NUCLEAR PHYSICS (17)

- Effective field theories for nuclear forces
- Nuclear structure (if there is time)

RECAP | How to describe nuclear forces

→ One boson exchange (OBE) model:

* Nuclear forces generated by exchange of light mesons ($\pi, \rho, \omega, \sigma, \dots$)

* $V_{NN}(\vec{r}) = \sum_{meson} V_{meson}(\vec{r}) = V_\pi + V_\rho + V_\omega + V_\sigma + \dots$

* ∀ meson had a job

Why the OBE model is not enough?

(Why look for alternative descriptions)

→ WANTED DEAD OR ALIVE:

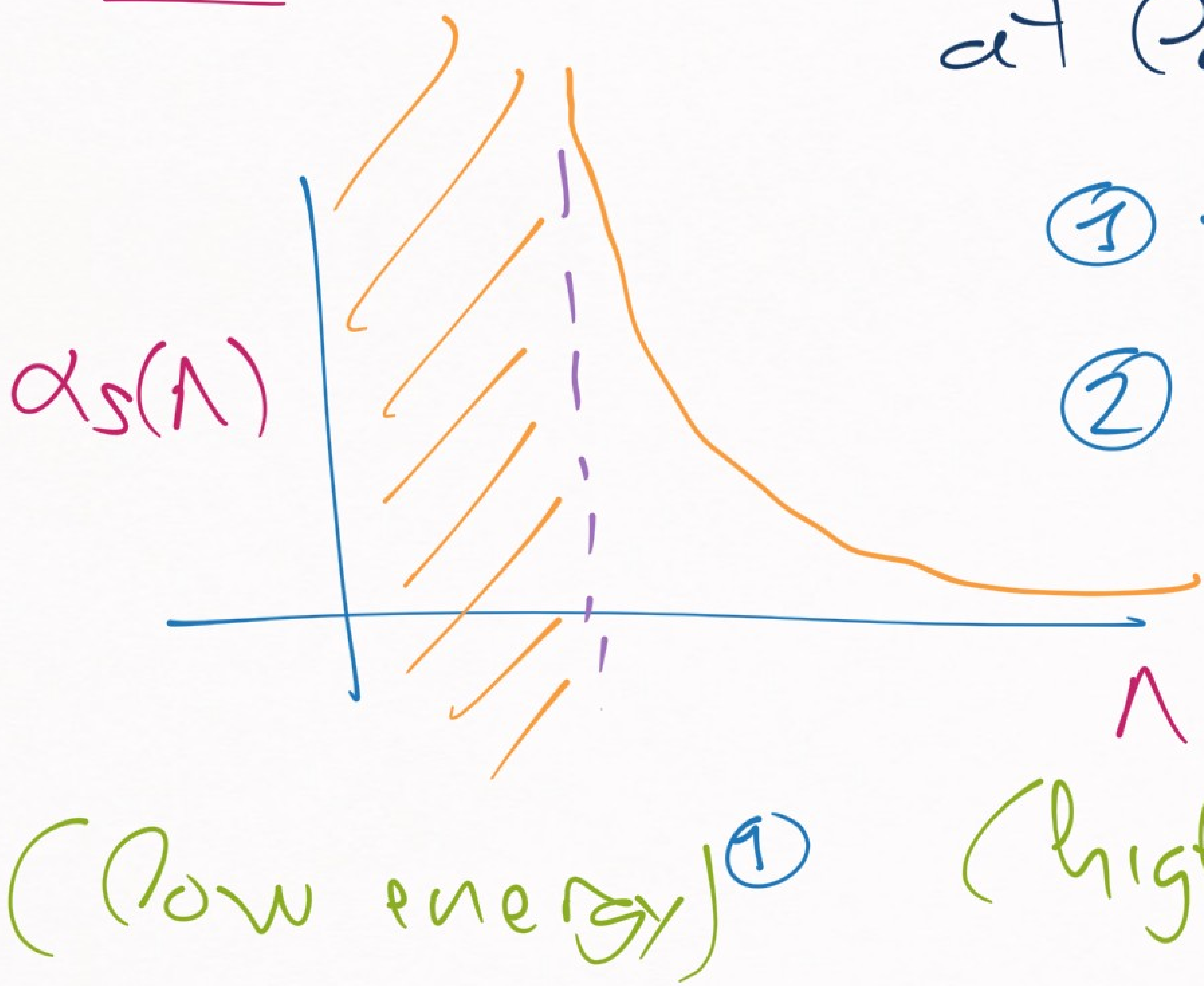
Explanation of the nuclear force based in QCD

→ We know that QCD is the fundamental theory of strong force!

$\Downarrow$

[We need an explanation of nuclear physics based on QCD]

PROBLEM → QCD is not solvable at low energies



① → nuclear physics

② → deep inelastic scattering

⇒ Different way of solving QCD

Lattice QCD

Solve QCD numerically in a computer

EFT

Find a RGE of QCD that is equivalent to QCD at low energies

Renormalization Group Evolution

EFT

- We can bypass the RGE condition (∃ an equivalent way to formulate it)

- Write down ∀ interactions involving the low energy degrees of freedom (e.g. N, π) and compatible w/ symmetries

EFT → Separation of scales

* Choose R_s (Separation scale or short-range scale)

* Include all physics at $r > R_s$

* Treat all physics at $r < R_s$ as unknown

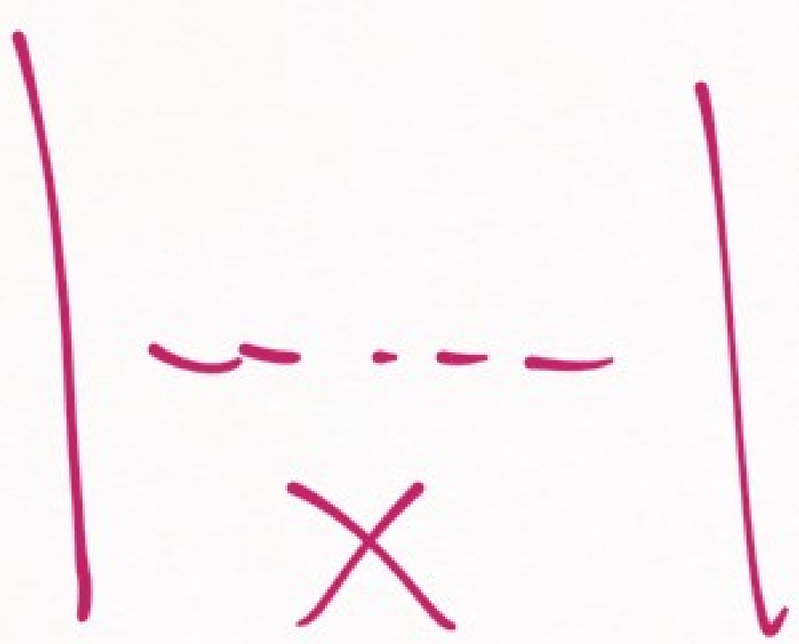
Depending on R_s → different nuclear EFTs

① $m_\pi R_s \sim 1$ ⇒ Pionless EFT ⊛

② $m_\rho R_s \sim 1$ ⇒ Pionful EFT

⊛ ⇒ Works because the deuteron is really big ($\gamma < m_\pi$)

[What do we do with R_s?]



"meson X"

$\rightarrow \frac{e^{-m_X r}}{r}$

1) $m_X R_s \gtrsim 1$

2) $m_X R_s \ll 1$

I Keep 

Depending on choice of R_S & mass of m_X:

1) "Collapse" this meson into a contact-range interaction ($m_X R_S \gtrsim 1$)

2) Keep this meson ($m_X R_S \ll 1$)

You can understand it from this point of view:

$$\left| \text{- - -}\overset{}{\underset{x}{}}\text{- -} \right| \sim \frac{1}{\vec{q}^2 + m_x^2} \begin{cases} ① \; \vec{q}^2 \ll m_x^2 \\ ② \; \vec{q}^2 \simeq m_x^2 \to \dfrac{1}{\vec{q}^2 + m_x^2} \end{cases}$$

$$① \quad \frac{1}{\vec{q}^2 + m_x^2} \simeq \frac{1}{m_x^2}\left(1 - \frac{\vec{q}^2}{m_x^2} + \frac{\vec{q}^4}{m_x^4} - \ldots\right)$$

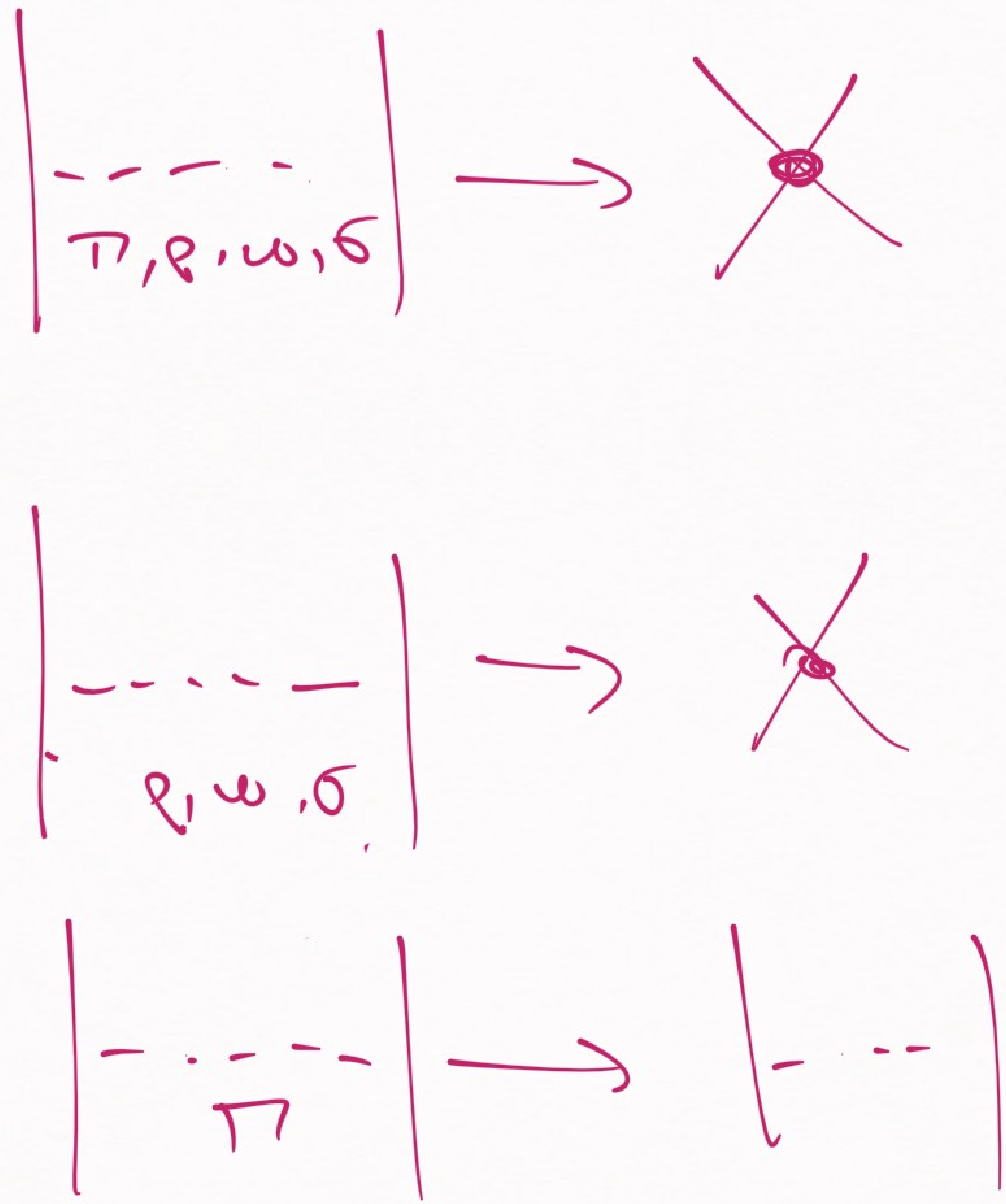
Expand → Sum of contact-range terms

Pionless

$m_\pi R_s \sim 1$

Pionful

$m_\rho R_s \sim 1$



only [illegible] contacts

contacts + pions

PIONLESS EFT → EFT($\not\pi$)

⇝ works for $k < m_\pi$

⇝ only contacts → very easy EFT

$$\langle p' | V_c | p \rangle = C_0 + C_2 (p^2 + p'^2) + C_4 (p^4 + p'^4) + C_4' p^2 p'^2 + \dots$$

$$\langle p' | U_c | p \rangle = C_0 + C_2(p^2 + p'^2) + C_4(p^4 + p'^4) + C_4' p^2 p'^2 + \ldots$$

C_0: $\mathcal{O}(p^0)$; $C_2(p^2 + p'^2)$: $\mathcal{O}(p^2)$; $C_4(p^4 + p'^4) + C_4' p^2 p'^2$: $\mathcal{O}(p^4)$

If p is small $\Rightarrow$ $C_0 > C_2(p^2 + p'^2)$

size of C_0, C_2

Convergence for $p \underline{\underline{<m_\pi}}$

$$|c_0| > |c_2(p^2 + p'^2)| \quad \text{for } p < m_\pi$$

$$\Rightarrow \left|\frac{c_0}{c_2}\right| \sim 2m_\pi^2$$

expected scaling

$$\left[c_0 \sim \frac{1}{m_\pi^2},\ c_2 \sim \frac{1}{m_\pi^4},\ c_4 \sim \frac{1}{m_\pi^6}\right]$$

