

# NUCLEAR PHYSICS (12)

→ GOLDSTONE THEOREM

→ CHIRAL SYMMETRY



RECAP

We studied the

LINEAR SIGMA MODEL  
(LSM)

→ Problem addressed  
by LSM:

Why is the plan so tight?

$$\frac{m_1}{m_p} \approx \frac{1}{8}$$

$$\frac{m_1}{m_2} \approx \frac{1}{17}$$



# LRM

1) Theory w/ massless nucleons & massive bosons

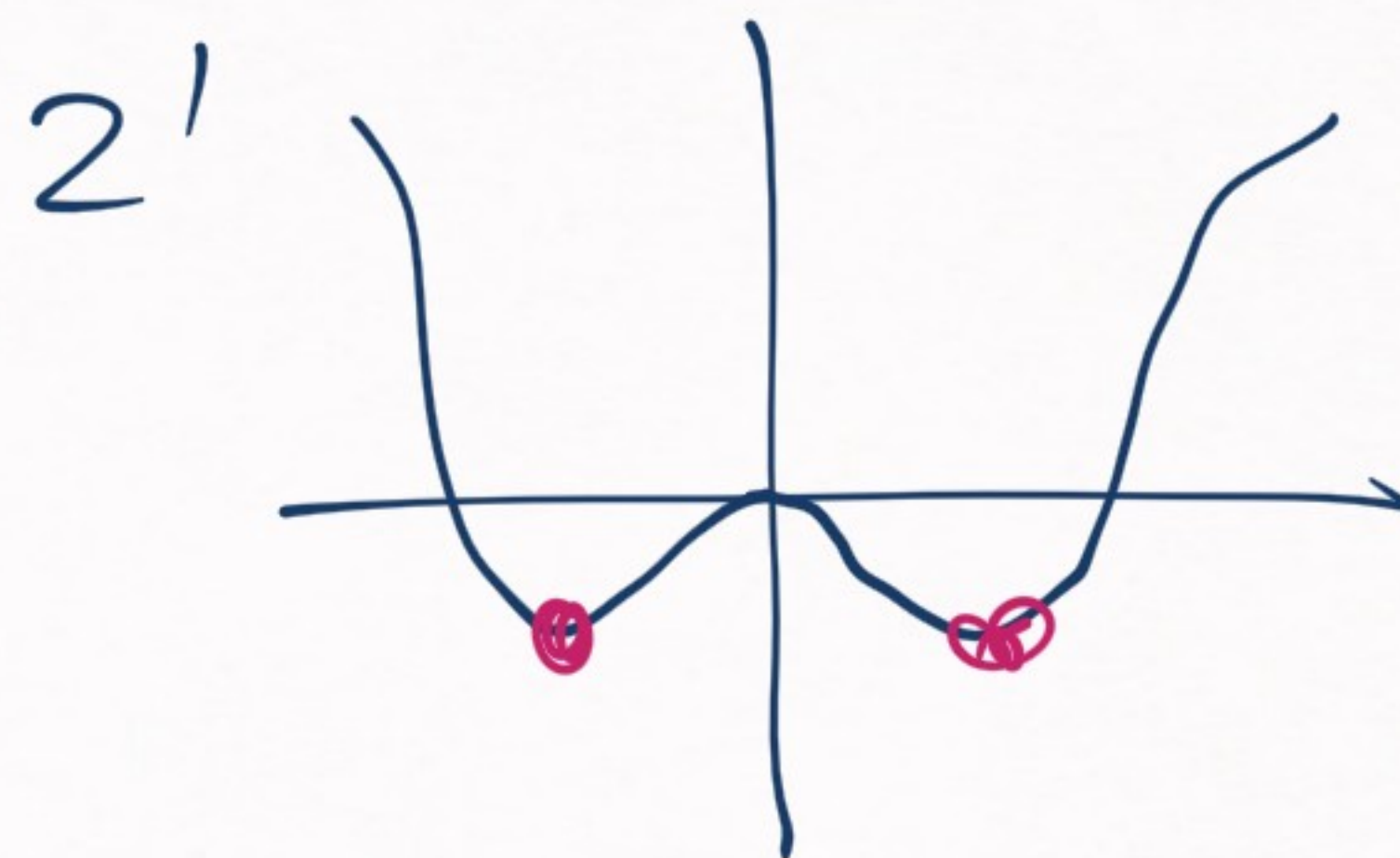
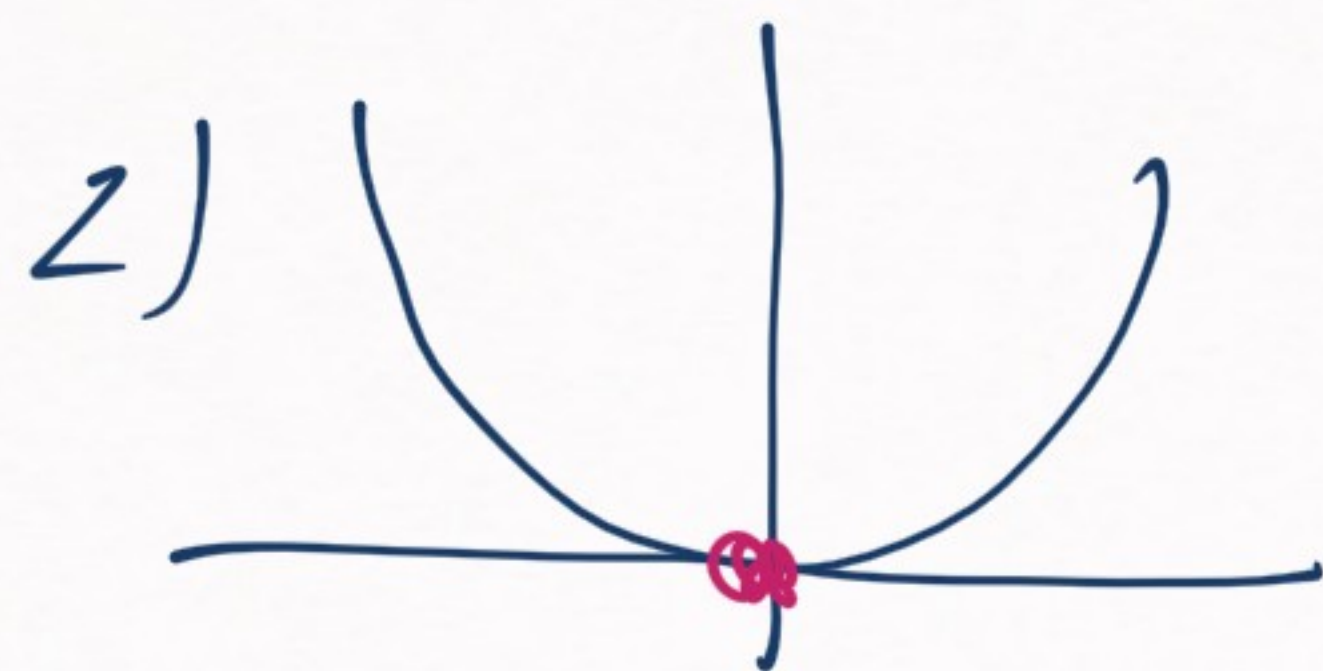
2) Quartic potential for the massive bosons

$$V(\alpha) = \frac{M^2}{2} \sum_i \alpha_i^2 + \frac{\lambda}{5} \left( \sum_i \alpha_i^2 \right)^2$$



2') Quartic potential is chosen w/  $\mu^2 < 0$

→ MEXICAN HAT POTENTIAL



3) Nature loves to minimize energy  
→ choice of ground state



→ Redefinition of the fields

MEYICAN  
HATI

$$\left\{ \begin{array}{l} \phi_i = 0 \text{ not minimum, but } \underbrace{|\vec{\phi}| = v} \\ \pi_i = \phi_i - v \quad / \quad |\vec{\pi}| = 0 \end{array} \right.$$

corresponds to the minimum

Why? → to be able to solve the OFT



(Note)  $\rightarrow$  in QFT, what we do is to somehow reduce the problem to a set of infinite harmonic oscillators  $\rightsquigarrow$  (h.o.)

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 \rightarrow \text{commutators}$$

( $\rightarrow$  creation/annihilation operators of h.o.)



4) End result is a theory w/

4.a) massive nucleons

4.b) massless pions

4.c) massive scalar boson ( $\sigma$ )

→ When L $\sigma$ M was formulated, the  $\sigma$  was not found experimentally  $\neq$



⇒ Formulate the non-linear  $\sigma$  model  
(end result similar but  
w/o  $\sigma$ )



(Later we found out that the  $\sigma$   
indeed exists)



Actually, the LGM is just one particular example of a more general result

→ **GOLDSTONE THEOREM**



# NAMBU-GOLDSTONE THEOREM

→ Full generalization of the idea behind  
the LOM

→ [SIMPLE DERIVATION]



(not very QFT-ish, but more  
(depend on group theory) QM-ish)



1) Hamiltonian  $H$  invariant under group  $G$

$$[H, G] = 0$$

2)  $H$  has a vacuum state (minimum energy or ground state)  $|0\rangle$

$$H|0\rangle = E_0|0\rangle, \quad |0\rangle \text{ ground/vacuum state}$$



3) Sometimes  $|0\rangle$  is not invariant  
under the group  $G$

$$[H, G] = 0 \quad \text{but}$$

$$G|0\rangle \neq |0\rangle$$

What we expect is that  $G|0\rangle = e^{i\alpha} |0\rangle$





4)  $\frac{1}{2}\langle e^{-1}, |0\rangle$  is invariant under a  
subgroup of  $G$

