NUCLEAR PHYSICS 2

→ UNDERSTANDING PHYSICS IN TERMS OF SCALES

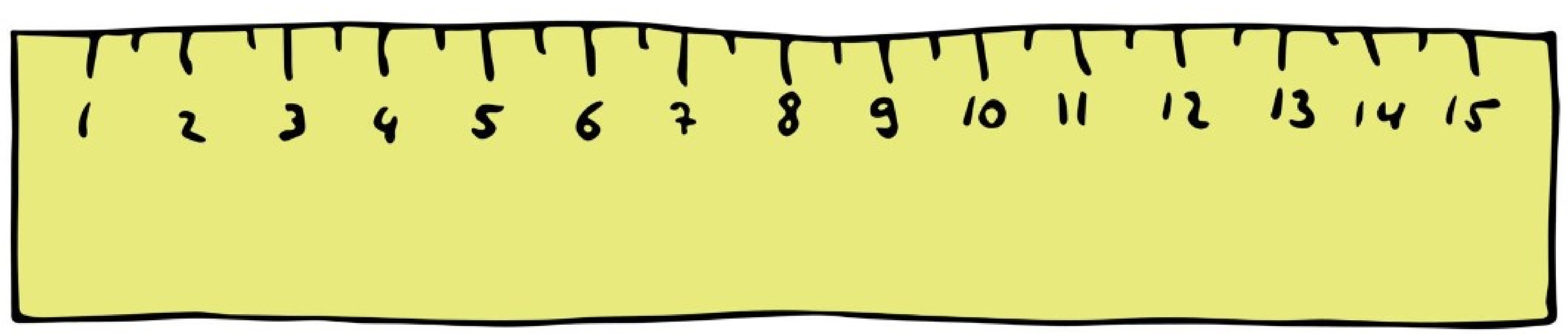
1) THE HYDROGEN ATOM

2) THE BOILING POINT OF CESIUM

RECAP OF PREVIOUS LESSON)

→ Our understanding of nature depends on the scale we are looking at

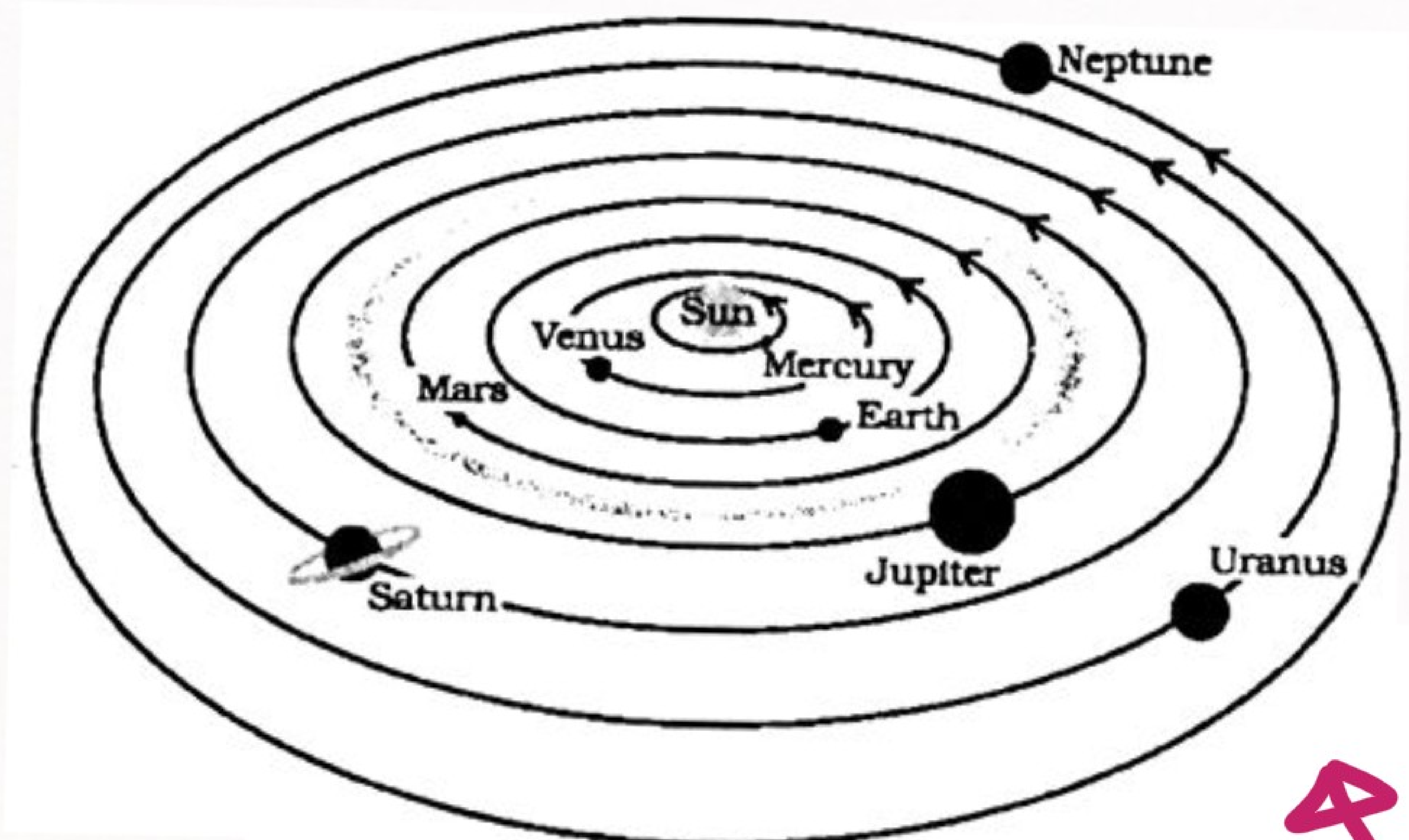
→ Which ruler are we using?

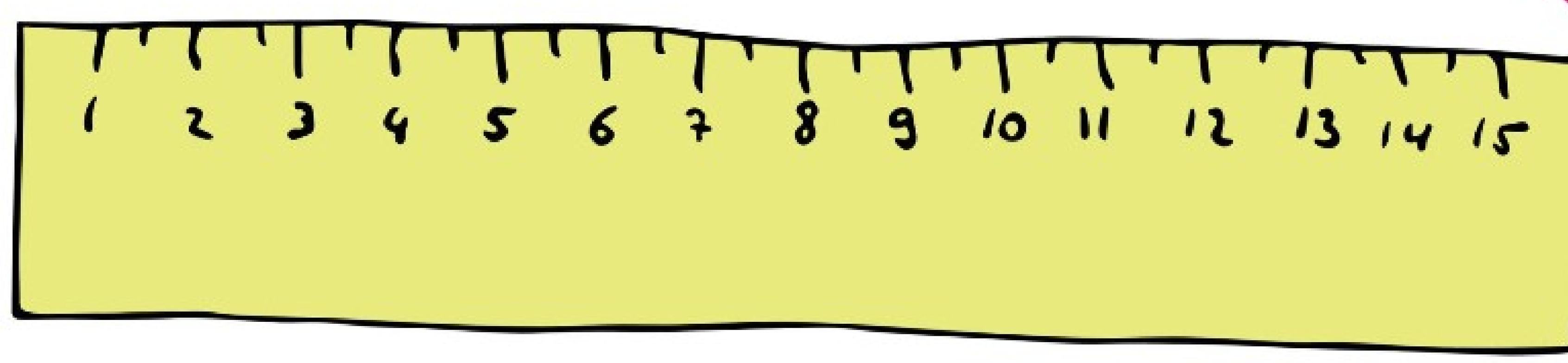


→ cm?

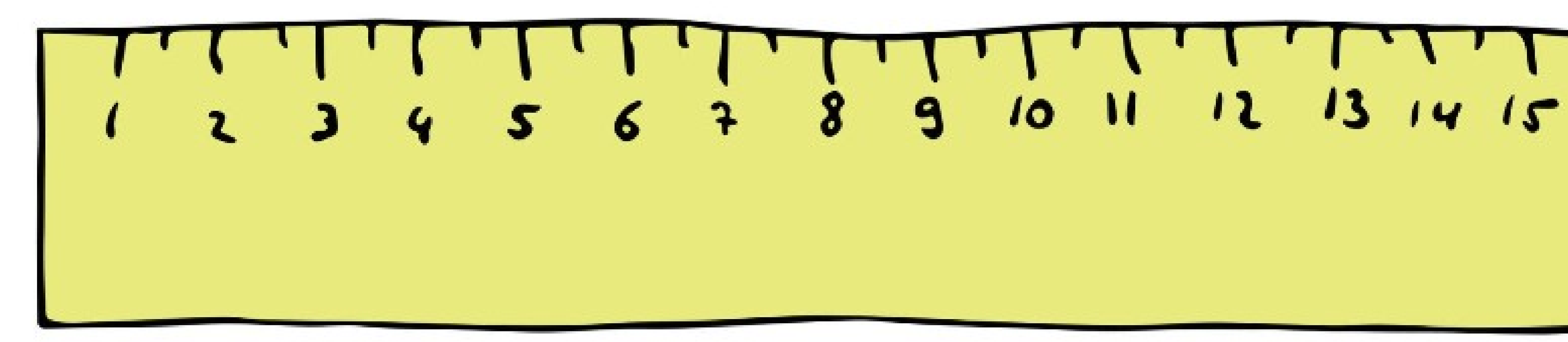
→ km?

→ light years?





ruler in a.u. (astronomical units)



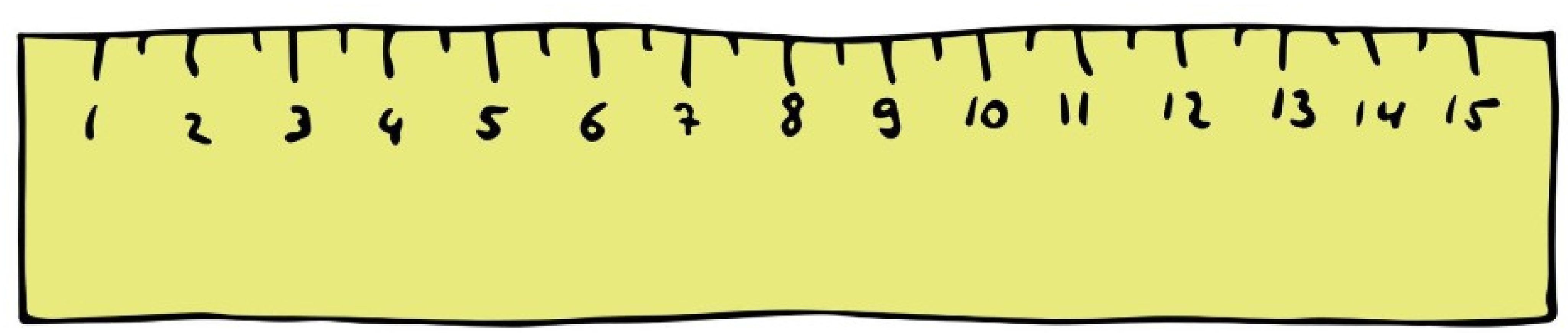
but if ruler in units of 10^4 light years

$\Downarrow$

$\Rightarrow$ then nature might look like that

then nature will look different

Accordingly, the type of physics we use depends on the size of our ruler



1m → classical mechanics

1 μm → microbiology

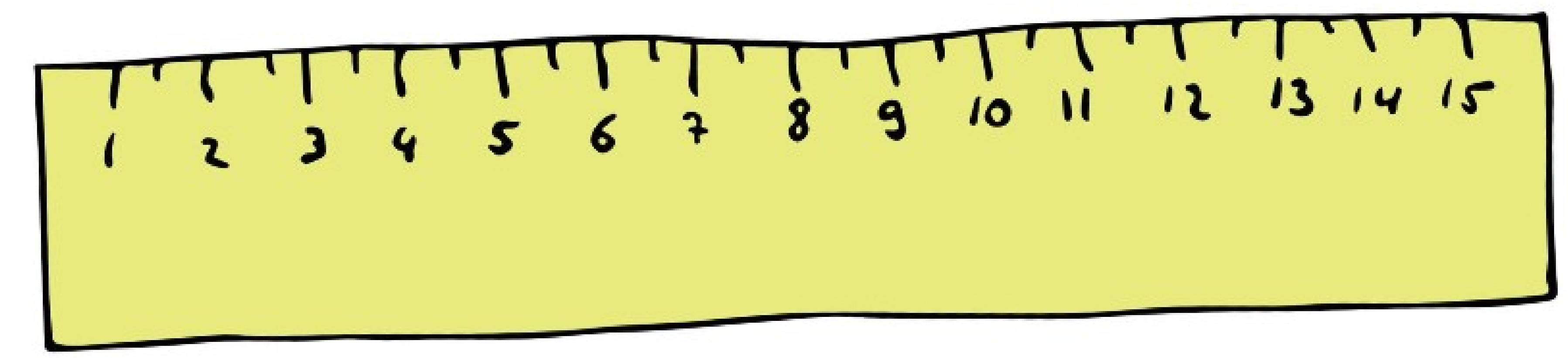
a few nm → chemistry

1 Å → atomic physics

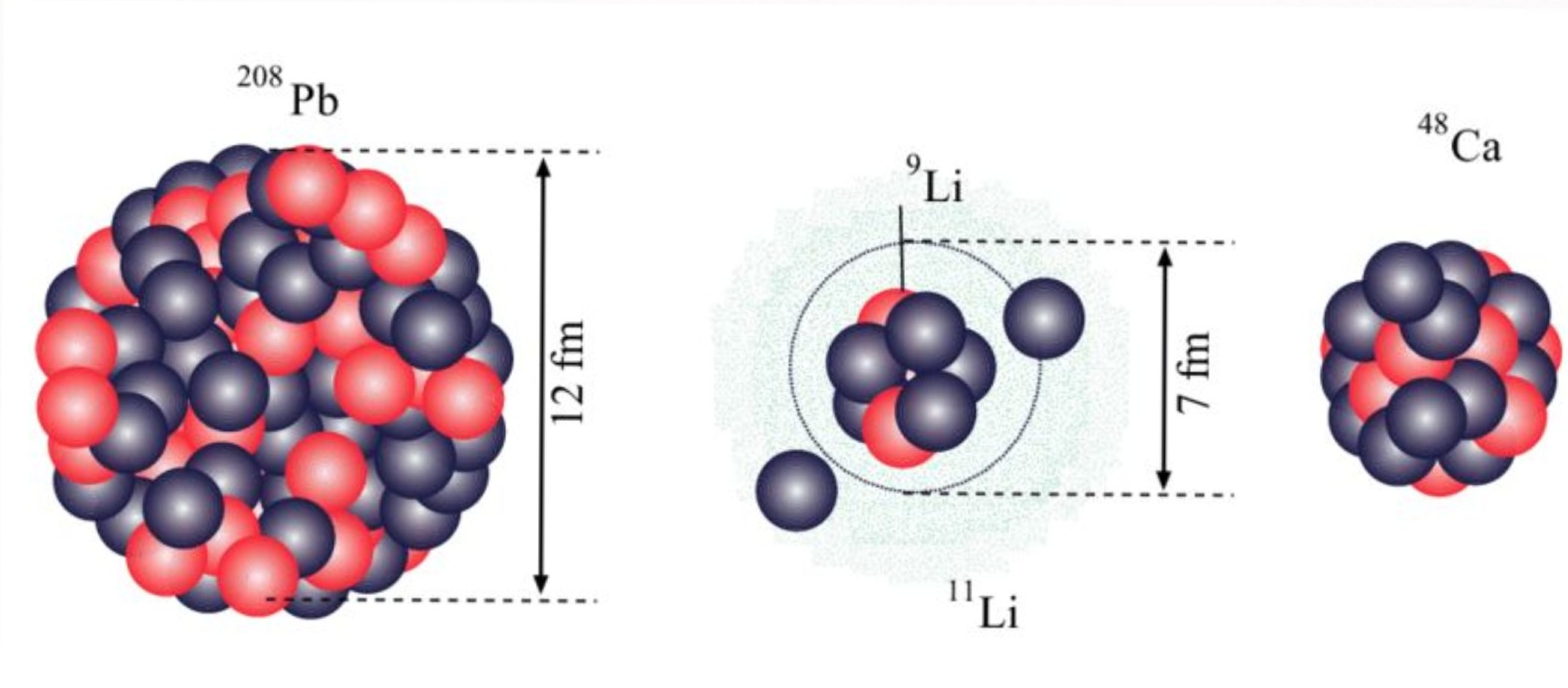
→ etc...

