NUCLEAR PHYSICS 8

Effective (field) theories

When using QFT

$\boxed{\text{RECAP}}$

Basic idea about Renormalization:

"Physics at long distances does not depend on the short distance details"

$\boxed{\text{Implications}} \Rightarrow$

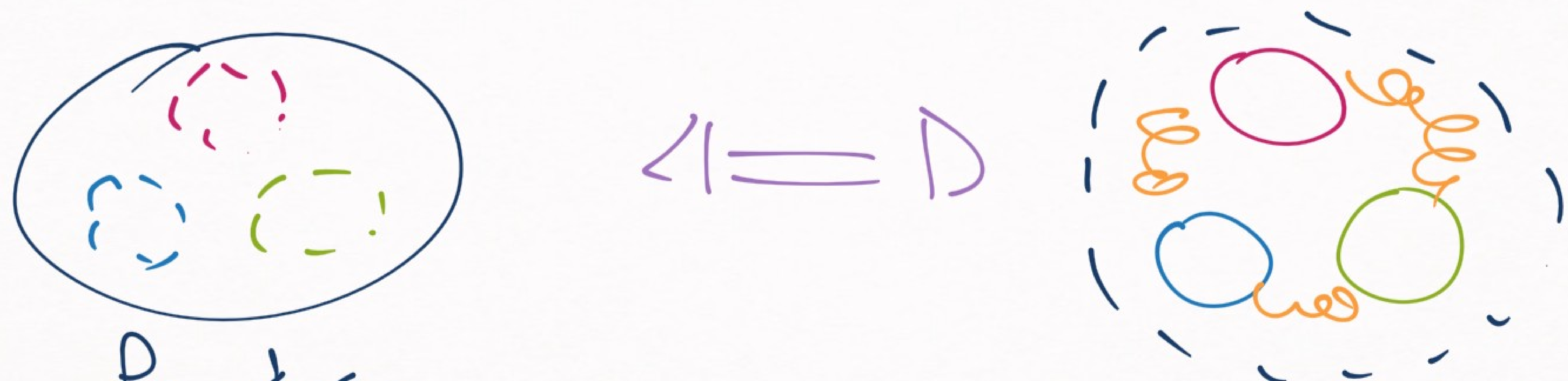
$\Rightarrow$ 1 $\exists$ of different fields of physics
(or different scientific disciplines)

general relativity / classical physics / quantum mechanics

(study them independently)

atomic physics / nuclear physics / particle physics

2



hadron (baryon/meson) $\Longleftrightarrow$ quarks & gluons

different understandings of nature depending on scale (but approx. independent)

$\boxed{\text{TOY EXAMPLE}}$ → TEACUPS & TEAPOTS

"Short-distance description"

"Long-distance description"

"Short-distance description"

→ Fourier equation for heat conduction

→ Equations for convection

⇝ Detailed & difficult description

"Long-distance description" ("EFT" approach)

SERIES OF STEPS:

1) Some dynamics Newton's law of cooling

$$\Delta T(t) = (T - T_{ext})(t) = (T_0 - T_{ext})\, e^{-\lambda(t-t_0)}$$

2) Find the relevant degrees of freedom (d.o.f.)

$S, \Sigma \rightarrow$ types of surface (liquid/solid)

3) Find a small expansion parameter

$$x = \frac{\Sigma}{\rho S}$$ (ρ is a numerical factor, probably $\rho \gg 1$)

$x \ll 1$ (or maybe just $x < 1$)

4) Write down the theory as an expansion

$$\lambda = S\left(c_0 + c_1 x + c_2 x^2 + \ldots\right)$$

low energy constants (LECs)

5) Choose some accuracy

$$\underbrace{\mathcal{O}(x)}_{\lambda = c_0 S + \mathcal{O}(x)},\ \mathcal{O}(x^2),\ \underbrace{\mathcal{O}(x^3)}_{\lambda = S(c_0 + c_1 x + c_2 x^2 + \mathcal{O}(x^3))},\ \ldots$$

6) Fit the LECs

to experimental data (or, if lucky,

$\Rightarrow$ deduce them from theory)

$\Rightarrow$ $\boxed{\text{EFT algorithm}}$

$\rightarrow$ Basic idea is simple

$\rightarrow$ Implementation could be complex

EFT $\Rightarrow$ $\exists$ expansion

$\Rightarrow$ (expansion is not unique)

Example of different expansions → CUP & POT

$$x = \frac{\Sigma}{\rho \zeta}$$

$x < 1$

$$\lambda_{cup} = \zeta \left(c_0 + c_1 x + c_2 x^2 + \dots \right)$$

→ Different expansions

$x > 1$

$$\lambda_{pot} = \Sigma \left(d_0 + \frac{d_1}{x} + \frac{d_2}{x^2} + \dots \right)$$

$\to$ $\exists$ more than one possible expansion

(we choose the best one)

$\Downarrow$

$\boxed{\text{TECHNICAL NAME}}$ $\longrightarrow$ $\boxed{\text{POWER COUNTING}}$

