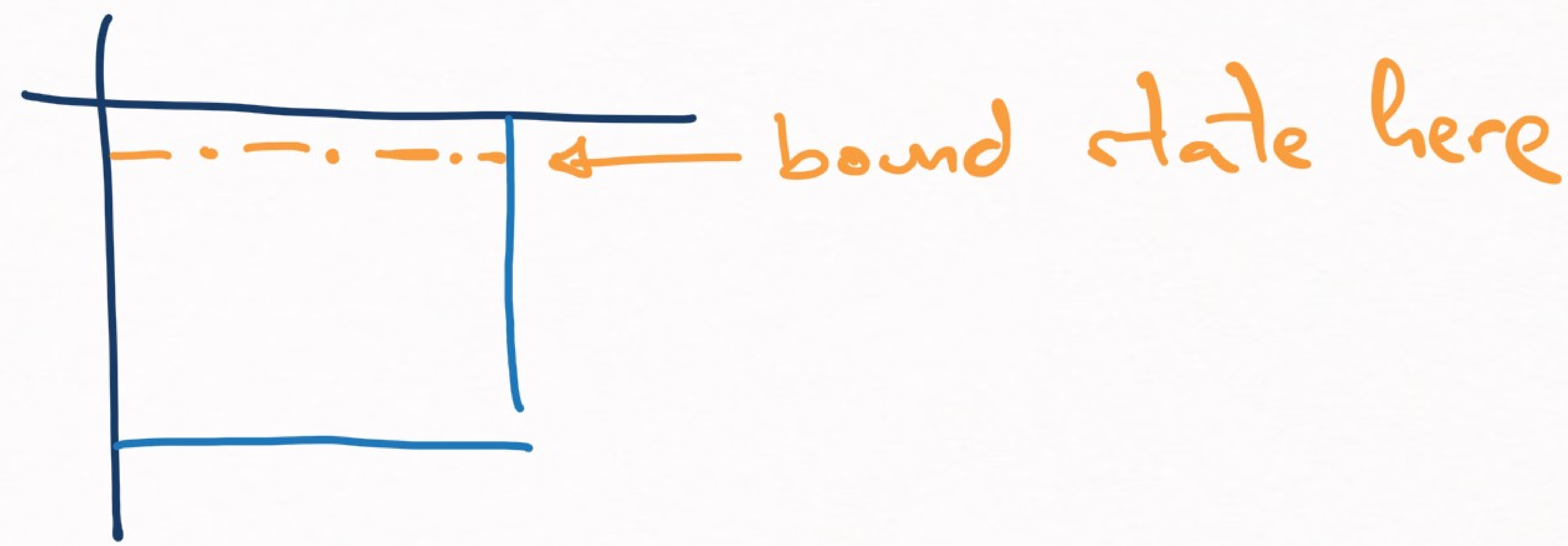
NATURAL



b) $B \ll O(V_0)$

(NOT NATURAL)

(HAPPENS RARELY)



Imagine you don't know anything
about the dynamics of the square well:

$$\boxed{\beta \in [0, V_0]}$$

→ MENTAL EXPERIMENT OF IMAGINING THAT β IS RANDOM

↓

[assuming a bound state exists]

⇒

Binding energy as a random variable (mental experiment) (*)

$\langle B \rangle \sim 0.5 V_0$ → NATURAL

$\langle B \rangle \leq 0.3 V_0$ → 30% of the time
not so rare, still natural

$\langle B \rangle \leq 0.01 V_0$ → 1% of the time
this is not the normal expectation, unnatural

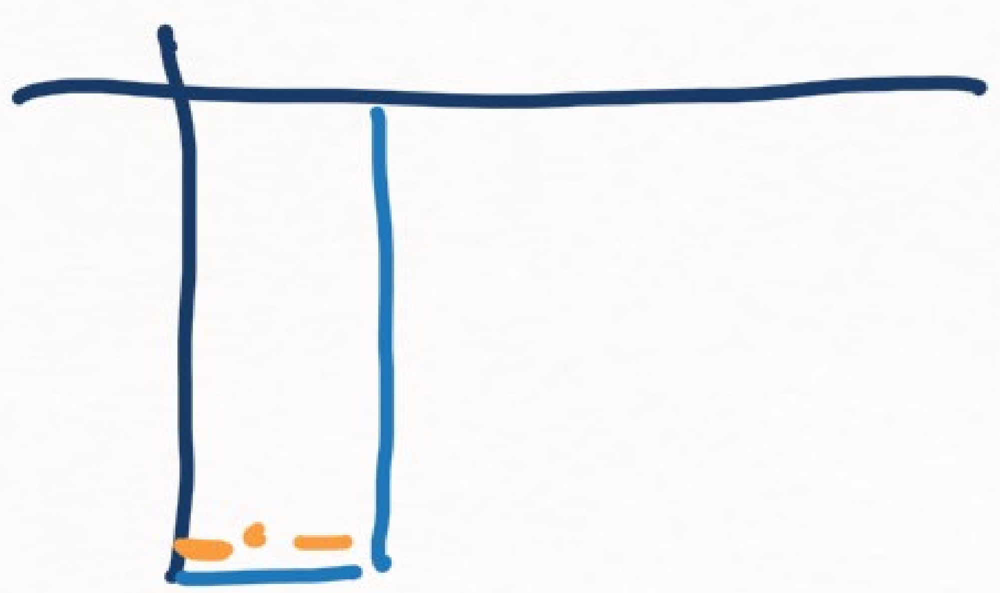
(*) → not reality

Natural & unnatural square wells : $R = \{R_s, a\}$

$R_s \rightarrow$ inverse strength

$a \rightarrow$ range

a) Natural means $B \sim \mathcal{O}(V_0)$

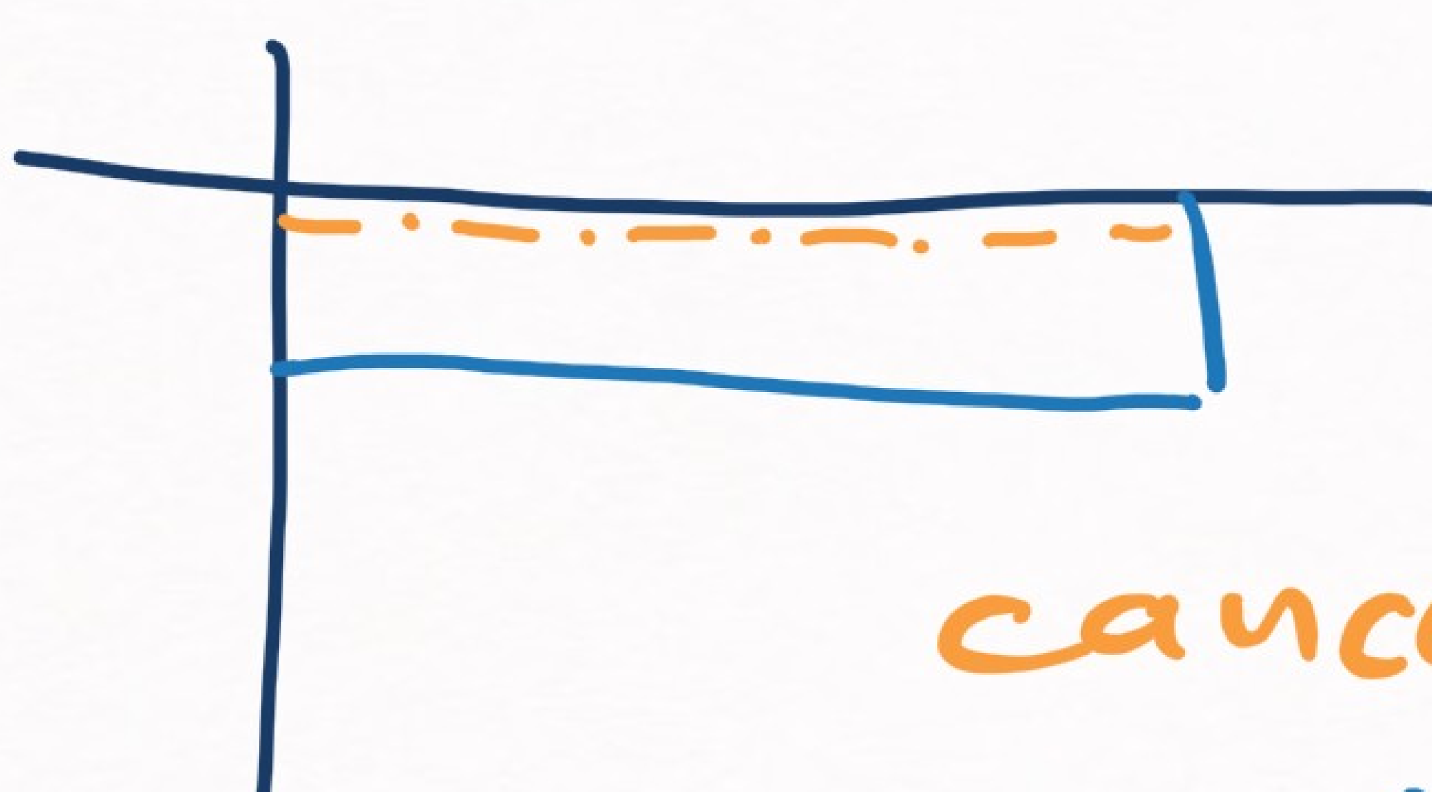


$\left.\begin{array}{l}\text{well is deep} \\ \Rightarrow B \simeq V_0\end{array}\right\} \rightarrow R_s$ is small $\rightarrow \boxed{R_s \ll a}$

