

NUCLEAR PHYSICS (20)

THE COLLECTIVE MODEL

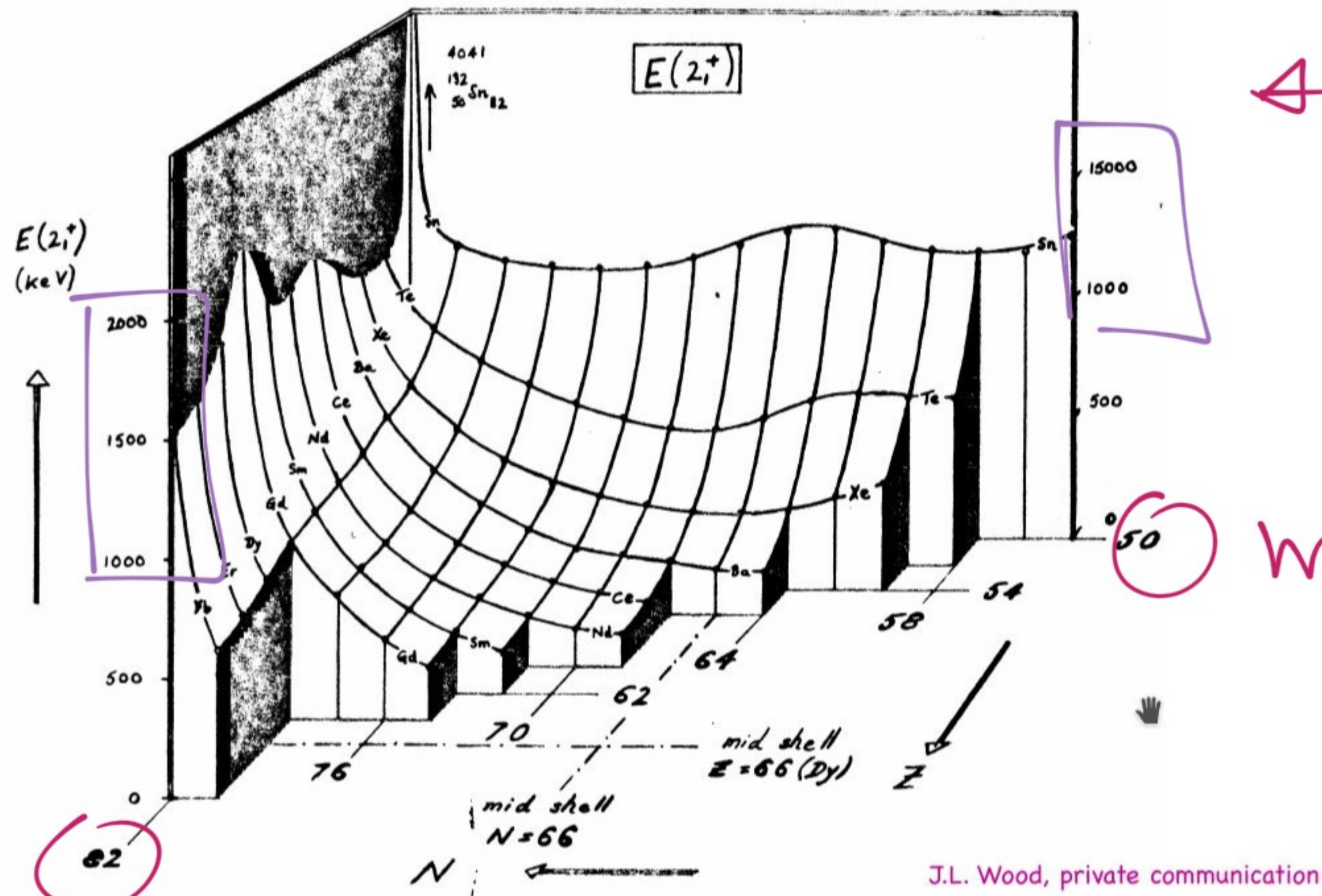


RECAP | Shell model

→ Nucleus as a collection of nucleons (fermions) in a mean-field

→ Magic numbers $Z, N = 2, 8, 20, 28, 50, 82, \dots$
(works better for Z, N close to these)

→ Residual interactions (improve over the mean field) pairing interaction



magic

← Energy of the 2_1^+ excited state

50 magic

Different
scalars involved
depending
on $N, Z = D \oplus$

④ \Rightarrow ① Close to N, Z magnetic,
 $E(z^+) - E(0^+)$ is about $(1-2) \text{ MeV}$

\leftarrow (check the \square boxes)

② Away from N, Z magnetic
 $E(z^+) - E(0^+)$ is much
smaller

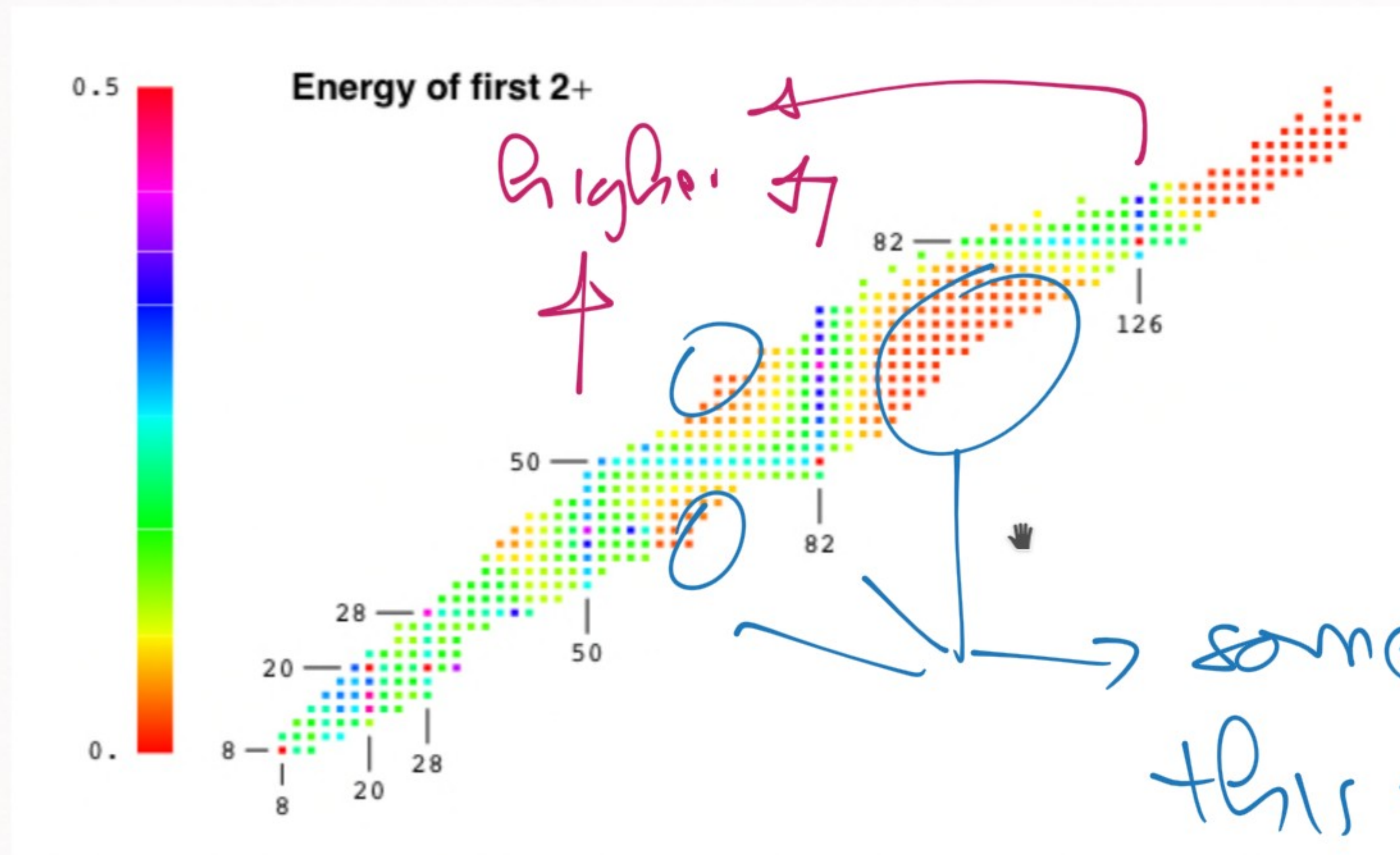
① → this is where shell model should work

② → here shell-model doesn't work
so well

① → $(1-2) \text{ MeV}$

② → $(0.1-0.2) \text{ MeV}$

} scales are
different



Higher $E(2+) - E(0+)$

for magic number

some cases where this $E(2+) - E(0+)$ is very small

Summary

(a) $\begin{array}{l} \text{--- } 2^+ \\ \text{--- } 0^+ \end{array} \left. \vphantom{\begin{array}{l} \text{--- } 2^+ \\ \text{--- } 0^+ \end{array}} \right\} \text{very common structure}$

(b) $E(2^+)$ is often too small for being explainable within the shell model

→ [GREAT TO HAVE EXPLANATION OF (a) & (b)] → DO

COLLECTIVE MODEL

↳ consider the nucleus as a whole
(ignore the nucleons)

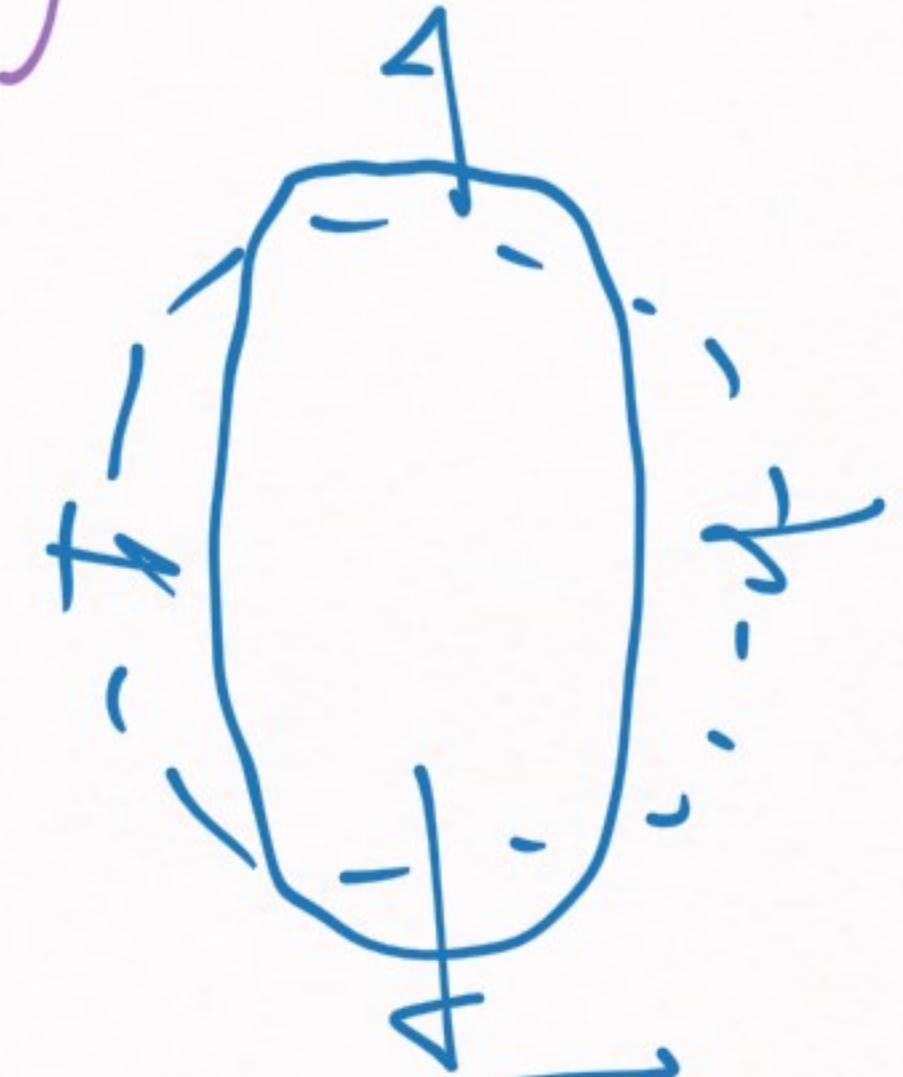
- 1) Liquid drop model → nucleus as a liquid
(\cong collective)
- 2) Shell-model → individual nucleons in V_{int}
(non collective)
- 3) Collective model → recover ideas from 1)

COLLECTIVE MODEL

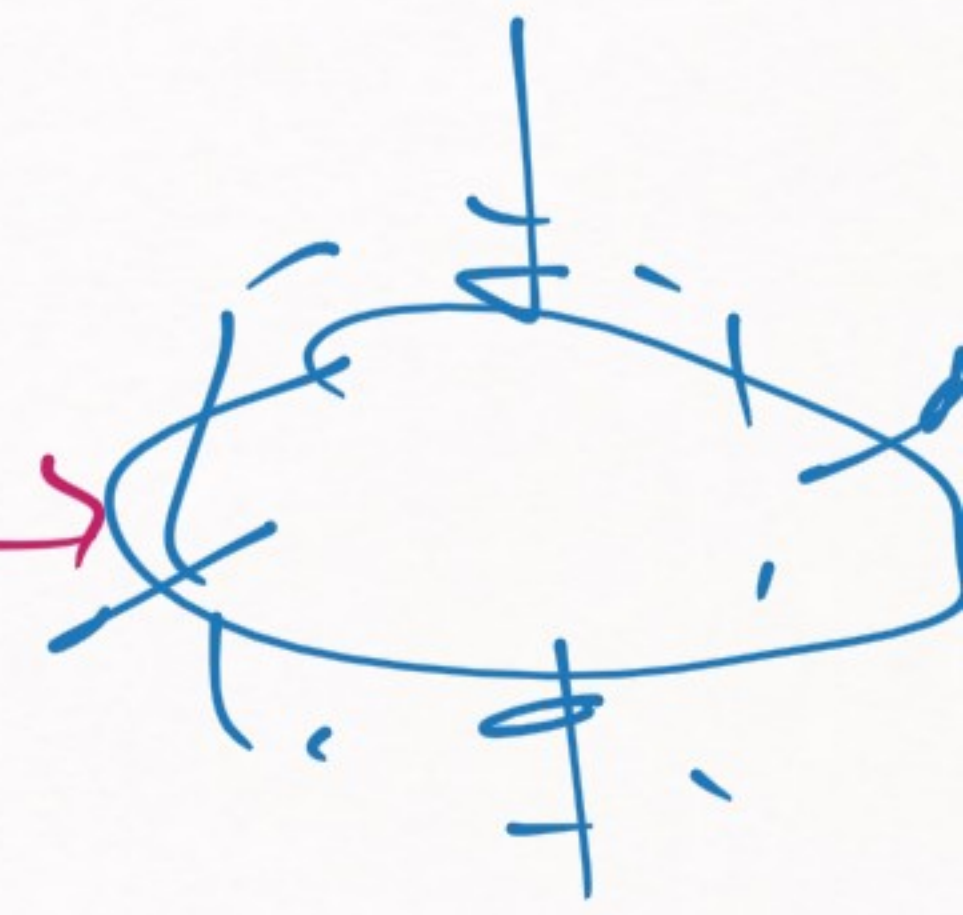
1) VIBRATIONS:
(nucleos as a liquid)



