[GO TO NEXT PAGE]

=D

NUCLEAR PHYSICS II

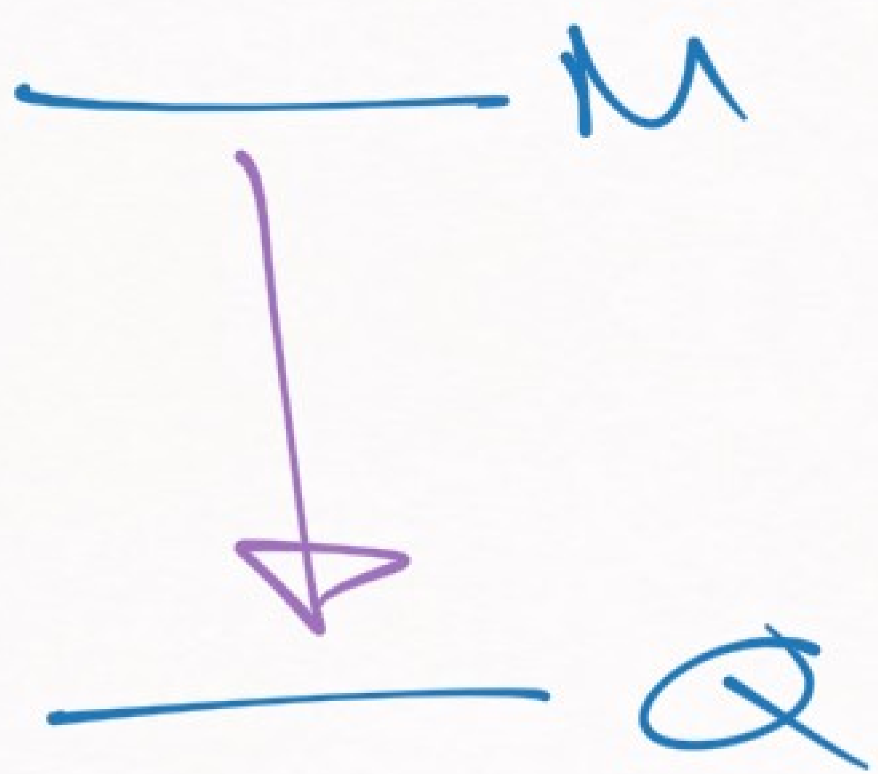
ISOSPIN SYMMETRY,

SU(3)-FLAVOR SYMMETRY,

CHIRAL SYMMETRY

(FIRST PART)

$\boxed{\text{RECAP}}$ → POWER COUNTING IN EFT



→ the point is to do an expansion in Q/M

[if $Q/M \ll 1 \Rightarrow$ this will converge quickly]

If we consider the most simple type of EFT

$$V_C(\vec{q}) = C_0 + C_2\,\vec{q}^{\,2} + C_4\,\vec{q}^{\,4} + \dots$$

Two types of counting

$$\begin{cases} C_0 \sim \dfrac{1}{M^2} & C_{2n} \sim \dfrac{1}{M^{2+2n}} \quad (2n > 0) \\[2ex] C_0 \sim \dfrac{1}{MQ} & C_{2n} \sim \dfrac{1}{Q^2 M^{2n}} \end{cases}$$

$\rightarrow$ $\exists$ a lot of ways to justify this result

$\overline{}\!\!\!\sum$

(Check previous lesson for a total of six justifications)

→ [ISOSPIN SYMMETRY]

(motivation) → n, p $\left\{ \begin{array}{l} m_p = 938.272 \text{ MeV} \\ m_n = 939.563 \text{ MeV} \end{array} \right.$

$$\boxed{m_n \simeq m_p}$$

$$\frac{m_n - m_p}{m_p} \sim \frac{1.3}{1000}$$ (tiny difference)

→ Is this fortuitous or a $\boxed{\text{symmetry}}$?

→ How to exploit this theoretically?

$m_p \simeq m_n$ → not the only instance

$m_{\pi^\pm} = 139.570$

$m_{\pi^0} = 135.977$

→ $\boxed{m_{\pi^\pm} \simeq m_{\pi^0}}$

Symmetry → we can use it to our advantage

n, p → two-different states of same particles

Analogous to e^- → $|e(\uparrow)\rangle$, $|e(\downarrow)\rangle$

different spins

try approach similar to spin

$\longrightarrow \boxed{\text{isospin}}$

$$|p\rangle = |N(\uparrow)_I\rangle = |\tfrac{1}{2}\ \tfrac{1}{2}\rangle_I$$

$$|n\rangle = |N(\downarrow)_I\rangle = |\tfrac{1}{2}\ {-\tfrac{1}{2}}\rangle_I$$

Analogy of $|s\ m_s\rangle \Rightarrow |I\ m_I\rangle_I$

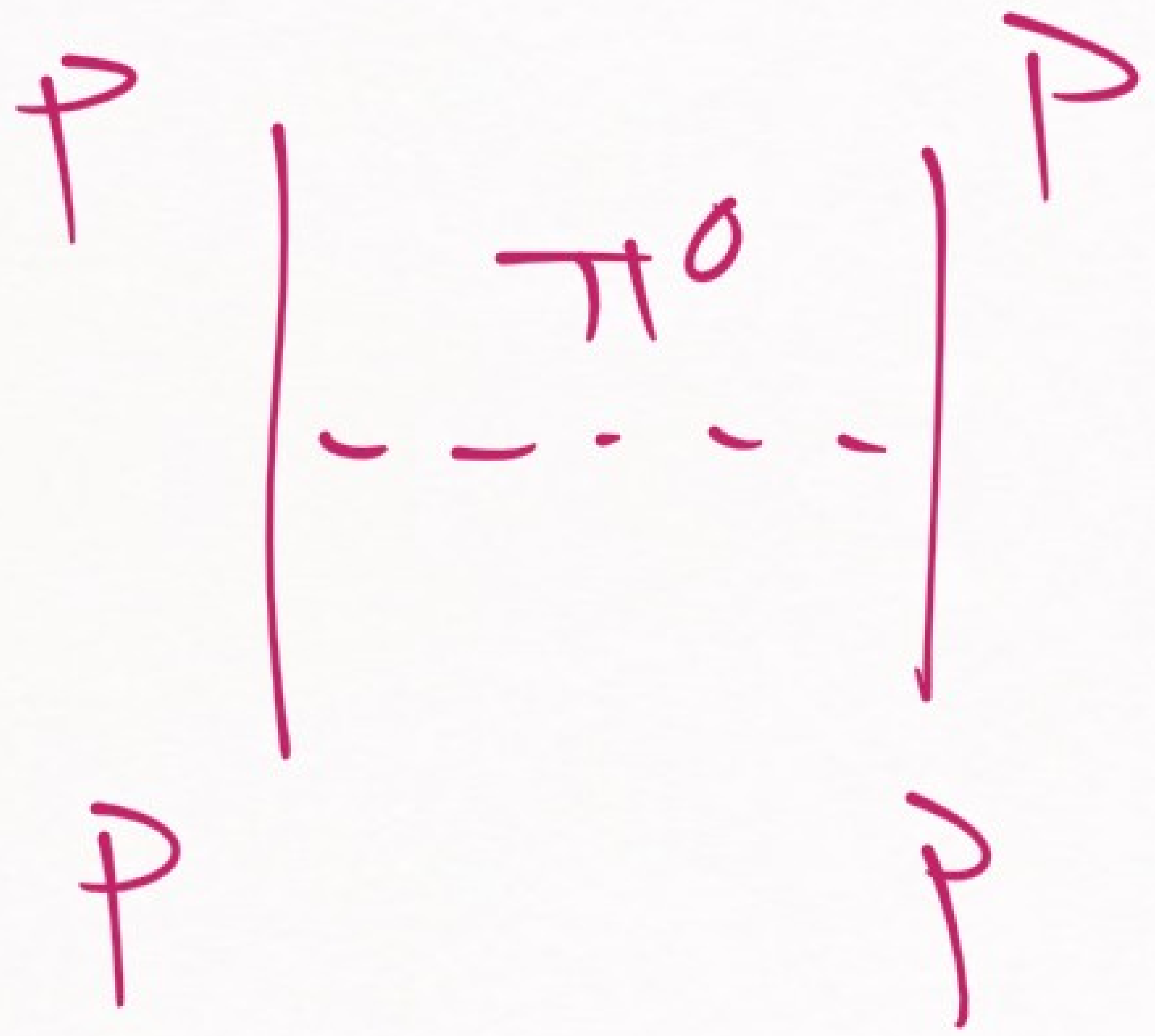
$$|\pi^+\rangle = |1\ 1\rangle_I$$
$$|\pi^0\rangle = |1\ 0\rangle_I$$
$$|\pi^-\rangle = -|1\ {-1}\rangle_I$$

