NUCLEAR PHYSICS 18

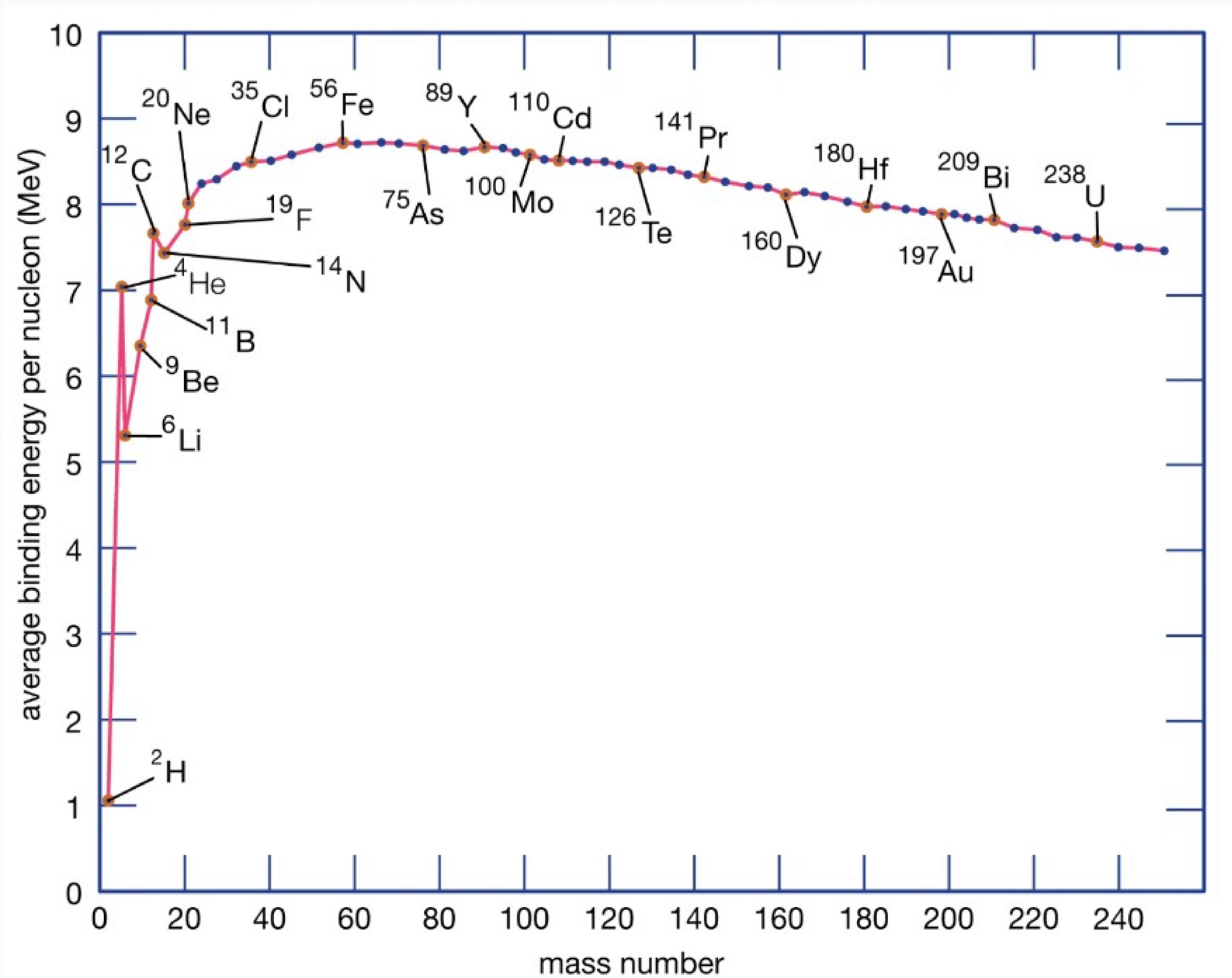
Nuclear properties
(part II)

RECAP → Description of nuclei

[What to include/describe?]

1) Binding energy → Last Lesson
2) Size of nuclei (we were at the middle of this)
3) J^P (angular momentum/parity)
4) Electromagnetic moments
5) Decays

1) BINDING ENERGY $\longrightarrow$ $\boxed{\frac{B}{A} \sim 8 \text{ MeV/nucleon}}$



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Saturation

~ mid-range attraction

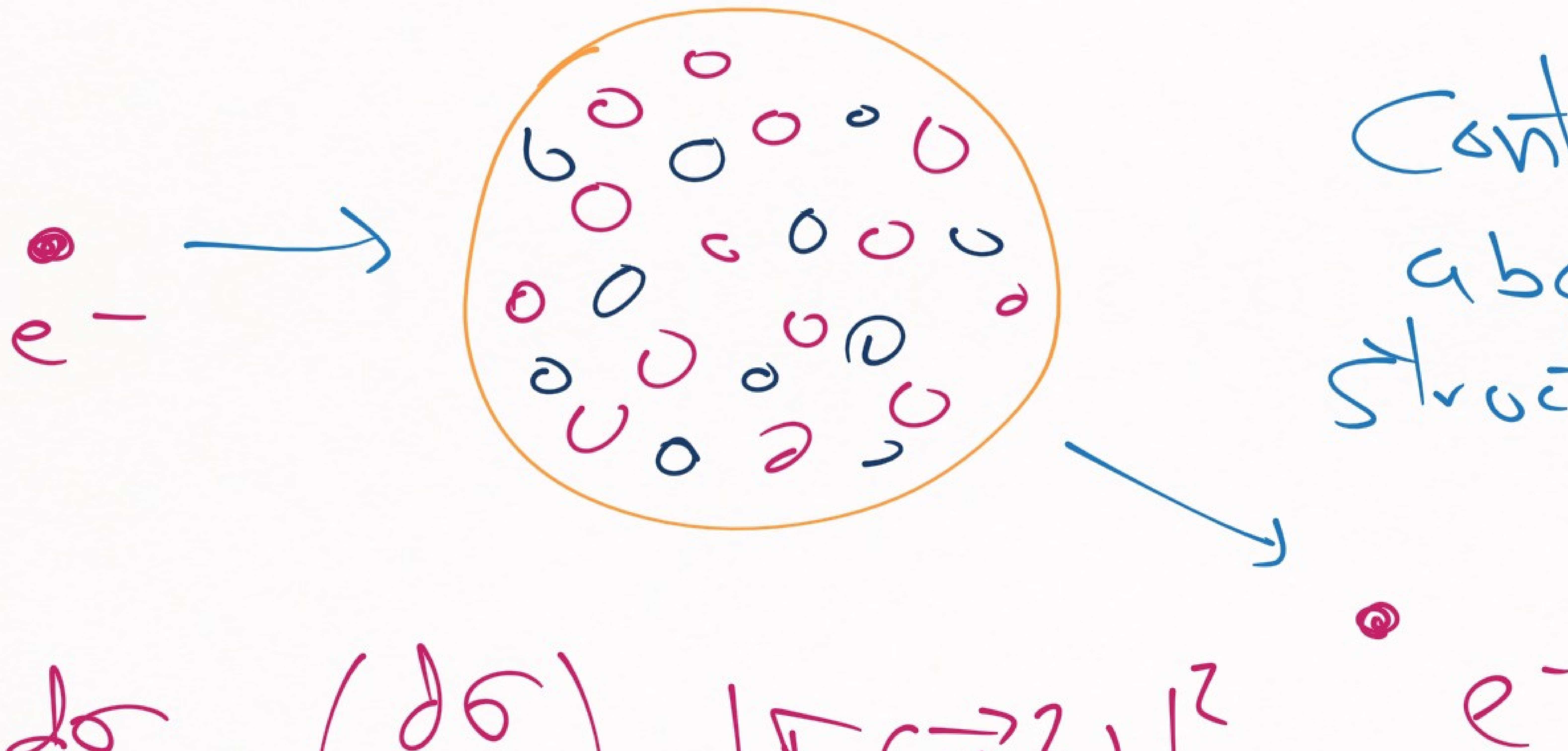
~ short-range repulsion

$\Longrightarrow$

LIQUID DROP MODEL

(explains B/A)

2) SIZE OF NUCLEI (AND DENSITY DISTRIBUTION)

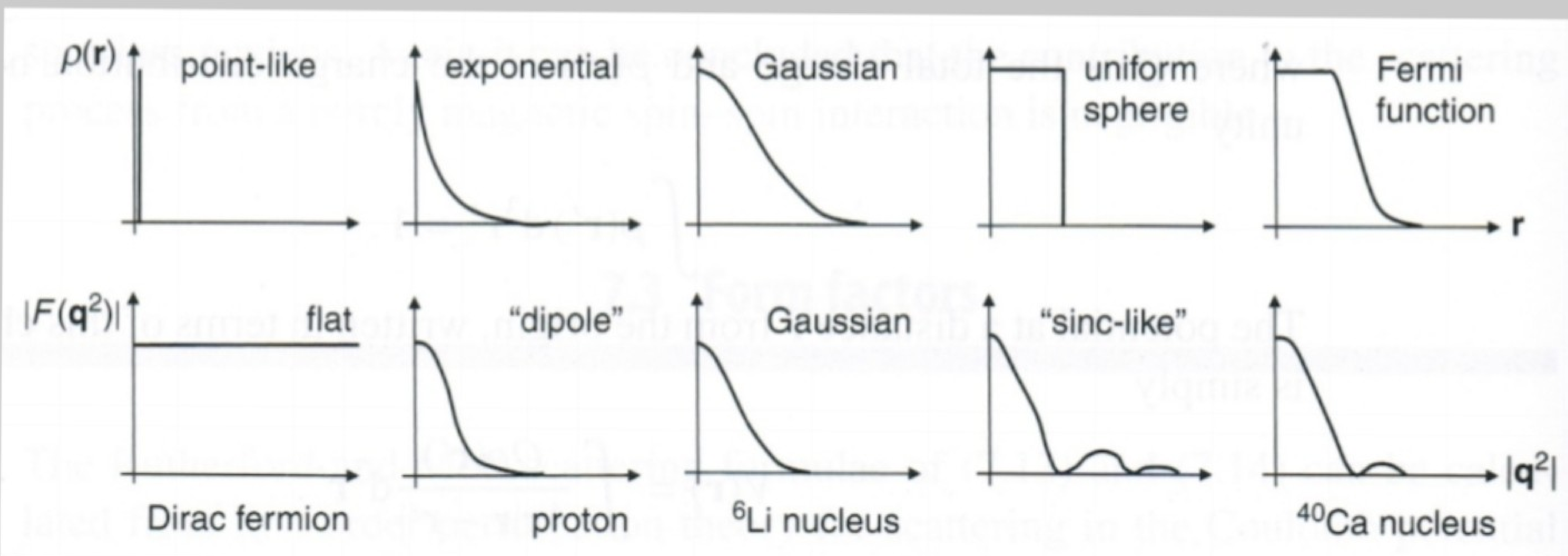


Contains info about the structure of nuclei

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{point-like}} |F(\vec{q}^{\,2})|^2$$

$\boxed{\text{Form factor}}$

$$F(\vec{q}^{\,2}) = \int d^3\vec{r}\, \underbrace{\rho(\vec{r})}_{\sim |\psi(\vec{r})|^2} e^{-i\vec{q}\cdot\vec{r}}$$



