NUCLEAR PHYSICS 3

→ MULTISCALE SYSTEMS &
THE PROBLEM OF FINE TUNING

1) WHY NUCLEAR PHYSICS IS DIFFICULT
2) IDENTIFYING SCALES IN 2-BODY SYSTEMS
3) FINE-TUNING IN THE SQUARE WELL

RECAP | In the previous lesson we learnt:

1) PHYSICAL SYSTEM $\longrightarrow$ CHARACTERISTIC SCALE

(a typical size or momentum of the system)

hydrogen atom: $a_B \simeq 0.5\,\text{Å}$

2) NATURAL SYSTEMS

(example)

2.a) $\exists$ a clear candidate for this typical scale that describes the system

we call it Q (for momenta) or R (for distance)

(or whatever other name we like)

→ 2.b) Observables of a natural system will be of $\mathcal{O}(1)$ when expressed in proper units of the typical scale

[$Q \equiv$ typical scale]

$$\langle\psi|\hat{O}|\psi\rangle = \underline{\underline{c}} \cdot \underline{\underline{Q}}^{\underline{\underline{d}}}$$

d } ⟶ canonical dimension (energy/momentum: $\underline{\underline{[\langle\psi|\hat{O}|\psi\rangle] = [M^d]}}$)

with $c = \underline{\underline{\mathcal{O}(1)}}$

For instance: → $\langle p\rangle \sim \underline{\underline{Q}}$ $(\sim 1/R)$ ←

→ $\langle r\rangle \sim \frac{1}{\underline{\underline{Q}}}$ $(\sim R)$ ←

$\left(Q \sim \frac{1}{R}\right)$

→ 2.c) In general, natural systems are easy

——— $\boxed{\text{BUT} \ldots}$ ——— $\exists$ always a but

3) MOST SYSTEMS HAVE MULTIPLE SCALES

→ $\{Q_1, Q_2, Q_3, \ldots\}$

[Today we will try to learn what to do in this case.]

By the way...

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Nuclear Physics, Spring 2023

The server is being updated

The slides are keep under the following directory, depending on the year:

- This year's slides → and you can find the slides
- Past year's slides
- Slides from 2021 (more or less same structure as this and past year's, so you can download them before the corresponding class for having a preview of the subjects we will be covering)
- Slides from 2020 (they are probably the best quality ones)

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and lessons 1 & 2 are already there

[MULTISCALE SYSTEMS :]

Why nuclear physics is more difficult than atomic physics?

MOST SYSTEMS $\rightarrow \{Q_1, Q_2, Q_3, \ldots\}$

USUALLY, WE HAVE THAT: $Q_1 \ll Q_2 \ll Q_3$ etc.

$$\boxed{Q_1 \ll \{Q_2, Q_3, Q_4 \ldots\}}$$

HYDROGEN ATOM

($Q \sim 1/R$ → whatever is more convenient)

$a_B \sim 5\cdot 10^4$ fm

$1/m_e \sim 400$ fm

$r_p \sim 1$ fm

$\{R_1, R_2, R_3, \dots\}$

$R_1 \gg R_2 \gg R_3 \gg \dots$

$R_1 \downarrow a_B$ (Bohr radius)

$R_2 \downarrow 1/m_e$ (QFT structure of electron)

$R_3 \downarrow r_p$ (size of the proton)

DEUTERON → here things will be different

→ what is the deuteron?

bound state of a proton & a neutron

it has a size about 5.4 fm

→ what is the force that binds the deuteron?

hydrogen atom → Coulomb force

$$V(r) = -\frac{\alpha}{r}, \quad \alpha \simeq \frac{1}{137}$$

deuteron → Nuclear force (Yukawa potential)

approximations

$$V(r) = -\frac{g^2}{4\pi}\frac{e^{-mr}}{r} = -\frac{\alpha_Y}{r}e^{-mr}$$

→ (similar to Coulomb) ← $\boxed{\text{how big is } \alpha_Y\text{?}}$

Deuteron $\rightarrow$ (neutron + proton) $\rightarrow$ $V_y(r) = -\frac{a_y}{r} e^{-mr}$

pion mass

Yukawa hypothesized that the nuclear force was the consequence of the exchange of a heavy particle (the pion)

$$\underbrace{\text{heavy particle}} \rightarrow V_y(r) \propto \frac{e^{-mr}}{r}$$