$$|\pi\rangle \sim |1\ m_I\rangle$$

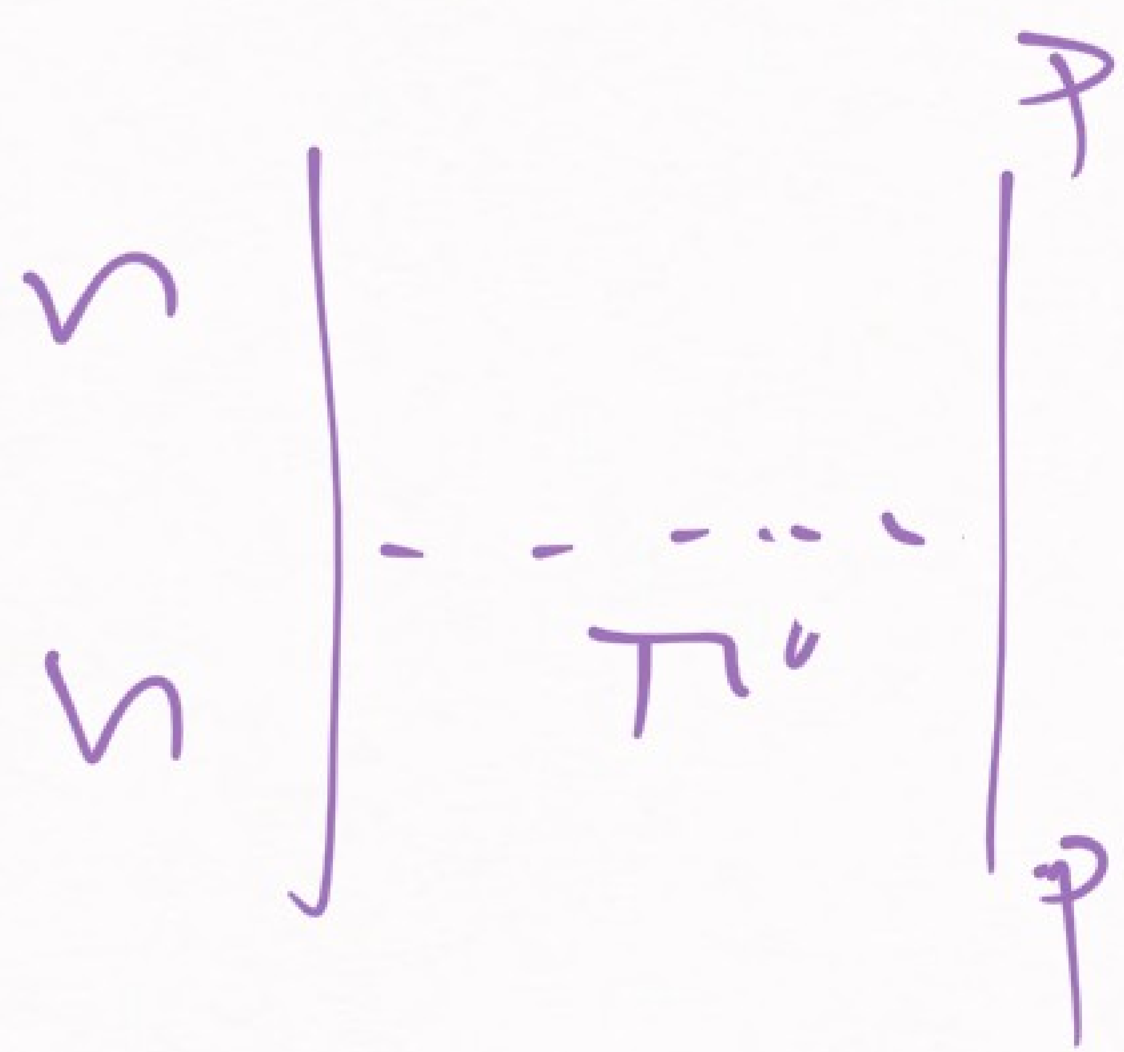
useful convention

(for other purposes)

[Consequences] → (NN Potential)

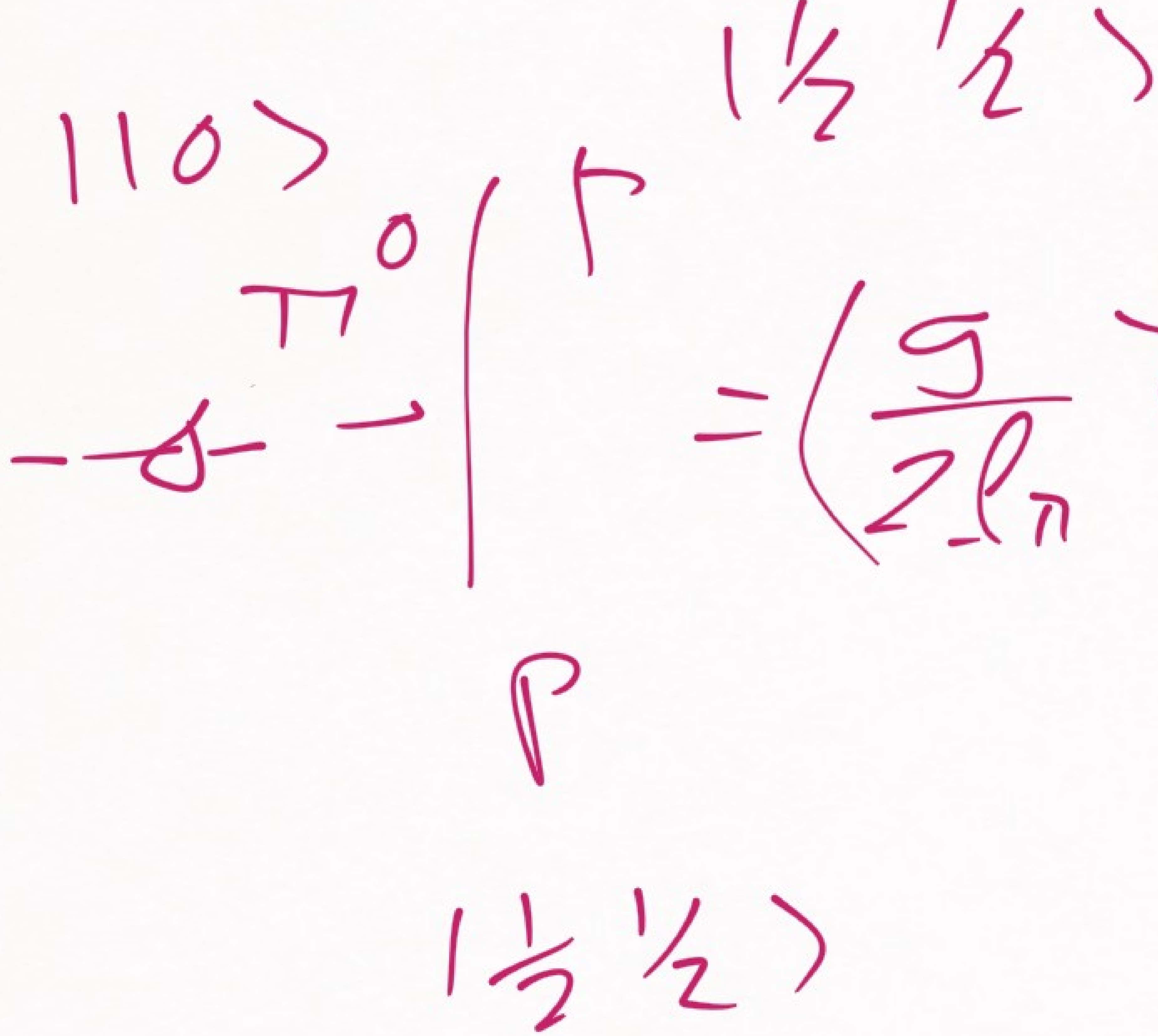


$$= V_0(\vec{q}) = -\frac{g^2}{4f_\pi^2}\frac{\vec{\sigma}_1\cdot\vec{q}\;\vec{\sigma}_2\cdot\vec{q}}{\vec{q}^2+m_\pi^2}$$



