NUCLEAR PHYSICS 7

→ QCD can't be solved analytically

What can we do to derive nuclear physics from QCD?

$\boxed{\text{RECAP}}$

→ QCD is very similar in structure to QED

Gauge theory → local symmetry of Dirac fields w/ respect to some group

QED → U(1)   QCD → SU(3)

→ structure indeed very similar

→ BUT ... $\boxed{\text{QED \& QCD behaviors are very different}}$

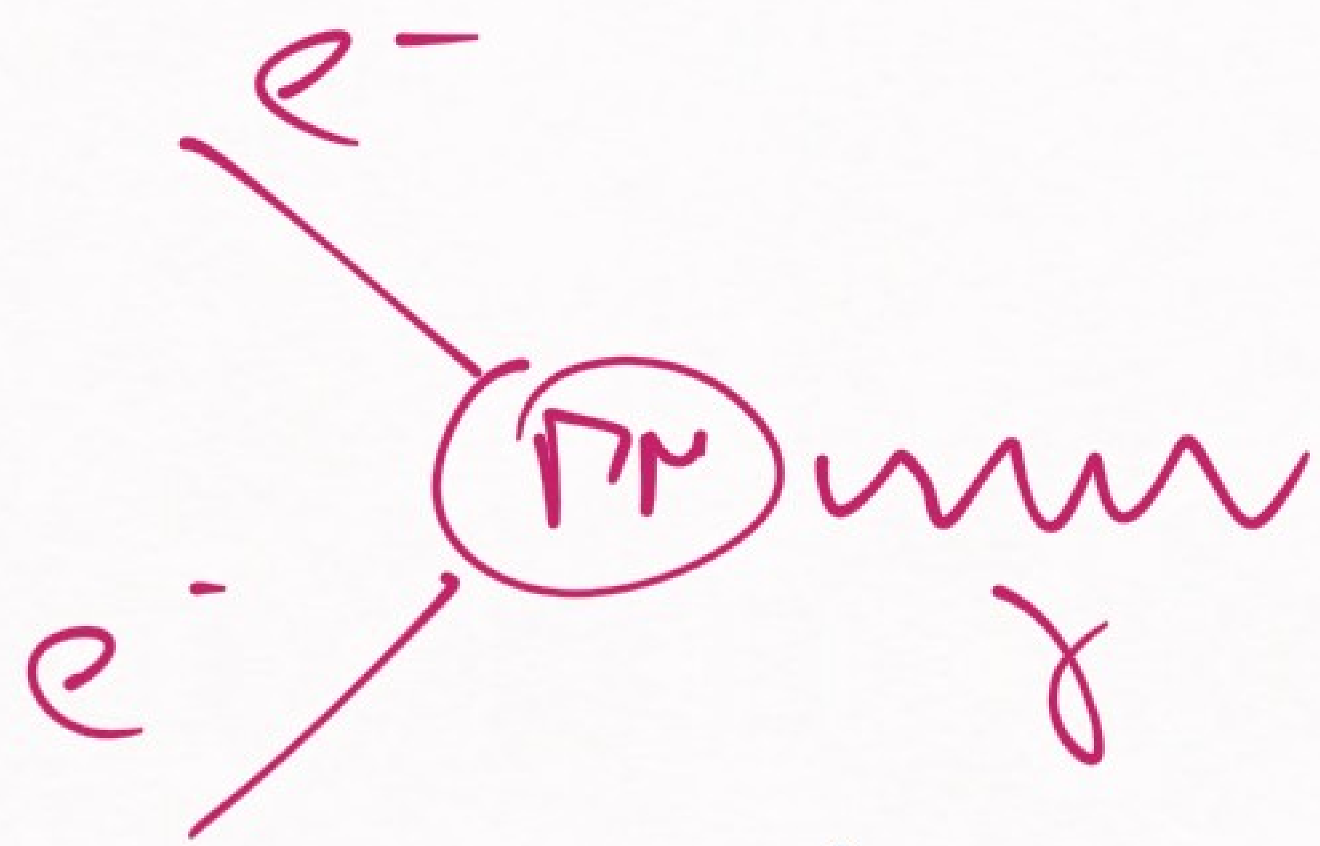
Why does this happen?