NUCLEAR PHYSICS $\longrightarrow$

(our course)



units $\rightarrow$ fm ($1\ \text{fm} = 10^{-15}\ \text{m}$)

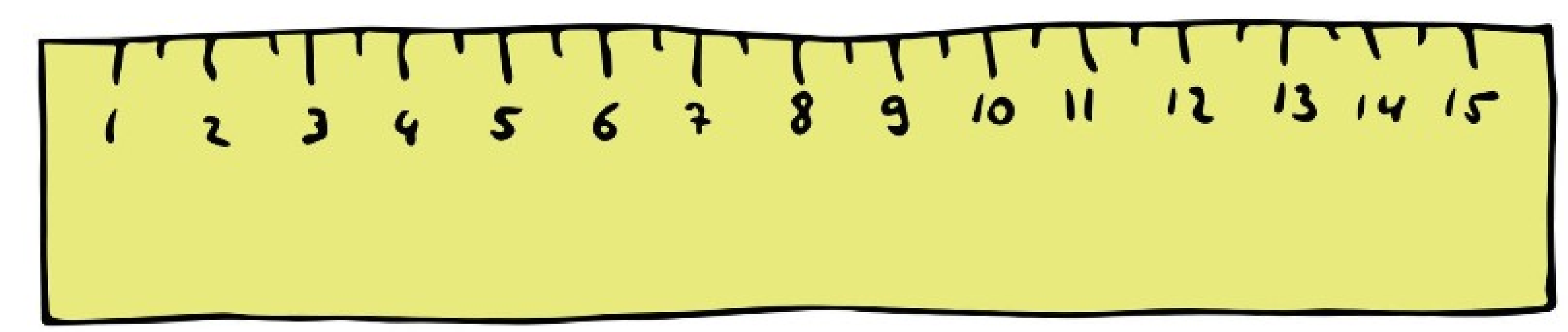


But before considering nuclear physics...

units: 1 Å $(= 10^{-10}\ \mathrm{m})$

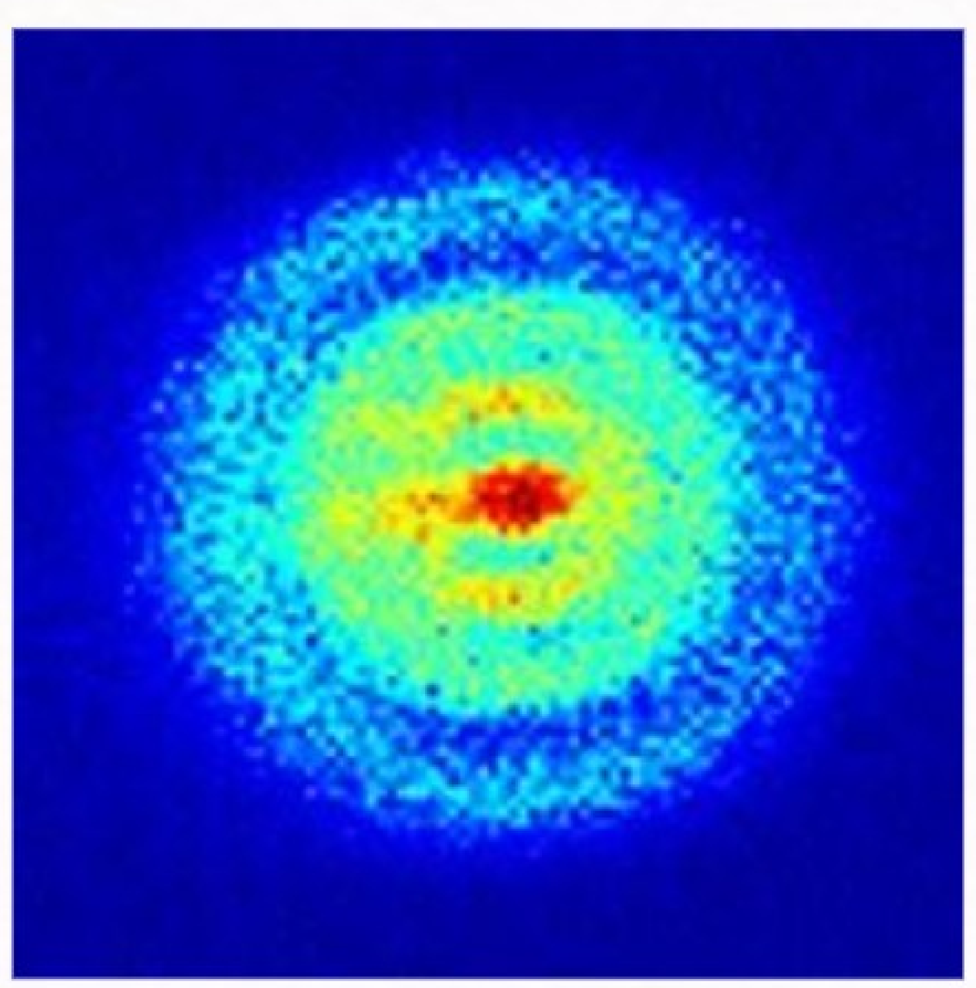
ATOMIC PHYSICS

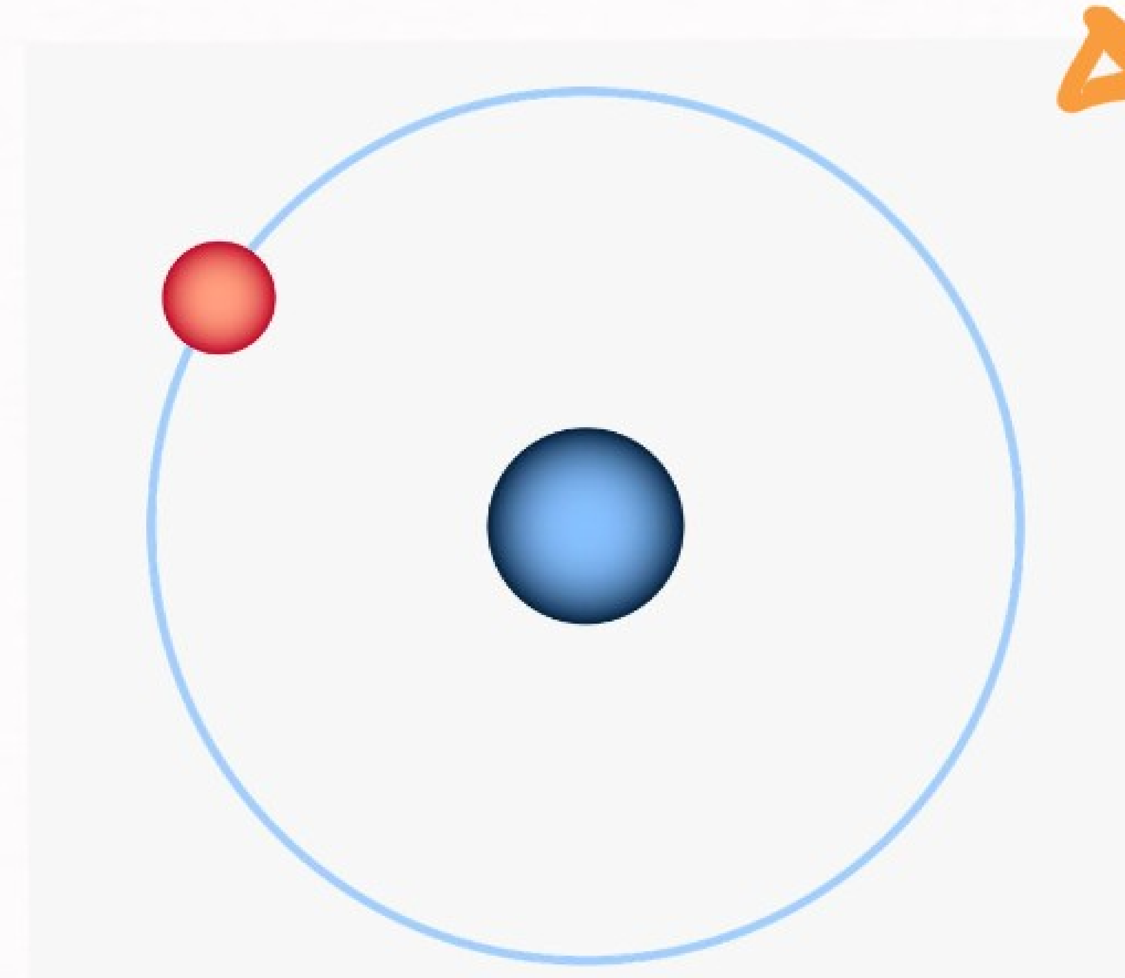
(introductory example)



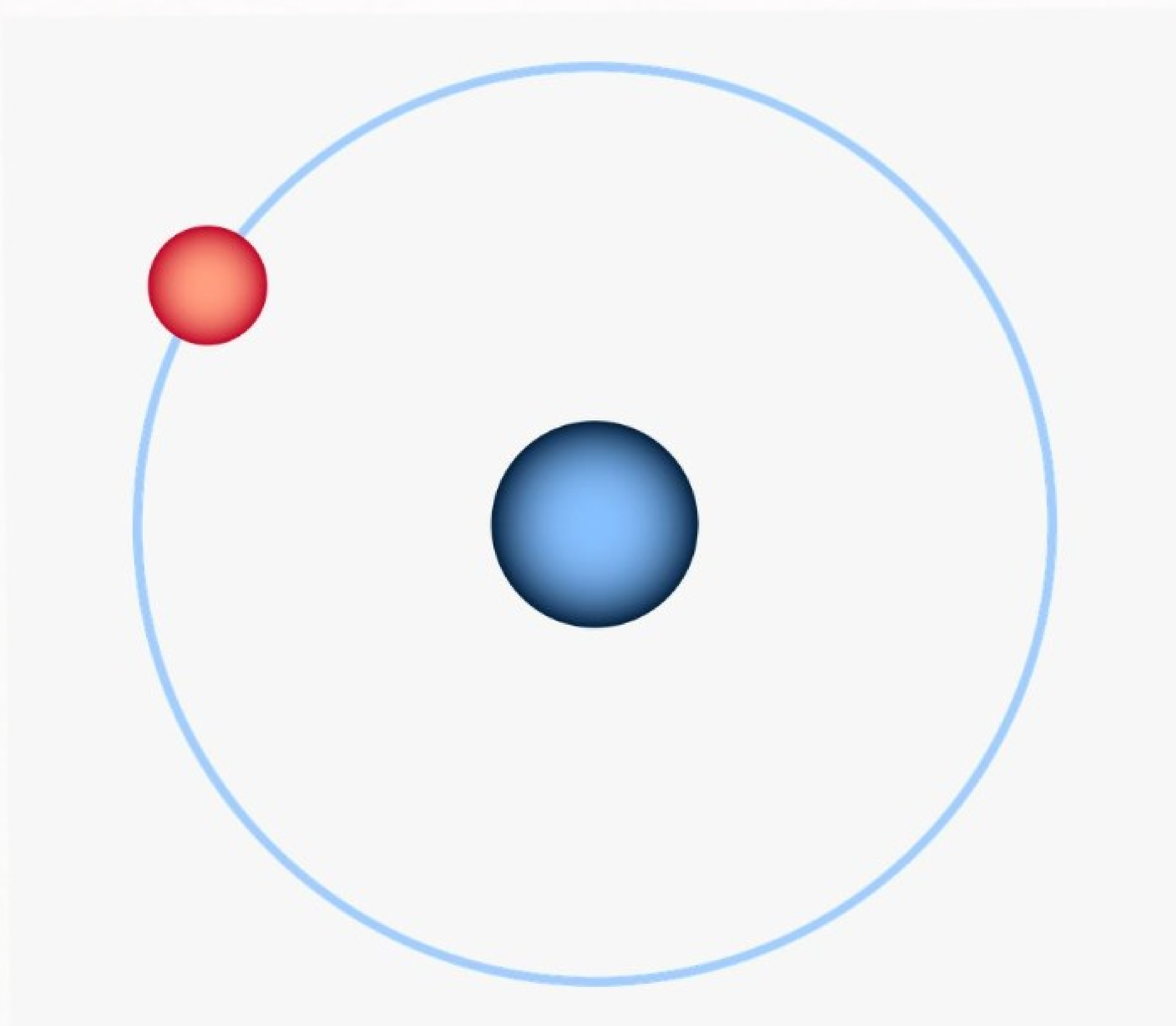
HYDROGEN ATOM

(easy: you have probably studied it before)





← (apparently a pic of a hydrogen atom)

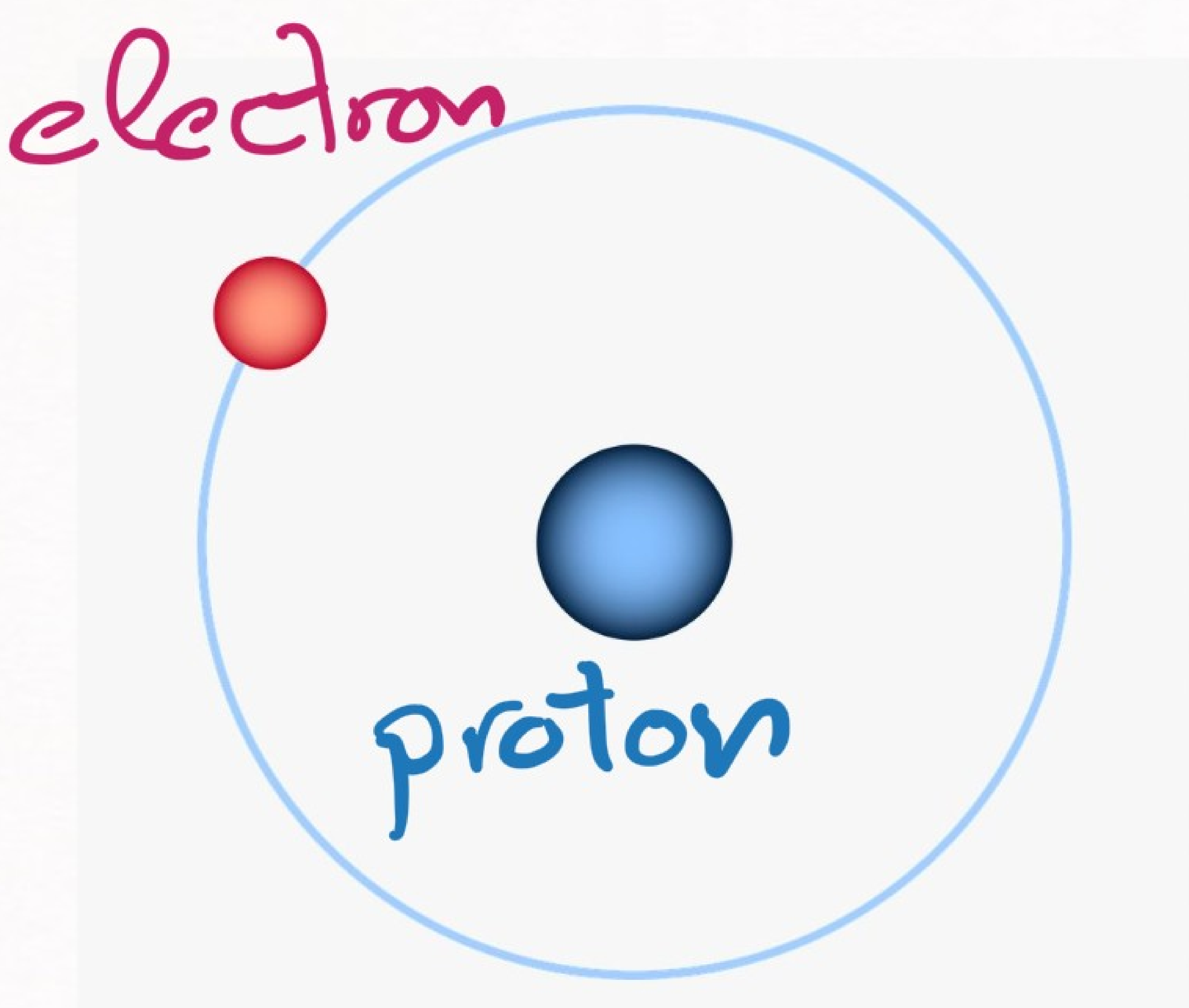


$\sim 1\,\text{Å}$

We want to understand the hydrogen atom in terms of [its natural scales]

What does that mean?

