NUCLEAR PHYSICS 10

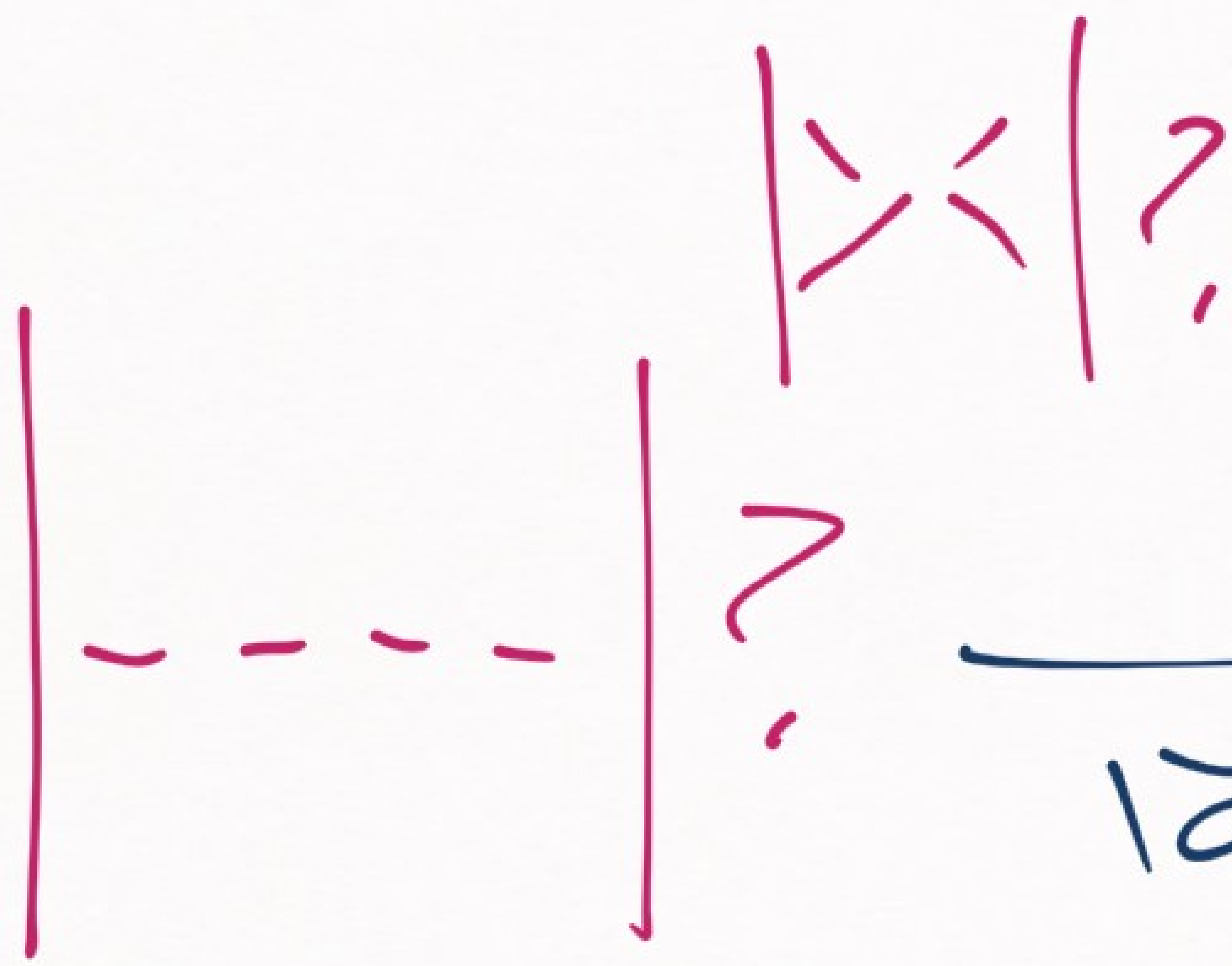
MORE ABOUT RENORMALIZATION

$\boxed{\text{RENT}}$ (basic idea) "Physics at long distances does not depend on short range details"

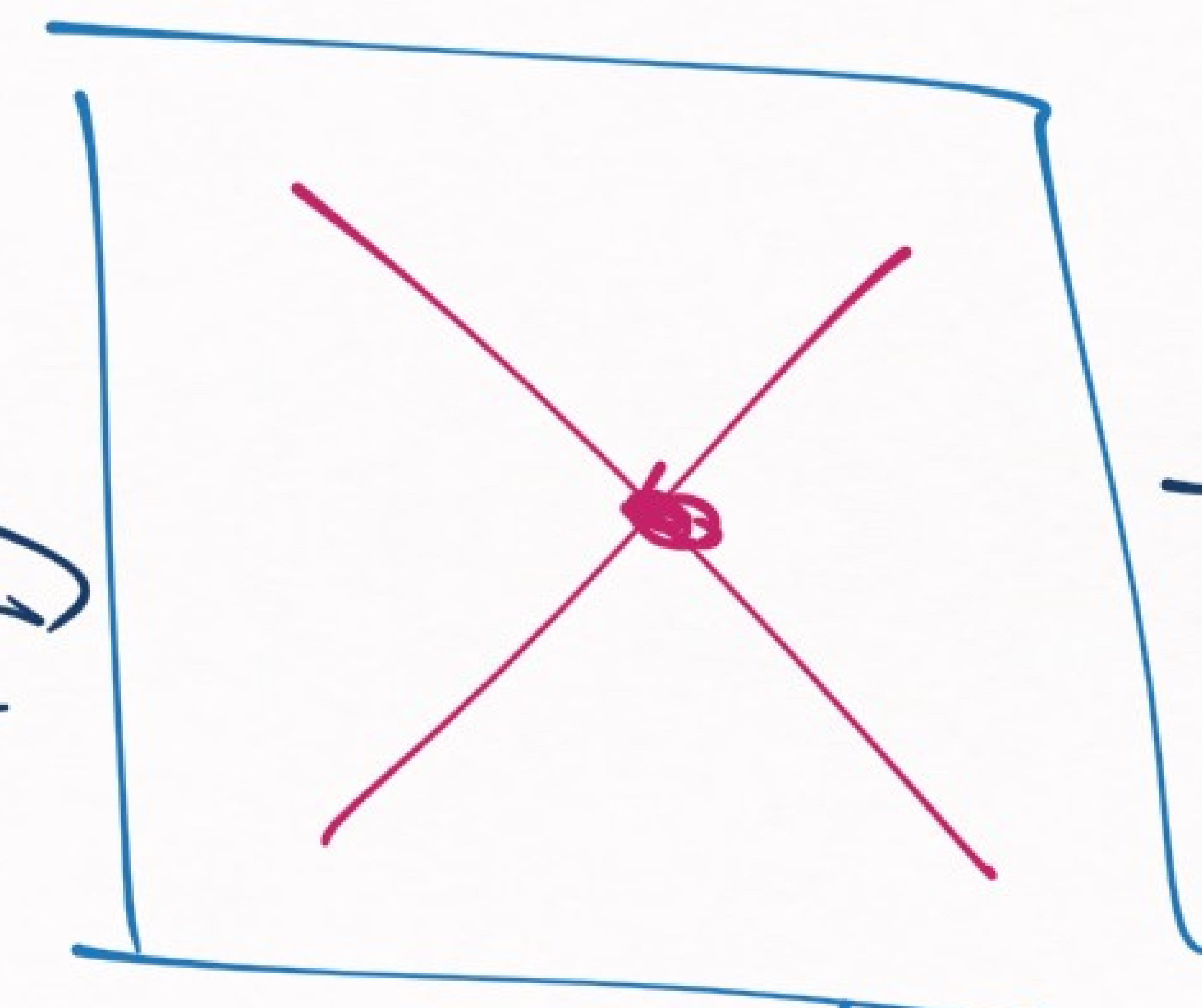
the way to do this $\longrightarrow$ a $\underbrace{\text{series of steps}}$

little review follows...