$$F \subseteq G \quad / \quad F|0\rangle = e^{i\alpha}|0\rangle$$

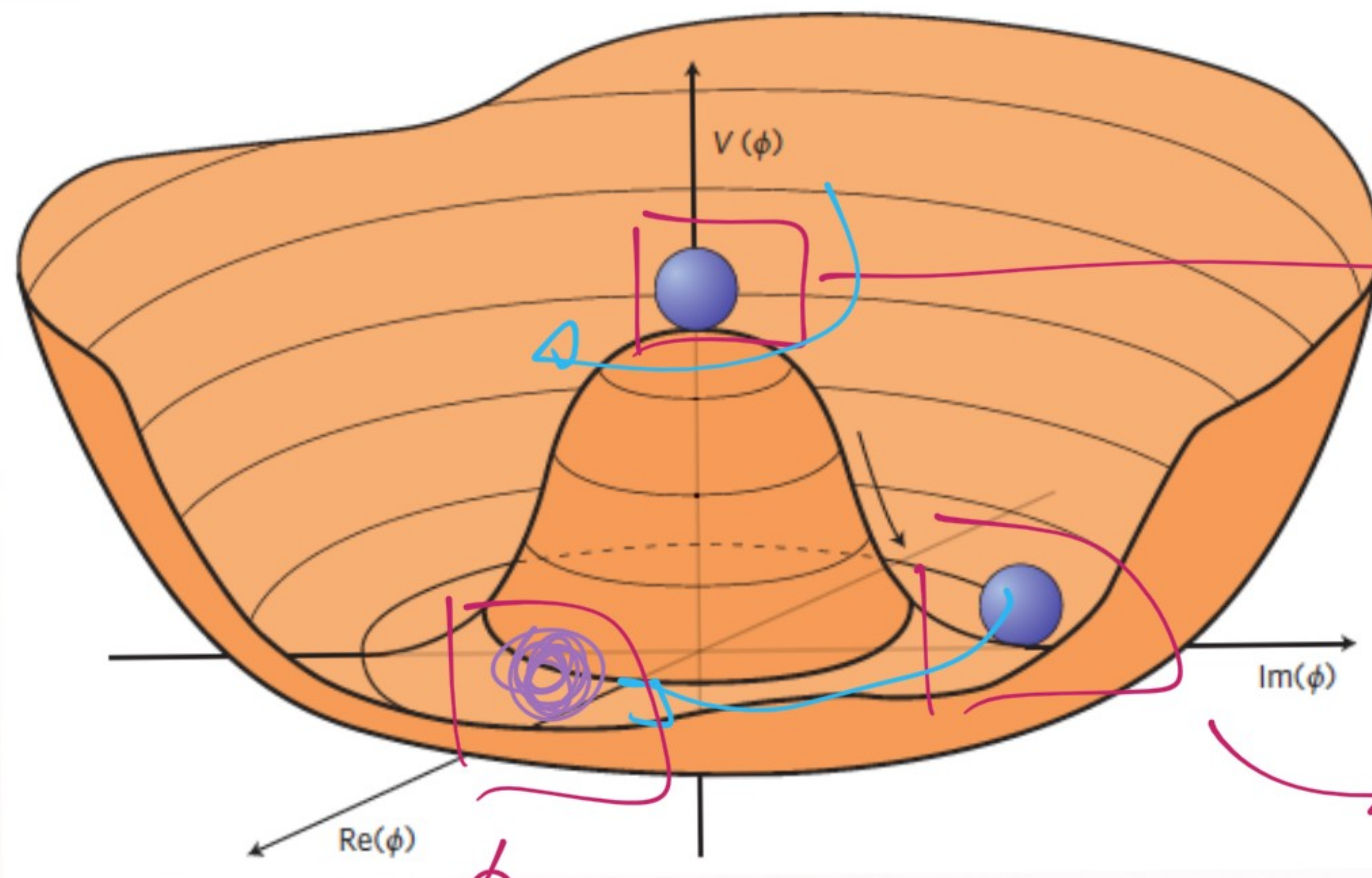




→ Let's go to 3)

$$G = O(2)$$

(rotations on the plane)



(maximum)

$$|\phi_1\rangle$$

$$R|\phi_1\rangle = e^{i\alpha} |\phi_1\rangle$$

$|\phi'_1\rangle$  (minimum)

$$R|\phi'_1\rangle = |\phi''_1\rangle \rightarrow \odot$$

$$|\phi_1'''\rangle \uparrow$$



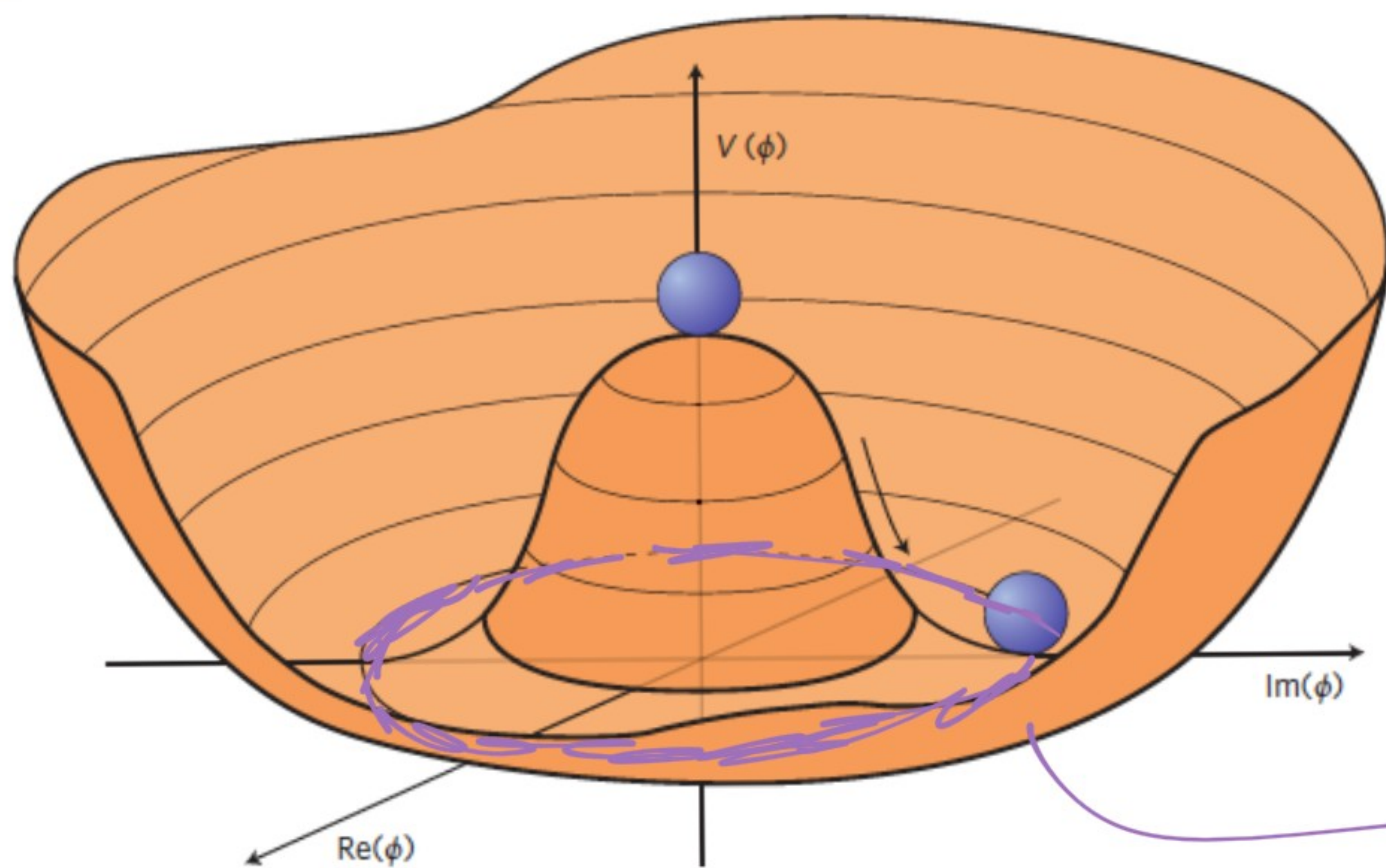
$$\textcircled{a} \rightarrow R|d_1'\rangle \neq |d_1'\rangle$$

that is,  $R|d_1'\rangle$  is linearly independent  
of  $|d_1'\rangle$

$\Downarrow$   
Vacuum (ground) state not invariant  
under rotations



In this case, this is because the minimum energy state is not unique



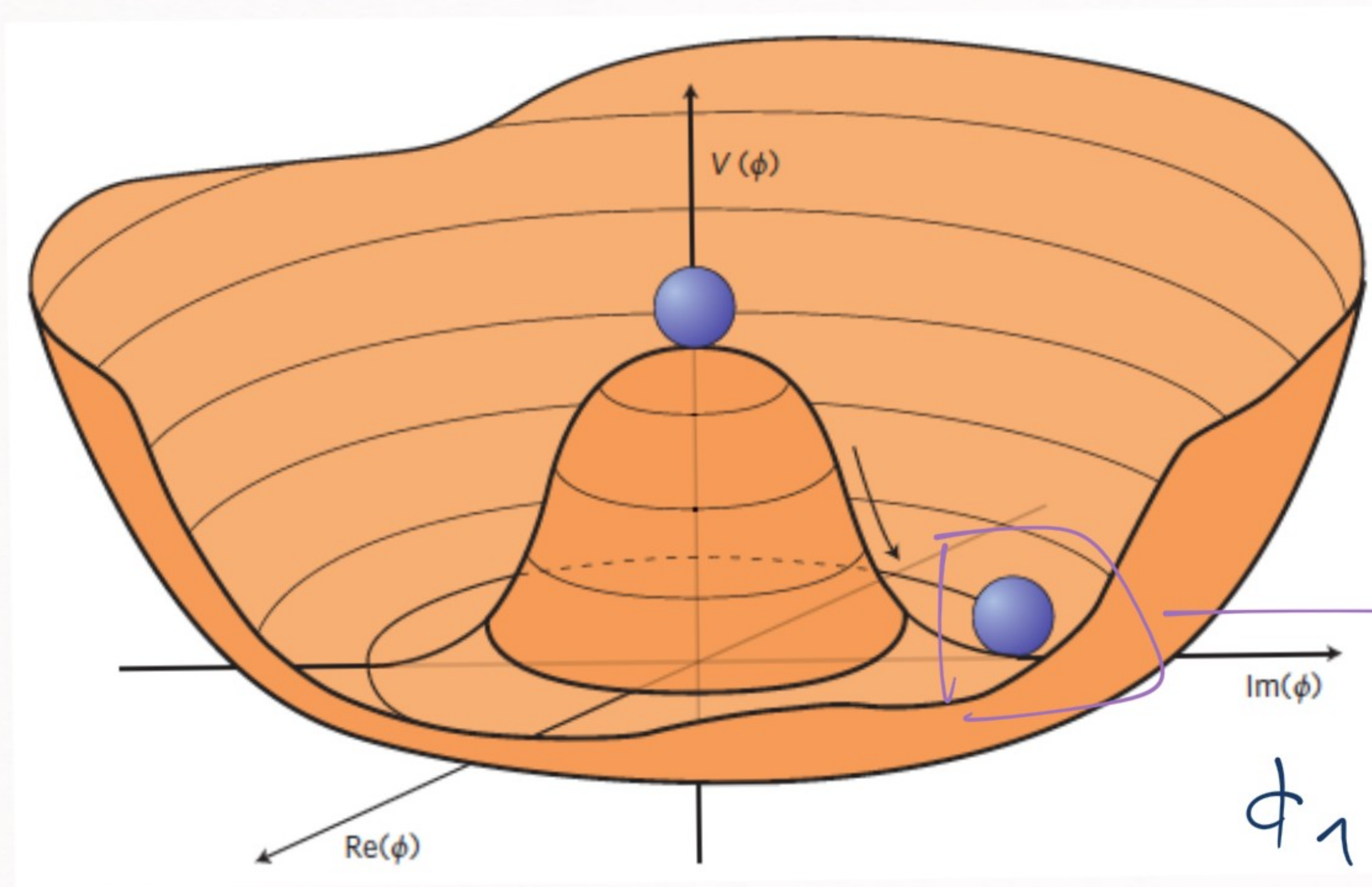
all minima



4)  $|0\rangle$  not unique (e.g. because there  
are many solutions to  $H|0\rangle = E_0|0\rangle$ )  
 $|0\rangle, |0'\rangle, |0''\rangle, \dots$