Coulomb $\rightarrow$ exchange of photon $\rightarrow$ $m = 0$ $\rightarrow$ $V_c(r) \propto 1/r$

$$V_Y(r) = -\frac{\alpha_Y}{r} e^{-mr} \qquad m \simeq 140 \text{ MeV}$$

Determine α_Y from the condition of reproducing the deuteron

$$\left[-\frac{1}{2\mu}\vec{\nabla}^2 + V_Y(r)\right]\Psi(\vec{r}) = -B_d \Psi(\vec{r}) \longrightarrow \alpha_Y$$

$B_d \simeq 2.2$ MeV

You can do the calculations (try it, please)

$$\boxed{\alpha_y \sim \frac{1}{3}}$$

compare w/ Coulomb:

$$\boxed{\alpha \sim \frac{1}{137}}$$

$\alpha_y \sim 45\ \alpha$

→ the nuclear force is much stronger than Coulomb

strong potential → less perturbative
+ more difficult

from the strength of the potential alone,
the deuteron should be more difficult
than the hydrogen atom

→ but there are more reasons

SCALES

1) Deuteron size

5.4 fm

2) Yukawa potential $\rightarrow$ finite range

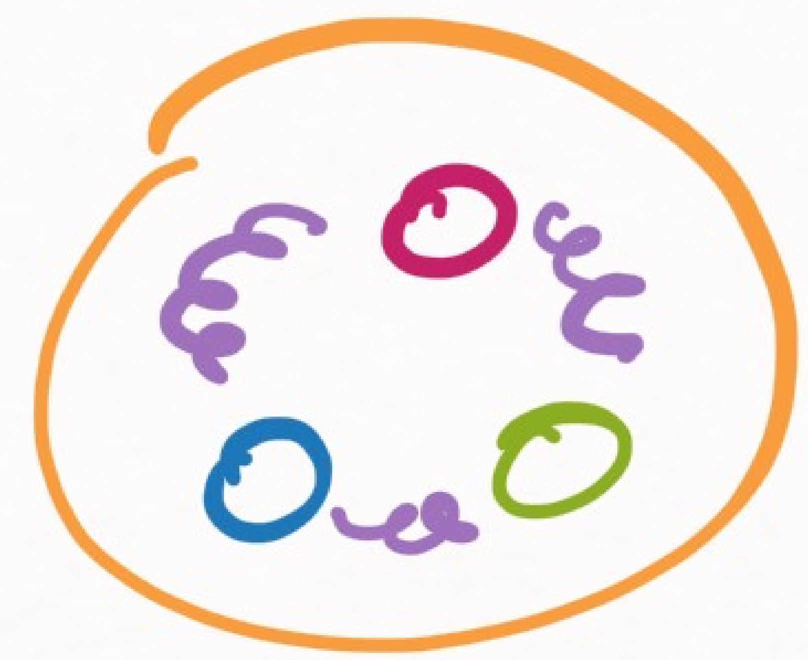
$$\frac{e^{-mr}}{r} \Rightarrow mr \gg 1, \quad V_Y(r) \rightarrow 0$$

only important for $mr \sim 1$

$$1.4\,\text{fm} \leftarrow m = 140\,\text{MeV}, \quad r \sim \frac{\hbar c}{140\,\text{MeV}} \sim 1.4\,\text{fm}$$