[two-body problem] $\longrightarrow$ described by a potential



$\xrightarrow[|\vec{q}|^2 \ll m^2]{}$



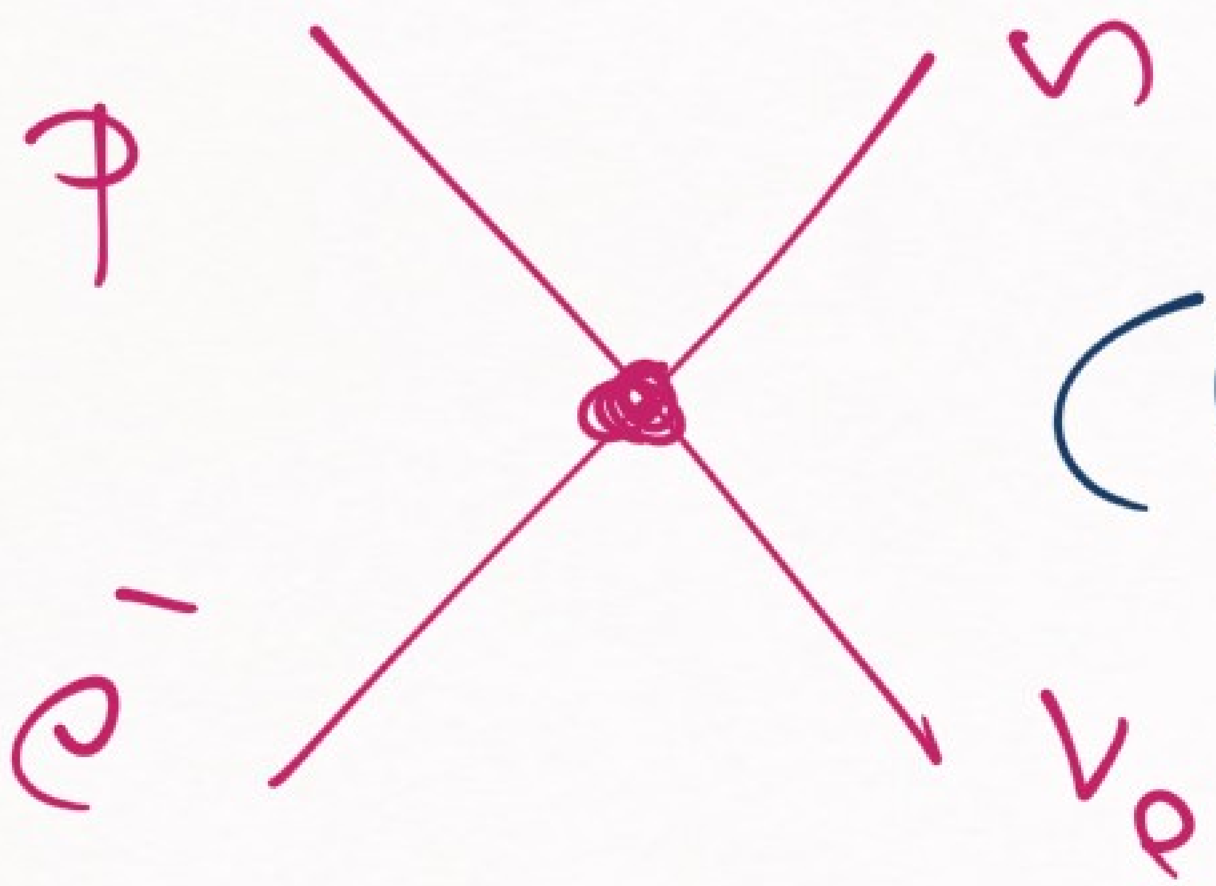
$\longrightarrow$ contact potential

what we see at long distances

$$V(\vec{q}) = -\frac{g^2}{\vec{q}^2 + m^2}$$

or $$V(\vec{q}) = \frac{\lambda^2}{M^2} f(\vec{q}/m)$$

A second example: zoom in

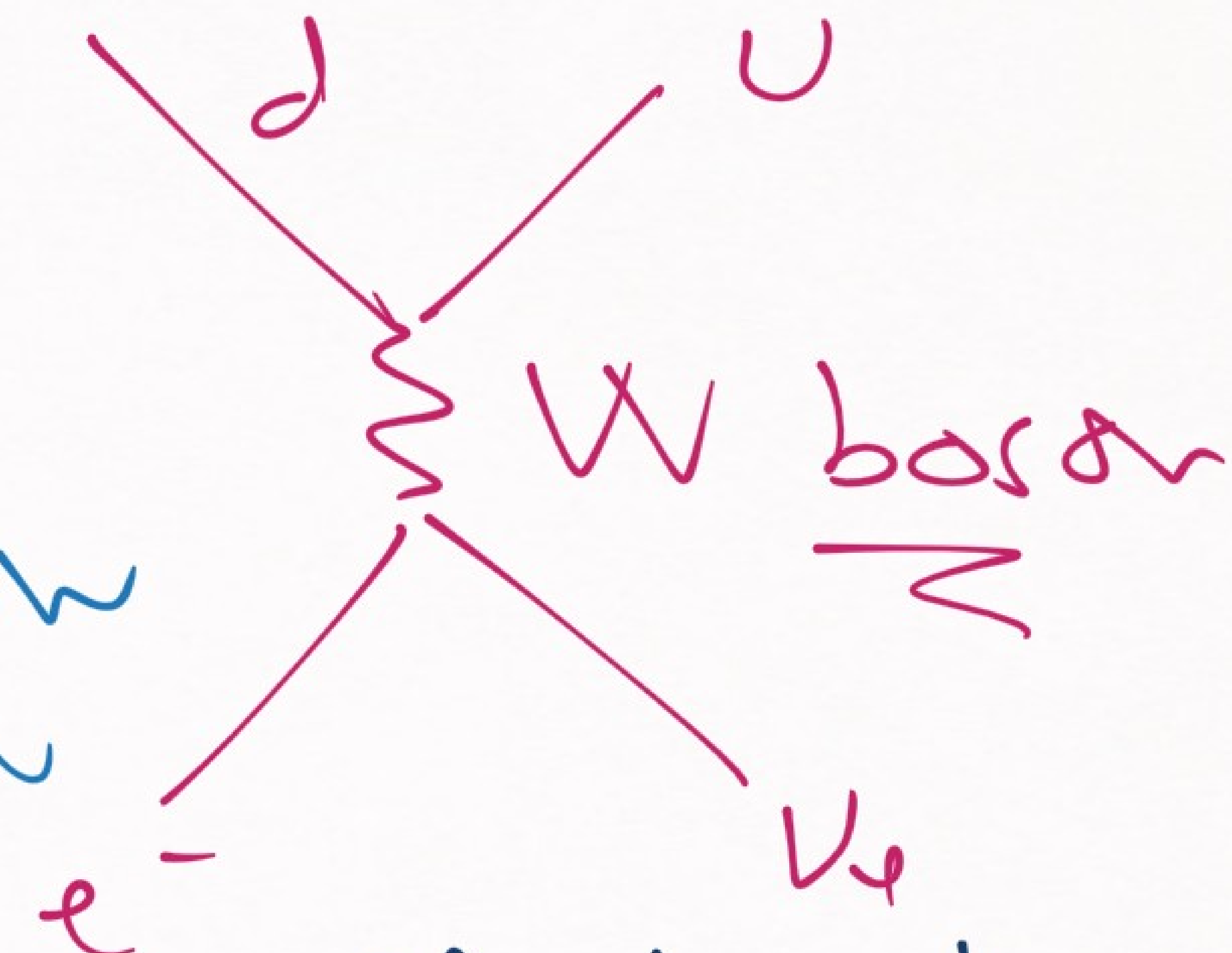


(long distances)