$\downarrow$

scale separation (check slide #2)

b) Unnatural means $B \ll \mathcal{O}(V_0)$



well is not so deep

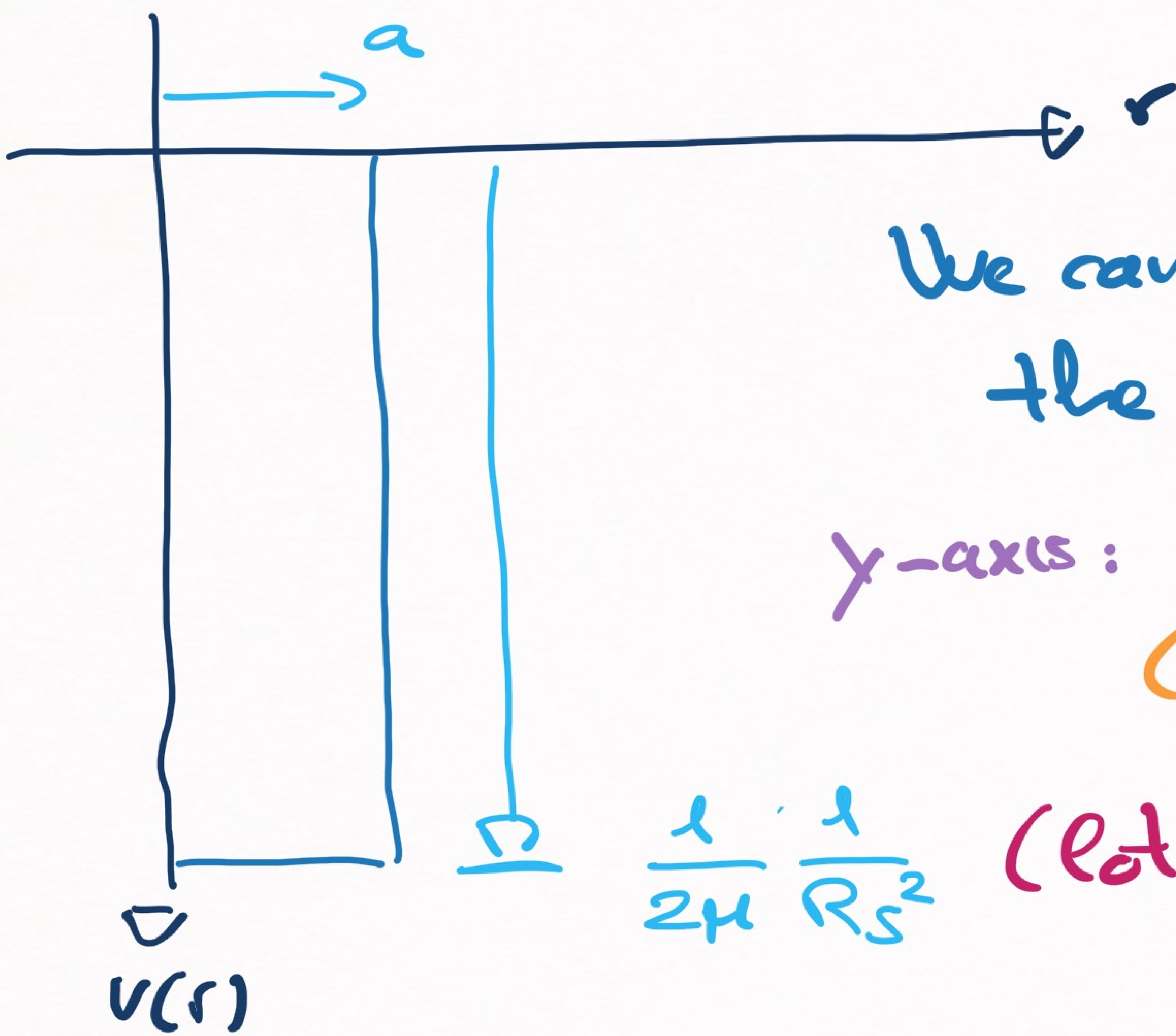
$B \simeq 0$

cancellation from R_s, a effects

$\boxed{R_s \sim a}$

slide #4

NATURAL WELL $\longrightarrow$ $\boxed{R_s \ll a}$ what does it mean?