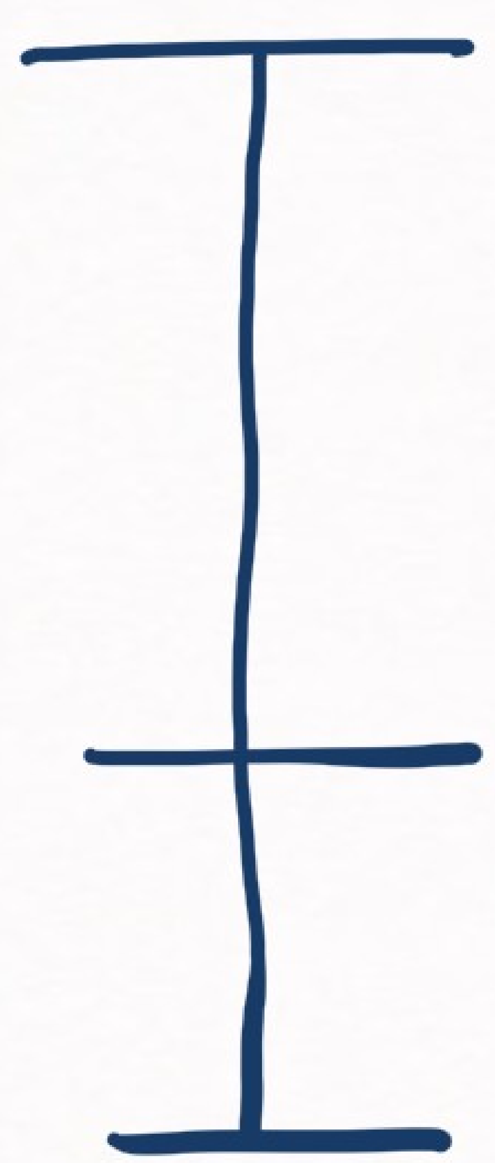
3) Size of neutron & proton

0.8 fm



proton → uud quarks

neutron → udd quarks



$r_d \sim 5.4$ fm

$s/m \sim 1.4$ fm

$r_p \sim 0.8$ fm

Deuteron

$r_d \sim 5.4\ \text{fm}$

$1/m \sim 1.4\ \text{fm}$

$r_p \sim 0.8\ \text{fm}$

Deuteron

$$\boxed{\frac{1}{m r_d} \sim \frac{1.4\ \text{fm}}{5.4\ \text{fm}} \sim 1/3}$$

(larger corrections)

$a_B \sim 54000\ \text{fm}$

$1/m_e \sim 400\ \text{fm}$

$r_p \sim 0.8\ \text{fm}$

Hydrogen atom

$$\boxed{\frac{1}{m_e a_B} \sim \frac{1}{137}}$$

(smaller corrections)

From this comparison $\rightarrow$

Corrections from shorter length scales in the deuteron are larger than the same corrections in the hydrogen atom