↓

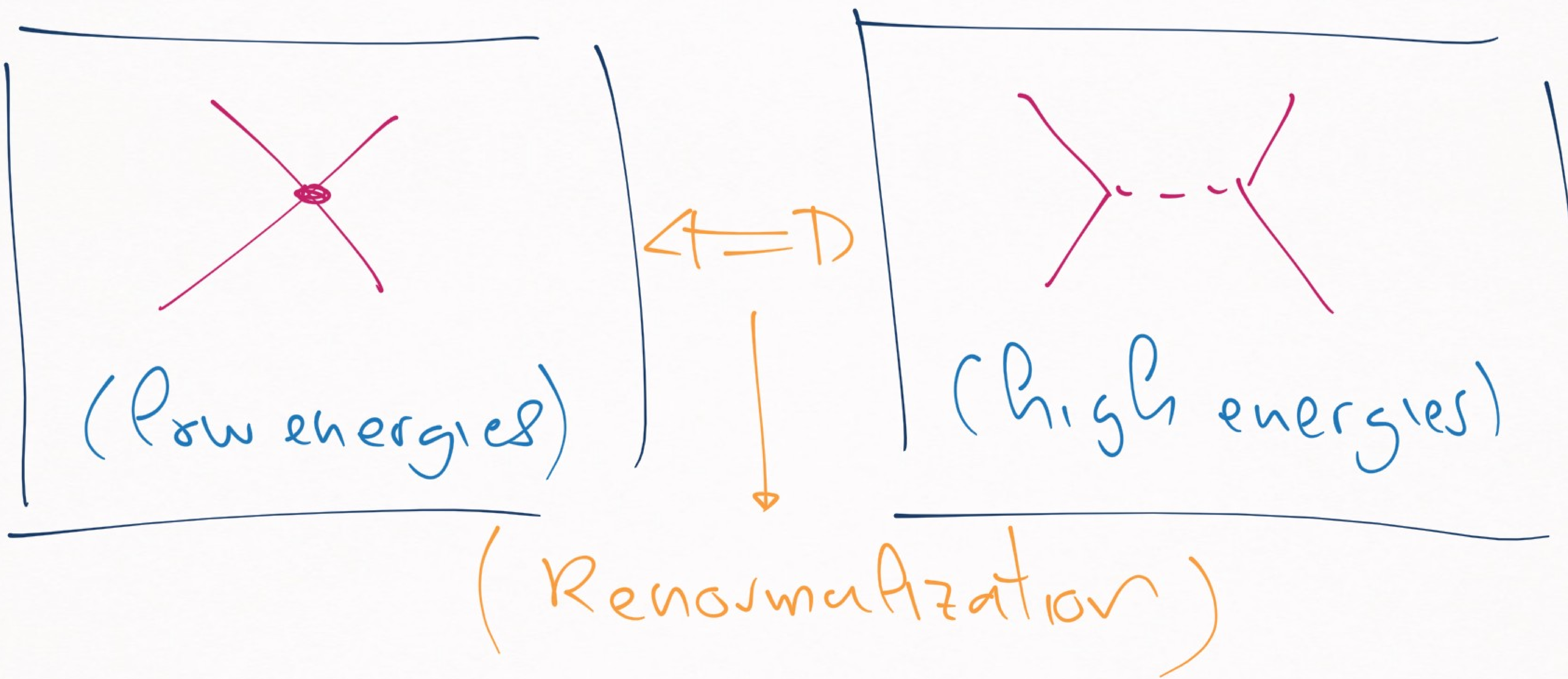
we don't know about the W

Fermi theory of weak interactions



(short distances)

For two-body potentials is the same...



[Renormalization]

basic idea → really simple

implementation → difficult part

→ divide the problem in several steps

Renormalization process

$\Rightarrow$

① Regularization

$$\cancel{V_c(\vec{r}) = C_0\,\delta^{(3)}(\vec{r})}$$

non-perturbatively problematic

Resolution scale (cutoff)

$$V_c(\vec{r}\,;R_c) = C_0 \boxed{\delta^{(3)}(\vec{r}\,;R_c)} \rightarrow \text{regular } (\neq \infty, |\vec{r}|=0)$$

② Renormalization (proper)

$$V_c(\vec{r};R_c) = \overbrace{C_0(R_c)}^{(b)}\,\underbrace{\delta^{(3)}(\vec{r};R_c)}_{a}$$

Include R_c-dependence in $C_0(R_c)$

R_c-dependence $\left[\begin{array}{l}\text{(a) potential form} \\ \text{(b) coupling}\end{array}\right]$ cancel each other

2 → generate complications

→ $C_0(R_c)$ can be determine w/ different methods

→ $\exists$ different solutions for $C_0(R_c)$ depending on physical situation

(Previous lesson) $\longrightarrow$ $\exists$ different solutions

$$V_c(\vec{r}) = C_0(R_c)\,\delta^{(3)}(\vec{r}; R_c)$$

$\to$ bound state solution

(non-perturbative systems)

$\to$ solution for perturbative interactions

example of scattering & Born term

[Non-perturbative solutions] → [$C_0(R_c)$ / reproduces a bound state]

In most cases requires a numerical calculation

But sometimes not ... (δ-shell regulator)

$$V_c(\vec{r}; R_c) = \frac{C_0(R_c)}{4\pi R_c^2}\,\delta(r - R_c)$$

$$\frac{1}{C_0(R_c)} \to \frac{\mu}{2\pi}\left(\gamma - \frac{1}{R_c}\right)$$

$\mu = \frac{m_1 m_2}{m_1 + m_2}$ (reduced mass)

$\gamma = \sqrt{2\mu B}$ (wave number)