$|0\rangle$  still invariant under  $\underline{F \subseteq G}$   
(a subgroup of  $G$ )





$\phi_1$

$$G = O(2)$$

or  $U(1)$

For

$$|\phi_1\rangle \dots$$

$\Gamma =$  trivial group

$$(2_1, C_1)$$



5) If  $[+1, G] = 0$ ,  $G|0\rangle \neq |0\rangle$

but  $F|0\rangle = e^{i\phi}|0\rangle$  ( $\omega/F \leq G$ )

$\Rightarrow$  There will be a series of

massless fields

in the theory

Their number will be

$$\omega G - nF$$

(here is the only  
time we have use  
 $\mathbb{Q}(F)$ )



$$\rightarrow \# \text{ of massless fields} = n_G - n_R$$

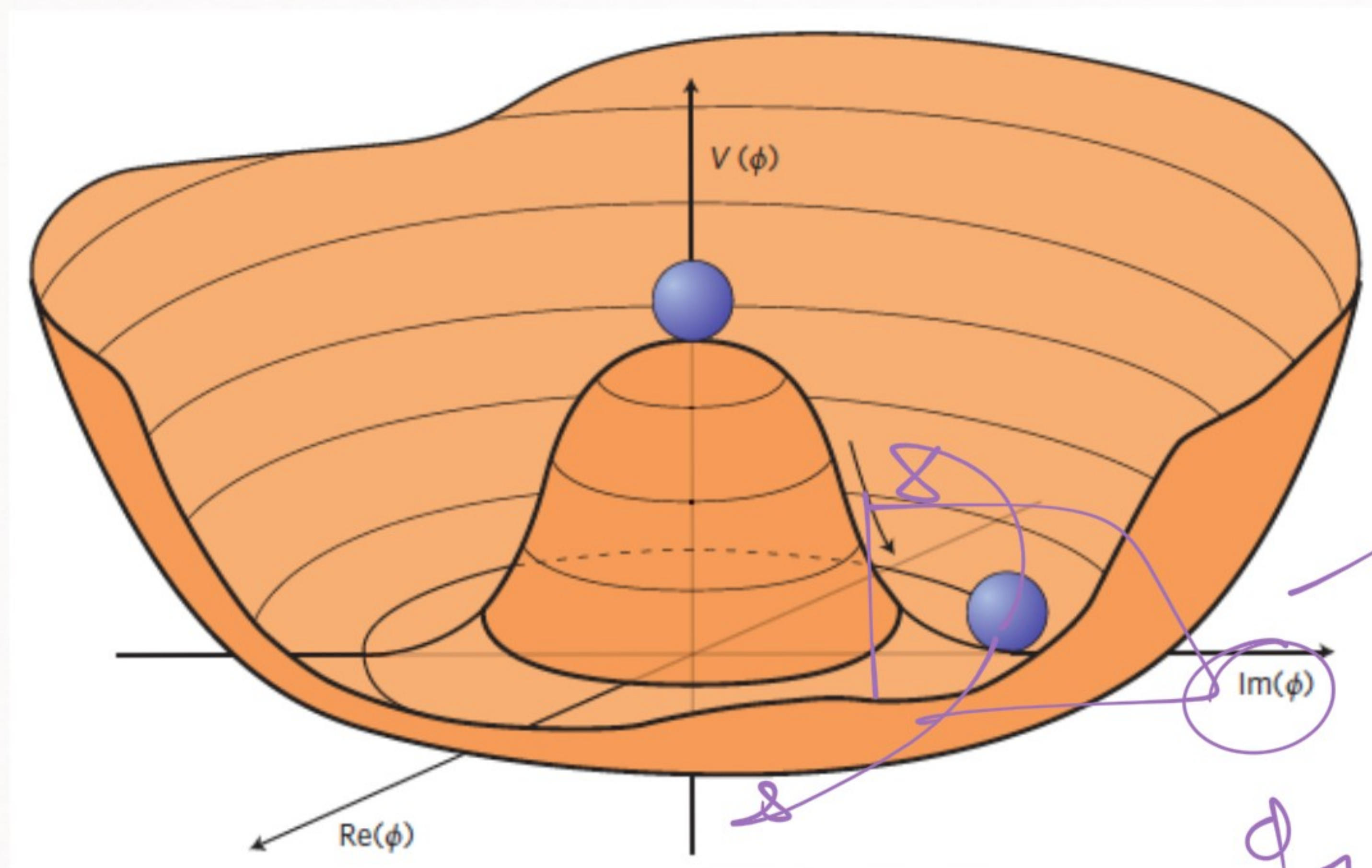
# of generators  
of group  $G$

# of generators  
of group  $R$

representation  
theory

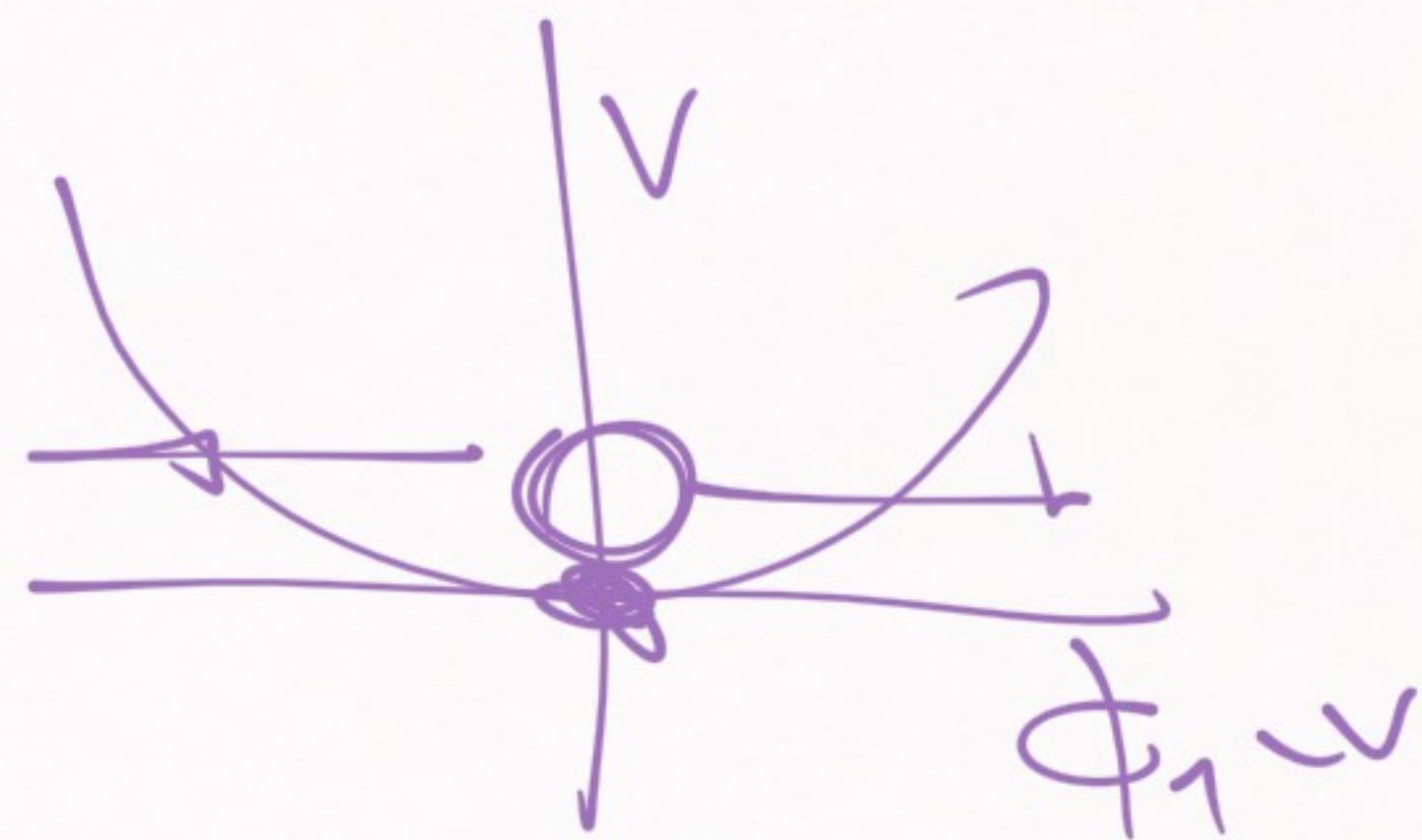