$\to$ count powers of $\underset{\sim}{x}$

$\to$ decide type of expansion

$\rightsquigarrow$ Power counting

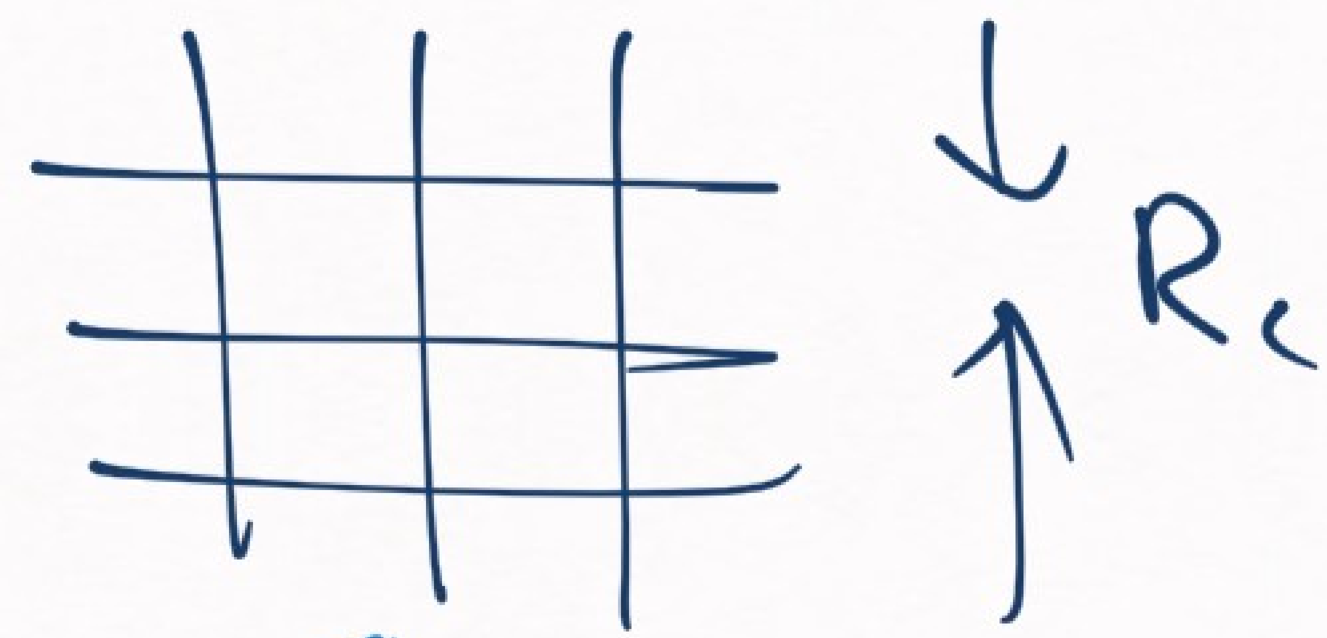
$\rightsquigarrow$ Universality classes

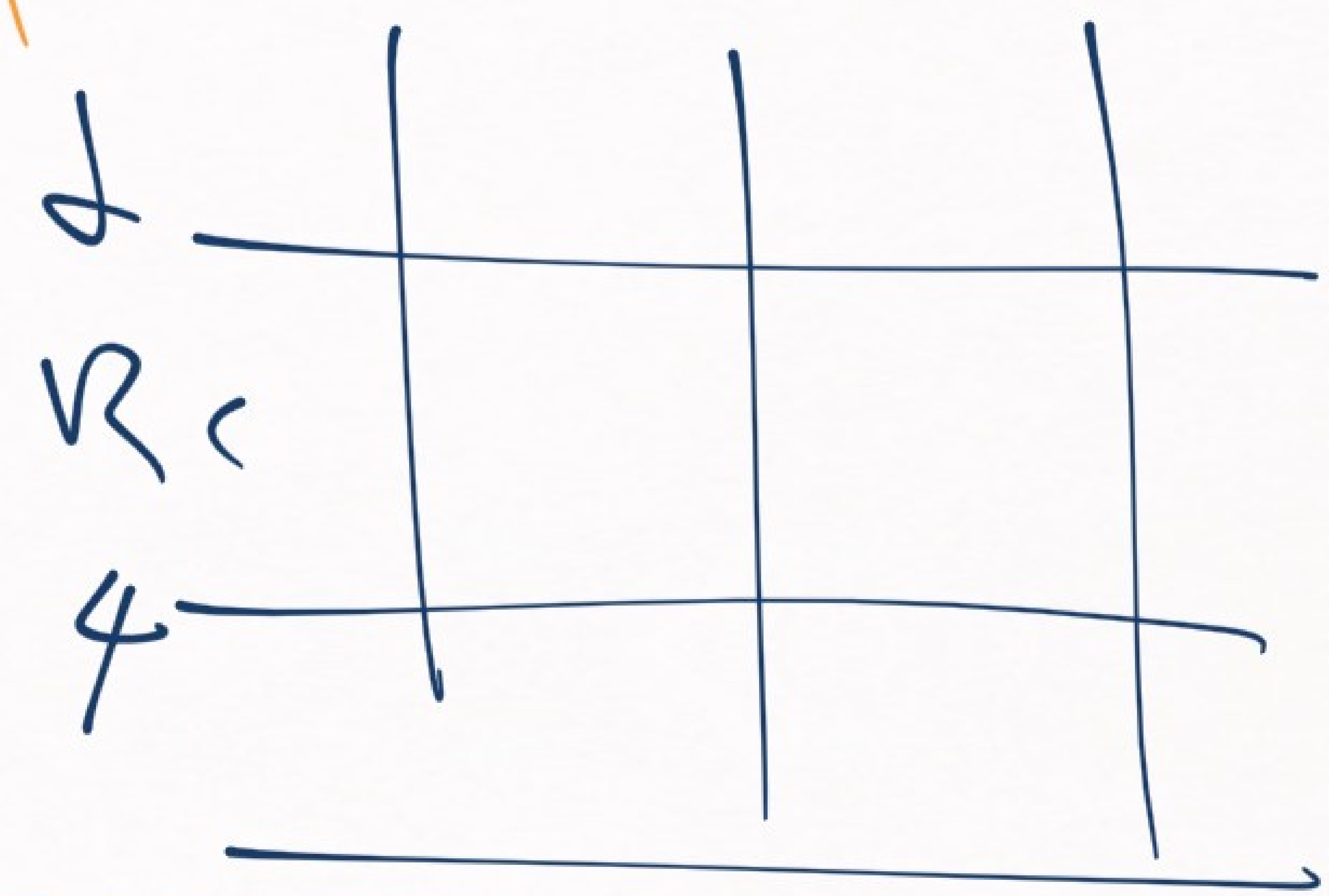
$\rightsquigarrow$ Infrared fixed points

Weird language of EFTs and RGE

$\boxed{\text{Two important concepts!}}$

Resolution scale





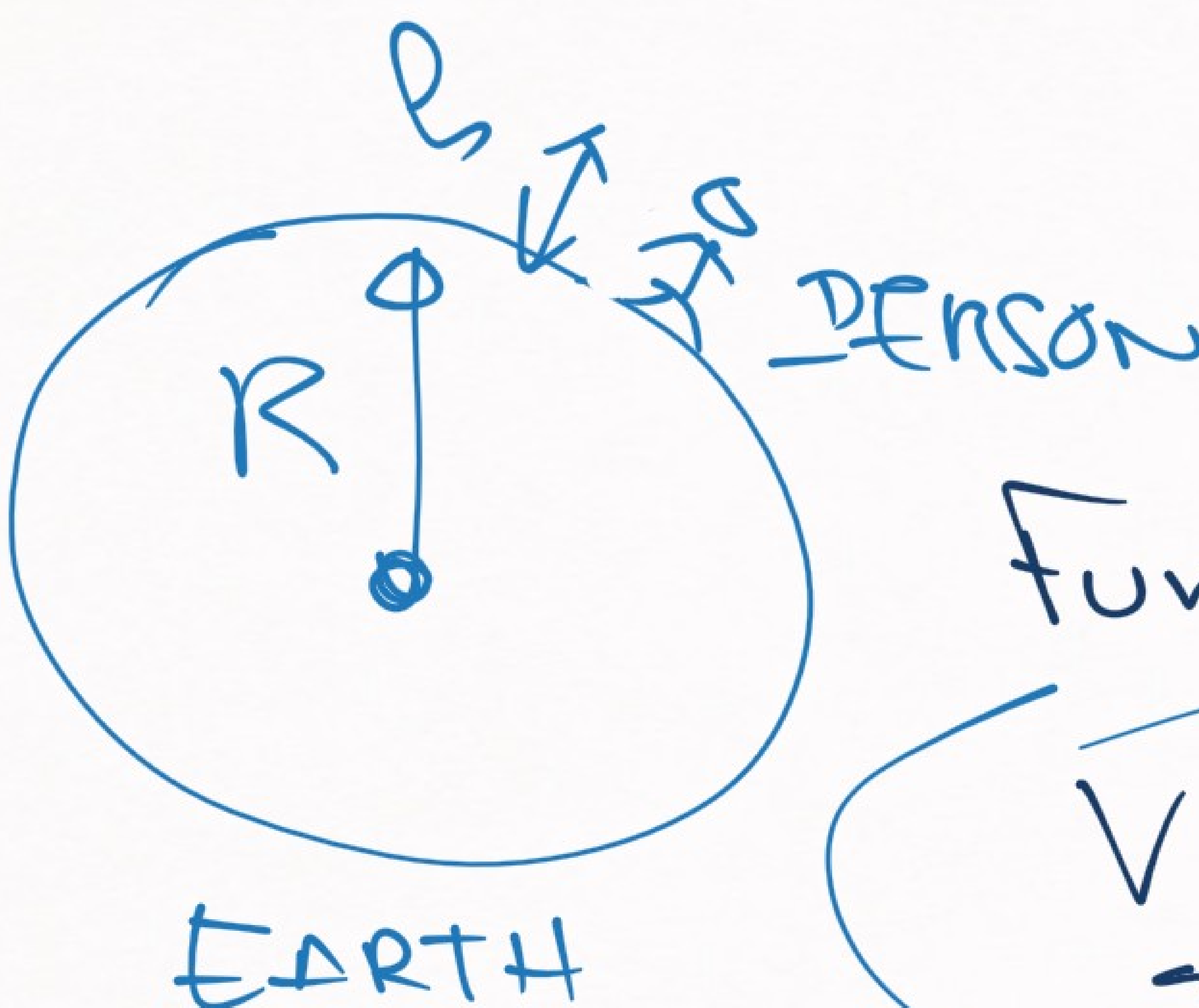
Infrared limit $\rightarrow$ $R_c \rightarrow \infty$

Ultraviolet limit $\rightarrow$ $R_c \rightarrow 0$

→ GENERAL IDEA OF EFT

Another easy example:

LECs



$\underline{\underline{R \gg h}}$

$$\Delta V_g = mgh\left(1 + c_1 \frac{h}{R} + c_2\left(\frac{h}{R}\right)^2 + \dots\right)$$

Fundamental theory:

$$V_g = -\frac{GmM}{R+h}$$ → solvable

SELECTED PAPERS ABOUT RENORMALIZATION

2. **The Renormalization group and the epsilon expansion**
(2700) K.G. Wilson (Princeton, Inst. Advanced Study & Cornell U., LNS), John B. Kogut
Published in **Phys.Rept. 12 (1974) 75-199**
DOI: 10.1016/0370-1573(74)90023-4
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record - Cited by 2700 records 1000+

→ classical manuscript (difficult to read unless you know a lot about solid state)

4. **Renormalization and Effective Lagrangians**
Joseph Polchinski (Harvard U.). Apr 1983. 27 pp.
Published in **Nucl.Phys. B231 (1984) 269-295**
HUTP-83-A018
DOI: 10.1016/0550-3213(84)90287-6
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
KEK scanned document
Detailed record - Cited by 1120 records 1000+

→ really good (background on ϕ^4 theory / basic QFT)

1. **Building light nuclei from neutrons, protons, and pions**
(45) Daniel R. Phillips (Ohio U.). Mar 2002. 54 pp.
Published in **Czech.J.Phys. 52 (2002) B49**
DOI: 10.1007/s10582-002-0079-z
To appear in the proceedings of Conference: C01-07-09.13 Proceedings
e-Print: **nucl-th/0203040** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record - Cited by 45 records

→ Good & easy (background: QM)

1. **Effective field theory for few nucleon systems**
(618) Paulo F. Bedaque (LBL, Berkeley), Ubirajara van Kolck (Arizona U. & RIKEN BNL). Mar 2002. 55 pp.
Published in **Ann.Rev.Nucl.Part.Sci. 52 (2002) 339-396**
DOI: 10.1146/annurev.nucl.52.050102.090637
e-Print: **nucl-th/0203055** | PDF
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record - Cited by 618 records 500+

→ Still easy (Review) (nuclear physics)