$R_c \to 0$

$R_c \to$ r-space

$\Lambda \to$ p-space

$$\boxed{C_0(R_c) \propto R_c}$$

for $R_c \to 0$

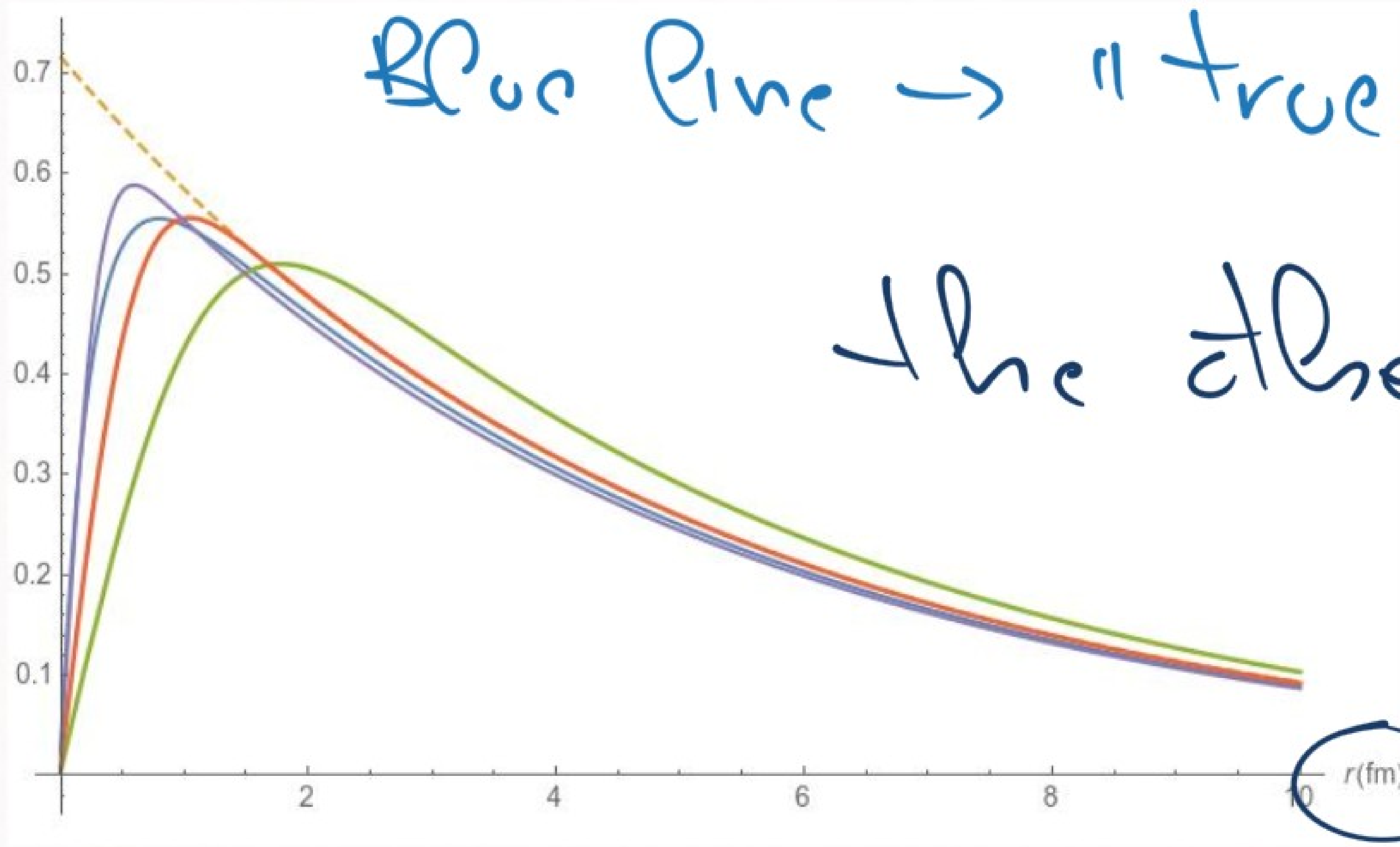
$\Lambda = D$

$$\boxed{C_0(\Lambda) \propto \frac{1}{\Lambda}}$$

for $\Lambda \to \infty$

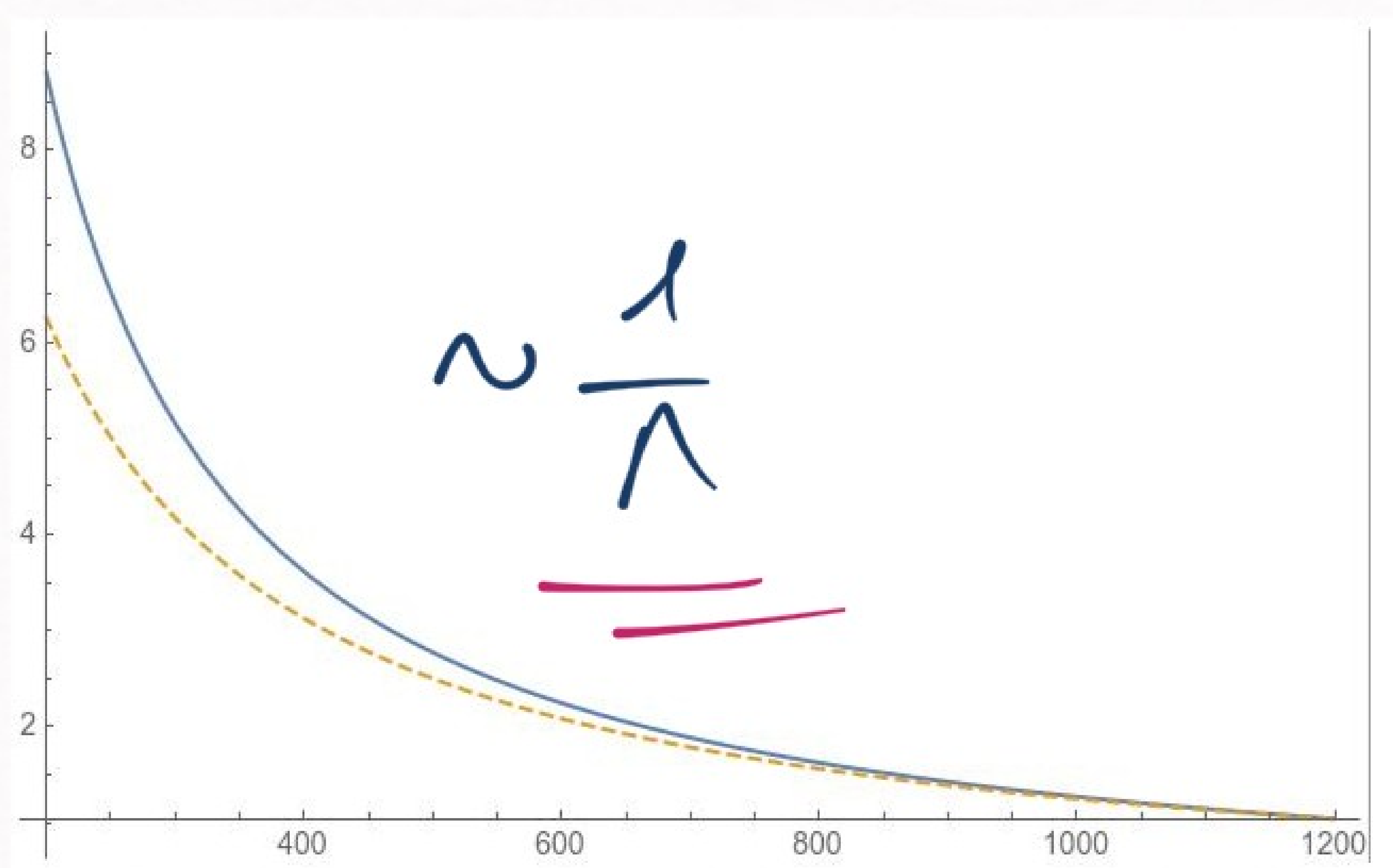
Blue line → "true wave function"

$\psi(r)$



The others → wave functions from a contact (w/ different Λ)

$C_0(\Lambda)$



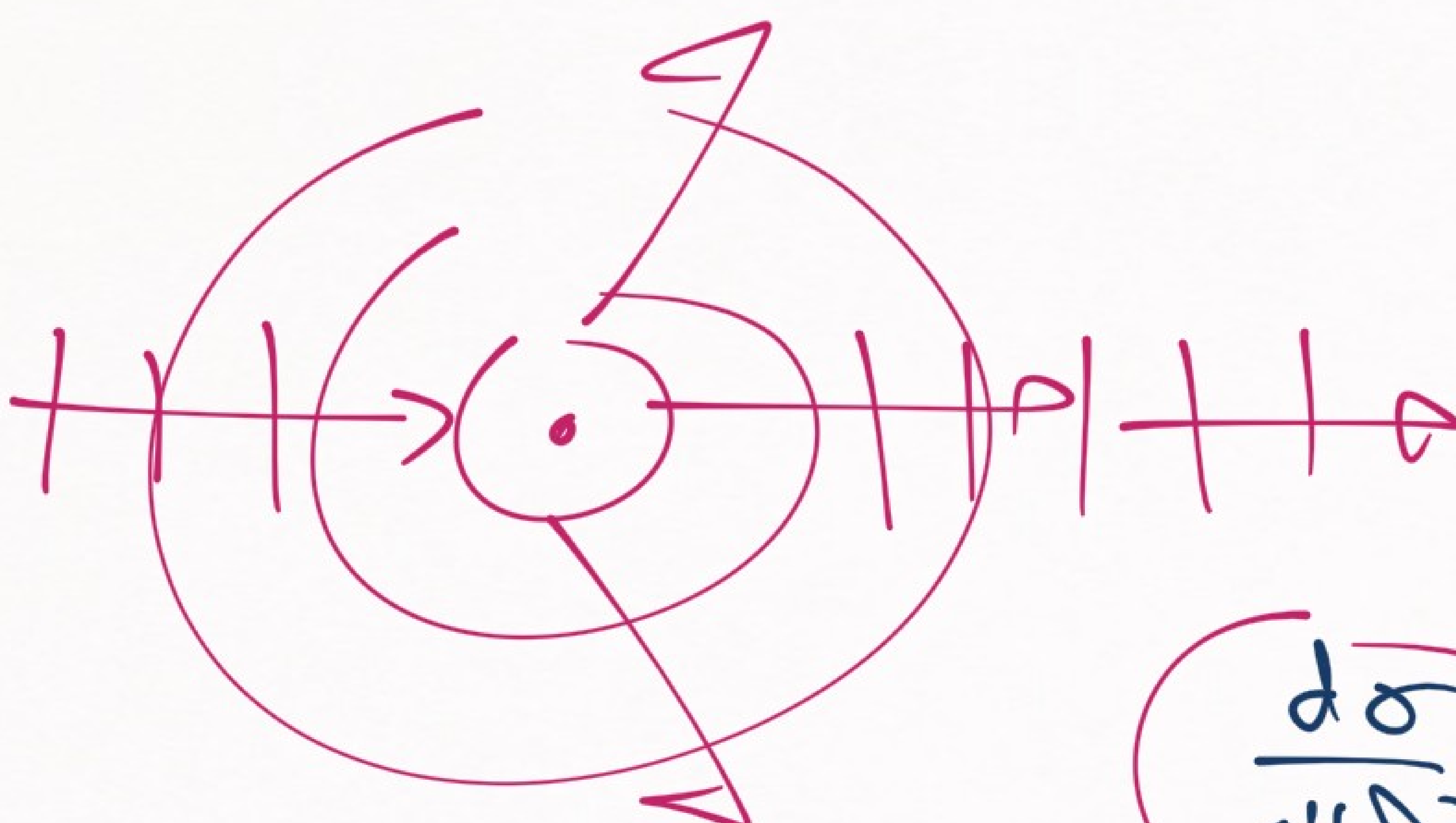
$\sim \frac{1}{\Lambda}$

→ expected running of $C_0(\Lambda)$

Λ

(check previous page)

[Perturbative solutions] → Example comes from scattering theory (if you don't know, we'll explain later)



$$\frac{d\sigma}{d\Omega} = |f(\hat{k})|^2$$

cross section

f → scattering amplitude

Scattering amplitude → Born approximation

(for perturbative systems)

$$f(\vec{k}) = -\frac{\mu}{2\pi}\int d^3\vec{r}\, V(\vec{r})\, e^{-i\vec{k}\cdot\vec{r}} + \ldots$$

$$= -\frac{\mu}{2\pi} V(\vec{k}) + \ldots$$

$$\frac{d\sigma}{d\Omega} \xrightarrow[|q|^2 \ll m^2]{} \left| \frac{\mu \overline{V(\vec{0})}}{2\pi} \right|^2$$

p-space
$(\vec{k} = \vec{0})$

$$\xrightarrow[V_c = C_0 \delta^{(3)}(\vec{r})]{} \left| \frac{\mu C_0}{2\pi} \right|^2 \rightarrow$$

this EFT description different than in bound state case

$\left[C_0(R_c) \simeq V(\vec{q}=\vec{0}) \right] \rightarrow$ very interesting

$\delta^{(3)}(\vec{r}) \quad \nabla^2 \delta^{(3)}(\vec{r})$

$$V_Y(\vec{q}) = -\frac{g_Y^2}{q^2+m^2} = -\frac{g_Y^2}{m^2}\left(1 + \frac{\vec{q}^2}{m^2} + \frac{\vec{q}^4}{m^4} + \dots\right)$$