→



→

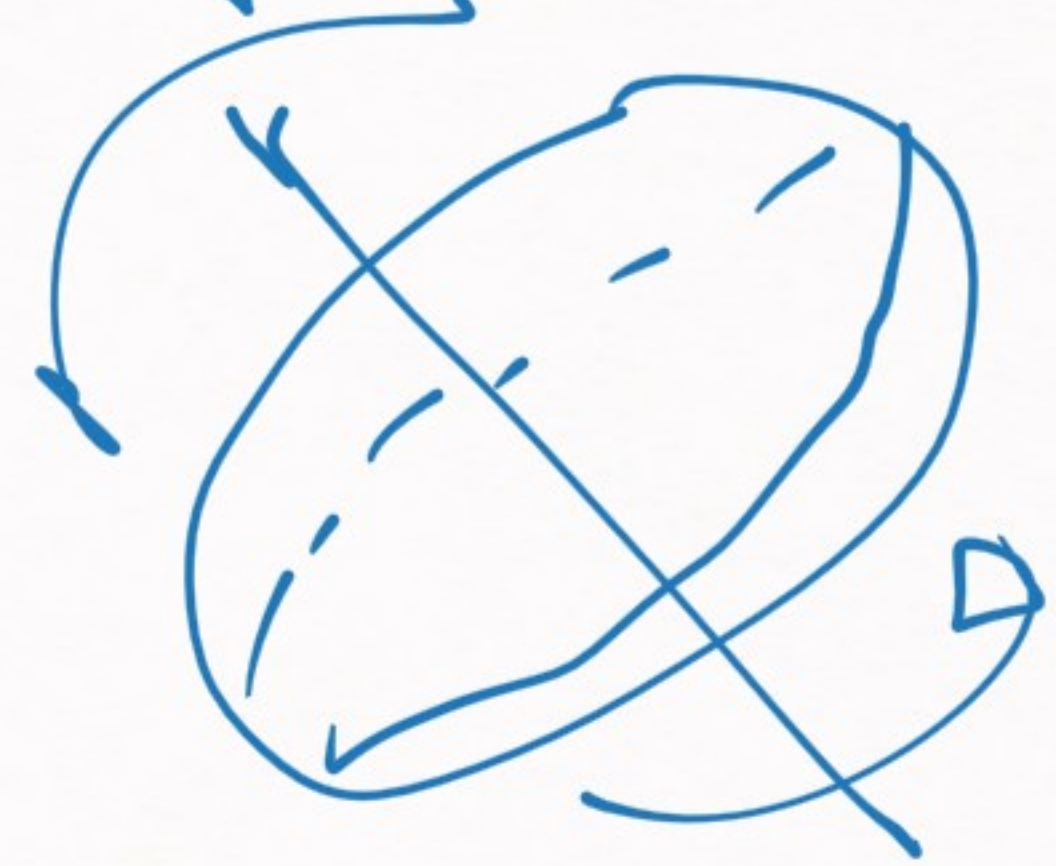


excited states (2)

2) ROTATIONS:
(nucleos as a solid)



→



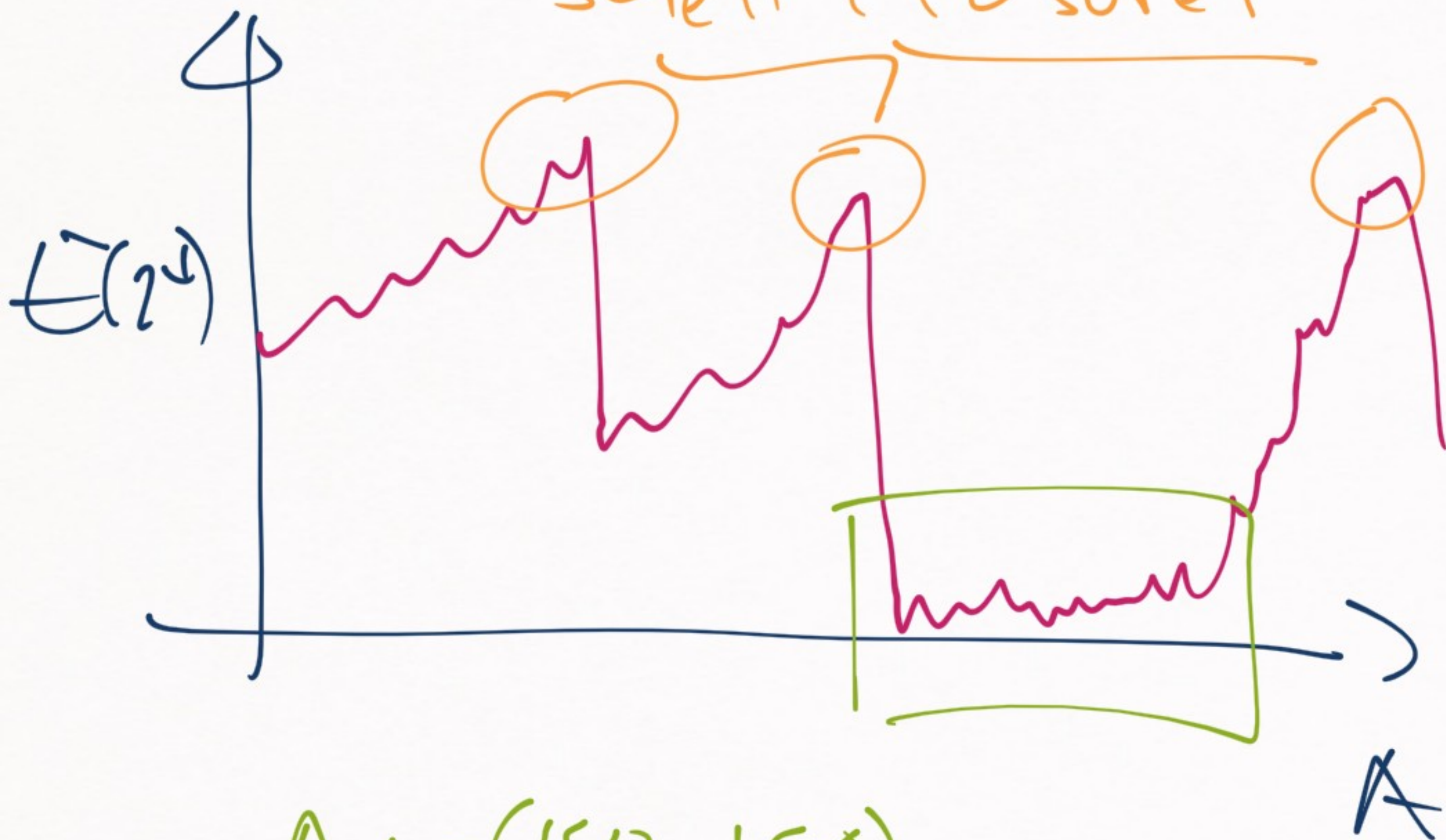
excited state (2)

ground state (deformed)

We begin w/ rotations

(nucleus as a rigid solid)

Shell closure



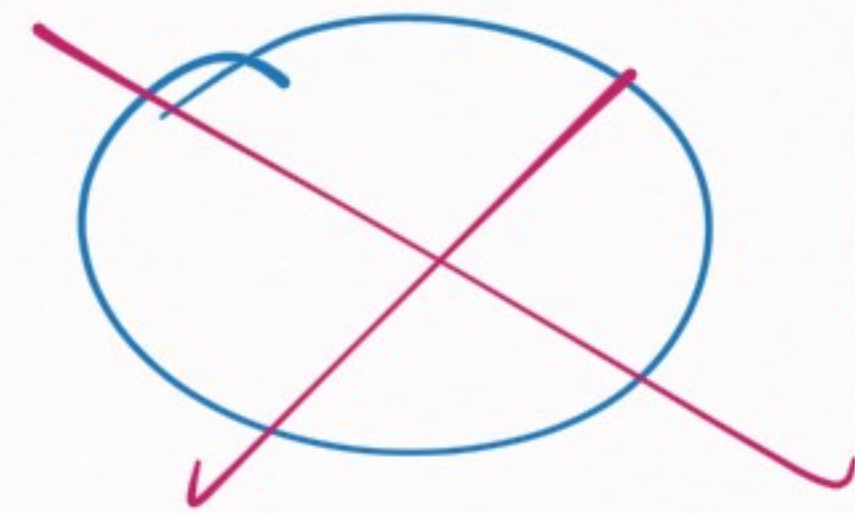
↓
very low energy
 $E(Z^A)$ excitations

$A \sim (150 - 190) \Rightarrow$ super small $E(Z^A)$ energies

\Rightarrow A \sim (150-190) behavior is unusual

these nuclei are very deformed $\underline{Q < 0}$
(large negative quadrupolar moment)

1) Vibrations? \rightarrow no



(Requires spherical nucleus)

2) Rotations? \rightarrow yes



→ We need to understand these rotations

QUANTUM ROTATOR

Solid object rotating in QM



(\exists a few tricks to explain it)

[QUANTUM ROTATOR]

Solid rigid & momenta of inertia
in classical physics



$$H = \frac{1}{2} I \omega^2 \rightarrow \text{rotation frequency}$$

high school physics ↙

momentum of inertia

→ This is classical, how do we quantize it?

1) $\boxed{\vec{L} = I \vec{\omega}}$ → angular momentum

(we know how \vec{L} is quantized)

2) $\boxed{H = \frac{1}{2} I \vec{\omega}^2} \Rightarrow \boxed{H = \frac{1}{2I} \vec{L}^2}$

this is easy to quantize

[QUANTUM ROTATOR]

$$H = \frac{1}{2I} L^2 \Rightarrow E(L) = \frac{\hbar^2}{2I} L(L+1)$$

$$L = 0, 1, 2, 3, \dots$$



in principle

(\exists constraints)

Invalid
up to this
point

Non-trivial part



the rotations of
the rotator
should be
non-trivial

The rotation

should change the quantum state

of the solid rigid

→ Simple, but very abstract!

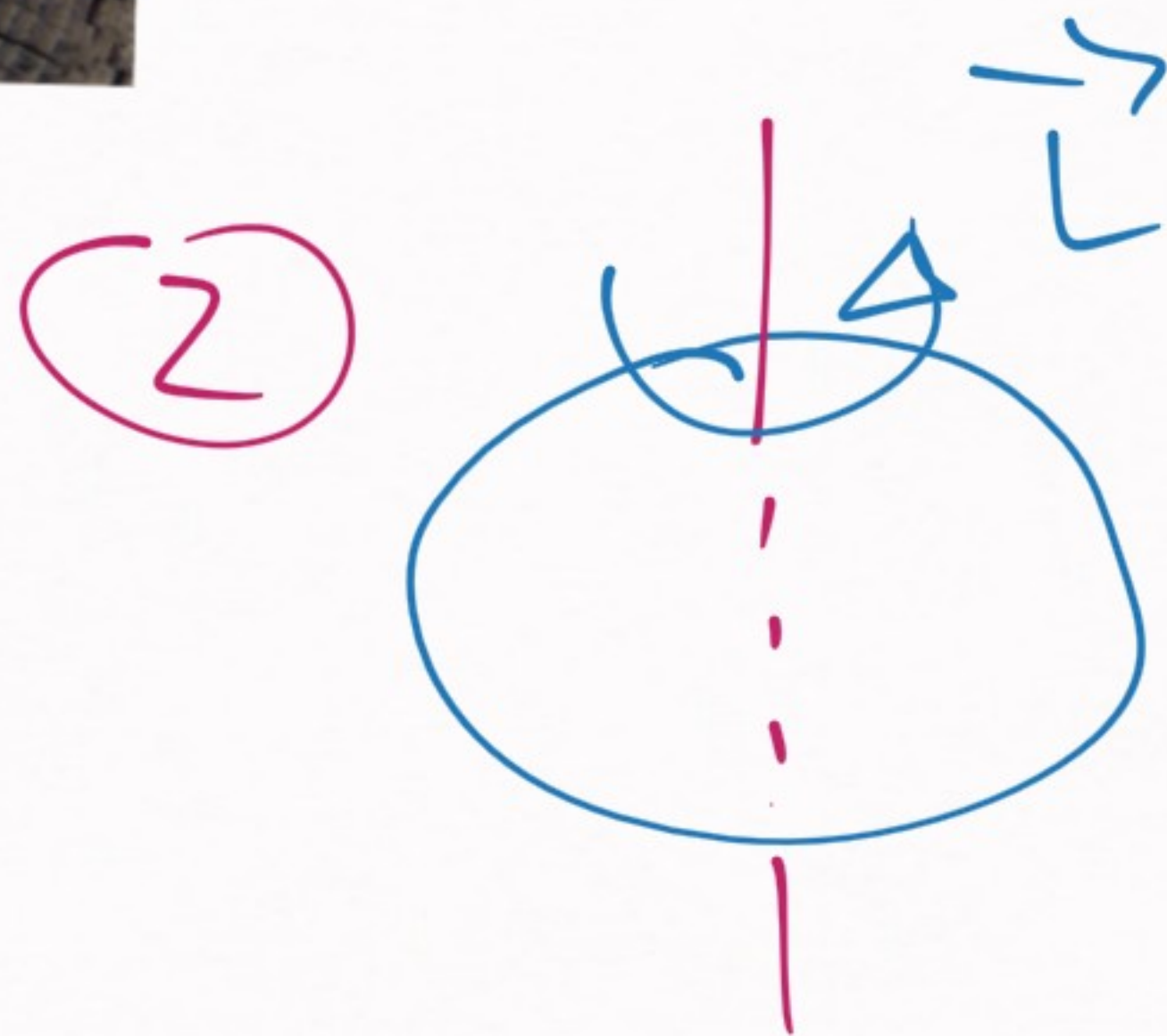
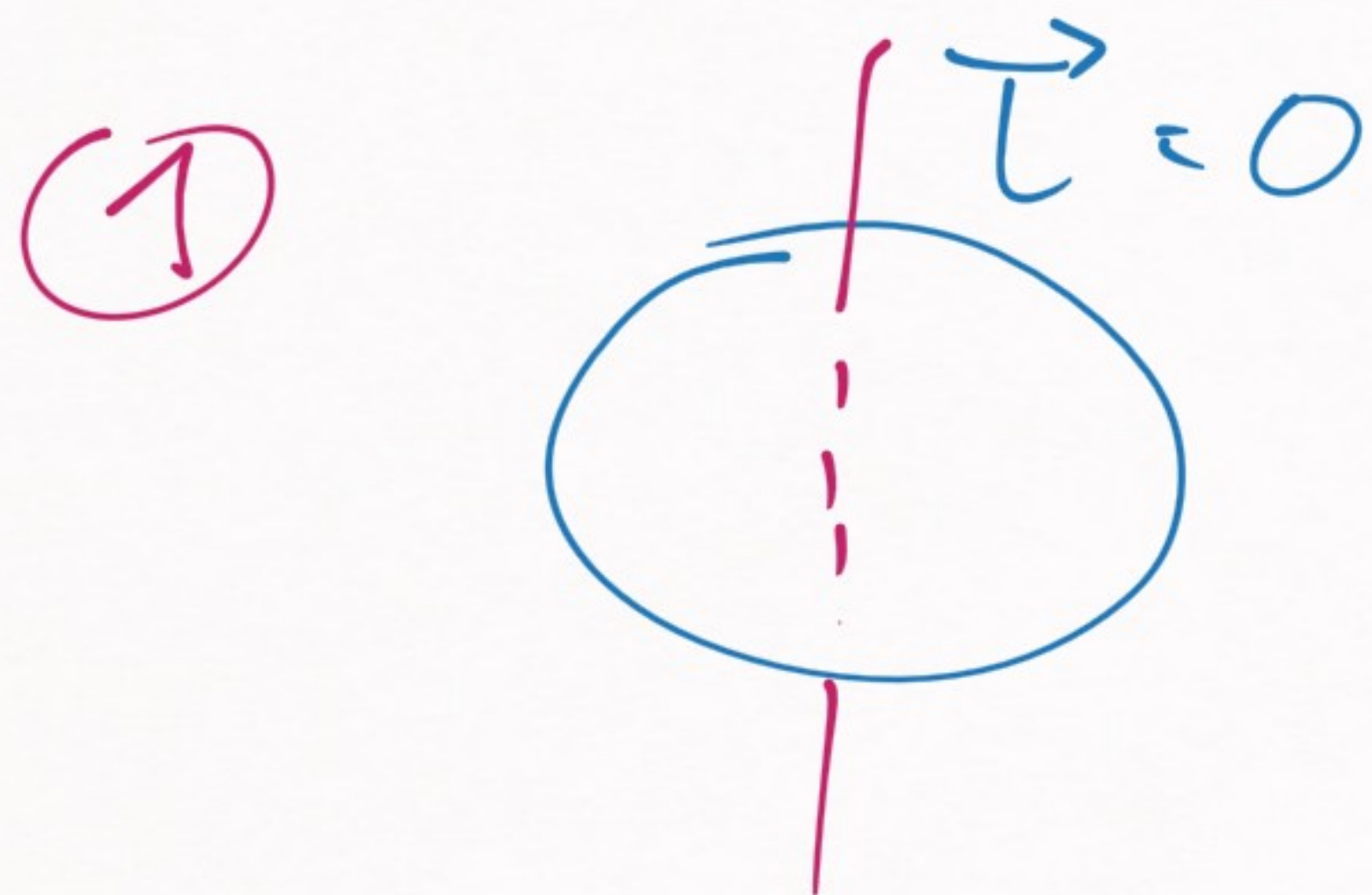