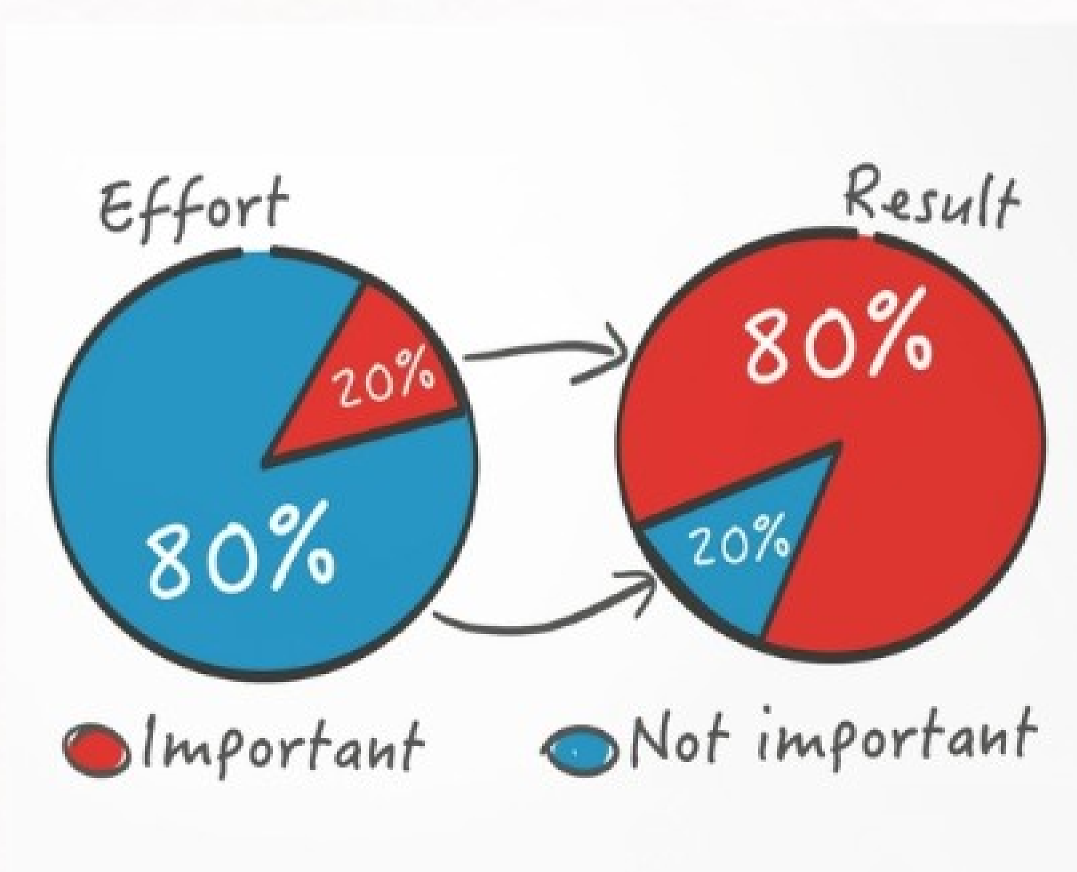
3. **A Renormalization group treatment of two-body scattering**
(196) Michael C. Birse, Judith A. McGovern, Keith G. Richardson (Manchester U.). Jul 1998. 4 pp.
Published in **Phys.Lett. B464 (1999) 169-176**
MC-TH-98-11
DOI: 10.1016/S0370-2693(99)00991-0
e-Print: **hep-ph/9807302** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record - Cited by 196 records 100+

→ Good (but difficult) very abstract (QM background)

Other "silly" examples of renormalization:

1) TEACUPS & TEAPOTS EXAMPLE

2) PARETO PRINCIPLE (a power counting argument in disguise)



" 20 % of effort gives you 80 % of the result "

$\lambda = 5\infty$

3) Art → IMPRESIONISMS

1) Big dots ($\Lambda \to Q$, $R_c \to \infty$)

2) No short-distance details (blobs of paints)

3) The result is great

→ $\boxed{\text{RENORMALIZATION IN QM \& NUCLEAR PHYSICS}}$

* We now know the basic ideas about renormalization
* We can try something more quantitative ⟹

$$\boxed{\frac{d}{dR_c}\langle\psi|\hat{O}|\psi\rangle = 0} \longrightarrow \text{Renormalization Group Equation (RGE)}$$

$\downarrow$

Find example(s)

Renormalization Group Equation (RGE)

$\downarrow$

Looks scary

(actually easy)

$\boxed{\text{QM PROBLEM}} \rightsquigarrow \boxed{\begin{array}{c}\text{TWO-BODY} \\ \text{PROBLEM} \\ \text{(Bound state)}\end{array}}$

$\left(\text{for } \frac{d}{dR_c}\langle\psi|\vec{O}|\psi\rangle = 0 \right) \leftarrow$

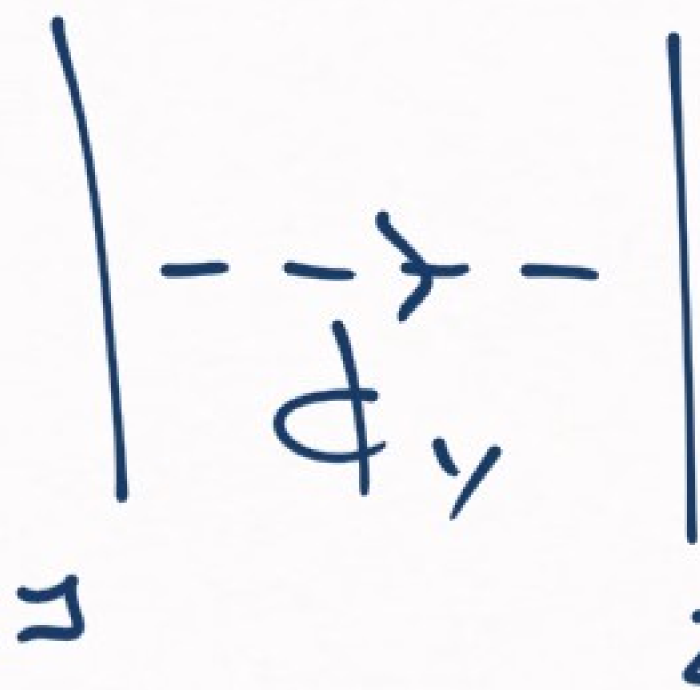
1) two-particles of mass M

2) potential (e.g. scalar boson of mass m)

$\longrightarrow$ Yukawa potential

3) $\left[-\frac{\vec{\nabla}^2}{M} + V(\vec{r}) \right] \psi(\vec{r}) = E\,\psi(\vec{r})$

$$V_y(\vec{r}) = -\frac{g_y^2}{4\pi}\frac{e^{-mr}}{r} \equiv$$



∃ condition for the existence of bound states w/ this potential:

$$\lambda = \frac{M}{m}\frac{g_y^2}{4\pi} \geq \lambda_c = 1.68\ldots$$

Two-body system:

1) $\lambda < \lambda_c$ → not enough attraction to form a bound state

2) $\lambda = \lambda_c$ → zero-energy bound state (unitary limit)

3) $\lambda > \lambda_c$ → binding

Solve numerically this sytem for $\boxed{\begin{aligned} M &= 1\,\text{GeV} \\ m &= 0.5\,\text{GeV} \end{aligned}}$