→ this case correspons w/ the idea of expandind $V(\vec{q})$ w a power series perturbative case $\longrightarrow$ more intuitive results for $C_0(R_c)/C_0(\Lambda)$

contact-range potential

$\simeq$ Taylor expansion of true potential

First observation $\longrightarrow$ $\exists$ different solutions for $C_0 = C_0(R_c)$

Second observation $\longrightarrow$ $\exists$ different views to understand the first observation

$\downarrow$

Now we will review them

OBSERVATION ABOUT THE CONTACT POTENTIAL

① Perturbative case

$$V_c(\vec{q}) = C_0 + C_2\vec{q}^{\,2} + C_4\vec{q}^{\,4} + \dots$$

$$V_Y(\vec{q}) = -\frac{g_Y^2}{\vec{q}^{\,2}+m^2} = -\frac{g_Y^2}{m^2}\left(1 + \frac{\vec{q}^{\,2}}{m^2} + \frac{\vec{q}^{\,4}}{m^4} + \dots\right)$$

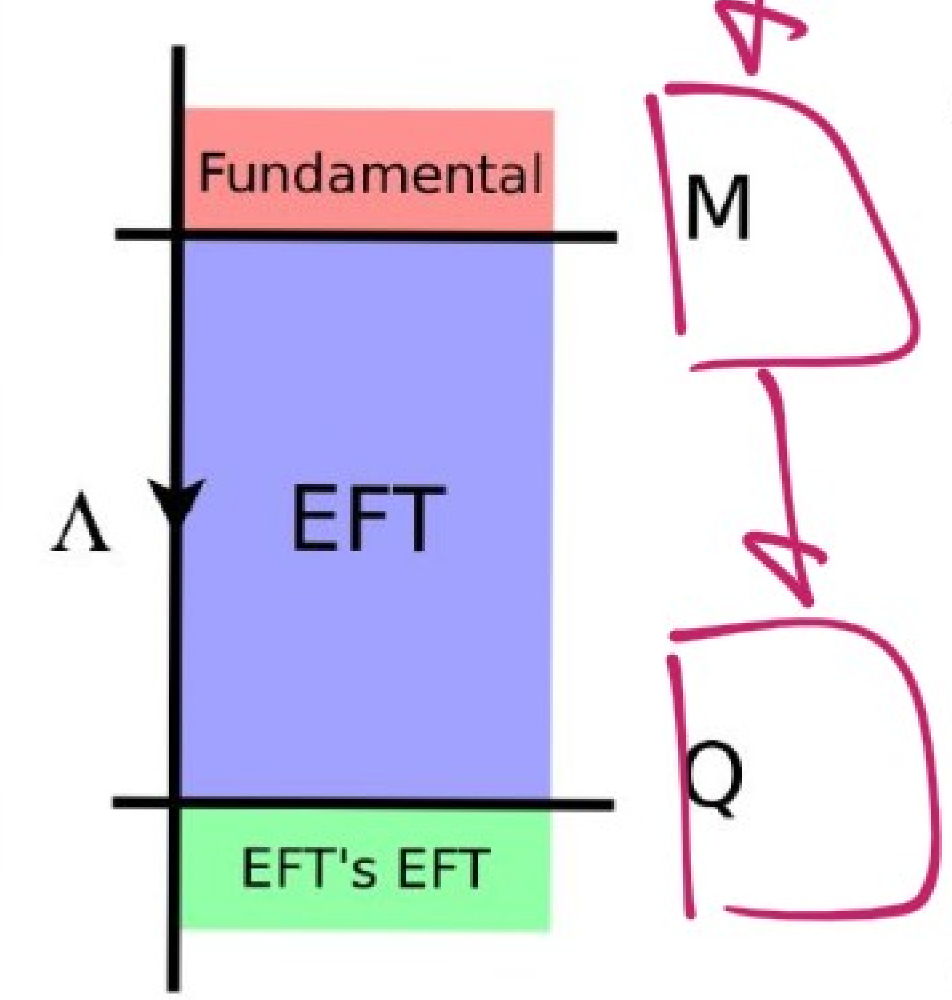
[CORRESPONDENCE] (Yukawa's example)

$$C_0(P_c) \sim -\frac{g_Y^2}{m^2}$$

this power goes up

$$C_2(P_c) \sim -\frac{g_Y^2}{m^4}$$

$$C_4(R_c) \sim -\frac{g_Y^2}{m^6}$$



Physics is unique, but choice of theory depends on resolution Λ:

- $\Lambda \geq M$: Fundamental
- $M \geq \Lambda \geq Q$: EFT

For equivalent descriptions:

$$\boxed{\frac{d}{d\Lambda}\langle\Psi|\mathcal{O}|\Psi\rangle = 0}$$

Renormalization group invariance

distinction between low/high energy scales

$$q \ll M$$

trick: write everything as an expansion in q/M

$$C_0 \simeq -\frac{g y^2}{m^2} \longrightarrow C_0 \sim \frac{1}{M^2}$$

$$Q \sim |\vec{q}\,|$$

$$M \sim m$$

$$C_2 \sim \frac{1}{M^4}$$

$$C_4 \sim \frac{1}{M^6}$$

$$C_{2n} \sim \frac{1}{M^{2+2n}}$$

orderings

$$V_c(\vec{q}\,) = \sum_{n=0}^{\infty} C_{2n}\, \vec{q}^{\,2n}$$

$$V_C(\vec{q}) = \sum_{n=0}^{\infty} C_{2n} |\vec{q}|^{n}$$

$$\Rightarrow \quad C_{2n} |\vec{q}|^{2n} \sim \frac{Q^{2n}}{M^{2n+2}}$$

$$\sim \frac{1}{M^2} \left(\frac{Q}{M}\right)^{2n}$$

$$\Rightarrow \boxed{\frac{C_{2n} |\vec{q}|^{2n}}{C_0} \sim \left(\frac{Q}{M}\right)^{2n}} \rightarrow \boxed{\text{each term is less important}}$$

$\to$ [This is called power counting]

perturbative case $\Big) \longrightarrow \dfrac{c_{2n}|q|^{2n}}{c_0} \sim \left(\dfrac{q}{M}\right)^{2n}$

each new term is suppressed by two orders in (q/M)
$\underline{\qquad 2 \qquad}$

② Non perturbative case:

$$\frac{1}{C_0(R_c)} \to \frac{M}{2\pi}\left(\gamma - \frac{1}{R_c}\right) \qquad \left[\begin{array}{l} Q \sim \gamma, |\vec{q}| \\ M \sim m, \mu \end{array}\right]$$

$$\boxed{C_0 \sim \frac{1}{MQ}} \to \text{different than in the perturbative case}$$

$$C_0 \sim \frac{1}{MQ}$$

$$\boxed{\begin{aligned} C_2 &\sim \frac{1}{M^2 Q^2} \\ C_{2n} &\sim \frac{1}{M^{2n} Q^2} \end{aligned}} \longrightarrow \text{when } \exists \text{ bound state}$$

Perturbative
Non-perturbative cases → different "power countings"

[we count the powers of αM
in different ways]

OBSERVATION IS DONE...