QED → U(1) → photon doesn't carry U(1) charge

QCD → SU(3) → gluons carry SU(3) charge (color)

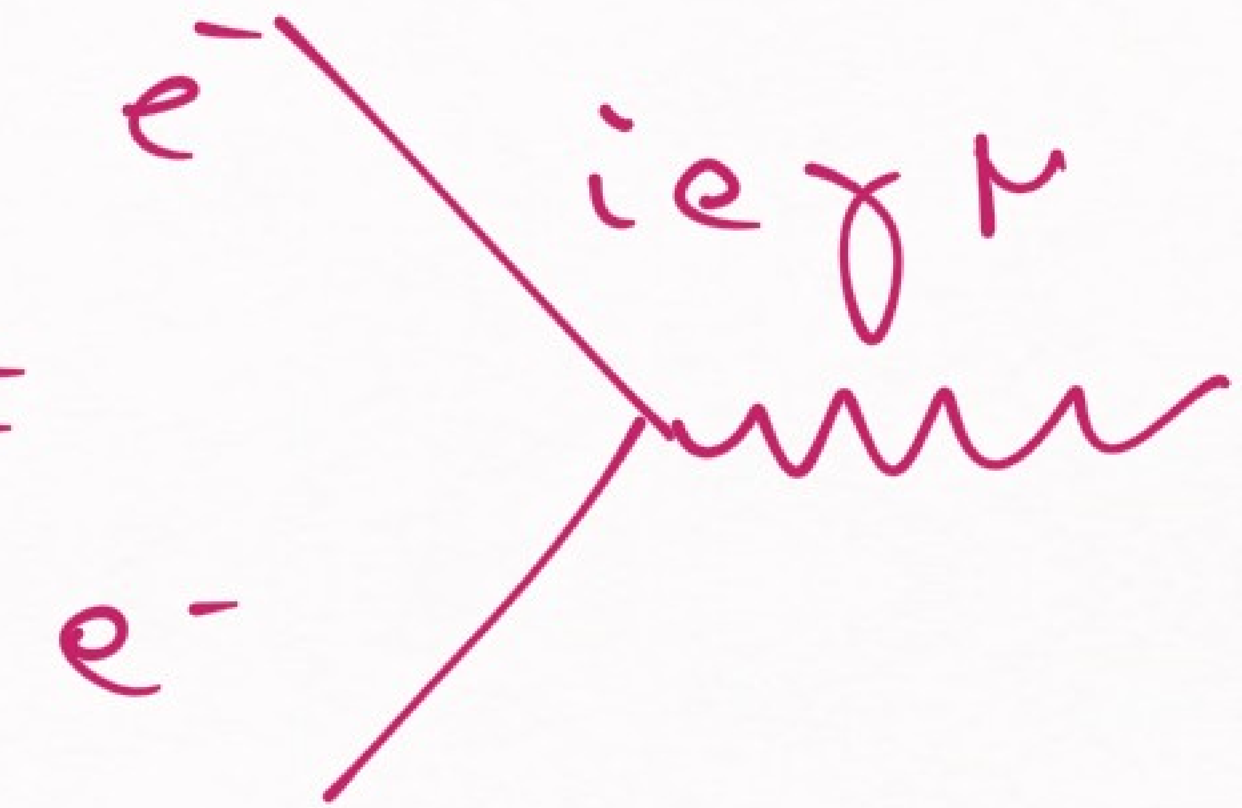
WHY THIS MAKES A BIG DIFFERENCE?