First, the hydrogen atom as you studied it in quantum mechanics 1



$$\underbrace{V(r) = -\frac{\alpha}{r}}_{\text{electron-proton potential}} \qquad \alpha \simeq \frac{1}{137}$$

(fine structure constant)

↳ Coulomb potential

2

→ We put the potential in the Schrödinger equation :

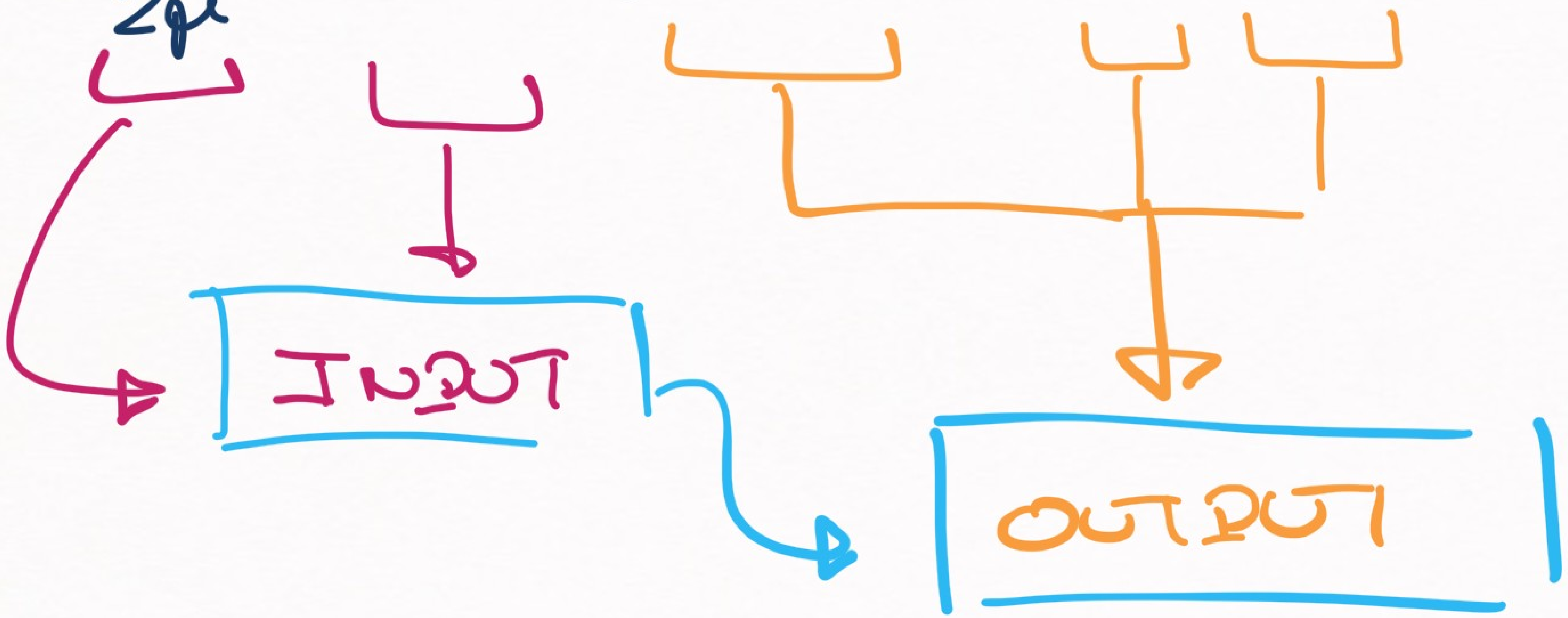
(We have to solve it)

$$\Big[-\frac{\vec{\nabla}^2}{\underbrace{2\mu}_{\text{reduced mass}}} + V(r)\Big]\underbrace{\Psi(\vec{r})}_{\text{the wave function}} = \underbrace{E_B}_{\text{energy of the bound states}}\Psi(\vec{r})$$

$$\frac{1}{\mu} = \frac{1}{m_e} + \frac{1}{m_p} \simeq \frac{1}{m_e}$$

$$\left[-\frac{\vec{\nabla}^2}{2\mu} + V(\vec{r})\right]\Psi(\vec{r}) = E_s\Psi(\vec{r})$$

3



A possible way to see this problem

(input → output)

Can you tell me what is mean square radius of the atom? (4)

$$\sqrt{\langle r^2 \rangle} = \int d^3\vec{r}\, |\Psi(\vec{r})|^2\, r^2 = \ldots$$

→ not immediately (first → solve Schrödinger, then → calculate $\sqrt{\langle r^2 \rangle}$)

Answer : eventually, yes, but only after a few calculations

5

Same comment applies to the binding energy

→ [But can we do it without a calculation?]

Q: can we do it w/ calculations?

A: yes, we can!

1

THE HYDROGEN ATOM

(FROM THE SCALE VIEWPOINT)

=D

[RETHINKING THIS PROBEM]

2

$$\underbrace{\left[-\frac{\nabla^2}{2\mu}-\frac{\alpha}{r}\right]}_{\text{INPUT}}\psi(\vec{r}) = \underbrace{E_B\,\psi(\vec{r})}_{\text{OUTPUT}}$$

We go back to the input/output way of thinking

$\boxed{\mu, \alpha}$ → in principle, two parameters ones

α → fine structure constant

μ → reduced mass

OUTPUT = f(INPUT) ← trivial observation (2)

(the output is a function of the input)

↓

FOR EXAMPLE :

$E_B = E_B(\mu, \alpha)$

binding energy : should be a function of μ & α

The binding energy will only depend on μ and α (they are the only parameters)

But we can simplify further: (3)

$$\left[-\frac{\nabla^2}{2\mu} - \frac{\alpha}{r}\right]\psi(\vec{r}) = E_B\,\psi(\vec{r})$$

$$\Big\downarrow \times(2\mu)$$

the equation will not change (*)

$$\left[-\nabla^2 - \frac{2\mu\alpha}{r}\right]\psi(\vec{r}) = 2\mu E_B\,\psi(\vec{r})$$

(*) → homogeneous differential equation

(INPUT → $2\mu\alpha$)

A few remarks :

1) $2\mu E_B = -\gamma^2$ ("the wave number") (just a definition)

$$E = \frac{k^2}{2\mu} \longrightarrow E = -\frac{\gamma^2}{2\mu}$$

(for positive center-of-mass energy) → (for negative center-of-mass energy)

A few remarks :

(5)

2) $\mu\alpha$ has units of inverse length ($[L]^{-1}$)

$$\boxed{\mu\alpha = \frac{\hbar c}{a_B}}$$

$\hbar c \longrightarrow$ conversion factor from energy/momentum to inverse length

$a_B \longrightarrow$ Bohr radius

$[E] = [M] = [p] = [L]^{-1} = [T]^{-1}$

conversion factor : $\hbar c$

$\boxed{\hbar c}$ → how it works? (conversion factor) (6)

$\hbar c = 1973.3\ \underline{eV} \cdot \underline{\mathring{A}}$ (eV → electron volt, $\mathring{A}$ → Ångstrom or 10^{-10}m)

(energy × length)

or

$$\boxed{\hbar c = 197.33\ MeV \cdot fm}$$

(MeV → megaelectronvolt, fm → fermi or 10^{-15} m)

We will mostly use this units for $\hbar c$

Example $\rightarrow$ $\mu\alpha$

$$\frac{1}{\mu} = \frac{1}{m_e} + \frac{1}{m_p} \simeq \frac{1}{m_e}$$

(reduced mass)

7

$$\alpha \simeq \frac{1}{137} \text{ (pure number)}$$

$$\underline{\underline{a_B}} = \frac{\hbar c}{\mu\alpha} \simeq \frac{197.33 \text{ MeV}\cdot\text{fm}}{\underbrace{0.5\text{ MeV}}_{\simeq m_e \text{ (electron mass)}}\cdot\left(\frac{1}{137}\right)}$$

$$\simeq 5.3\cdot 10^{4} \text{ fm} \quad \text{or} \quad \boxed{0.53\text{ Å}}$$

Putting all the pieces together:

⑧

1) $2\mu E_B = -\gamma^2$ 2) $\mu\alpha = \frac{1}{a_B}$ (the usually not indicated)

$$\Rightarrow \left[-\nabla^2 - \frac{2}{a_B r}\right]\psi(\vec{r}) = -\gamma^2\psi(\vec{r})$$

this is the only input parameter

($a_B \simeq 0.53$ Å)

This means that we can write everything as powers of $\underline{\underline{a_B}}$ } only input parameter

a

Example: $\gamma \to$ units of momentum ($[L^{-1}]$)

$$\Rightarrow \boxed{\gamma = \frac{c}{a_B}} \quad \text{w/ } c \text{ a number}$$

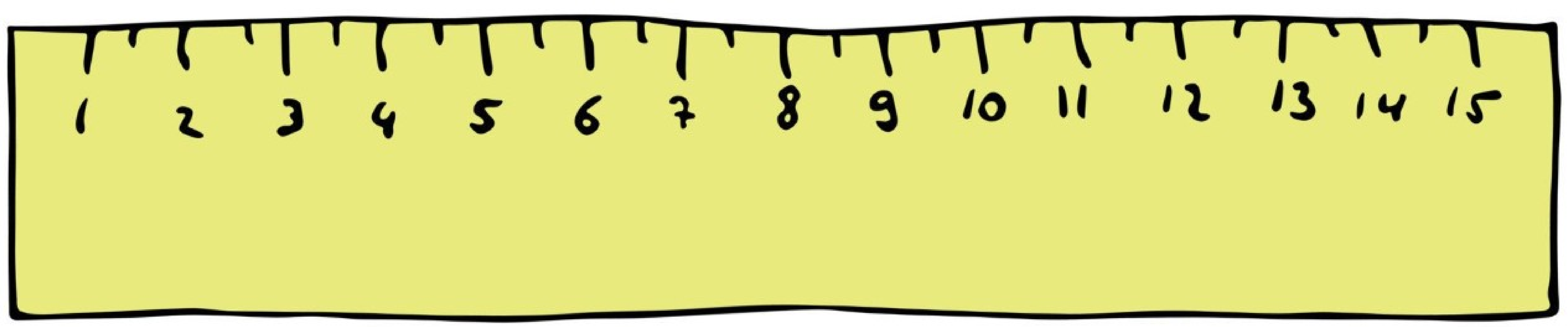
$\to \sqrt{\langle r^2 \rangle} \to$ units of length

$$\Rightarrow \sqrt{\langle r^2 \rangle} = d\, a_B \quad \text{w/ } d \text{ a number}$$

Then, observables of the hydrogen atom can be expressed as a $\underbrace{\text{number} \times (\overset{\text{dimensions}}{a_B})}_{\text{very easy}}$

10

ruler for the hydrogen atom



$1\ \underline{\underline{a_B}} = \left[1 \text{ a.u. (atomic unit)} \right]$

→ our ruler for the hydrogen atom has units of $\underline{\underline{a_B}}$

But which is the size of the number ?

11

$$\gamma = \frac{c}{a_B}, \quad \langle\sqrt{r^2}\rangle = d\, a_B$$

how large are c & d ?

$\exists$ HYPOTHESIS $\rightarrow$ $\boxed{\text{NATURALNESS}}$

$\forall$ these numbers are of $\mathcal{O}(1)$

$c \sim \mathcal{O}(1)$, $d \sim \mathcal{O}(1)$

FOR EXAMPLE :

$$\gamma \simeq \frac{1}{a_B} \Rightarrow E_B \simeq -\frac{1}{2\mu}\left(\frac{1}{a_B}\right)^2 \simeq -\frac{m_e}{2}\left(\frac{1}{137}\right)^2$$

$$\boxed{\simeq -13.6\ \text{eV}}$$

our dimensional estimation (naturalness)

AND, EXPERIMENTALLY WE FIND :

$$\boxed{E_B = -13.6\ \text{eV}}$$

no calculation involved in this

Another possibility is to solve Schrödinger: 13

$$\left[-\vec{\nabla}^2 - \frac{2}{a_B r}\right]\psi(\vec{r}) = -\gamma^2 \psi(\vec{r})$$

SOLUTION: $\gamma = \frac{1}{a_B}$

$$\boxed{\psi(\vec{r}) = \frac{1}{\sqrt{4\pi}} \frac{2}{a_B^{3/2}} e^{-r/a_B}}$$

HYDROGEN ATOM :

→ Do dimensional estimations to guesstimate observables] already done it for r (now $\sqrt{\langle r^2 \rangle}$)

→ Or we can do it the traditional way, that is, solving Schrödinger and then calculating observables

With this we can now compare what happens w/ $\sqrt{\langle r^2 \rangle}$

NATURALNESS : $\boxed{\sqrt{\langle r^2 \rangle} \approx a_B}$

Pretty close

FULL CALCULATION:

$$\langle r^2 \rangle = \int d^3\vec{r}\, |\psi(\vec{r})|^2 r^2 = \frac{4}{a_B^3} \int_0^\infty dr\, r^4 e^{-2r/a_B}$$

$$= \frac{a_B^2}{8} \int_0^\infty \underbrace{dx\, x^4 e^{-x}}_{4!} = 3 a_B^2$$

$$\boxed{\sqrt{\langle r^2 \rangle} = \sqrt{3}\, a_B}$$

To summarize :

$$\left[\; r_B = \frac{c}{a_B} \;,\; \sqrt{\langle r^2 \rangle} = d\, a_B \;\right] \quad \text{NATURALNESS}$$

$$c = 1 \qquad\qquad d = \sqrt{3} \quad \Big] \quad \text{FULL CALCULATION}$$

$$\underbrace{= \mathcal{O}(1) \qquad\qquad = \mathcal{O}(1)}$$

Expectation & Reality similar

(naturalness) (full calculation/experiment)

← When this happens we have

$\boxed{\text{A NATURAL PROBLEM}}$

1) There is a characteristic scale

EXAMPLE → in the hydrogen atom

momentum scale

(electron mass × fine structure constant)

$$Q_B \simeq m_e \alpha = 3.7\,\text{keV} \rightarrow a_B = \frac{\hbar c}{Q_B} = 0.53\,\mathring{A}$$

2) Everything else is $O(1)$ in terms of the characteristic scale

$$\langle \hat{O} \rangle = \langle \psi | \hat{O} | \psi \rangle = [L]^n$$

NATURALNESS SAYS THAT

$$\boxed{\langle \hat{O} \rangle \simeq O(1)\, a_B^n}$$

Additional example → speed of an electron ⊛ inside a hydrogen atom

Hydrogen atom → [wave number (∼ binding energy) ; mean square radius]

natural

=

⊛ → $\langle v \rangle = \left\langle \frac{p}{m_e} \right\rangle = \langle \psi | \frac{1}{m_e} p | \psi \rangle$

about right

$\langle v \rangle \sim \frac{Q_B}{m_e} \sim \frac{3.7\,\text{keV}}{511\,\text{keV}} \sim 7 \cdot 10^{-3}\, c \sim 2200\ \text{km/s}$

NATURAL PROBLEM :

$$\gamma \simeq \frac{1}{a_B} \quad \longleftrightarrow \quad \gamma = \frac{1}{a_B}$$

$$\sqrt{\langle r^2 \rangle} \simeq a_B \quad \longleftrightarrow \quad \sqrt{\langle r^2 \rangle} = \sqrt{3}\, a_B$$

$$\langle v \rangle \simeq \frac{\alpha_B}{m_e} \simeq \frac{1}{m_e a_B} \longleftrightarrow \sqrt{\langle \sqrt{v^2} \rangle} = \frac{1}{m_e a_B}$$ (please check)

MORAL : NATURAL PROBLEMS ARE EASY

Let's see a more complex example ...