→ TRY TO UNDERSTAND $C_d = C_d(R_c)$
FROM DIFFERENT VIEWS

VIEW 1 $\longrightarrow$ what we have done until now

$\boxed{C_0(R_c)}$ $\longrightarrow$ fit it to some quantity

perturbative case $\rightarrow$ $\frac{d\sigma}{d\Omega} \rightarrow |a_0|^2$

$$\frac{d\sigma}{d\Omega} = \left|\frac{\mu C_0}{2\pi}\right|^2 \qquad \boxed{C_0 = \frac{2\pi}{\mu} a_0}$$

scattering length

non-perturbative case $\Big) \rightarrow$ δ-shell + bound state

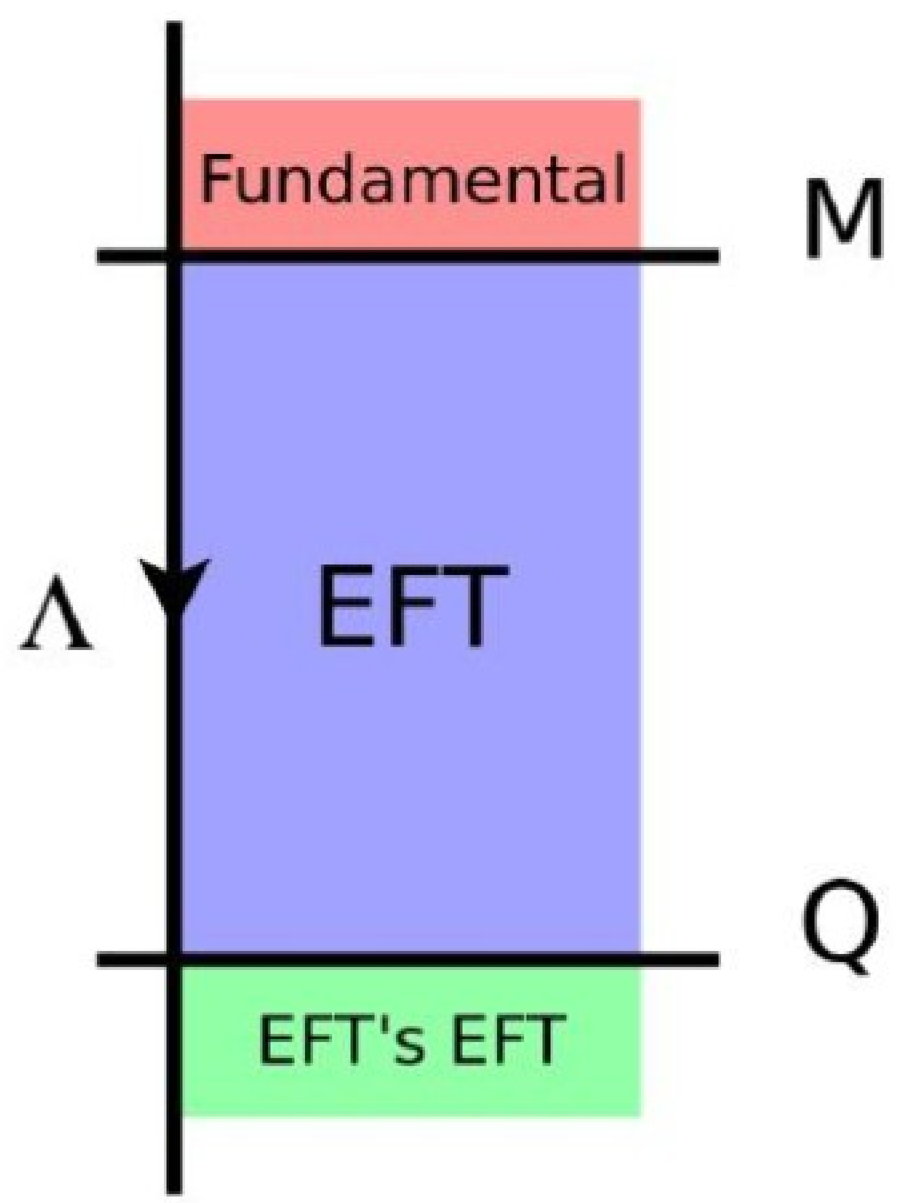
$$\frac{1}{C_0(R_c)} \rightarrow \frac{M}{2\pi}\left(\gamma - \frac{1}{R_c}\right)$$

$$\sum$$

→ most direct understanding

→ only requires to calculate things

View 2 → Renormalization group equation (RGE)



Physics is unique, but choice of theory depends on resolution Λ:

- $\Lambda \geq M$: Fundamental
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For equivalent descriptions:

$$\boxed{\frac{d}{d\Lambda}\langle\Psi|\mathcal{O}|\Psi\rangle = 0}$$

Renormalization group invariance

[Observables will be independent of R_c]

(more abstract)

WHAT IS REQUIRED FOR THIS VIEW?

2.a → Choice of regulator

2.b → Choice of observables

2.c → Assumption for the wave function

(2.a) $\rightarrow$ $V_c(r) = C_0(R_c) \frac{\delta(r-R_c)}{4\pi R_c^2}$

(2.b) $\rightarrow$ $\hat{O}$ = (scattering amplitude / Hamiltonian)

Short-cut: use the potential itself

(V is not an observable, at least in QFT)

(not totally correct, but it's good enough)

2.2 → wave function

perturbative system → free wave function

$$\langle \vec{r} | \psi \rangle = e^{i\vec{k}\cdot\vec{r}}$$

non-perturbative system → bound wave function

$$\langle \vec{r} | \psi \rangle = \frac{\Delta_r}{\sqrt{4\pi}} \frac{e^{-\gamma r}}{r}$$

perturbative $\rightarrow$ $\langle \psi | V_c | \psi \rangle \simeq C_0(R_c)$

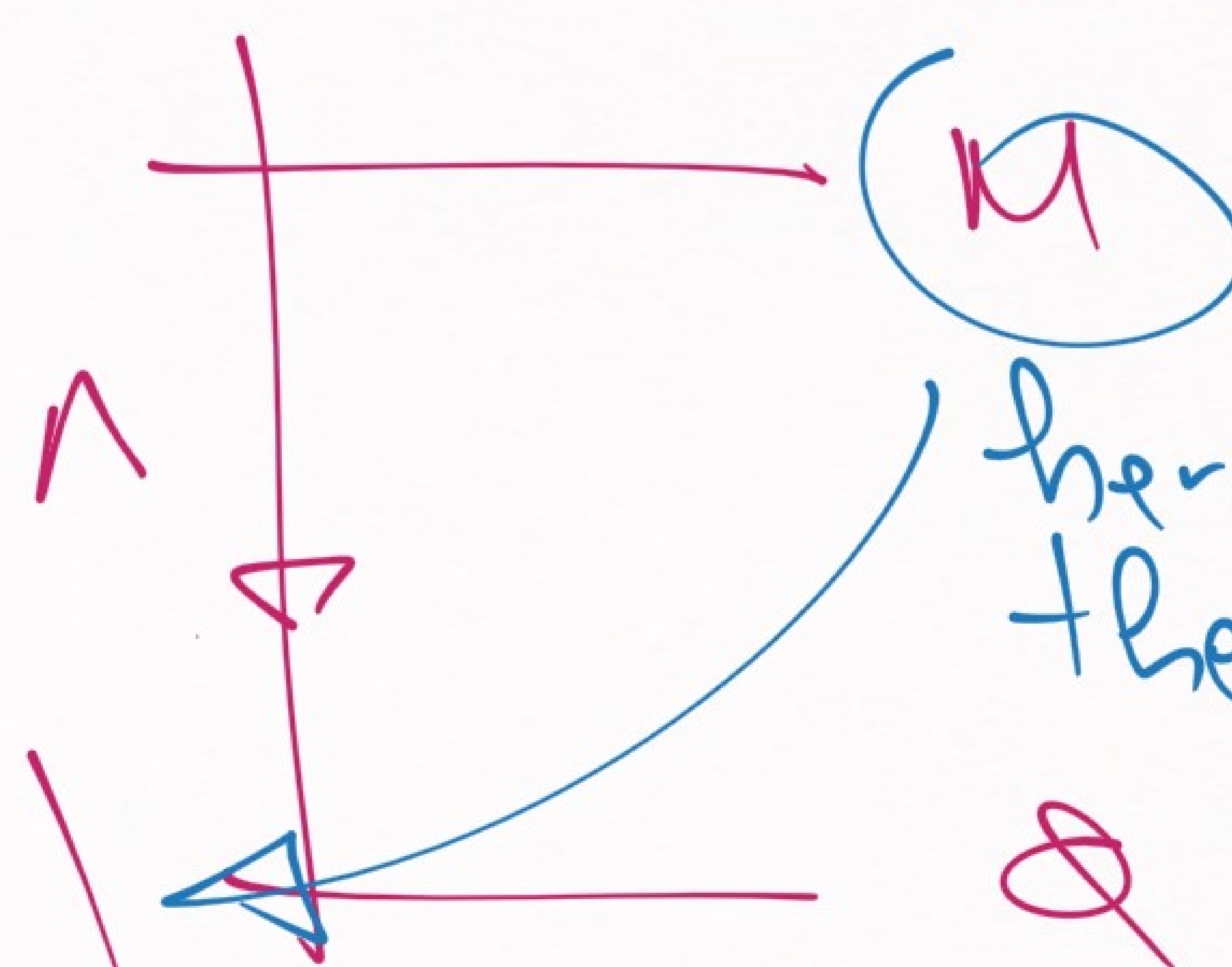
$$\frac{d}{dR_c}\langle \psi | V_c | \psi \rangle = \frac{d}{dR_c}\left[C_0(R_c)\right] = 0$$