QED

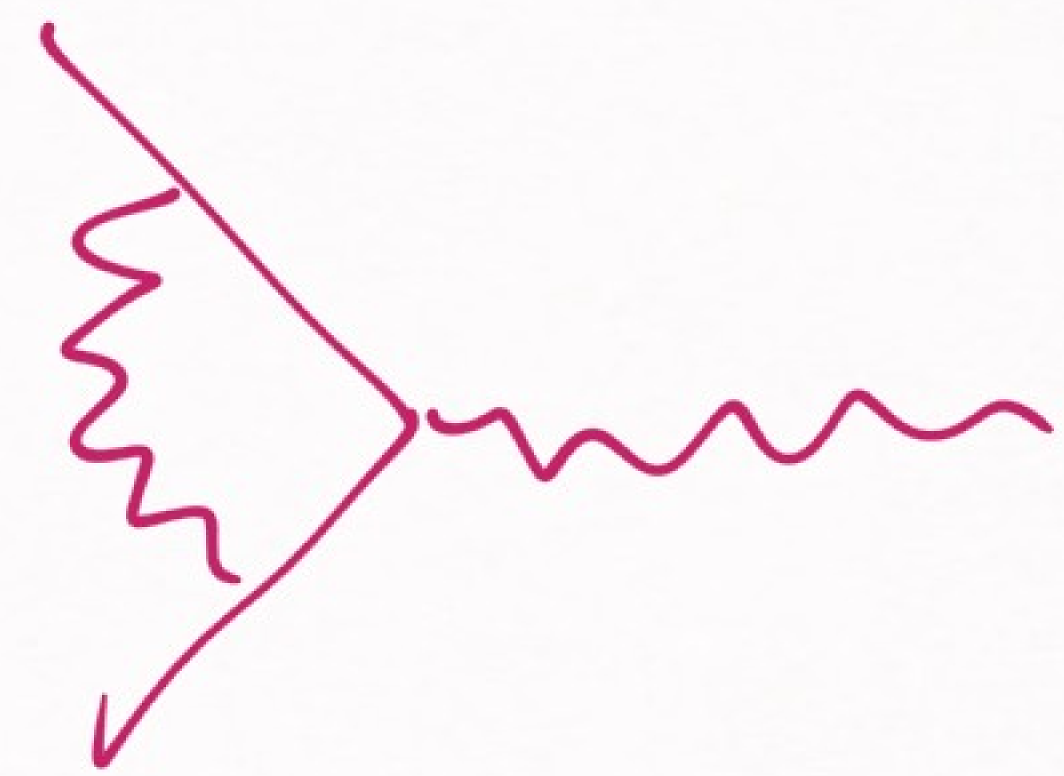


(vertex function)

$=$



$+$



$+ \ \ldots$

$\Rightarrow$

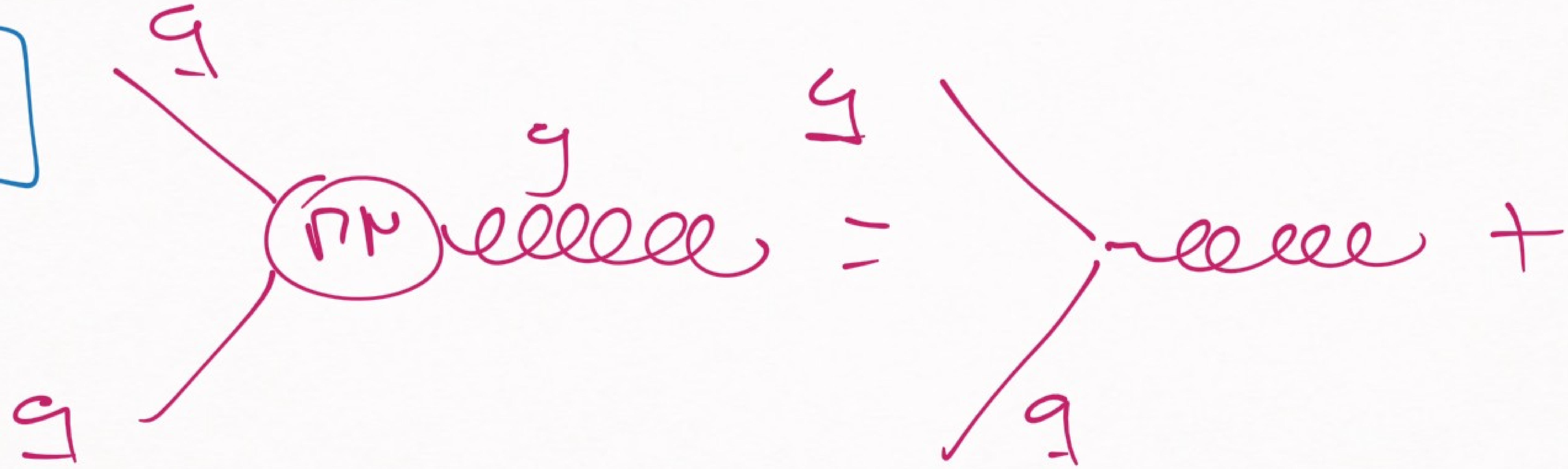
$\Rightarrow \quad e \to e_R(Q^2) \qquad \alpha = \frac{e^2}{4\pi} \longrightarrow \underline{\underline{\alpha(Q^2)}}$

$$\boxed{\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi}\log\left(\frac{Q^2}{\mu^2}\right)}}$$

e.m. strength increases w/ energy

$\hookrightarrow$ if $Q^2 > \mu^2$, $\alpha(Q^2) > \alpha(\mu^2)$

$\boxed{\text{QCD}}$



gluon has color

$$\boxed{\alpha_s(Q^2) = \frac{12\pi}{(33 - 2n_f)\log\left(\frac{Q^2}{\Lambda^2_{QCD}}\right)}}$$