→ Imagine that we rotate this

Classically,
no problem

But in QM

$|1\rangle \neq |2\rangle$



[Perfect sphere] \rightarrow $| \text{non-rotating} \rangle$
 $= | \text{rotating} \rangle$
wave functions identical

$H | \text{non-rotating} \rangle = H | \text{rotating} \rangle$
no difference in energy

If we had a perfect sphere rotating

$$E(L) = \frac{1}{2I} L(L+1) \quad , \quad \boxed{L=0}$$

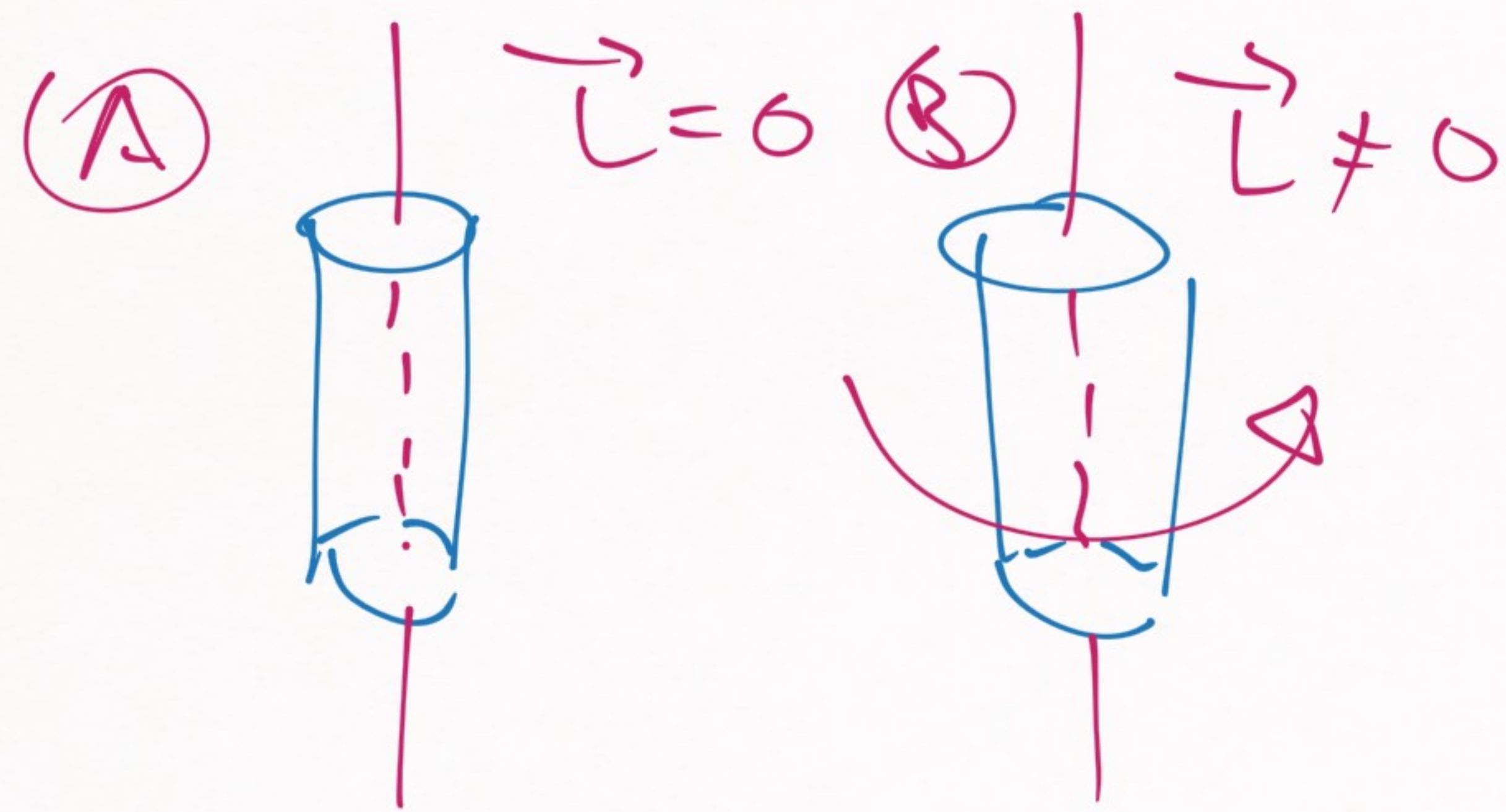
↓
we have strong
constraint
from symmetry

SPHERICAL NUCLEUS \Rightarrow ~~ROTATIONAL STATES~~
(forbidden by symmetry)



NON-SPHERICAL NUCLEUS
 \Rightarrow [A FEW ROTATIONAL STATES ARE POSSIBLE]

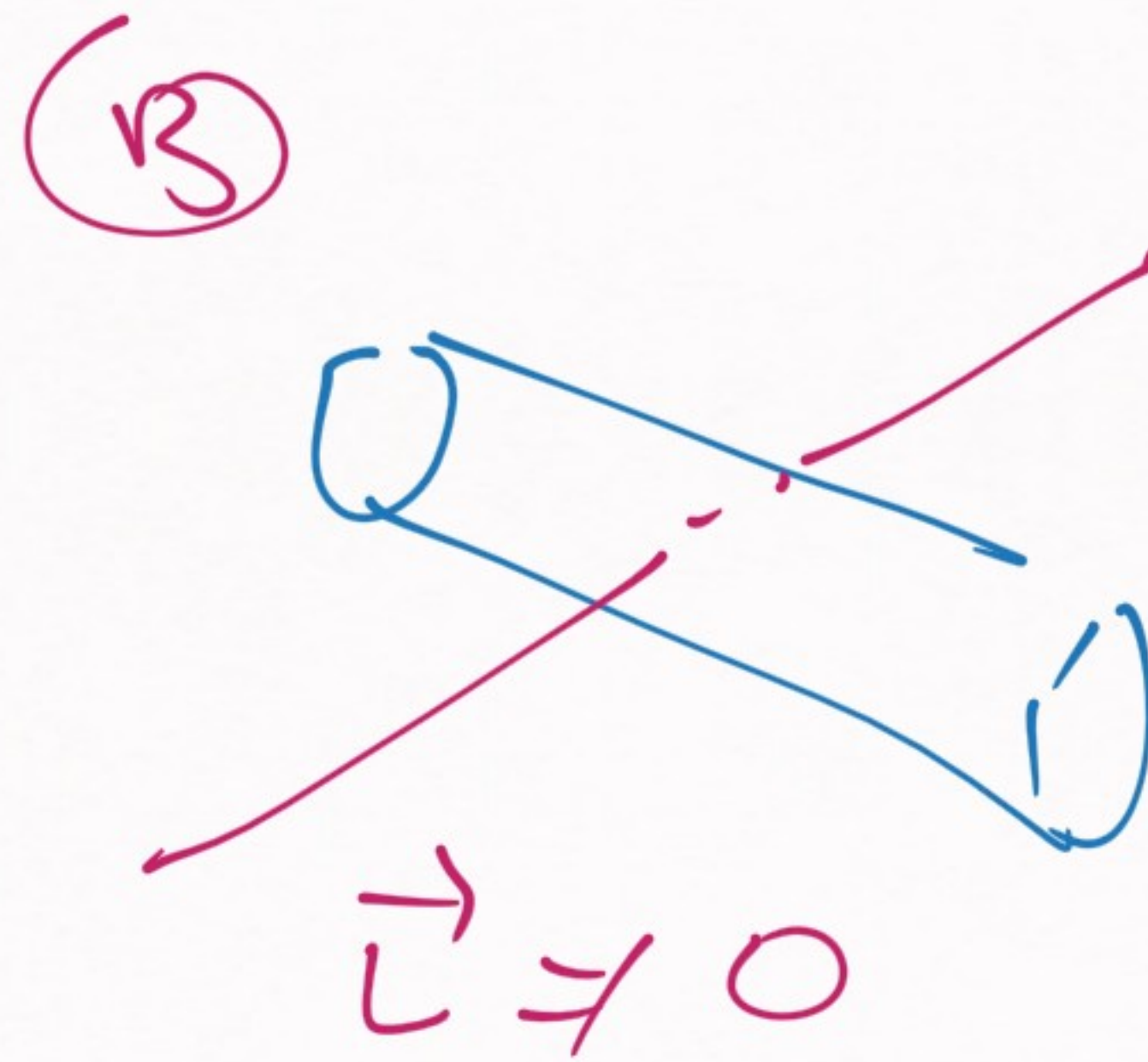
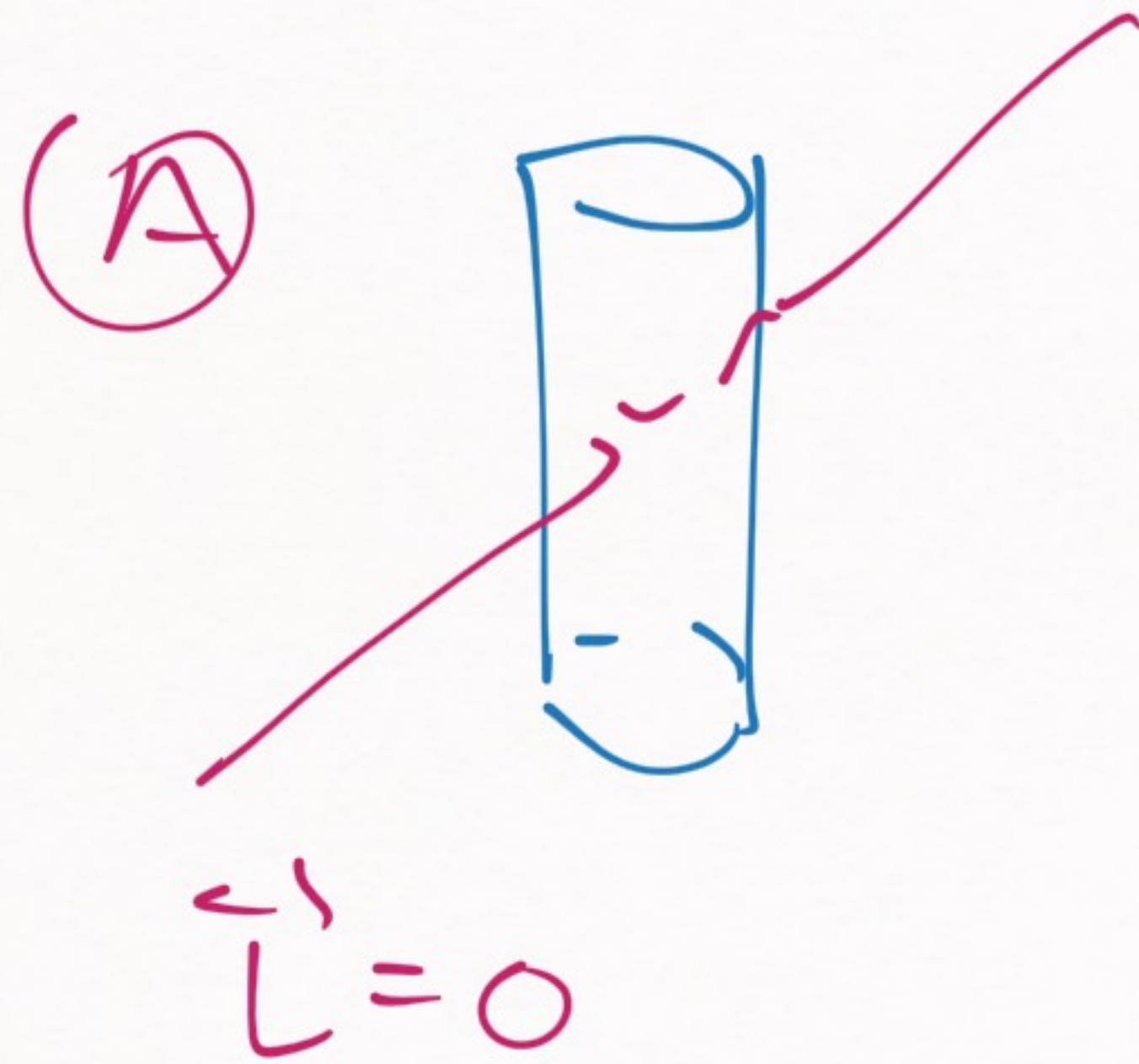
EXAMPLE → QUANTUM ROD



$$|A\rangle = |B\rangle$$

↓
this relation
is trivial
(no energy)

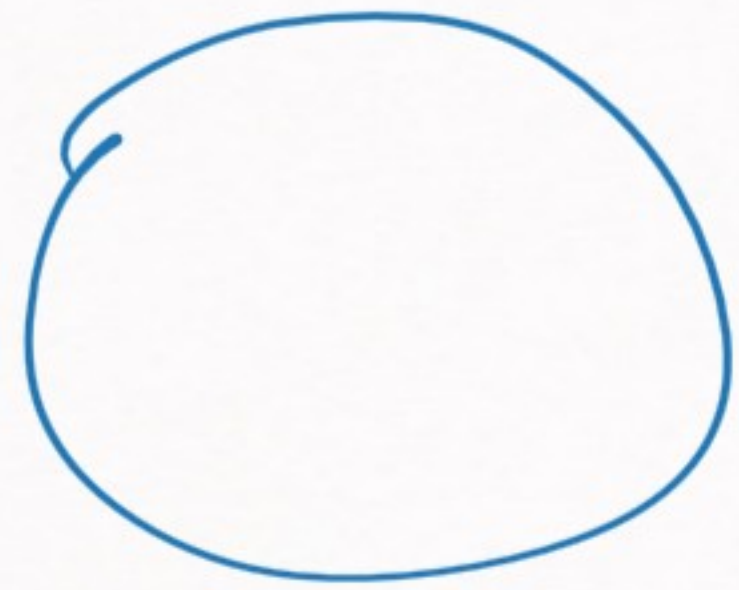
But with a different rotation...