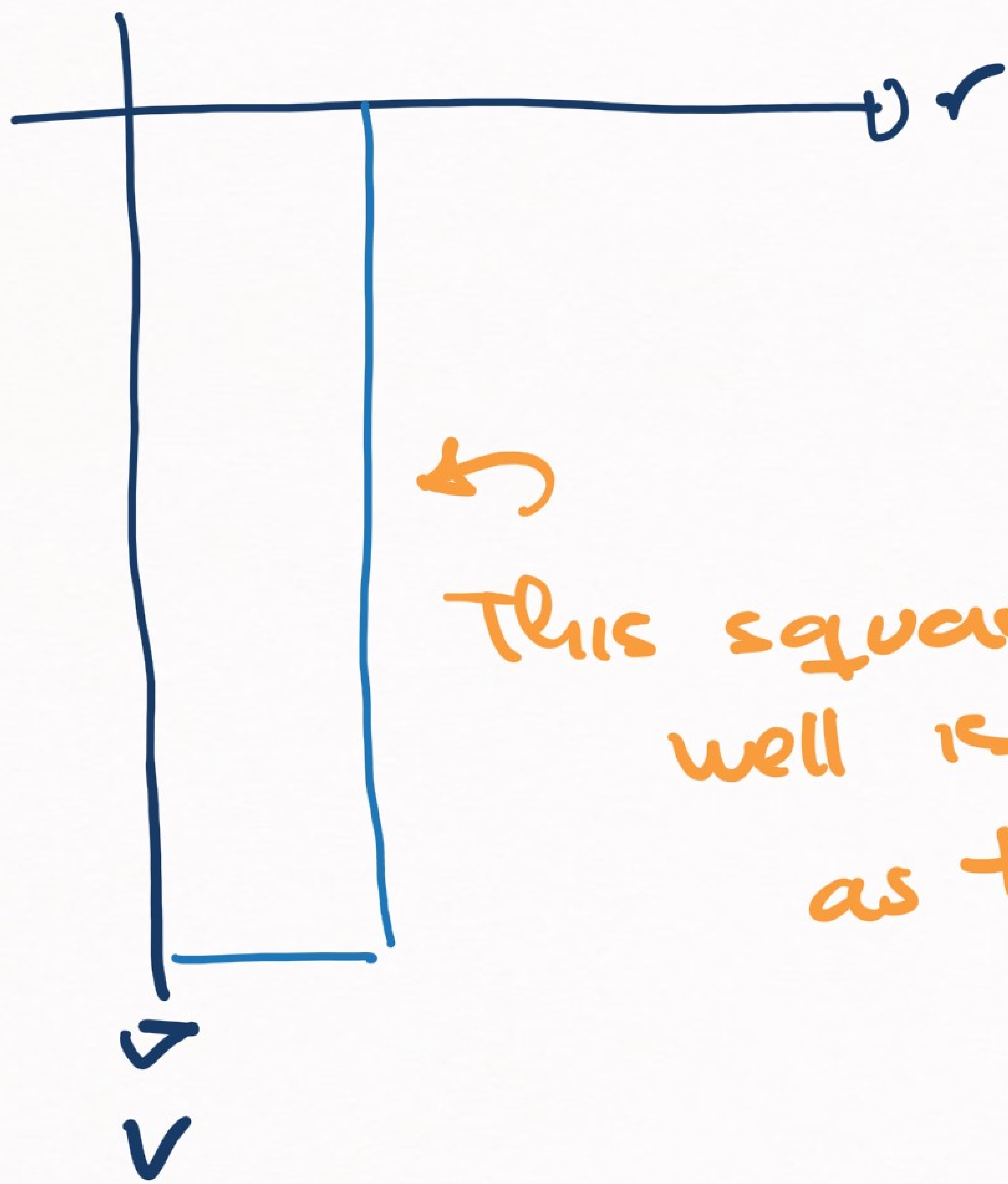
We can always change the origin of V

y-axis: $V \longrightarrow V - V_0$

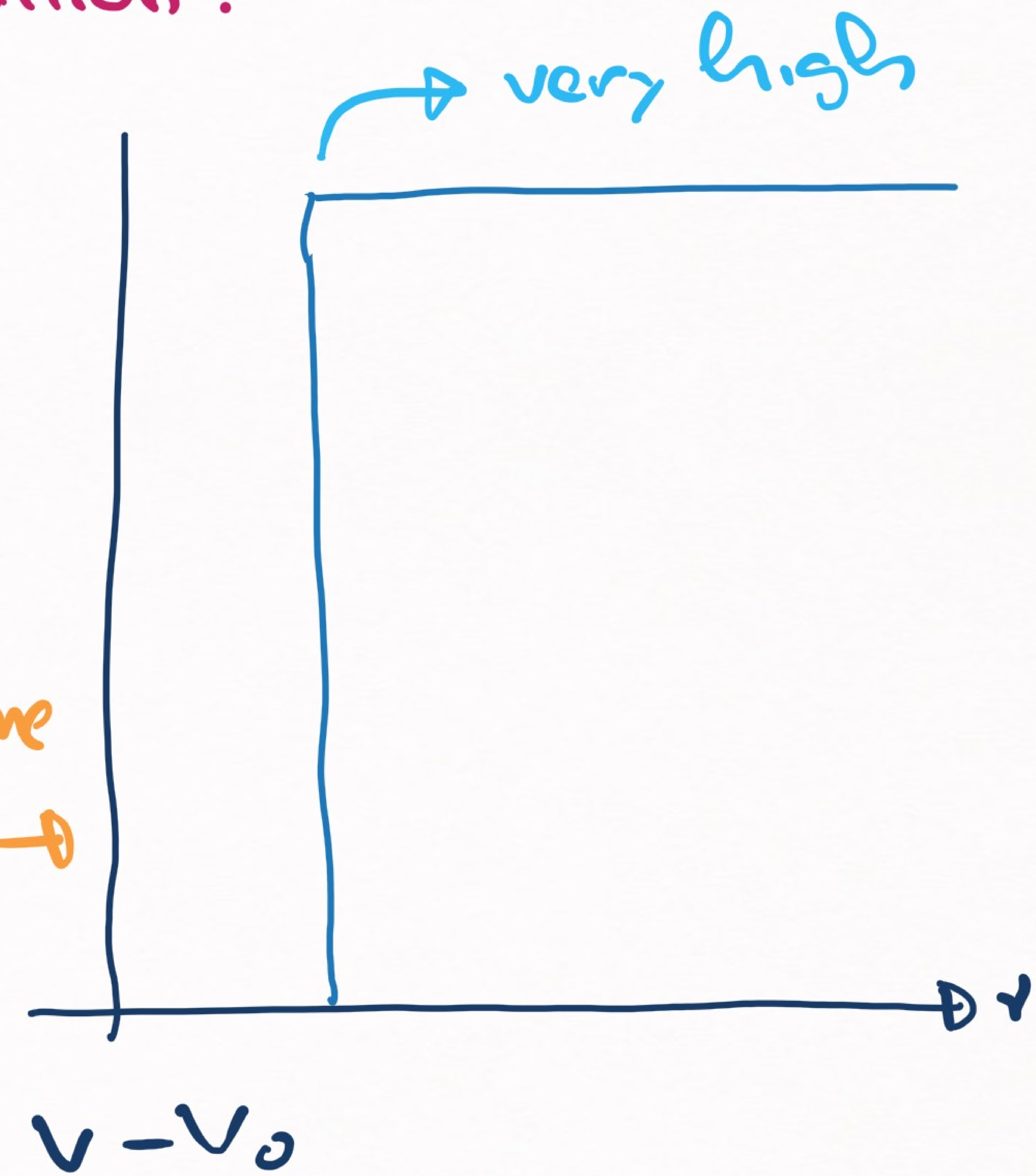
($V_0 > 0$ by convention)

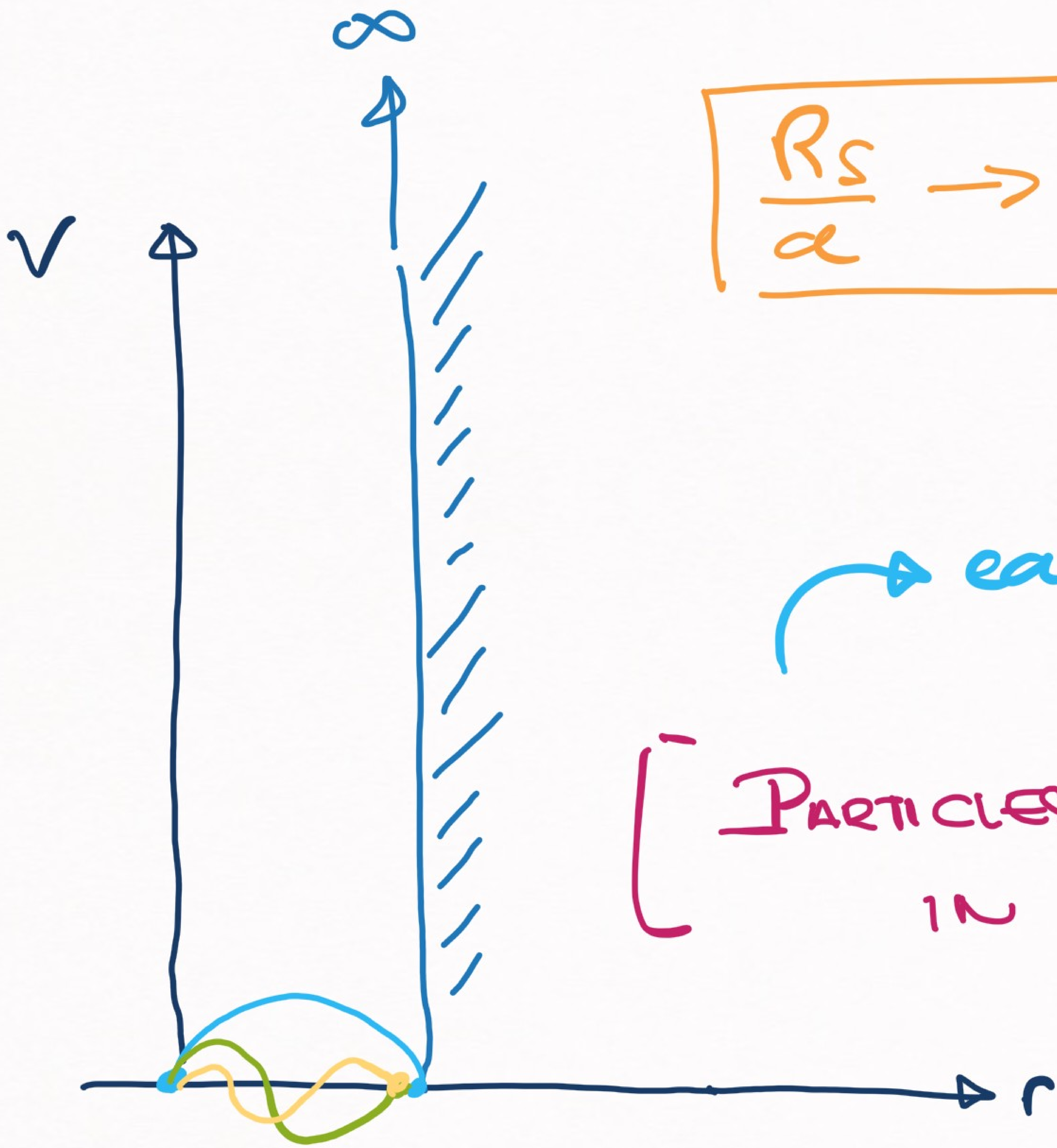
$\frac{1}{2\mu} \frac{1}{R_s^2}$ (lot of depth : R_s is small)