→ strong force becomes weaker at high energies

→ ⊛

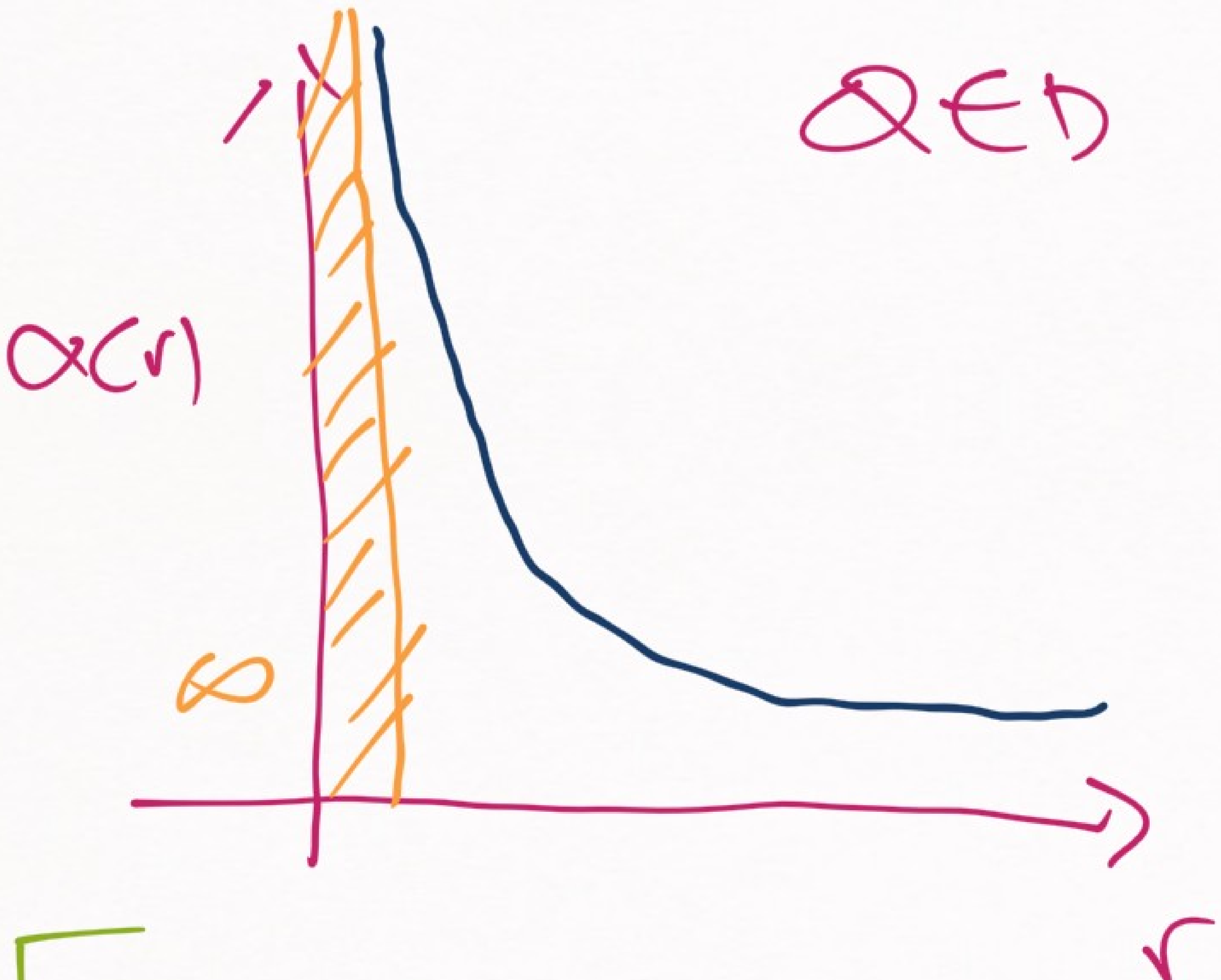
n_f: # of flavors (# of quark types, u, d, s, c, b, t)