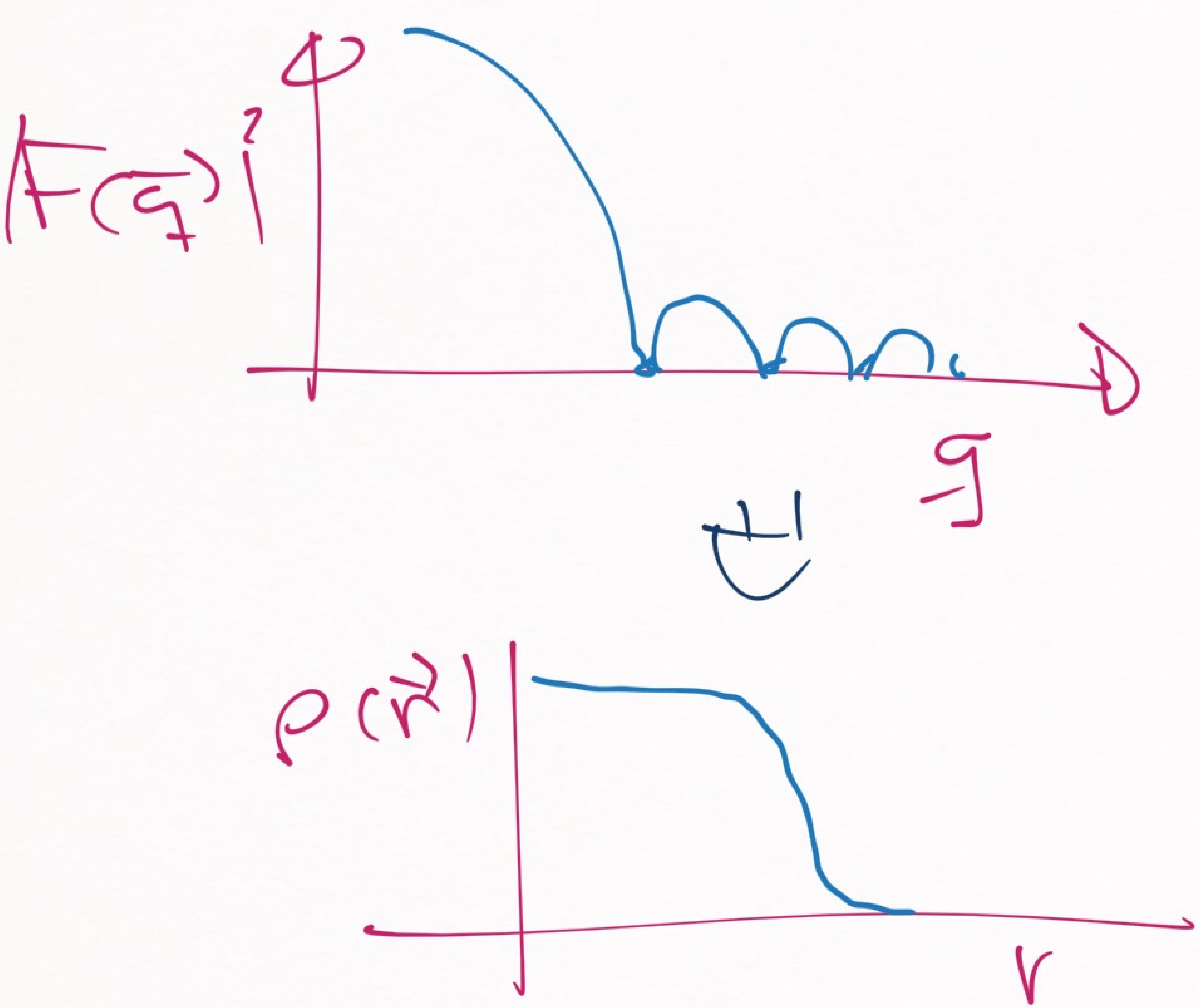
taken from ref. [2]

$\rho(\vec{r})$

$\downarrow$

$F(\vec{q}^{\,2})$

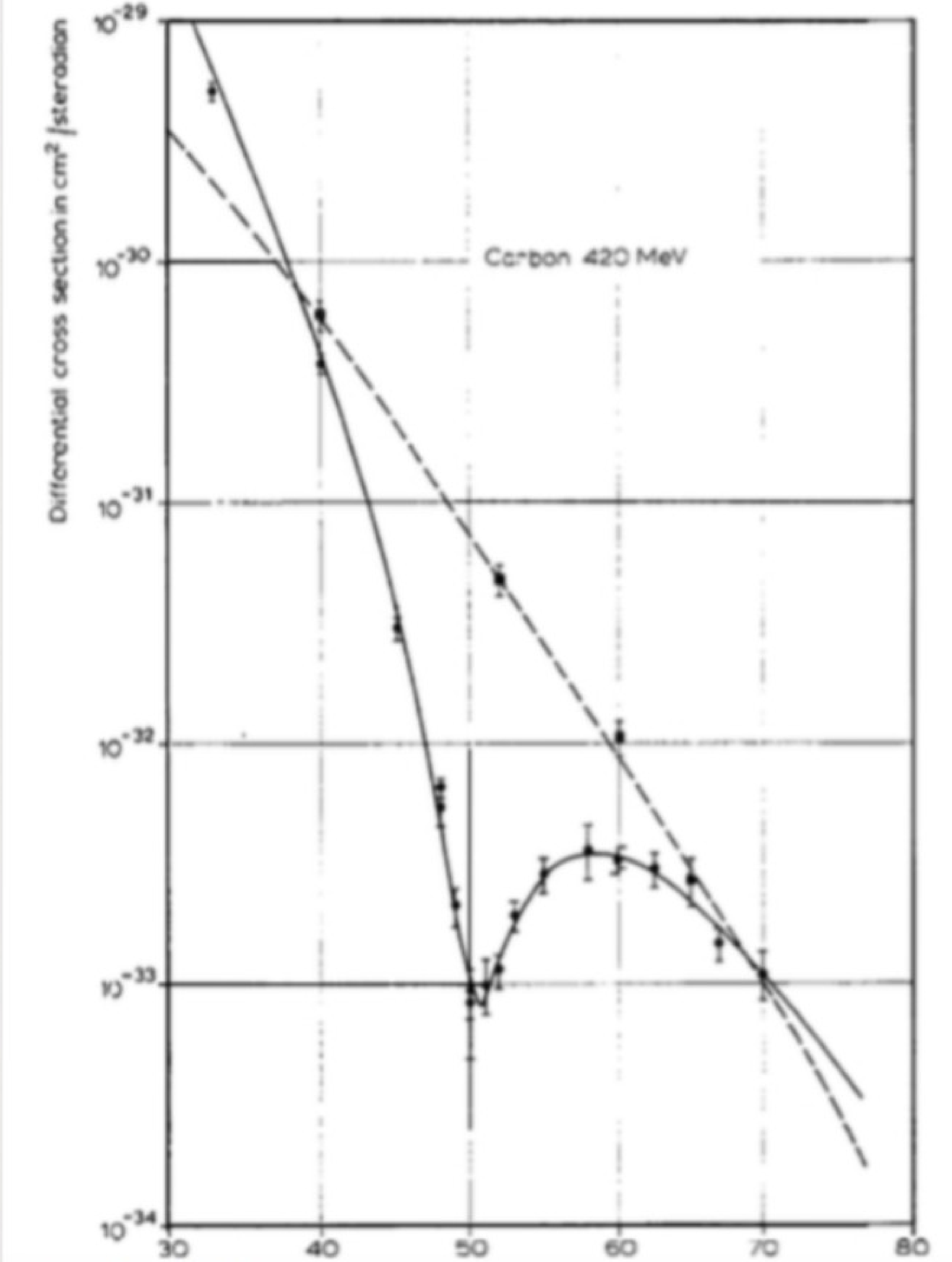
Experiments see this:



$\Rightarrow$ only possible ρ:

1) constant density nucleus

2) density falls sharply near the surface



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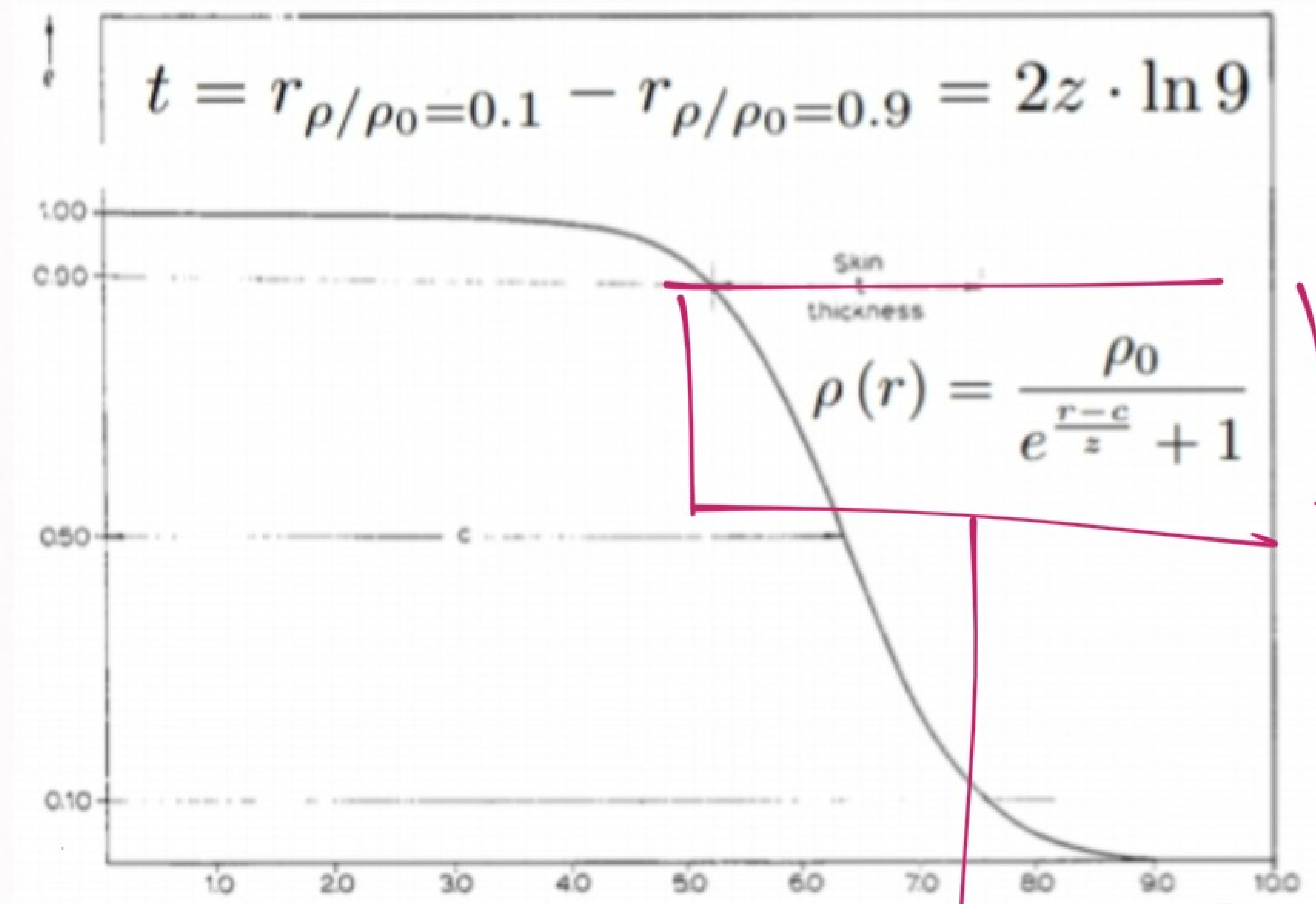


taken from ref. [5]