Interpretation of this condition :



This square well is the same as this one



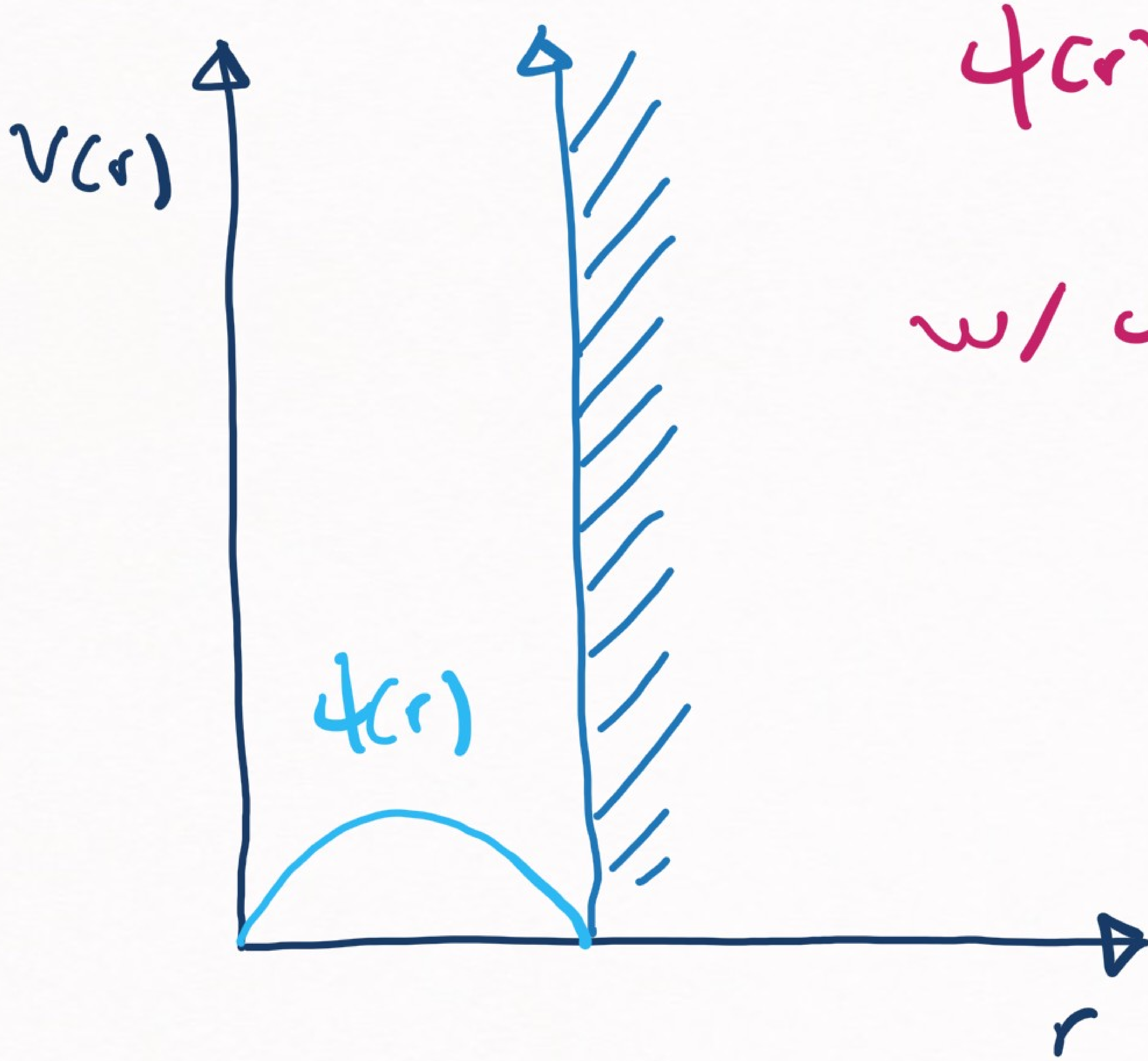


$$\boxed{\frac{R_s}{\alpha} \to 0} \to \text{approximation}$$

→ easy to solve problem

[PARTICLES TRAPPED IN AN INFINITE WELL]

And we can calculate the spectrum:



$$\psi(r) = \frac{u(r)}{r} \Rightarrow \underbrace{\begin{cases} u(0) = 0 \\ u(a) = 0 \end{cases}}_{\text{two conditions}}$$

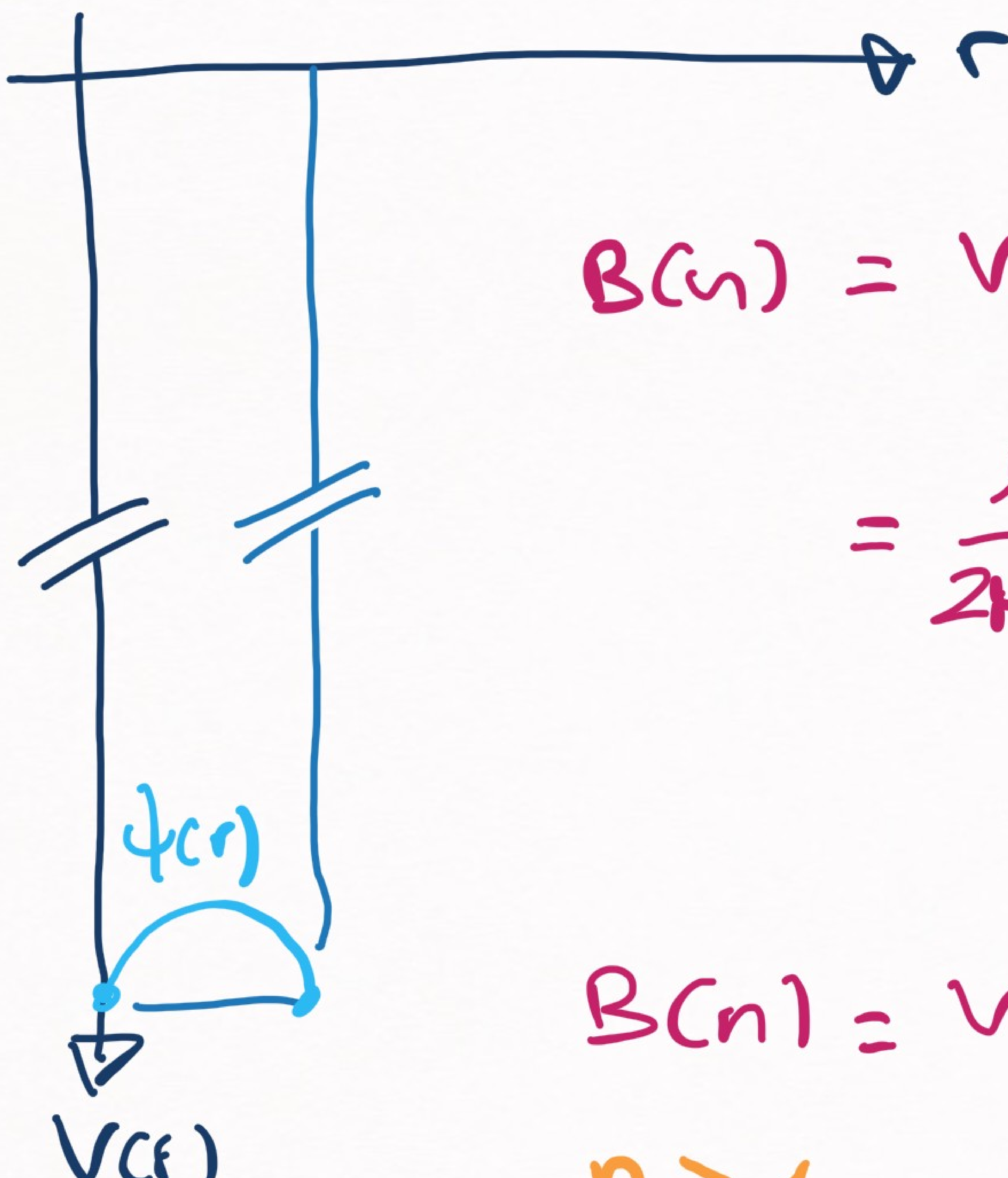
w/ $u(r) = \sin(kr)$

1) $u(0) = 0 \rightarrow$ trivial for $\sin(kr)$

2) $u(a) = 0 \rightarrow$ energy levels

$\sin(ka) = 0 \qquad ka = n\pi$

Now we change back the energy origin:



$E_B = -B$

infinite well approximation

$$B(n) = V_0 - \frac{k_n^2}{2\mu}$$

$$= \frac{1}{2\mu}\left(\frac{1}{R_s^2} - \frac{(n\pi)^2}{a^2}\right)\left[1 + O\left(\frac{R_s}{a}\right)\right]$$

or

$$B(n) = V_0\left(1 - \frac{R_s^2}{a^2}(n\pi)^2 + O\left(\frac{R_s^3}{a^3}\right)\right)$$

$n \geq 1$

(Taylor series)

If natural $(R_s \ll a) \Rightarrow$ $B(n) = V_0 \left[1 - \mathcal{O}\left(\frac{R_s^2}{a^2}\right)\right]$

$\rightarrow \left[B(n) = V_0 + \text{corrections}\right] \leftarrow$ $B \sim \mathcal{O}(V_0)$

Notice that the answer can be written as a series:

$$B_n = V_0 \left[1 - \frac{R_s^2}{a^2}(n\pi)^2 + 2\frac{R_s^3}{a^3}(n\pi)^2 + \mathcal{O}\left(\frac{R_s^4}{a^4}\right)\right]$$

$\rightarrow$ try to calculate it

power series in $\frac{R_s}{a}$

What have we learnt here? $\underbrace{\text{meaning}}$ of natural in scales

→ If we have a natural ($R_s \ll a$) square well:

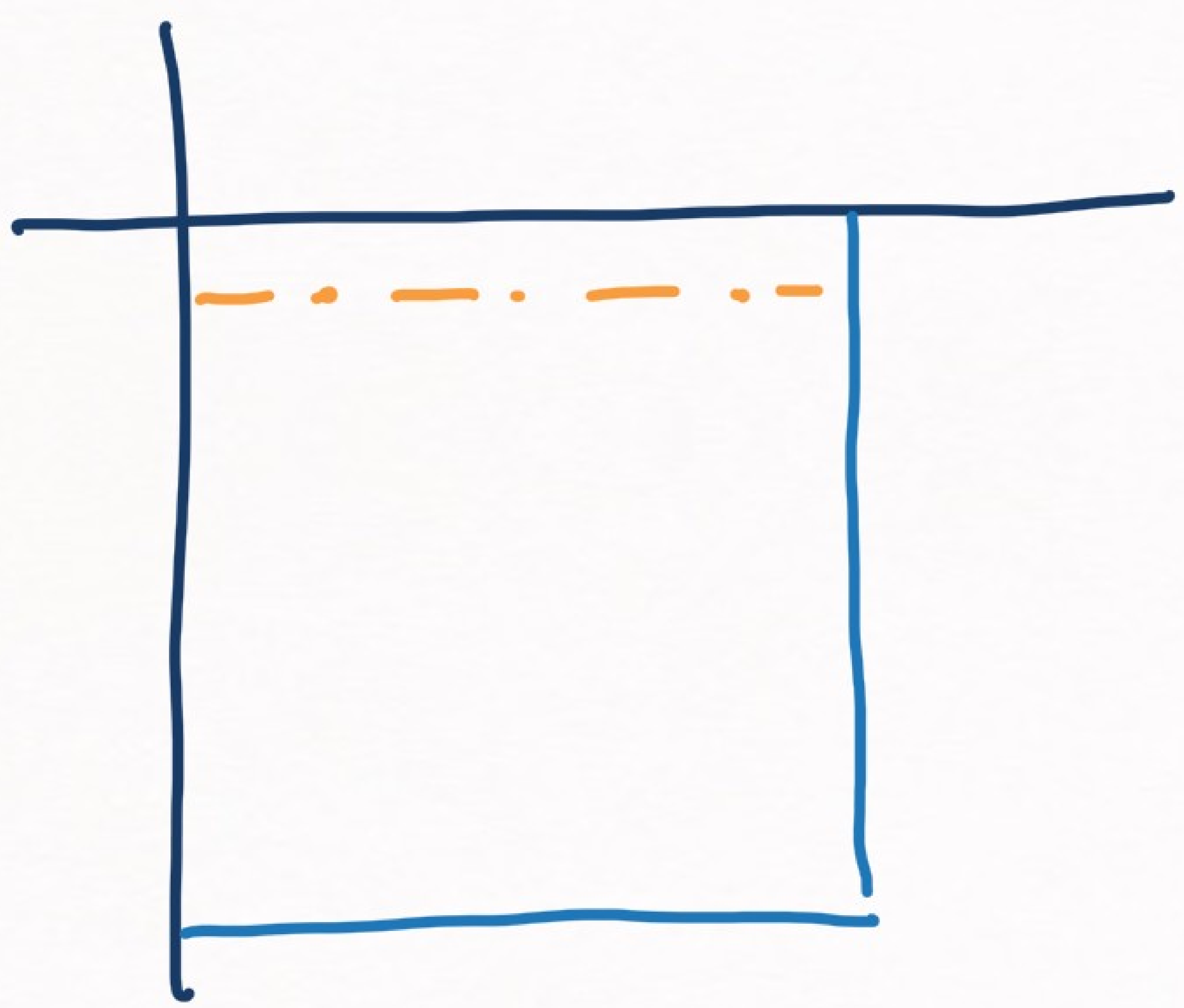
1) Scale separation → $\frac{R_s}{a}$ small expansion parameter
(by definition of natural)

2) Low energy physics given by $\underline{\underline{R_s}}$

$$B = -\frac{1}{2\mu}\frac{1}{R_s^2} + \dots \;/\; \gamma = \frac{1}{R_s}\left(1 + \mathcal{O}\left(\frac{R_s}{a}\right)\right)$$

3) Corrections in terms of $\frac{R_s}{a}$
(check the expansion before)

But the well might be unnatural: $\boxed{B \ll V_0}$



$$\underbrace{B \ll \mathcal{O}(V_0)}_{\text{why this happens}}$$

$\Downarrow$

$\rightarrow$ Check the solution of the square well (check the energy levels)

For this we use the eigenvalue equation:

$$\cot(R_s a) = 0 + \text{corrections}$$

$$\left[\, \kappa \cot(\kappa a) = -\gamma \,\right]$$

$$\kappa = \sqrt{2\mu(V_0 - B)} = \sqrt{\frac{1}{R_s^2} - \gamma^2}$$

$$\gamma \to 0$$

UNNATURAL CASE

$$\kappa \to \frac{1}{R_s}$$

$$\begin{aligned} \kappa R_s \cot(R_s a) &= -R_s \gamma \\ &= \mathcal{O}(R_s \gamma) \\ &= 0 + \mathcal{O}(R_s \gamma) \end{aligned}$$

That is, we have that:

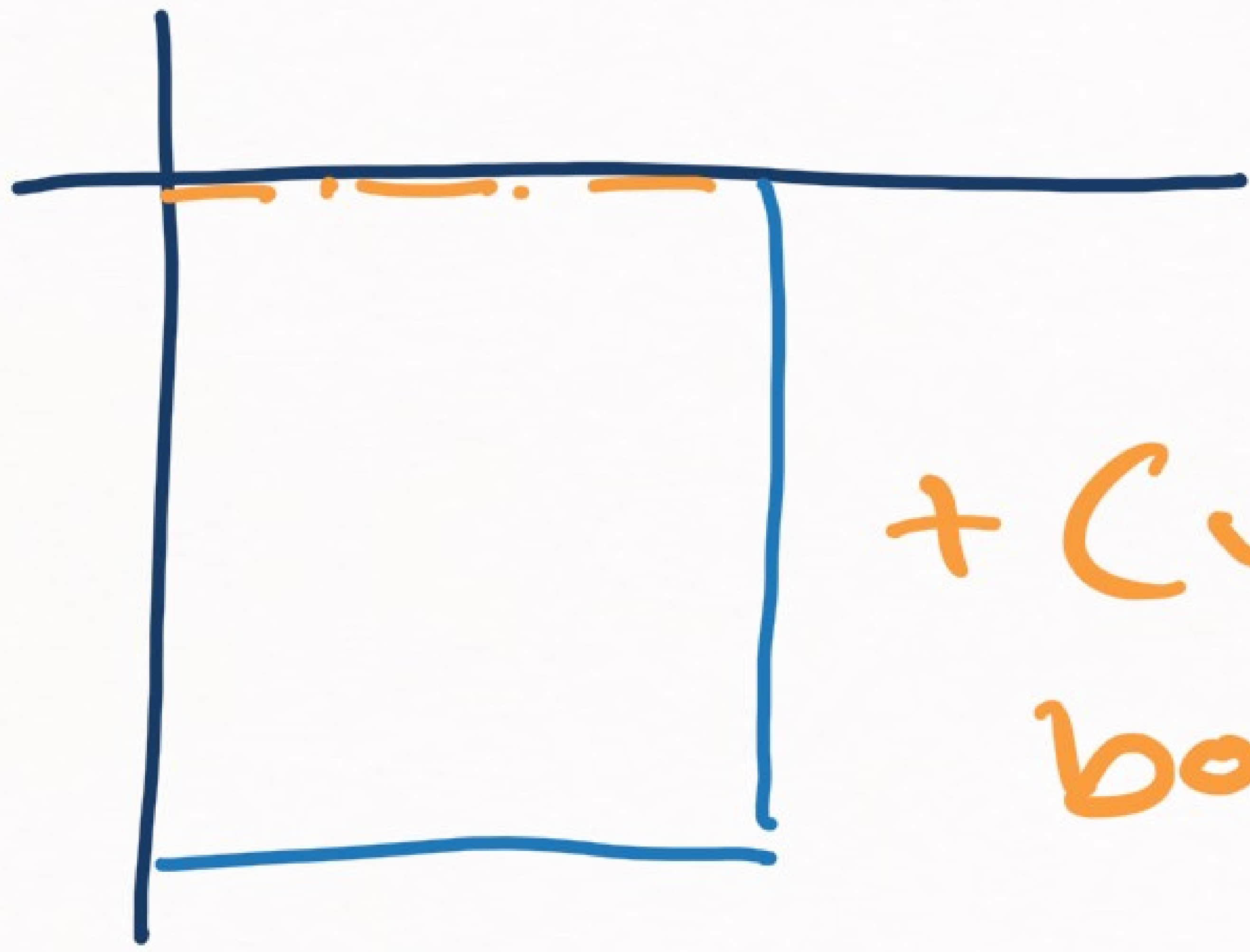
$\kappa \cot(\kappa a) = -\gamma$ can be expanded: $\underline{\underline{\gamma R_s << 1}}$

$\kappa = \frac{\ell}{R_s}\left(1 + O(\gamma R_s)\right) \rightarrow \underline{\underline{\cot(R_s a) = 0 + O(\gamma R_s)}}$

$\left[\kappa a = (2n+1)\frac{\pi}{2} + \underline{\underline{O(\gamma R_s)}} \right]$

if $n > 0$, then $\exists$ several bound states
(but only the less bound has $\gamma \to 0$)

Fine-Tuning (3)



$\boxed{B \simeq 0} \Rightarrow \boxed{\kappa a = \frac{\pi}{2}}$ (n=0)

+ (unique bound state)

or, equivalently:

$$\boxed{\frac{a}{R_s} = \frac{\pi}{2}}$$

$$\Downarrow$$

$B \simeq 0$ requires this ratio of scales

$\{R_s, a\}$ & $R_s \sim a$

(situation of slide #4)

FINE-TUNING 2

$\boxed{\gamma \to 0} \Rightarrow \boxed{R_s \sim a}$

(two scale system w/ poor separation of scales)

Notice one thing:

→ the condition for $\gamma \to 0$ is stronger than just $R_s \sim a$ Σ

→ there is a very specific ratio $\frac{R_s}{a} \simeq \frac{\pi}{2}$ for which $\gamma \to 0$ (fine tuning)

FINE-TUNING 3

What if we vary $\frac{a}{R_s} = \frac{\pi}{2}(1+x)$?

We will calculate γ for different values

of x → how small x needs to be

for $\gamma \ll \frac{1}{R_s}$

FINE-TUNING ④

✓ → UNNATURAL

✗ → NATURAL

$x = 0 \Rightarrow \gamma = 0 \quad (\gamma \ll \frac{1}{R_s})$ ✓

$x = 0.1 \Rightarrow \gamma \simeq \frac{0.16}{R_s} \quad (\gamma \ll \frac{1}{R_s})$ ✓

$x = 0.2 \Rightarrow \gamma \simeq \frac{0.37}{R_s}$ ($\gamma < \frac{1}{R_s}$ but also $\gamma \sim 1/R_s$) $\approx$

$x = 0.3 \Rightarrow \gamma \simeq \frac{0.51}{R_s} \quad (\gamma \sim 1/R_s)$ ✗

$x = 0.4 \Rightarrow \gamma \simeq \frac{0.73}{R_s} \quad (\gamma \sim 1/R_s)$ ✗

$\boxed{\text{MORAL}}$ If we want to have an unnatural system, $Q_1 \simeq Q_2$ is not enough

$\Rightarrow$ a very specific relation between Q_1 & Q_2

$\boxed{\text{EXAMPLE}}$ $\rightarrow$ $\frac{a}{R_S} \simeq \frac{\pi}{2}$ (within about a 20% window)

[HUGE DEPENDENCE
ON A VERY SPECIFIC CONDITION]

$$\frac{a}{R_s} \simeq \frac{\pi}{2} \longrightarrow$$ this ratio needs to be fine-tune if we want to have an unnatural system

If we observe

$$\gamma << \frac{1}{R_s} \Rightarrow$$ fine-tuning

—— o ——

RECAP → SQUARE WELL IS A GOOD MODEL OF NATURAL & UNNATURAL TWO-BODY SYSTEMS

THE SQUARE WELL $\longrightarrow$ What have we learnt?

1) System with two scales $\{R_s, a\}$

2) If $\exists$ a good scale separation $R_s \ll a$
then the system is natural $(\gamma \sim 1/R_s)$

3) If scale separation poor $R_s \sim a$
the system might be unnatural

unnatural $\rightarrow$ fine tuning $\underline{\underline{\dfrac{a}{R_s} \simeq \dfrac{\pi}{2}}}$