$\rightarrow$ [100% compatible w/ what we know]

[Yukawa example] $\rightarrow$ $C_0(R_c) = -\frac{g_y^2}{m^2}\left(1 + \mathcal{O}(R_c m)\right)$

$$\frac{d}{dR_c}\left[C_{2n}(R_c)\right]=0 \Rightarrow C_{2n}(R_c)\sim \text{constant}$$

$$[C_{2n}]\sim\frac{1}{E^{2n+2}}$$



$$\boxed{C_{2n}(\Lambda\sim M)\sim\frac{1}{M^{2n+2}}}$$

here is where the RGE begins

RGE: $M \to \mu$

$$C_{2n} \sim \frac{1}{M^{2n+2}}$$ $\longrightarrow$ what we already knew

non-perturbative case

$$\langle \psi | V_c | \psi \rangle \sim \frac{C_{2n}(R_c)}{R_c^{\,2}}$$

$$\longrightarrow \quad \frac{d}{dR_c}\langle \psi | V_c | \psi \rangle \simeq \frac{d}{dR_c}\left[\frac{C_{2n}}{R_c^{\,2}}\right] = 0$$

$$C_{2n}(\Lambda \sim M) \sim \frac{1}{M^{2n+2}}$$

$$\frac{d}{d\Lambda}\left[\Lambda^2 C_{2n}(\Lambda)\right]$$

$$C_{2n}(\Lambda \sim Q) \sim \frac{1}{M^{2n}} \frac{1}{Q^2} \rightarrow \boxed{\text{what we already knew}}$$

(page 27)

Only tricky part $\rightarrow$ $\frac{d}{d\Lambda}\langle\psi|V_c|\psi\rangle = 0$

$$\boxed{\begin{gathered} C_0 \simeq \frac{1}{MQ} \\ \frac{d}{d\Lambda}\left[\Lambda C_0(\Lambda)\right] \simeq 0 \end{gathered}}$$

what we have seen

$\Downarrow$

$$C_0 \simeq \frac{1}{Q^2}$$

$$\frac{d}{d\Lambda}\left[\Lambda^2 C_0(\Lambda)\right] \simeq 0$$

$\Rightarrow$ $\boxed{\text{REASON}}$ $\longrightarrow$ approximations

$\boxed{\text{why } O(\Lambda) \text{ fails}}$

$V_c \rightarrow$ not observable

$$\boxed{\frac{d}{d\Lambda}\langle\psi|V_c|\psi\rangle \simeq 0} \longrightarrow \boxed{\frac{d}{d\Lambda}\langle\psi|H|\psi\rangle \simeq 0}$$

However, $C_{2n}(\Lambda)$ for $n \geq 1$ works!

$$H = T + V = \underbrace{\boxed{T + V_0}}_{\text{non-perturbative}} + \delta V \longrightarrow \text{perturbative}$$

$$\langle\psi|H|\psi\rangle = \langle\psi|(T+V_0)|\psi\rangle + \langle\psi|\delta V|\psi\rangle$$

$$\frac{d}{d\Lambda}(\cdots) = 0 \;\Leftarrow\; \frac{d}{d\Lambda}\langle\psi|T+V_0|\psi\rangle = 0$$

$$\quad \& \; \frac{d}{d\Lambda}\langle\psi|\delta V|\psi\rangle = 0$$

Outcome $\rightarrow$ $$\boxed{\frac{d}{d\lambda}\langle\psi|\delta V|\psi\rangle = 0}$$

this is a good approx.

(provided it's a perturbative potential)

View ② example

not easy to read

3. **A Renormalization group treatment of two-body scattering**
(196) Michael C. Birse, Judith A. McGovern, Keith G. Richardson (Manchester U.). Jul 1998. 4 pp.
Published in **Phys.Lett. B464 (1999) 169-176**
MC-TH-98-11
DOI: 10.1016/S0370-2693(99)00991-0
e-Print: **hep-ph/9807302** | **PDF**

References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service

Detailed record - Cited by 196 records 100+

$$\boxed{\frac{d}{d\Lambda}\langle\psi|\hat{O}|\psi\rangle = 0}$$

$\boxed{\text{View } 3}$ $\longrightarrow$ directly counting powers of Q & M

Example

$\frac{d\sigma}{d\Omega} \to |a_0|^2$

$f(\vec{k}) = -\frac{M}{2\pi}\int d^3\vec{r}\, V(\vec{r})\, e^{-i\vec{k}\vec{r}} + \mathcal{O}(V^2)$

$\frac{d\sigma}{d\Omega} = |f(\vec{k})|^2$

$f(\vec{k}) \xrightarrow[k \to 0]{} -a_0$

$\rightarrow \boxed{a_0 \sim \frac{1}{M}} \longrightarrow$ for natural problems

$$C_0 = \frac{2\pi}{\mu} a_0 = \begin{cases} \mu \sim M \\ a_0 \sim \frac{1}{M} \end{cases} \sim \frac{1}{M^2}$$

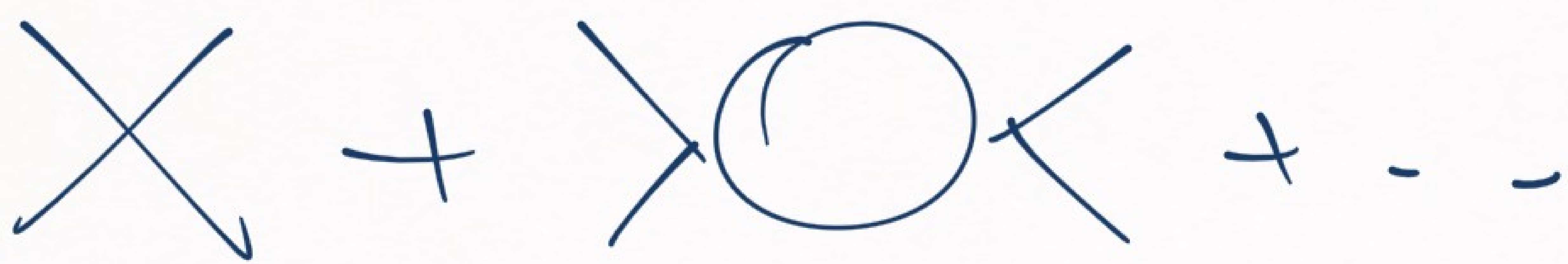
WHAT ABOUT NON-PERTURBATIVE PHYSICS?

→ Diagrammatic representation of Schrödinger equation

→ this has to be justified in terms of power counting

$$\mathcal{O}\left(\overline{\underline{V}}\right) = \mathcal{O}\left(\overline{\underline{V \quad V}}\right)$$

If you don't know about this, don't panic



$$C_0 + \underbrace{C_0^2 J_0(k)}_{\text{loop function}} + \dots$$

$$J_0(k) = \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{1}{E - \frac{\vec{q}^{\,2}}{2\mu}} f^2\left(\frac{\vec{q}}{\Lambda}\right)$$

$$\boxed{\mathcal{I}_0(k) \sim MQ}$$

$$\parallel$$

$$\int \frac{d^3\vec{q}}{(2\pi)^3} \frac{1}{E - \frac{\vec{q}^2}{2\mu}} = \left\{ \begin{array}{l} \vec{q} \sim Q \\ \vec{k} \sim Q \\ \mu \sim M \end{array} \right\} \sim \frac{Q^3}{Q^2} M$$

$$\sim MQ$$

$$E = \frac{k^2}{2\mu}$$

$$C_0 \sim C_0^2 J_0 \sim M\alpha\, C_0^2$$

$$1 \sim M\alpha\, C_0$$

$$\boxed{C_0 \sim \frac{1}{M\alpha}}$$

This should be the counting for non-perturbative systems

This type of arguments are used here

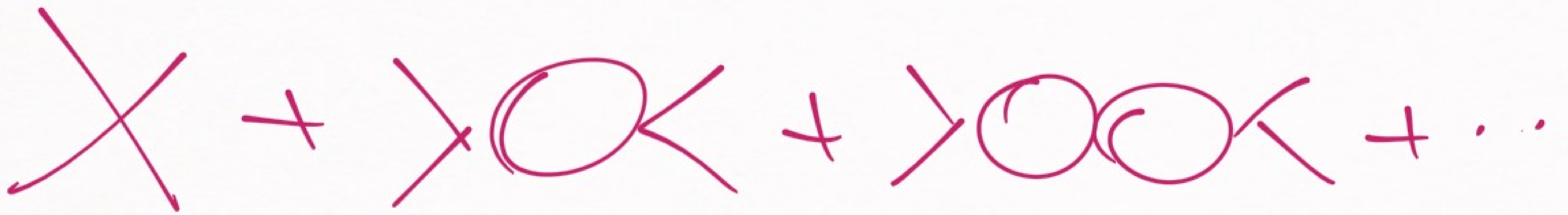
3. **Effective field theory of short range forces**
U. van Kolck (Caltech, Kellogg Lab & Washington U., Seattle). Aug 1998. 38 pp.
Published in **Nucl.Phys. A645 (1999) 273-302**
KRL-MAP-230, NT-UW-98-01
DOI: 10.1016/S0375-9474(98)00612-5
e-Print: **nucl-th/9808007** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record - Cited by 331 records 250+