QFT → we reduce everything to harmonic oscillator



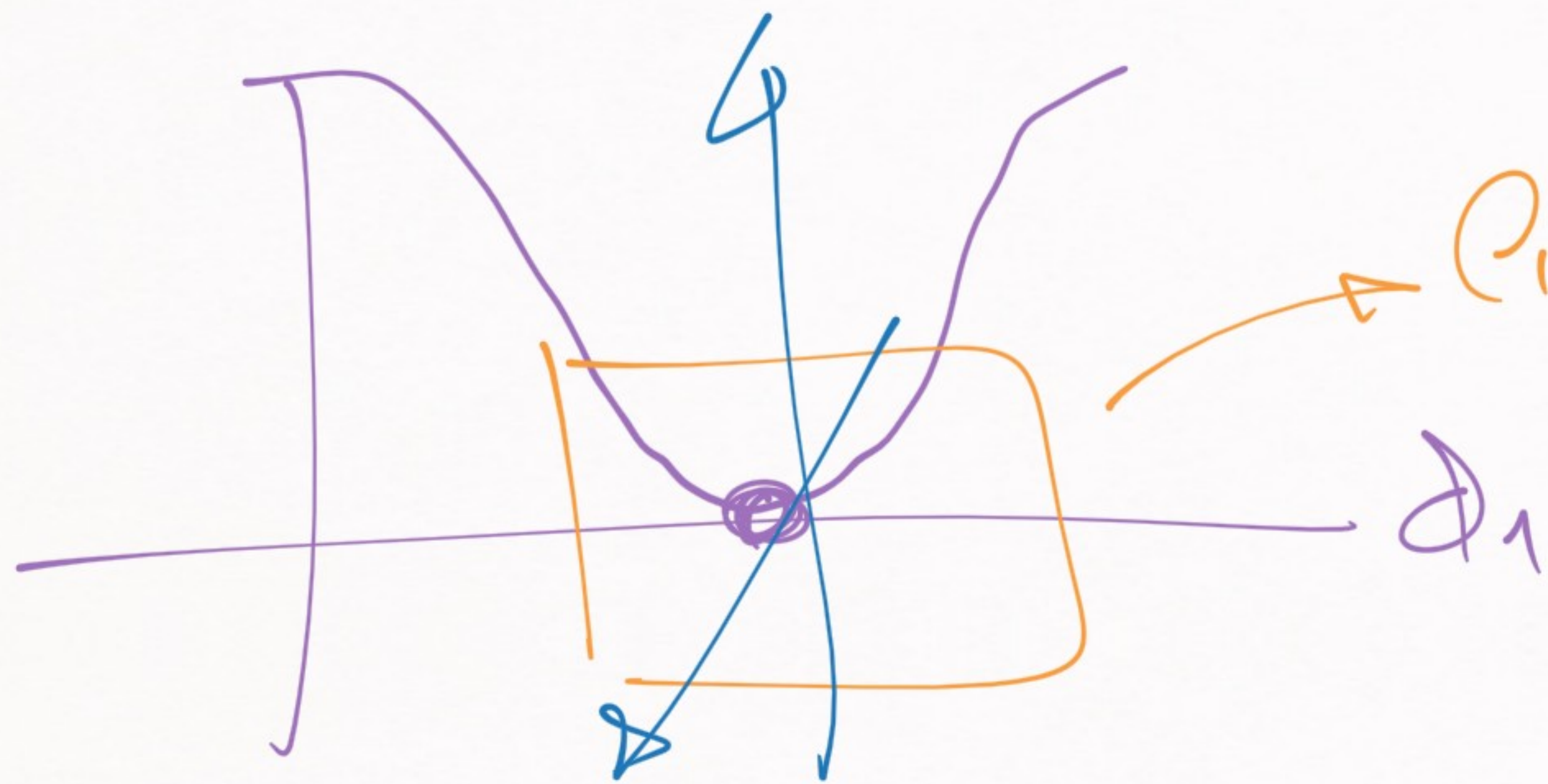
$q_1$

$q_1$



potential  
 $\propto (q_1 - v)^2$



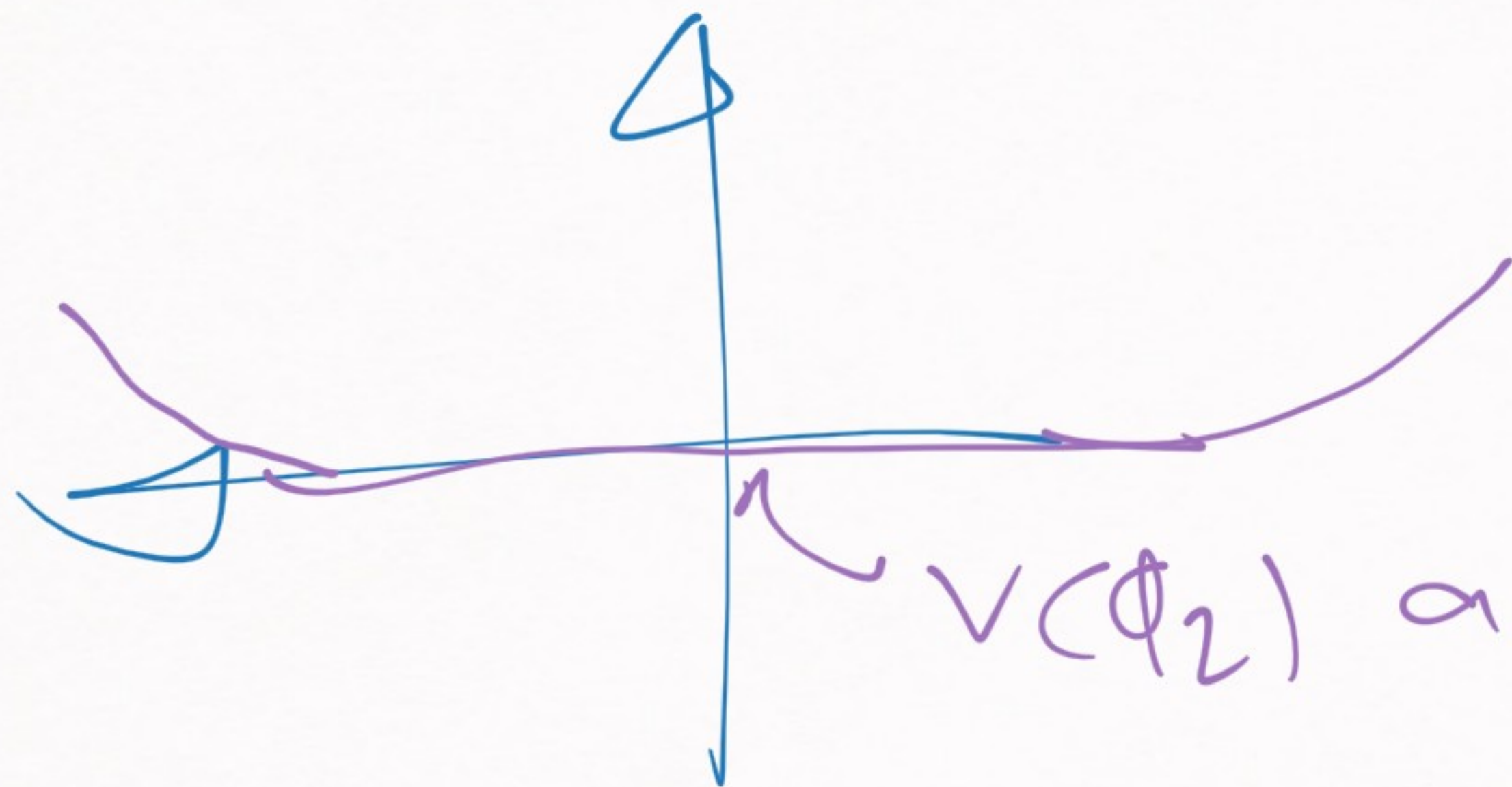


like h.o. potential

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi$$

$$- \frac{1}{2} m^2 \phi^2$$

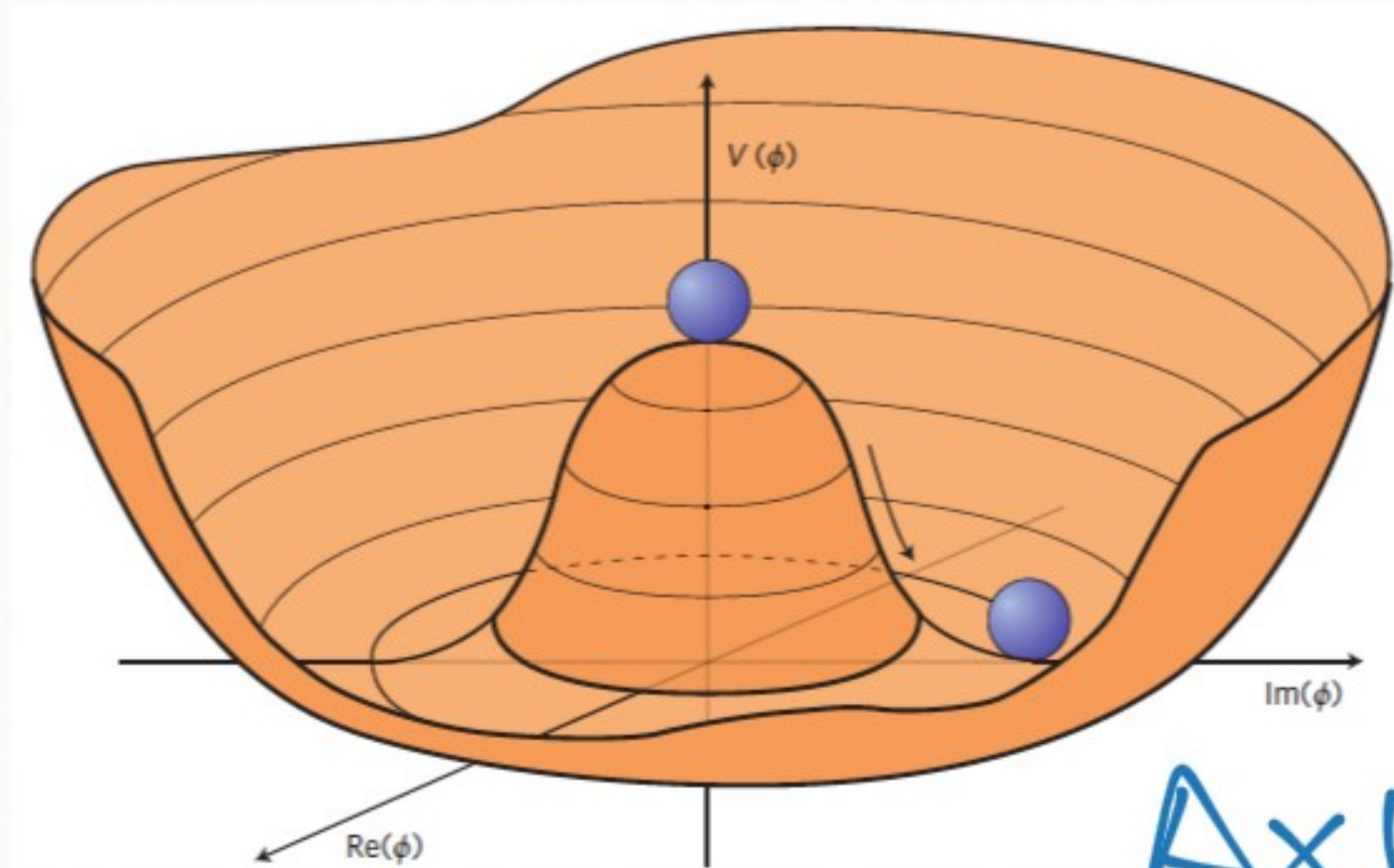
quadratic potential  
 $\ll m a g$



$V(\phi_2)$  around here  $\ll 0$



$|\phi_1\rangle$  here can be excited in two ways



1) around axis 1

→ the excitation will be massive

2) around axis 2

→ the excitation will be massless

Axis 2

Axis 1

Goldstone theorem



→ Excitations around " $ax + UZ$ "  
correspond to the massless  
bosons in Goldstone theorem