Woods-Saxon distro

Hofstadter experiment

→ Woods-Saxon : typical for nuclei

$$\boxed{\rho(r) = \frac{\rho_0}{1 + e^{(r-R_0)/a_0}}}$$

Typical parameters

$\rho_0 \simeq 0.17\ \mathrm{fm}^{-3}$

$a_0 \simeq 0.54\ \mathrm{fm}$

$R_0 \simeq 1.128\, A^{1/3}\ \mathrm{fm}$

$(1.1 - 1.2)$

→ Saturation

→ constant density

→ mid/short range V_{NN}

3) ANGULAR MOMENTUM & PARITY (J^P)

$J^P(\pi) = 0^-$

$J^P(\rho) = 1^-$

$J^P(\sigma) = 0^+$

$P: \vec{r} \longrightarrow -\vec{r}$

$\vec{a} \longrightarrow -\vec{a}$ $\quad \boxed{J^P : 1^-}$

$\vec{a} \wedge \vec{b} \longrightarrow (-\vec{a}) \wedge (-\vec{b}) = \vec{a} \wedge \vec{b}$ $\quad \boxed{J^P = 1^+}$

$\vec{a} \cdot \vec{b} \longrightarrow (-\vec{a}) \cdot (-\vec{b})$ $\quad \boxed{J^P = 0^+}$

$\vec{a} \cdot (\vec{b} \wedge \vec{c}) \longrightarrow (-\vec{a}) \cdot ((-\vec{b}) \wedge (-\vec{c}))$ $\quad \boxed{J^P = 0^-}$

$J^P(n) = \frac{1}{2}^{+}$ → convention

— · —

$J^P(d) = 1^+$ d → deuteron

$J^P(^3H/^3He) = \frac{1}{2}^+$

$J^P(^4He) = 0^+$

it's easy to measure

Why JP ? → JP of excited states of a nucleus

Clues about nuclear structure

a few examples:

^4He

0^+ $\boxed{B(^4He^*) \sim 8 \text{ MeV}} \rightarrow \oplus$

0^+ $B(^4He) \sim 28$ MeV

$\oplus \rightarrow$ "BREATHING MODE" OF THE COLLECTIVE MODEL

Later, we will see a few models

→ Shell-model

→ Collective model $\begin{cases} \text{Vibrational} \\ \text{Rotational} \end{cases}$

Each one predicts different J^P for excited states

1) ^4He

0^+

0^+

⊂ vibrational

(breathing)

2) ^{41}Ca

$3/2^+$

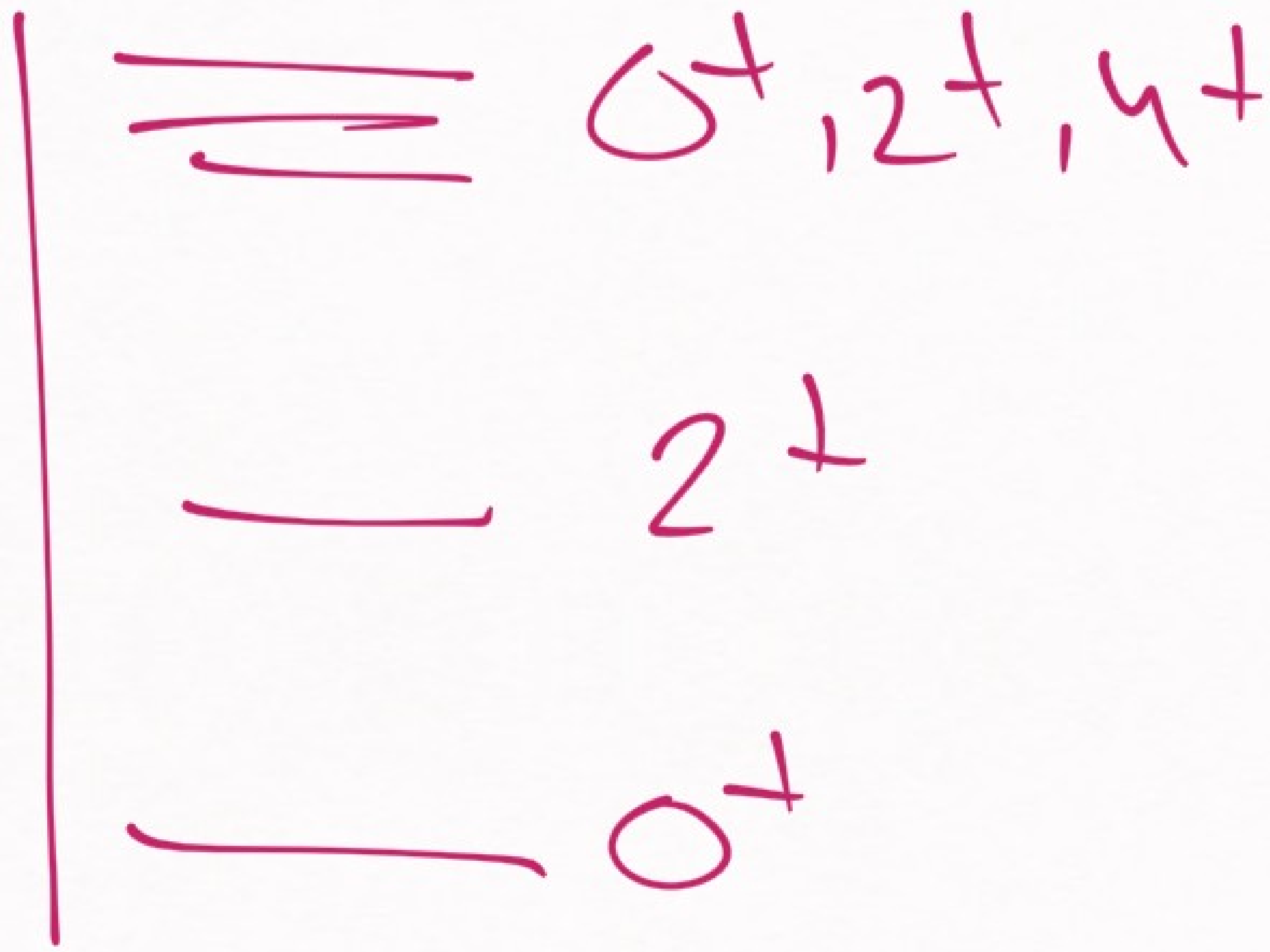
$3/2^-$

$7/2^-$

⊂ shell-model

3)

^{122}Te



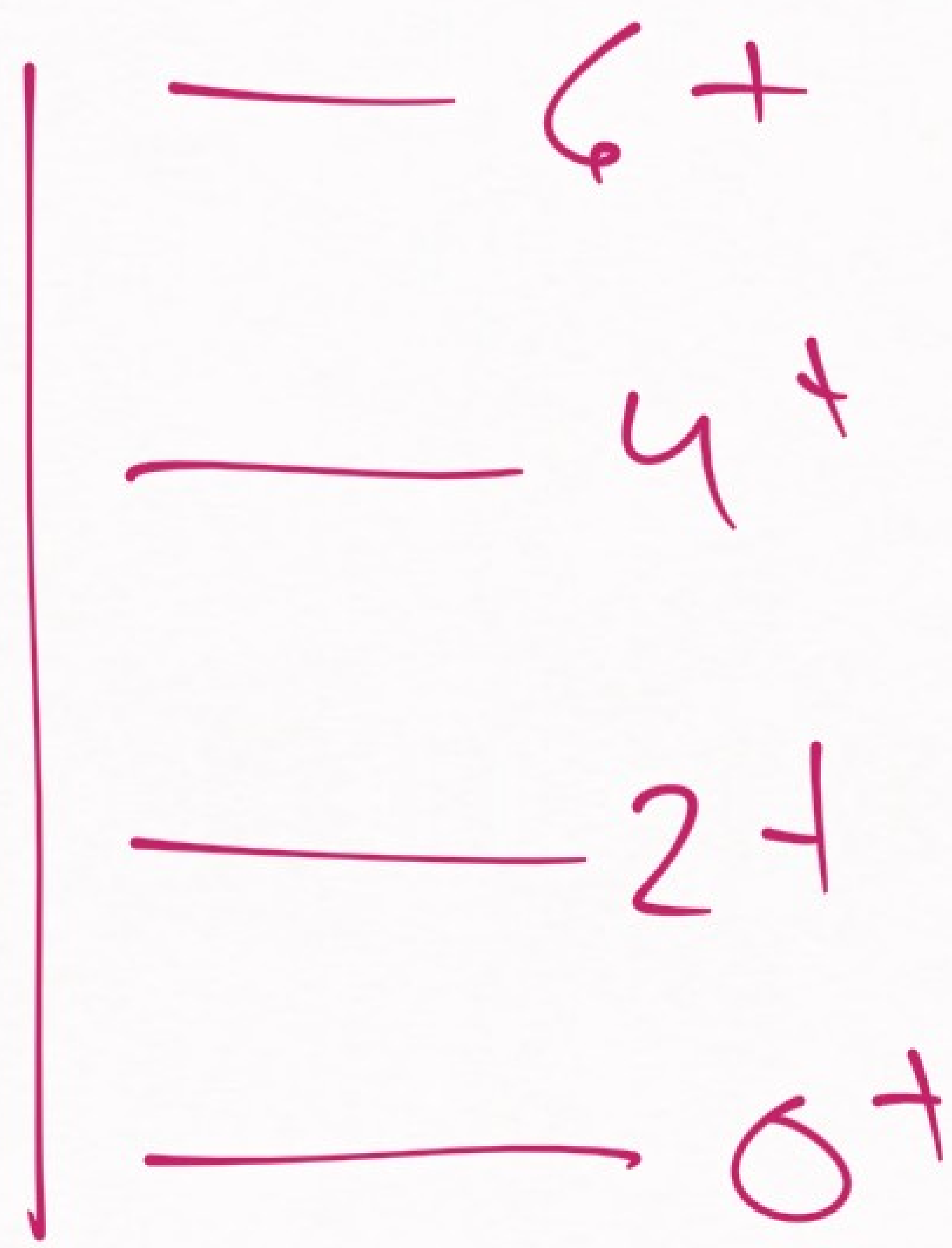
← vibrational

(quadrupole excitations)

[H
^4He]

4)