We can truncate the expansion depending on desired accuracy

1) $\langle p' | V_c | p \rangle = C_0 + \frac{1}{m_\pi^2} \mathcal{O}\left(\frac{p^2}{m_\pi^2}\right)$

2) $\langle p' | V_c | p \rangle = C_0 + C_2(p^2 + p'^2) + \frac{1}{m_\pi^2} \mathcal{O}\left(\frac{p^4}{m_\pi^4}\right)$

and so on (only keep a few terms dep. on accuracy goal)

EFT → We can renormalize it as we like

⇙

We are free to choose our favorite regulator

Example

$$\langle p'|V_c|p\rangle = \left[C_0 + C_2(p^2+p'^2) + \ldots\right] f\left(\frac{p}{\Lambda}\right) f\left(\frac{p'}{\Lambda}\right)$$

(p-space)

↕ on-shell equivalent

$$V_c(r) = \frac{C_k(R_c)}{4\pi R_c^2}\,\delta(r - R_c) \quad / \quad C_k = C_0 + C_2 k^2 + \ldots$$

(r-space)

→ We are free to use our favorite regularization

↳ If we use the δ-shell,

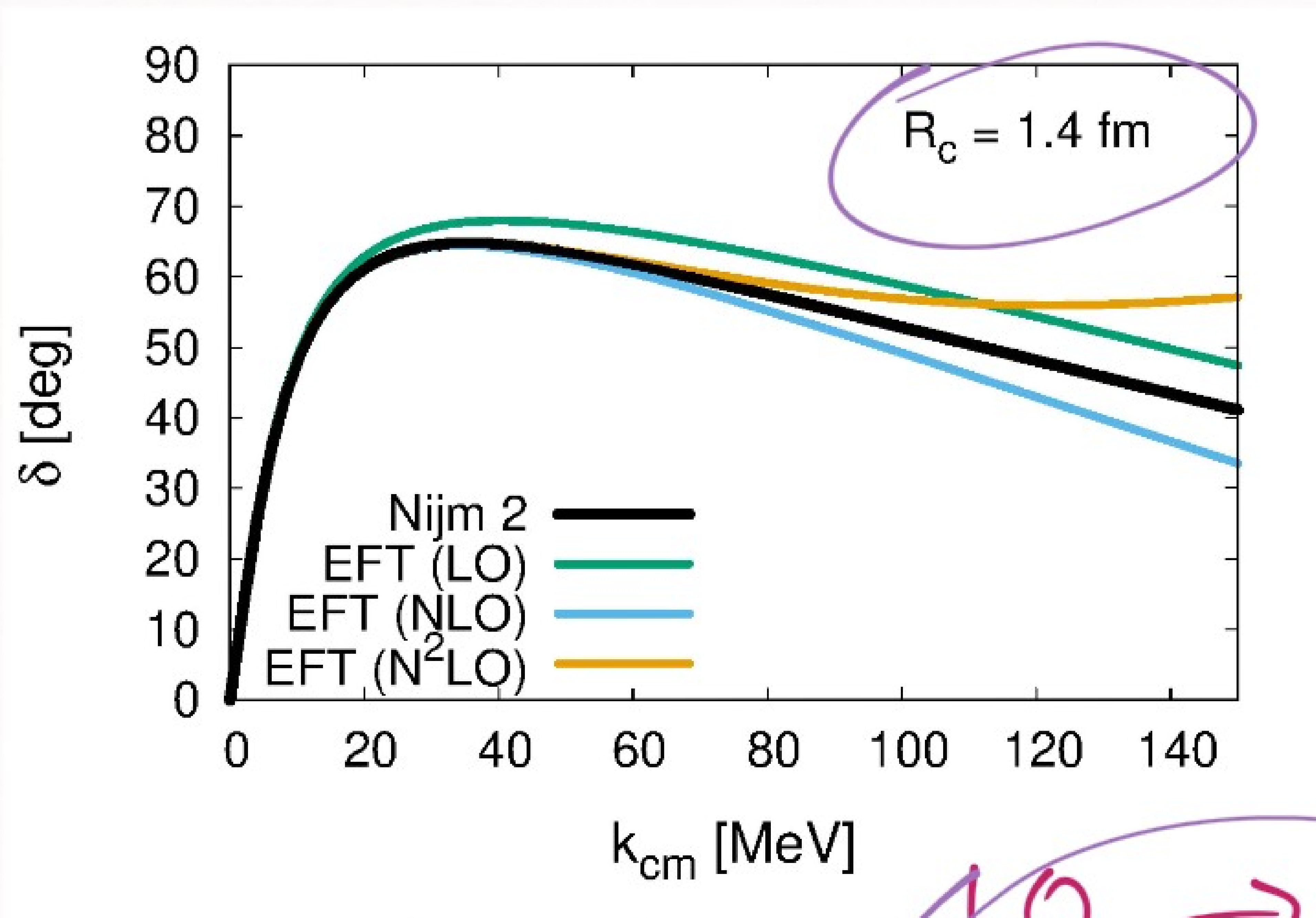
∃ analytical solution ↗ ⊕

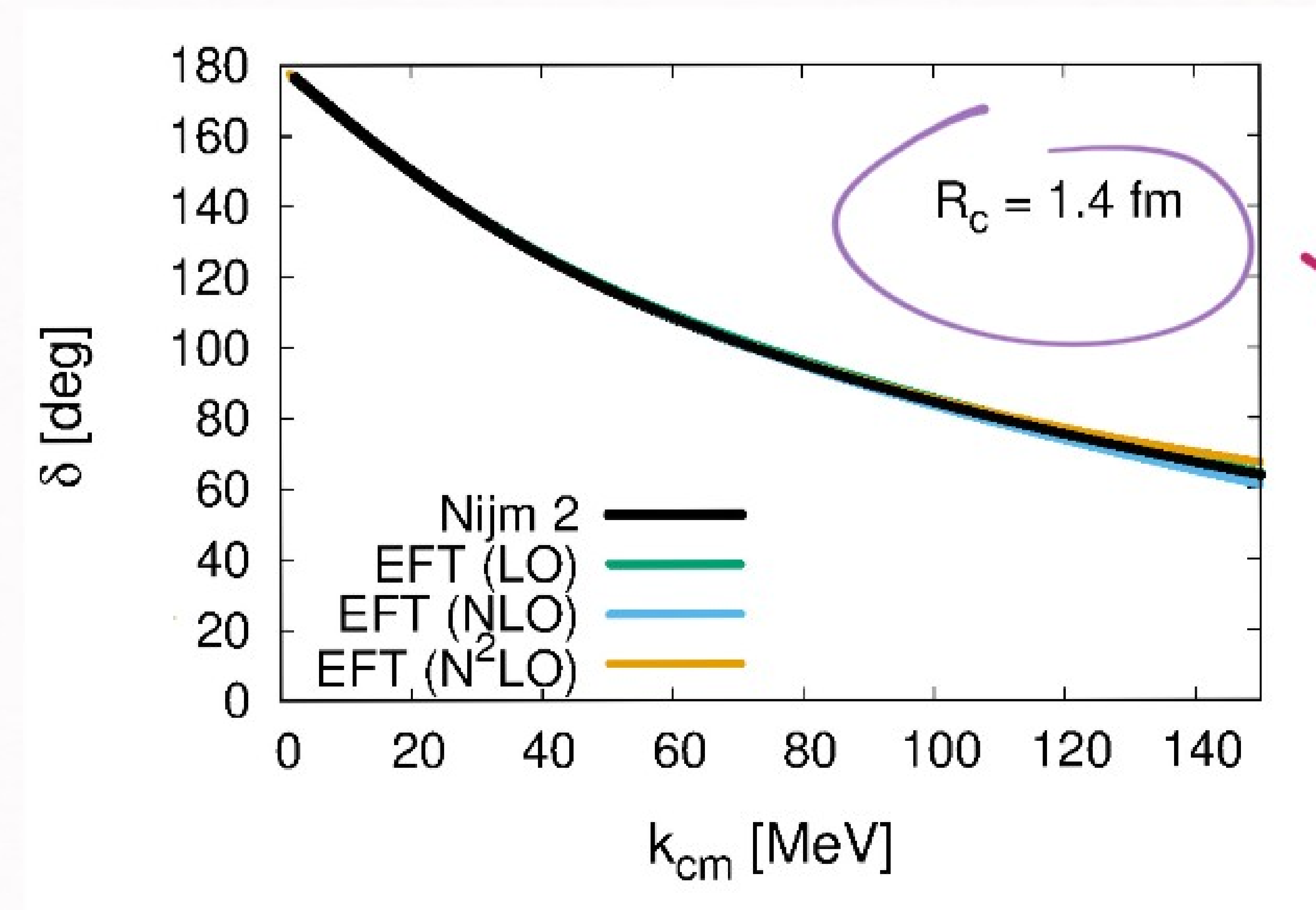
$$k\cot(kR_c+\delta) - k\cot\delta = 2\mu\,\frac{C_0(R_c)}{4\pi R_c^2}$$

→ explicit solution to pionless

⊛ ⇒ We can do calculations

Remember





R_c is not R_S

$^1S_0$

LO → C_0

NLO → + C_2

N^2LO → + C_4 → careful w/ names

$^3S_1$

[A few comments...]

→ EFT(π̸) happens to be equivalent to the effective range expansion (at least in the two-body sector)

Why? → $$k\cot(kR_c+\delta) - k\cot(kR_c) = \frac{2\mu\, C(R_c)}{4\pi R_c^2}$$

Take the $R_c \to 0$ limit

$$\lim_{R_c \to 0} k \cot\delta - \frac{1}{R_c} = \frac{2\mu}{4\pi R_c^2}\left[C_0(R_c) + C_2(R_c)k^2 + C_4(R_c)k^4 + \ldots\right]$$

$$\downarrow$$

$$\lim_{R_c \to 0} k \cot\delta = \left[\hat{C}_0(R_c) + \frac{1}{R_c}\right] + \hat{C}_2(R_c)k^2 + \hat{C}_4(R_c)k^4 + \ldots$$

$$\hat{C}_{2n}(R_c) = \frac{2\mu}{4\pi R_c^2} C_{2n}(R_c)$$

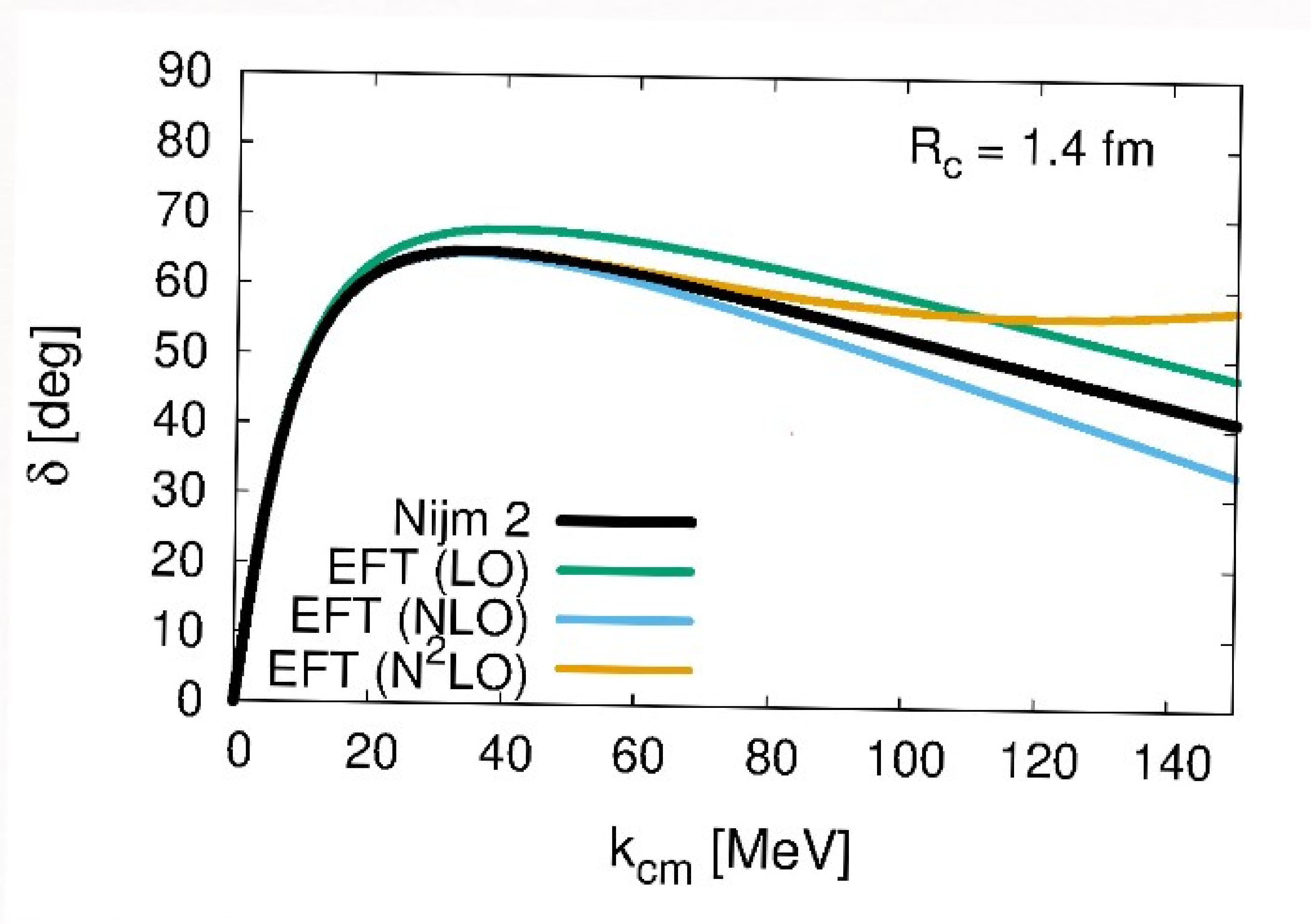
$$\text{EFT: } k\cot\delta \rightarrow \left(C_0(R_c) + \frac{1}{R_c}\right) + C_2(R_c)\,k^2 + C_4(R_c)\,k^4 + \ldots$$

$$\text{ERE: } k\cot\delta = -\frac{1}{a_0} + \frac{1}{2} r_0 k^2 + v_2 k^4 + \ldots$$