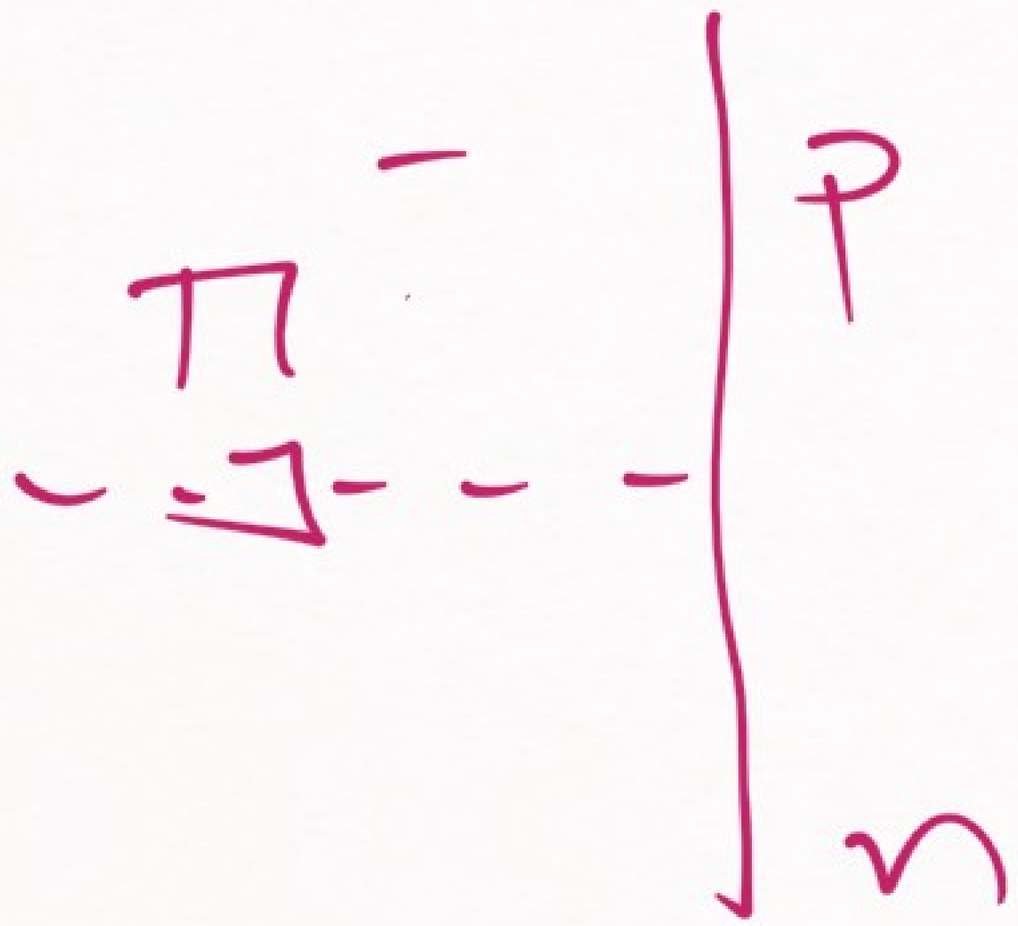
? ⇒

$\Rightarrow$ Isospin formalism answers these questions



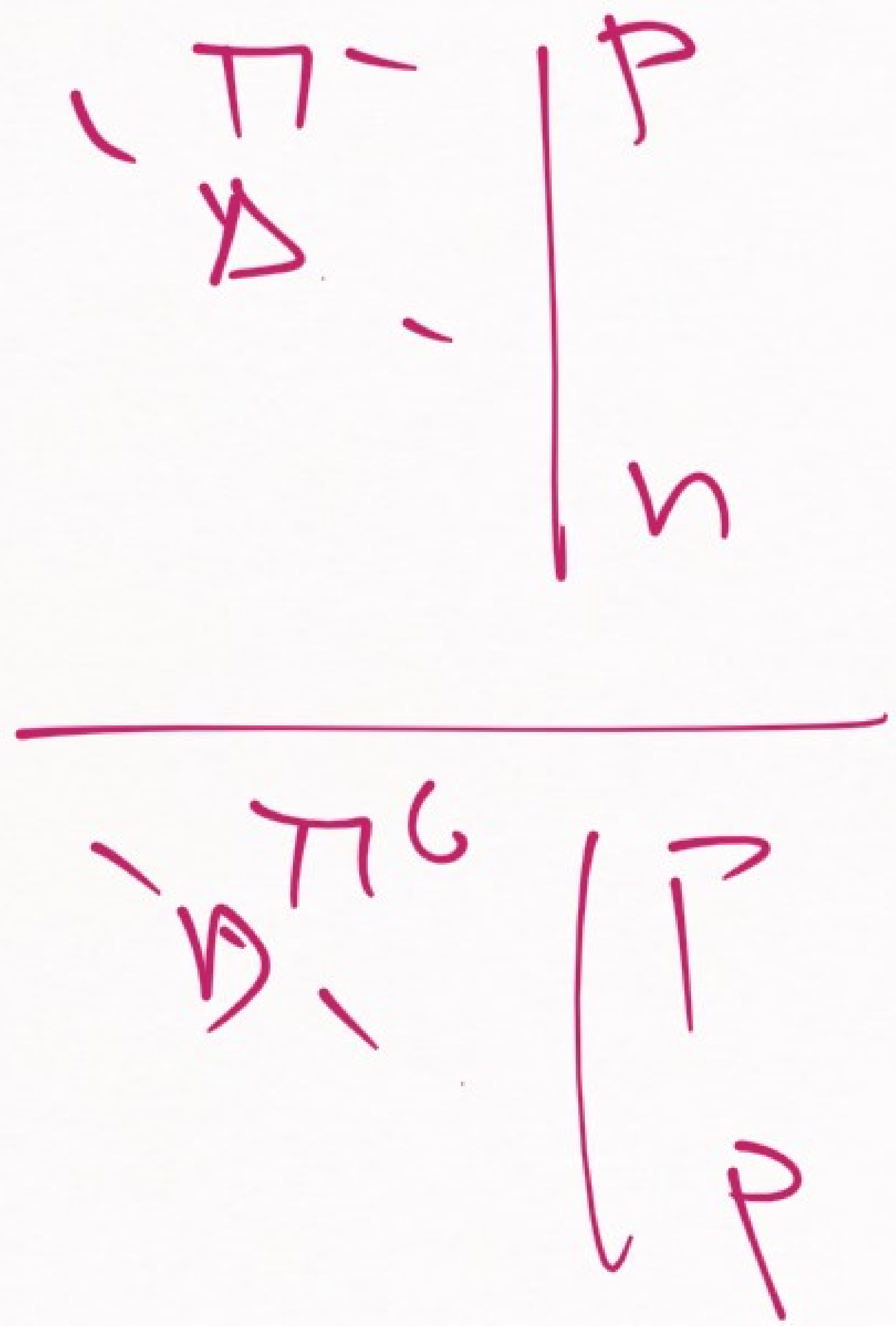
$$= \left(\frac{g}{2 m_\pi}\, \vec{\sigma} \cdot \vec{q}\right) = (\text{vertex}(pp\pi^0))$$

$$= \lambda \underbrace{\langle \tfrac12\, \tfrac12\, 1\, 0 \,|\, \tfrac12\, \tfrac12 \rangle}_{\text{analogy to SU(2) spin}}$$



$$= \lambda \left\langle 1\,-1\,\tfrac{1}{2}\,\tfrac{1}{2} \,\middle|\, \tfrac{1}{2}\,-\tfrac{1}{2} \right\rangle_I$$

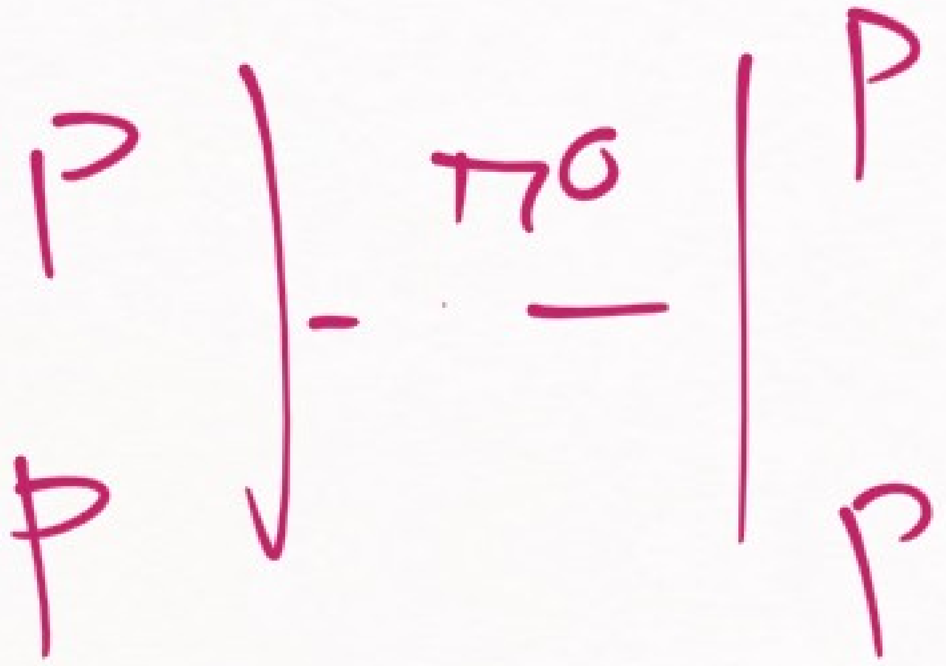
Reminder → $\langle s_1 m_1 s_2 m_2 | s m \rangle$ are the Clebsch-Gordan coefficients



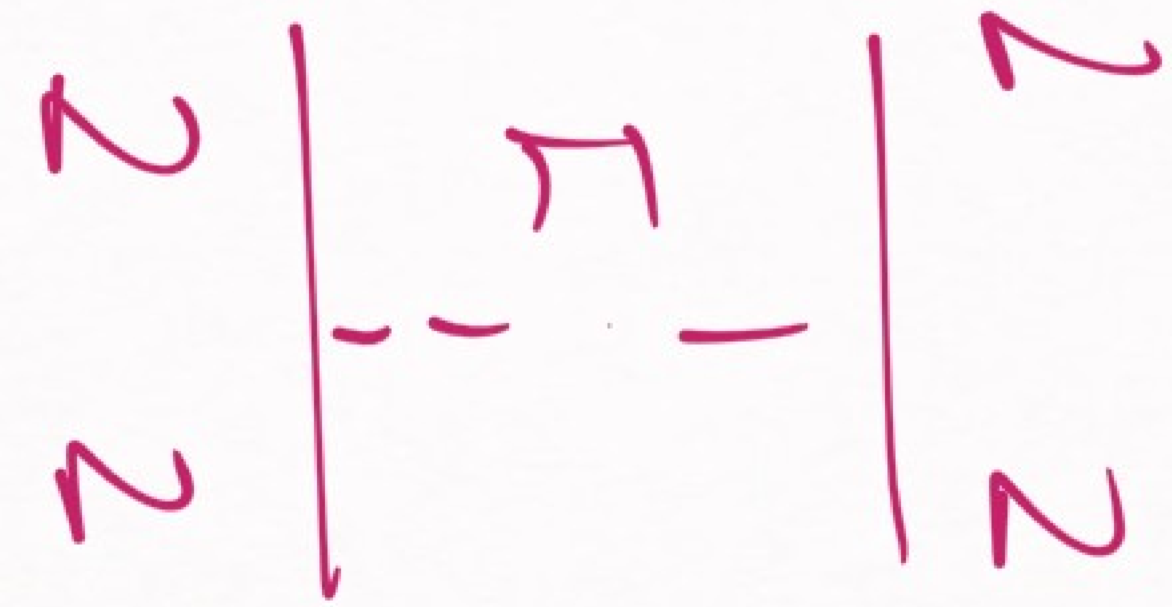
$$\sim \frac{\langle 1\,-1\,\tfrac{1}{2}\,\tfrac{1}{2} \,|\, \tfrac{1}{2}\,-\tfrac{1}{2}\rangle}{\langle 1\,0\,\tfrac{1}{2}\,\tfrac{1}{2} \,|\, \tfrac{1}{2}\,\tfrac{1}{2}\rangle} \sim \pm\sqrt{2}$$

→ this is how we can apply isospin here

w/ isospin



$\Rightarrow$



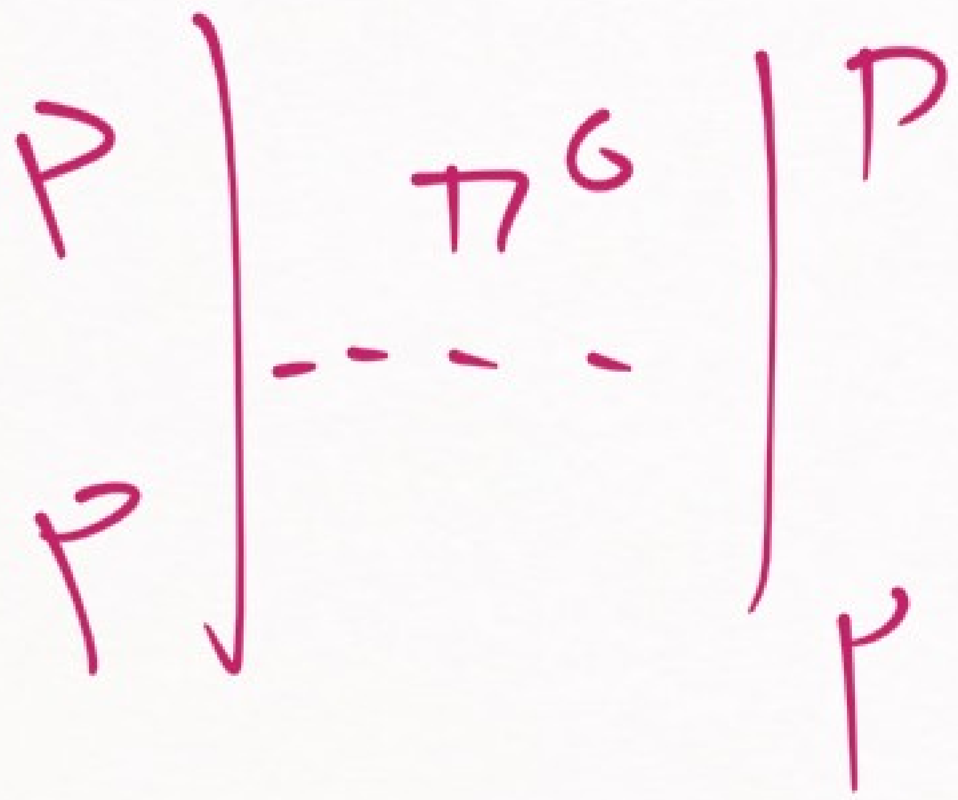
(any combination of n, p, π^+, π^-, π^0)