^{166}Er



rotational

$\boxed{1, 2, 3, 4}$ $\rightarrow$ show how JP will connect w/ nuclear models

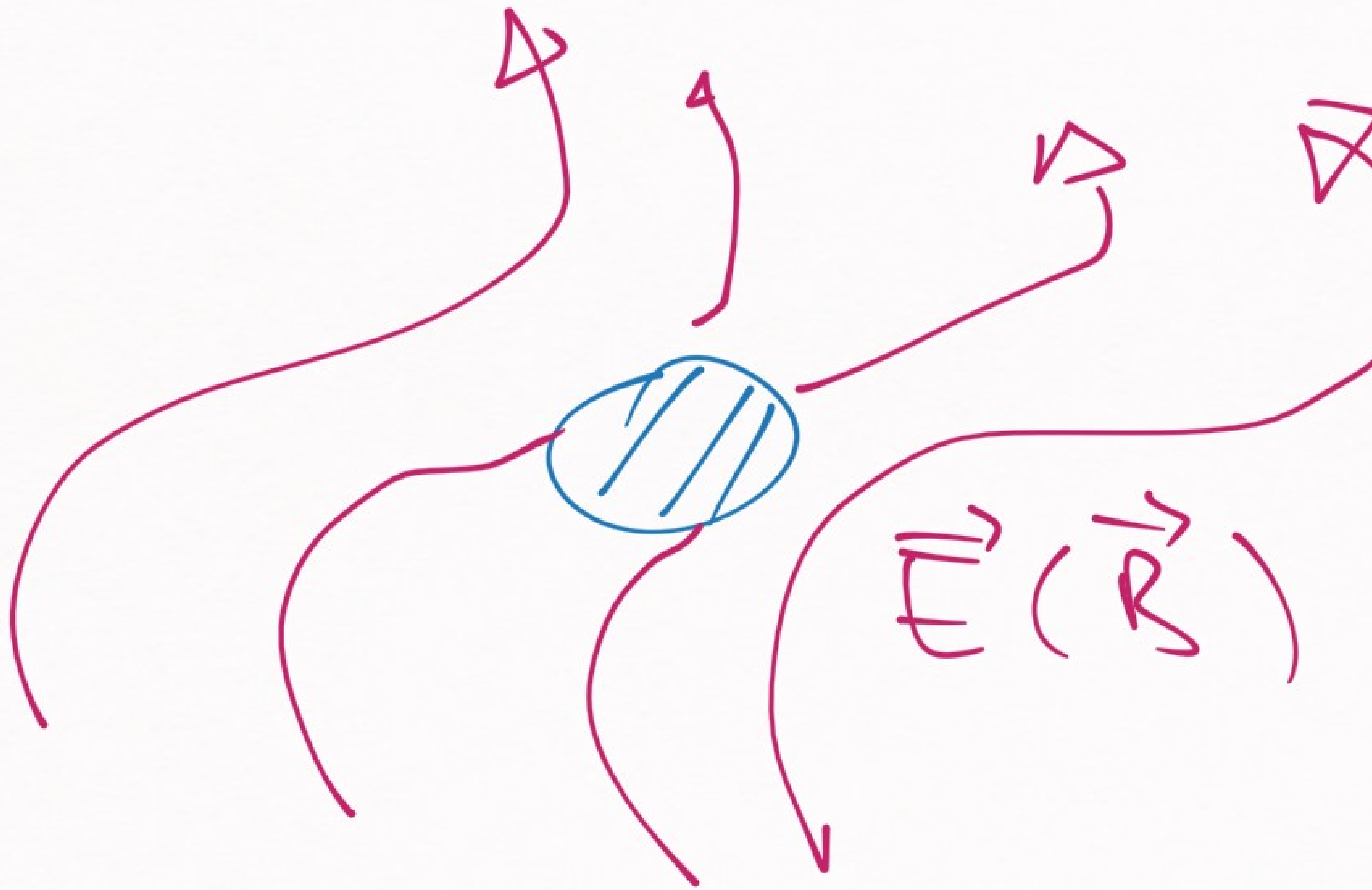
(we will study this connection soon)

4) ELECTROMAGNETIC PROPERTIES OF NUCLEI

(magnetic dipole moment & electric quadrupole moment)

→ What are these? =D

What are "moments"? $\longrightarrow$ DESCRIBE THE POTENTIAL ENERGY FOR $\rho(\vec{r})$ IN A COMPACT WAY



 $\longrightarrow$ CHARGE DISTRIBUTION $\rho(\vec{r})$

$\rho(\vec{r}) \rightarrow$ [maybe really complex] $\rightarrow$ [we use "moments" instead]

ELECTRIC MOMENTS

MAGNETIC MOMENTS

$\Rightarrow \oplus$

⊛ ⇒ ELECTRIC MOMENTS

$$V = \int d^3\vec{r}\; \underbrace{\rho(\vec{r})}_{\text{charge distro}}\; \Phi(\vec{r})$$

scalar potential

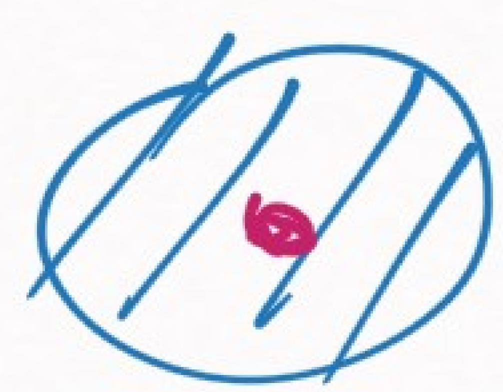
$$\vec{E} = -\vec{\nabla}\Phi$$

TAYLOR EXPANSION

$$\Phi(\vec{r}) = \Phi(\vec{0}) + r_i \partial_i \Phi(\vec{r})\big|_{\vec{r}=0} + \frac{1}{2} r_i r_j \partial_i \partial_j \Phi\big|_{\vec{r}=0} + \ldots$$

(as many terms as we want)

ASSUMPTION 1:



$\vec{r} = \vec{0}$

ASSUMPTION 2:

$\vec{E}$

$$V = \Phi(\vec{0}) \int d^3\vec{r}\, \rho(\vec{r}) + \partial_i \Phi \big|_{\vec{0}} \int d^3\vec{r}\; r_i\, \rho(\vec{r})$$

$$+ \frac{1}{2} \partial_i \partial_j \Phi \big|_{\vec{r}=0} \int d^3\vec{r}\; r_i r_j\, \rho(\vec{r}) + \ldots$$

$$\partial_i \partial_j \partial_k \ldots \Phi \longleftrightarrow \int d^3\vec{r}\, (r_i r_j r_k \ldots)\, \rho(\vec{r})$$

Every moment will be given a name:

$$\underline{q} = \int d^3\vec{r}\, \rho(\vec{r}) \longrightarrow \text{charge}$$

$$\vec{d} = \int d^3\vec{r}\, \vec{r}\, \rho(\vec{r}) \longrightarrow \text{dipole moment}$$

$$Q_{ij} = \int d^3\vec{r}\, (3 r_i r_j - \delta_{ij} r^2)\, \rho(\vec{r}) \longrightarrow \underline{\underline{\text{quadrupole moment}}}$$

$$\langle r^2 \rangle = \int d^3\vec{r}\, r^2 \rho(\vec{r})$$

$$V = q \cdot \Phi - \vec{d} \cdot \vec{E} - \frac{1}{6} Q_{ij} \partial_i E_j + \dots$$

$\rightarrow$ Expansion for the potential energy of a charge distribution within an electric field

MAGNETIC FIELD

$\vec{\mu}(\vec{r}) \rightarrow$ magnetic moment distribution

$$V = -\int d^3\vec{r}\; \vec{\mu}(\vec{r}) \cdot \vec{B}(\vec{r})$$

$$\vec{B}(\vec{r}) = \vec{B}(\vec{0}) + \vec{r} \cdot \vec{\nabla}\vec{B}\Big|_{\vec{r}=0} + \dots \Rightarrow \otimes$$

$\oplus \Rightarrow \quad V = -\vec{\mu}\cdot\vec{B}\big|_{\vec{0}} - \frac{1}{6} Q_{\mu ij}\, \partial_i B_j \big|_{\vec{0}} + \ldots$

magnetic dipole magnetic quadrupole

$$\vec{\mu} = \int d^3\vec{r}\, \vec{\mu}(\vec{r})$$

$$Q_{\mu ij} = \int d^3\vec{r} \left[\frac{3}{2} r_i \mu_j(\vec{r}) + \frac{3}{2} r_j \mu_i(\vec{r}) - \vec{r}\cdot\vec{\mu}(\vec{r}) \right]$$

→ We have well-defined moment,

[SIMPLIFICATION] → Electric dipole

For ∀ nuclei

$$\boxed{\begin{aligned} \vec{d}_E &= \vec{0} \\ Q_{My} &= 0 \end{aligned}}$$

→ WHY?

magnetic quadrupole

$$\rho(\vec{r}) = |\psi(\vec{r})|^2$$

$\vec{r} \to -\vec{r}$ (parity transformation)

$$\psi(\vec{r}) \longrightarrow (-)^P \psi(\vec{r})$$

$$\rho(\vec{r}) \longrightarrow \rho(\vec{r})$$

$$\int d^3\vec{r}\ \underbrace{\rho(\vec{r})\,\vec{r}} = \vec{0}$$

$\vec{r} \rightarrow -\vec{r}$, the integral should be the same

$$\vec{d} \rightarrow -\vec{d} \implies \vec{d} = -\vec{d} \Rightarrow \vec{d} = 0$$

$\rightarrow$ $\forall$ electric dipole moments are zero (only possibility of $\vec{d}_E \neq \vec{0}$ is that the neutron/proton have intrinsic $\vec{d}_E \neq 0$)

$\rightarrow$ Beyond the standard model physics

This is why $\exists$ experiments trying to find $\vec{d}_E \neq \vec{0}$

$\rightarrow \boxed{\vec{d} = \vec{0}}$ for $\forall$ nuclei

Repeating the same arguments:

$$\boxed{Q_{\mu y} = 0}$$

(if we have transition,
they can be $\neq 0$)

In general, we only worry about:

1) MAGNETIC DIPOLE MOMENT

2) ELECTRIC QUADRUPOLE MOMENT

($\exists$ ALSO CHARGE, BUT IT'S TRIVIAL)

Sometimes $\rightarrow$ magnetic octupole moment
(8-pole)

electric hexadecapole moment
(16-pole)

(but these are usually small & not particularly important)

What can we learn from μ, & Q ?

magnetic dipole

electric quadrupole

Let's see $\boxed{\text{the deuteron}}$ $\rightarrow$ $\underline{\underline{\mu, Q}}$

DEUTERON'S QUADRUPOLE & MAGNETIC MOMENTS

$$\rho(\vec{r}) = \langle \Psi_A | \sum_{k=1}^{A} e_k | \Psi_A \rangle$$

$$= \langle \Psi_A | \sum_{k=1}^{A} e_k \, \delta(\vec{r} - \vec{r}_k) | \Psi_A \rangle$$

$$Q_{ij} = \int d^3\vec{r} \, (3 r_i r_j - \delta_{ij} r^2) \, \rho(\vec{r})$$

1) Deuteron $\quad Q_d = 0.286\ e\,fm^2$

2) Triton, ^3He $\quad Q = 0$

3) ^4He $\quad Q = 0$ → Why is this?

$$\boxed{Q = \langle JJ | Q_{33} | JJ \rangle}$$ → Definition

Trick for the moments :