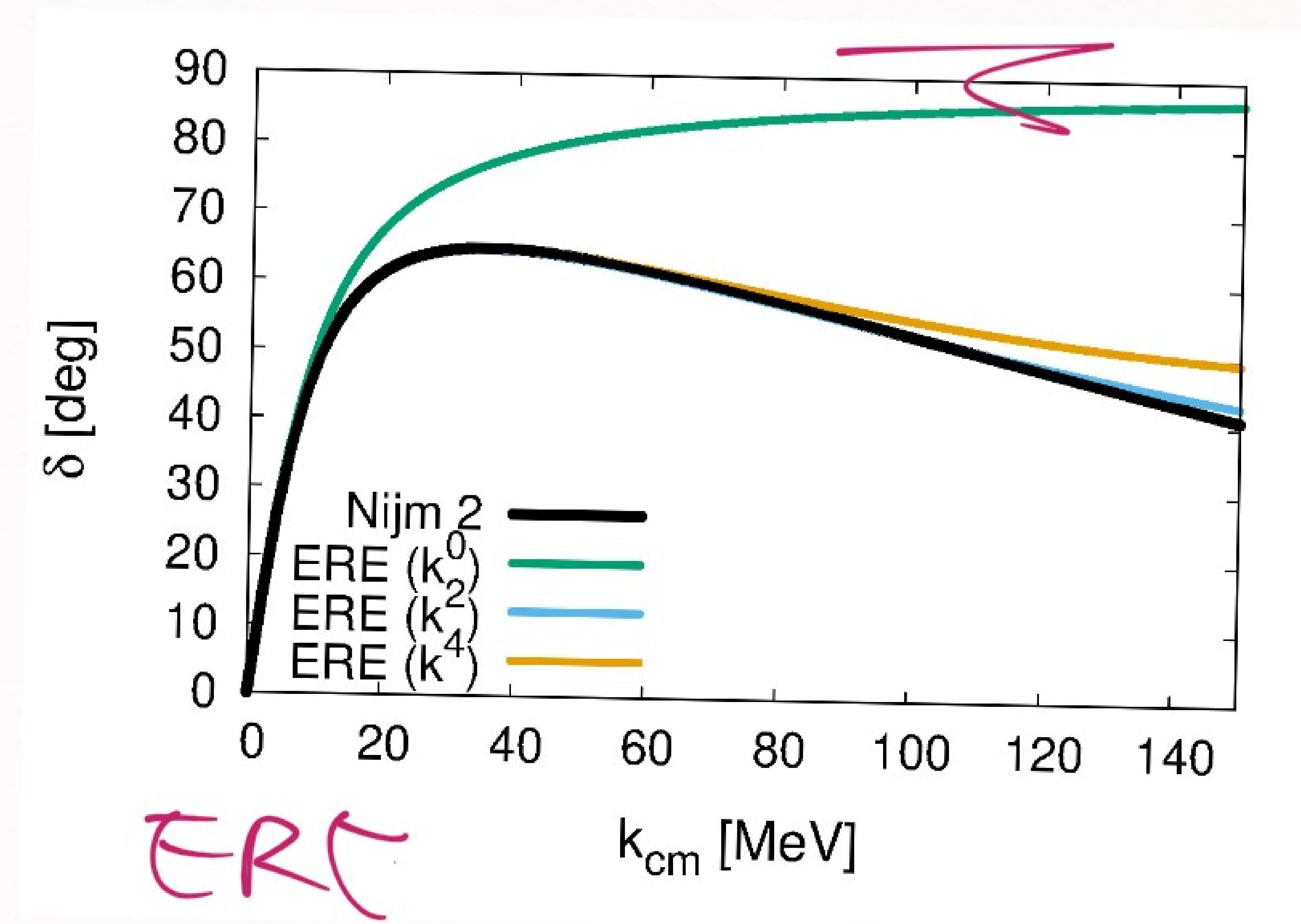
Term-by-term equivalence

First comment → EFT($\not\pi$) ≃ ERE

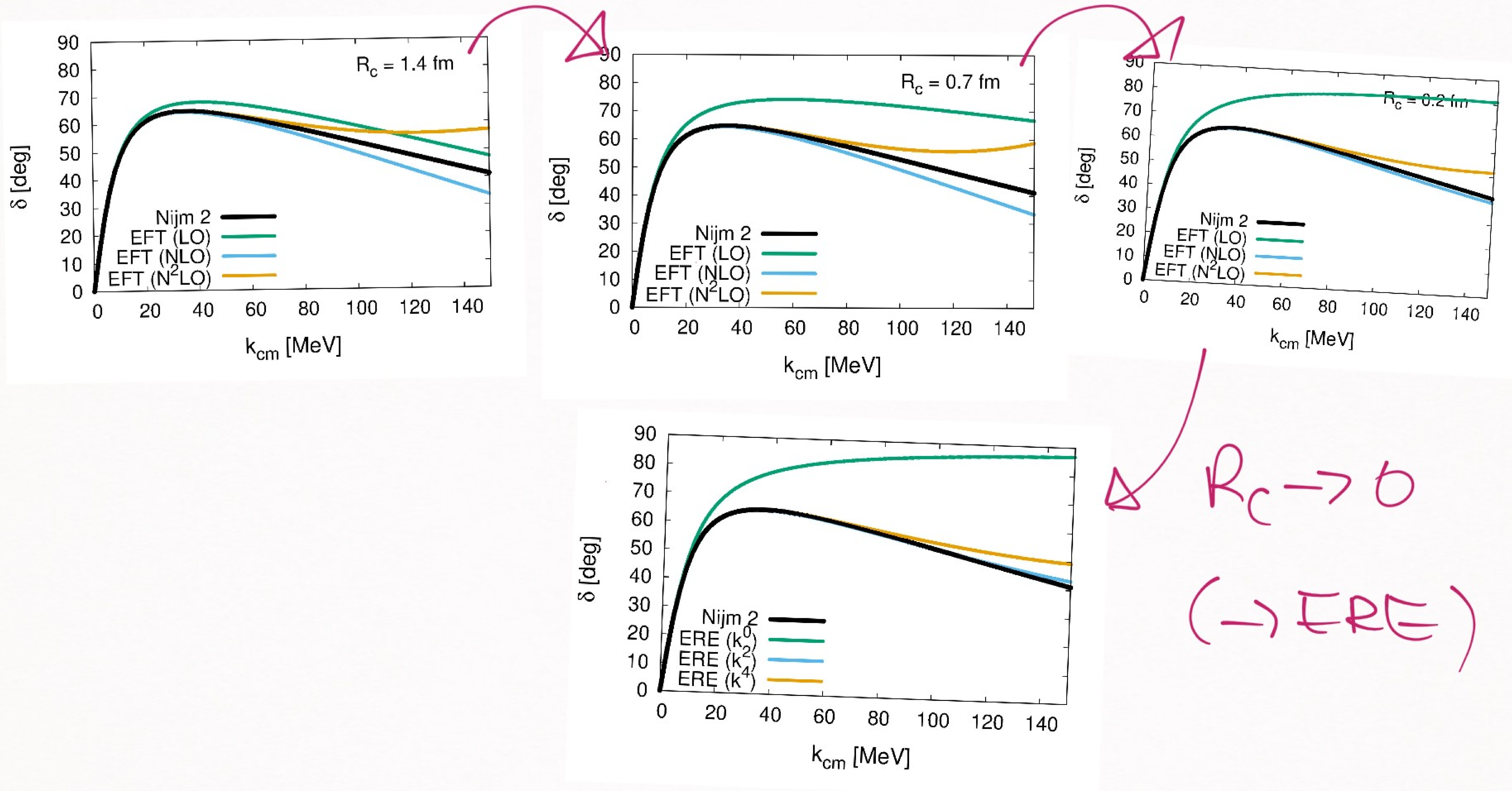
(in two-body sector)



EFT($\not\pi$)



ERE



R_c = 1.4 fm
δ [deg]
k_{cm} [MeV]
Nijm 2
EFT (LO)
EFT (NLO)
EFT (N^2LO)
R_c = 0.7 fm
δ [deg]
k_{cm} [MeV]
Nijm 2
EFT (LO)
EFT (NLO)
EFT (N^2LO)
R_c = 0.2 fm
δ [deg]
k_{cm} [MeV]
Nijm 2
EFT (LO)
EFT (NLO)
EFT (N^2LO)
δ [deg]
k_{cm} [MeV]
Nijm 2
ERE (k^0)
ERE (k^2)
ERE (k^4)
$R_c \to 0$
($\to$ ERE)

Second comment) → EFT($\not\pi$) we still have some residual cutoff dependence

cutoff dependence

1) Bad type → divergence

$\lim_{R_c \to 0} \frac{1}{R_c} \to \infty$ Bad !!

2) Good type → no divergence

$\lim_{R_c \to 0} R_c \to 0$ Harmless

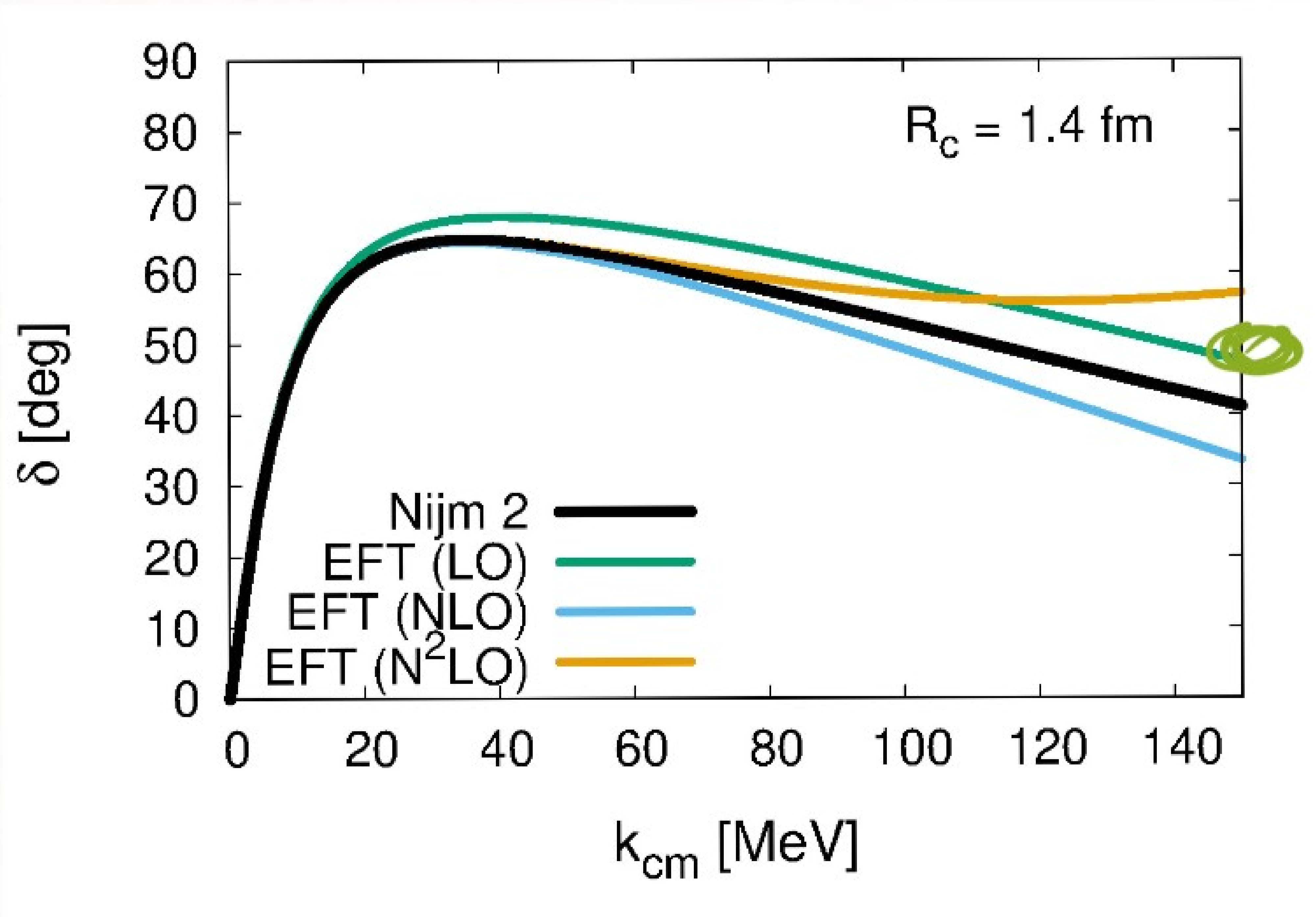
EFT ($\not\pi$)