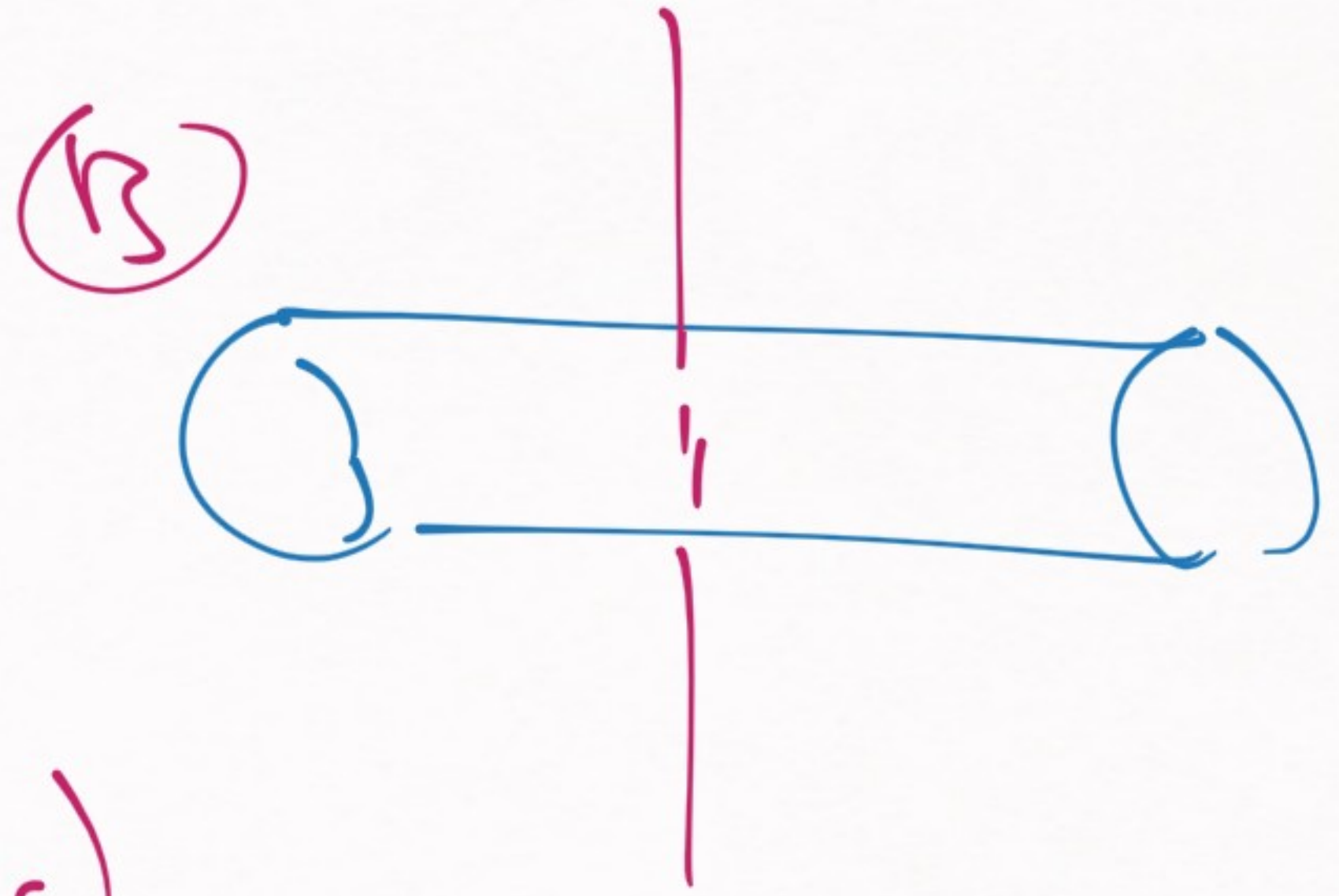
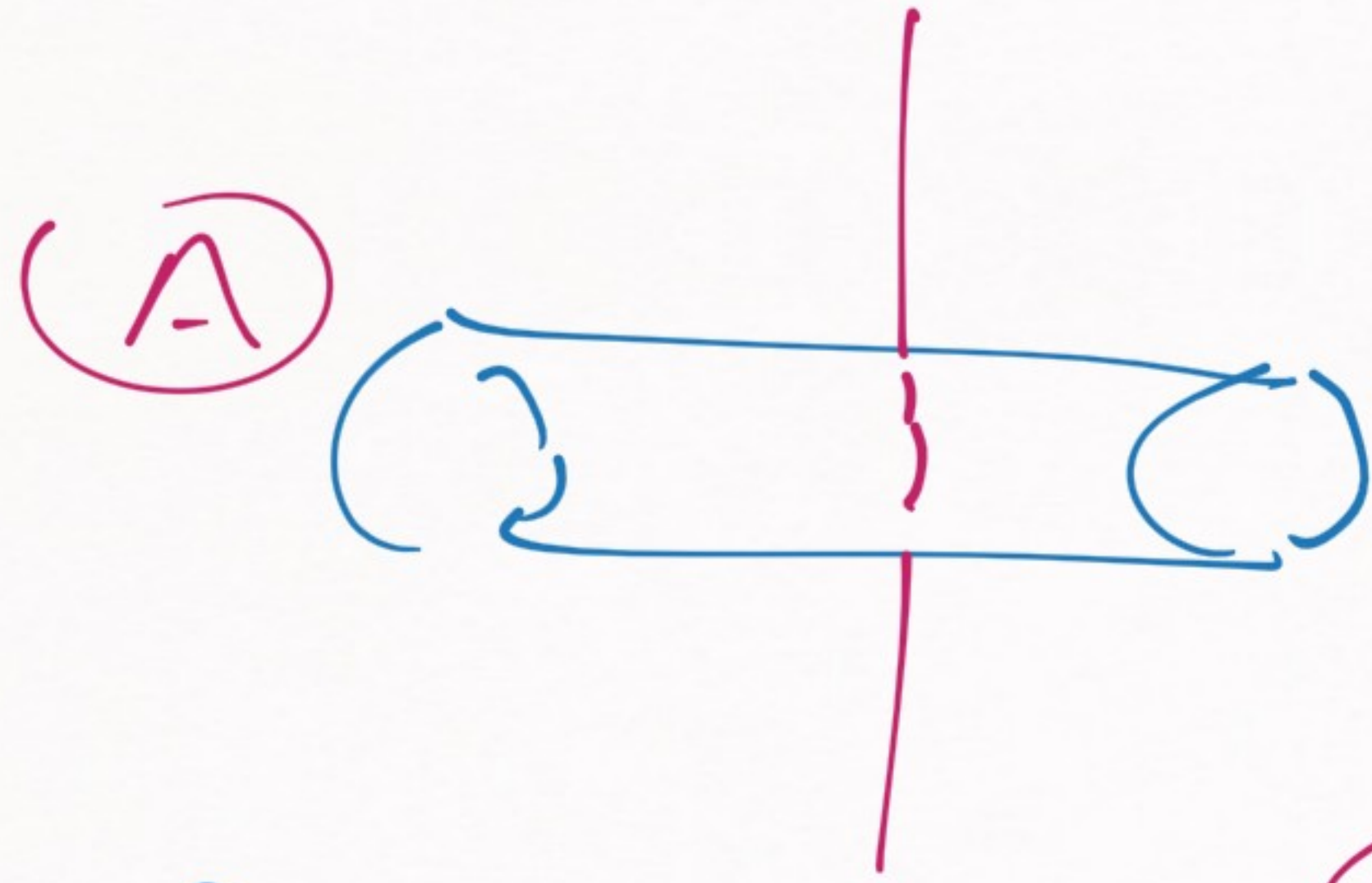
$|A\rangle \neq |B\rangle$
} this rotation counts
//

→ depending on axis of rotation,
the rotation might be non-trivial

→ DEPENDING ON SLIDE,
∃ MORE CONSTRAINTS



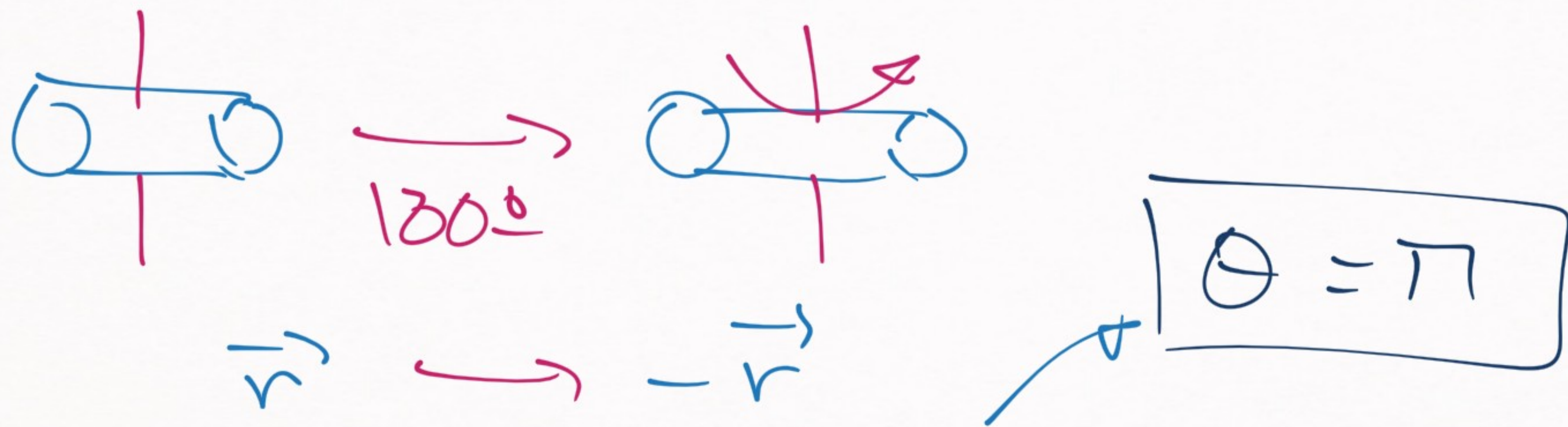
$\rightarrow \forall$ rotations are trivial



π
(180° degrees)

If we have
an object $\rightarrow |A\rangle = |B\rangle$
that is parity-symmetric

(Per 180 degree
rotation)



$$|A\rangle \longrightarrow e^{i\Theta L} |A\rangle = |B\rangle$$

$$|A\rangle = |B\rangle$$

only possible
 $\Rightarrow e^{i\pi L} = (-1)^L = 1$

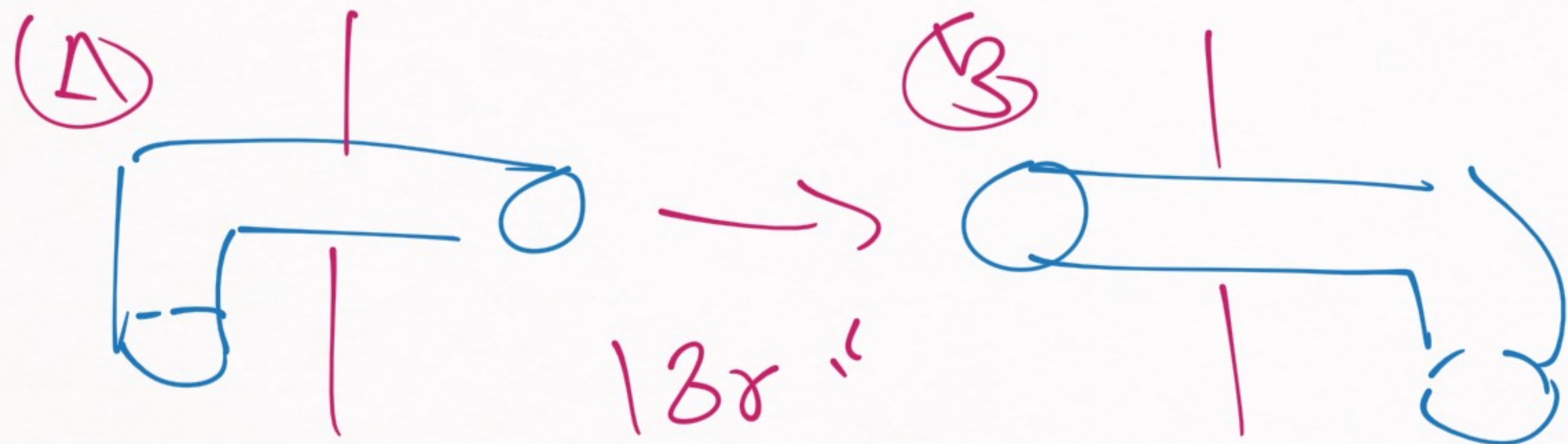
If the object has mirror symmetry (parity)

$$\Rightarrow \boxed{e^{i\pi l} = +1}$$

\Downarrow

$$\boxed{l = 0, 2, 4, \dots}$$

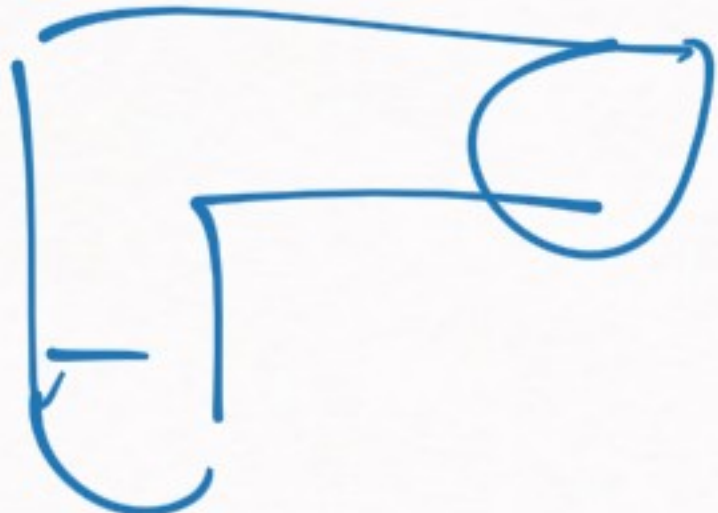
→ these are
the only
allowed
rotations



$A \neq R \Rightarrow$ with this shape,
 there will be no constraint
 on L
 $\Rightarrow L = 0, 1, 2, 3, 4, \dots$

Nuclei are symmetric under parity

 → nuclei could be like this

 → nuclei will never be like this

 chiral object (breaks parity)