↳ Easy to follow if you understand QM well both in r-space & p-space

$\boxed{\text{VIEW } 4}$ $\longrightarrow$ divergencies / infinities

(usual view you see in old QFT textbooks)

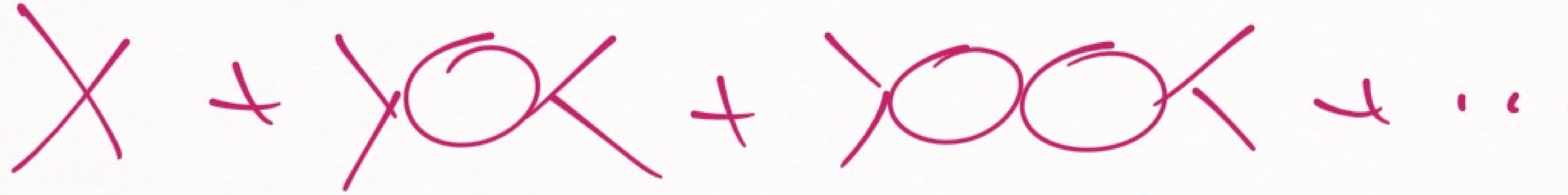
You have to renormalize to get rid of infinities



$$f(\vec{k}) = -\frac{\mu}{2\pi}\Big[\int d^3\vec{r}\, V(\vec{r})\, e^{i\vec{k}\cdot\vec{r}}$$

$$+ \int d^3\vec{r}\, d^3\vec{r}\,'\, V(\vec{r})\, V(\vec{r}\,')\, e^{-i\vec{k}\cdot\vec{r}}\, e^{i\vec{k}\cdot\vec{r}\,'}\, G(\vec{r}, \vec{r}\,') + \ldots\Big]$$

Green function
(loop function)



$$C_0 + C_0^2 I_0(k;\Lambda) + C_0^3 I_0^2(k,\Lambda) + \cdots$$

$$I_0(k,\Lambda) = \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{\Theta(\Lambda - |\vec{q}|)}{\frac{k^2}{2\mu} - \frac{q^2}{2m}} \simeq \frac{M}{2\pi}(\Lambda + ik) + \ldots$$

$\rightarrow \infty$

(for $\Lambda \rightarrow \infty$)

$$C_0 + C_0^2 I_0 + C_0^3 I_0^2 + \dots$$

$$\sim \qquad \sim \qquad \sim$$

$$C_0 \qquad \underbrace{C_0^2(\Lambda + ik)}_{\Lambda \text{ divergence}} \qquad \underbrace{C_0^3(\Lambda + ik)^2}_{\Lambda^2 + 2ik\Lambda}$$

→ absorbed in δC_0 $\qquad$ $\overline{C_2}$

→ add new contacts as to absorb new divergences

→ what determines power counting in this view

$\boxed{\text{VIEW (5)}}$ ⟶ RESIDUAL CUTOFF DEPENDENCE

$\frac{d\sigma}{d\Omega}$ ⟶ $V_c(\vec{r}) = C_0 \frac{\delta(r - R_c)}{4\pi R_c^2}$

$\rightarrow \quad f(\vec{k}) = -\frac{\mu}{2\pi} C_0(R_c) \left[\frac{\sin(kR_c)}{kR_c} \right]^2 \Longrightarrow ⊗$

$$\circledast \Rightarrow \frac{d}{dR_c}\rho(k) \neq 0$$

$$\frac{d}{dR_c}\rho(k) = 0 + \underbrace{\mathcal{O}\left((gR_c)^2\right)}$$

you may want to remove this $\rightarrow$ new counterterms / couplings

$$f(x) = -\frac{\mu}{2\pi} C_0(R_c)\left[1 - \frac{1}{3}(gR_c)^2 + O((gR_c)^4)\right]$$

If I add a $C_2(R_c)\, g^2$ term

$\Rightarrow$ I can remove

this $(gR_c)^2$ term

in $f(x)$

I construct the expansion as to gradually remove the higher order cutoff dependence

Σ → NN power counting using same arguments from residual cutoff dependence

1. **Renormalizing Chiral Nuclear Forces: Triplet Channels**
(68) Bingwei Long (Jefferson Lab), C.J. Yang (Arizona U. & Ohio U., Inst. Nucl. Part. Phys.). Nov 2011. 20 pp.
Published in **Phys.Rev. C85 (2012) 034002**
JLAB-THY-11-1464, INT-PUB-11-038
DOI: 10.1103/PhysRevC.85.034002
e-Print: **arXiv:1111.3993** **[nucl-th]** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service; OSTI.gov Server; JLab Document Server
Detailed record - Cited by 68 records 50+

$\boxed{\text{VIEW } 6}$ → matrix elements of short range potential

$V_S(r)$ → true short-range potential

$\boxed{\langle \Psi_L | V_S | \Psi_L \rangle}$ → how does this look like?

Ψ_L → long-range wave function

$$\langle \psi_L | V_S | \psi_L \rangle \begin{cases} 1)\ V_S \\ 2)\ \psi_L \end{cases}$$

how they scale

$V_S \rightarrow$ example

$$\boxed{V_Y(r)} = -\frac{g^2}{4\pi}\frac{e^{-mr}}{r} = \circledcirc$$

$$\circledcirc = \boxed{m\, f(mr)}$$

$$V_S(r) = M\, f(Mr)$$

M S a

$$f(x) = -\frac{g^2}{4\pi}\frac{e^{-x}}{x} \quad \text{(dim less)}$$

$\psi_L \rightarrow$ $\langle \vec{r} | \psi_L \rangle = e^{i\vec{k}\cdot\vec{r}}$

$\psi_L \sim e^{iQr}$

~ 1

$|e^{iQr}| \sim 1$

$$\langle \vec{r} | \psi_L \rangle = \frac{A_s}{\sqrt{4\pi}} \frac{e^{-\gamma r}}{r}$$

$$\sim (\#) \frac{e^{-Qr}}{(Qr)}$$

$$\sim (Qr)^{-1}$$

$$\boxed{\psi_L \sim (Qr)^n}$$

6.a) $V_S(r) \sim M \rho(Mr)$

6.b) $\psi_L(r) \sim (Qr)^n$

$\rightarrow$ $\boxed{\text{how does } \langle \psi_L | V_S | \psi_L \rangle \text{ scales?}}$

$$\langle \psi_L | V_S | \psi_L \rangle \sim \left(\frac{Q}{M}\right)^{2n} \underbrace{J_S(2n)}_{\text{dimless}} \frac{1}{M^?}$$

$$J_S(k) = \int_0^\infty dx\, \rho(x)\, x^k$$

$$\langle \psi_L | V_S | \psi_L \rangle \sim \left(\frac{Q}{M}\right)^{2n} I_S(2n) \frac{1}{M^2}$$

scaling of short-range potential in a given system

perturbative $\rightarrow$ $n=0$ $\rightarrow$ $\langle \psi_L | V_S | \psi_L \rangle \sim \frac{1}{M^2}$

short-range physics has its expected importance non-perturbative $\rightarrow$ $\boxed{n=-1}$

$$\langle \psi_L | V_S | \psi_L \rangle \sim \left(\frac{M}{a}\right)^2 \frac{1}{M^2}$$

$\rightarrow$ short-range physics are enhance by a factor of $(M/a)^2$

(caveat: $\langle \phi | V | \psi \rangle$ vs $\langle \psi | H | \psi \rangle$)

→ this reproduces what we obtain

from $\frac{\partial}{\partial a}\langle \psi | V | \psi \rangle = 0$

$\boxed{\text{BOTTOM-LINE}}$ → a lot of ways to understand EFT & Renormalization

FRIDAY $\rightarrow$ ISOSPIN, SU(3),

CHIRAL SYMMETRY