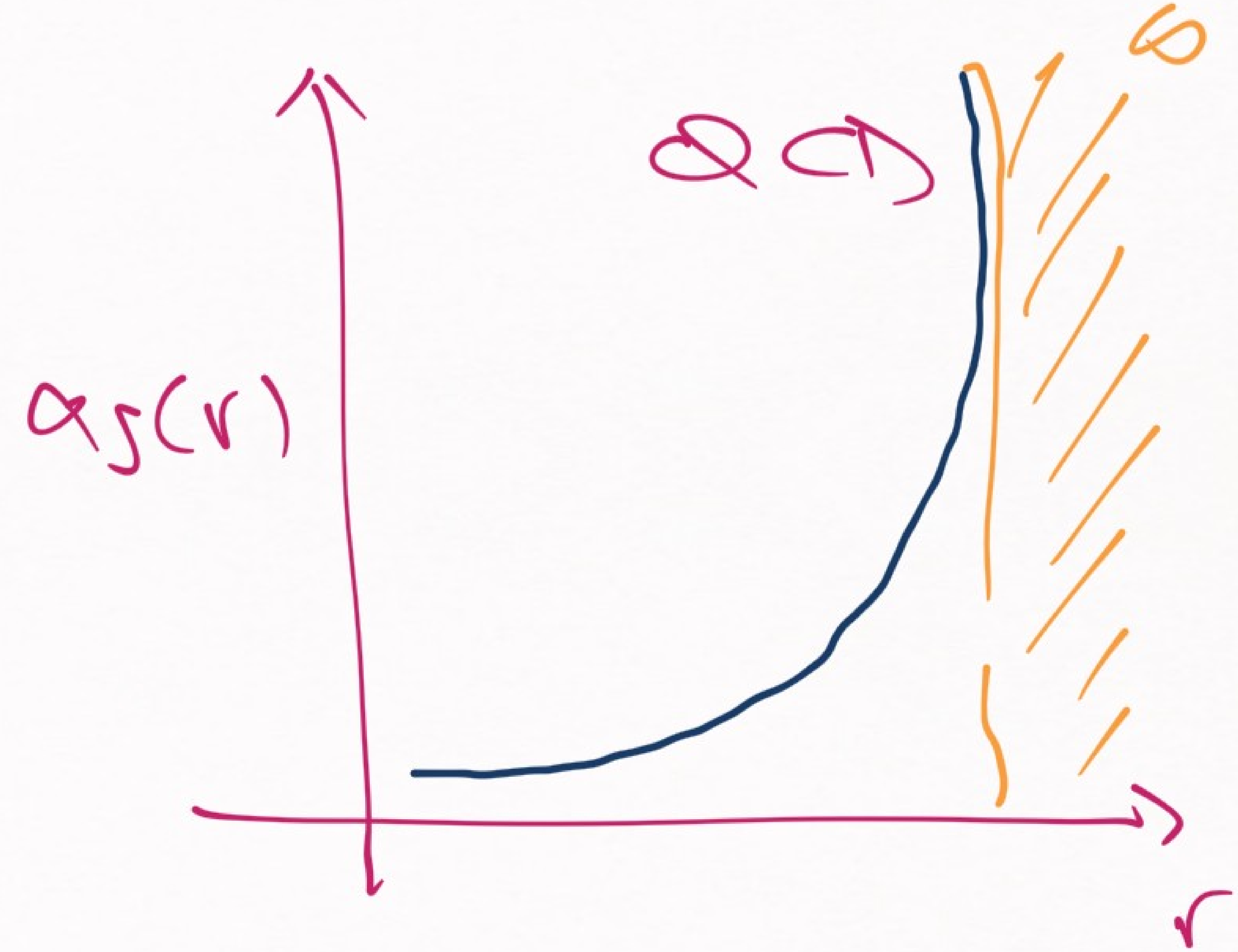
⇓

If $Q^2 > \mu^2$, then $\alpha_s(Q^2) < \alpha_s(\mu^2)$

⊛ → $\Lambda_{QCD} \sim (200-350)$ MeV

QED vs QCD





$$\left[\, r \leq \frac{1}{\Lambda_0} \,,\ \Lambda_0 \sim \underline{10^{280}}\ \mathrm{MeV} \right]$$