RECAP

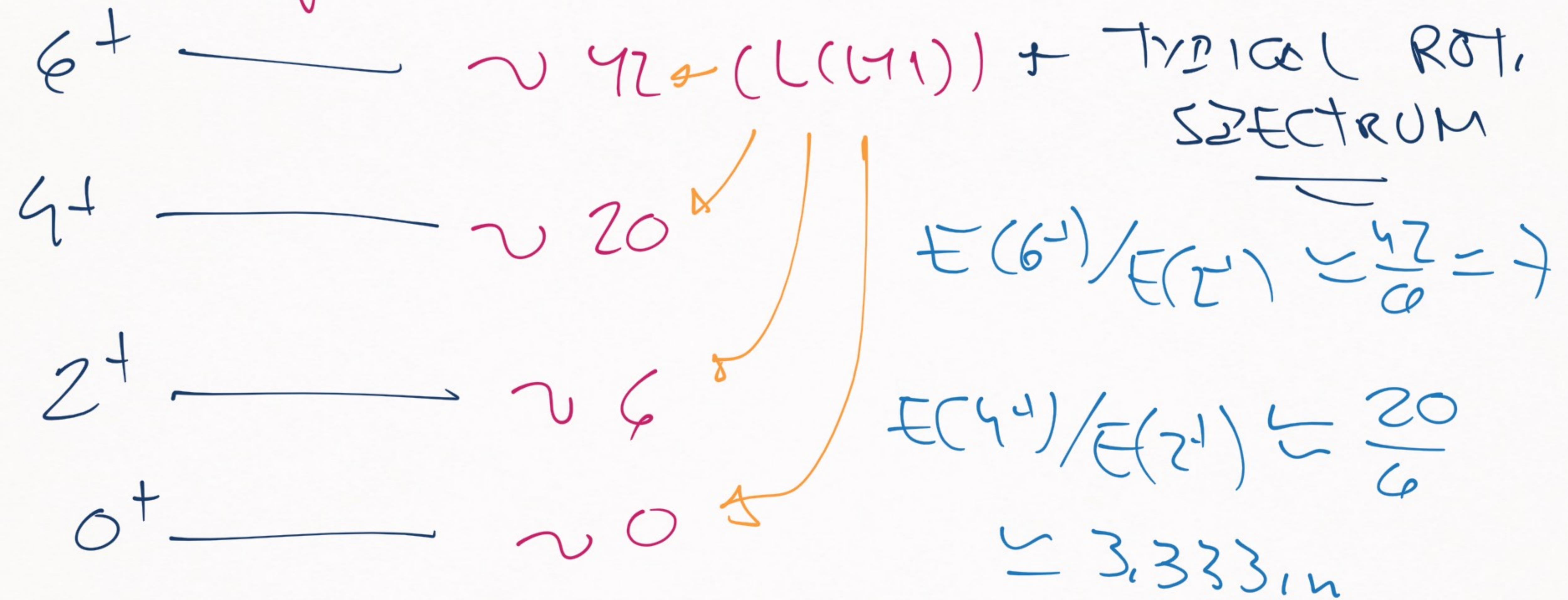
ROTATIONAL LEVELS IN NUCLEI

- 1) Only for non-spherical nuclei
- 2) Nuclei \uparrow have good parity

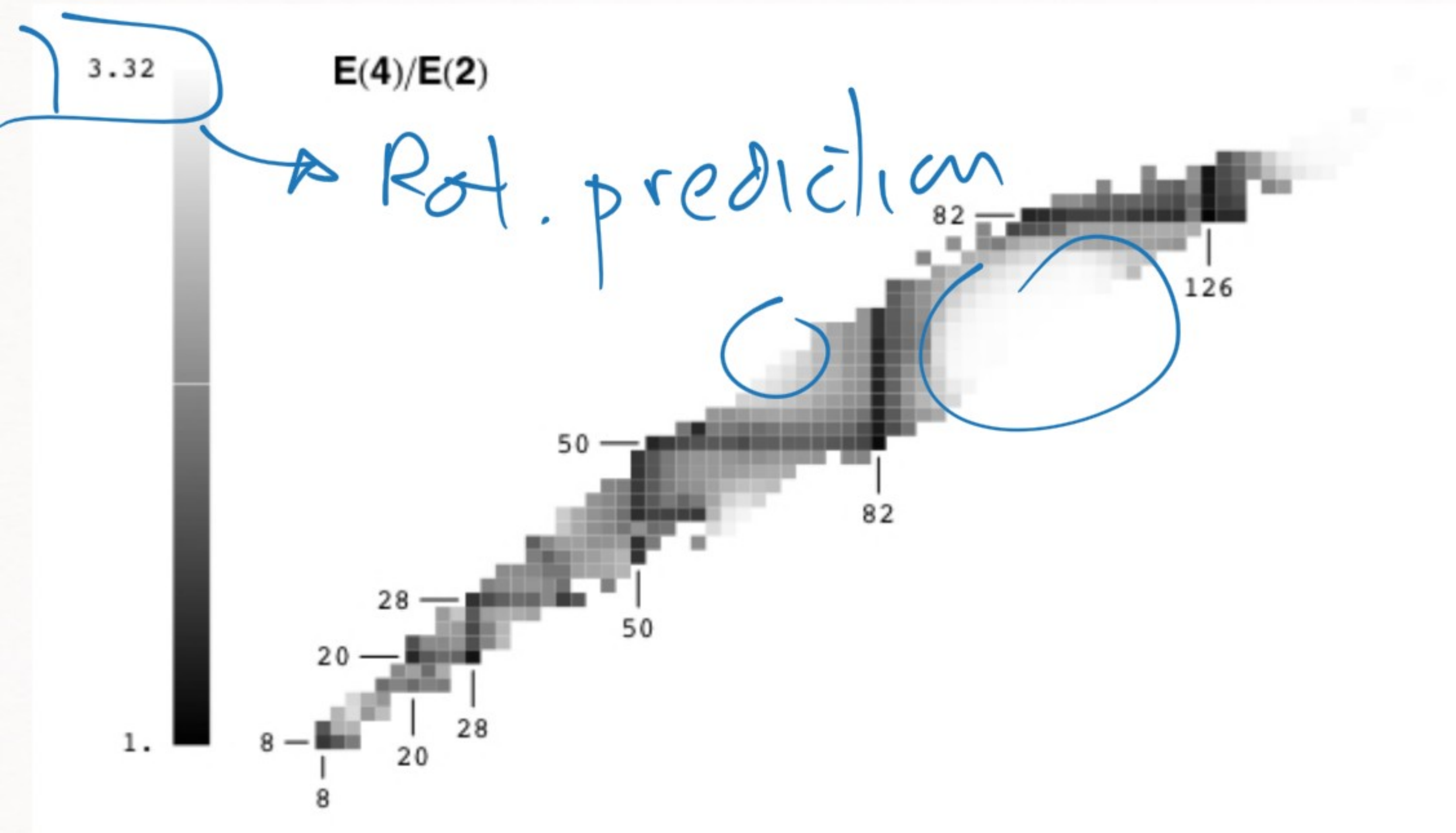
$$\Rightarrow \boxed{L = 0, 2, 4, \dots}$$

$$\left[E(L) = \frac{1}{2I} L(L+1), \quad L \text{ even} \right]$$

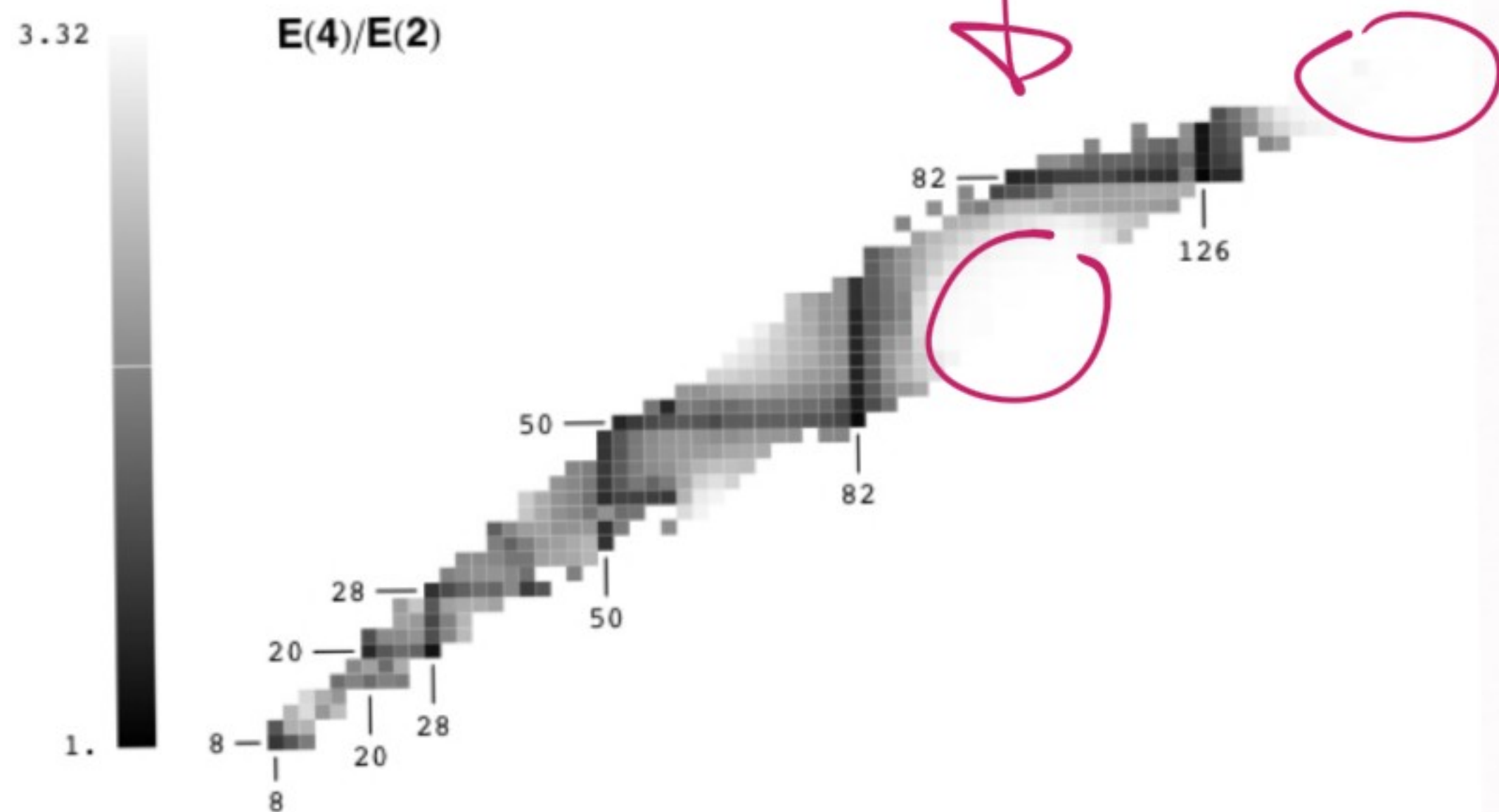
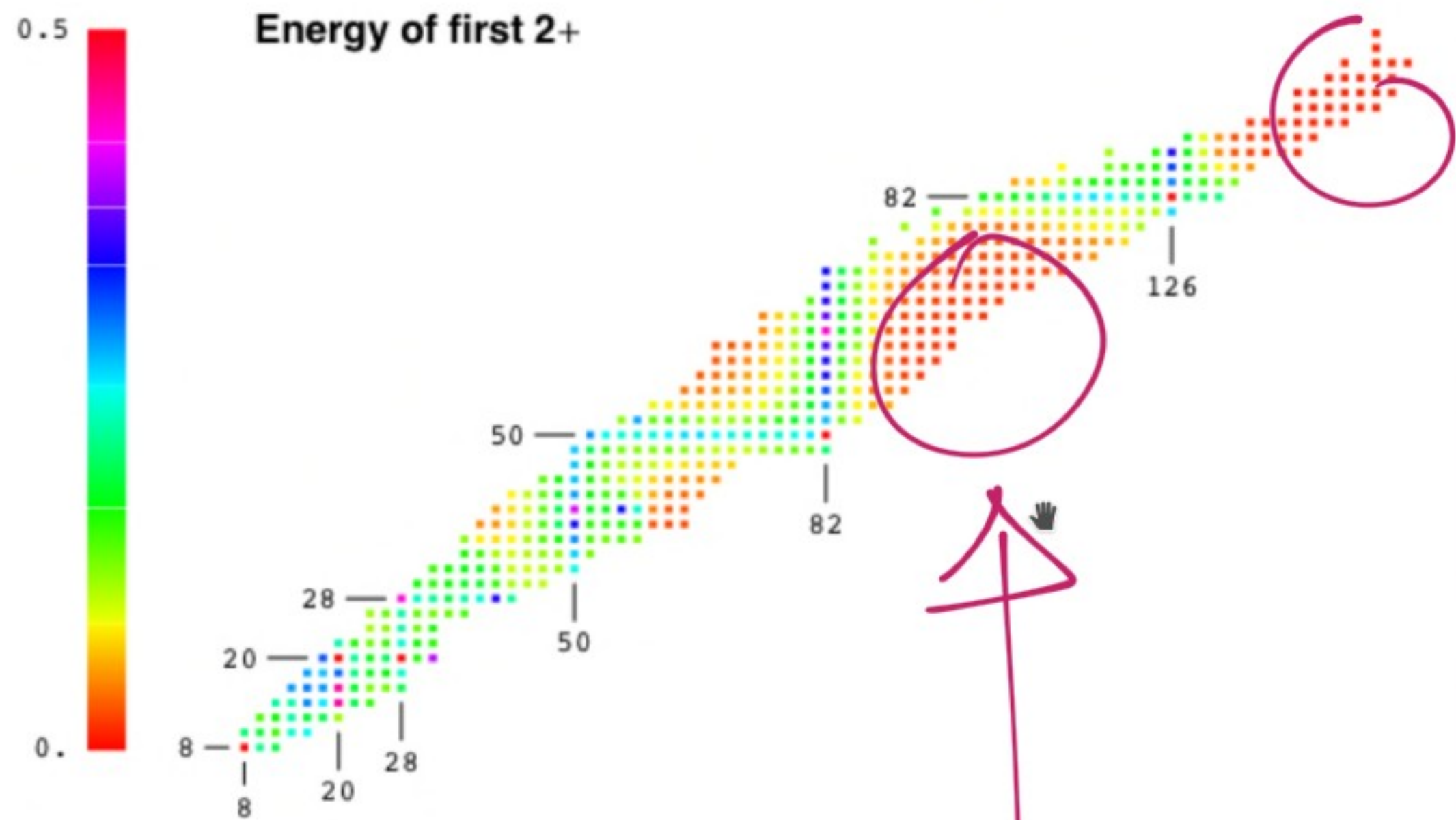
Δ deformed nuclei will have the following spectrum:



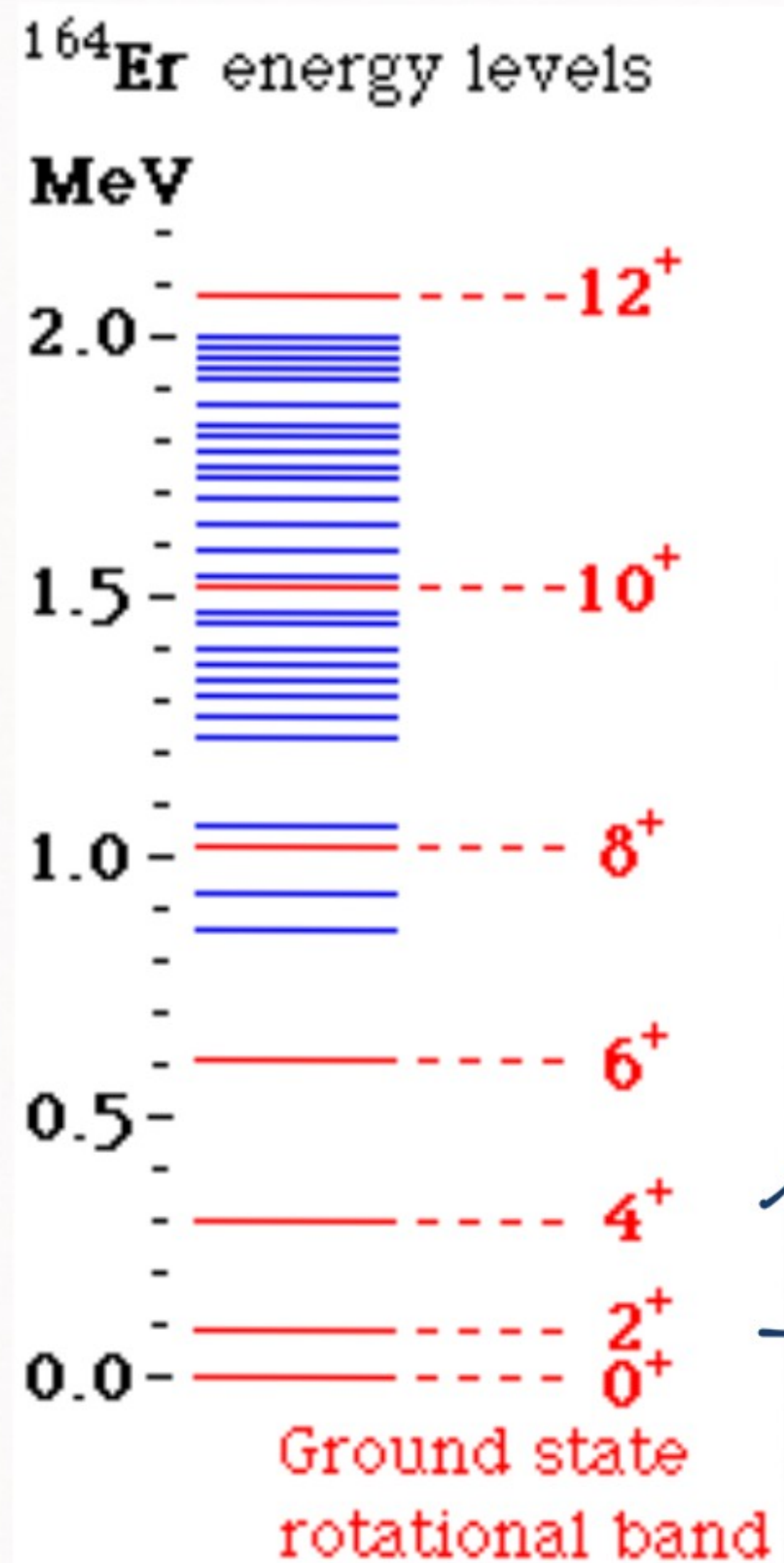
Plot of $E(4^+)/E(2^+)$ for a few nuclei;



$A \sim (150 - 190)$
 nuclei indeed
 have this type
 of rotational
 spectrum



Rotational energy
 levels
 hence small
 excitation
 energies



$A \leq 164$ (belong to rotational nuclei)

a set of non-rotational levels (at higher energies)

$E(2^+) \approx 0.13 \text{ MeV}$
 $E(4^+) \approx 0.1 \text{ MeV}$

Lower energy levels
 \downarrow
 ROTATIONAL

[REVIEW OF THE SCALES INVOLVED]

1) Shell-model $\sim (1-2) \text{ MeV}$

2) Rotational levels $\sim (0.1-0.2) \text{ MeV}$

$$E(4^+) / E(2^+) \sim 3.3?$$

3) Vibrational levels (now) $\sim 0.1 \text{ MeV}$

$$E(4^+) / E(2^+) \sim 2$$

→ [VIBRATIONS IN THE COLLECTIVE MODEL]