EXPECTATION → DESCRIPTION OF THE DEUTERON MORE DIFFICULT THAN DESCRIPTION OF HYDROGEN ATOM

3

GALILEAN GRAVITY & NEWTONIAN GRAVITY

← another example of how to exploit a separation of scales

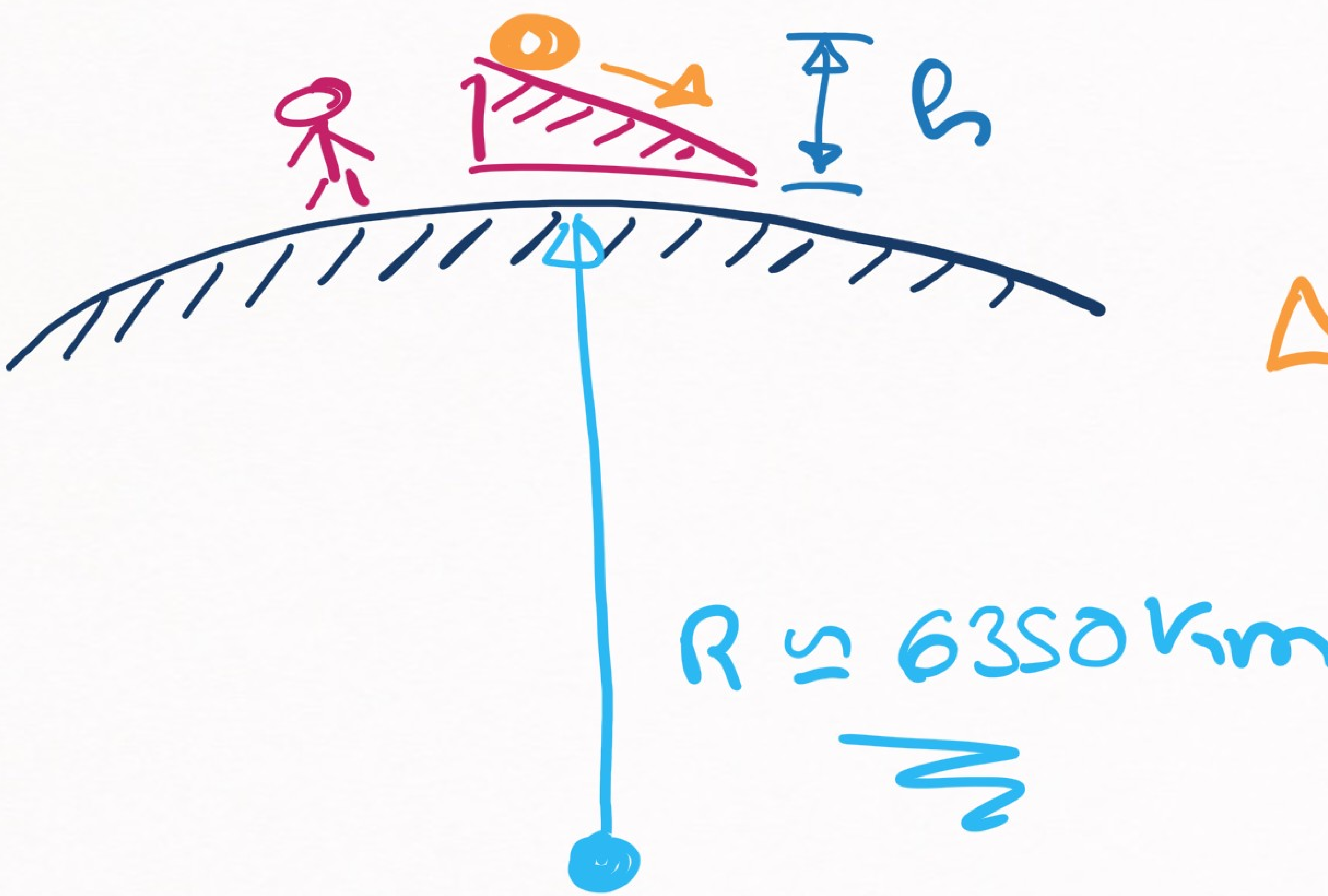
What do I mean by Galilean gravity?

approximation that $\underbrace{\Delta V_{gravity}}_{\text{change in the potential}} = \underbrace{m}_{\text{mass}}\,\underbrace{g}_{g = 9.8\, m/s^2}\,\underbrace{h}_{\text{height}}$

$$\boxed{\Delta V = mgh} \longrightarrow$$ this is a good approximation if we ignore the size of the earth

$\Rightarrow$

$(h \ll R_{earth})$



$$\Delta U = mgh$$

(comes from this:)

$$\Delta U = \frac{GMm}{R} - \frac{GMm}{R+h}$$

G : Newton's constant

M : mass of the earth

R : radius of the earth

$$\Delta V = \frac{GMm}{R} - \frac{GMm}{R+h} \longrightarrow \text{expansion on } \frac{h}{R}$$

Two scales $\{h, R\}$, $\underline{h \ll R}$

$R \sim 6400\,\text{km}$

$h \sim 3\,\text{m}$

huge separation of scales

Power series:

$$\Delta V = \frac{GMm}{R} - \frac{GMm}{R+h} = \overbrace{\frac{GM}{R^2}}^{g} mh \underbrace{\left[1 - \left(\frac{h}{R}\right) + \left(\frac{h}{R}\right)^2 - \left(\frac{h}{R}\right)^3 + \ldots\right]}_{\text{Expansion in } \frac{h}{R}}$$