$\boxed{\text{THE BOILING POINT OF CESIUM}}$

Using only information about the scales!

$\Rightarrow$

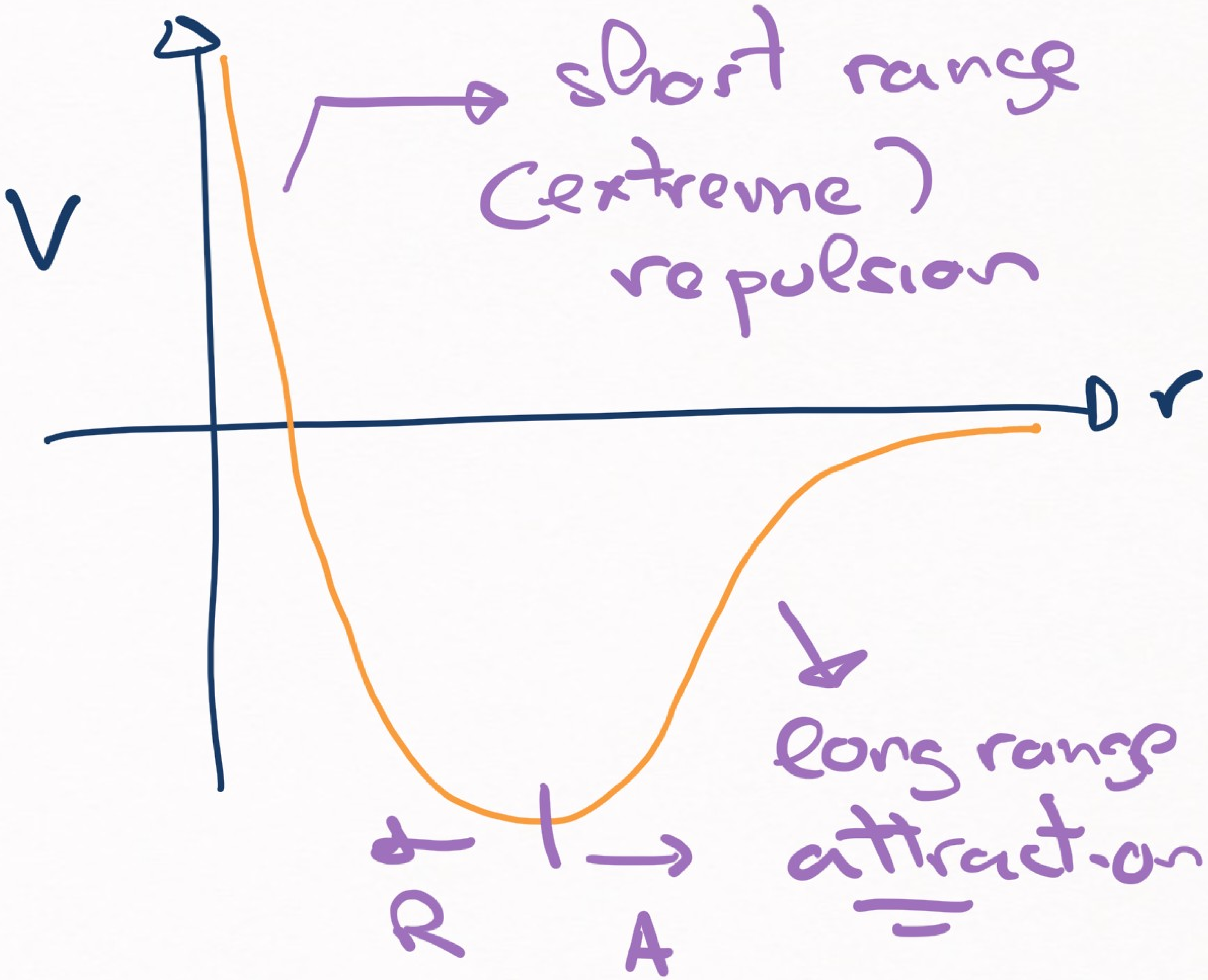
Useful to have more examples of naturalness

[SOME INFORMATION ABOUT CESIUM]

Cs $\rightarrow$ atomic number 133 $\rightarrow$ $m(Cs) \simeq 130$ GeV

(most common isotope in experiments)

Cs - Cs potential



Qualitative features:

it becomes difficult to squeeze them too much

1) Short-range repulsion

→ exact form is not well-known

→ but details are not that important

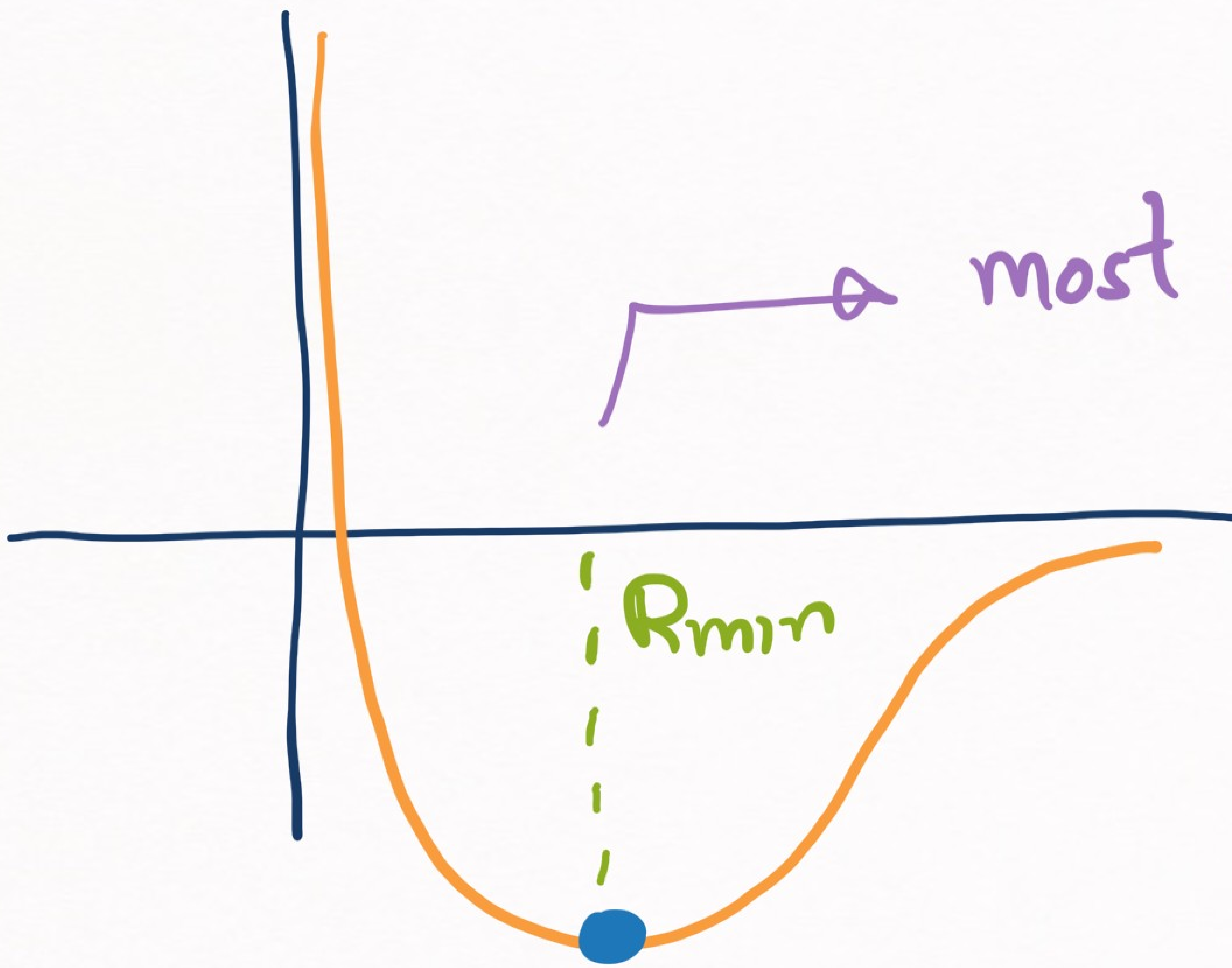
2) Long-range attraction → van der Waals force

$$V_{vdW}(r) = -\frac{C_6}{r^6} = \boxed{-\frac{1}{2m}\frac{R_{vdW}^4}{r^6}}$$

→ different way of writing the potential

we know this constant

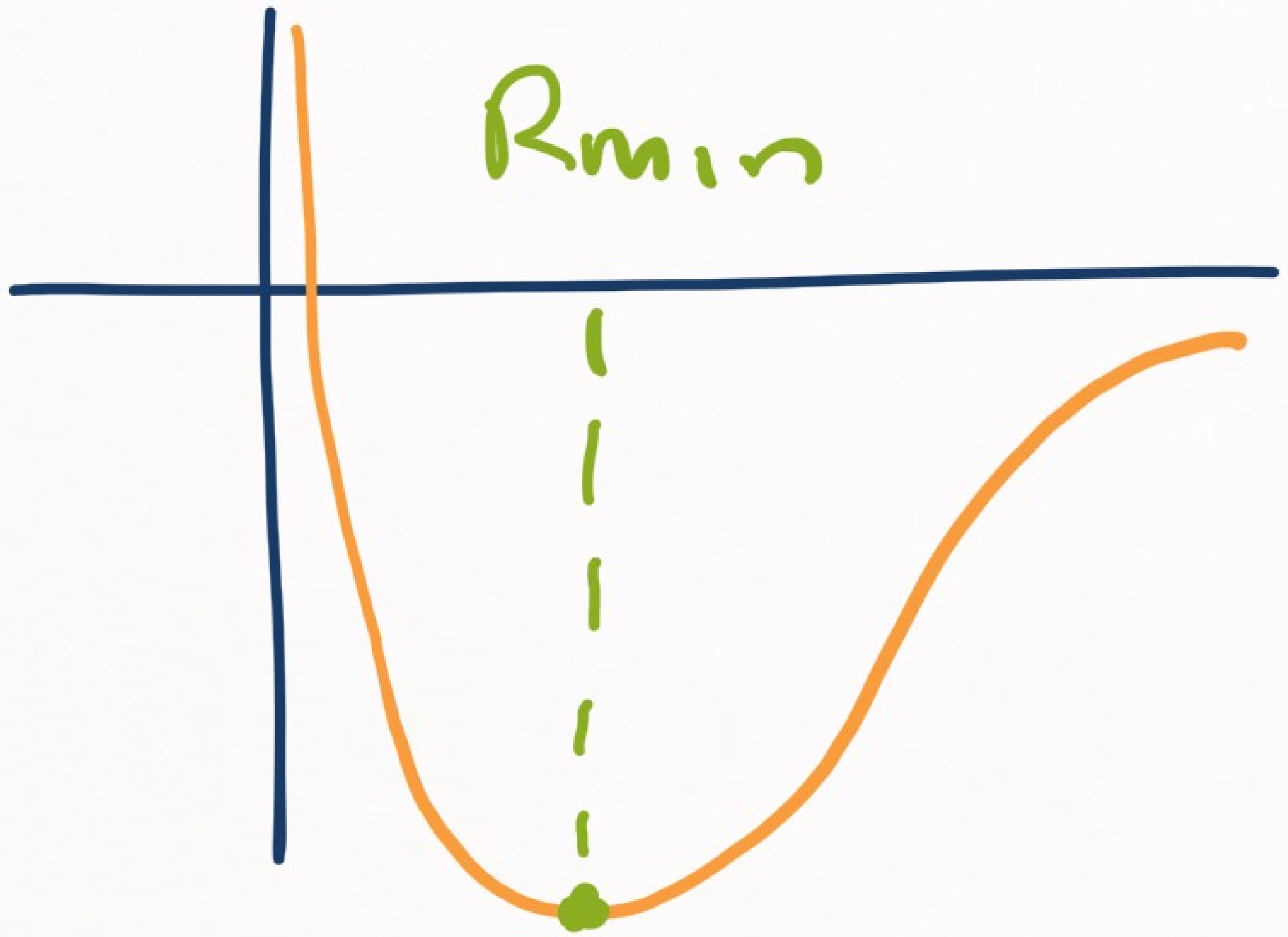
How to do the calculation?



most atoms will be packed around the minimum

minimum of the potential

$R_{min} \simeq$ average separation of Cs atoms



(virial theorem)

$$\rightarrow \underbrace{|T_{min}| \simeq |V_{min}|}_{\text{similar kinetic \& potential energies}}$$

(only interested in order of magnitude approx.)

We want to know the kinetic energy

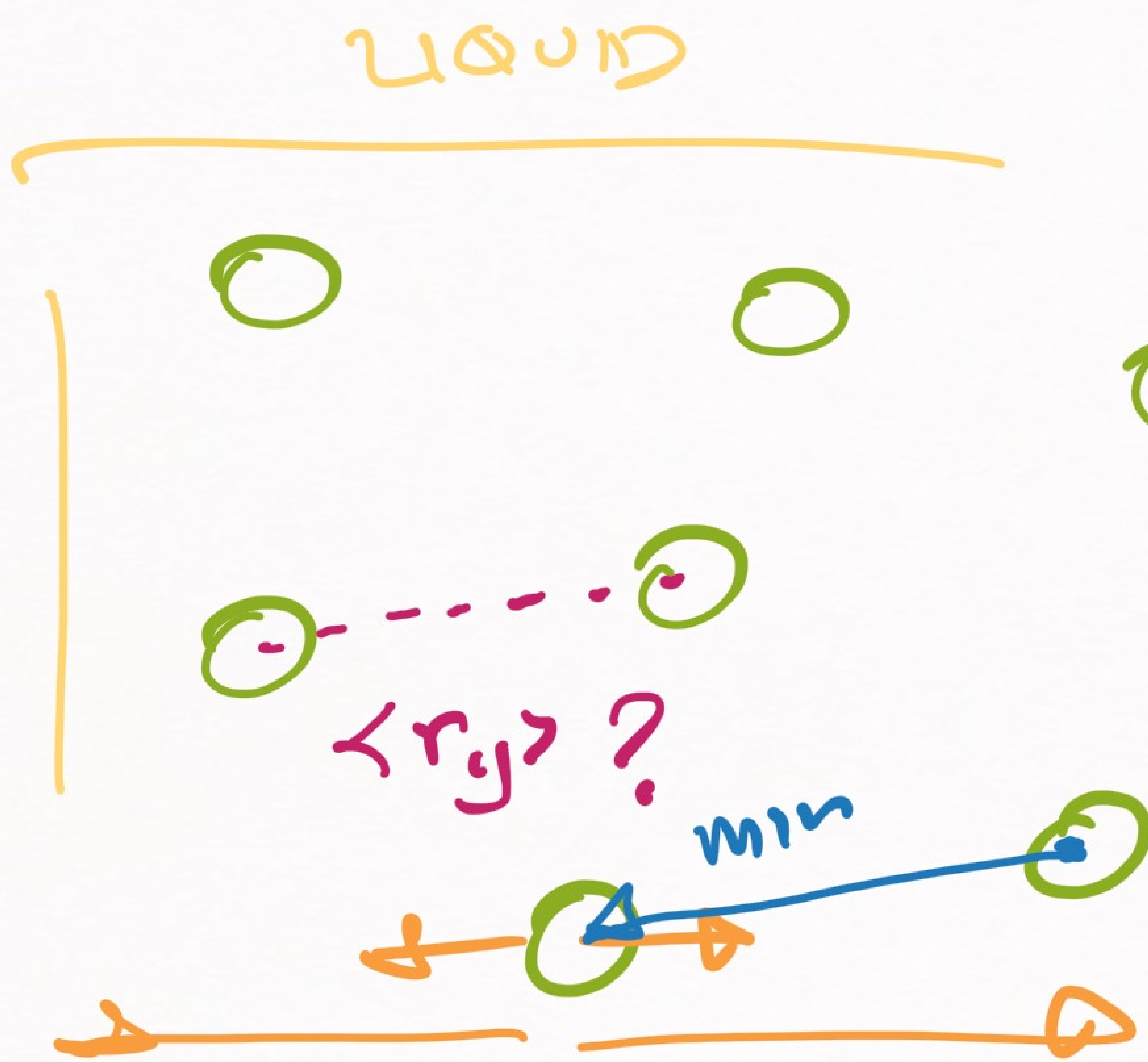
$$T_{min} = \frac{1}{2m}\left(\frac{1}{R_{Cs}}\right)^2$$