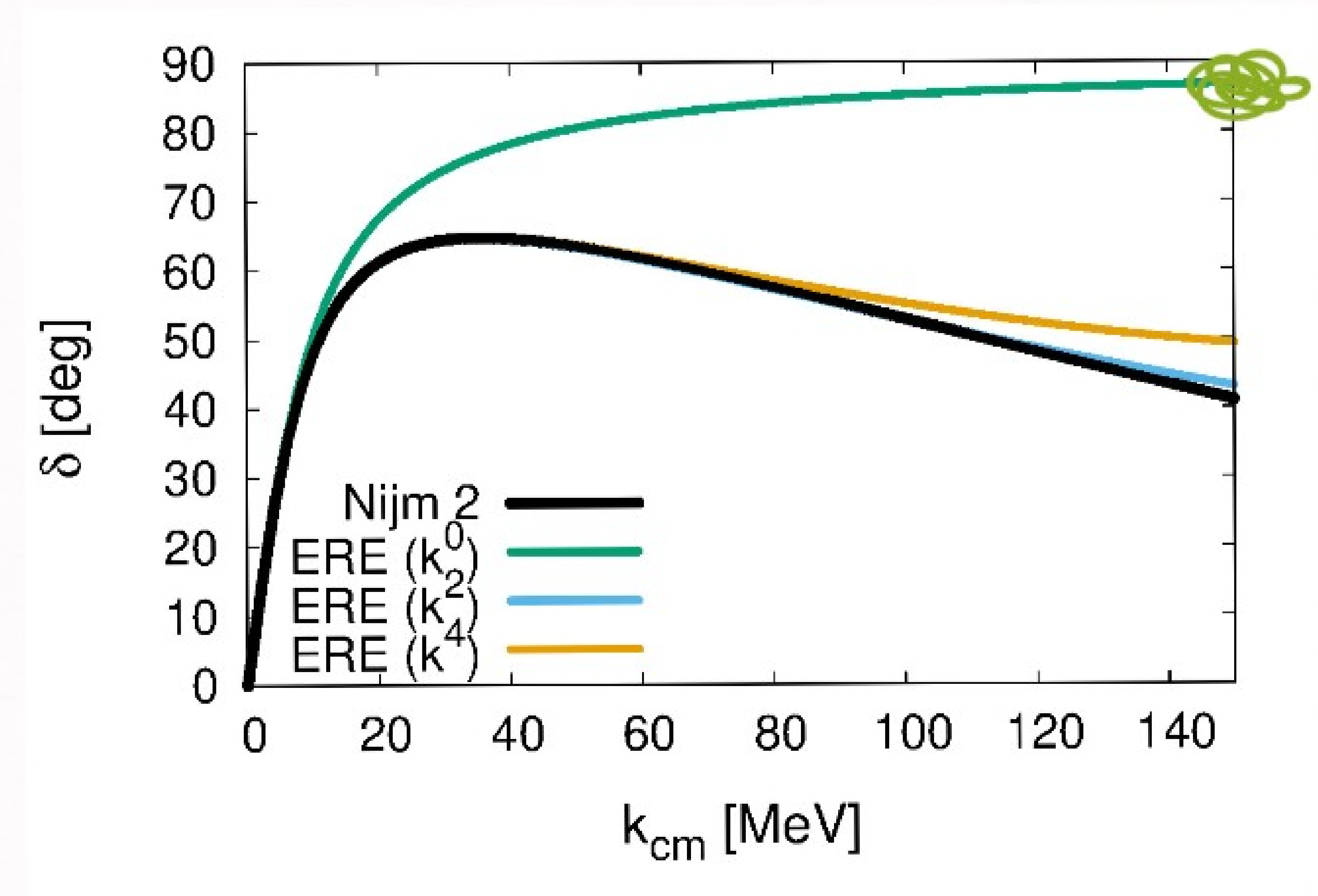
$$k\cot\delta\Big|_{LO} = -\frac{1}{a_0} + \lambda R_c k^2 \quad (1)$$

(1), (2) very similar for good choices of R_c [$\to 0$ for $R_c \to 0$]

(A) $\to$ We can use this to our advantage

$$k\cot\delta\Big|_{NLO} = -\frac{1}{a_0} + \frac{1}{2} r_0 k^2 + \lambda' R_c^3 k^4 \quad (2)$$





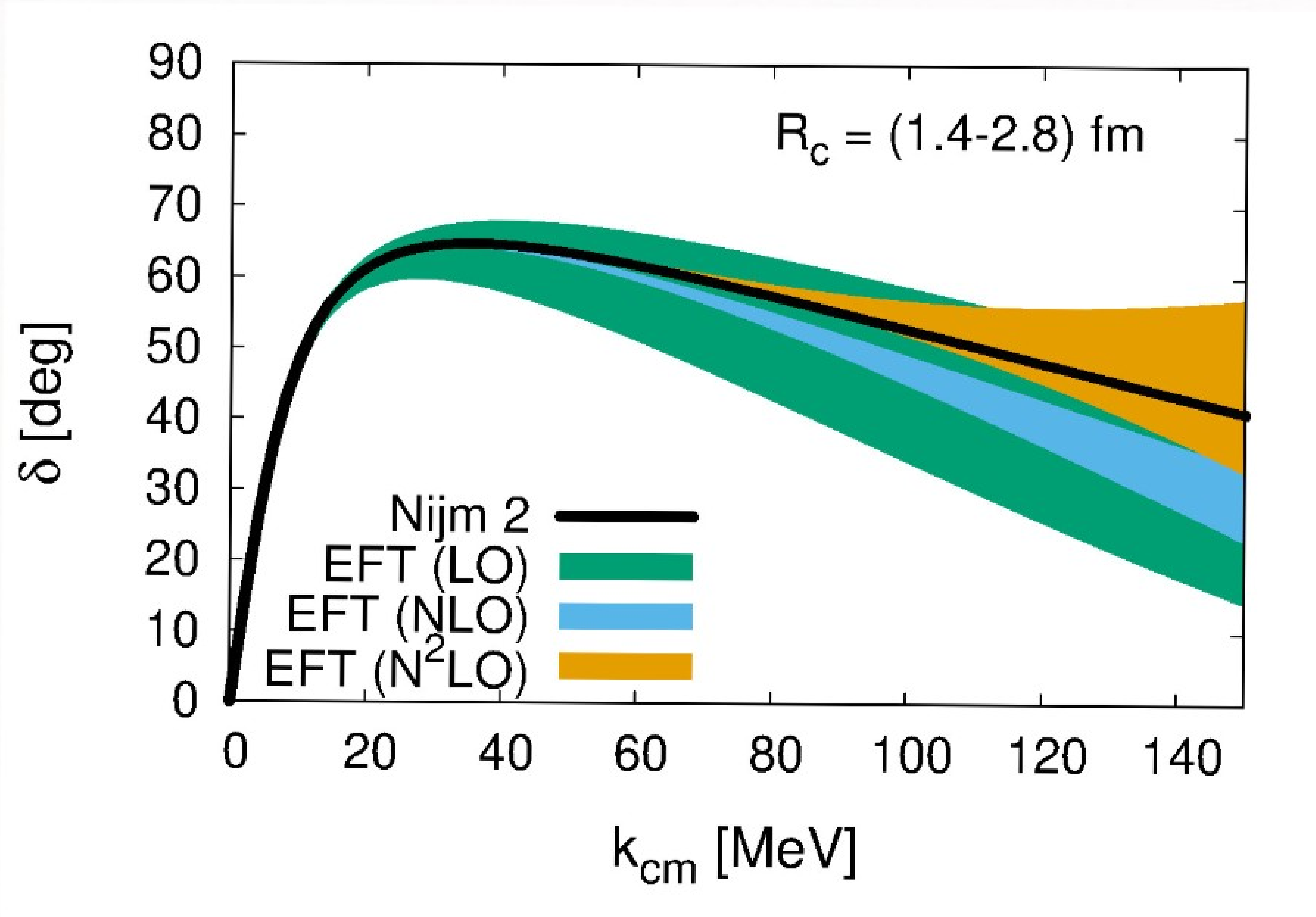
LO w/ R_c = 1.4 fm better than ERE (k^0)

Third comment) → We can use this residual cutoff dependence to estimate the error of our EFT calculation

(Why?)

Residual R_c dep

→ higher order effect



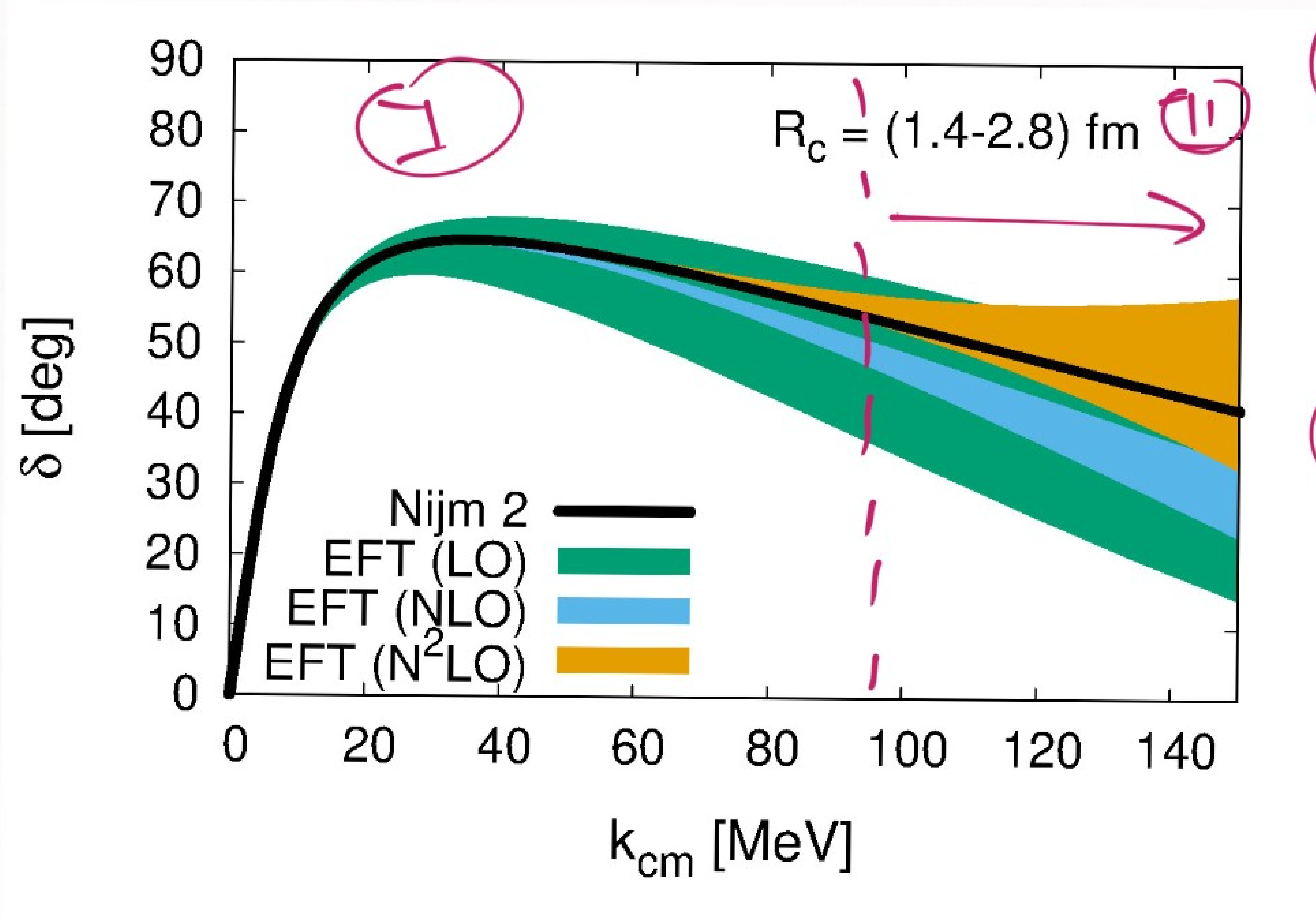
→ I vary the cutoff

→ Experimental data usually inside my "error band"