Galilean gravity:

$$\Delta V = mgh\left[1 + \mathcal{O}\left(\frac{h}{R}\right)\right]$$

correct up to $\mathcal{O}\left(\frac{h}{R}\right)$ corrections

Make improvements:

$$\Delta V = mgh\left[1 - \frac{h}{R} + O\left(\frac{h^2}{R^2}\right)\right]$$

we can continue up to achieving our desired accuracy

General relativity
(Einstein's gravity) $\xrightarrow[v \ll c]{}$ Newtonian gravity

Newtonian gravity $\xrightarrow[h \ll R]{}$ Galilean gravity

$\boxed{\text{CLEAR HYERARCHY OF SCALES}}$

(which can be used to our advantage)

$$\boxed{\text{STANDARD MODEL OF PARTICLE PHYSICS}}$$

↳ We believe that there are new physics beyond the standard model

(typical example → SUSY)

New scale : $\boxed{M_{BSM}}$

If the scale separation is good:

$$\underbrace{M_{BSM}}_{\substack{\text{scale of} \\ \text{new physics}}} \longrightarrow \underbrace{\mathcal{O}\left(\frac{Q}{M_{BSM}}\right)}_{\substack{\text{small corrections of} \\ \text{the physics that we know}}}$$

→ this is why ∃ many experiments trying to find deviations w/ standard model

SUMMARY

1) Physical systems have typical scales
(usually many scales)

2) There might be a separation of these scales (some scales bigger / smaller than others)

→ Formulate expansions / easier descriptions of a system

$\rightarrow$ $\boxed{\text{HOW DO WE IDENTIFY SCALES IN A TWO-BODY SYSTEM}}$

Starting point $\rightarrow$ Schrödinger equation for a two-body system

$$\left[-\frac{1}{2\mu}\vec{\nabla}^2 + V(\vec{r})\right]\psi(\vec{r}) = E\psi(\vec{r})$$

(similar to what we already did for hydrogen)

$$\Big[-\frac{1}{2\mu}\vec{\nabla}^2 + \underbrace{V(\vec{r})}_{\text{potential}}\Big]\underbrace{\psi(\vec{r})}_{\text{wave function}} = \underbrace{E}_{\text{center-of-mass energy}}\,\psi(\vec{r})$$

$$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$$

(reduced mass)

$$E = \frac{1}{2\mu}k^2 \quad (E>0)$$

$$= -\frac{1}{2\mu}\gamma^2 \quad (E<0)$$

We can consider the dimensions:

$$[\mu] = [E]$$

$$[\vec{\nabla}] = [E^2]$$

$$[V] = [E]$$

$$\underbrace{[\psi] = [E^{3/2}]}$$

for a normalized wave function (usually negative energy solutions only)

$$\int \underbrace{d^3\vec{r}}\, |\psi(\vec{r})|^2 = 1$$

$$[E^{-3}] \times [E^3] = [1]$$

Non-relativistic system $\rightarrow$ multiply everything by 2μ

$$\left[-\frac{\nabla^2}{2\mu} + V(\vec{r})\right]\psi(\vec{r}) = E\,\psi(\vec{r})$$

$\times(2\mu)$

I will assume $E < 0$

$$\Big[-\vec{\nabla} + \underbrace{2\mu V(\vec{r})}_{\text{Reduced potential}}\Big]\psi(\vec{r}) = -\gamma^2\,\psi(\vec{r})$$

$[2\mu V] = [E^2]$

or $[L^{-2}]$

REDUCED POTENTIAL → ALLOWS US TO FIND THE TYPICAL SCALES

$\Rightarrow$

1) $V(r) = -\frac{\alpha}{r} \longrightarrow 2\mu V(r) = -2\mu\alpha \frac{1}{r}$

$\boxed{a_B} \leftarrow \quad = -\frac{2}{a_B r} \quad [L^{-2}]$

2) $V(r) = -\frac{C_6}{r^6} \longrightarrow 2\mu V(r) = -\frac{2\mu C_6}{r^6}$

$R_{vdW} = (2\mu C_6)^{1/4} \leftarrow \quad = -\frac{R_{vdW}^4}{r^6} \quad [L^{-2}]$

NEXT LESSON → MORE EXAMPLES

SEE YOU ON FRIDAY

$\boxed{15:50}$