$\Rightarrow$ modulo signs

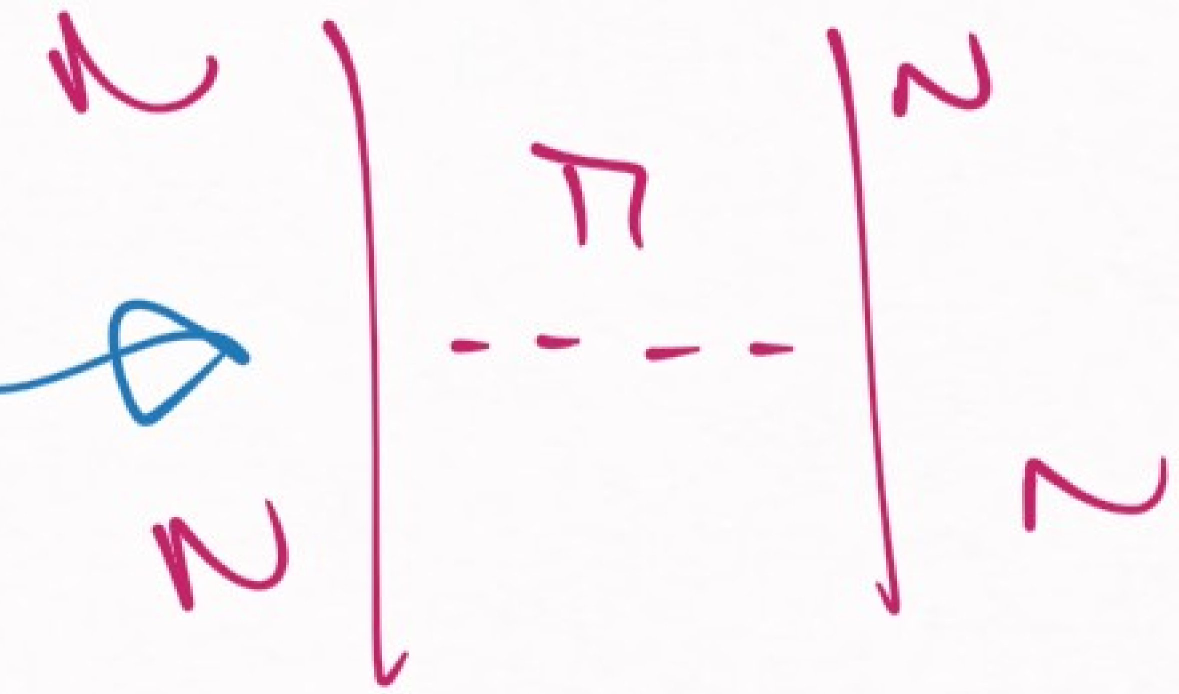
Best way to do this:

$\pi^a \to \tau^a$ (in the diagrams)

$\Rightarrow$ $a = 1, 2, 3$ (cartesian basis) $\left(\pi^\pm = \mp \frac{\pi^1 \pm i\pi^2}{\sqrt{2}}\right)$



$= V_0(\vec{q})$ $\rightsquigarrow$



$= \vec{\tau}_1 \cdot \vec{\tau}_2 V_0(\vec{q})$

$$\vec{\tau}_1 \cdot \vec{\tau}_2 = \begin{cases} 1 & I = 1 \\ -3 & I = 0 \end{cases}$$

→ Exactly the same as $\vec{\sigma}_1 \cdot \vec{\sigma}_2$ w/ spin

⊛ $\Rightarrow$ Isospin coupling (like spin)

two spin-1/2 particles $\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$

$$\left(\begin{aligned} |1\,1\rangle &= |\tfrac{1}{2}\,\tfrac{1}{2}\rangle|\tfrac{1}{2}\,\tfrac{1}{2}\rangle \\ |1\,0\rangle &= \frac{1}{\sqrt{2}}\Big(|\tfrac{1}{2}\,\tfrac{1}{2}\rangle|\tfrac{1}{2}\,{-\tfrac{1}{2}}\rangle \\ &\qquad + |\tfrac{1}{2}\,{-\tfrac{1}{2}}\rangle|\tfrac{1}{2}\,\tfrac{1}{2}\rangle\Big) \end{aligned} \right)$$

etc

same thing for isospin

$$|NN\rangle_I \Rightarrow \frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$$

$$|NN(I=0)\rangle = \frac{1}{\sqrt{2}}\left(|pn\rangle - |np\rangle\right) \rightarrow \boxed{\text{deuteron}}$$

$$|NN(I=1, M_I=0)\rangle = \frac{1}{\sqrt{2}}\left(|pn\rangle + |np\rangle\right)$$

etc. ..

Extended Fermi-Dirac statistics:

$n, p \rightarrow$ fermions (antisymmetric wf)

pp, s-wave $\Rightarrow S = 0$
$(L=0)$

$$|pp(S=0)\rangle = \frac{1}{\sqrt{2}}\left(|p\uparrow p\downarrow\rangle - |p\downarrow p\uparrow\rangle\right)$$

(same w/ nn)

If we have $n\,p$ $\Rightarrow$ in principle no symmetry constraints

(bc different type of particles)

But...

... if I add NOSPM $\Rightarrow$ $\boxed{\begin{array}{c} n, p \text{ same} \\ \text{type} \\ \text{of particle} \\ (N) \end{array}}$

$\Downarrow$

$\otimes$

⊕ ⇒ Isospin ⇒ N ⇒ if two-nucleons, then they should be antisymmetric

permutation

$$P_{12} |N_1 N_2 (L, S, I)\rangle = (-)^L (-)^{S-1} (-)^{I-1} |N_2 N_1 (L, S, I)\rangle$$

⇓

new constraints

Σ

⊕ $|S_1 S_2 (SM)\rangle = (-)^{S-S_1-S_2} |S_2 S_1 (SM)\rangle$

$\Rightarrow$ $P_{12}|NN\rangle = -|NN\rangle$

(fermions)

$(-)^{L+S+I} = (-1)$ $\quad$ $L+S+I$ = odd

$(-)^{L+S+I+1} = 1$

$$\boxed{L+S+I = 1, 3, 5, \ldots}$$

$$\boxed{(-1)^{L+S+1} = (-1)} \Rightarrow \text{Implications}$$

$$NN \; (\text{S-wave } (L=0)) \Rightarrow \boxed{S+I=1}$$

$$S=0 \; (\text{singlet}) \Rightarrow I=1$$

$$\left[S=1 \; (\underbrace{\text{triplet}}_{I}) \Rightarrow I=0 \right] \rightarrow \boxed{\text{Deuteron}}$$

(DEUTERON) $\rightarrow$ $S=1$

$$|d\rangle = \frac{1}{\sqrt{2}}\left(|pn\rangle - |np\rangle\right)$$

(SINGLET) $\rightarrow$ $S=0$

$$|{}^1S_0(m_J=0)\rangle = \frac{1}{\sqrt{2}}\left(|pn\rangle + |np\rangle\right)$$

$$|{}^1S_0(m_J=1)\rangle = |pp\rangle$$

$$|{}^1S_0(m_J=-1)\rangle = |nn\rangle$$

[More examples of isospin symmetry]

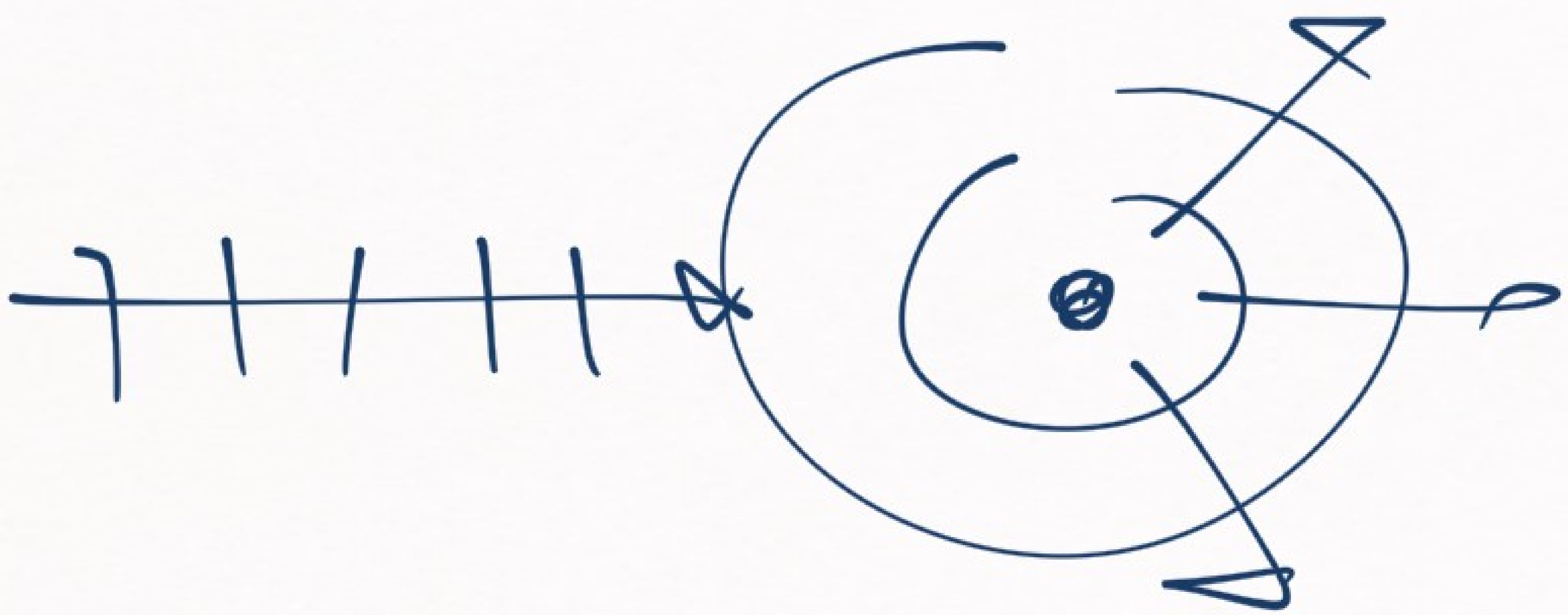
Singlet → 3 isospin states ($I = 1$)