Fourth comment) → EFTs have a radius of convergence

↳ only work at low energies

$$kR_s \lesssim 1$$

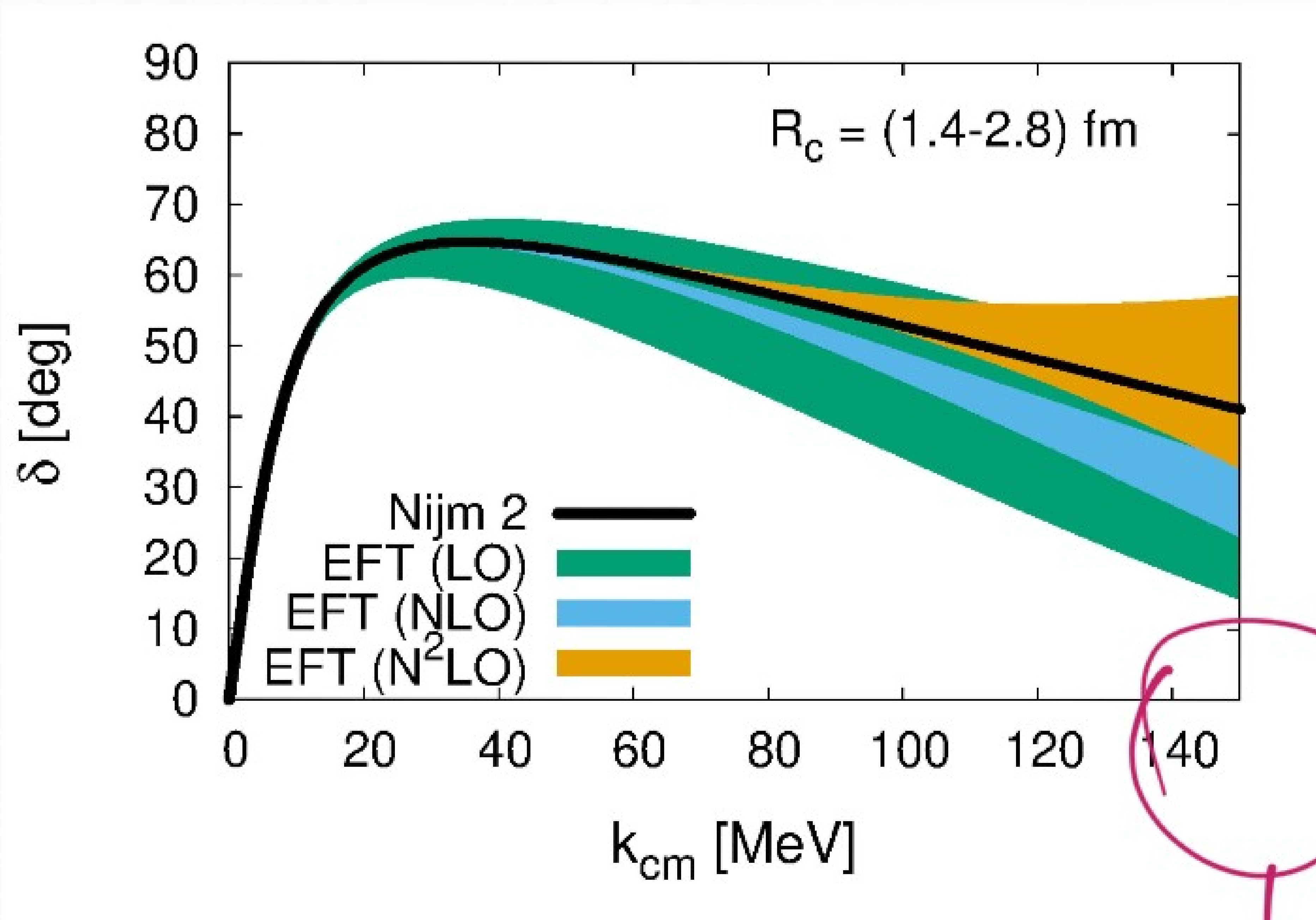


(I) → no apparent problem

(II) → error bands grow w/ order (this should not happen)

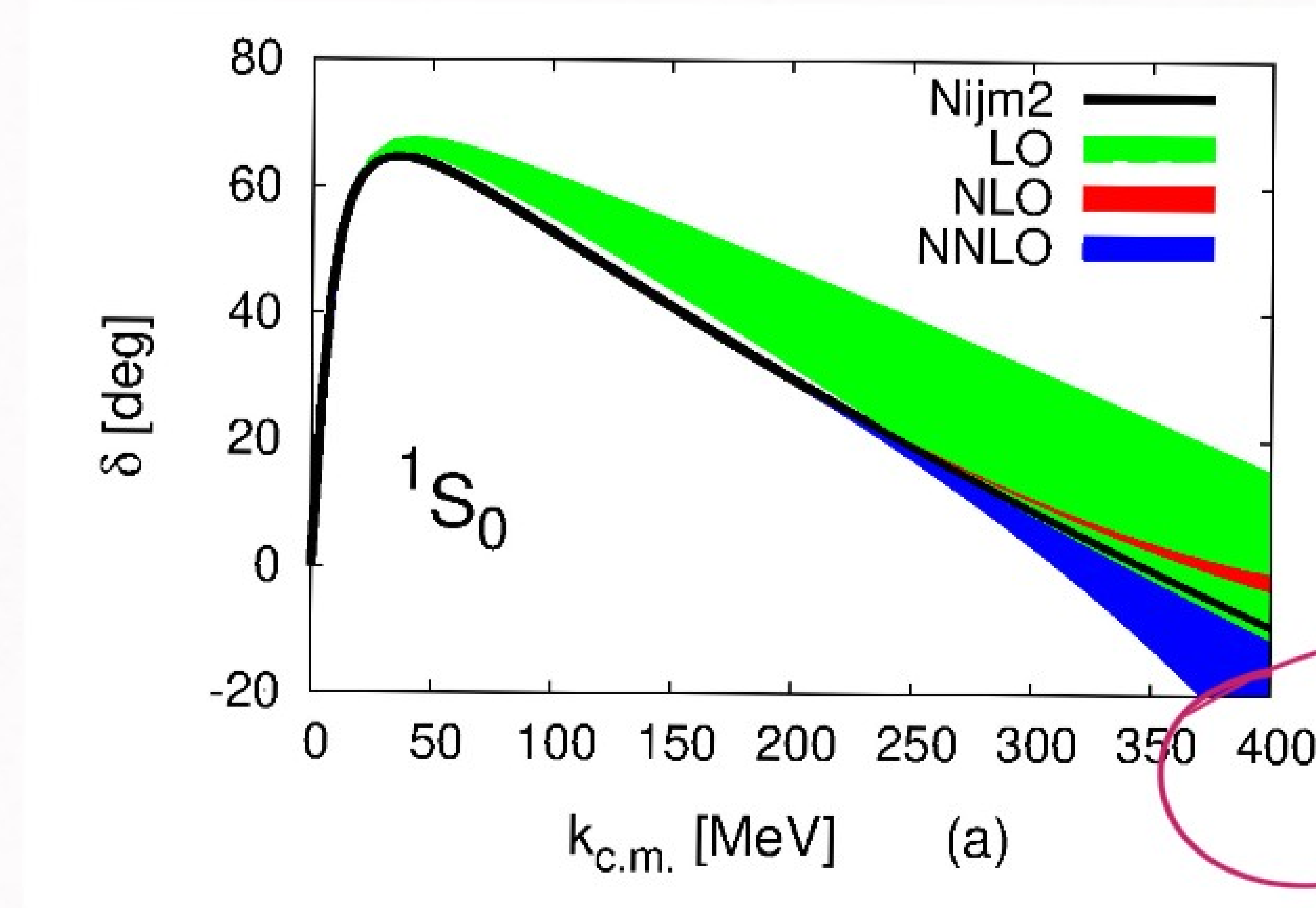
If I went to k = 200 MeV → it will look bad

EFT($\not\pi$)



$k < m_\pi$

EFT(χ)



$k < m_\rho/2$

→ You can only use EFT within its range of validity

(if you want to extend this range of validity ⇒ you have to use a different EFT)

PIONFUL EFT

1) Pionless EFT → only valid at very small momenta

We have to go beyond this limitation

→ **INCLUDE PIONS**

2) Include pions → increase the range of validity of the EFT

↙

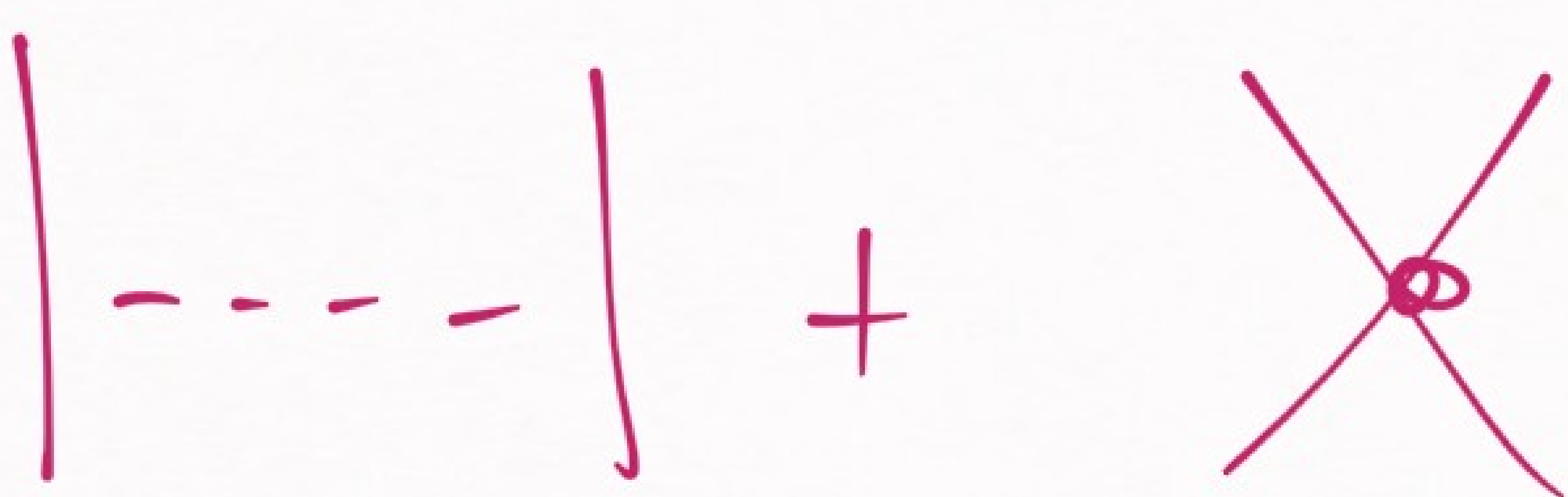
generate a lot of difficulties w/ renormalization

EFT(π) is a more difficult theory

→ [BIRD'S EYE VIEW OF EFT(χ)]

① Still an expansion: LO, NLO, N^2LO, etc.

② LO is easy to formulate:



ODE + a contact

$$V_{LO} = V_c^{(0)} + V_F^{(0)}$$

$$V_c^{(0)} = C_S + C_T \vec{\sigma}_1 \cdot \vec{\sigma}_2 \quad (\simeq C_0)$$

$$V_F^{(0)} = -\frac{g_A^2}{4 f_\pi^2} \vec{\tau}_1 \cdot \vec{\tau}_2 \frac{\vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q}}{q^2 + m_\pi^2}$$

(3) We have to regularize & renormalize

p-space $\quad V_{LO}(\vec{q})$ or $\langle \vec{p}\,' | V_{LO} | \vec{p} \rangle$

$$\longrightarrow \quad V_{LO}(\vec{q})\, f\left(\frac{|\vec{q}|}{\Lambda}\right) \text{ or } \langle \dots \rangle\, f\left(\frac{p}{\Lambda}\right) f\left(\frac{p'}{\Lambda}\right)$$

r-space $\quad V_{LO}(\vec{r}, R_c) = V_{OPE}(\vec{r})\, \Theta(r - R_c)$

$$+ \left(C_S + C_T\, \vec{\sigma}_1 \cdot \vec{\sigma}_2\right) \frac{\delta(r - R_c)}{4\pi R_c^2}$$

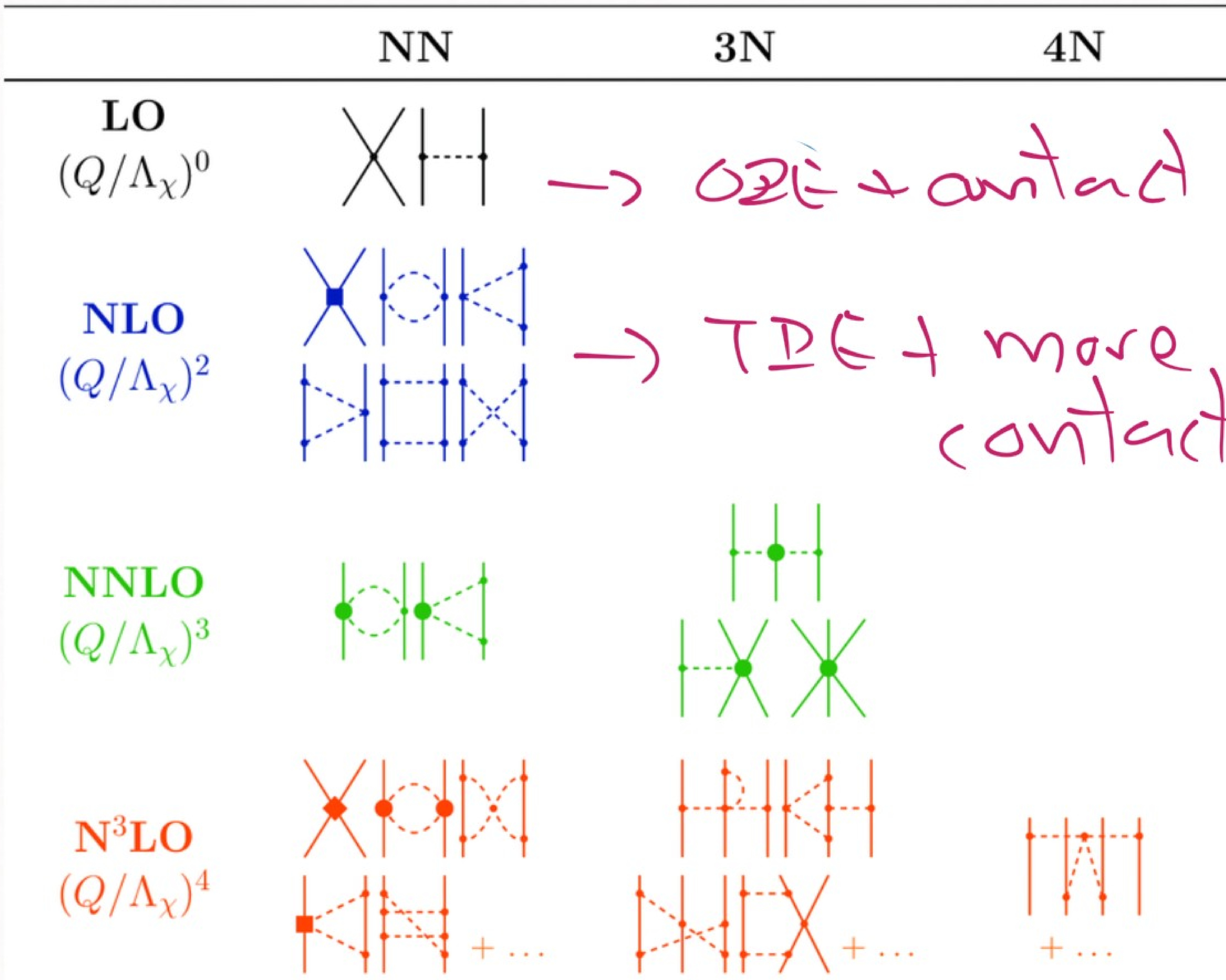
(3') There will be surprise

(coming from the pions)