$J \rightarrow \exists (2J+1)$ moments only

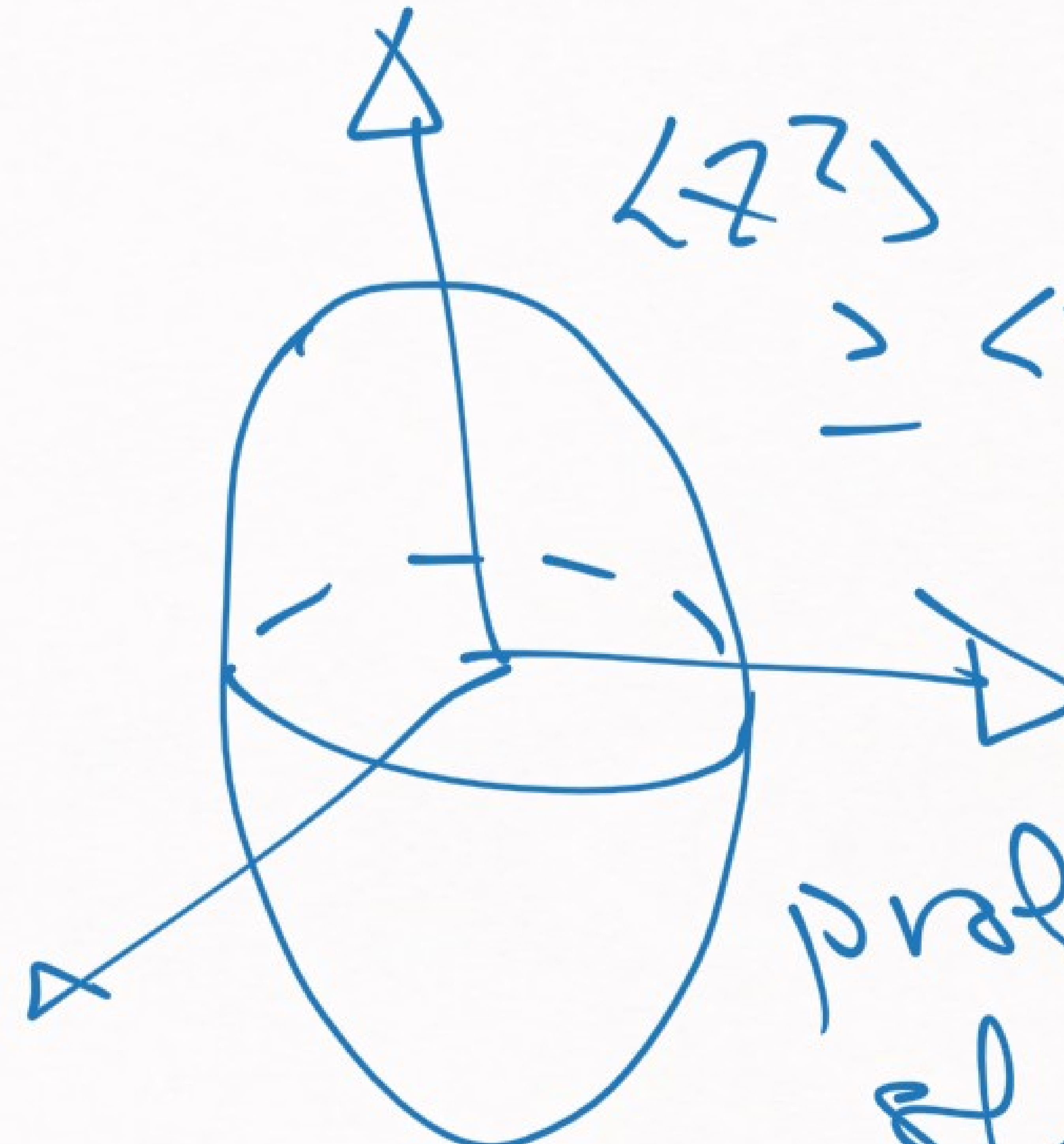
$$\left.\begin{array}{l} 1)\ J = 0 \rightarrow \text{charge} \\ 2)\ J = 1/2 \rightarrow \text{charge}, \mu \end{array}\right\} {}^3He, {}^3H, {}^4He \quad \boxed{Q = 0}$$

3) $J = 1 \rightarrow$ charge, μ, Q

DEUTERON'S Q

Deuteron → $\boxed{Q_d > 0}$

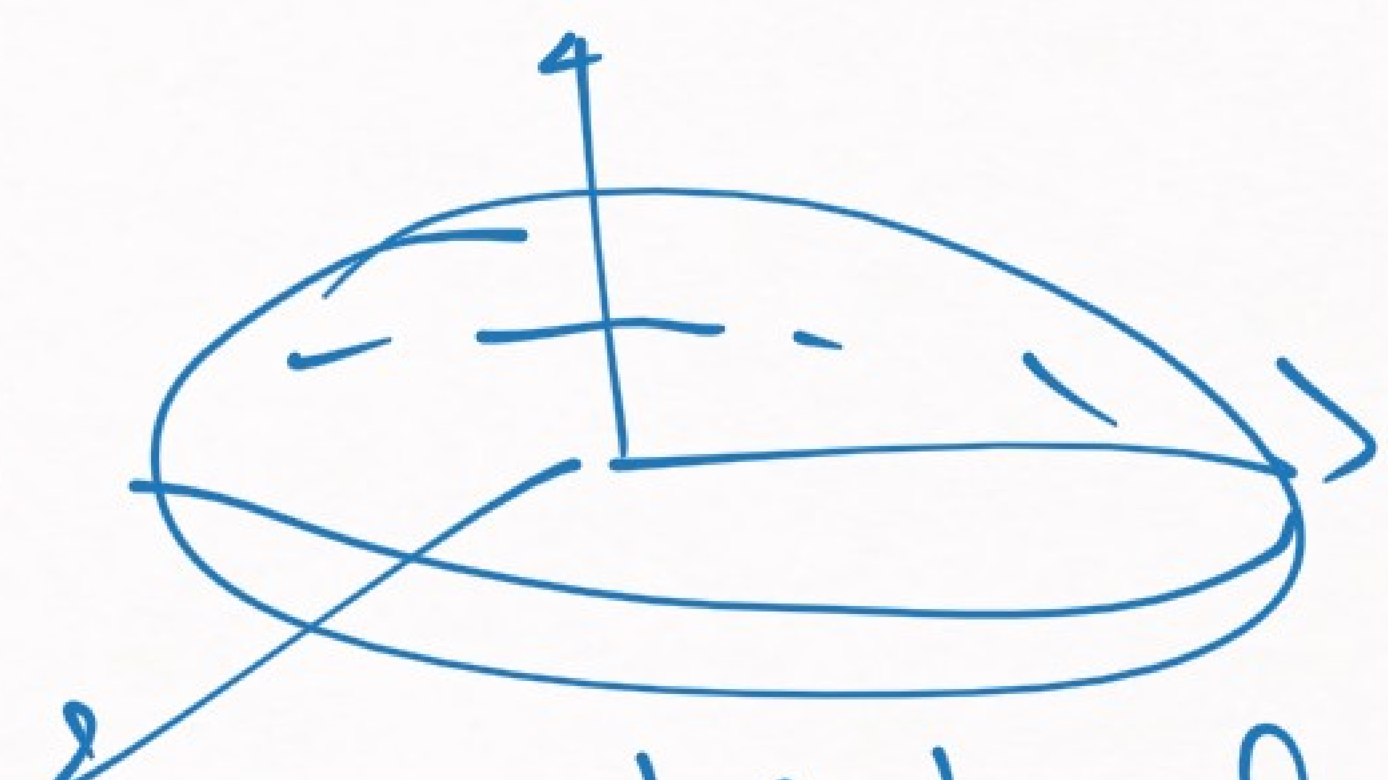
↳ pion is 0^-



$\langle z^2 \rangle \geq \frac{\langle x^2 \rangle + \langle y^2 \rangle}{2}$

prolate shape

Most nuclei (large A) → $\boxed{Q_d < 0}$

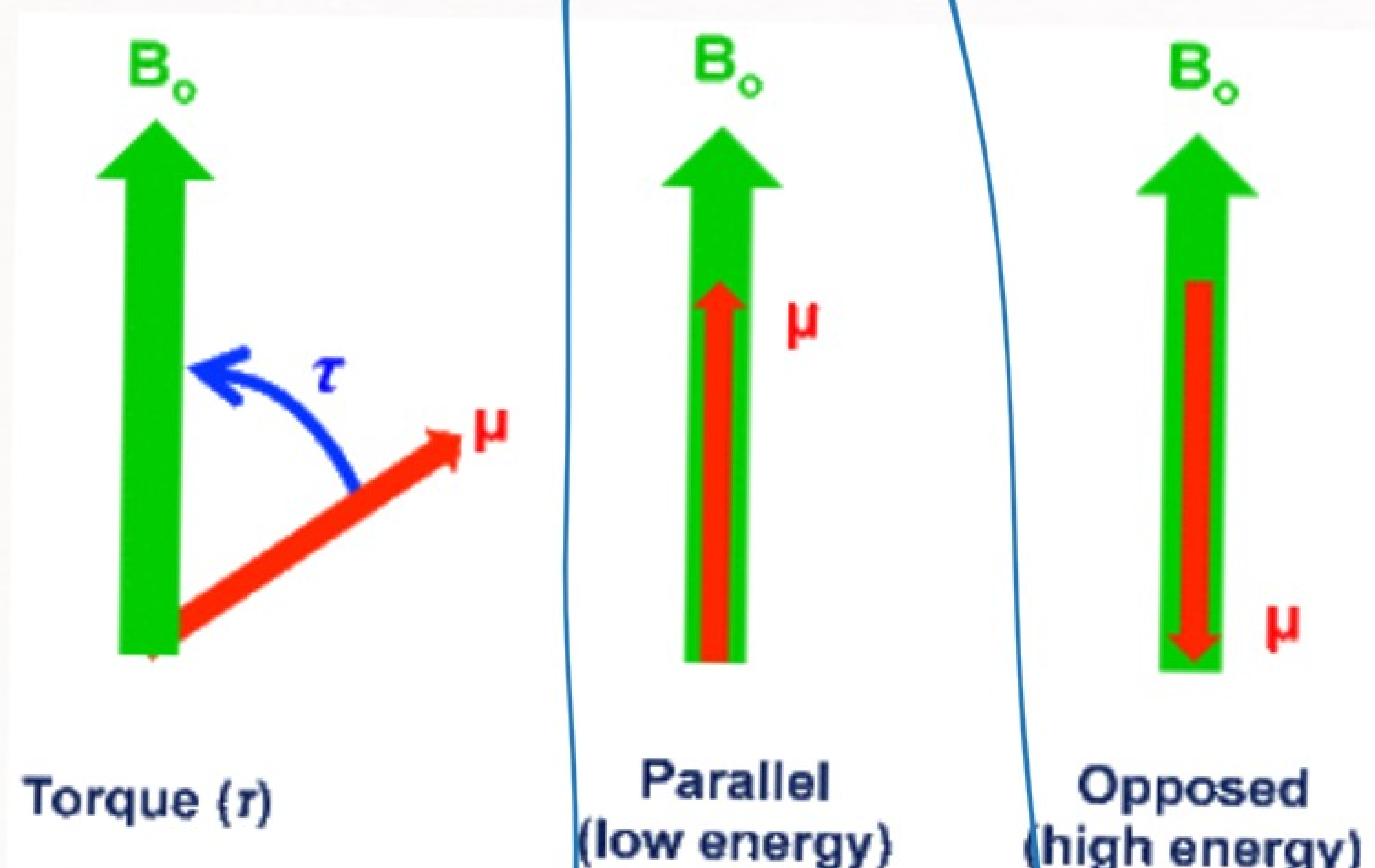


oblate shape

MAGNETIC MOMENT

$H = -\vec{\mu}\cdot\vec{B}$

min. energy happens for $\vec{\mu} \parallel \vec{B}$



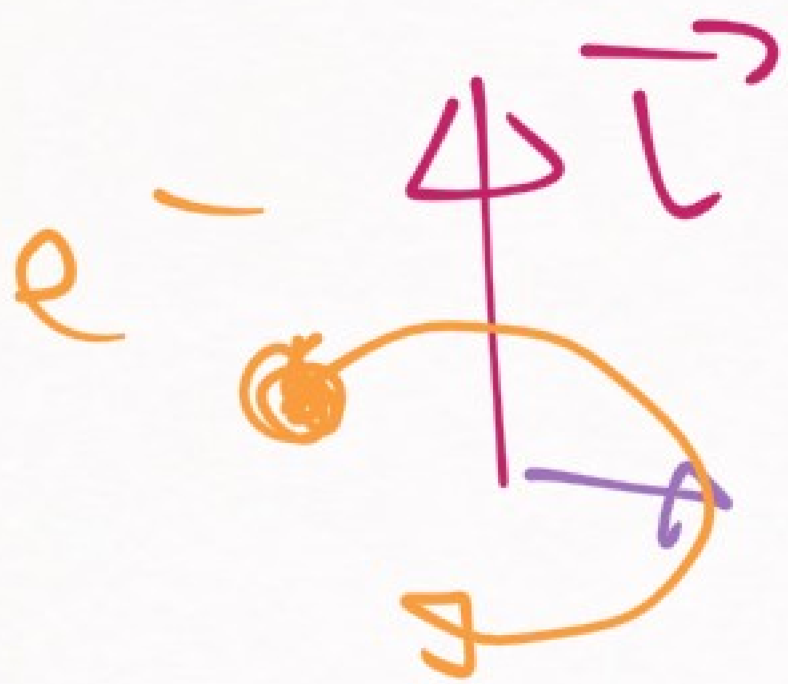
GROUND STATE

$\vec{\mu}$ → pseudovector

(reason why compasses also work in southern hemisphere)

How to calculate $\vec{\mu}$?