1) $\begin{matrix} \longrightarrow Z^+ \\ \longrightarrow 0^+ \end{matrix}$

2) $\begin{matrix} E(2^+) \\ \text{SCAPE} \end{matrix}$

3) $\begin{matrix} E(4^+) \\ \hline E(2^+) \end{matrix}$

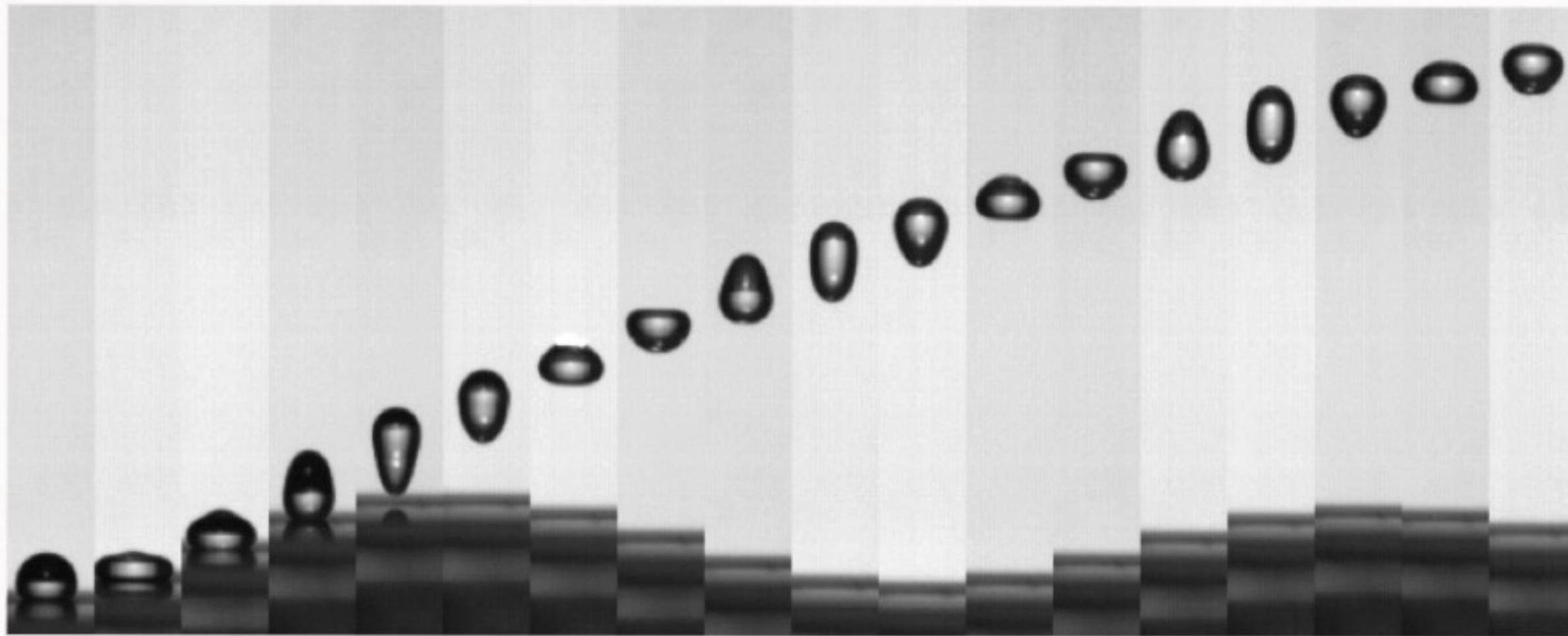
most even-even
nuclei like
these


$\boxed{1, 2, 3}$ → require
different explanation

(COLLECTIVE MODEL)

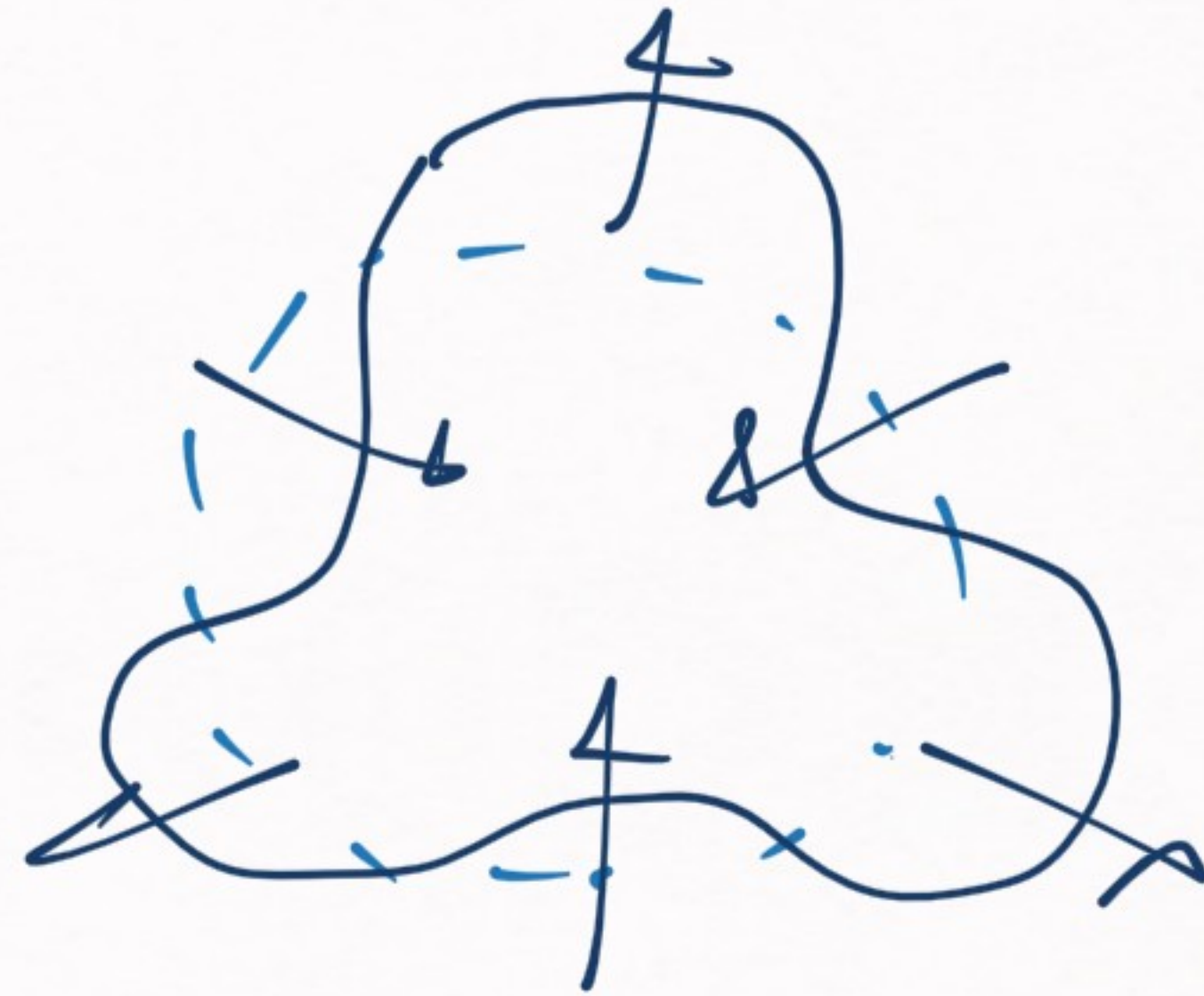
- 1) ROTATIONAL \rightarrow non-spherical (solid)
- 2) VIBRATIONAL \rightarrow spherical (liquid)

CLASSICAL VIBRATIONS OF A LIQUID DROP



\rightarrow An version
of this


(BASIC IDEA)



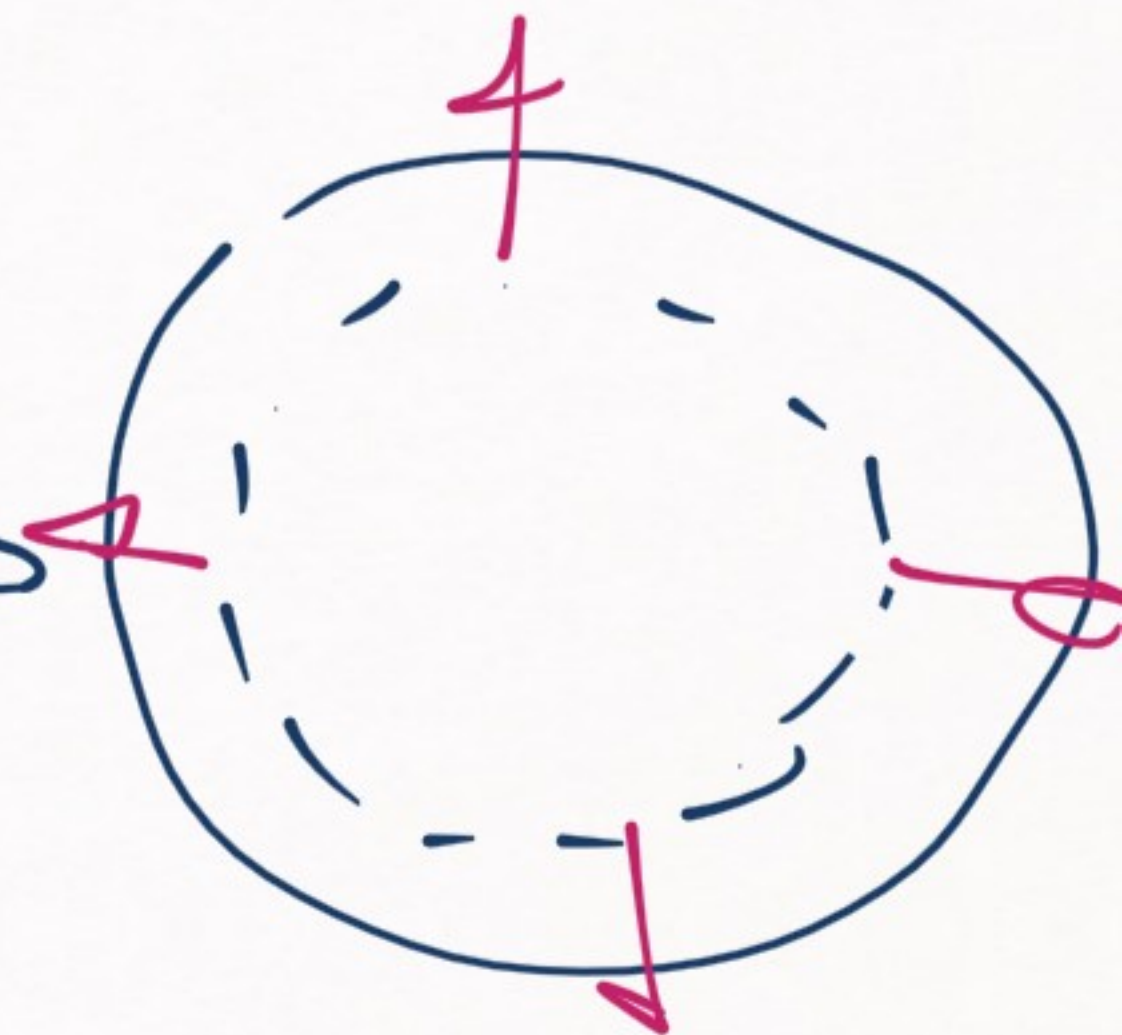
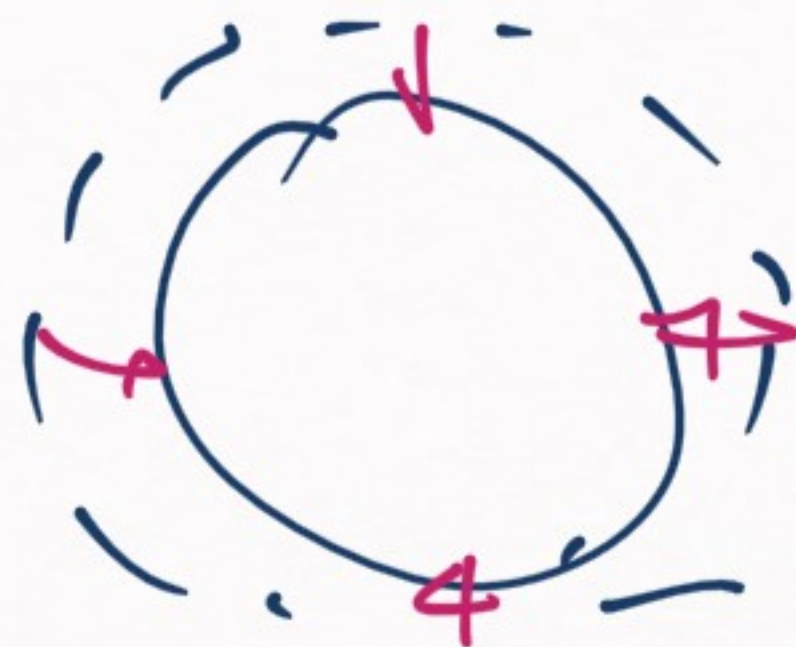
Which vibrations
are
possible



EXCITED STATE
(QUANTIZED
VIBRATION)

TYPES OF VIBRATION

①) $L=0$



"breathing mode"



very high energy

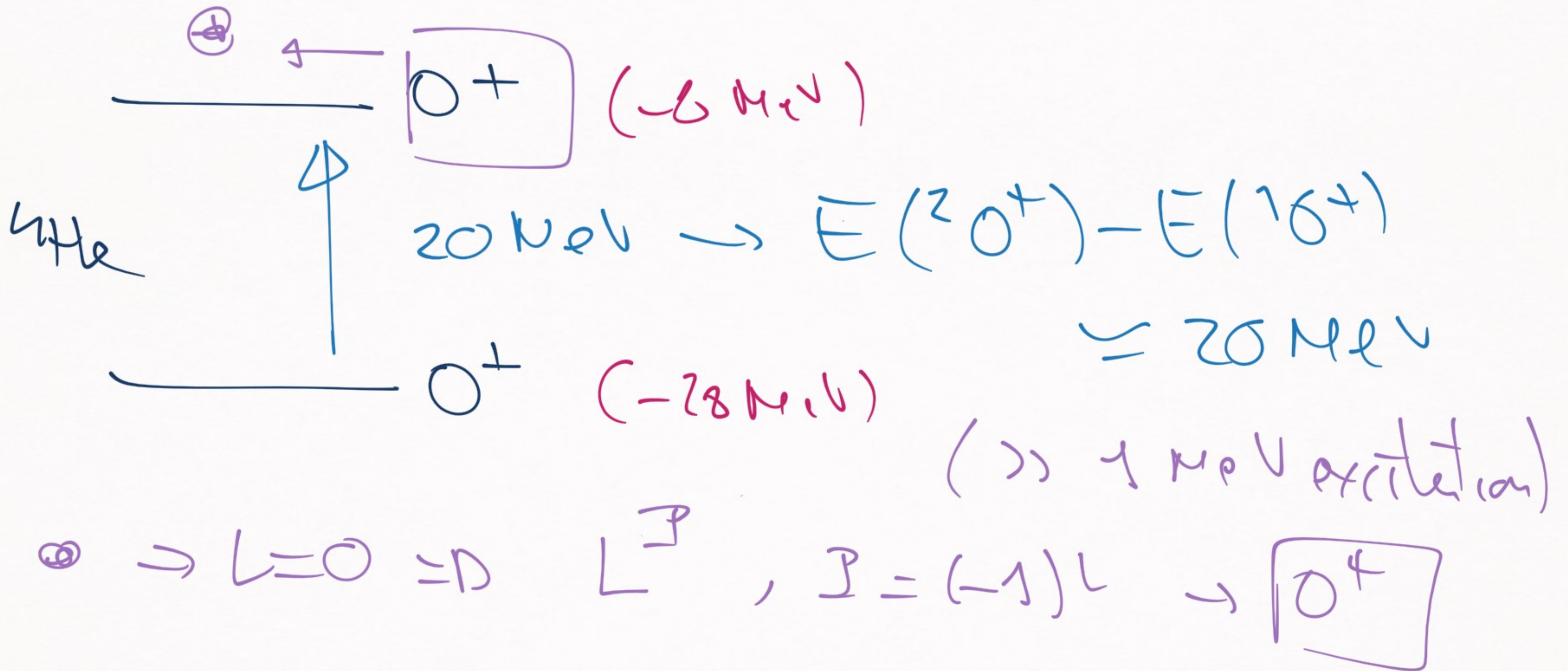


why?