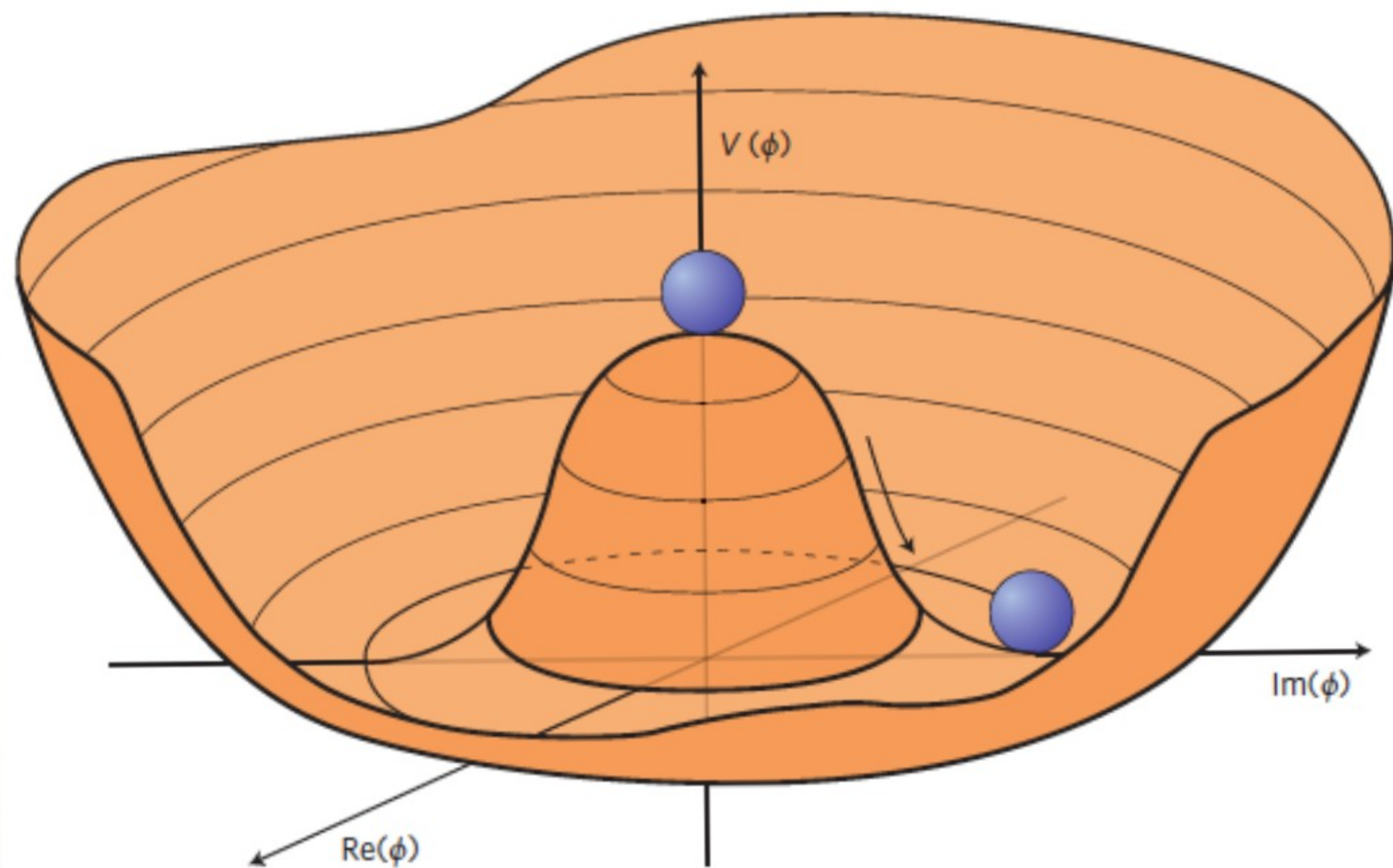
↙  
[  $n_0 - n_{\pm 1}$  massless bosons fields ]



$$G = O(2)$$

$$F = \mathbb{Z}_2 / C_1 / SO(1)$$

} → (Group theory tells us)



$$G = O(n)$$

$$n_G = \frac{n(n-1)}{2}$$

$$G = \text{trivial}$$

$$n_G = 0$$



→ In the previous case

$$\begin{aligned} n_G &= 1 \\ n_F &= 0 \end{aligned}$$

$$G = O(2)$$

$$F = SO(1)/\mathbb{Z}_2/\dots$$

(trivial group)

$n_G - n_F$  massless fields

1 massless field  
(correct)



SECOND EXAMPLE  $\rightarrow$  LOM

$$V(\phi) = \frac{\mu^2}{2} \sum_{i=0}^3 (\phi_i^2) + \frac{\lambda}{4} \left( \sum_{i=0}^3 \phi_i \right)^2$$

$G = O(4)$  (if we assume  $\phi_0, \phi_1, \phi_2, \phi_3$  real)

For  $\mu^2 < 0 \Rightarrow F = O(3) \rightarrow$  Let's check this



Vacuum in LSM was  $|0\rangle = \{ \phi_0 = v \}$

$$\{ \phi_0, \vec{\phi} \} \quad \underline{\underline{|\vec{\phi}|^2 = 0}}$$

$$\boxed{F = \mathcal{O}(3)}$$



$\Rightarrow$  We count the generators:

$$G = O(4) \rightarrow n_G = \frac{n(n-1)}{2} = 6$$

$$F = O(3) \rightarrow n_F = \frac{n(n-1)}{2} = 3$$

$$\Rightarrow n_G - n_F = 3 \text{ massless fields}$$





$\Rightarrow d_i, i=1,2,3 \Rightarrow \pi$

$\Rightarrow$  [ GOLDSTONE THEOREM  
explains the findings of  
the LSM ]



Application  $\rightarrow$  CHIRAL SYMMETRY

$\leadsto$  We already said a bit about  
this symmetry

(e.g. the correct pion interaction

$\otimes \rightarrow$  derivative interaction

was  $\mathcal{L} = \frac{g_A}{2f_\pi} \bar{\psi} \gamma_5 \gamma_\mu \vec{\tau} \cdot \vec{\pi} \overleftrightarrow{\partial}_\mu \psi$  (2)



Quarks  $\rightarrow$   $u, d, s$

(especially)  $\downarrow$

CHIRAL SYMMETRY

$m_q \ll \Lambda_{QCD}$

$$\frac{m_u}{\Lambda_{QCD}}, \frac{m_d}{\Lambda_{QCD}}, \frac{m_s}{\Lambda_{QCD}} \ll 1$$

We can apply  $\rightarrow$   
GOLDSTONE THEOREM

SPONTANEOUSLY  
BROKEN



What is CHIRAL SYMMETRY?

Quantum mechanical analog of chiral symmetry

Helicity

→ Will help us understand this chiral symmetry



1) Helicity  $\rightarrow$

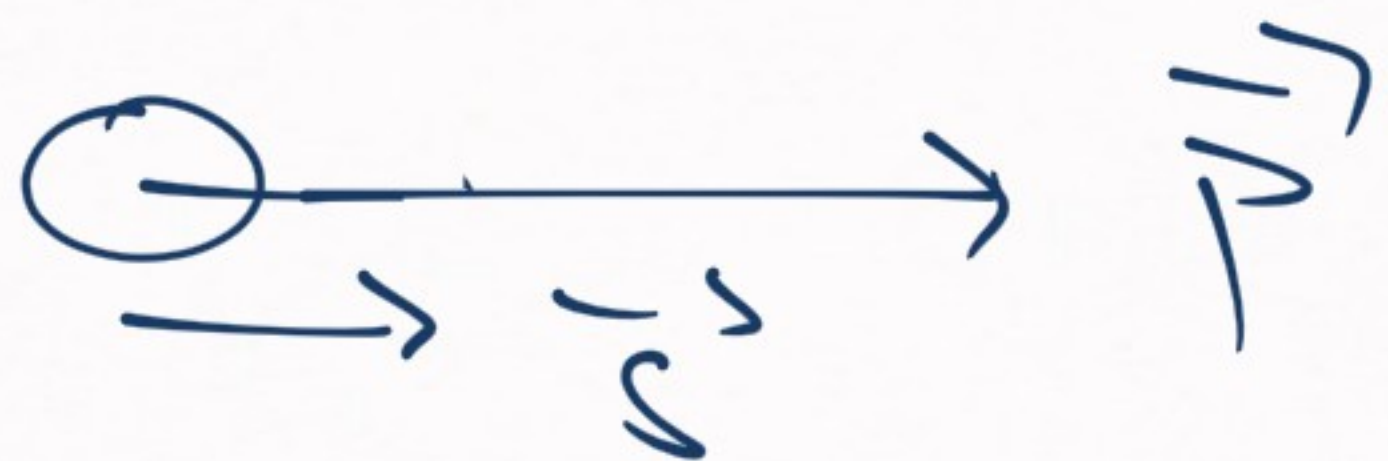
$$\lambda = \frac{\vec{S} \cdot \vec{p}}{|\vec{p}| |\vec{S}|}$$

(See spin-1/2  
in this  
example)