1) CLASSICAL MECHANICS



$$\boxed{\vec{\mu}_L = \frac{e}{2m}\vec{L}}$$

Classical definition

2) QUANTUM MECHANICS → extra contrib.

→ intrinsic spin

$\boxed{\text{QM}}$ → $\vec{J} = \vec{L} + \vec{S}$ ⟶ $\vec{\mu} = \vec{\mu}_L + \vec{\mu}_S$

1) FIRST GUESS WOULD BE: $\vec{\mu}_S = \frac{e}{2m}\vec{S}$

But it's wrong

2) CORRECT ANSWER IS:

$$\boxed{\vec{\mu}_S = g_S \frac{e}{2m}\vec{S}}$$

new factor

$$\vec{\mu}_s = g_s \frac{e}{2m} \vec{S}$$

gyromagnetic factor

(depends on type of particle)

electron → pointlike ⇒ ⊕

(*) $\Rightarrow$ ELECTRON

1) DIRAC EQUATION $g_s(e^-) = 2$

(QM + special relativity)

2) QED CORRECTIONS ("g-2" of electron)

$$g_s(e^-) = 2\left[1 + \frac{\alpha}{2\pi} + \mathcal{O}(\alpha^2)\right]$$

$\exists \mathcal{P}$ $\exists$ structure, $g-2 \neq 0$

$\rightarrow$ neutron, proton are composite !!

$g_p = 5.586$
$g_n = -3.826$ $\Big\} \rightarrow$ $g_p \neq 2$, $g_n \neq 0$

point-like neutrons proton

$$\vec{\mu}_s = g_s \left|\frac{e_p}{2m_p}\right| \vec{S}$$

$\boxed{\text{SUMMARY}}$ $\rightarrow$ $\vec{\mu} = \mu_N (\vec{l} + g_s \vec{s})$

$\mu_N = \frac{e}{2m_N}$ (nuclear magneton)

$\exists$ a second definition

$\vec{s} = \vec{\sigma}/2$

$\vec{\mu} = \mu_N (\vec{l} + \mu_s \vec{\sigma})$

$\mu_s = \frac{g_s}{2}$

$\mu_S(p) = +2.793$ $\quad (\mu_p = +2.793\,\mu_N)$

$\mu_S(n) = -1.913$ $\quad (\mu_n = -1.913\,\mu_N)$

more usual in other contexts

$$\mu = \langle JJ | \hat{\mu}_z | JJ \rangle$$

operator

[MAGNETIC MOMENT] → STRUCTURE OF A NUCLEUS

→ [DEUTERON] →

$$\boxed{\mu_d = 0.8573\,\mu_N} \rightarrow \text{Experiment}$$

→ GIVES INFO ABOUT DEUTERON'S D-WAVE

$$\mu = \langle JJ | \hat{\mu}_z | JJ \rangle$$

$$\hat{\vec{\mu}} = \sum_{i=1}^{A} \hat{\vec{\mu}}_i \quad , \quad \hat{\vec{\mu}}_i = \mu_N \left(e_i \vec{L} + \mu_S(i) \vec{\sigma} \right)$$

(DEUTERON) $\hat{\vec{\mu}}_d = \mu_N \underline{\vec{L}_p + \mu_p \vec{\sigma}_p} + \mu_n \vec{\sigma}_n$

$$\mu = \langle 11 | (\hat{\mu}_a)_3 | 11 \rangle \qquad |11\rangle = |\tfrac{1}{2}\tfrac{1}{2}\rangle_p |\tfrac{1}{2}\tfrac{1}{2}\rangle_n$$

Let's imagine deuteron is pure S-wave

$$\mu_d = \langle 11 | \overbrace{\mu_N L_3}^{0\ (\text{S-wave})} + \mu_p \underbrace{\sigma_{p3}}_{1} + \mu_n \underbrace{\sigma_{n3}}_{1} | 11 \rangle$$

$$= \mu_p + \mu_n \simeq 0.88\ \mu_N$$

$$\mu_{d,exp} \simeq 0.86\ \mu_N$$

Deuteron is not pure S-wave

$$\mu_d^{S\text{-wave}} \simeq 0.88\,\mu_N$$

$$\mu_{d,exp} \simeq 0.86\,\mu_N$$

$\rightarrow$ Difference require a D-wave

from $Q_d \Rightarrow \exists$ D-wave ✓

$\downarrow$

(change μ_d)

$$|\psi_d\rangle = a_s|{}^3S_1\rangle + a_d|{}^3D_1\rangle$$

$$|a_s|^2 + |a_d|^2 = 1$$

$$\mu({}^3S_1) = 0.88\,\mu_N$$

$$\mu({}^3D_1) = 0.31\,\mu_N \rightarrow \text{if } a_d \neq 0,$$

$$\mu_d < \mu({}^3S_1)$$

$$\mu_d = |a_S|^2 \mu(^3S_1) + |a_D|^2 \mu(^3D_1)$$

$$\mu_{d,exp} \simeq 0.86\, \mu_N$$

$\Rightarrow$ Deuteron has a D-wave probability of about $\underline{\underline{3\text{-}4}}$ %.

$$\left.\begin{array}{l} P_S = |a_S|^2 \simeq 0,96 \\ P_D = |a_D|^2 \simeq 0.04 \end{array}\right\} \Rightarrow \boxed{\text{Md, exp}}$$