$V_{OPE}(r^4) \xrightarrow[r \to 0]{} \frac{1}{r^3}$ → singular potential

→ makes renormalization difficult

(4)



Contributions at different orders

becomes more complex

→ Enough for a bird's eye view

↓

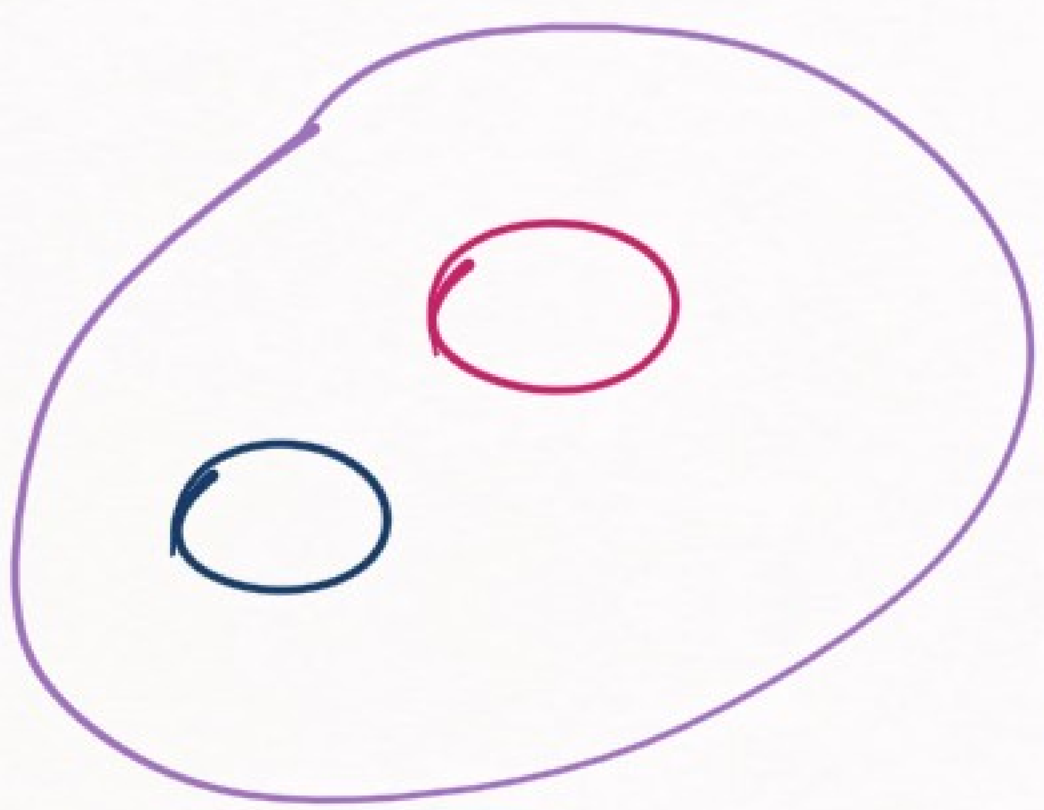
→ Complex topic (a few of you have some expirience)

NUCLEAR STRUCTURE

→ What happens when we have many nucleons?

HOW TO DESCRIBE THIS?

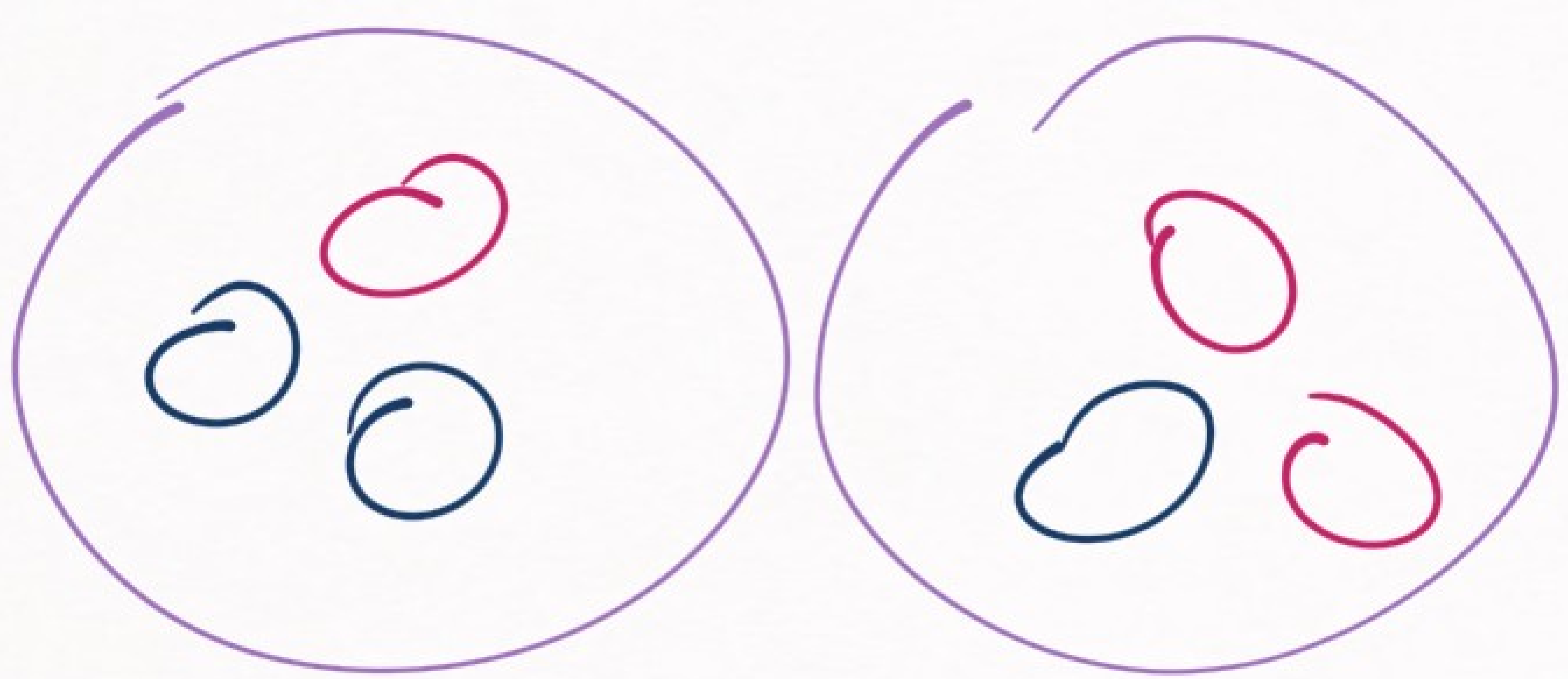
Why do we need to study nuclear structure?



A = 2 nucleons → DEUTERON

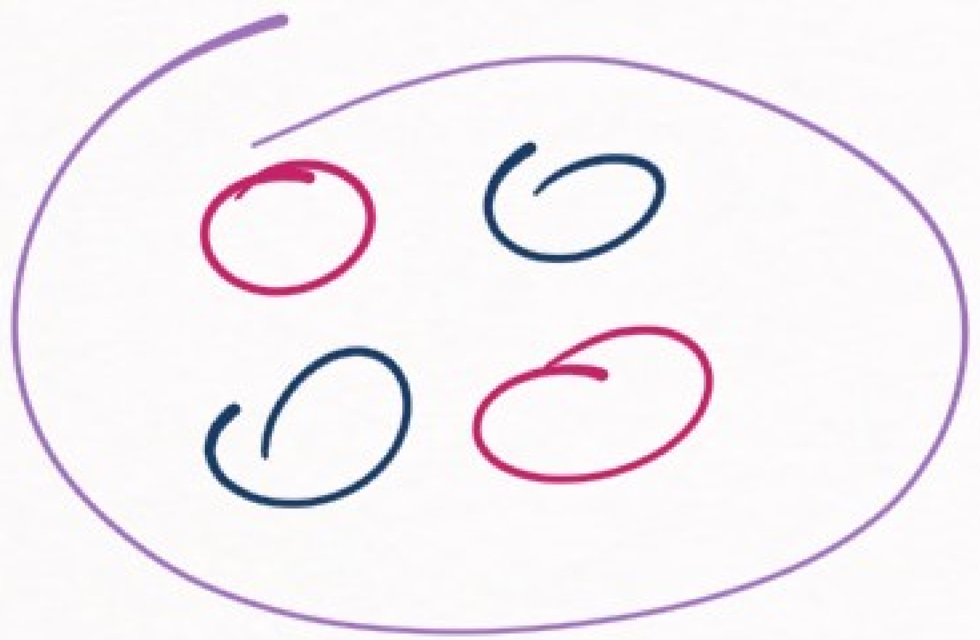
(Descriptions → two-body wave functions)

^3H $\quad$ ^3He



A = 3 nucleons

(solve Faddeev, get wave function, compute things)



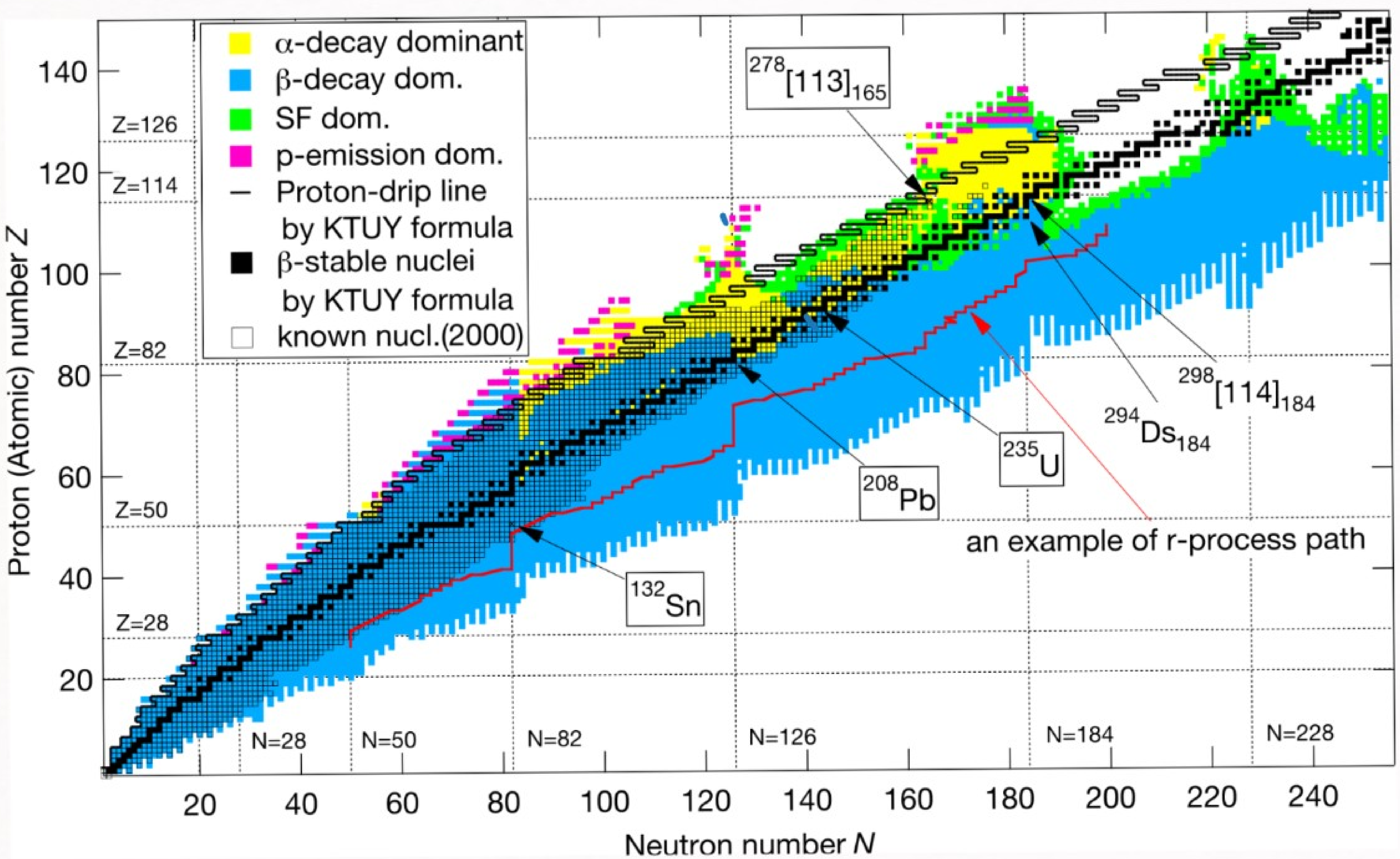
^4He

→ $A = 4$, still solvable (Faddeev-Yakubovsky)

How long can we do this?