$\rightarrow$ R_{Cs} the characteristic scale for Cs clusters

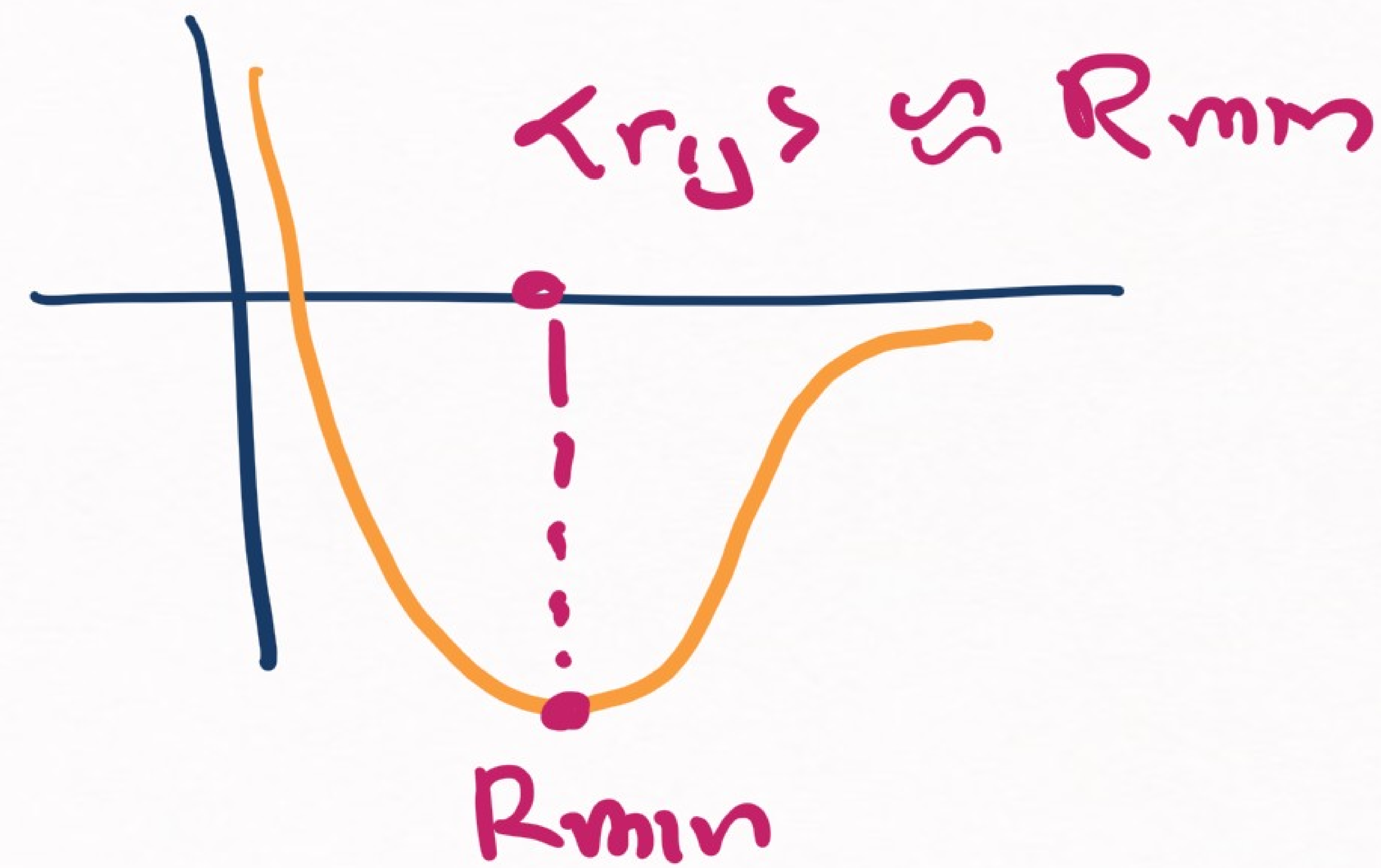
$$T_{min} = \frac{1}{2m}\left(\frac{1}{R_{cs}}\right)^2 = \underbrace{\frac{1}{2} m \langle v_{cs} \rangle^2}_{\text{the average speed of Cesium atoms}}$$

Then when $\langle v_{thermal} \rangle$ is larger than $\langle v_{cs} \rangle$, it will be impossible for the atoms to cluster around the minimum

(if we don't give heat to the system)



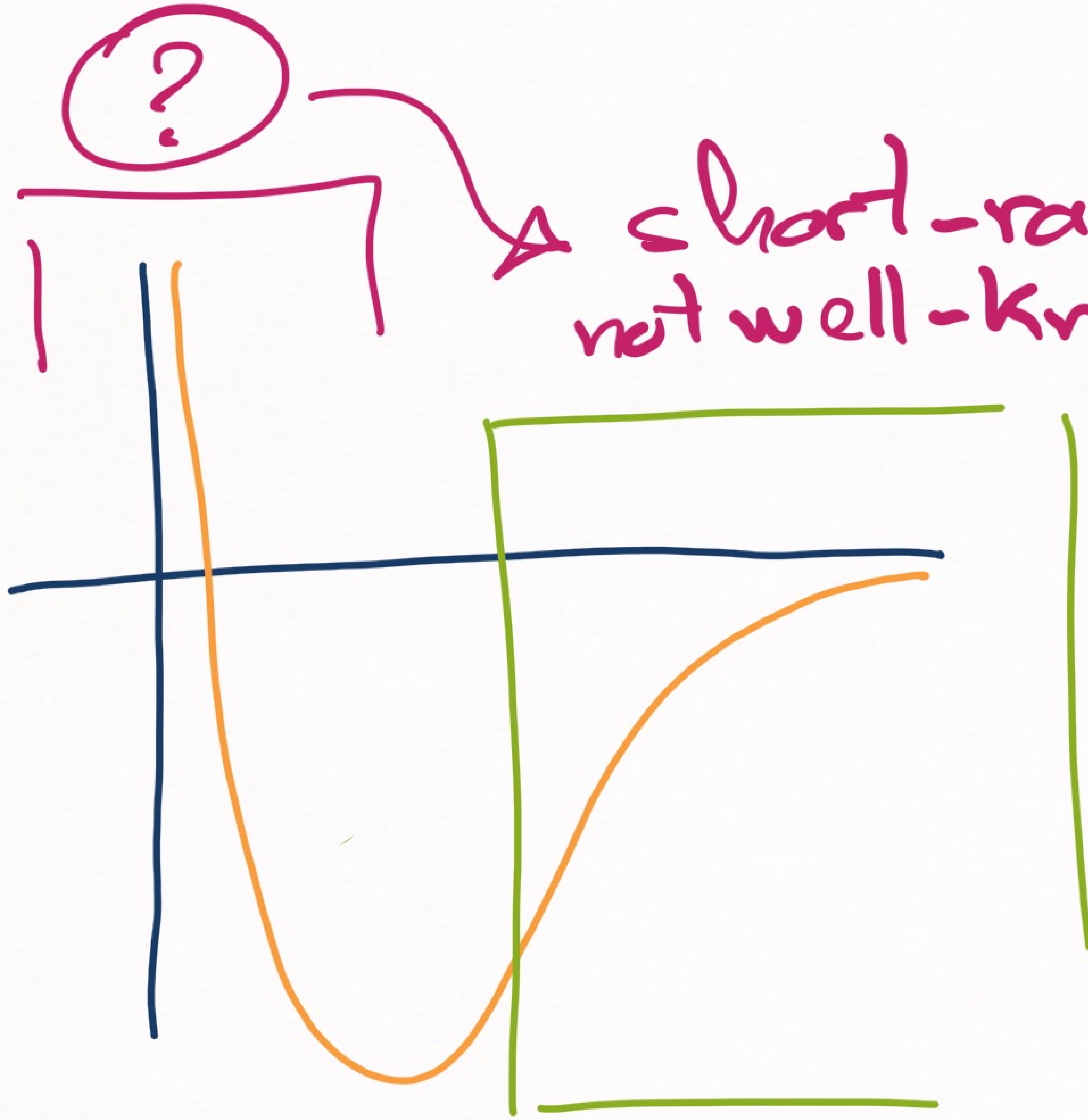
AVERAGE SEPARATION



$T = 0°K \rightarrow \langle E_{kin} \rangle \simeq \langle V(R_{min}) \rangle$

$T \uparrow\uparrow \rightarrow \langle E_{th} \rangle > \langle V(R_{min}) \rangle \rightarrow \boxed{\text{GAS}}$

HOW TO CALCULATE V_{mn} ? (STEP 1 OF OUR CALCULATION)



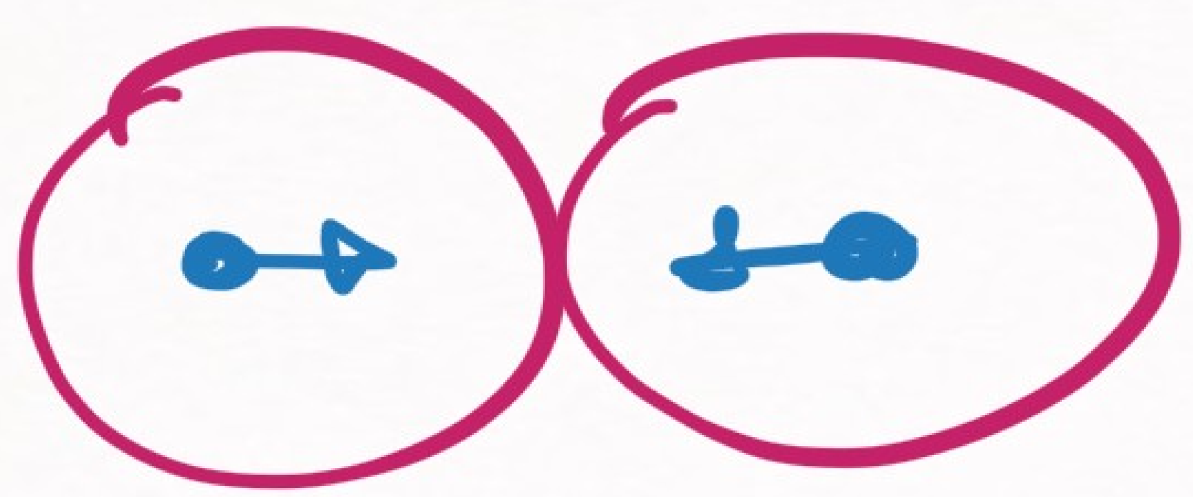
short-range
not well-known

van der Waals
forces

We only know the tail
of the potential!