→ a few of you will be QM purists

→ the wave function is not an observable

→ P_S, P_D should not be observables

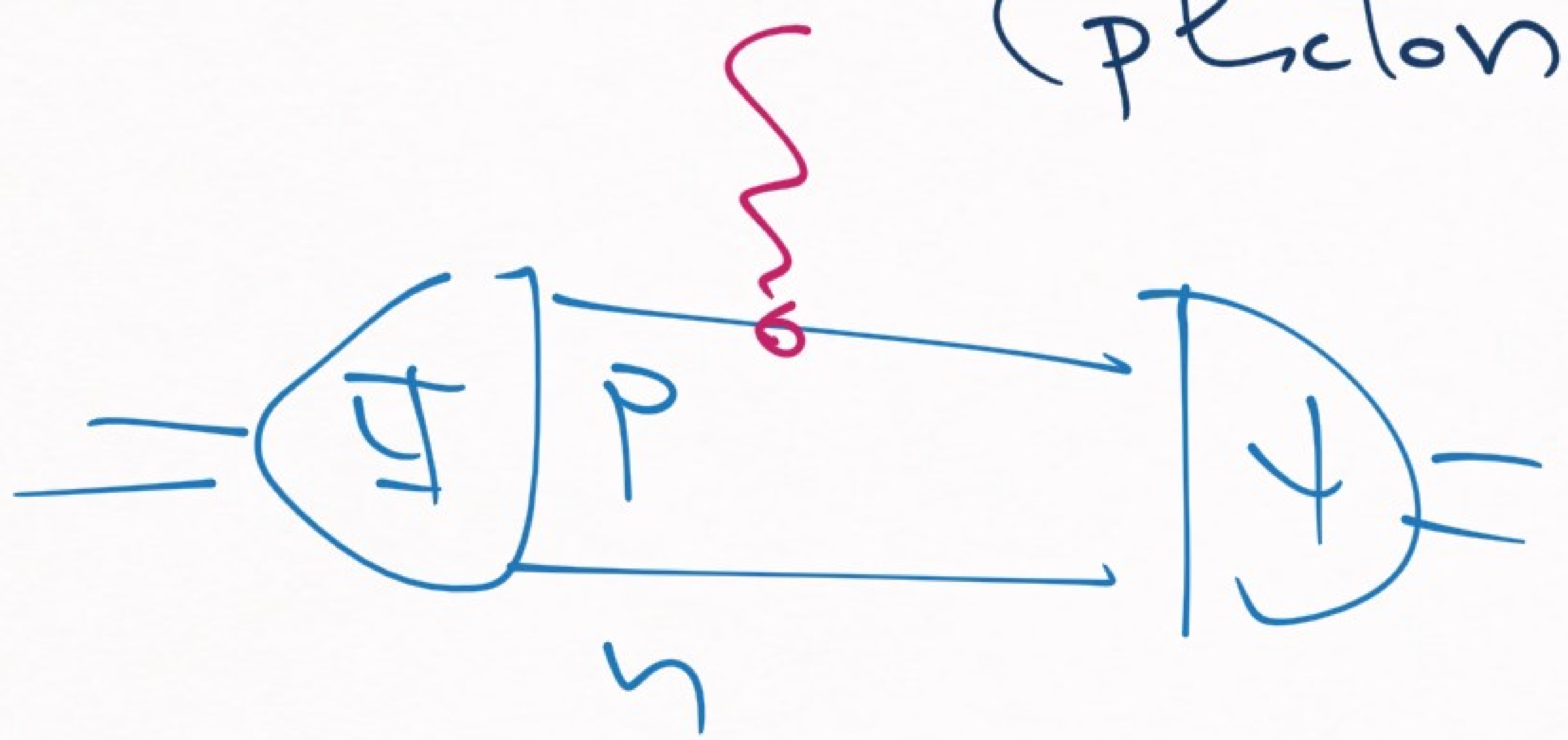
→ Our argument is incomplete

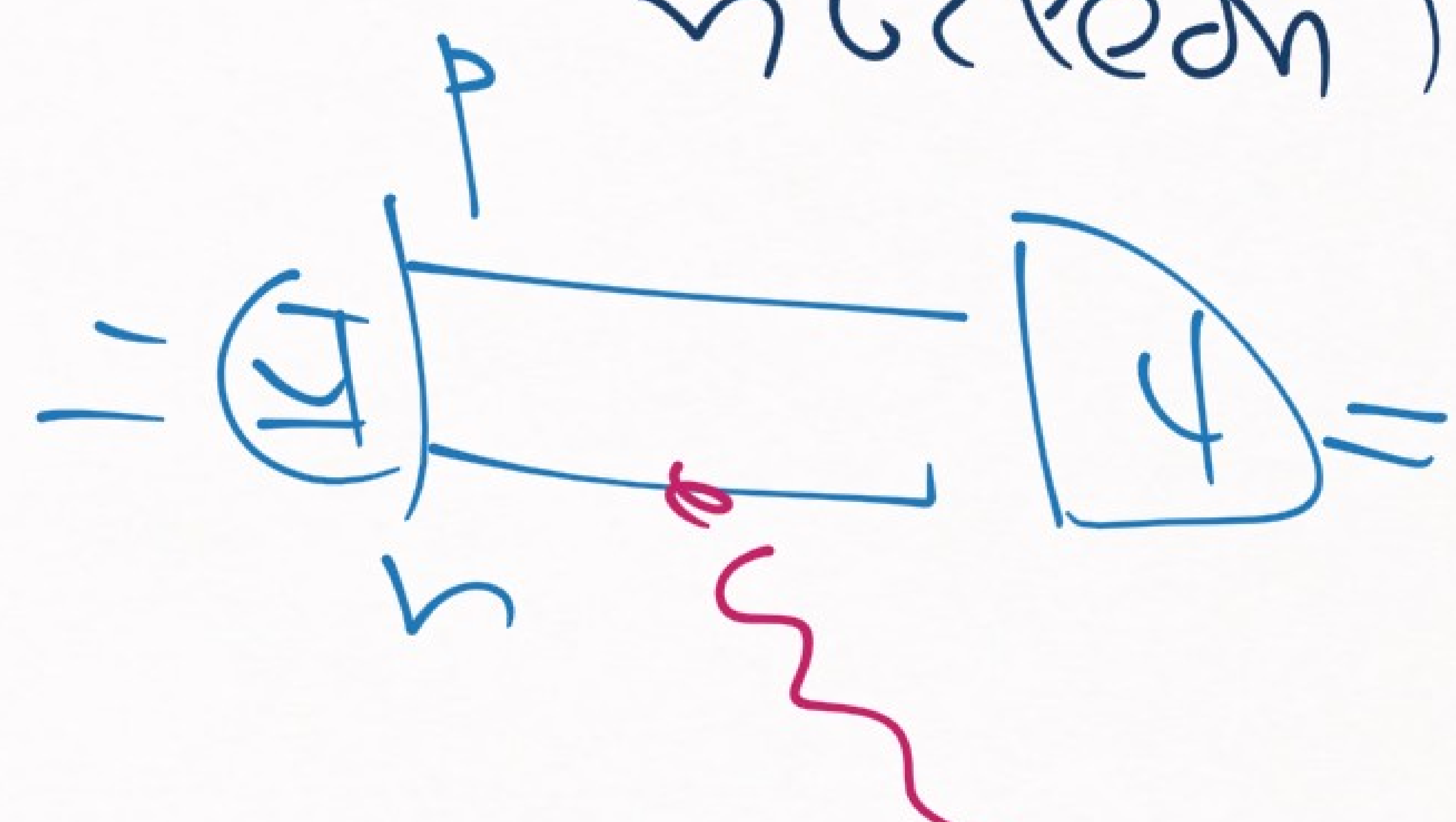
→ We are missing something in our calculation

→ [TWO-BODY CURRENTS]

$P_S \simeq 0.96$
$P_D \simeq 0.04$ → ∃ IMPULSE ASSUMPTION

(photon only hits a nucleon).





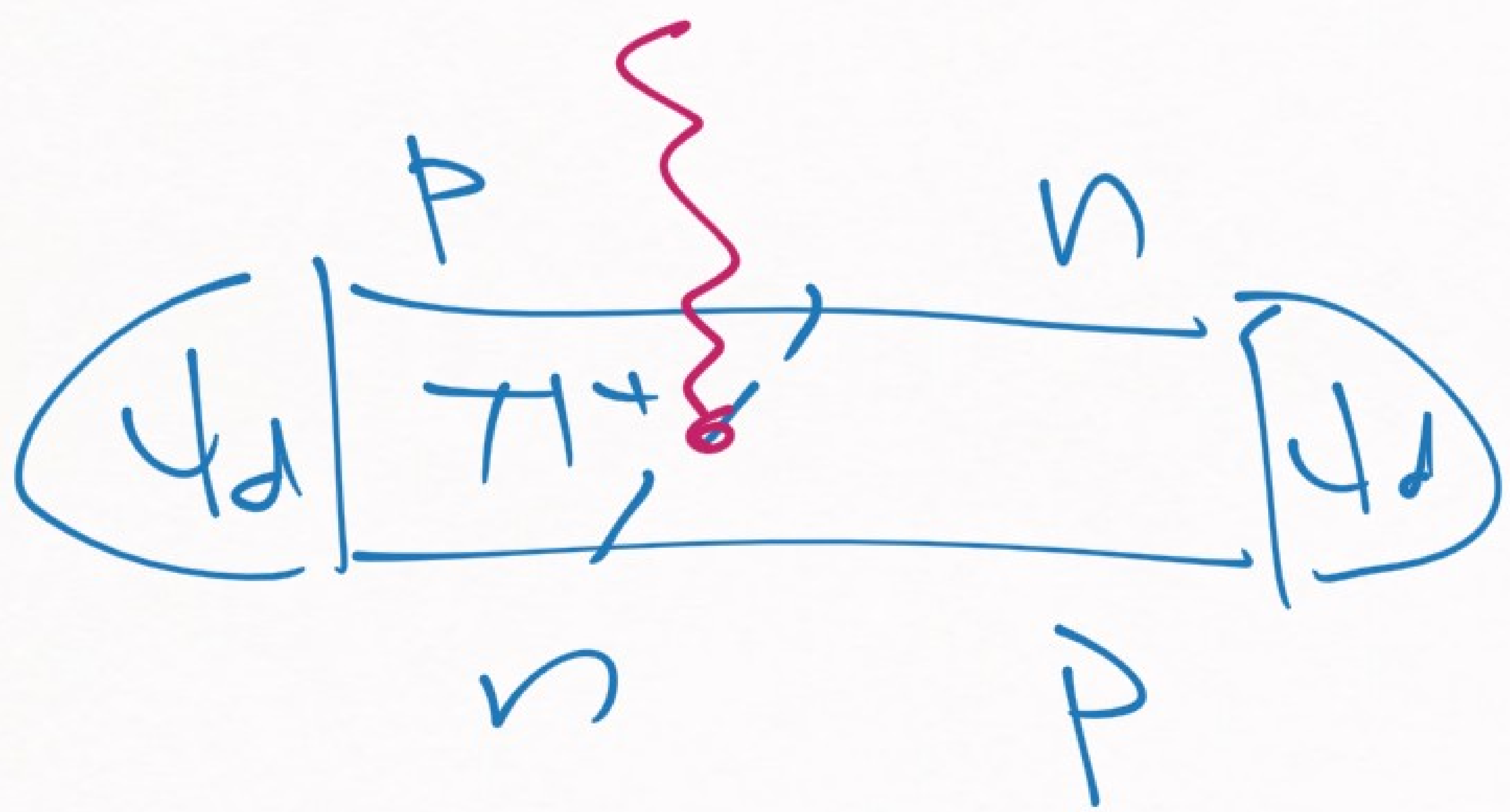
⊗ → ONE-BODY CURRENT

(the em current involves only one particle)

Deuteron → potential → an exchange particle

can be hit by a photon

[TWO-BODY CURRENT]



$\Rightarrow$ contribution to μ_d

$\boxed{\text{TWO BODY CURRENT}}$ $\longrightarrow$

1) Not so easy to calculate

2) $\mu \neq \mu(S\text{-wave}) + \mu(D\text{-wave})$

$\exists$ interference

3) P_S, P_D now can't be extracted from μ_d

TWO BODY CURRENTS → small corrections to electromagnetic moments

↓

Deuteron quadrupole moment

$$Q_d^{exp} = 0.2859(3)\ e\,fm^2$$

$$Q_d^{1B} = 0.276\ e\,fm^2, \quad Q_d^{2B} = 0.010\ e\,fm^2$$

$$\boxed{|Q_d^{1B}| \gg |Q_d^{2B}|}$$

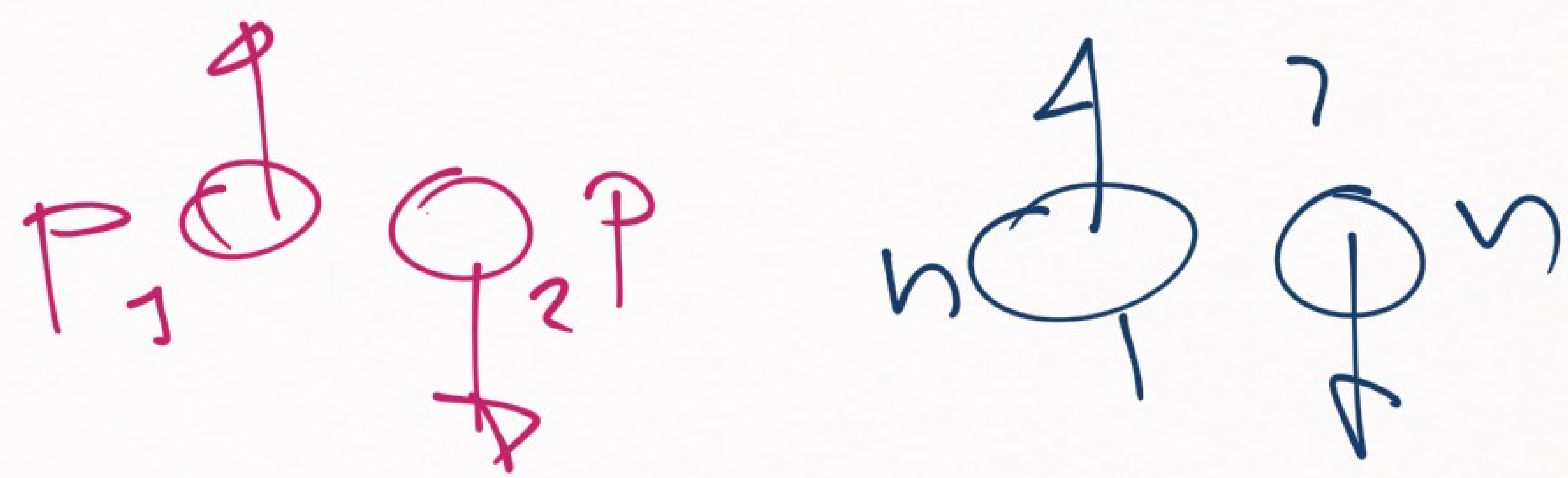
$$\mu_d = \underbrace{\mu_d^{1B}}_{\text{main contribution}} + \underbrace{\mu_d^{2B}}_{\text{small}}$$

[MAGNETIC MOMENTS OF NUCLEI]

even-even nuclei $\longrightarrow$ $\boxed{\text{PAIRING}}$

$\downarrow$ even Z (protons)

$\downarrow$ even N (neutrons)