→ $A < 10-12$?

→ We won't get far



→ For most nuclei, we can't solve Schrödinger

→ We need SIMPLIFICATIONS

A small $\Rightarrow$ Exact methods

(some version of Schrödinger)

A big $\Rightarrow$ We have to exploit properties /structure of these nuclei

(→ aim: doing simplifications)

WHAT DO WE WANT TO DESCRIBE?

1) Binding energy of nuclei

2) Size

3) Angular momentum & parity

4) Magnetic dipole moments (e.m. moments)

5) Stability / decay

[NUCLEAR PROPERTIES]

1) Binding energy (definition)

Deuteron → $B_d = (m_p + m_n) - m_d \simeq 2.2\ \text{MeV}$

Triton → $B_t = (m_p + 2m_n) - m_t \simeq 8.5\ \text{MeV}$

↱ # protons

$\Rightarrow B(Z,N) = (Zm_p + Nm_n) - M(Z,N)$

↳ # neutrons

$$B(Z,N) = (Z m_p + N m_n) - M(Z,N)$$

Interesting property:

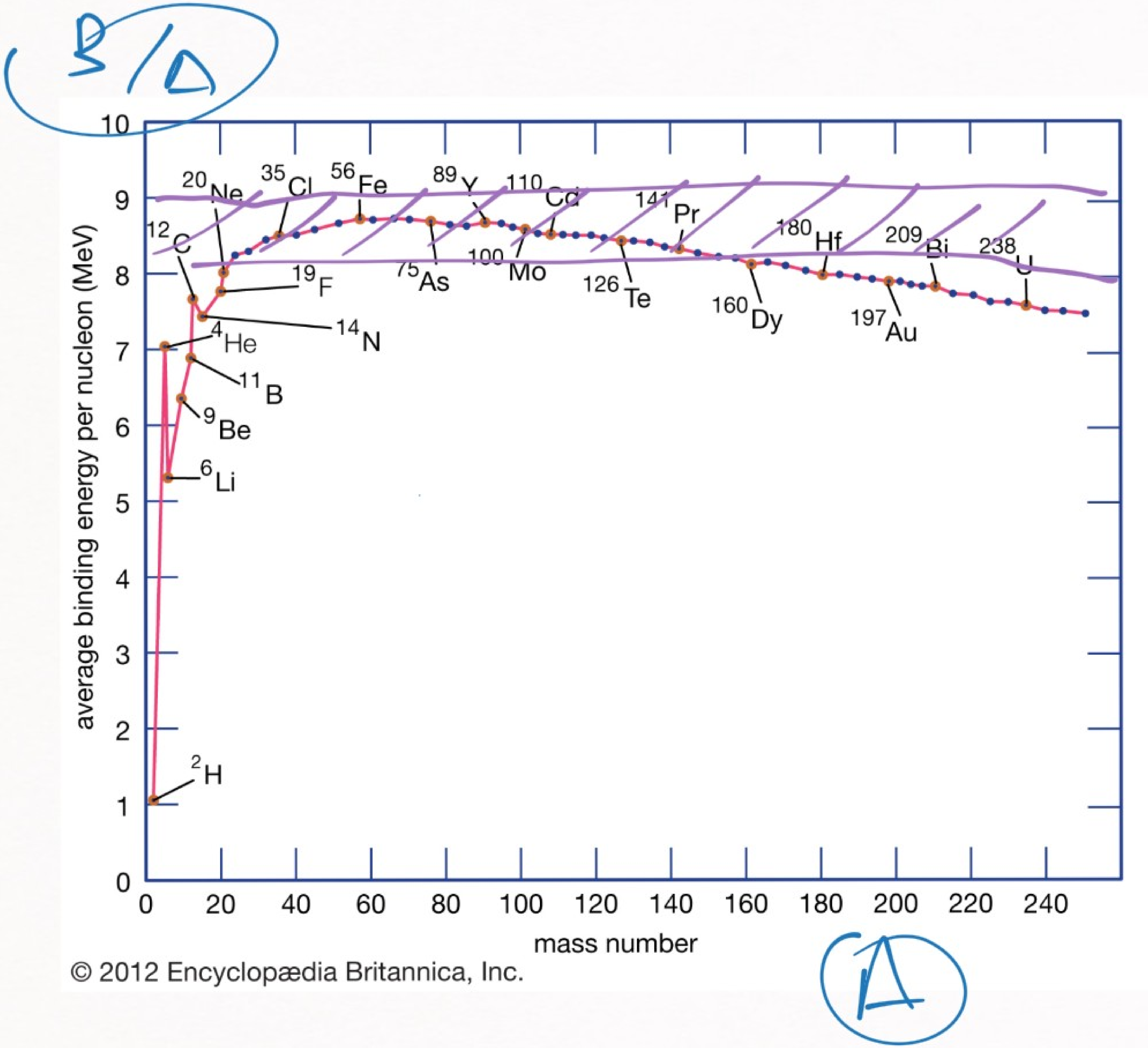
$\frac{B}{A} \sim (8-9)$ MeV / nucleon

$A \gtrsim (20-30)$ nucleon

SATURATION

$A = N + Z$

# of nucleons



→ <u>Observation</u>

B/A has a maximum around 56 Fe (approx.)

→ <u>Consequence</u>

A < 56 → fusion possible

A > 56 → fission possible

Separation energy → energy require to knock off a nucleon

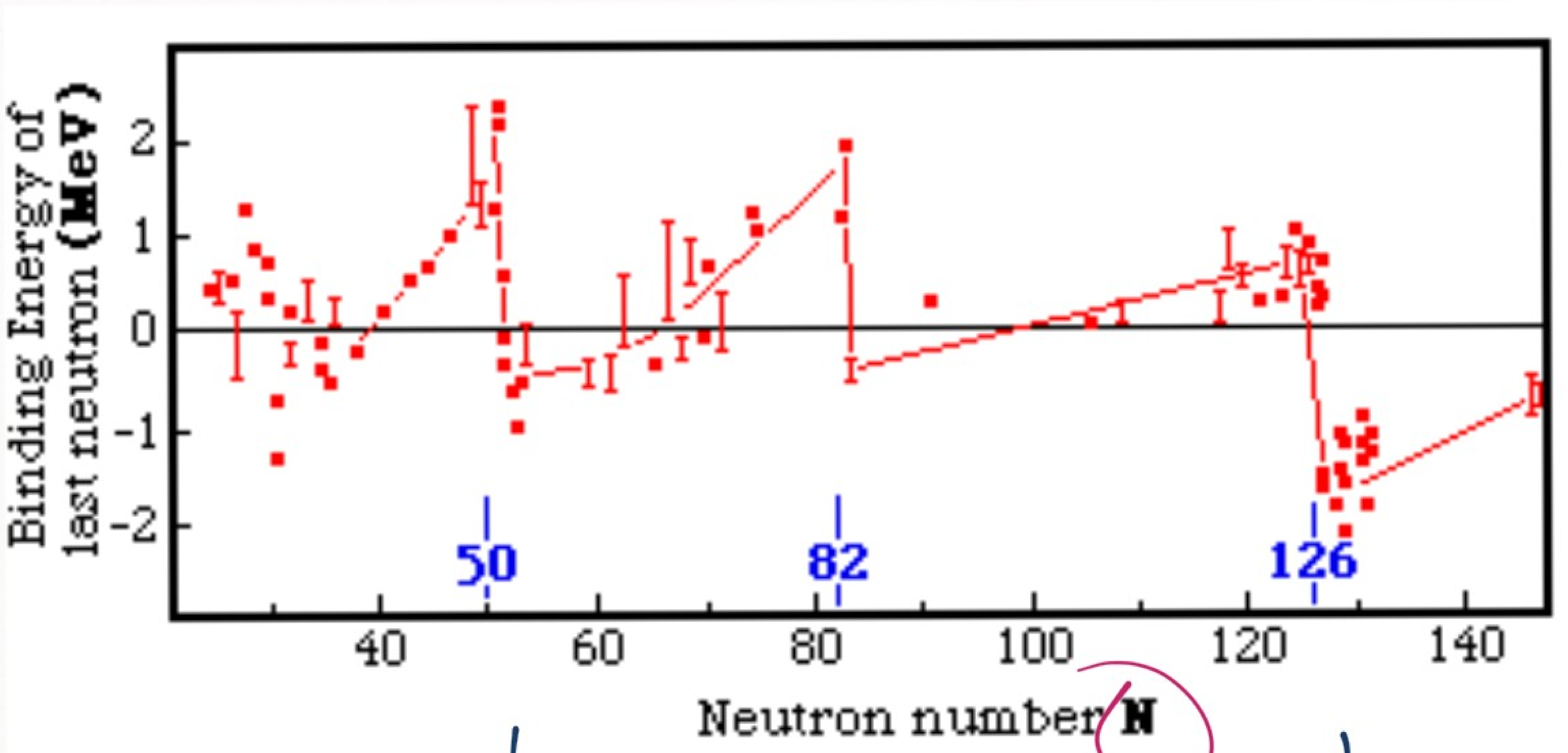
⇓

$$S_p(Z,N) = B(Z,N) - B(Z-1,N)$$

$$S_n(Z,N) = B(Z,N) - B(Z,N)$$

} easy definition

→ ∃ a pattern in S_p & S_n



→ $S_n(Z,N)$

∃ N / S_n is larger than usual

(same thing happens w/ S_p)

Magic numbers → $N(Z) = 2, 8, 20, 28, 50, 82, 126, \ldots$

neutron & proton numbers that give very stable nuclear configuration

→ Hint of the existence of the SHELL MODEL

Separation energy for α-particles

(α-particles → ^4He nucleus)

$$S_\alpha(Z,N) = B(Z,N) - B(Z-2,N-2) - B(2,2)$$

→ energy require to knock off an α-particle from a nucleus

$A > 150 \rightsquigarrow \boxed{S_\alpha < 0}$

↙

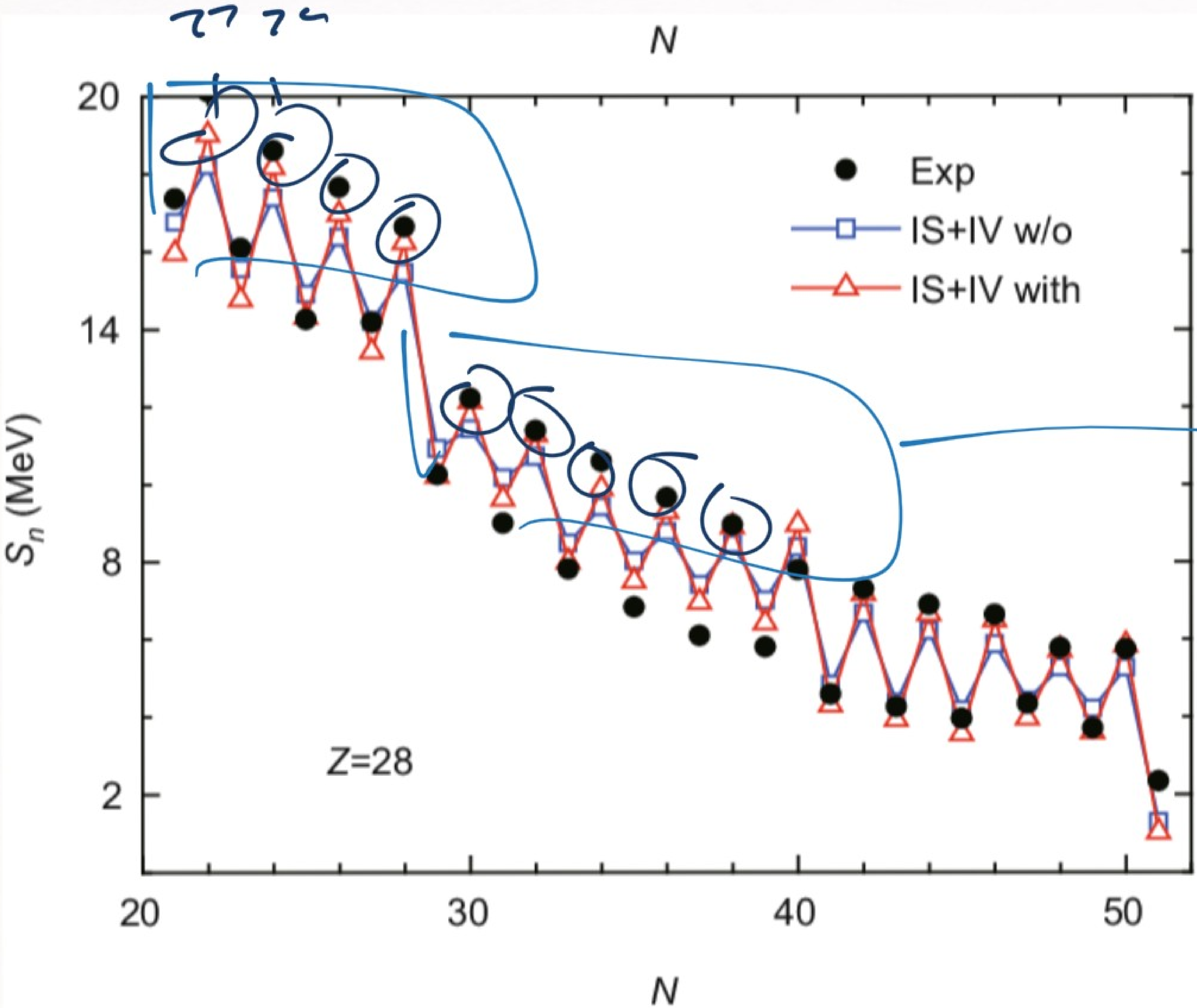
α-particles will be spontaneously emitted in these nuclei