$\Rightarrow$

But we also know that the short-range repulsion is related to the finite size



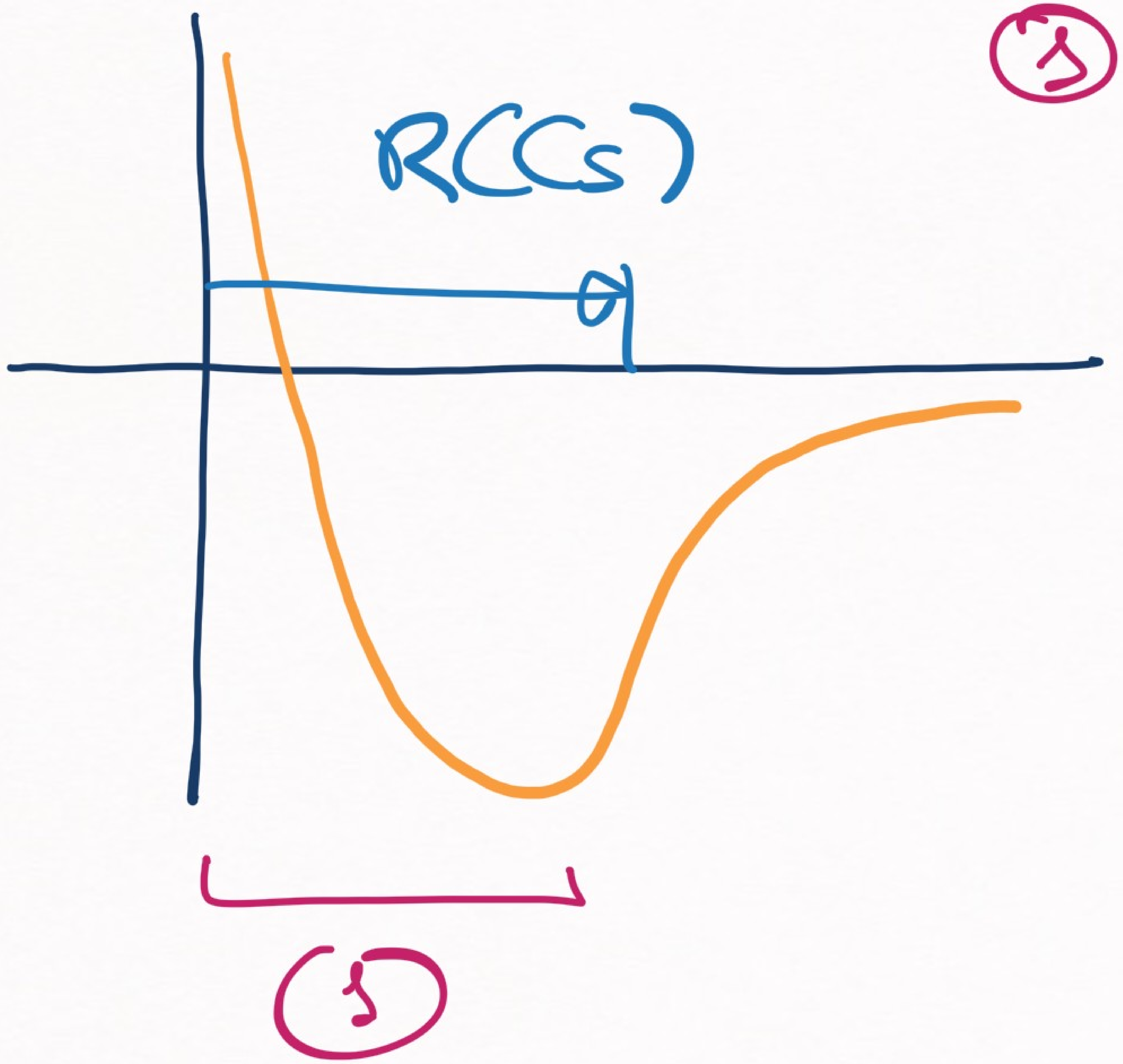
Cs Cs

it's difficult to squeeze them for $R < \boxed{R(Cs)}$

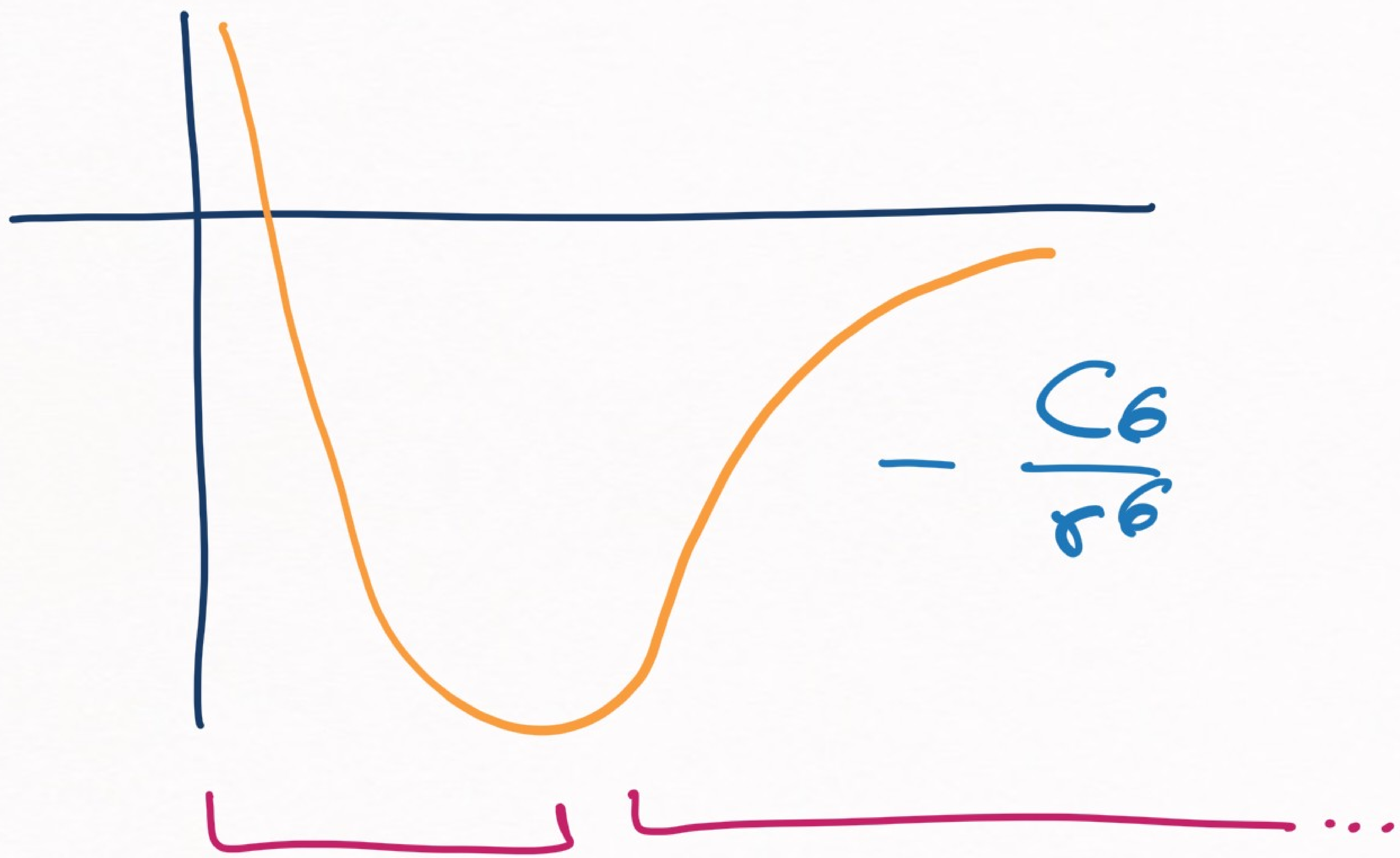
Radius of Cesium atom

$(\simeq 7\, a_B)$

And this means that:



1 → the repulsive region
there is a consequence
of the Cesium
atom finite size



(3)
Repulsion
(unknown)

(2)
Attraction →

$$\boxed{V(r) = -\frac{C_6}{r^6} = -\frac{1}{2\mu}\frac{R_{vdW}^4}{r^6}}$$

C_6 / RvdW is known:

$$\boxed{R_{vdW} = 203\ a_B}$$

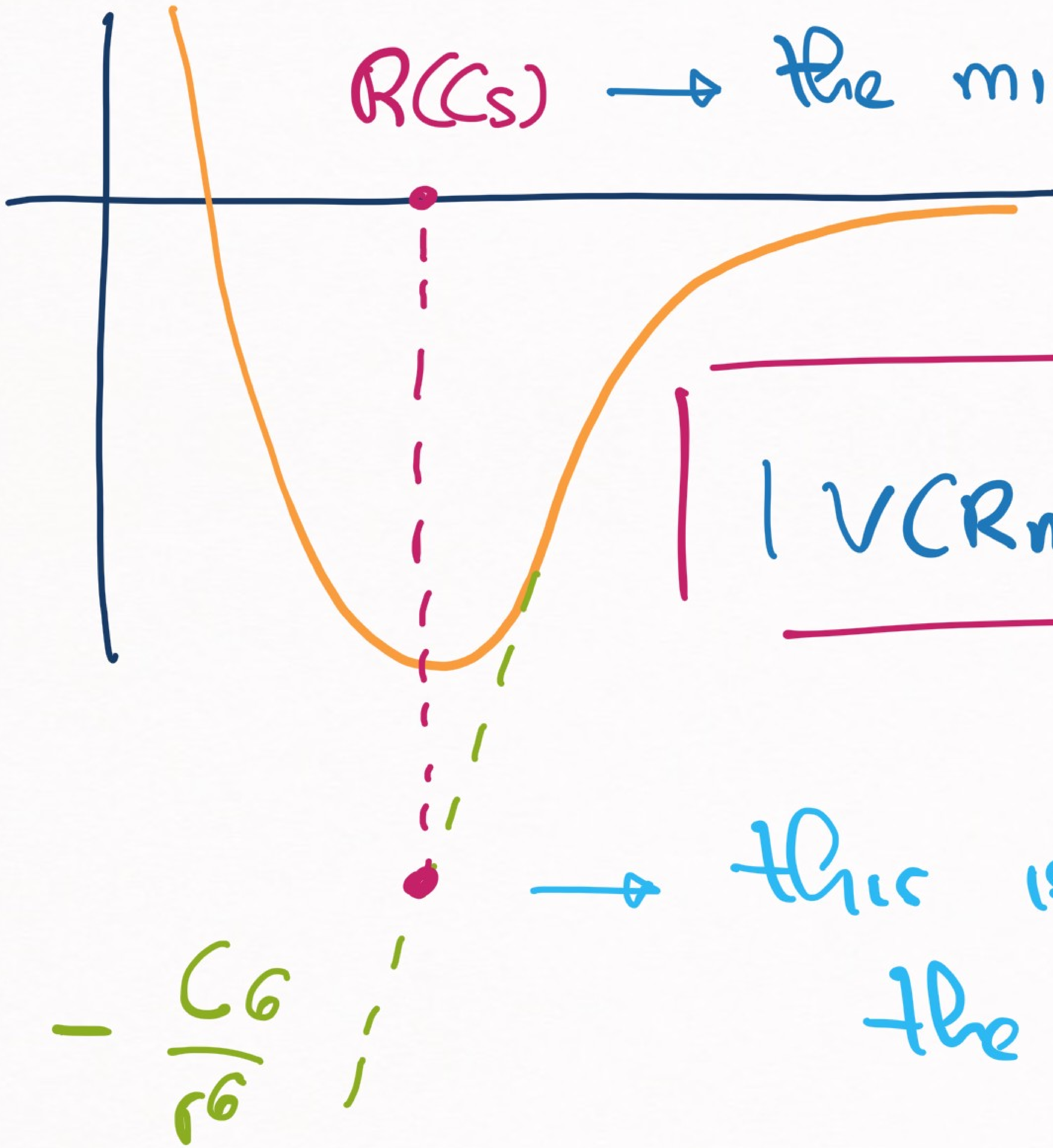
Two scales for the moment:

$R(Cs) \simeq 7\,a_B$

$R_{vdW} \simeq 203\,a_B$

But we still need to estimate V_{mn}

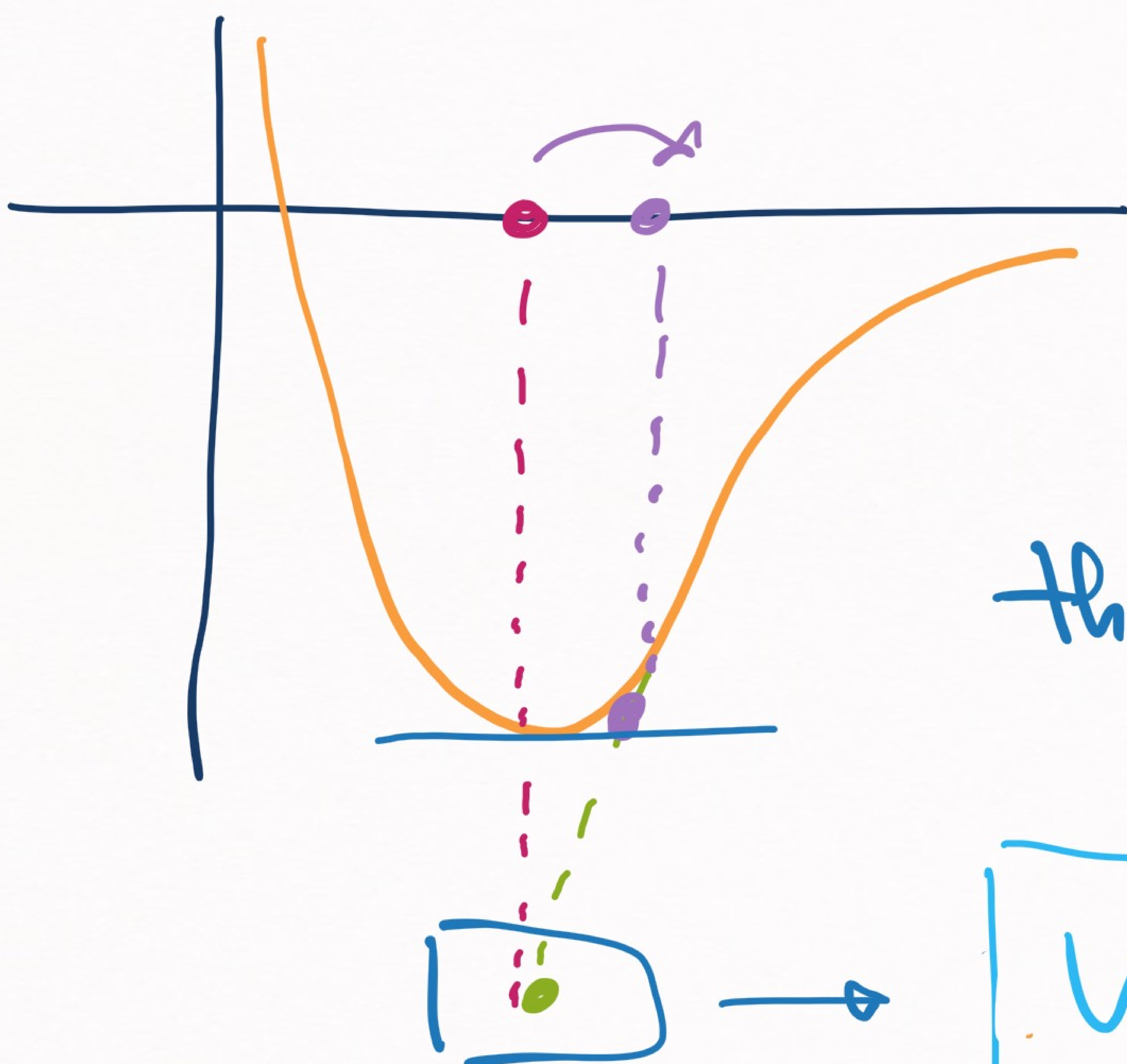
$\boxed{\text{How?}}$



→ the minimum probably coincides w/ the size of the atoms

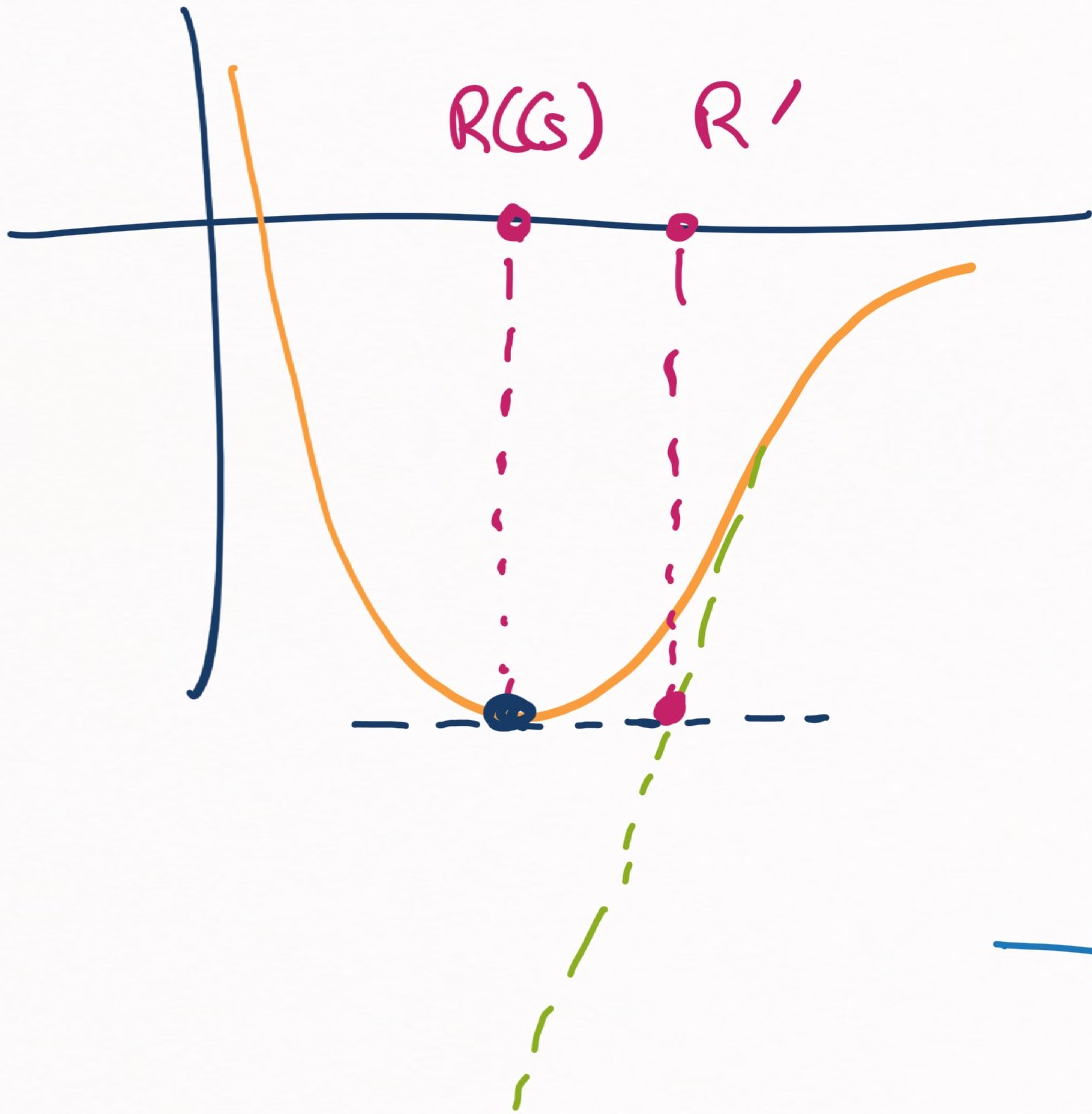
$$\boxed{|V(R_{min})| < |V_{vdW}(R_{min})|}$$

→ this is $V_{vdW}(r)$, the part of the potential we know



this approximation will be too small

$$\boxed{V_{vdW}(R(C_s)) < V_{min}}$$



But if we use R' a bit larger than R(Cs)

⇓

We might guess V_{min} right from

$$\boxed{V_{vdW}(R') \simeq V_{min}}$$

$R(Cs) \simeq 7\, a_B \longrightarrow R' \simeq 10\, a_B$ looks like a good approx.