$\lambda > \lambda_c \Rightarrow E_B = -\frac{\gamma^2}{M}$

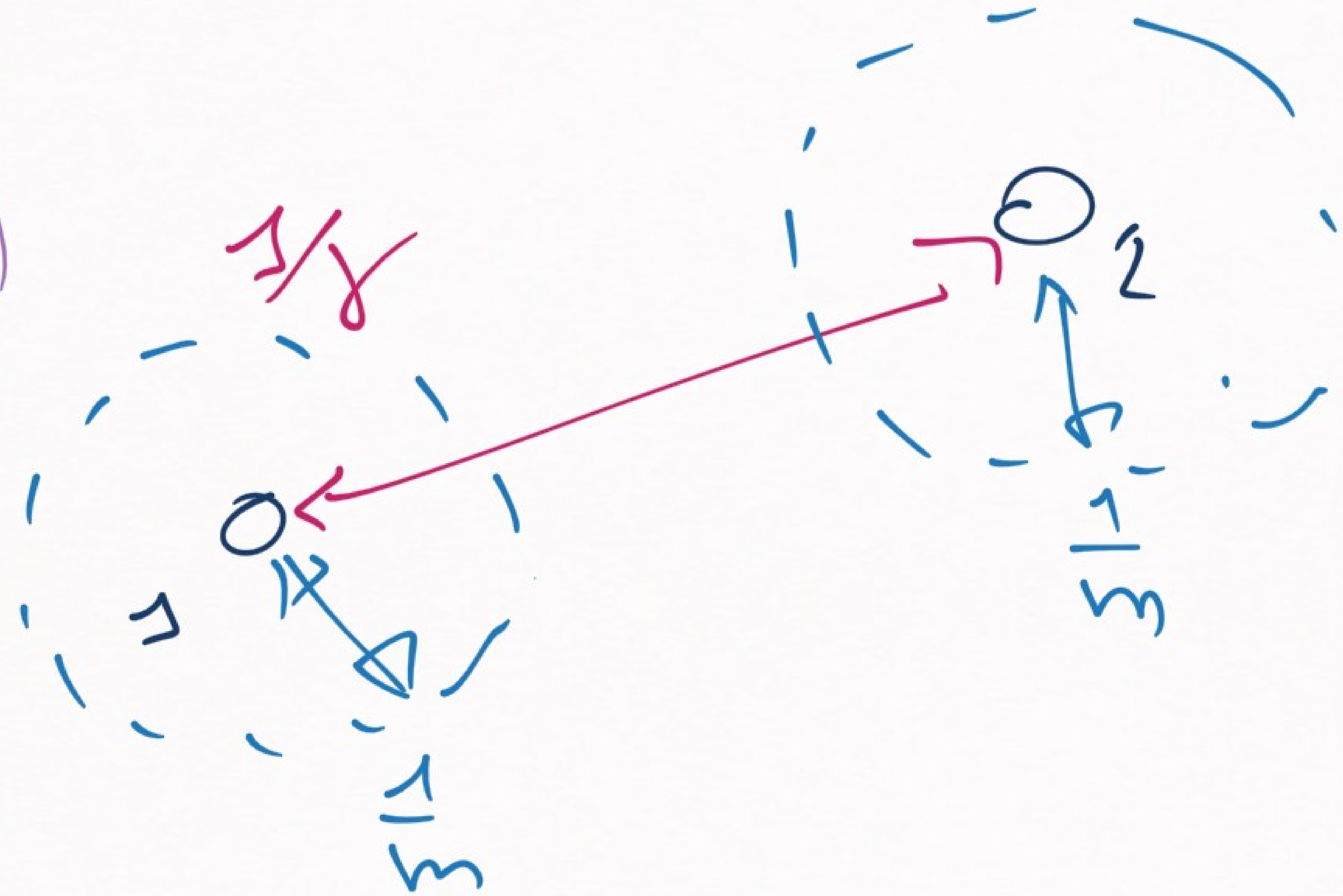
$\boxed{\gamma \to \text{wave number}}$

$\boxed{\frac{1}{\gamma} \sim \text{size of the bound state}}$

EFT

$\frac{1}{\gamma} > \frac{1}{m}$

$\boxed{\text{EFT}}$



$$\boxed{\frac{\gamma}{m} < 1}$$

Here size of bound state larger than range of the potential

[Results of numerics for V_Y :]

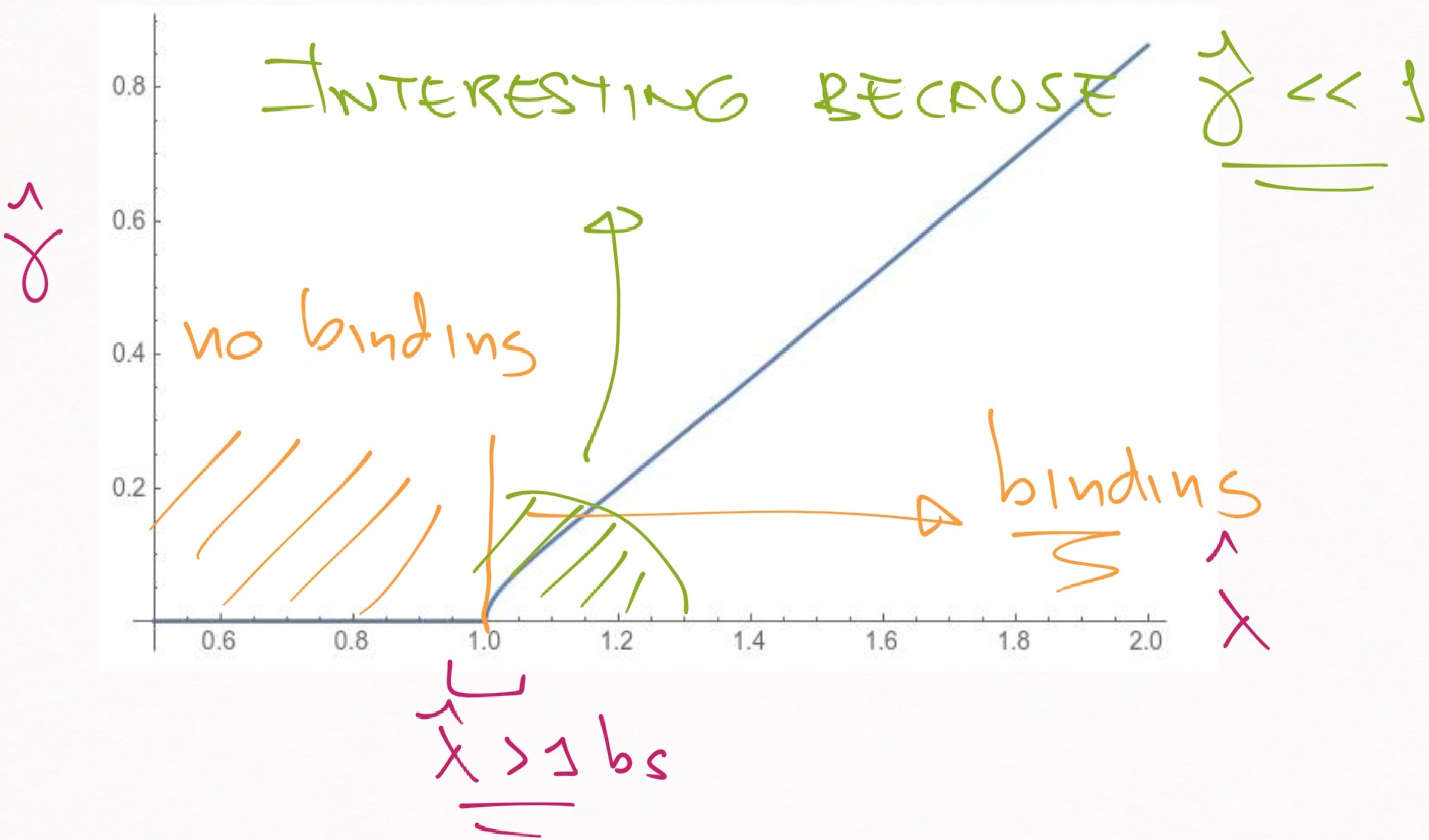
$$\gamma = \gamma(\lambda)$$

$$\lambda = \frac{M}{m}\frac{g_Y^2}{4\pi}$$

This is good, but it's even better to use rescaled qualities (dimensionless quantities)

$$\gamma \to \hat{\gamma} = \frac{\gamma}{m} \quad (\hat{\gamma} < 1 \to \text{EFT}) \qquad \hat{\lambda} = \frac{\lambda}{\lambda_c} \quad (\hat{\lambda} \geq 1 \text{ bound state})$$

Calculate numerically $\hat{\gamma} = \hat{\gamma}(\hat{\lambda})$

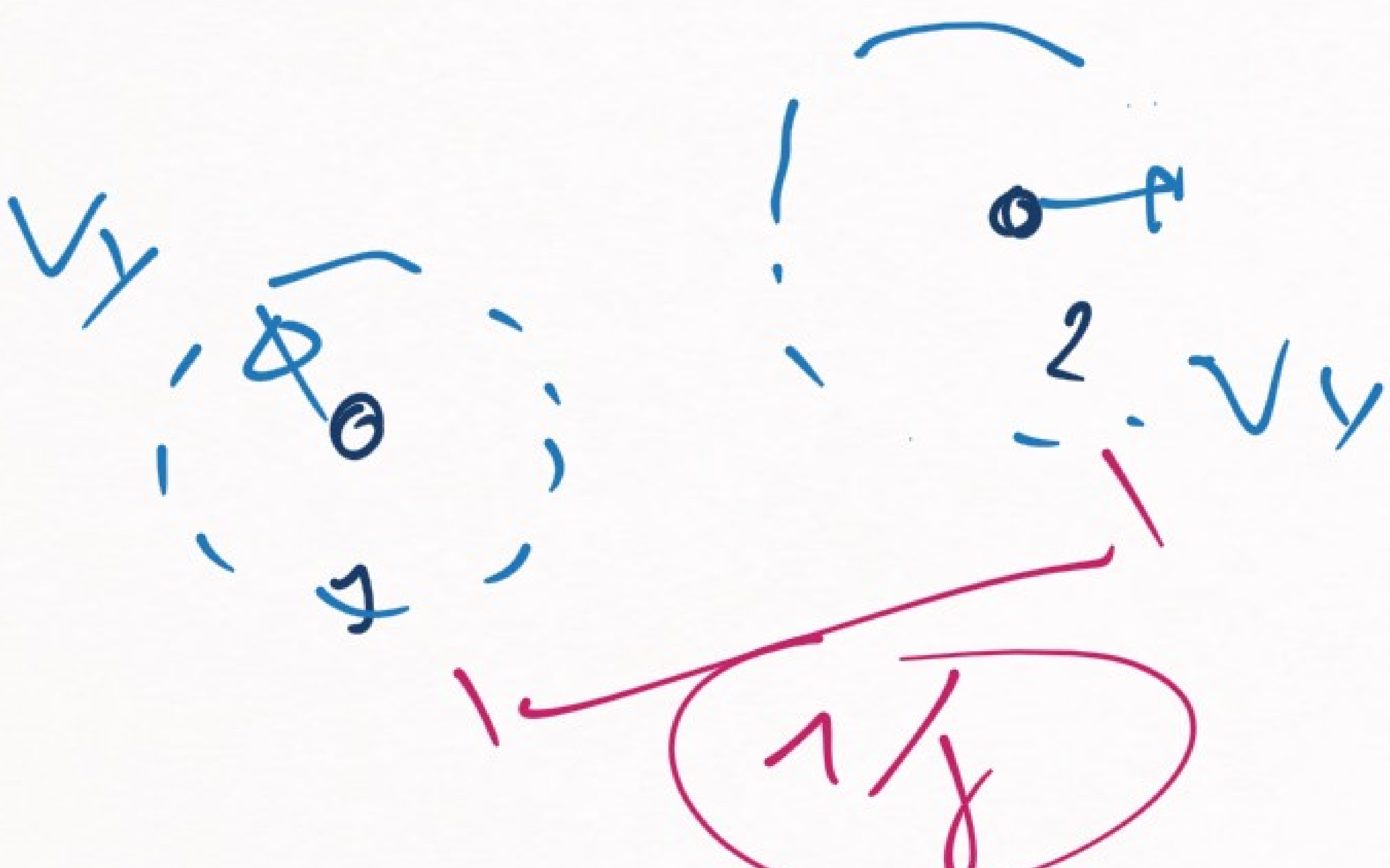


[WHY THIS IS AN INTERESTING $\underbrace{\text{REGION}}_{\hat{\lambda} = 1 + \delta}$?]