This looks trivial, but has many applications

$$\boxed{\text{FINE-TUNING IN THE TWO NUCLEON SYSTEM}} \to \circledast$$

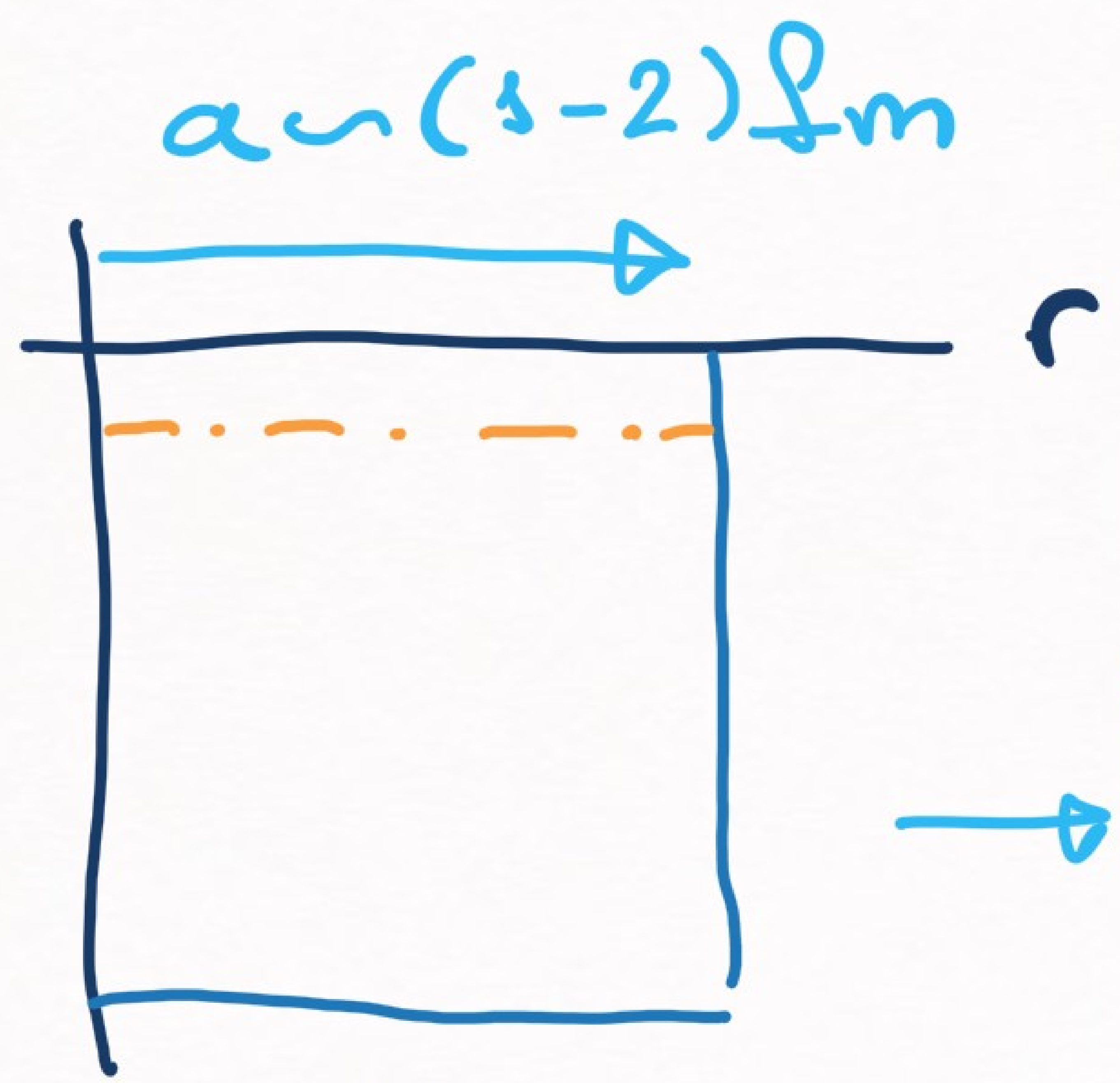
↓

We can understand this w/ square well

$\circledast \to$ neutron + proton systems

DEUTERON (FINE TUNING)

neutron + proton, $\underline{\underline{S = 1}}$



$$E_B = -\underline{\underline{B}} = -2.2 \text{ MeV}$$

$$\left[E_B = \langle \underline{\underline{H}} \rangle = \langle T \rangle + \langle V \rangle \right]$$

kinetic energy

potential energy (*)

(V ~ 50 MeV)

V(r)

(*) $\underline{\underline{V \sim 50 \text{MeV}}}$

DEUTERON (FINE TUNING)

$$|E_B| = |\langle T\rangle + \langle V\rangle|$$

$\downarrow$

$$2\,\text{MeV} = |\underbrace{\langle T\rangle}_{\simeq 48\,\text{MeV}\ \sim\ 50\,\text{MeV}} - 50\,\text{MeV}|$$

E_B is small

$\langle T\rangle, \langle V\rangle$ are big

$\langle T\rangle, \langle V\rangle$ cancel out at a level of $\frac{E_B}{|\langle T\rangle|}$ or $\frac{E_B}{|\langle V\rangle|}$

$\sim \frac{1}{25} \rightarrow (*)$

$(*) \rightarrow$ Sometime we say that there is a fine-tuning of

$\underline{\underline{1/25}}$

VIRTUAL STATE (EVEN MORE FINE-TUNING)

Remarks about the neutron-proton system

(What is this virtual state?)

a) $S = 0 \rightarrow$ It is not bound, but just by a little

b) $S = 1 \rightarrow$ Deuteron (bound)

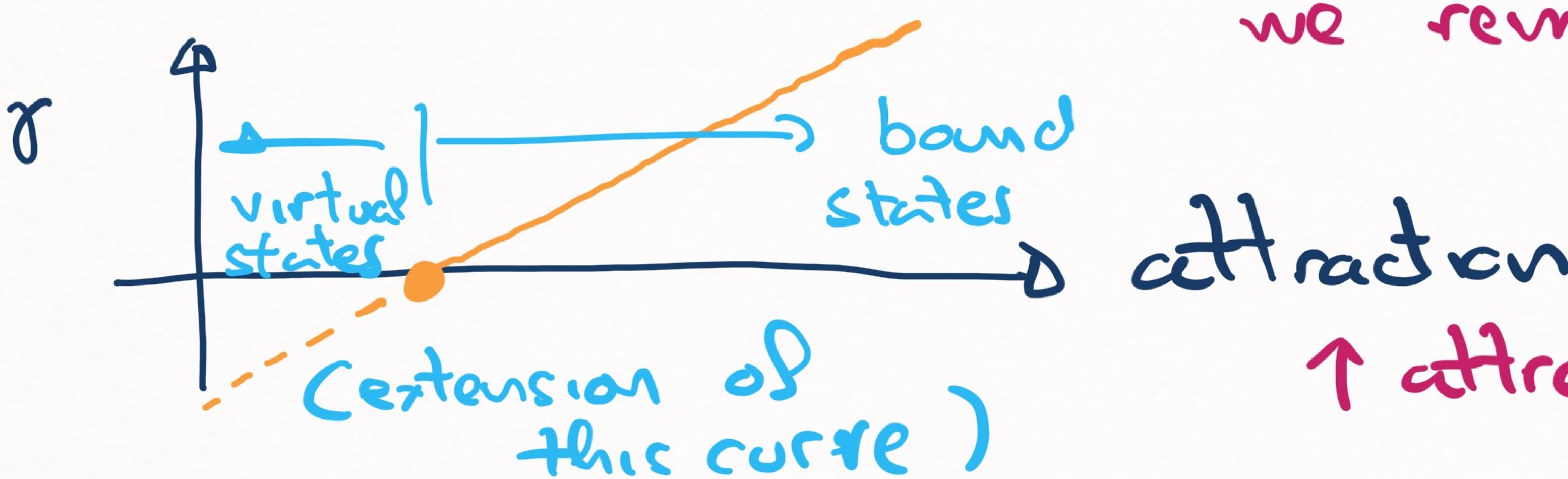
$n \rightarrow S = 1/2$, $p \rightarrow S = 1/2 \Rightarrow np$, $S = 0$ or 1

What is a virtual state?

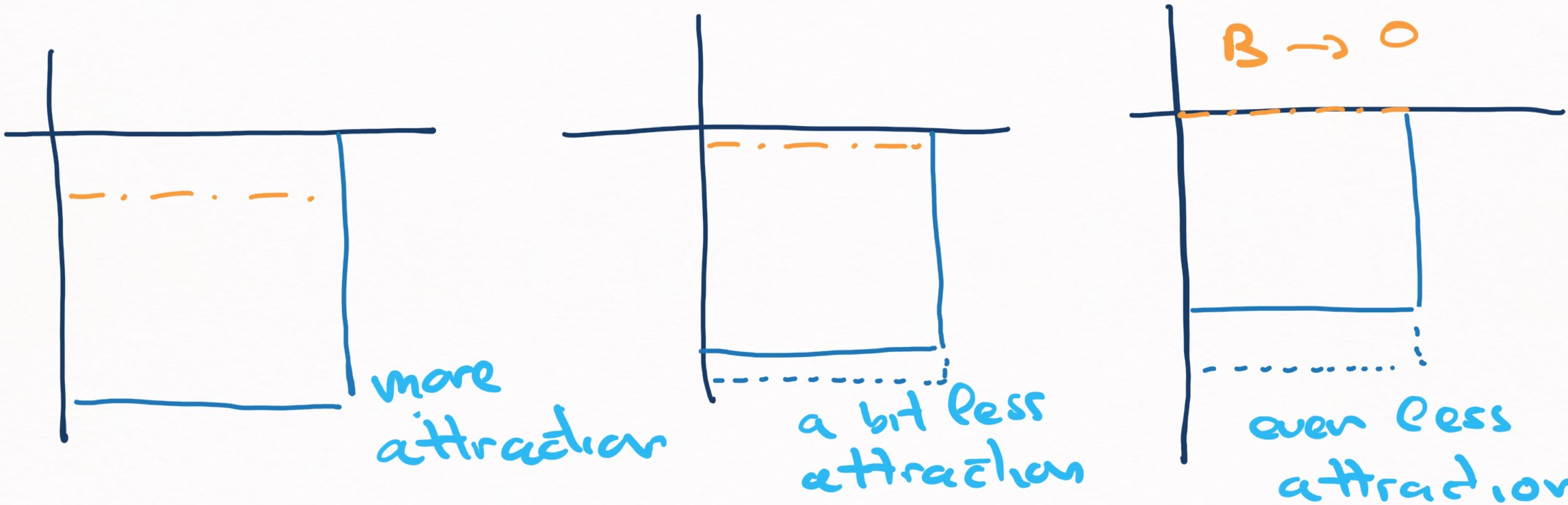
1) Bound state → $[-\nabla^2 + 2\mu V(\vec{r})]\psi(\vec{r}) = -\gamma^2 \psi(\vec{r})$

b.s. wave function $\Big\}$ $\psi(\vec{r}) \sim \dfrac{e^{-\gamma r}}{r}$ $(r \to \infty)$ (Schrödinger → solve it)

2) Virtual state → What happens when we remove attraction?



$\uparrow$ attra $\Rightarrow$ $\uparrow \gamma$



And then what?