$$\boxed{V_{min} = -\frac{1}{2\mu}\frac{R_{vdW}^4}{(R')^6} \quad \checkmark}$$

And now we can try to obtain T_{min}:

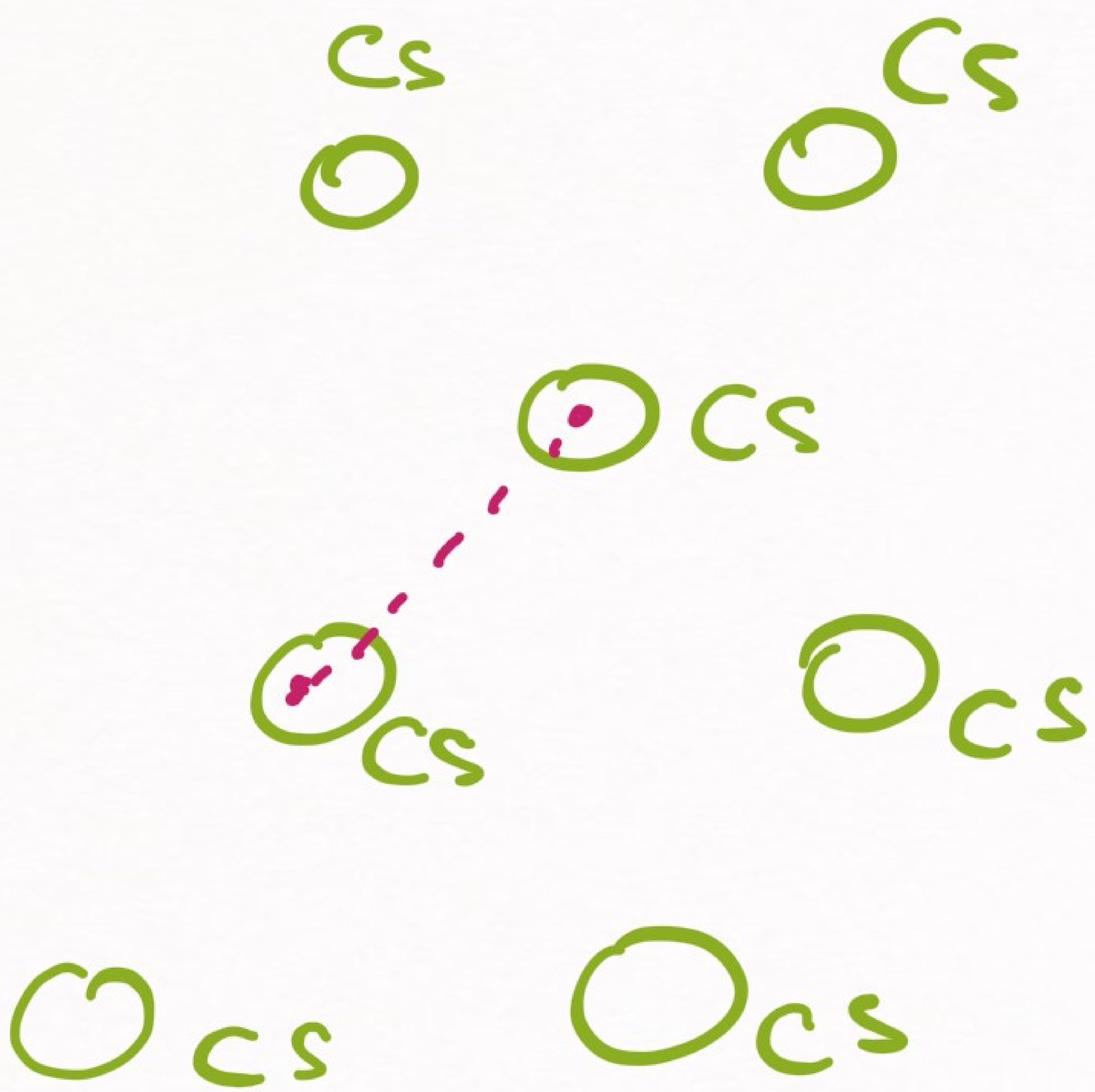
$$\underbrace{\frac{1}{2\mu} Q_{cs}^2}_{T_{min}} \simeq \underbrace{\frac{1}{2\mu} \frac{R_{vdW}^4}{(R')^6}}_{V_{min}} \qquad R' \simeq 10\, a_B$$

$(Q_B \simeq 3.7\,\text{keV})$

$$\Rightarrow \boxed{Q_{cs} \sim 20\, Q_B \sim \frac{20}{a_B}}$$

Then we check the expected speed of Cs atoms

(at zero temperature)



$\langle r \rangle \sim R'$ (actually R_{min})

$$\langle T \rangle_{kin} \sim \frac{1}{2M} Q_{cs}^2$$

$$\langle v \rangle \sim \frac{Q_{cs}}{m_{cs}} \sim \underline{\underline{350\,m/s}}$$

($Q_{cs} \simeq 70$ keV)

When $\underbrace{\langle v_{thermal} \rangle}_{\text{how to calculate this?}} \geq 350\ m/s$ $\Rightarrow$ the Cs atoms will disperse (boiling point)

$$\boxed{E_{thermal} = k_B T}$$

↳ k_B : Boltzmann constant

T : temperature (in kelvin)

$$k_B \sim \frac{1\,eV}{11000\,°K} \longrightarrow$$ Room temperature :

$$T \sim 300\,°K$$

$$\boxed{\Rightarrow E_T \sim 50\,meV}$$

Center of the sun:

$$T \sim 15 \cdot 10^{6}\,°K$$

$$E_T \sim \frac{1\,eV}{11 \cdot 10^{3}} 1.5 \cdot 10^{7} \sim 1.4 \cdot 10^{3}\,eV$$

$$(1.1 \cdot 10^{4})$$

$$\sim \underline{\underline{1.4\,keV}}$$

$$\frac{1}{2} m \langle v_{thermal} \rangle^2 = k_B T$$

$$\rightarrow T = \frac{1}{2} \frac{m \langle v \rangle^2}{k_B}$$

If $\langle v \rangle \simeq 350\ m/s \Rightarrow \underbrace{T(Cs) \simeq 1000\,^\circ K}$

$$\boxed{\begin{array}{c}\text{Reality:} \\ T(Cs) \simeq 944\,^\circ K\end{array}}$$

this would be our estimation of the Cs boiling point

In short...

$\boxed{\text{Cs boiling point}}$

Dimensional argument:

$T(Cs) \simeq 1000°K$

Experiment:

$T(Cs) \simeq 944°C$

$\boxed{\text{IT WORKS REALLY WELL!}}$

CONCEPT → $\boxed{\text{NATURALNESS}}$

→ very easy approximations for what is the value of observables in a few systems

→ hydrogen atom (easiest example)

→ boiling point of Cs (more elaborate example)

MORAL:

NATURAL PROBLEMS ADMIT REALLY DUMB APPROXIMATIONS

... AND THEY WILL WORK!

(PROBLEM → WHAT HAPPENS IF THERE ARE MULTIPLE SCALES?)

NEXT

→ NATURAL PROBLEMS ARE EASY

(usually only 1 relevant scale)

→ HOWEVER, MOST PHYSICAL PROBLEMS INVOLVE SEVERAL SCALES

(this will trigger corrections)

$$\boxed{\text{HOW DO WE DEAL W/ CORRECTIONS COMING FROM OTHER SCALES ?}}$$

Let's see a well-known example:

[THE HYDROGEN ATOM]

$\rightarrow$

HYDROGEN ATOM, SCALES:

1) most important scale is $\boxed{a_B \left(\simeq \frac{1}{m_e \alpha}\right)}$

2) but $\exists$ other scales electron & proton

proton is not point-like (composed of quarks)

$\hookrightarrow$ it has a size

electron is fundamental, but still will have some structure

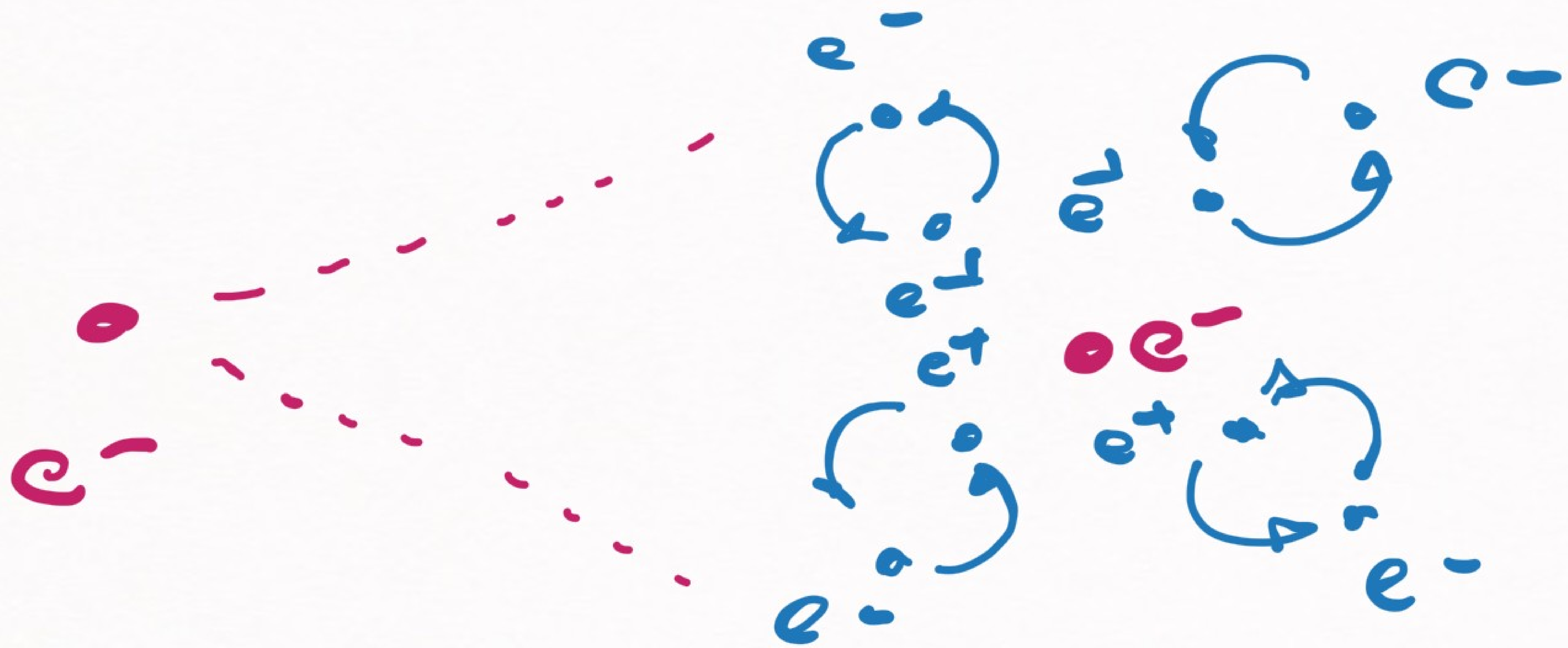
THE TWO OTHER SCALES :

1) Electron size → fine structure

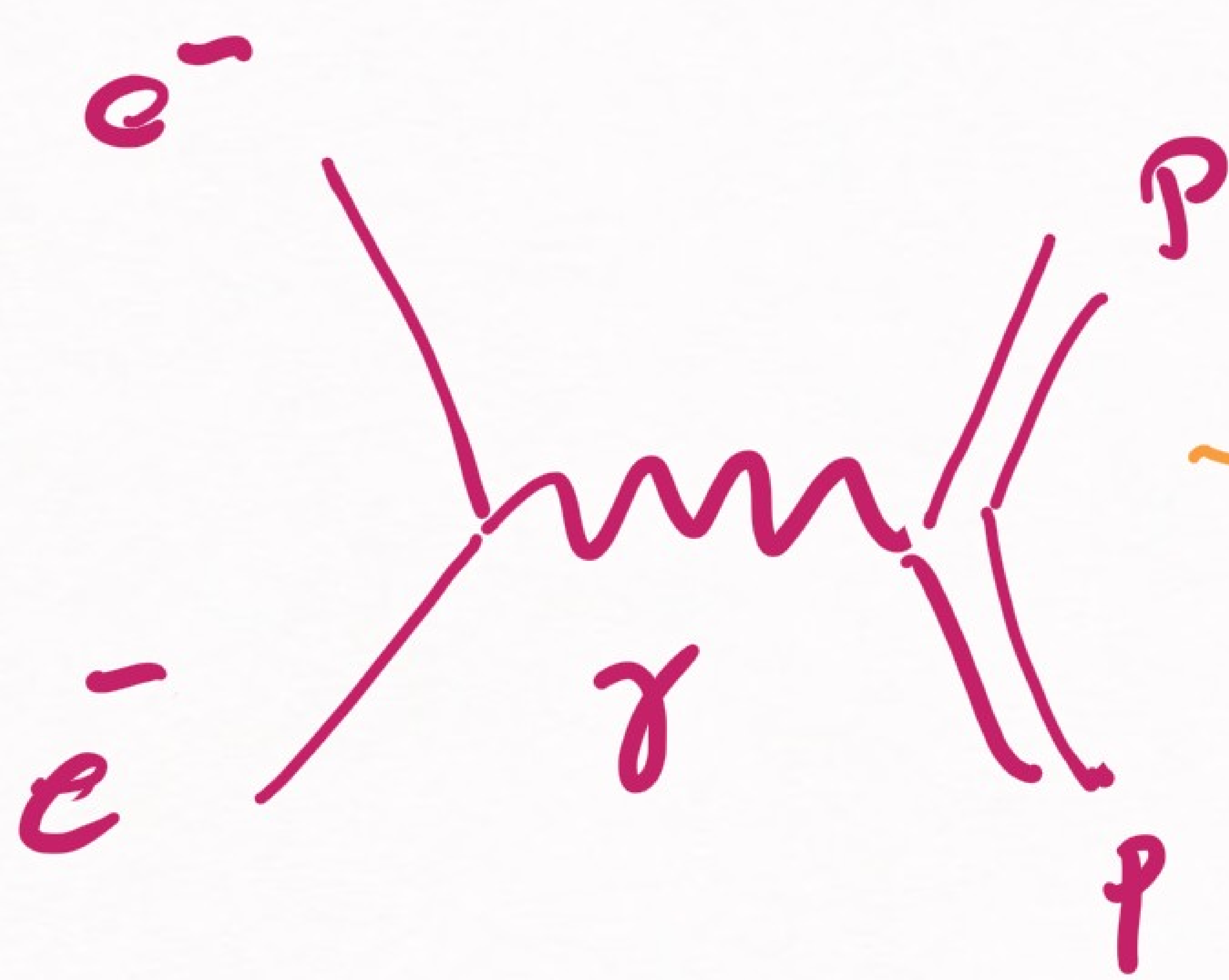
2) Proton size → hyperfine structure

But, electron size?