$(\delta > 0, \delta \ll 1)$

Reason:

If $\hat{\lambda} = 1 + \delta \Rightarrow \boxed{\gamma \ll 1}$



$\longrightarrow$ most of the wave function lies outside the range of the potential

→ Simplification of the wave function

If $mr \gg 1 \Rightarrow V_Y \simeq 0 \Rightarrow \boxed{\nabla^2 \psi = +\gamma_B^2 \psi}$

$$V_Y = -\frac{g^2}{4\pi} \frac{e^{-mr}}{r}$$

$$u(r) = A_s \, e^{-\gamma r}$$

$$\psi_B(\vec{r}) = \frac{1}{\sqrt{4\pi}} \frac{u(r)}{r} = \frac{A_s}{\sqrt{4\pi}} \frac{e^{-\gamma r}}{r}$$

$$\psi_B(\vec{r}) \xrightarrow[m|\vec{r}| \gg 1]{} \frac{1}{\sqrt{4\pi}} \frac{\Delta_s e^{-\gamma r}}{r} \quad \psi_L(\vec{r})$$

1) If $\gamma \sim m$, then ψ_L is only a small part of the wave function

$$P_S > P_L$$

$$\int |\psi_B(\vec{r})|^2 d^3\vec{r} = 1 = \int_{m|\vec{r}| \leq 1} |\psi_B(\vec{r})|^2 d^3\vec{r} + \int_{m|\vec{r}| > 1} |\psi_B|^2 d^3\vec{r}$$

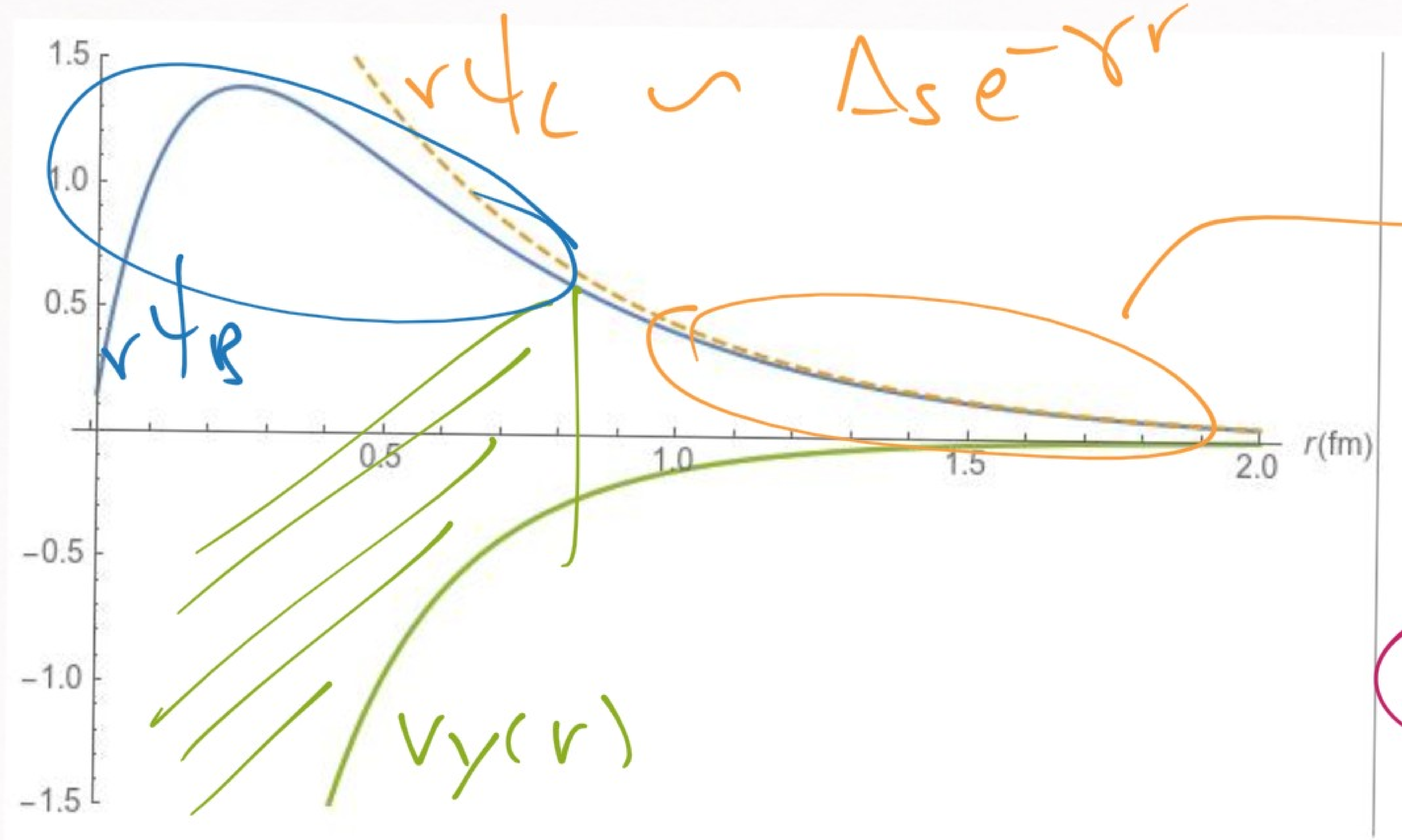
2) If $\gamma \ll m$, then most of the wf lives outside the range of potential

$$\int_{m|\vec{r}| \lesssim 1} |\psi_B(\vec{r})|^2 d^3\vec{r} \ll \int_{m|\vec{r}| > 1} |\psi_B(\vec{r})|^2 d^3\vec{r}$$

2) $\longrightarrow$ $\psi_B(\vec{r}) = \frac{1}{\sqrt{4\pi}} \frac{A_S e^{-\gamma r}}{r}$

good approx.

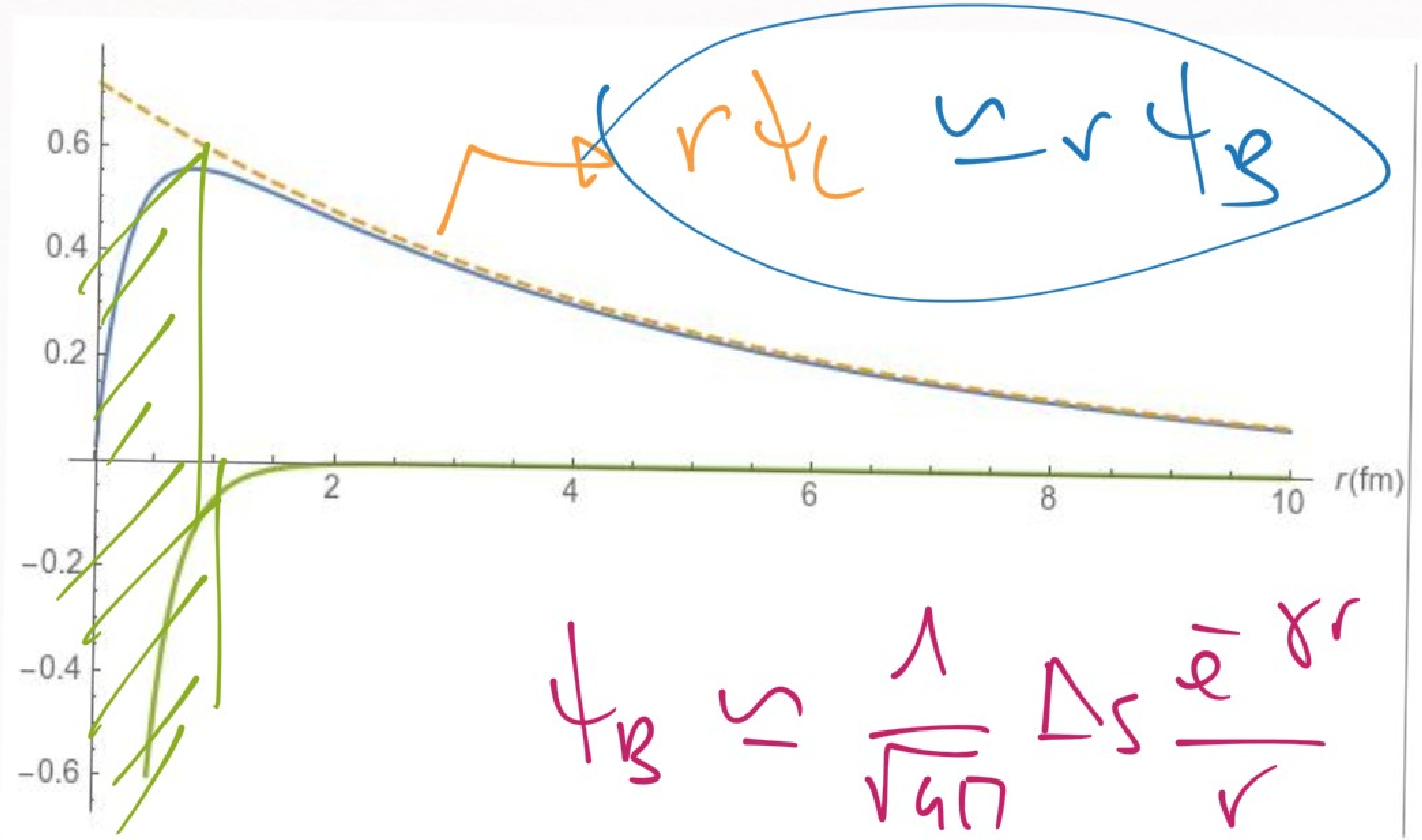
1) $\lambda = 2\lambda_c$ ($\hat{\lambda} = 2$), $\gamma = 0.86\, m$ ($\gamma \sim m$)