↳ does not bind

↳ scattering observable

↳ scattering length $\Rightarrow \oplus$

@ $\Rightarrow$ What is this scattering length? $\rightarrow$ units of $[L]$



$$\sigma = |f(\vec{k})|^2$$

(cross-section)

Low energies $k \to 0$) $\rightarrow$ $\sigma \rightarrow 4\pi |a_0|^2$ $\rightarrow$ units of $[L]^2$

$$\frac{d\sigma}{d\Omega} \rightarrow |a_0|^2$$

$$a_0^S(pp) \simeq -17.3\ \text{fm}$$

$$a_0(np) \simeq -23.7\ \text{fm}$$

$$a_0(nn) \simeq -18.6\ \text{fm}$$

In general, similar

$$(a_0)_{natural} \sim \frac{1}{m_\pi} \sim 1.4\ \text{fm}$$

notice we have extremely large values

→ another example of isospin symmetry

(NEXT EXAMPLE) ^3H , ^3He

$|nnp\rangle$ $|npp\rangle$

$$|^3H\rangle = |\tfrac{1}{2}\, -\tfrac{1}{2}\rangle_I$$

$$|^3He\rangle = |\tfrac{1}{2}\, +\tfrac{1}{2}\rangle_I$$

→ same isospin ⇒ state

$$\Rightarrow \quad B(^3H) = 8.48\,\text{MeV}$$

$$B(^3He) = 7.72\,\text{MeV}$$ (pp electromagnetic repulsion)

$$\sim$$

$$\downarrow$$

$$B(^3He) \simeq B(^3He)$$ (but w/ differences coming from Coulomb)

(Other example) $\longrightarrow$ ^6He, ^6Li, ^6Be

(^4He core + nn, np, pp in the singlet)

$|^6He\rangle = |1\ {-1}\rangle_I$

$|^6Li\rangle = |1\ 0\rangle_I$

$|^6Be\rangle = |1\ {+1}\rangle_I$

$B(^6He) \simeq 29.27$ MeV

$B(^6Li) \simeq 31.99$ MeV

$B(^6Be) \simeq 26.92$ MeV

pretty similar

$\boxed{\text{Little RECAP}}$ → isospin symmetry works pretty well

NEXT THING → $\boxed{\text{WHERE DOES THIS SYMMETRY COME FROM?}}$

→ ORIGINS OF ISOSPIN IN QCD

muimd

Very light quarks

$\boxed{\text{QCD}}$ → 6 types of quarks $\boxed{\begin{matrix} u & d & s \\ c & b & t \end{matrix}}$

$$\left.\begin{array}{l} m_u \sim 3\,\mathrm{MeV} \\ m_d \sim 5\,\mathrm{MeV} \end{array}\right\} \quad m_u, m_d \ll \Lambda_{QCD}$$

$$\sim (200-350)\,\mathrm{MeV}$$

$$p \to uud \qquad \boxed{m_p \gg 2m_u + m_d}$$

$$n \to udd \qquad \boxed{m_n \gg m_u + 2m_d}$$

$$\left[\begin{array}{l} m_p \sim 3\Lambda_{QCD} \\ m_n \sim 3\Lambda_{QCD} \end{array}\right]$$

QCD → good scale separation for u,d

$$\boxed{\frac{m_u}{\Lambda_{QCD}}, \frac{m_d}{\Lambda_{QCD}} \ll 1}$$

→ In practical terms, u,d quarks are equivalent

$\Rightarrow$ $\boxed{\begin{matrix} u \leftrightarrow d \\ \text{(symmetry)} \end{matrix}}$ → SU(2)$_I$

[ISOSPIN IS A CONSEQUENCE OF $m_u, m_d \ll \Lambda_{QCD}$]

$\rightarrow$ We can notice here that $\exists$ another "light" quark $\rightarrow$ s $\nearrow$ $\boxed{\frac{m_s}{\Lambda_{QCD}} < 1}$

$m_s \sim 95\ \text{MeV}$ $\quad \frac{m_s}{\Lambda_{QCD}} \sim \frac{1}{2} - \frac{1}{3}$

→ We can in principle extend this symmetry to the s-quark

→ SU(3)-flavor symmetry

$u, d \rightarrow SU(2)$

$u, d, s \rightarrow SU(3)$

$\boxed{SU(3)_F}$ $\rightarrow$

(baryon octet)

$|p\rangle = |uud\rangle$

$|n\rangle = |udd\rangle$

$\rightarrow$ other similar baryons w/ s-quarks

$|\Sigma^+\rangle = |uus\rangle$

$|\Xi^0\rangle = |uss\rangle$

$|\Lambda^0\rangle = |uds\rangle$

$SU(3)_F$ partners of n, p

$\boxed{SU(3)_F}$ $\longrightarrow$

1) Works well w/ couplings

2) Works so-so w/ masses

$n, p \longrightarrow m_n \simeq m_p \simeq \underline{\underline{940\,\text{MeV}}}$

$\Sigma \longrightarrow m_\Sigma \simeq 1193\,\text{MeV}$ (20% heavier)

$\Xi \longrightarrow m_\Xi \simeq 1318\,\text{MeV}$ (30% heavier)

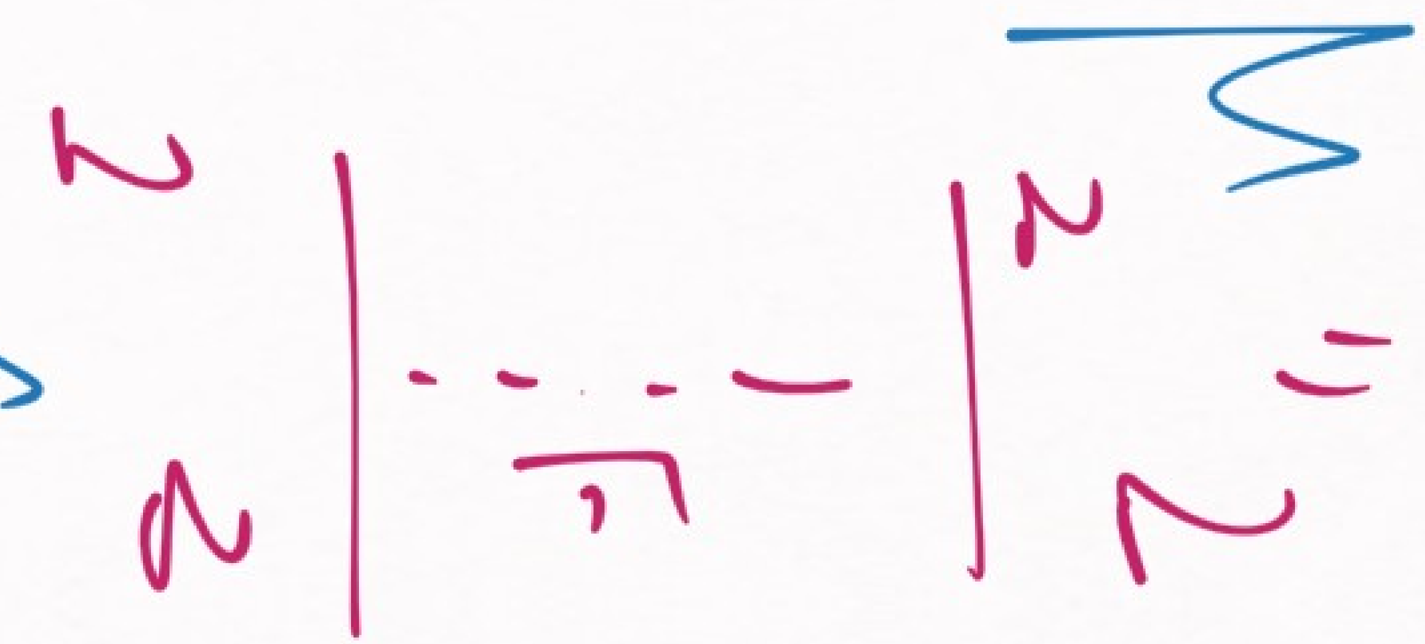
→ masses → $\boxed{\begin{array}{l} m_s \text{ is not so small} \\ (m_s \sim 100\ \mathrm{MeV}) \end{array}}$