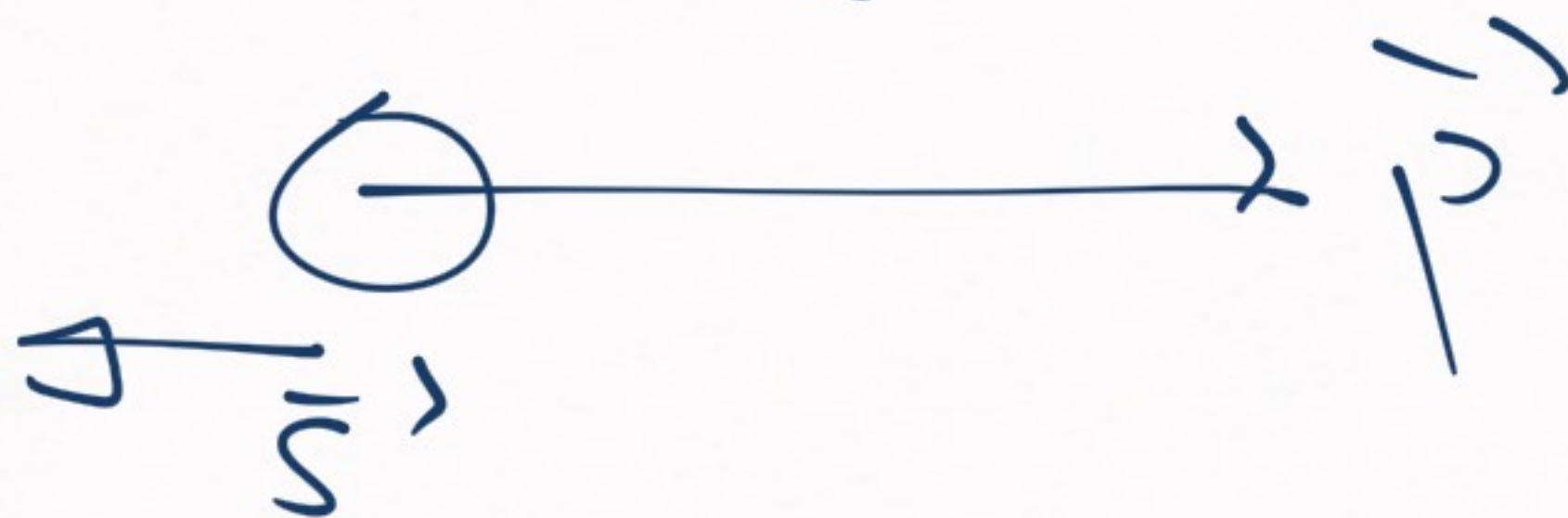
two eigenvalues

$$\lambda = \pm 1$$

$$\lambda = +1$$

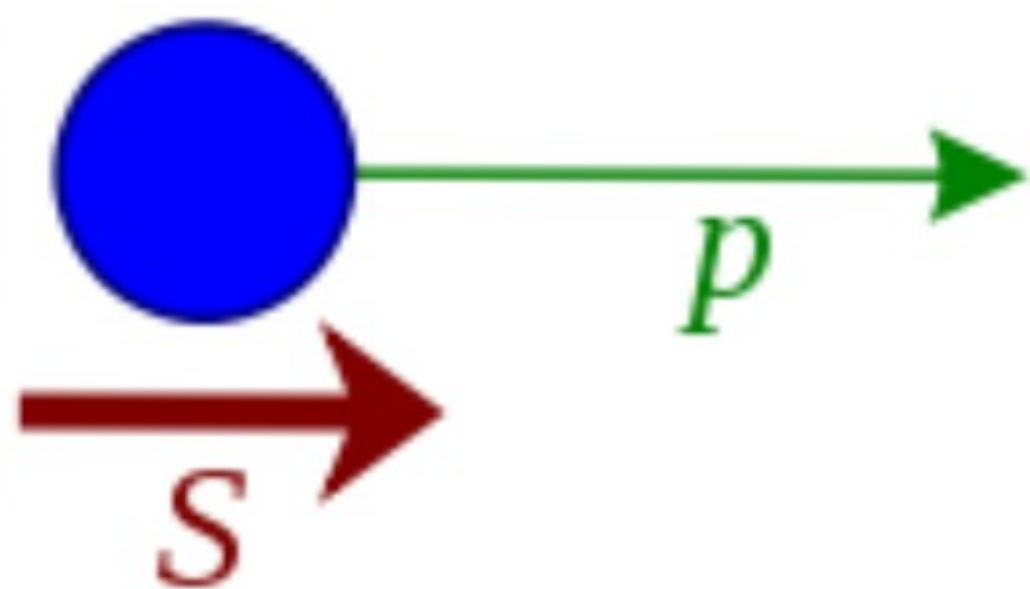


$$\lambda = -1$$





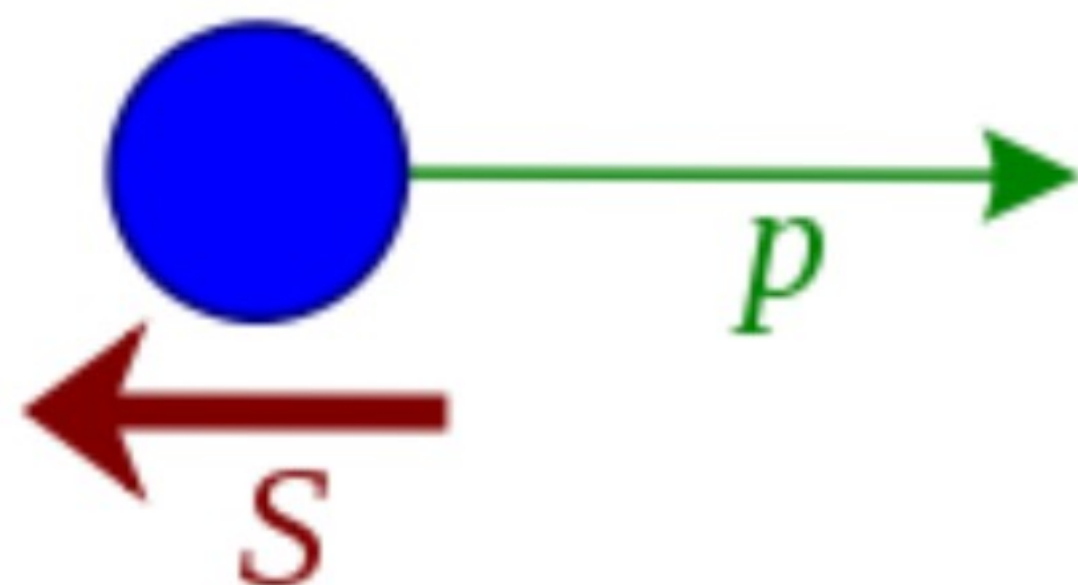
Right-handed:



$$\lambda = +1$$

(Right handed)

Left-handed:



$$\lambda = -1$$

(Left handed)

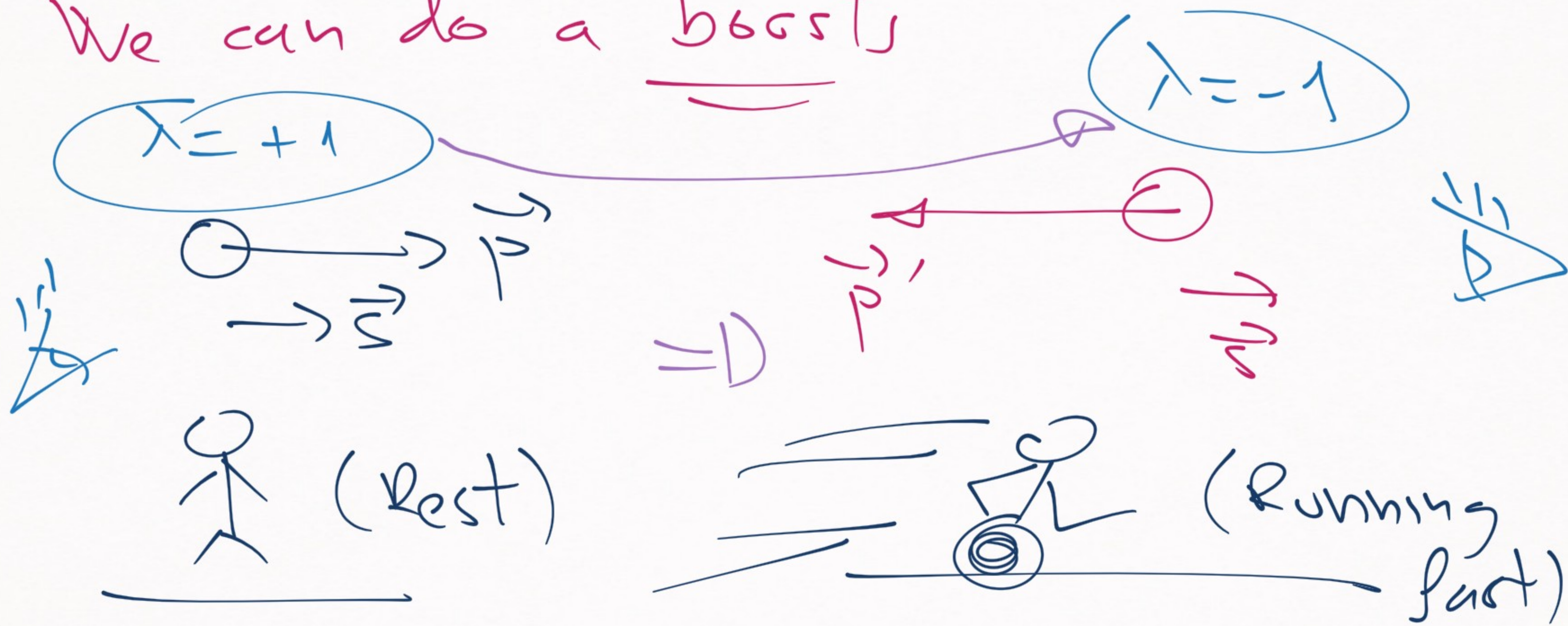
For  $m \neq 0$ ,  
helicity  
is not  
a conserved  
quantity,

→ why →



→ Why  $\lambda$  not conserved for  $m \neq 0$

We can do a boost





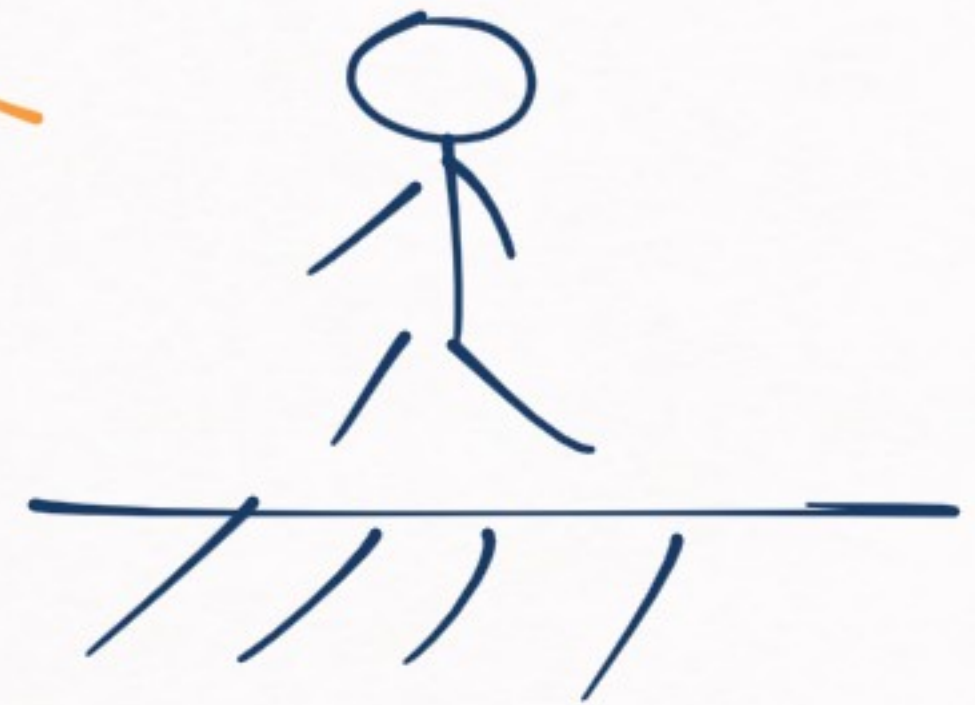
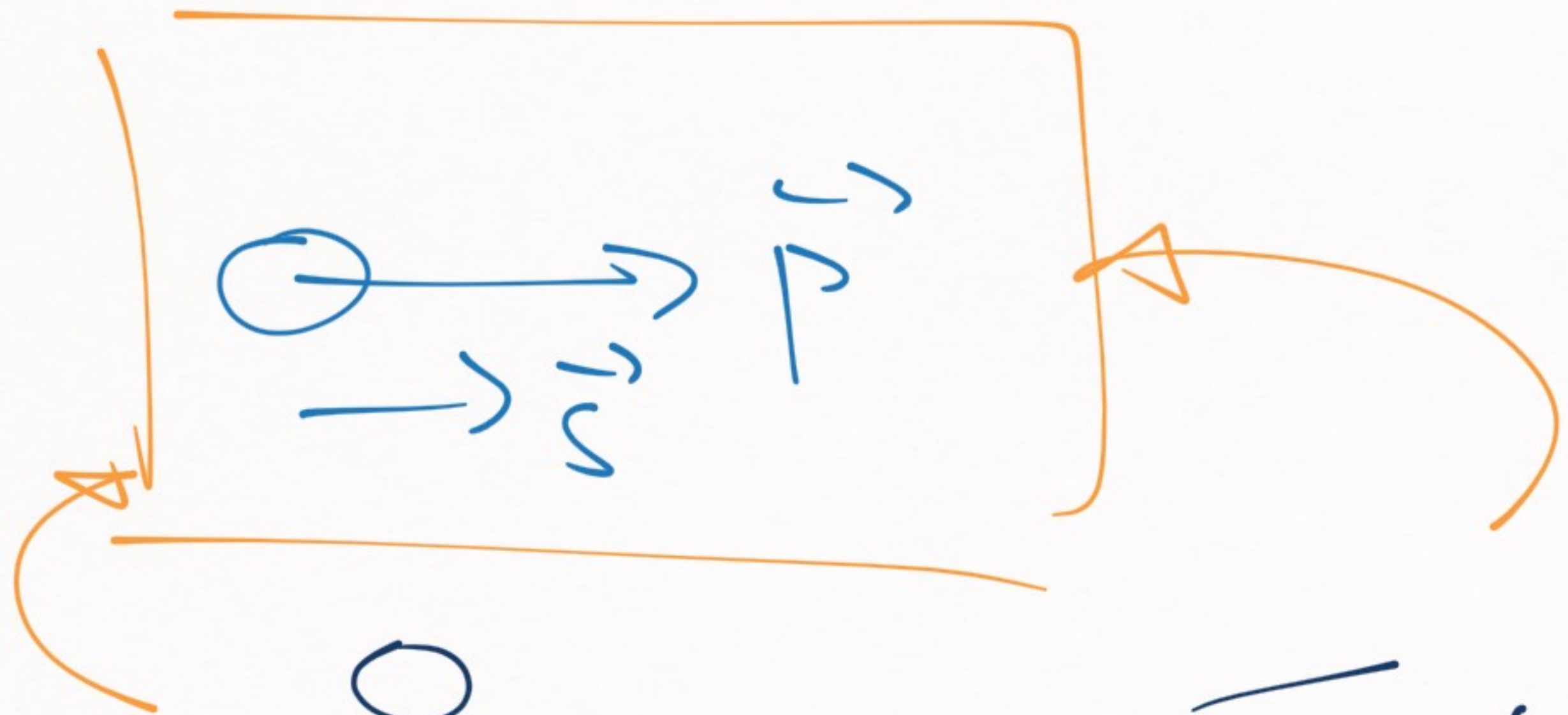
If  $m \neq 0 \Rightarrow \lambda = +1$  can become  $\lambda = -1$   
depending on the choice  
of frame of reference  
(boosted / non-boosted)