liquids are not easily compressed

BREATHING MODE EXAMPLE \rightarrow ${}^4\text{He}$



1) $L=1$ dipolar vibration (not physical)



Something weird here \rightarrow transition

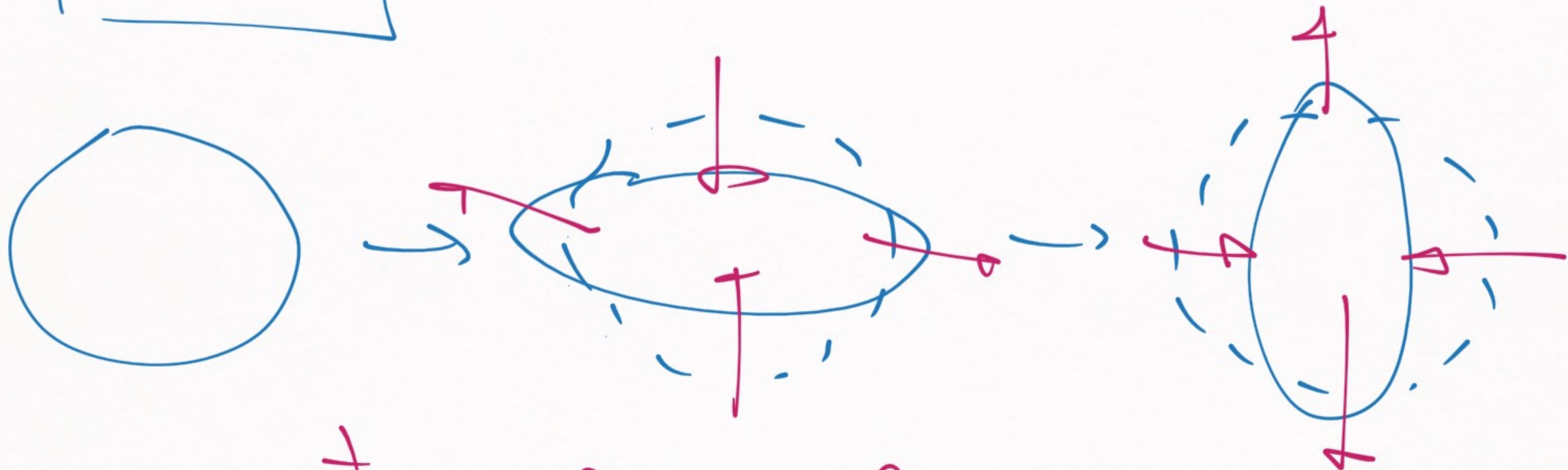
\rightarrow not an internal vibration

\rightarrow should not change the energy

FIGURE
 $L=1$

2) $L=2$

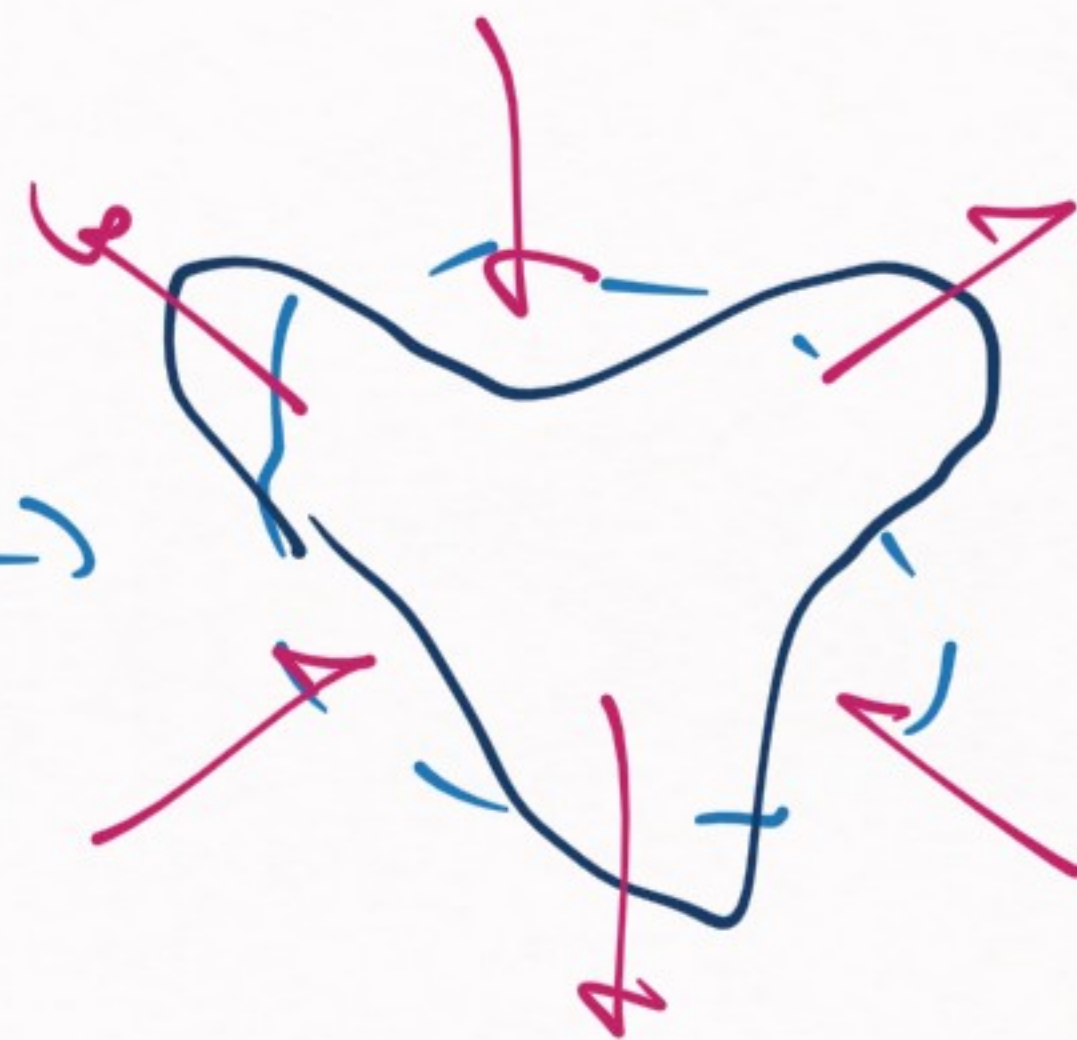
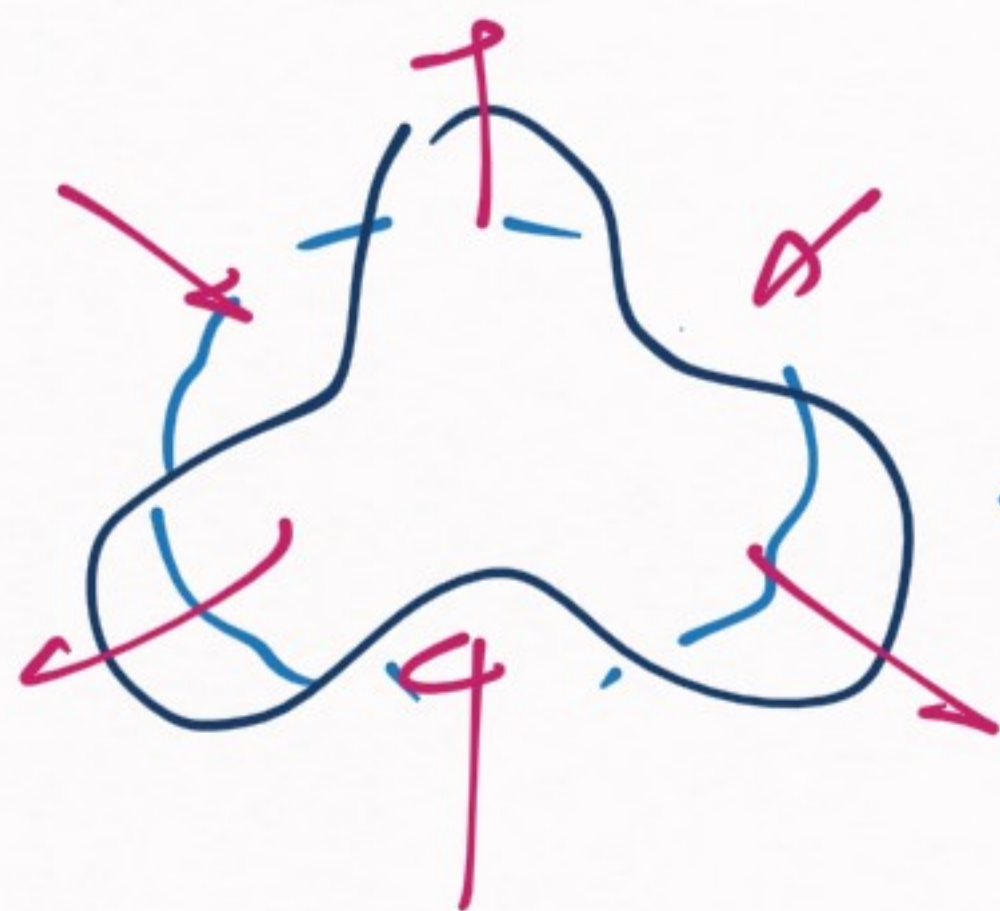
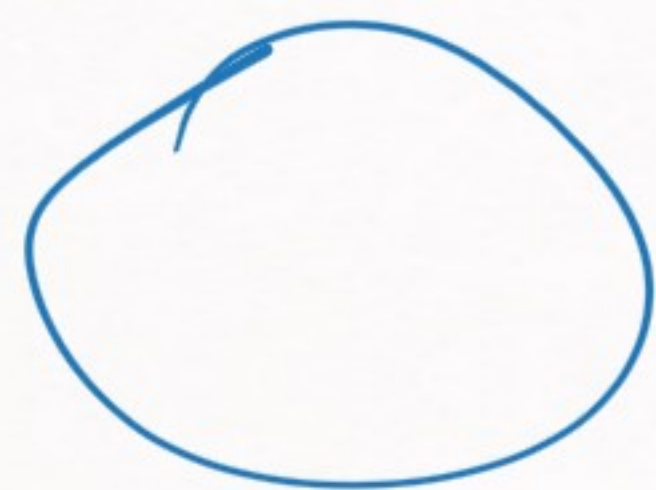
QUADRUPOLE VIBRATION



→ non-trivial & low energy

→ MOST IMPORTANT TYPE OF VIBRATION

3) $L=3$ OCTUPOLAR



$$\omega_3 \leq (2-3)\omega_1$$



[$L=3$ vibrational energy]



$L=7$ energy

4) $l = 4, 5, 6, \dots$

→ higher energy

→ usually not important

(except in certain cases)

RECAP



$L = (0, 2, 3, \dots)$ VIBRATIONS



Too high energy



most common ones



QM DESCRIPTION

→ Second quantization: harmonic oscillator
 $|0\rangle, (a^\dagger)^n |0\rangle \leq |n\rangle$

→ Quantized vibrational state
is called a "phonon"

(maybe you've seen it in statistical mechanics)

Practical terms:

Vibrational state \rightarrow \swarrow # of octupolar excitations

$= |n_0, n_2, n_3, n_4, \dots\rangle$

\swarrow
of breathing excitations

\downarrow
of quadrupolar excitations

$$H |n_0, n_1, n_2, \dots\rangle =$$

$$\hbar (\omega_0 n_0 + \omega_1 n_1 + \omega_2 n_2 + \dots)$$

$$|n_0, n_1, n_2, \dots\rangle$$

ignore
this

+ corrections owing to
phonon-phonon interactions
(small, interacting boson model)

First approximation: $(\hbar=1 \text{ for simplicity})$

$$|1 n_0, n_1, n_2, \dots\rangle = (n_0 \omega_0 + n_1 \omega_1 + n_2 \omega_2 + \dots) |n_0, n_1, n_2, \dots\rangle$$

→ just count the numbers of each type of excitations

$J\pi$

\rightarrow

monopole \rightarrow

0^+

quadrupolar \rightarrow

2^+

octupolar \rightarrow

3^-

more than one excitation

\rightarrow coupled their angular
momenta


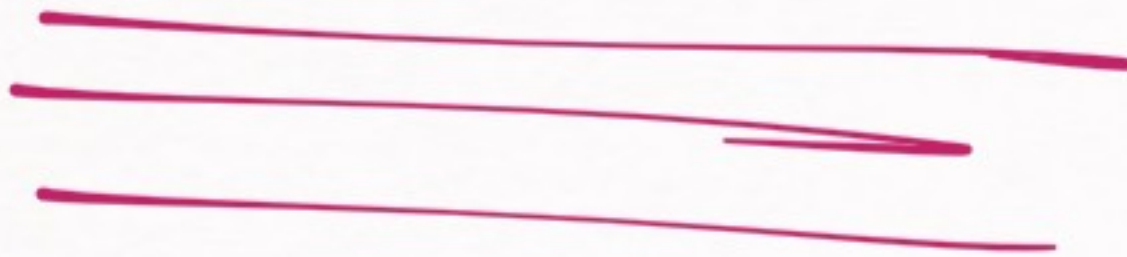


2 quadrupolar phonons.

$$2^+ \oplus 2^+ = 0^+ \oplus 2^+ \oplus 4^+$$

(no 1^+ or 3^+ \rightarrow exercise)

(is really simple)

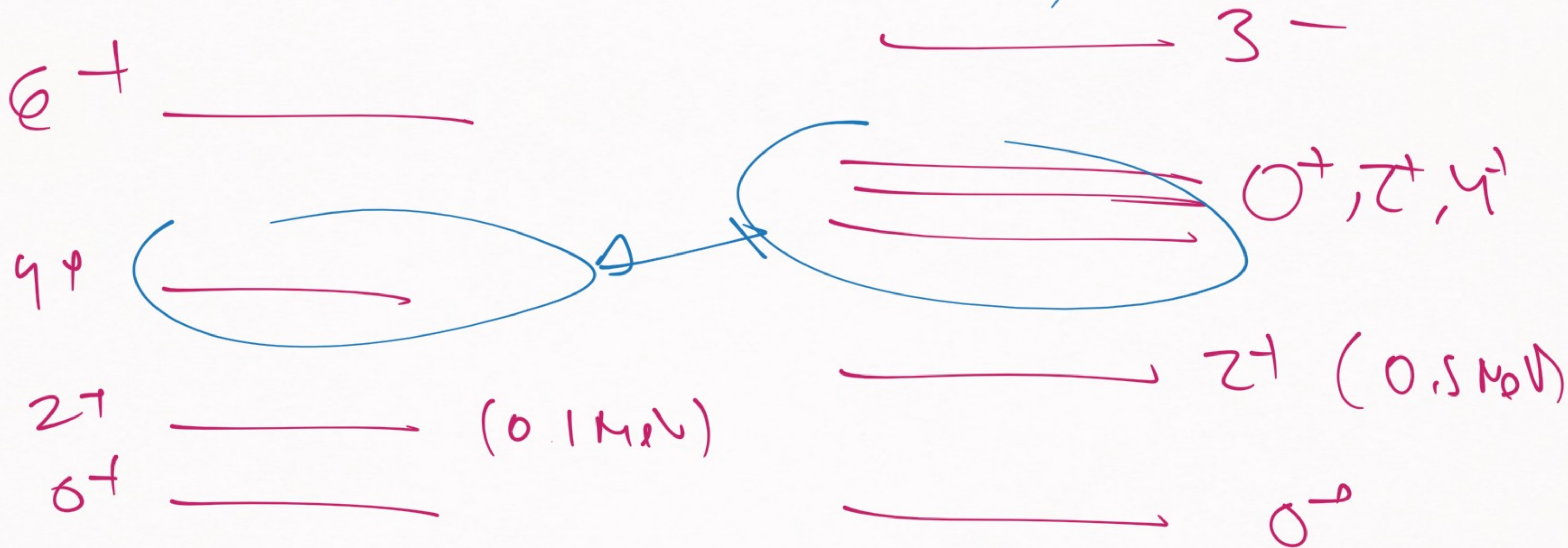
TYPE OF SPECTRUM PREDICTED :