good approx. for $V_Y \simeq 0$

(I) → we really are sensitive to details in V_Y

2) $\lambda = 1.05\,\lambda_c$, $\gamma = 0.075\,m$



$r\psi_L \simeq r\psi_B$ → We don't need to know the details of the potential V_Y

$\psi_B \simeq \frac{1}{\sqrt{4\pi}} \Delta_S \frac{e^{-\gamma r}}{r}$

really good approximation

WHAT DO WE LEARN FROM THIS EXAMPLE :

1) Natural system ($\gamma \sim m$)

1.a) Wave function/potential $\rightarrow$ same size

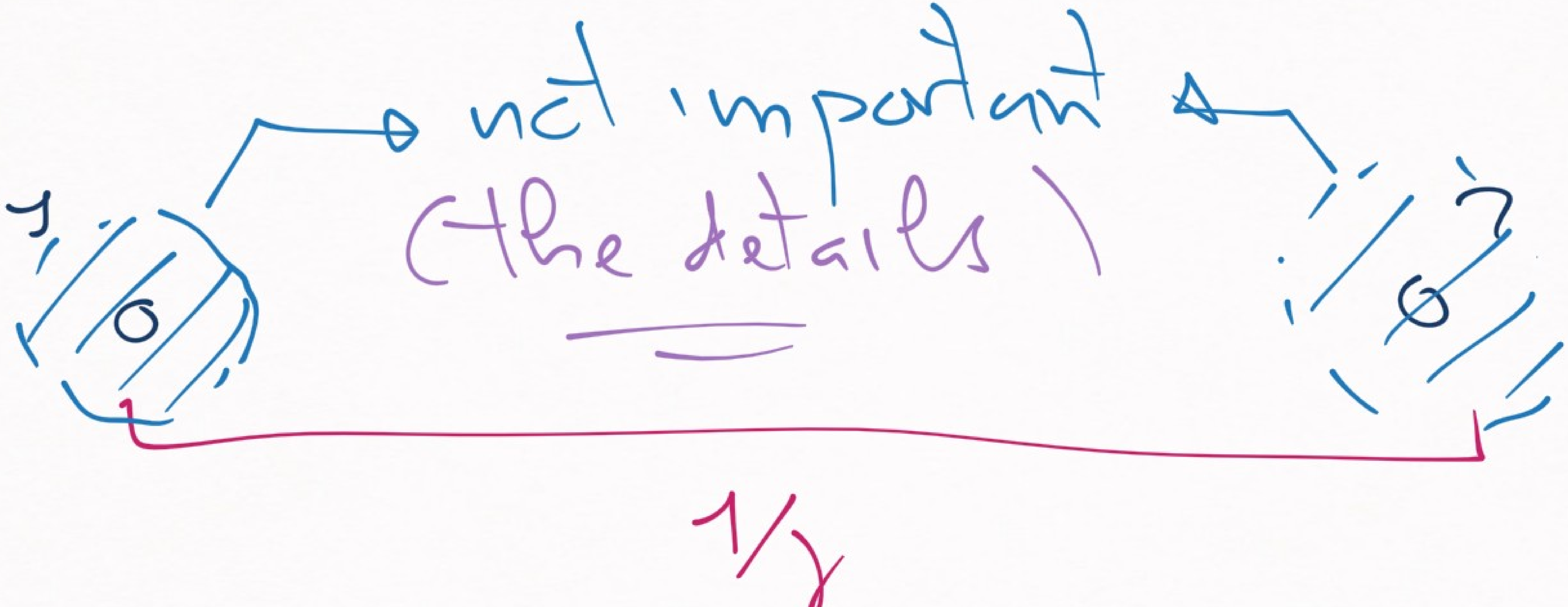
1.b) Form of wave function depends on the form of the potential

2) Unnatural system ($\gamma \ll m$)

2.a) wf $\gg$ potential $\rightarrow$ much larger

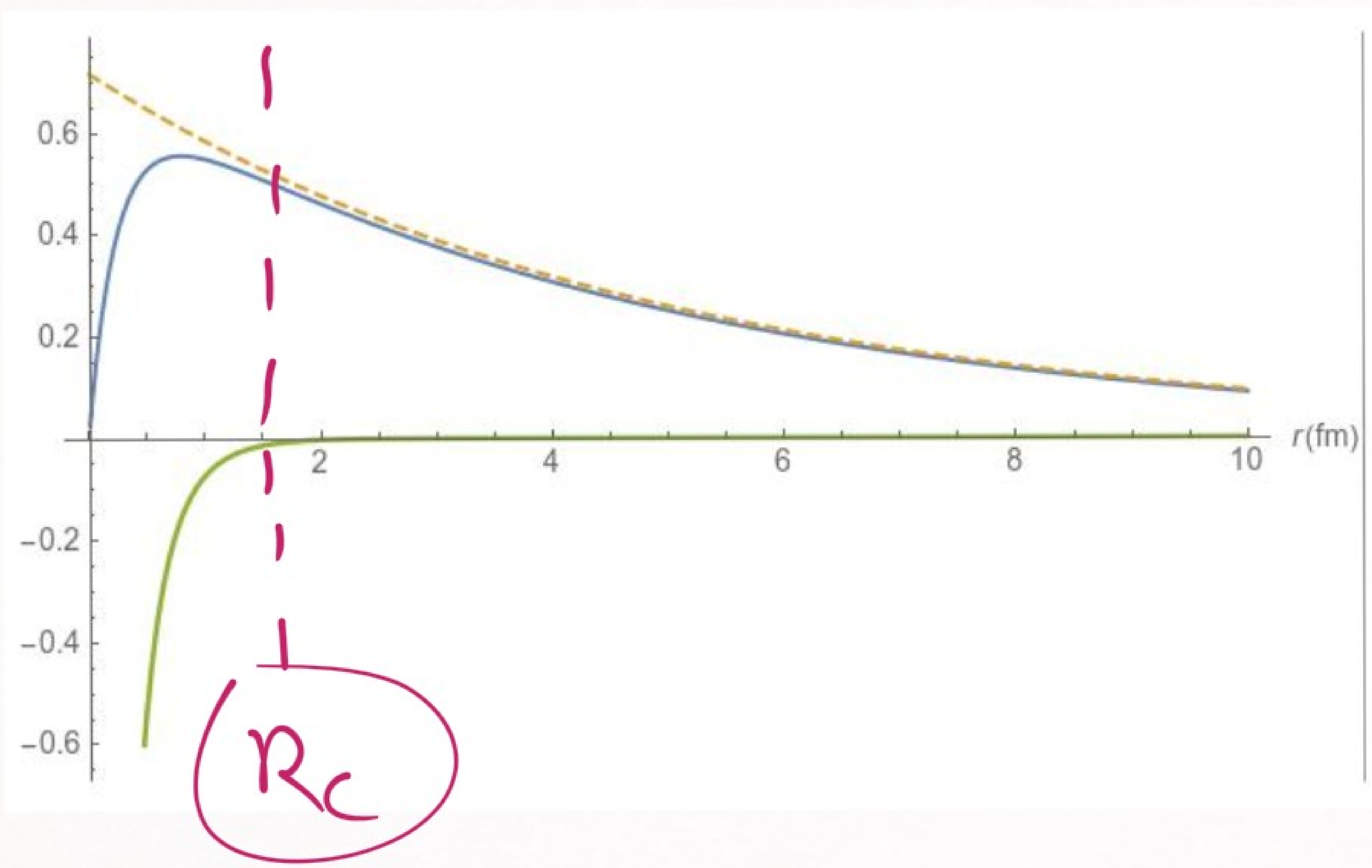
2.b) wf not sensitive to pot. details

(2) → "Physics at long distances does not depend on short distance details"