→ not true if  $m = 0$



If  $m=0 \Rightarrow \boxed{v=1}$

the particle will always travel at the speed of light



$\Rightarrow$



$|v_{ours}| < 1$

$(\frac{v}{c})$



massless  
particles

→ Helicity is conserved



(except for parity)



[The QFT analog of helicity]

$$\mathcal{L} = \bar{\psi} (\not{\partial} - m) \psi$$

→ Dirac field

kinetic term



mass term



⇒ global U(1) symmetry



Global symmetry  $\rightarrow$   $U(1)$  symmetry

$$\left. \begin{aligned} \psi(x) &\rightarrow e^{i\alpha} \psi(x) \\ \psi^\dagger(x) &\rightarrow e^{-i\alpha} \psi^\dagger(x) \end{aligned} \right\} \mathcal{L} \rightarrow \mathcal{L}$$

$\Downarrow$

$$\bar{\psi}\psi = \psi^\dagger \gamma^0 \psi \rightarrow \psi^\dagger \gamma^0 \psi$$



→ there might be more symmetries here

why?

→

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}$$

→ four components

→ Further differentiation  
of this field:

$$\psi = \psi_L + \psi_R$$

$$\psi_L = \mathcal{P}_L \psi = \frac{1}{2} (1 - \gamma_5) \psi$$

$$\psi_R = \mathcal{P}_R \psi = \frac{1}{2} (1 + \gamma_5) \psi$$



$$\begin{aligned}\psi &= \psi_L + \psi_R \\ \psi_L &= P_L \psi = \frac{1-\gamma_5}{2} \psi \\ \psi_R &= P_R \psi = \frac{1+\gamma_5}{2} \psi\end{aligned}$$

$\psi_L \rightarrow$  left-handed Dirac field.  
 $\psi_R \rightarrow$  right-handed

$$\begin{aligned}P_L^2 &= P_L \\ P_R^2 &= P_R \\ P_L^\dagger &= P_L \\ P_R^\dagger &= P_R\end{aligned}$$

$\rightarrow$  they are projectors  
( $P_L + P_R = 1$ )



$$\psi = \psi_L + \psi_R \rightarrow \text{this separation is possible}$$

extra symmetries? (maybe)

$$\psi_L \rightarrow e^{i\alpha_L} \psi_L$$

$$\psi_R \rightarrow e^{i\alpha_R} \psi_L$$

Is it a symmetry?

analogous to

$$\psi \rightarrow e^{i\alpha} \psi$$



$$\psi_L \rightarrow e^{i\alpha_L} \psi_L \Rightarrow U_L(1)$$

$$\psi_R \rightarrow e^{i\alpha_R} \psi_R \Rightarrow U_R(1)$$

Do THEY WORK?

(it depends)

$$\mathcal{L} = \bar{\psi} (\not{\partial} - m) \psi$$

$$= \bar{\psi} (\not{\partial} \psi) \text{ kinetic}$$

$$- m \bar{\psi} \psi \text{ mass}$$

terms

||



① Kinetic

$$\bar{\psi} \not{\partial} \psi = \bar{\psi}_L \not{\partial} \psi_L + \bar{\psi}_R \not{\partial} \psi_R$$

$$\xrightarrow{U_L} \bar{\psi}_L \not{\partial} \psi_L + \bar{\psi}_R \not{\partial} \psi_R$$

$$\xrightarrow{U_R} \bar{\psi}_L \not{\partial} \psi_L + \bar{\psi}_R \not{\partial} \psi_R$$

→ Kinetic term has both  $U_L(1)$  &  $U_R(1)$

symmetry  
↔



## ② mass term

$$m \bar{\psi} \psi = m \bar{\psi}_L \psi_R + m \bar{\psi}_R \psi_L \quad (\text{mixes L \& R terms})$$

$\xrightarrow{U_3(L)}$

$$m e^{i\alpha_L} \bar{\psi}_L \psi_R + m e^{i\alpha_L} \bar{\psi}_R \psi_L$$

$\xrightarrow{U_1(R)}$

$$m e^{i\alpha_R} \bar{\psi}_L \psi_R + m e^{-i\alpha_R} \bar{\psi}_R \psi_L$$



$$\mathbb{F}_4 \xrightarrow{\times} \mathbb{F}_4$$

$$U_L(1), U_R(1)$$

} not invariant  
under  
these two  
transformations

$\cong$



1)  $\mathcal{L}_{\text{Dirac}}, m \neq 0$

→ Global symmetry is  $U_{\underline{L+R}}(1)$

$$\psi = \psi_L + \psi_R, \quad \psi \rightarrow e^{i\alpha} \psi$$

2)  $\mathcal{L}_{\text{Dirac}}, m = 0$

CHIRAL SYMMETRY

→ Global symmetries are  $U_L(1), U_R(1)$



How does chiral symmetry apply to QCD?

$$\mathcal{L}_{\text{QCD}} = \underbrace{-\frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu}}_{\text{Gluons (not important for the moment)}} + \underbrace{\bar{q} i \not{D} q}_{\text{quark kinetic}} - \underbrace{\bar{q} M q}_{\text{quark mass}}$$

terms



$$g = \begin{pmatrix} c \\ v \\ u \\ s \\ t \end{pmatrix}, \quad M = \begin{pmatrix} m_u & & & & \\ & m_d & & & \\ & & m_s & & \\ & & & m_c & \\ & & & & m_t \end{pmatrix}$$

Question  $\rightarrow$  can we ignore a few of these masses?

(e.g. by considering them as perturbations)



→ in certain cases we can approximate

$$m_g \approx 0$$

Comparison

this has to do w/ QCD  
natural scale

$$[\Lambda_{\text{QCD}} \approx (200 - 350) \text{ MeV}]$$



$u, d, s, c, b, t$

1)  $\frac{m_u}{\Lambda_{QCD}}, \frac{m_d}{\Lambda_{QCD}} \ll 1$   $\left( \frac{m_s}{\Lambda_{QCD}} < 1 \right)$  ✓

$u, d$  ( & maybe  $s$  ) quarks  $\rightarrow$

$m_u, m_d \lesssim 0$   
 $(m_s \lesssim 0)$

2)  $\frac{m_c}{\Lambda_{QCD}}, \frac{m_b}{\Lambda_{QCD}}, \frac{m_t}{\Lambda_{QCD}} \gg 1$  ✗



# Reminder

$$m_u \sim 2 \text{ MeV}$$

$$m_d \sim 5 \text{ MeV}$$

$$m_s \sim 95 \text{ MeV}$$

→ chiral symmetry  
might be applicable

$$m_c \sim 1.2 \text{ GeV}$$

$$m_b \sim 4.2 \text{ GeV}$$

→ but not for these  
quarks



$(\Phi \text{CD}) \rightarrow$

light quarks

3

heavy quarks



chiral symmetry

other symmetries



two approximations

a)  $u, d$  light  $\rightarrow n=2$

b)  $u, d, s$  light  $\rightarrow n=3$

(heavy quark symmetry)



a)  $m_g = 0 \rightarrow$  chiral transformations

$U_L \quad SU_R$

1-type of  
fermion field

$$\left\{ \begin{array}{l} \psi_L \rightarrow e^{i\alpha_L} \psi_L \\ \psi_R \rightarrow e^{i\alpha_R} \psi_R \end{array} \right.$$

2-types of  
fermion fields

$$\Rightarrow \psi_{L,R} \rightarrow \begin{pmatrix} \psi \\ \alpha \end{pmatrix}_{L,R}$$



$$\begin{pmatrix} u \\ d \end{pmatrix}_L \rightarrow (2 \times 2 \text{ matrix}) \begin{pmatrix} u \\ d \end{pmatrix}_L$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_R \rightarrow (2 \times 2 \text{ matrix}) \begin{pmatrix} u \\ d \end{pmatrix}_R$$

what type of matrix

$$(u \ \bar{d})_L \begin{pmatrix} u \\ d \end{pmatrix}_L \rightarrow (u \ \bar{d})_L \begin{pmatrix} u \\ d \end{pmatrix}_L$$



Since  $u, d$  are complex

$\Rightarrow$   $2 \times 2$  matrices need to  
belong to  $U(2)$

$U_L(2)$

$U_R(2)$





$$\begin{pmatrix} u \\ d \end{pmatrix}_{L/R} \rightarrow V_{L/R} \begin{pmatrix} u \\ d \end{pmatrix}_{L/R}$$

$$\left. \begin{array}{l} V_L \in U_L(2) \\ V_R \in U_R(2) \end{array} \right\} U_L(2) \otimes U_R(2) \text{ chiral symmetry}$$

$$g_L \rightarrow V_L g_L$$

$$g_R \rightarrow V_R g_R$$



b) if we add  $s$ :

$$\begin{pmatrix} c \\ d \\ s \end{pmatrix}_{L,R} \rightarrow V_{UR} \begin{pmatrix} c \\ d \\ s \end{pmatrix}_{L,R}$$

$$V_L \in U_L(3)$$

$$V_R \in U_R(3)$$

Symmetry group for

$$G = U_L(n) \otimes U_R(n)$$

$n$  types  
of light  
quarks



Result 1) For  $n$  species of light quarks  
Lac invariant  $G = U(n) \otimes U(n)$

except that this is not true

(something unexpected happens)