$\Rightarrow$ what happens if there is less attraction?

We can still try to solve Schrödinger:

$$\left[-\nabla^2 + 2\mu V(\vec{r})\right]\psi(\vec{r}) = -\gamma^2 \psi(\vec{r})$$

$$\boxed{\psi(r) \sim \frac{e^{-\gamma r}}{r}} \xrightarrow[\text{attraction}]{\text{reduce}} \boxed{\psi(r) \sim \frac{e^{+\gamma r}}{r}}$$

(bound state solution)

(normalizable: $\int d^3\vec{r}\, |\psi(\vec{r})|^2 < \infty$)

virtual state solution

(not normalizable: $\int d^3\vec{r}\, |\psi(\vec{r})|^2 \to \infty$)

But, does this make sense? mathematically yes (but physically)

⇐ Scattering (Colissions)

For $E_{cm} \to 0$, the cross section goes to: (σ)

↳ observable we measure when colliding two particles

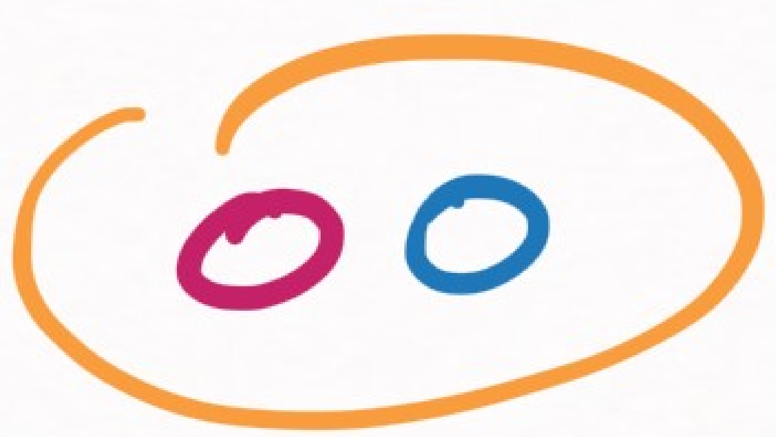
$$\boxed{\sigma \to \frac{4\pi}{\gamma^2}}$$

bound/virtual state energy

$\gamma > 0$, $\gamma < 0$ → no difference

Low energy scattering → no distinction between bound / virtual state

From scattering alone we will not be sure whether there is a bound state



$s = 0$

we know it's not a bound state, but a virtual state

VIRTUAL STATE → [THE REASON WHY THERMAL NEUTRONS ARE BAD FOR YOUR HEALTH]

(not only in scatterings)

Radiative neutron capture

$$n + p \to d + \gamma$$

prob $\propto |\underline{\underline{M_f}}|^2$

high prob.

$$M_f = \int \underbrace{e^{-\gamma_B r}}_{\text{wf of deuteron}} \underbrace{(1 - \gamma_V r)}_{\text{wf of np for } S=0 \text{ at zero energy}} dr \sim \frac{1}{\sqrt{\gamma_B}} \left(\frac{1}{\gamma_B} - \frac{1}{\gamma_V} \right)$$

Large

VIRTUAL STATE

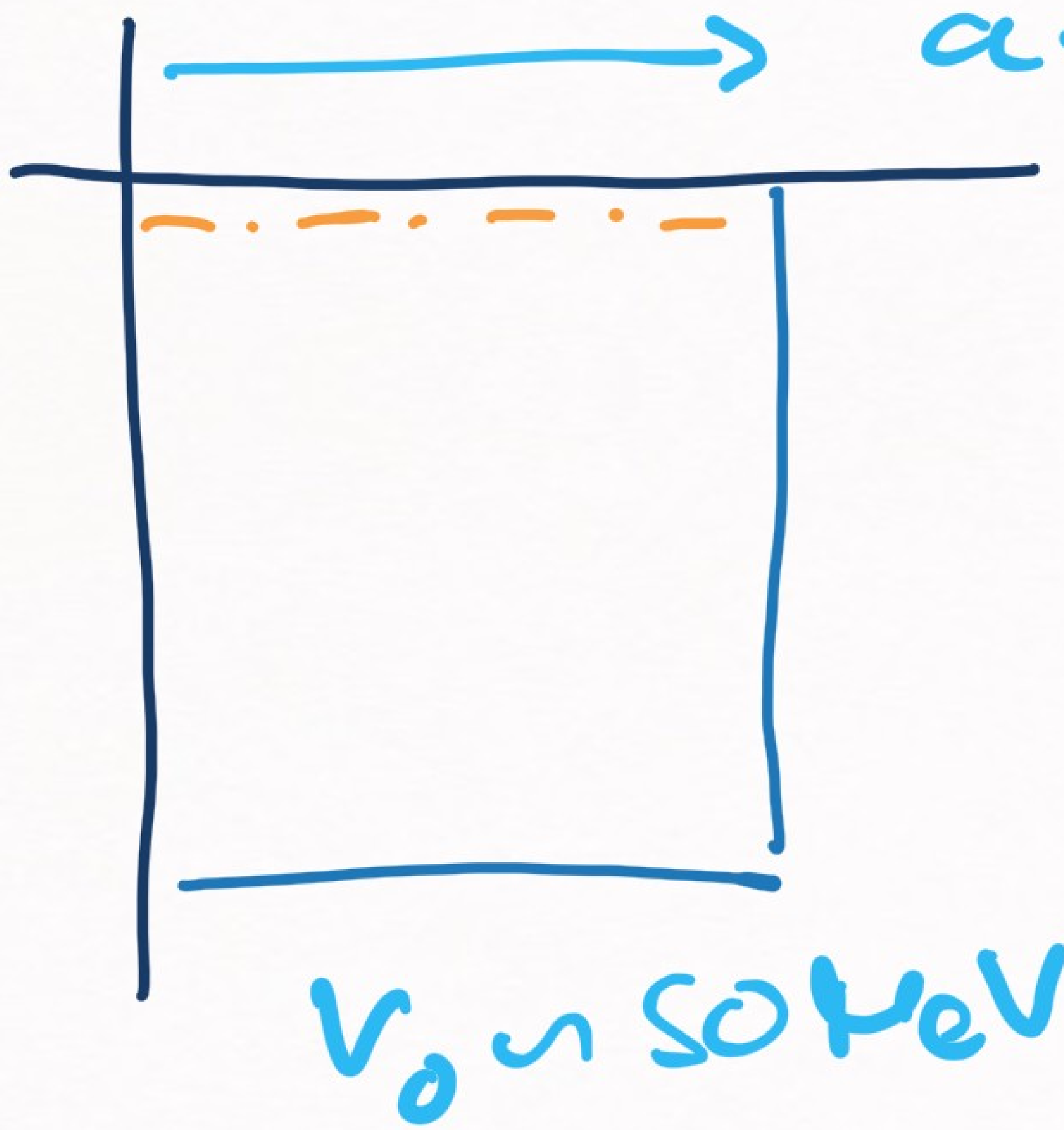
$B_V \sim \underline{\underline{0.07}}\ \text{MeV}$ $\left(B_V = \dfrac{\gamma_V^2}{2\mu} \right)$

$B_B \sim 2.2\ \text{MeV}$

$$\left.\begin{array}{l} \gamma_V \sim 8\ \text{MeV} \\ \gamma_B \sim 45\ \text{MeV} \end{array}\right\}$$

not such a large diff. when compared w/ the <u>binding</u> energies

VIRTUAL STATE $\longrightarrow$ HOW MUCH FINE TUNING?



$$E_v = \underbrace{\langle T \rangle + \langle V \rangle}_{}$$

$\downarrow$

$0.07\,\mathrm{MeV}$

$$|\langle T \rangle|,\ |\langle V \rangle| \sim 50\,\mathrm{MeV}$$

fine tuning of $\dfrac{0.07}{50} \sim 0.0035 \sim \dfrac{1}{700}$

RECAP

NUCLEAR PHYSICS

1) DEUTERON : np bound state w/ $S=1$

fine tuning of 1 part in 25

2) VIRTUAL STATE : np scattering w/ $S=0$

fine tuning of 1 part in 700

(you can check it by reproducing bound/virtual with a square well of $a \sim (1\text{-}2)\,\mathrm{fm}$)

NEXT LESSON | → Why is there fine-tuning?

Two possibilities:

a) BY CHANCE

b) BECAUSE THERE IS A CONSPIRACY

(CONSPIRACIES IN PHYSICS MEANS THAT THERE IS A SYMMETRY)

SEE YOU TUESDAY 15:50