	3^- (one octupole phonon)	$(\rightarrow (2-3) \omega_2)$ ω_3
	$0^+, 2^+, 4^+$ (two quadrupole phonons)	$2\omega_2$
	2^+ (one quadrupole phonon)	ω_2
	0^+ (zero phonons)	0

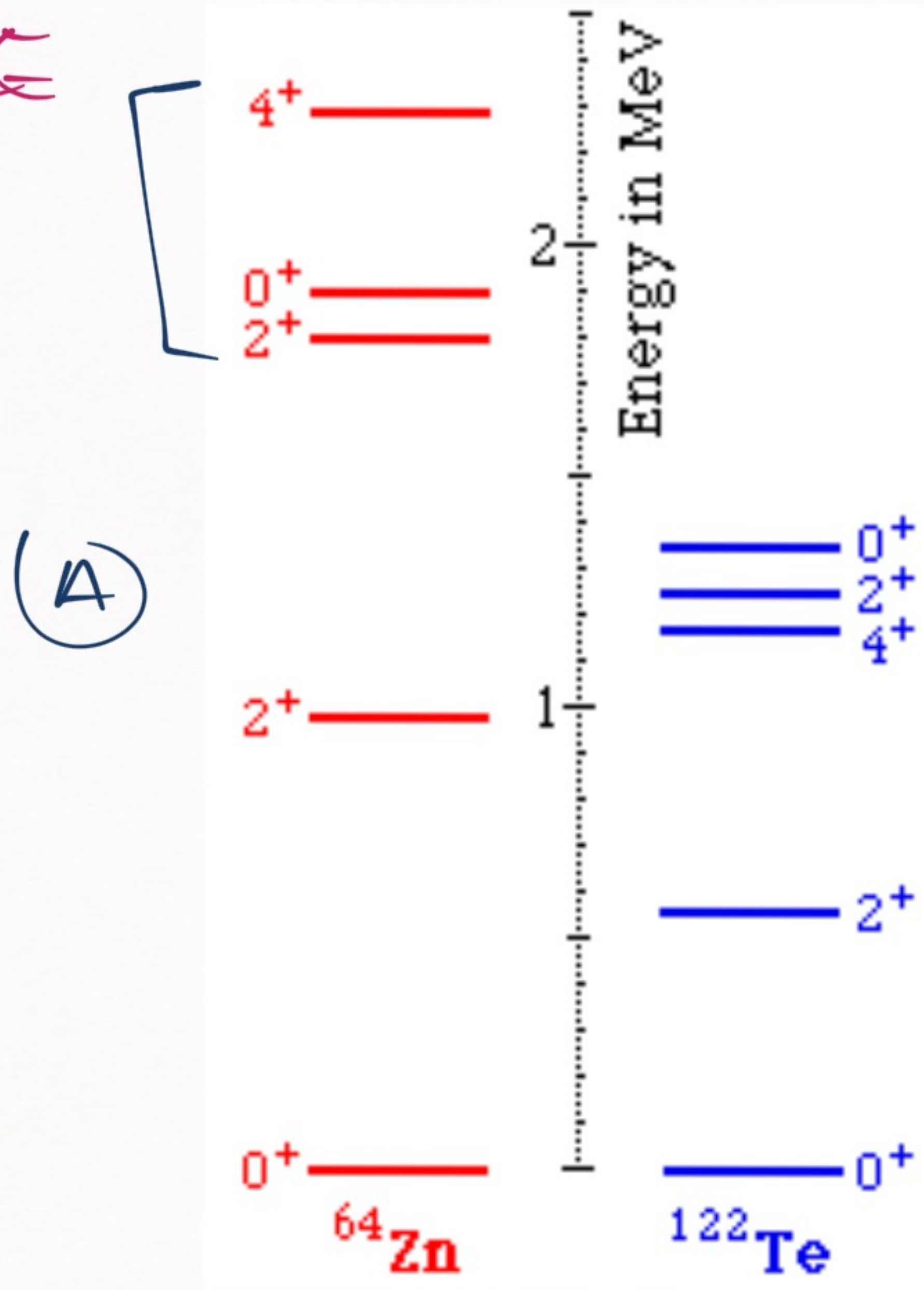
ROTATIONS

VIBRATIONS (DIFFERENT)

(not same scale)



EXAMPLE

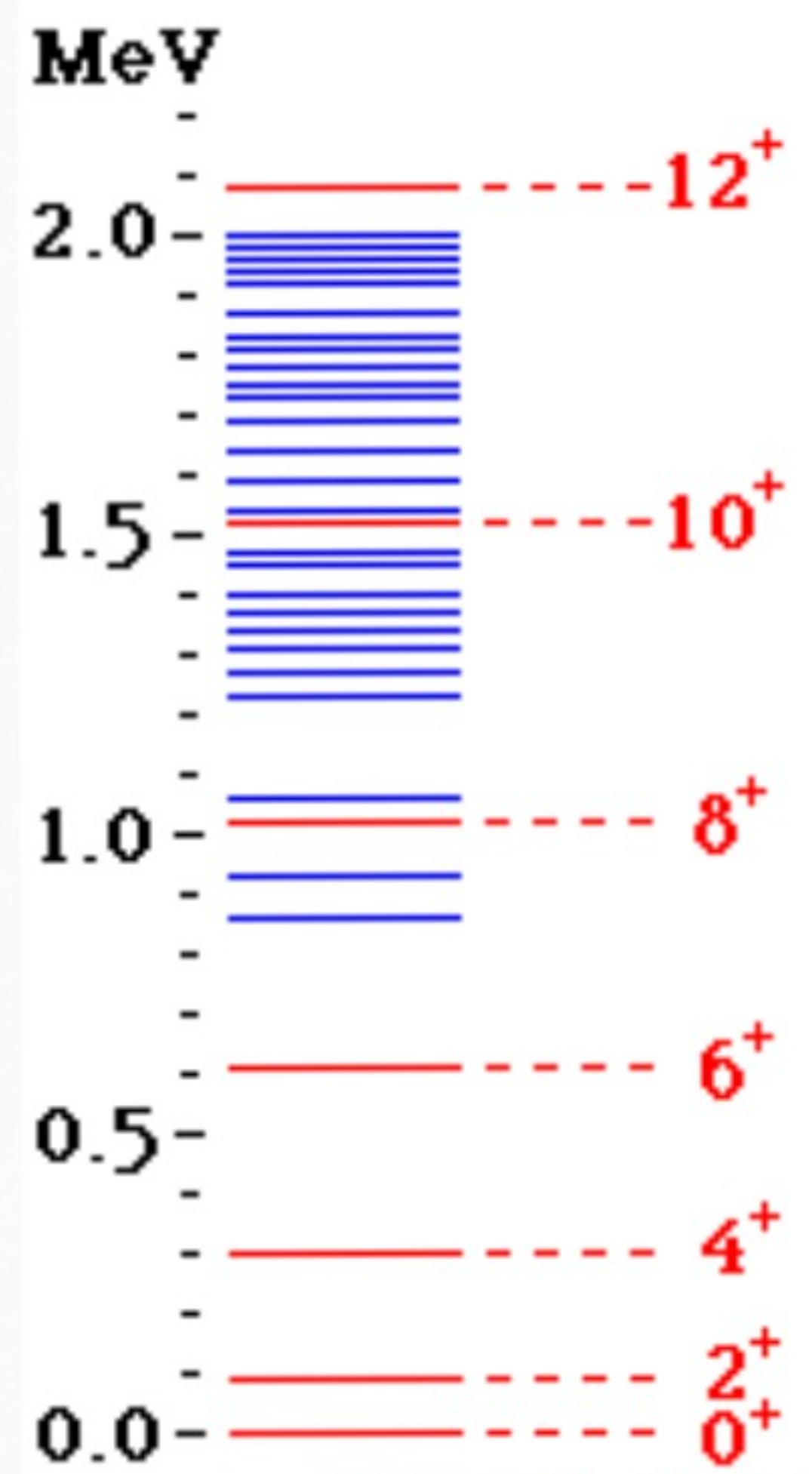


(A) → transition between vibrations & shell model

(B) → typical vibrational spectrum

(B)

^{164}Er energy levels



Ground state rotational band

(R)

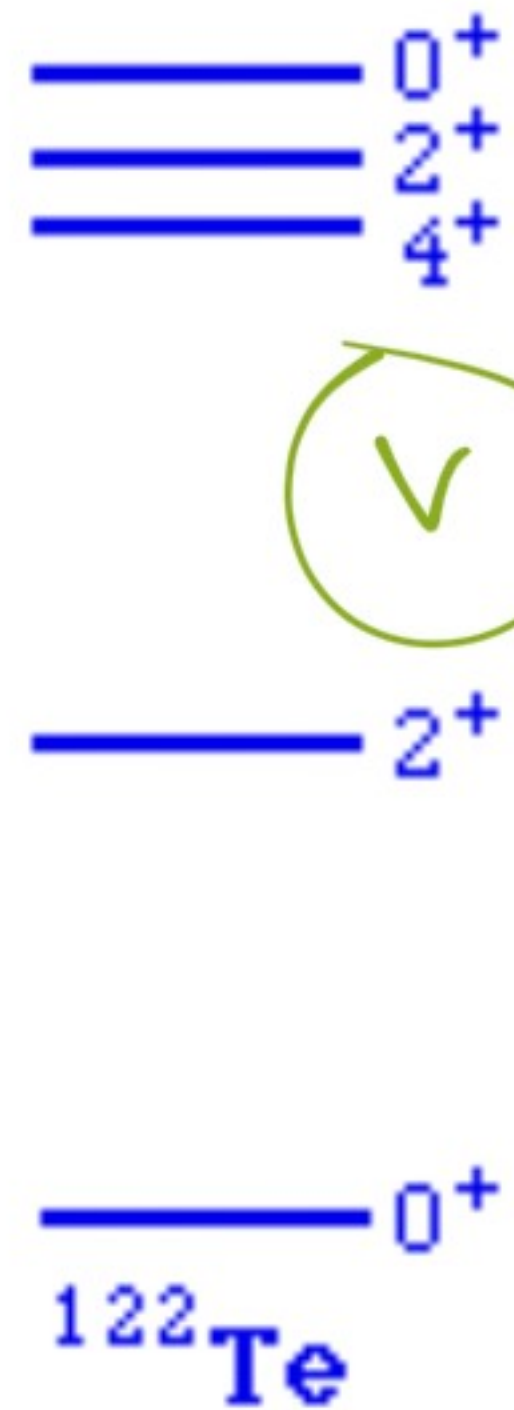
$E(4^+) / E(2^+) \approx 3.33$



^{64}Zn

(S)

Energy in MeV

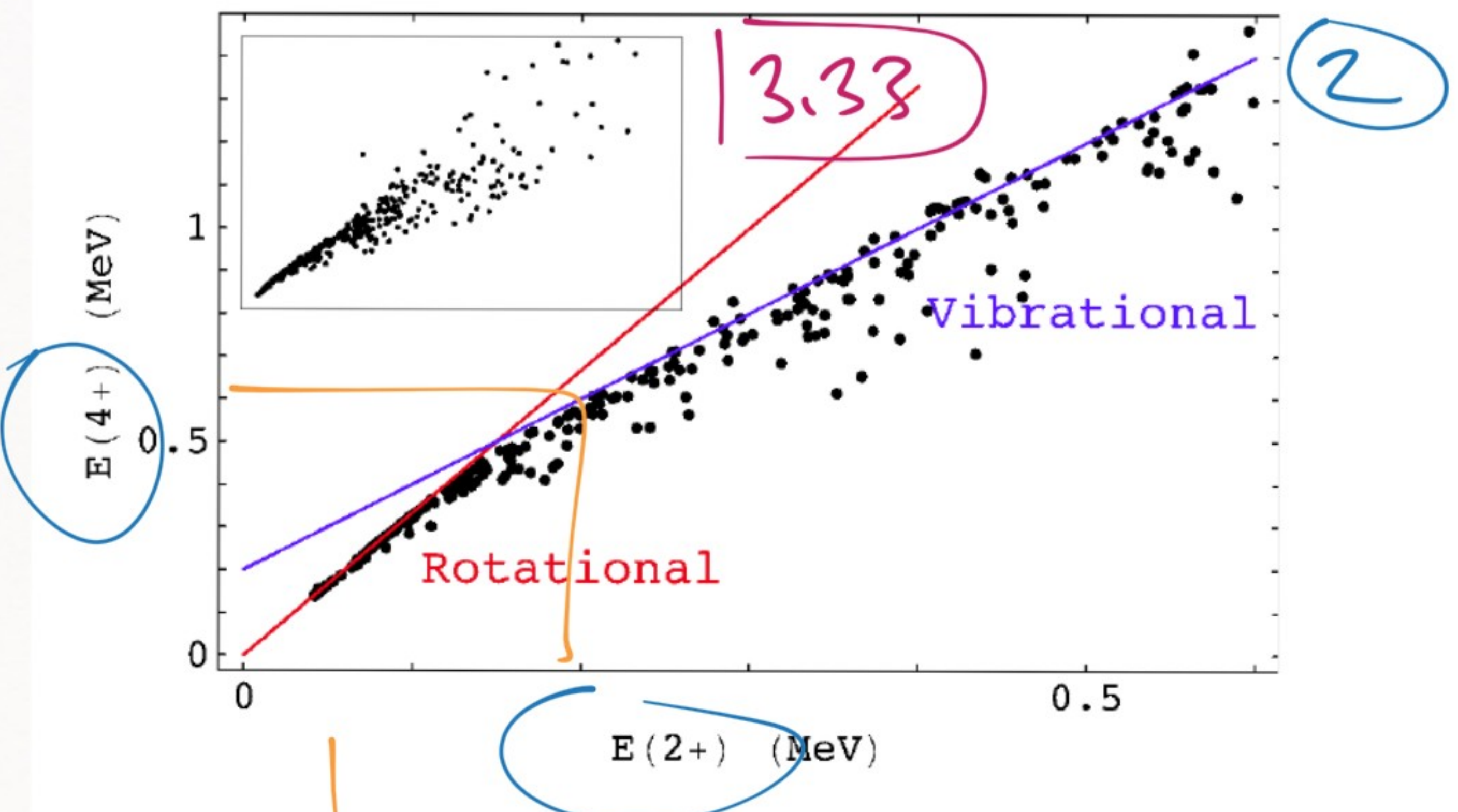


^{122}Te

(v)

R \rightarrow Rotational
 v \rightarrow vibrational
 S \rightarrow shell model


$$\frac{E(4^+)}{E(2^+)} \approx 2$$



N.V. Zamfir *et al.*, Phys. Rev. Lett. 72 (1994) 3480

transition to rotational pattern

→ COLLECTIVE MODEL
IS RELATIVELY EASY



→ [SHELL-MODEL]

→ RESIDUAL INTERACTION

(beyond the pairing interaction)

GOOD THINGS ABOUT SHELL MODEL:

- 1) magic numbers (close to magic)
- 2) separation energies about right

Review the basic assumptions:

1) MEAN-FIELD POTENTIAL

$$H = \sum_0 T_i + \sum_{(ij)} v_{ij} + \sum_{(ijk)} V_{ijk} + \dots$$

$$\rightarrow H = \sum_i (T_i + V_i^{\text{MF}}) + \Delta V$$

Residual

2) NUCLEONS ARE FERMIONS

3) FILL SHELLS TILL THEY ARE FULL

1) JP near closed shells well predicted

(160 → 150, 170) ✓

2) Excited spectra near a closed shell

(94 Cu, 38 Ni) ✓

3) Pairing interaction to explain deviation for high J shells

(203 Tl, 207 Pb) ✓

→ very basic features

→ a few big things were left
unexplained

1) How do we define $V_{MF}(s)$?

Before, we have just used an
ad-hoc potential that works

$$V_{MF} = \frac{1}{2} m \omega^2 r^2 - \sum \vec{e}_i \cdot \vec{r}$$

(ad-hoc, but explained the shell order)

Other options that allow to calculate

V_{MF} (Hartree-Fock, Skyrme, Gogny...)

2) How to deal with ΔV ?

→ Pairing interaction (most simple example of ΔV)

→ \exists much more complicated way (more accurate) ways to do this

→ SEE YOU ON TUESDAY