For 2 we can describe the system without knowing anything about $V_X(r)$

$\Rightarrow$

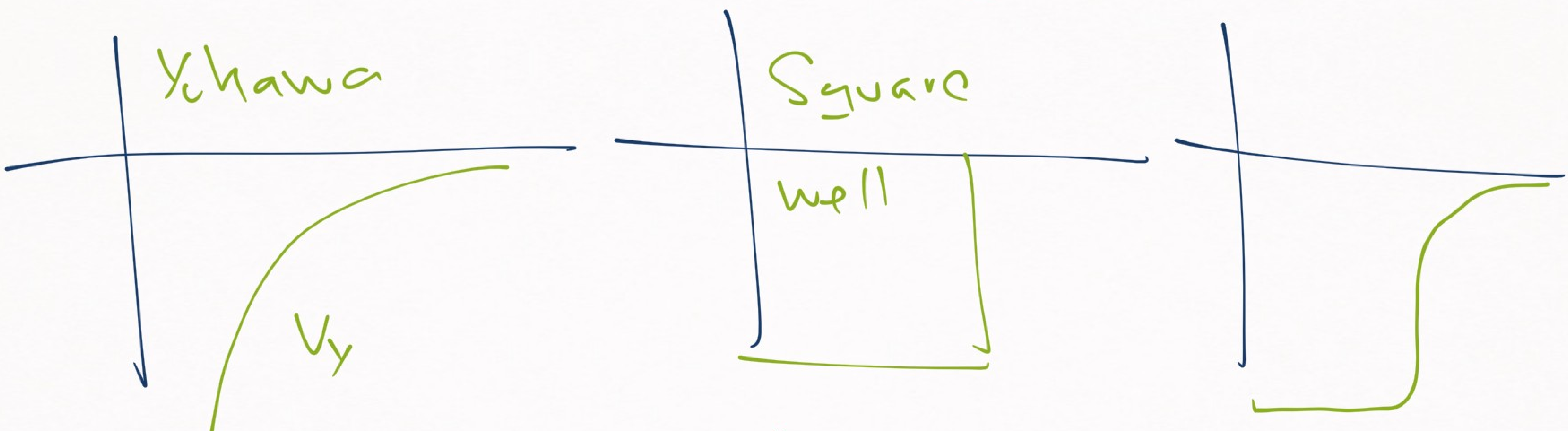


If $r > R_c$

$$\Rightarrow \psi_D \simeq \frac{1}{\sqrt{4\pi}} \frac{A_S e^{-\gamma r}}{r}$$

$\Rightarrow$

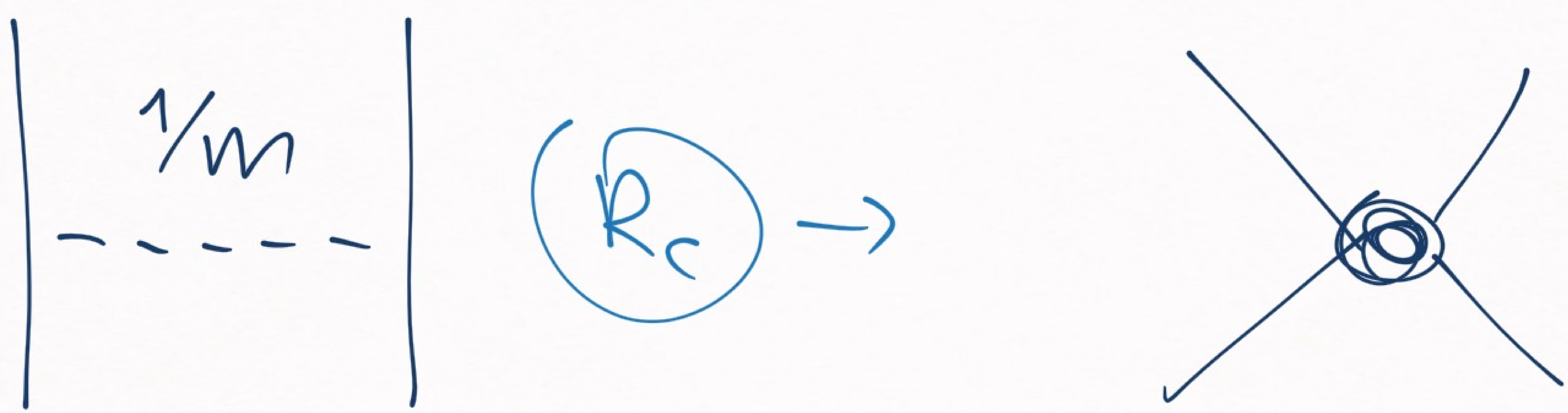
For $\gamma \ll m \Rightarrow$ It doesn't matter the form of the potential



$\rightarrow$ We always obtain same wf

→ We can try a description in terms of an arbitrary potential of range R_c

If $mR_c \gg 1$



REVIEW $\rightarrow$ ① Bound state w/ γ

② Potential w/ range R_H

$\Rightarrow$ [If $\gamma R_H \ll 1$, then it does not matter what is the form of the potential]

→ We don't have to know the exact V

Two-body system ⇒ $\psi_B(\vec{r}) = \frac{A_S}{\sqrt{4\pi}} \frac{e^{-\gamma r}}{r}$

(trivial)

Yet, it would be nice to be able to do

standard QM calculation w/o

(non-trivial) Knowing V

→ Let's see how to do the nontrivial part

Fundamental potential $\xrightarrow{|\vec{q}|^2 \ll m^2}$ Effective potential

$$V_Y(\vec{q}) = -\frac{g_Y^2}{\vec{q}^2 + m^2} \longrightarrow V_Y(\vec{q}) = -\frac{g_Y^2}{m^2}\left(1 - \frac{\vec{q}^2}{m^2} + \frac{\vec{q}^4}{m^4} - \frac{\vec{q}^6}{m^6} + \dots\right)$$

For $|\vec{q}|^2 \ll m^2$, we can use an "effective" V

$$V_{eff}(\vec{q}) = C_0 + C_2|\vec{q}|^2 + C_4|\vec{q}|^4 + \cdots$$
$$= \hat{C}_0 + \hat{C}_2\left|\frac{\vec{q}^2}{m^2}\right| + C_4\left|\frac{\vec{q}^4}{m^4}\right|^2 + \cdots$$

$\Rightarrow \exists$ an expansion of V in terms of a small quantity

(LOW ENERGIES) $\rightarrow$ (GENERIC POTENTIAL AS A POWER SERIES IN $|\vec{q}|^2$)

$\Rightarrow$

$$\boxed{V(\vec{q}) = C_0 + C_2 \vec{q}^{\,2} + C_4 \vec{q}^{\,4} + \dots}$$

If $|\vec{q}|^2 \ll m^2$, then the first term will be enough:

"Leading order potential"

$\mathcal{O}(q^2)$ or higher is ignored

$$\boxed{V_{LO}(\vec{q}) = C_0}$$

But...

$$\boxed{V_{LO}(\vec{q}) = C_0} \rightarrow \text{problematic}$$

$\Downarrow$ Fourier-transform

$$\boxed{\begin{aligned} V_{LO}(\vec{r}) &= \int \frac{d^3\vec{q}}{(2\pi)^3} V_{LO}(\vec{q})\, e^{-i\vec{q}\cdot\vec{r}} \\ &= C_0\, \delta^{(3)}(\vec{r}) \end{aligned}}$$

REMINDER

$$\delta(x) = \begin{cases} \infty & x = 0 \\ 0 & x \neq 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(x)\, dx = 1$$

$$\sum$$

Most generic low-energy potential is:

$$\boxed{V_{LO}(\vec{r}) = C_0\,\delta^{(3)}(\vec{r})}$$

If $C_0 < 0$ (attractive) $\longrightarrow$ variational estimation of the binding energy

$$E_B \le \underbrace{\langle \psi_{trial} | H | \psi_{trial} \rangle}$$

$$\langle \psi_{trial} | V_0 | \psi_{trial} \rangle = C_0 \underbrace{|\psi_{trial}(\vec{0})|^2}_{\infty}$$

$$\psi_{trial}(\vec{r}) \to \frac{1}{r} \qquad \underbrace{< 0 \quad \infty}$$

$$E_B \le -\infty$$

$$\Rightarrow \left[V_{LO}(\vec{r}) = C_0\, \delta(\vec{r}) \quad / \; C_0 < 0 \right]$$

→ does not have a fundamental state

(system that collapses)

Σ

STRANGE BECAUSE $G\,\delta(\vec{r})$ SHOULD BE A GENERIC POTENTIAL FOR LOW-ENERGY SYSTEMS

→ WHAT'S WRONG HERE?