$\sum$

Another type of sep. energy

$$\delta_n = S_n(Z,N) - S_n(Z,N-1)$$

→ difference between nuclei w/ even or odd numbers of neutron or proton



more stable

⇕

nuclei w/ even # of n have larger Sn

(δn) → Configurations w/ even numbers of neutrons or protons are **more stable**

Nuclei w/ odd # neutrons / # protons are less stable & **more rare**

[Binding energies] → obtained a lot of info

→ Saturation

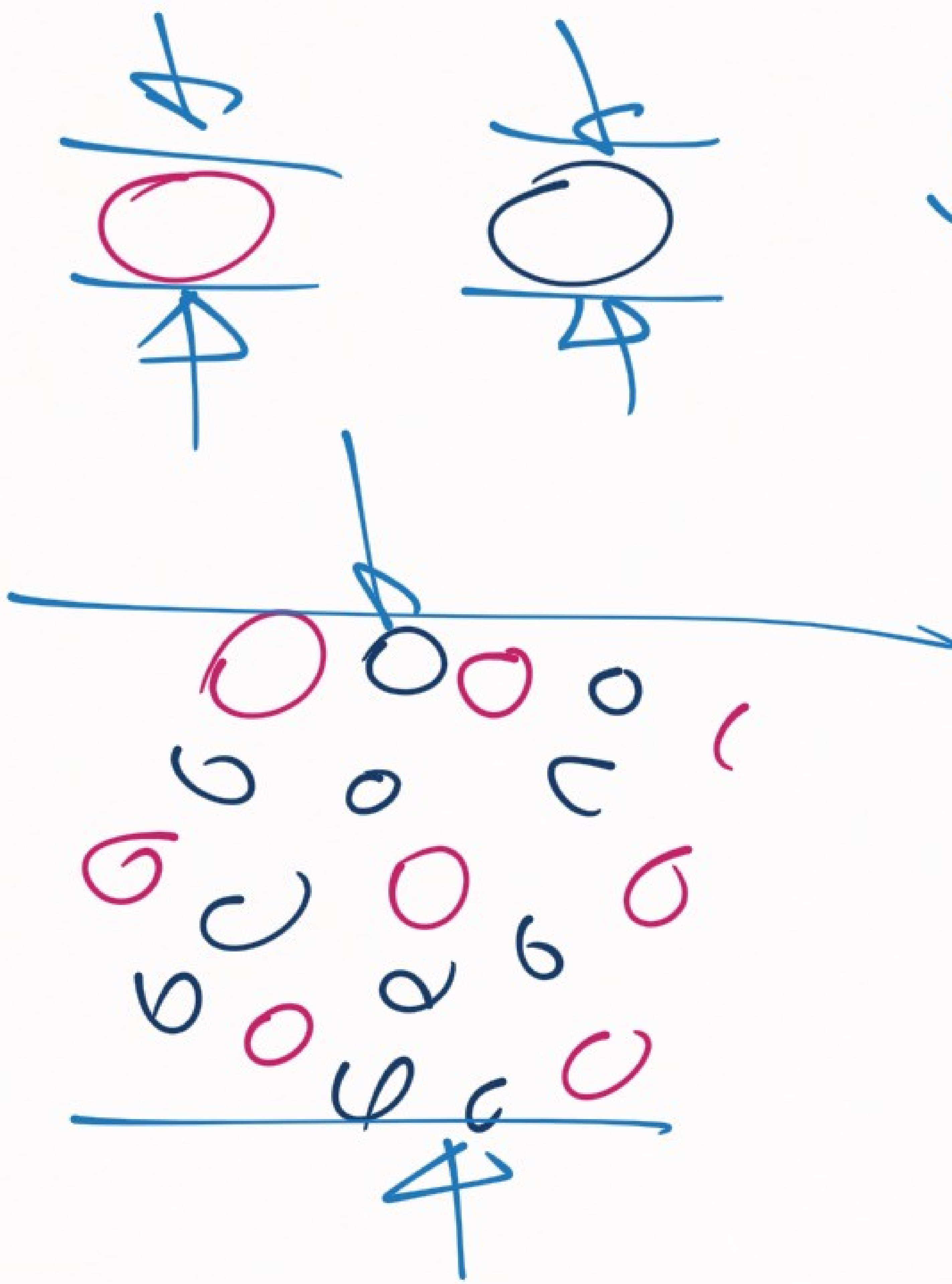
→ Magic numbers

→ α-decay of heavy nuclei

→ "pairing energy" → even(A)/even(Z) nuclei are more stable

Next nuclear property:

2) NUCLEAR SIZE



$\sqrt{\langle r^2 \rangle_{n,p}} \sim (0.5 - 1.0)\,\mathrm{fm}$

$R^3 \sim A\, r_0^3$

w/ $r_0 \sim (1.2 - 1.3)\,\mathrm{fm}$

r_0 = avg seperation of n,p

$$R \sim A^{1/3} r_0$$

$r_0 \sim (1.2 - 1.3)\, fm$

→ size of most nuclei

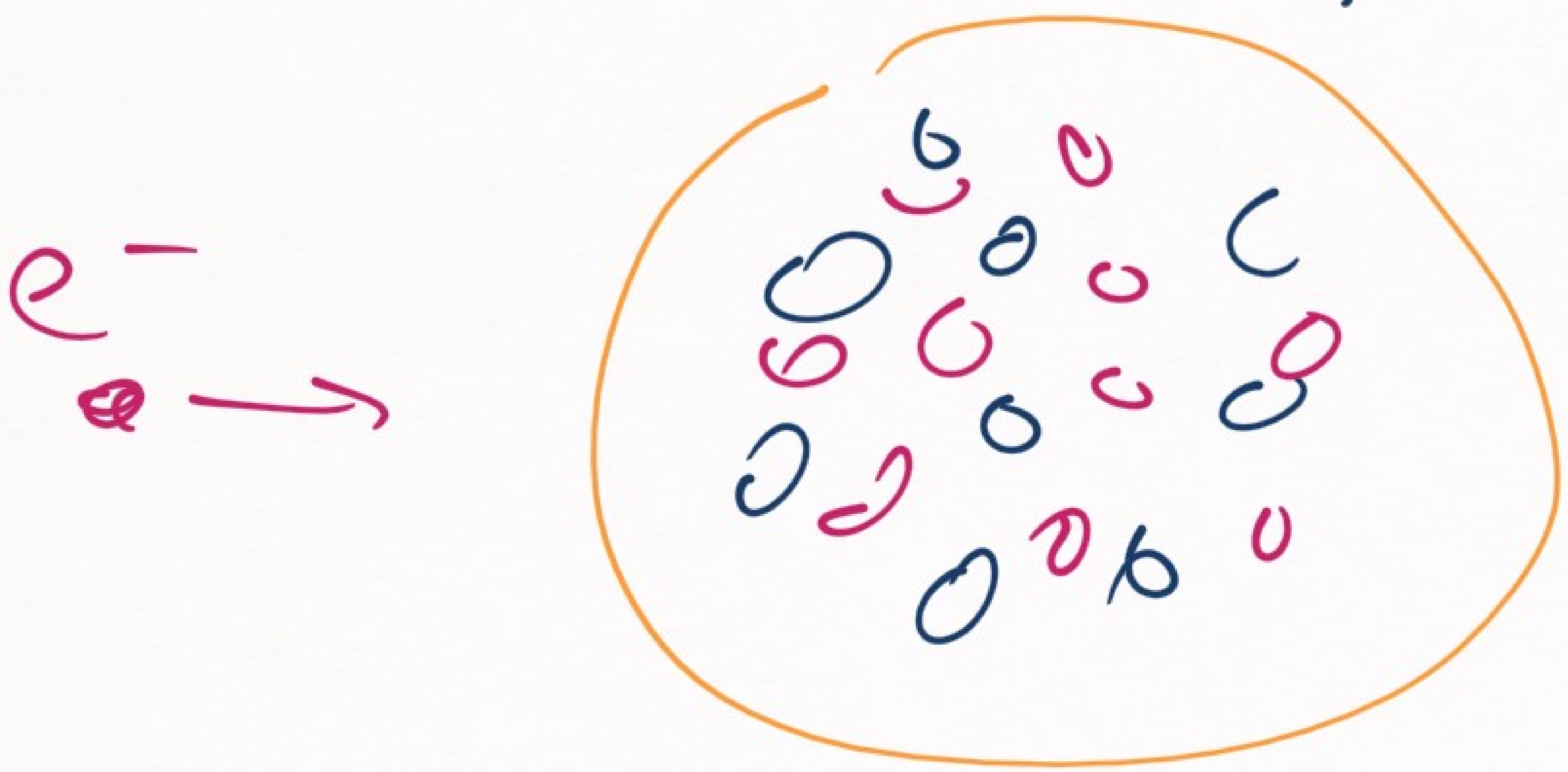
How do we know this?

→ [Form factors of nuclei]

↓

(?)

FORM FACTOR → difference between scattering w/ point-like particles & extended objects

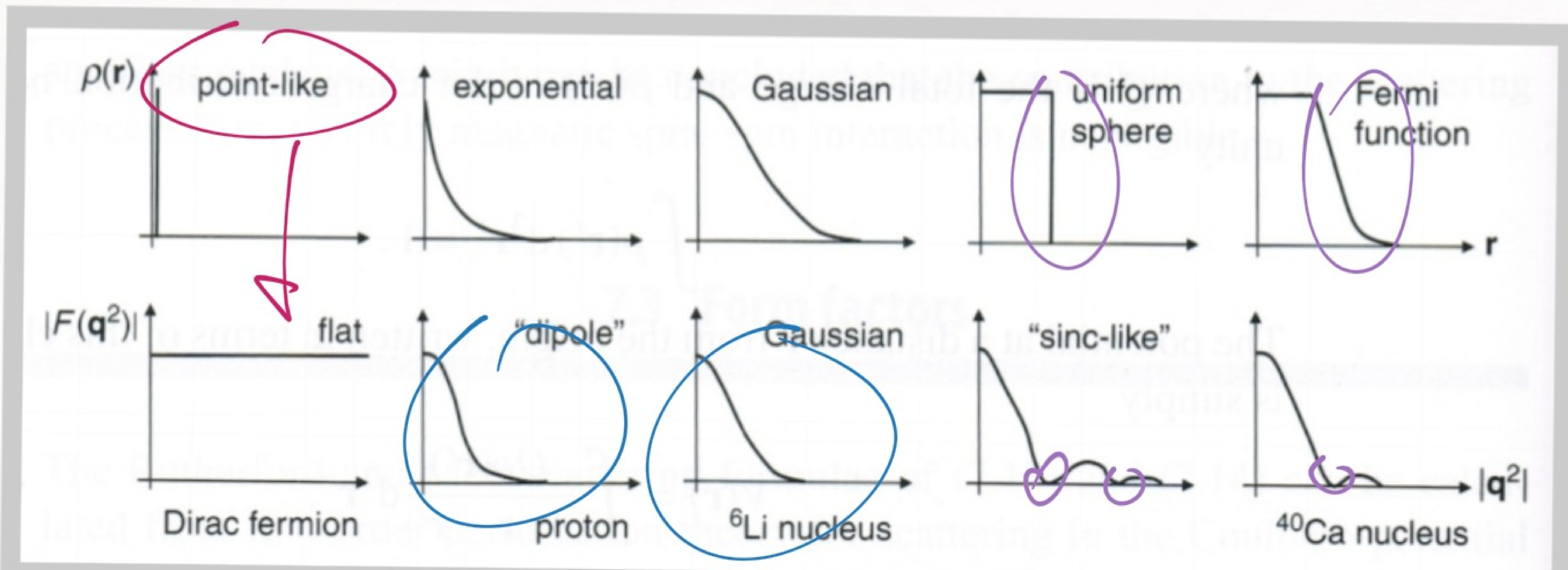


$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{point-like}} |F(\vec{q})|^2$$

$F(\vec{q})$ → form-factor → internal structure nucleus

$$F(\vec{q}) = \int d^3\vec{r}\, |\psi(\vec{r})|^2 e^{-i\vec{q}\cdot\vec{r}}$$

$\rho(\vec{r}) \rightarrow$ charge distribution



taken from ref. [2]

$\rightarrow \rho$

$\rightarrow F(|\vec{q}|^2)$

→ Actually, experiments show zeros on the Form factors

⇒ Density in nuclei is more or less constant and then drops quickly

SEE YOU ON FRIDAY