$\vec{S}_n = \vec{0}$ $\qquad$ $\vec{S}_n = \vec{0}$

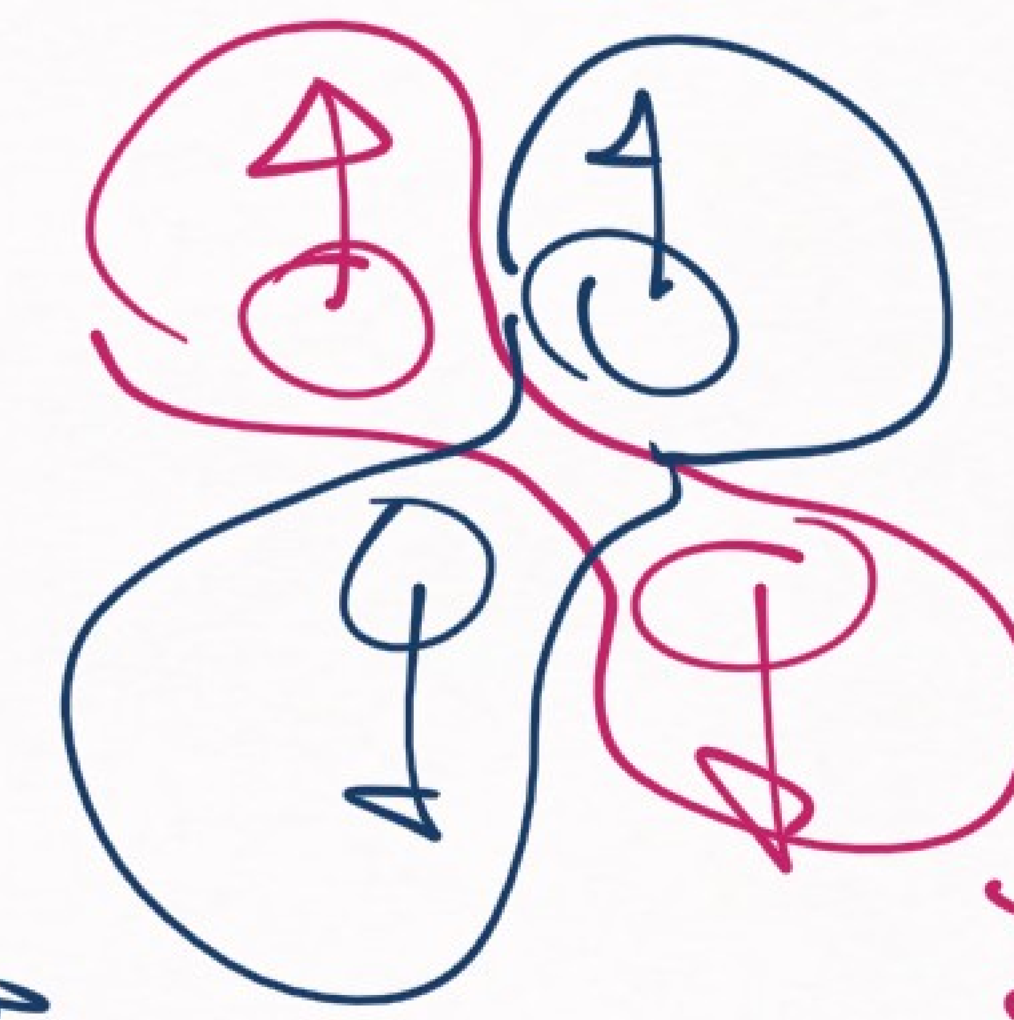
$\rightarrow \forall$ we have a pair, $\underline{\underline{\vec{\mu}_{pair} = \vec{0}}}$

$$\vec{\mu}_{pair} = \vec{\mu}_1 + \mu_2 = \mu_{n/p}\underbrace{(\vec{\sigma}_1 + \vec{\sigma}_2)}_{\vec{0}}$$

$$\boxed{\mu(\text{even-even}) = 0}$$

$$\boxed{J(\text{even-even}) = 0}$$

$$\boxed{{}^4He}$$



GREAT SIMPLIFICATION

$\vec{S}_{n's} = 0$ $\quad$ $\vec{S}_{p's} = 0$

$$\vec{S}_{^4He} = \vec{S}_{p's} + \vec{S}_{n's} = \vec{0}$$

even-odd nuclei → impaired nucleon

→ even-even core + single unpaired nucleon

$$\vec{J}(A) = \cancel{\vec{J}_{core}} + \vec{J}_N = \vec{J}_N$$

$\cancel{\vec{J}_{core}} \to \vec{0}$

$$\mu(A) = \cancel{\mu_{core}} + \mu_N = \mu_N$$

$\cancel{\mu_{core}} \to 0$

1) $\hat{\vec{\mu}}_N = \mu_N (e_N \vec{l} + \mu_s \vec{\sigma})$ even-odd nuclei

2) $S = 1/2$, $J = L \pm 1/2$, L is given

3) SCHMIDT VALUES

$$\mu_N (J = L + 1/2) = g_L (J - 1/2) + \frac{1}{2} g_s$$

$$\mu_N (J = L - 1/2) = g_L \frac{J(J + 3/2)}{J+1} - \frac{g_s}{2} \frac{J}{J+1}$$

odd-odd nuclei → case-by case basis

(5 stable odd-odd nuclei)

^{2}H, ^{6}Li, ^{10}Be, ^{14}N, ^{180}Ta

weird one (not totally sure if completely stable or not)

$\boxed{\text{RECAP}}$ (magnetic moments)

1) Deuteron contains D-wave $\mu_d \neq \mu_d(\text{Schmidt})$

2) Nuclei allow for $(P_D \neq 0)$

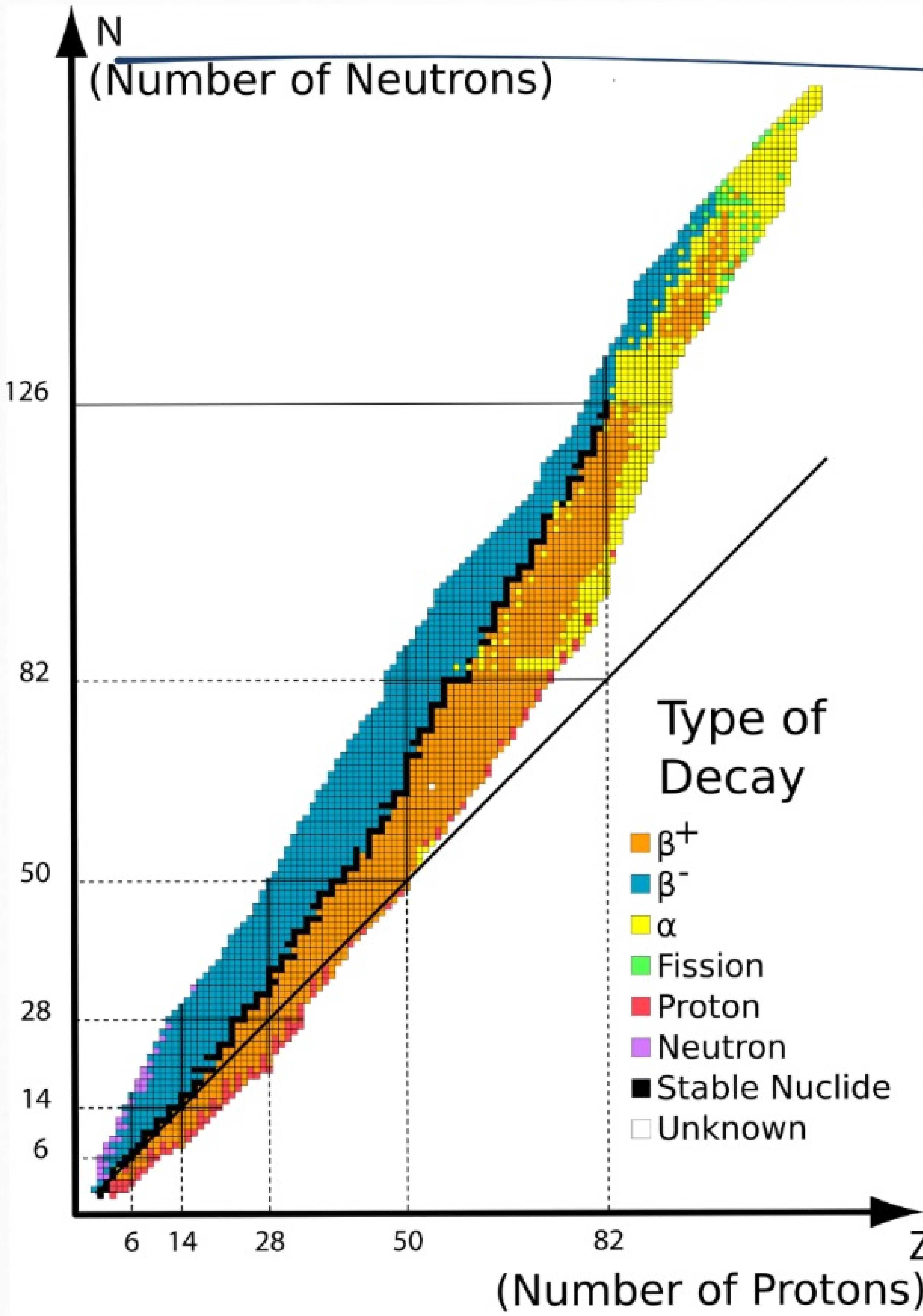
simplifications (pairing) Schmidt values

2.a) even-even nuclei $\mu = 0$

2.b) even-odd nuclei $\mu = \mu_N(\text{unpaired})$

2.c) odd-odd nuclei case-by-case

DECAYS & STABILITY



(α, β, γ decays)

Which nuclei will decay?

→ any nuclei for which decay is energetically allowed

→ if $\nexists$ symmetry forbidding the decay

& the decay generates energy

⇒ it will happen

E(initial nucleus) > E(final nucleus + decay products)

⇒ it will happen

1) α-decay $\;{}^{A}_{Z}X_{N} \rightarrow {}^{A-4}_{Z-2}X_{N-2} + \alpha$

Energitacally possible if:

$$\left[\; S_\alpha(Z,N) = B(Z,N) - B(Z-2,N-2) - B(2,2) > 0 \;\right]$$

2) β^+ & β^- decays: (emission of positron or electron)

β^-: ${}^{A}_{Z}X_{N} \rightarrow {}^{A}_{Z+1}X_{N-1} + e^- + \bar{\nu}_e$

β^+: ${}^{A}_{Z}X_{N} \longrightarrow {}^{A}_{Z-1}X_{N+1} + e^+ + \nu_e$

Simplest case: $n \rightarrow p + e^- + \bar{\nu}_e$

$Q = -(m_p + m_e + m_\nu - m_n)$

$= 1.29\,\text{MeV} > 0$

$\rightarrow$ $n \to p + e^- + \bar{\nu}_e$ is easy to compute

$$\tau = \frac{\hbar c}{\Gamma} \frac{1}{c} \simeq 930 \text{ seconds}$$

Next example $\rightarrow$ triton, ^3H | $\boxed{\tau \sim 10 \text{ years}}$

$$^3H \to {}^3He + e^- + \bar{\nu}_e \qquad Q \simeq 0.0186 \text{ MeV}$$

$$Q = +B(^3H) + (m_n - m_p) - B(^3He) - m_e \geq 0$$

3) γ-Decay

$$\underbrace{{}^{A}_{Z}X_N^{*}}_{\text{excited state}} \rightarrow \underbrace{{}^{A}_{Z}X_N}_{\text{less-excited or ground state}} + \gamma$$

it happens super fast

Next lesson

$\rightarrow$ $\boxed{\text{Nuclear models}}$