↙

this will change
our nave expetation
that all masses are the same

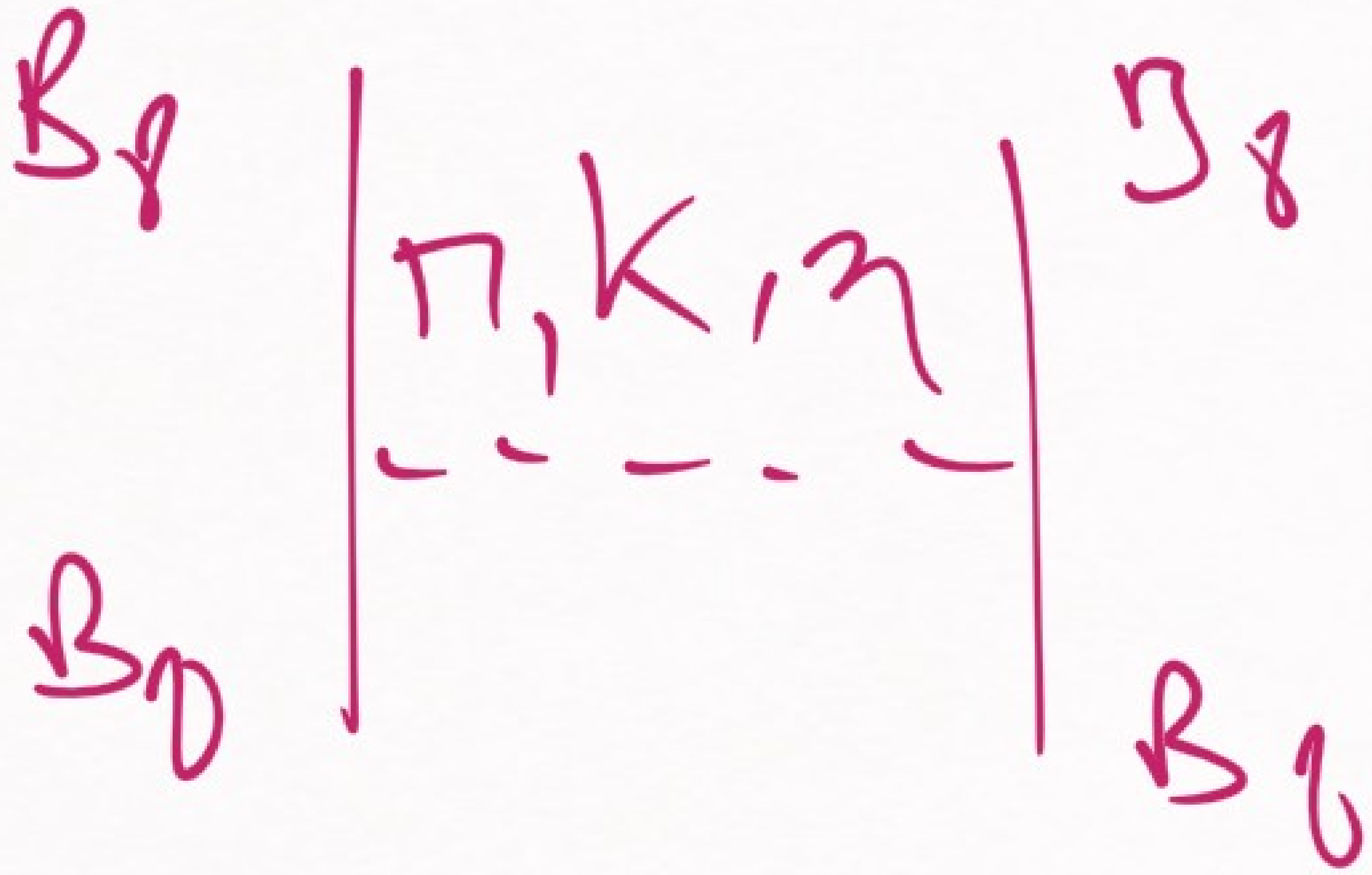
Couplings / potentials / ets $\rightsquigarrow$ works better

$= V_0(\vec{q}) \longrightarrow$



$= \vec{\tau}_1 \cdot \vec{\tau}_2 \, V_0(\vec{q})$

more complicated than



$= (SU(3)_F\text{-factor}) \times V_0(\vec{q})$

$B_8 = N, \Lambda, \Sigma, \Xi$

but works well

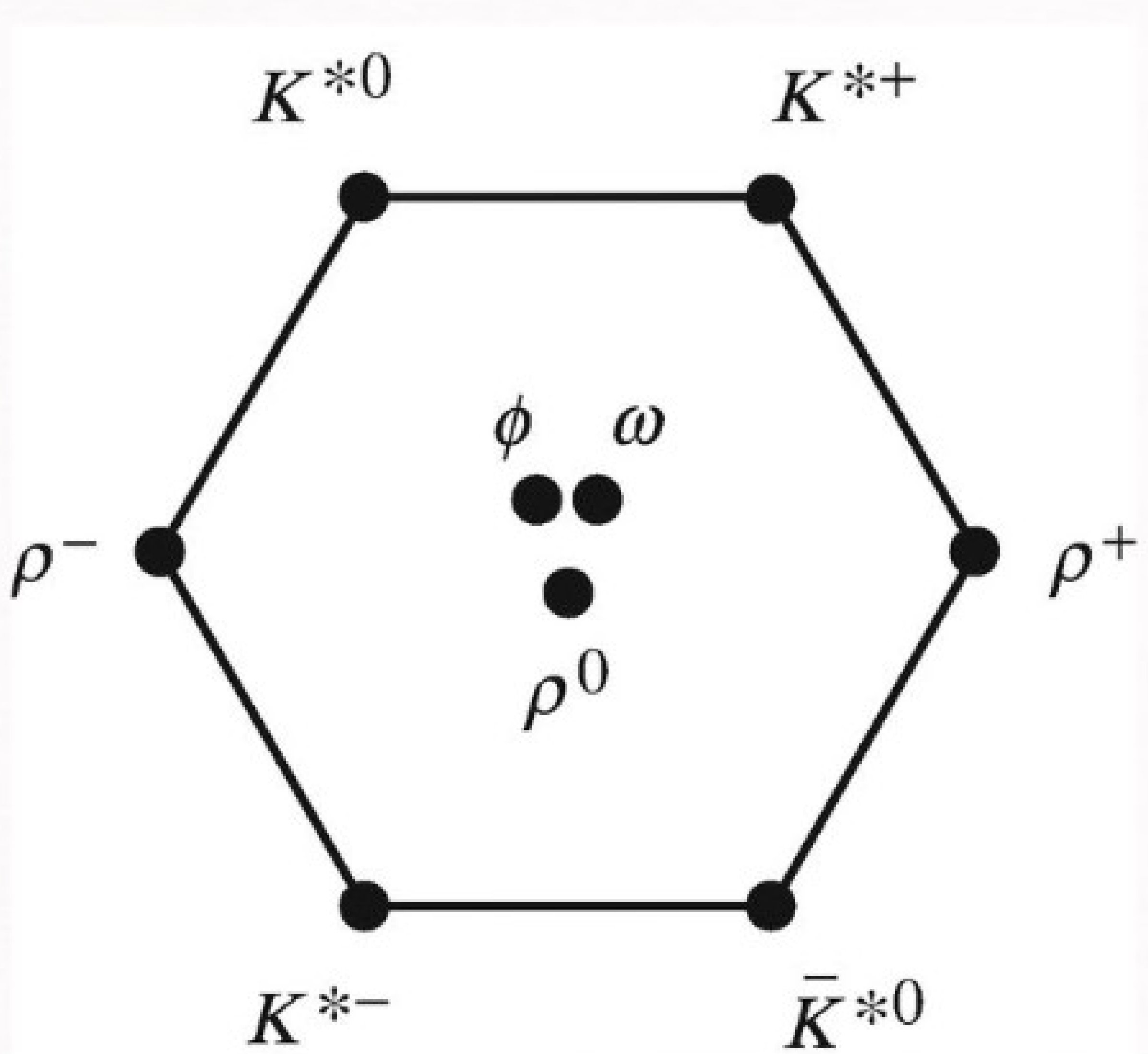
→ we will not cover these factors in detail