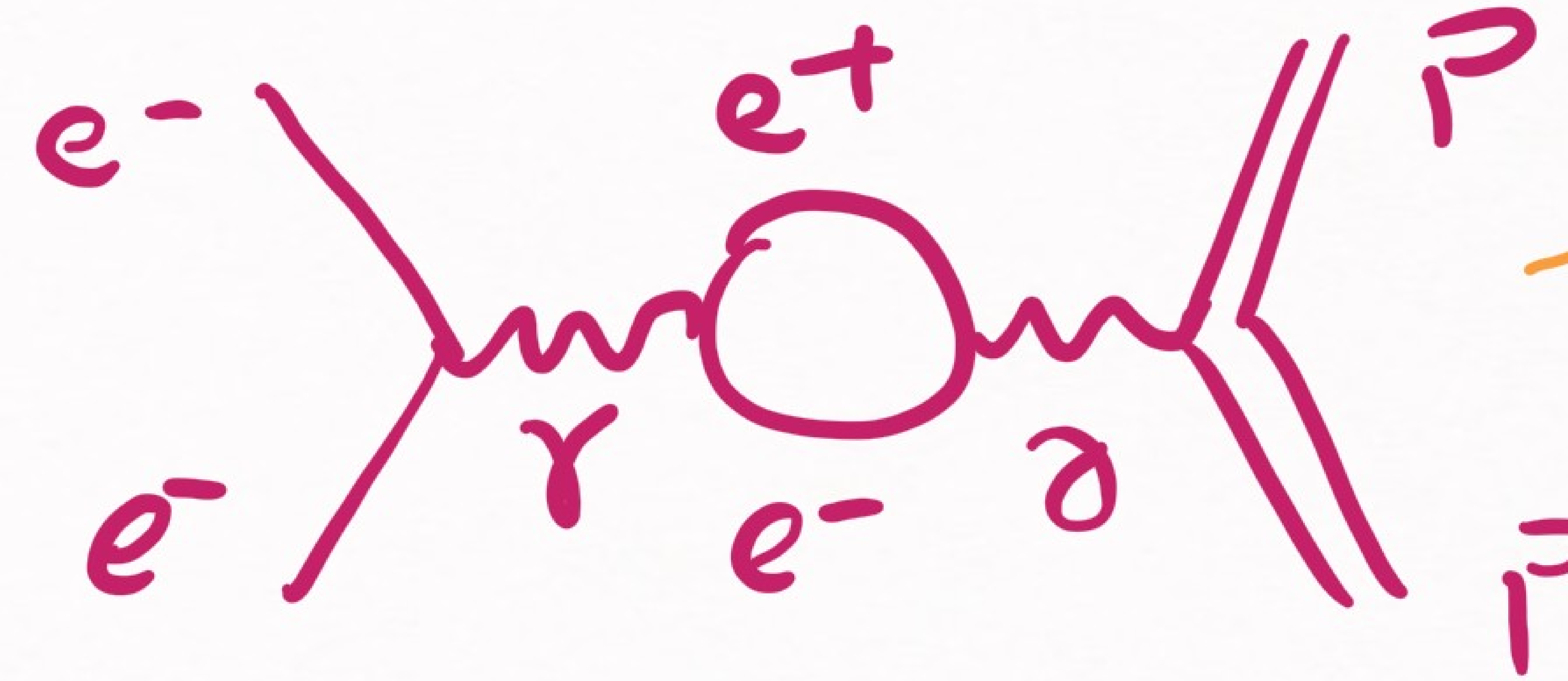
ELECTRON SIZE → VACUUM POLARIZATION



Corrections to the Coulomb potential



Coulomb potential



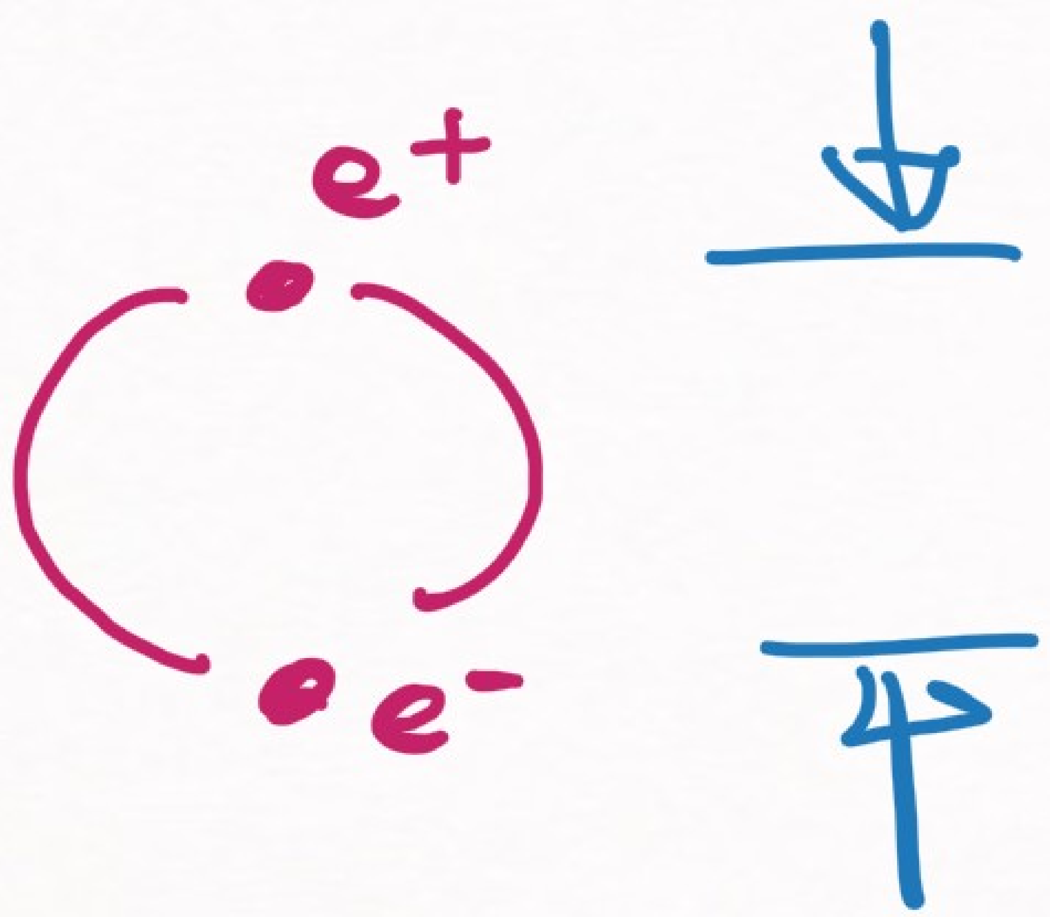
→ What is vacuum polarization?

→ effect coming from QFT (relativistic + quantum effect)

→ What is the size of this effect?

→ $\Delta x \Delta p \sim \hbar c \qquad \Delta p \sim m_e$

$$\Delta x \sim \frac{\hbar c}{m_e} \Rightarrow \boxed{r_e \sim \frac{1}{m_e}}$$



which size?

$\Delta x \, \Delta p \sim \hbar$ (uncertainty principle)

$\Delta x \sim \dfrac{\hbar c}{\Delta p}$, $\Delta p \sim m_e$

$$\Rightarrow \frac{1}{m_e} \left(= \frac{\hbar c}{m_e} \right) \simeq 400 \text{ fm}$$

$$a_B = \frac{1}{m_e \alpha} \simeq 137 \frac{1}{m_e} \simeq 54000 \text{ fm}$$

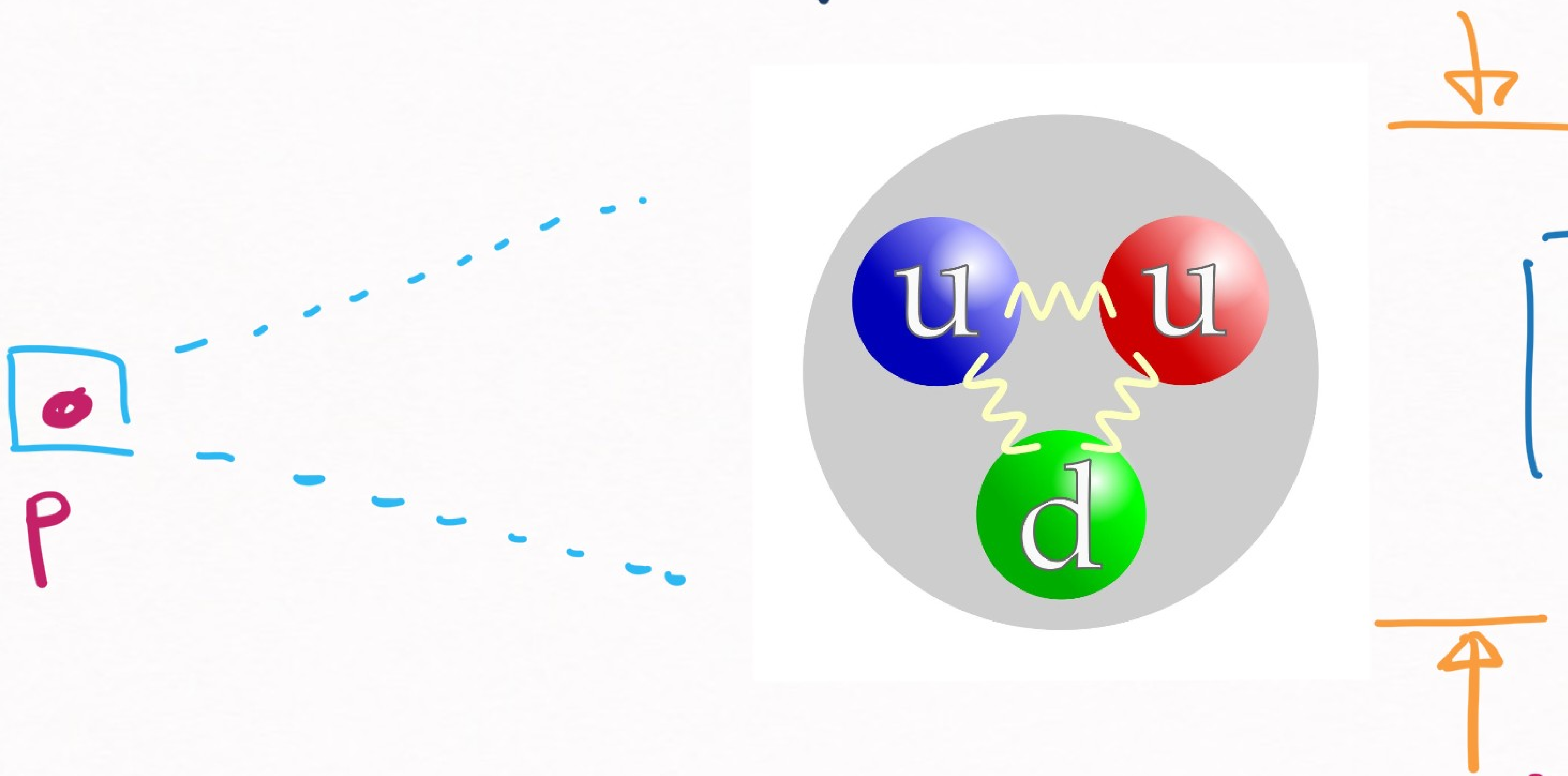
$$\boxed{a_B \simeq 54000\ \text{fm}}$$

137 times bigger than this

$$\boxed{\frac{1}{m_e} \simeq 400\ \text{fm}}$$

smaller than a_B

next: size of the proton



$$\boxed{r_p \simeq 1 \text{ fm}}$$

this is unlike vaccuum polarization
because it represents the compositeness
of the proton

HYDROGEN ATOM $\longrightarrow$ MULTISCALE PROBLEM

$a_B \simeq 54000$ fm $\longrightarrow$ VERY BIG

[VERY SMALL CORRECTIONS]

$\left.\begin{array}{l} 1/m_e \sim 400 \text{ fm} \quad \left(\sim \dfrac{a_B}{137}\right) \\ r_p \sim 1 \text{ fm} \quad \left(\sim \dfrac{a_B}{54000}\right) \end{array}\right\} \longrightarrow$ VERY SMALL

HOW DO WE DEAL WITH MULTIPLE SCALES?

1) BIGGER SCALE → identify it

→ give us the bulk properties of the system

2) SMALLER SCALES

→ gives us small corrections

→

WHAT ARE THE SIZE OF THESE CORRECTIONS?

→ Naively, we have R_L & R_S

R_L → Bigger scale

R_S → Smaller scale

and expect corrections of size $\mathcal{O}\left(\frac{R_S}{R_L}\right)$

THIS WORKS FOR HYPERFINE CORRECTIONS:

1) PROTON SIZE EFFECTS: $\frac{r_p}{a_B} \sim 2 \cdot 10^{-5}$

BUT NOT FOR FINE-STRUCTURE CORRECTIONS:

2) ELECTRON SIZE → not a normal size, but a combination of quantum & relativistic effects

$E_{kin} = \frac{p^2}{2m}\left(1 + O\left(\frac{p^2}{m^2}\right)\right)$ $\left(\frac{r_e}{a_B}\right)^2 \sim 5 \cdot 10^{-5}$

HOW DO THIS COMPARES W/ REALITY?

1) FINE STRUCTURE EFFECTS

Expectation: $\boxed{5 \cdot 10^{-5}}$

$2P$ $\left(-\dfrac{13.6\,eV}{4}\right)$

$10.2\,eV$

$1S$ $(-13.6\,eV)$

$2P_{3/2}$

$4.5 \cdot 10^{-5}\,eV$

$2P_{1/2}$

$$\frac{4.5 \cdot 10^{-5}}{10.2\,eV} \sim 5 \cdot 10^{-6}$$

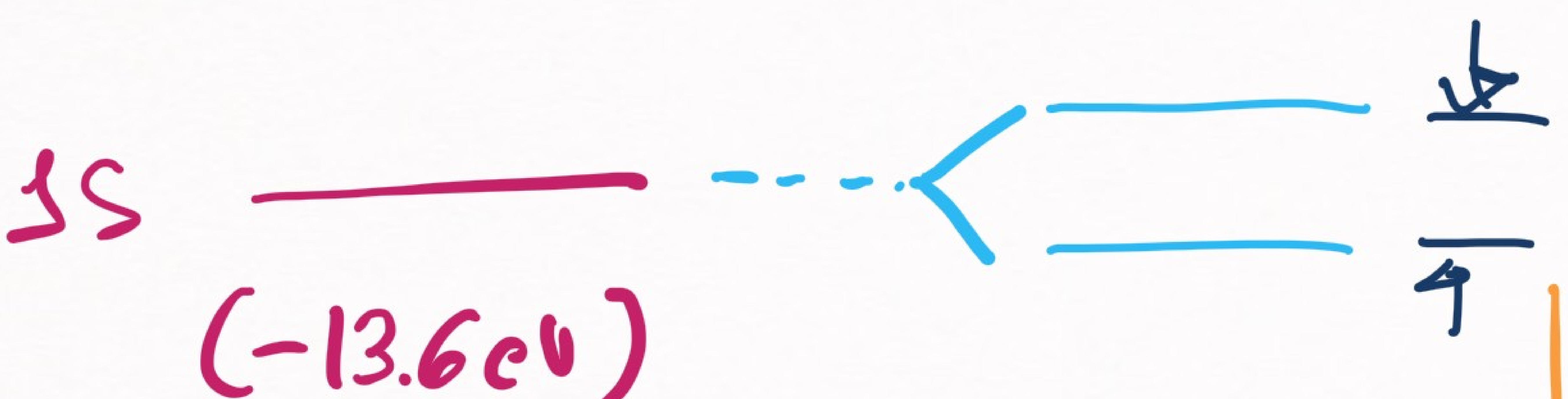
$$3\,\frac{4.5 \cdot 10^{-5}}{10.2\,eV} \sim 1.5 \cdot 10^{-5}$$

NOT PERFECT,

BUT NOT BAD EITHER

2) HYPERFINE STRUCTURE $\boxed{\left(\frac{r_p}{a_B}\right) \sim 2\cdot 10^{-6}}$

(Remember → the nature of this effect is different from the "electron size")



$5.9\cdot 10^{-6}\,\text{eV}$

$$\boxed{\frac{5.9\cdot 10^{-6}\,\text{eV}}{13.6\,\text{eV}} \sim 5\cdot 10^{-7}}$$

TAKE-HOME MESSAGE:

PHYSICAL SYSTEMS THAT ARE

1) NATURAL ✓

2) HAVE A GOOD SEPARATION OF SCALES ✓

ARE VERY EASY TO DEAL WITH

3

COROLARY SYSTEMS THAT

1) are not natural
2) have a poor separation of scales

ARE GOING TO BE A NIGHTMARE TO DEAL WITH

→ Very difficult (not a bad thing, because they are good research projects)

NEXT LESSON

→ NUCLEAR PHYSICS

1) IS NOT NATURAL
(deuteron size is very large)
2) HAS A POOR SEPARATION OF SCALES
size of the proton ~ 0.8 fm
range of the potential ~ 1.4 fm

SEE YOU ON TUESDAY

AT 15:50

Z