→ RENORMALIZATION ENTERS

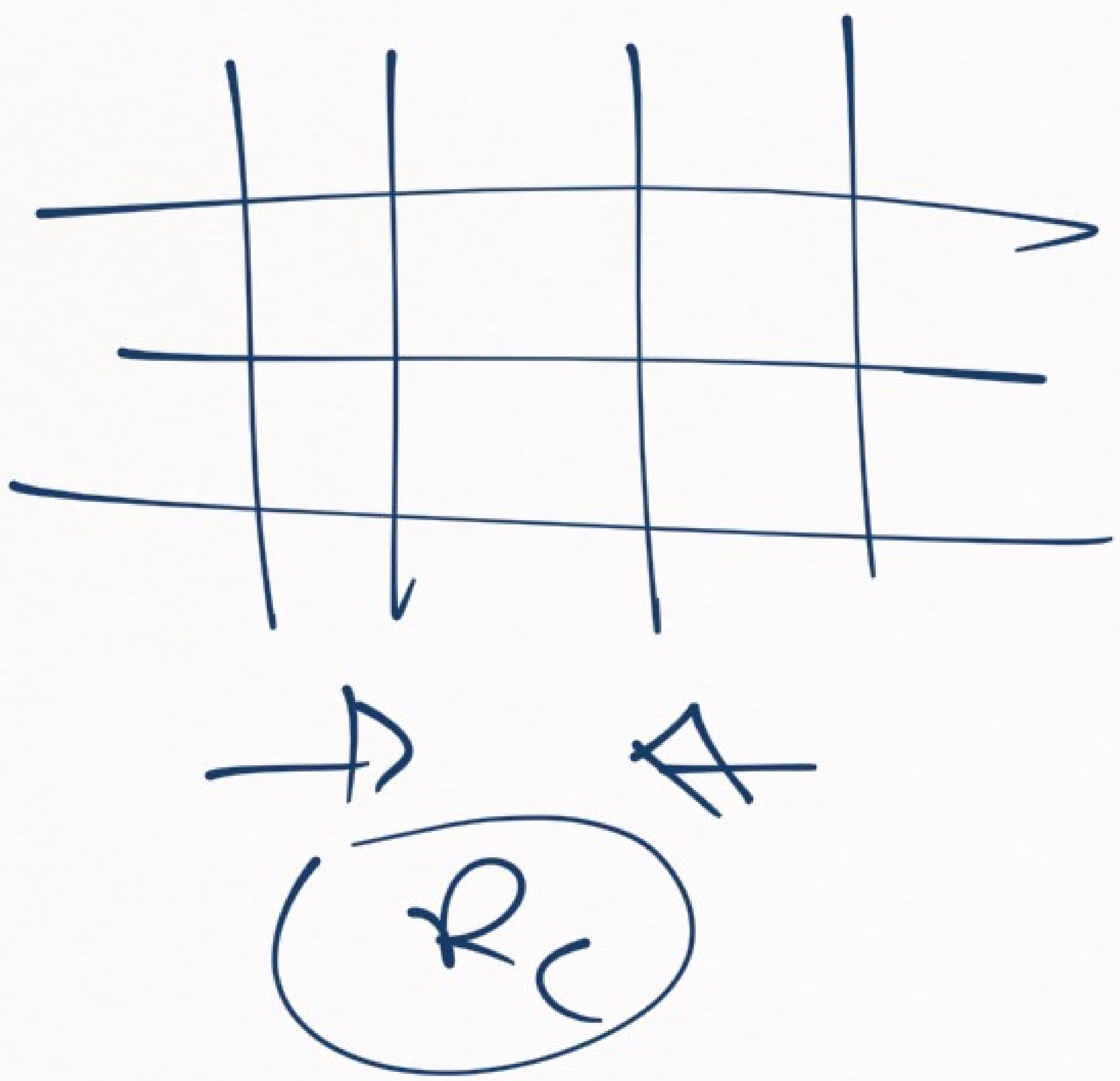
$V_{LO}(\vec{r}) = C_0 \, \delta(\vec{r}) \rightarrow$ Renormalize it

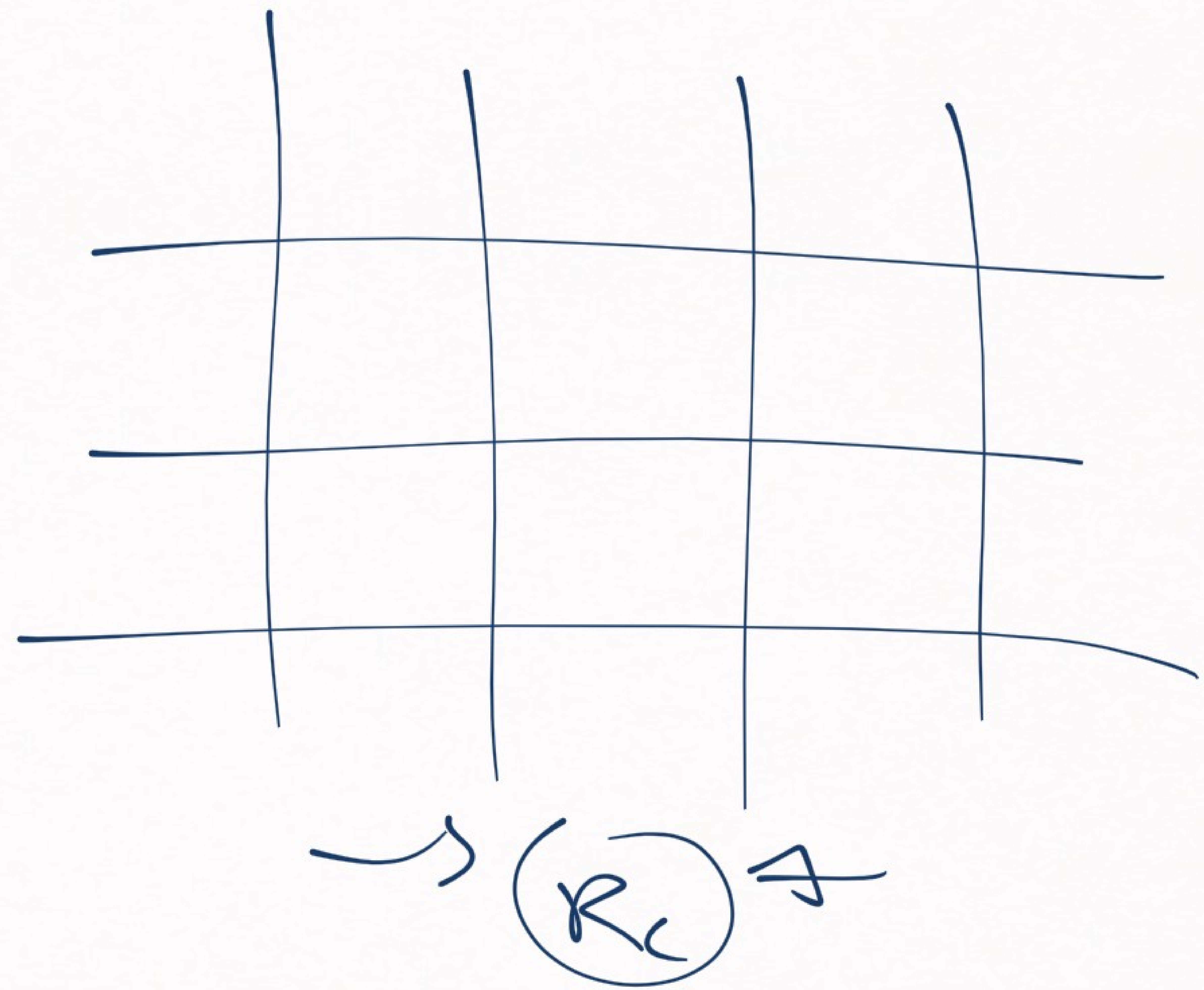
Two steps

$\boxed{1}$ REGULARIZATION

$\boxed{2}$ RENORMALIZATION (PROPER)

$\boxed{\text{REGULARIZATION}}$ → CUTOFF / RESOLUTION SCALE





1) p-space :

$$V(\vec{q}) = C_0 \xrightarrow{\Lambda} \begin{cases} V(\vec{q}) = C_0 \, e^{-\vec{q}^2/\Lambda^2} \\ V(\vec{q}) = C_0 \, \Theta(\Lambda - |\vec{q}|) \\ V(\vec{q}) = C_0 \dfrac{\Lambda^2}{\vec{q}^2 + \Lambda^2} \end{cases}$$

any option / $V(|\vec{q}| \to 0) \to C_0$ is correct

$V(|\vec{q}| \to \infty) \to 0$

2) r-space

$$V(\bar{q}) = C_0 e^{-q^2/\Lambda^2} \xrightarrow{\text{FT}} V(\vec{r}) = C_0 \frac{\Lambda^3}{8\pi^{3/2}} \times e^{-\frac{1}{4}\Lambda^2 r^2}$$

$$V(\vec{r}) = C_0 \delta^{(3)}(\vec{r}) \longrightarrow C_0 \frac{\theta(R_c - r)}{4\pi R_c^3}$$

$$C_0 \frac{\delta(r - R_c)}{4\pi R_c^2}$$

→ #1 $\boxed{\text{REGULARIZE}}$

#2 → RENORMALIZE → $\boxed{\text{NEXT LESSON}}$

$V_\omega(\vec{r}; R_c)$

[Result should be independent of R_c]