Baryons $SU(3) \to N, \Lambda, \Sigma, \Xi$

↓

Mesons $SU(3) \to \rho, \omega, K^*, \phi$

$\bar{u}s$, $\bar{d}s$; $s\bar{s}$; $u\bar{s}$, $d\bar{s}$

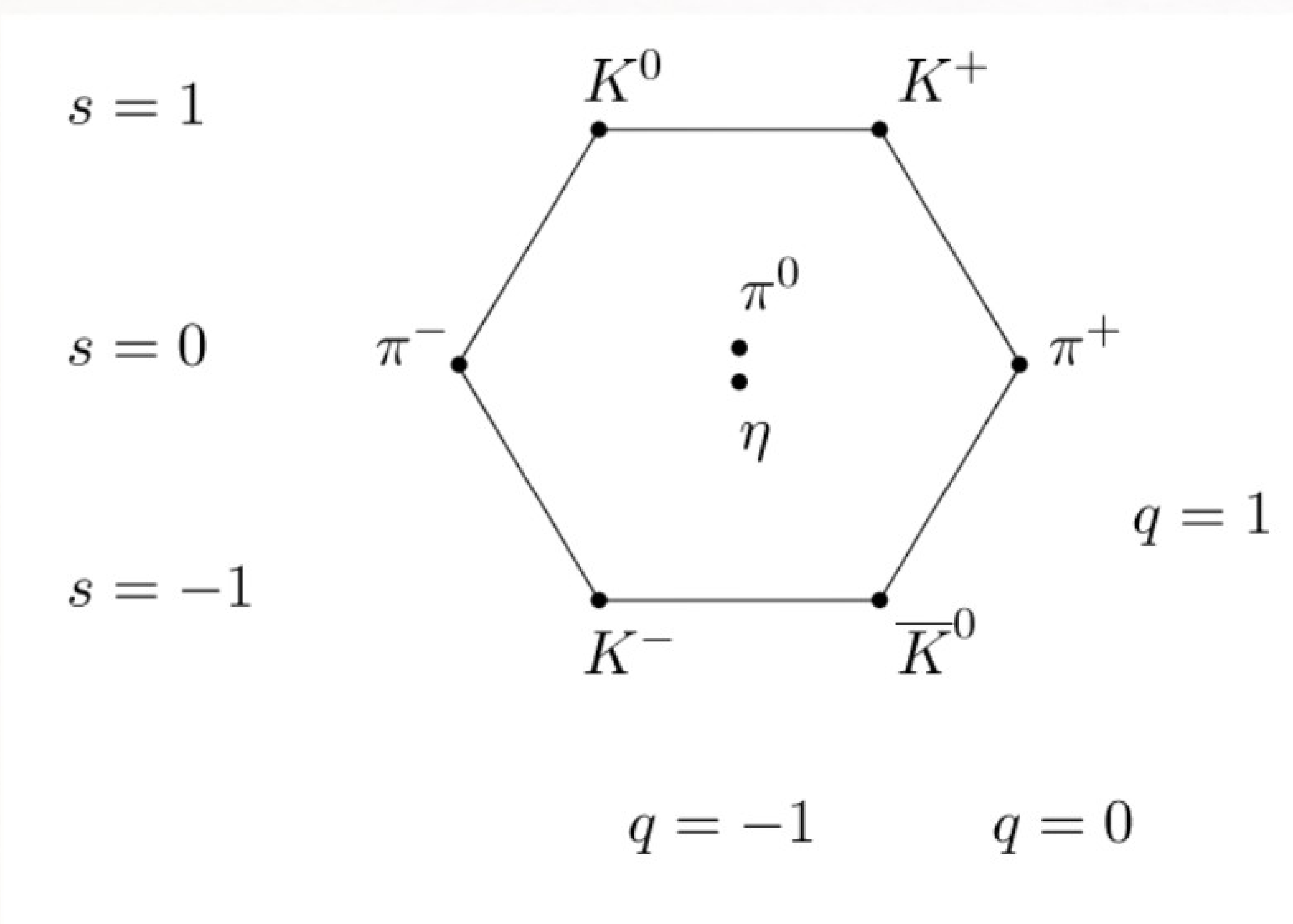


→ meson nonet

$m_\rho \simeq m_\omega \simeq 770-780\,\text{MeV}$

$m_{K^*} \simeq 890\,\text{MeV}, \; m_\phi \simeq 1020\,\text{MeV}$

→ SUDDENLY THINGS BECOME STRANGE WHEN CONSIDERING THE PION SU(3)$_K$ - PARTNERS



→ pion, eta, kaon

$$\left.\begin{aligned} m_\pi &\simeq 138 \text{ MeV} \\ m_K &\simeq 495 \text{ MeV} \\ m_\eta &\simeq 548 \text{ MeV} \end{aligned}\right\} \text{not very symmetric}$$

→ $\pi, \underline{K}, \eta$ → $SU(3)_F$ ~~-flavor~~ failing?

→ [or $\exists$ another effect or something]

$\boxed{\text{CHIRAL SYMMETRY}}$

↳ π, K, η are special

$$\frac{m_u}{\Lambda_{QCD}}, \frac{m_d}{\Lambda_{QCD}}, \frac{m_s}{\Lambda_{QCD}} < 1 \Rightarrow SU(3)_F \text{ symmetry}$$

$$\Downarrow$$

[Broken chiral symmetry] $\Rightarrow$ $\boxed{m_\pi = m_K = m_\eta = 0}$

these mesons are special

(not the case, but

$m_\pi \ll m_\rho$ $\boxed{m_K < m_{K^*}}$ $\boxed{m_\eta < m_\omega}$)

strange quarks, $\frac{m_s}{\Lambda_{QCD}} \sim \frac{1}{2} - \frac{1}{3}$

[HOW TO EXPLAIN WHAT IS CHIRAL SYM.]

→ not trivial, requires elaboration
(INTERMEDIATE STEPS)

→ the linear sigma model (step 1)

→ Goldstone theorem (step 2)

→ Chiral symmetry (step 3)

$$m_\pi \sim 140\,\text{MeV}$$

$$\frac{m_\pi}{m_N} \sim \frac{1}{7} \qquad \frac{m_e}{m_N} \sim 0.8 \sim \mathcal{O}(1)$$

$\rightarrow$ we have to explain why m_π so small

Σ

(previous 3 steps)

60's → GELL-MANN & LEVI → $\boxed{\text{LINEAR } \sigma\text{-MODEL}}$

(pion very light)

1 theory w/ nucleons, 1 scalar field,

3 pseudoscalar fields

→ massless nucleons + massive meson

$N \to I = 1/2$, $\phi_0 \to I = 0$, $\vec{\phi} = \{\phi_1, \phi_2, \phi_3\}$

↳ $I = 1$

↳ I write a lagrangian:

$$\mathcal{L}_{LSM} = \underbrace{i\bar{N} \not{\partial} N}_{\text{(N kinetic)}} + g\underbrace{\bar{N}(\phi_0 + i\gamma_5 \vec{\tau}\cdot\vec{\phi})N}_{N \leftrightarrow \phi \text{ interaction}}$$

$$+ \frac{1}{2} \sum_{i=0}^{3} \underbrace{\partial_\mu \phi_i \, \partial^\mu \phi^i}_{\phi \text{ kinetic}} - \underbrace{V(\phi)}_{\phi \text{ mass + other things}}$$

$$V(\phi) = \underbrace{\frac{1}{2}\mu^2 \sum_{i=0}^{3} \phi_i^2}_{\phi\text{ mass}} + \underbrace{\frac{\lambda}{4}\left(\sum_{i=0}^{3} \phi_i^2\right)^2}_{\phi\text{ interactions}}$$

$\rightarrow$ $\boxed{N \text{ massless} \;/\; \phi^0, \vec{\phi} \text{ massive } (\mu)}$

$\Rightarrow$ $\boxed{N \text{ massive}, \phi_0 \text{ massive}, \vec{\phi} \text{ massless}}$

ν massless, $\bar{d}$ massive

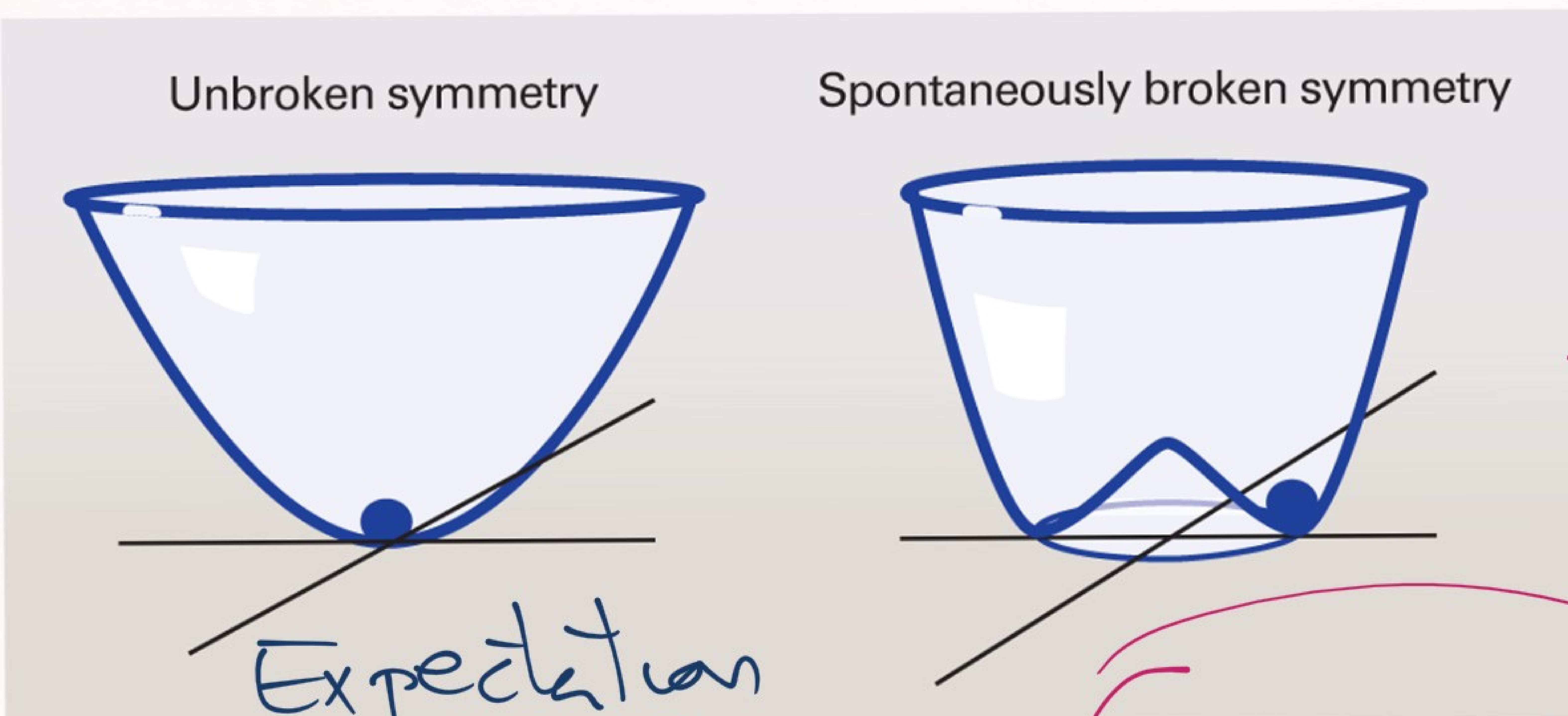
$\Downarrow$

ν massive, $\vec{d}$ massless

$\boxed{\text{QFT trick}} \longrightarrow \boxed{\text{Mexican-hat potential}}$

MEXICAN HAT POTENTIAL

$$V(\phi) = \frac{1}{2}\mu^2\left(\sum_i \phi_i^2\right) + \frac{\lambda}{4}\left(\sum_i \phi_i^2\right)^2 \rightarrow \mu^2, \lambda$$



Expectation

$$\boxed{\begin{array}{c}\mu^2 > 0 \\ \lambda > 0\end{array}}$$

$\mu^2 < 0, \lambda > 0$ → Allowed by math

But mass should not be negative, right?