↳ Chiral anomaly



RECD?  $\rightarrow$  anomaly is a classical symmetry,  
broken by the quantization  
process

CHIRAL ANOMALY  $\rightarrow$   $U_L \times U_R(1)$  will be  
violated

$$g_L \rightarrow e^{i\alpha_L} g_L$$

$$g_R \rightarrow e^{i\alpha_R} g_R$$

$$\left. \begin{array}{l} g_L - g_R \\ \rightarrow e^{i\alpha_L - \alpha_R} \end{array} \right\} \text{(if } n > 1 \text{)}$$



Result 1')

$$G = U_L(n) \otimes U_R(n)$$



chiral anomaly

on  $U_{L-R}(1)$

$$G = SU_L(n) \otimes SU_R(n) \otimes U_{L+R}(n)$$



$q_L \rightarrow \psi_L q_L$   
 $\psi_L \in SU(n)$

$q_R \rightarrow \psi_R q_R$   
 $\psi_R \in SU(n)$

$l \rightarrow e, \nu$   
 $\psi \in U(1)$



Confront theory w/ reality

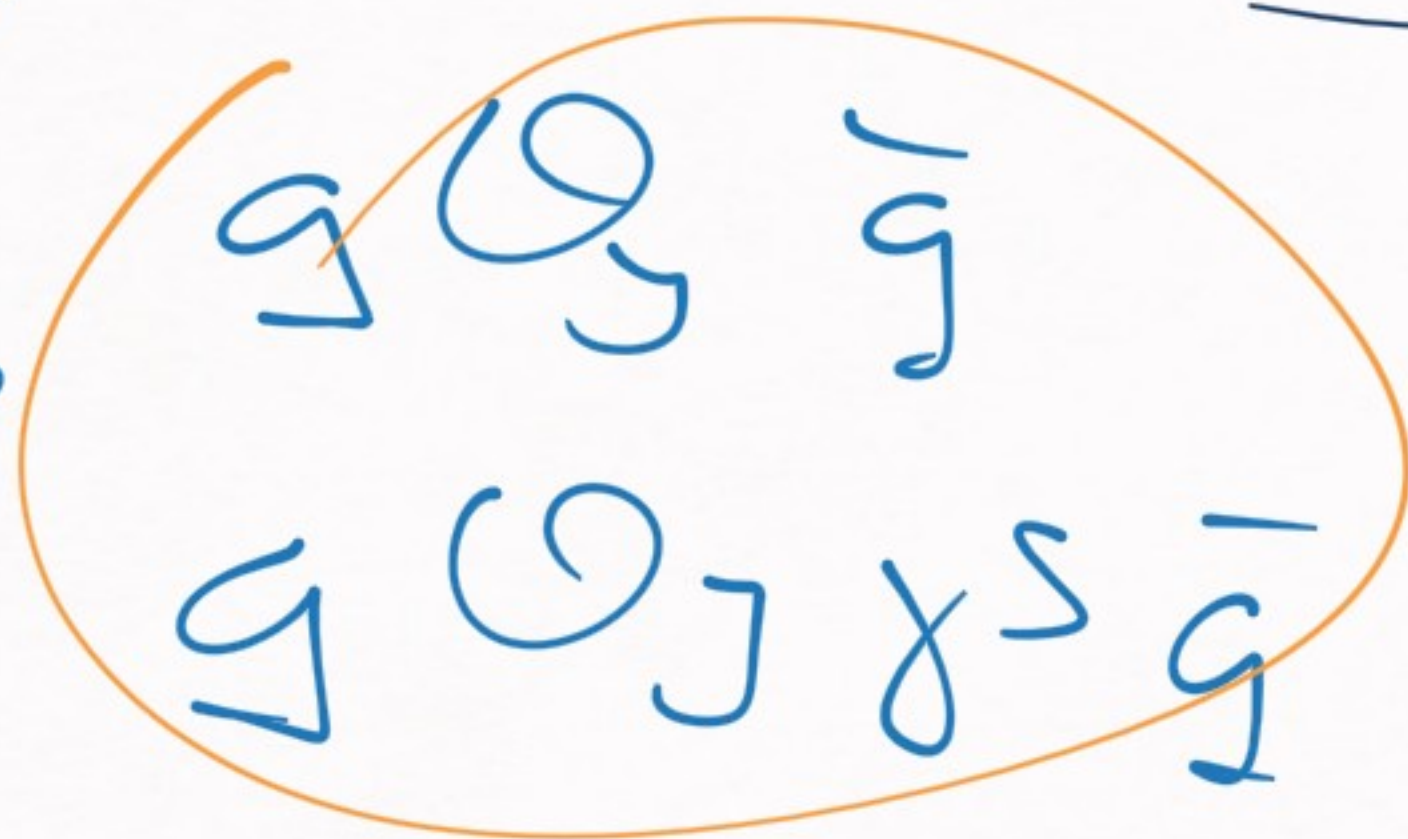
$$G = SU_L(n) \otimes SU_R(n) \otimes U_{L+R}(1)$$

Prediction

$$m(H, J^+) = m(H, J^-)$$

Meson

$q\bar{q} \rightarrow$



$H \rightarrow$  hadron

opposite parities ( $\gamma$ )



What happens is :

$$1) \text{ } \rho(770) \rightarrow \begin{matrix} m_{\pi^0} \approx 140 \text{ MeV} \\ m_{\pi^{\pm}} \approx 140 \text{ MeV} \end{matrix} \quad m_{\pi^0} \neq m_{\pi^{\pm}}$$

$$2) \text{ } \rho(770), \rho(1700) \quad m_{\rho} \neq m_{\rho^*}$$
$$J = 1^- \quad 1^-$$

$$3) \text{ } \rho(770), \rho(1700) \quad m_{\rho} \neq m_{\rho^*}$$
$$J = 1^- \quad 1^-$$



→ GROUND STATE OF QCD IS NOT  
INVARIANT UNDER

$$G = SU_L(N) \otimes SU_R(N) \otimes U(1)_{LR}$$

Why?

→ positive & negative  
parity states

F

← have different masses  
~



→ Opportunity to use GOLDSTONE THEOREM

RECAP

$$H / [H, G] = 0$$

$$|0\rangle / G |0\rangle \neq |0\rangle$$

but still  $F \subseteq G$

$$F|0\rangle = ()|0\rangle$$

$$\boxed{|\alpha\rangle}$$

$$G = SU_L(N) \times (U_R(N)$$

$$\times U(1)$$

$$G |mN\rangle \rightarrow |mN\rangle$$

not vacuum



What is the symmetry of the lower mass hadrons?

$\downarrow$   
 $\uparrow$

$m(p) < m(u_1)$

$\downarrow$  → L+R  $g_1, g_2, g_3$

$\downarrow$  → L-R  $g_1, g_2, g_3$

$$\begin{aligned}
 F &= SU_{L+R}(n) \\
 &\otimes U(1)_{L+R}(n) \\
 &= U_{L+R}(n)
 \end{aligned}$$



GOLDSTONE  
THEOREM

1)  $G = SU_L(n) \otimes SU_R(n) \otimes U(1)$   
 $U(1)$   
 $U(1)$

$SU(n) \rightarrow n^2 - 1$  generators

$n_G = 2n^2 - 1$

2)  $F = U_{LR}(n)$

$U(n) \rightarrow n^2$  generators

$n_F = n^2$



(3+7)

→

$n^2 - 1$  massless bosons

(they will not be massless once we include  $\int g \mu g$  as a perturbation)



a)  $n=2$  chiral symmetry (u, d)

→  $n^2 - 1 = 3$  massless bosons

→ pions ( $m_{\pi^{\pm}}, m_{\pi^0} \approx 140 \text{ MeV}$ )

$J^P?$

$\pi/G \sim SU_{L-R}(n)$

$9 \otimes 3 \rightarrow 0$

→ pseudoscalars



b)  $n=3 \rightarrow n^2-1 = 8$  "massless" bosons

$\rightarrow \underbrace{\pi^{\pm}, \pi^0}_3, \underbrace{K^{\pm}, K^0, \bar{K}^0}_4, \underbrace{\eta}_1$

$0^- \quad 0^+$   
 $m_{\pi} \ll m_{\eta}$

$m_K \ll m_{K^0}$

$m_{\eta} \ll m_{a_0}$

$m_K = 495 \text{ MeV}$

$m_{\eta} = 550 \text{ MeV}$

$\Sigma$



By applying the GOLDSTONE THEOREM

→  $n^2 - 1$  massless bosons

$m_u, m_d, m_s \rightarrow 0$

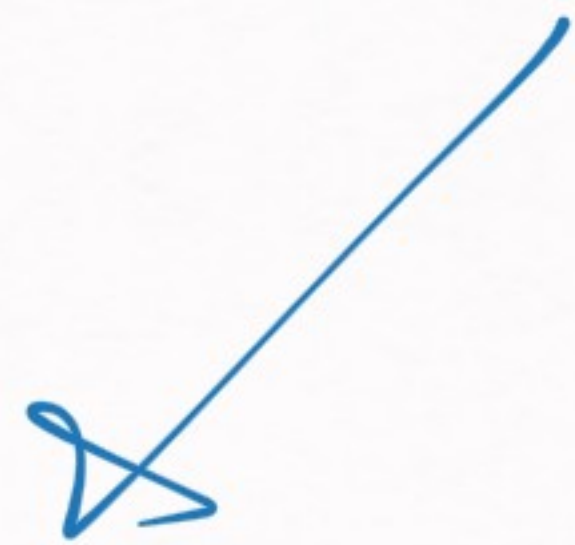
$\bar{q}Mq$  is a perturbation

→ explains why pions are so light



Second consequence of  
(not really explained  
why here)

GOLDSTONE  
THEOREM



pion interactions are derivative



w/o chiral symmetry :

$$\mathcal{L}_{\pi NN} = g_{\pi NN} \bar{\psi} \gamma_5 \vec{\tau} \cdot \vec{\pi} \psi$$

(no derivatives)

w/ chiral symmetry

$$\mathcal{L}_{\pi NN} = \frac{g_A}{2P_+} \bar{\psi} \gamma_5 \gamma_\mu \vec{\tau} \cdot \vec{\pi} \partial_\mu \psi$$

forbidden

allowed



→ changes the nature of pion interaction

$$\frac{g_A}{f_\pi} = \frac{g_{\pi NN}}{M_N}$$

→ Goldberger-Treiman  
relation

→ pion interactions will be weaker  
at low energies



→ important for calculation  
of two-pion exchange  
potential

(w/o chiral symmetry & derivative  
interactions, these calculations  
fail)



→ WE CONTINUE  
ON FRIDAY