MEXICAN HAT POTENTIAL

1) Standard quartic potential

$$V(\phi) = \frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$$

2) We change sign of mass term

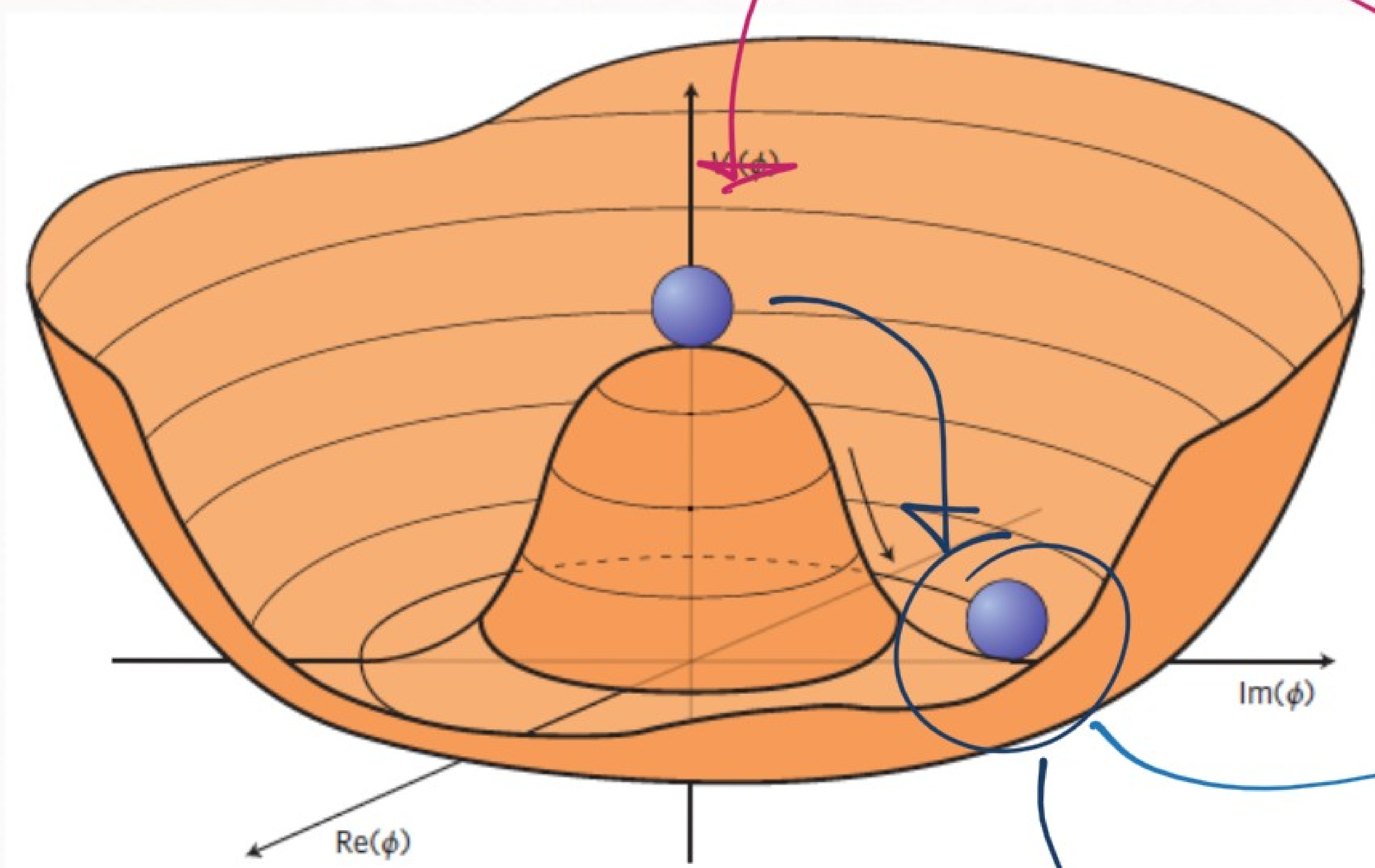
$$\mu^2 < 0 \,, \quad \lambda > 0$$

this will no longer be the mass of ϕ field

Why μ is not a mass? → nature likes minimum energy states

$(\phi^0, \vec{\phi}) = (0, \vec{0})$

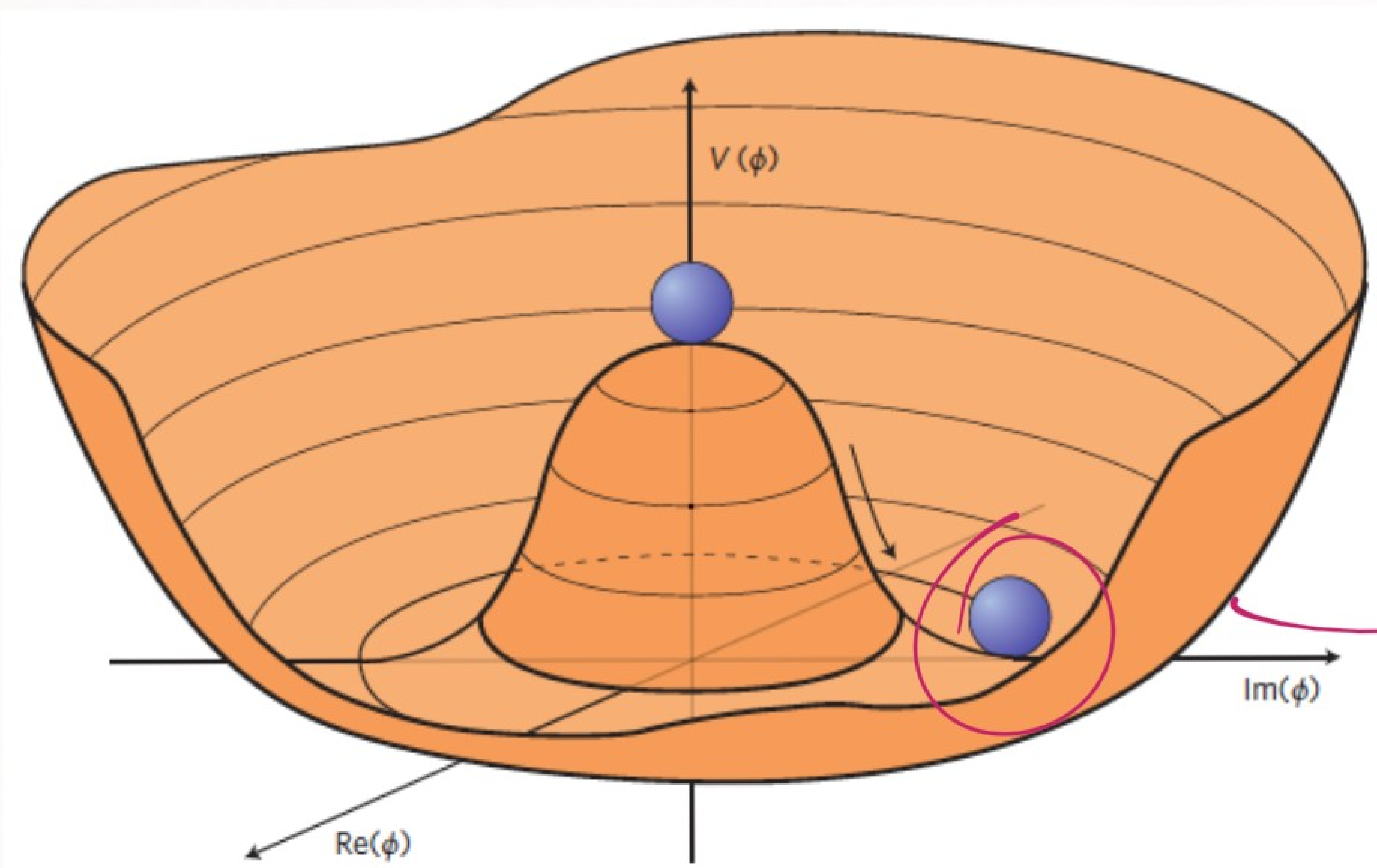
↳ this is not the ground state of this QFT



this will be the ground state

· min energy

1) GROUND STATE → STATE THAT MINIMIZES $V(\phi)$



$$\boxed{\frac{dV}{d\phi^0} = 0}$$

$$\phi^0 = \sqrt{-\frac{\mu^2}{\lambda}} = v$$

(min. for mexican hat potential)

2) REDEFINE THE FIELDS / NEW QFT

HAS IT'S MINIMUM AT $\Phi_{new} = 0$

$\to$ Change of variables

$(\bar{\sigma}, \vec{\pi})_{min} = (0, \vec{0})$

$(\phi_0, \vec{d})_{min} = (v, \vec{0})$

$\hookrightarrow (\sigma, \vec{\pi}), \quad \vec{\sigma} = \phi^0 - v, \quad \vec{\pi} = d$

$$(\alpha_0, \vec{\alpha}) \rightarrow (\sigma, \vec{\pi})$$

$$\mathcal{L}_{L\sigma M}(\alpha) \rightarrow \mathcal{L}_{L\sigma M}(\sigma, \vec{\pi}) = \underbrace{\bar{N}(i\partial\!\!\!/ - gV)N}_{1}$$

$$+ \underbrace{g\bar{N}(\sigma + i\gamma^5 \vec{\tau}\cdot\vec{\pi})N}_{2}$$

$$+ \underbrace{\frac{1}{2}(\partial_\mu \sigma\, \partial^\mu \sigma) - \frac{1}{2}(\lambda v^2)\sigma^2}_{3}$$

$$+ \underbrace{\frac{1}{2}\partial_\mu \vec{\pi}\, \partial^\mu \vec{\pi}}_{4} - V(\vec{\pi}, \vec{\sigma})$$

WHAT IS THE MEANING OF THESE TERMS?

1 → mass term for the nucleon

$$\boxed{m_N = gv}$$ ✓

2 → the coupling of the new fields σ & $\vec{\pi}$ are identical

$g_{\pi NN} \simeq 13$
$g_{\sigma NN} \simeq 10$
30% error

$$\boxed{g_{\pi NN} = g_{\sigma NN}}$$ ✓

③ → the sigma is massive

$$\boxed{m_\sigma^2 = \lambda v^2 = -\mu^2}$$ ✓ $m_\sigma \sim 500\,\text{MeV}$

④ → the pion will be massless

$$\boxed{m_\pi = 0}$$ ✓ $m_\pi \ll m_\sigma, m_N$

→ (VERY INTERESTING MODEL)

→ explains large mass of nucleon

→ explains small mass of pions

✓

small pion mass

$$V(\phi) \to V(\phi) + \delta V(\phi) \, , \quad \delta V \propto -\frac{\epsilon}{v} \phi_0$$

$$\epsilon \ll 1$$

However, in the 60's people didn't observe the sigma → weak point of LσM (back then)

Gell-Mann & Levy → non-linear σ-model (does not contain sigma)

INTERESTING POINT

$\exists$ mechanisms in QFT for the pion to be very light

LSM & Mexican hat potential

→ specific example Goldstone theorem

1 Goldstan theorem

+

2 Breaking of a QCD approx. symmetry called chiral symmetry

NEXT LESSON

1 & 2

1 + 2 $\longrightarrow$ Explain the physics of pions

SEE YOU ON TUESDAY