(Landau pole)

$$\left[\, r \geq \frac{1}{\Lambda_{QCD}} \,\right]$$

$\boxed{\text{QCD}}$ → **Problem:** Nuclear physics happens at distances where we cannot apply QCD

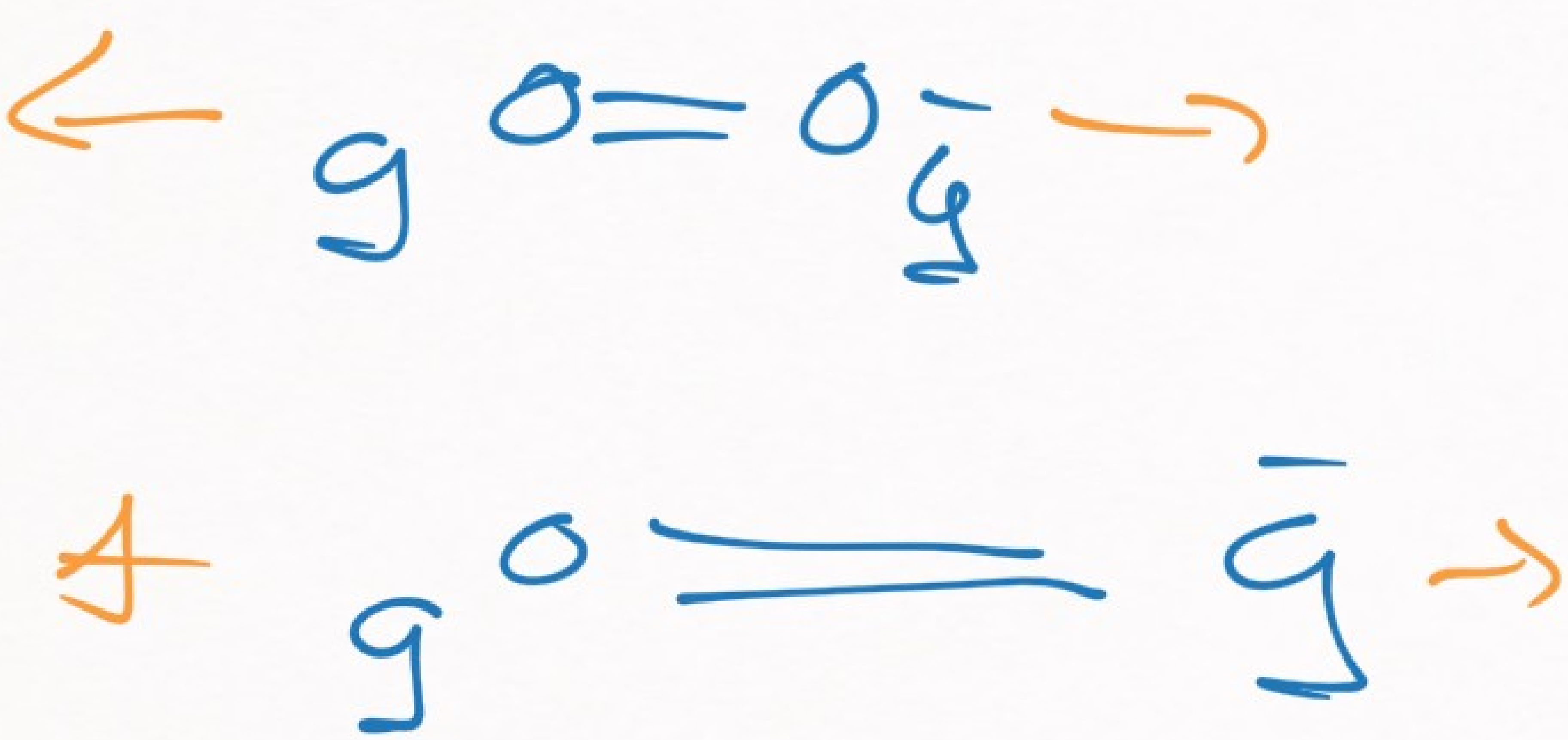
RELATED TO CONFINEMENT
(NO FREE QUARKS)

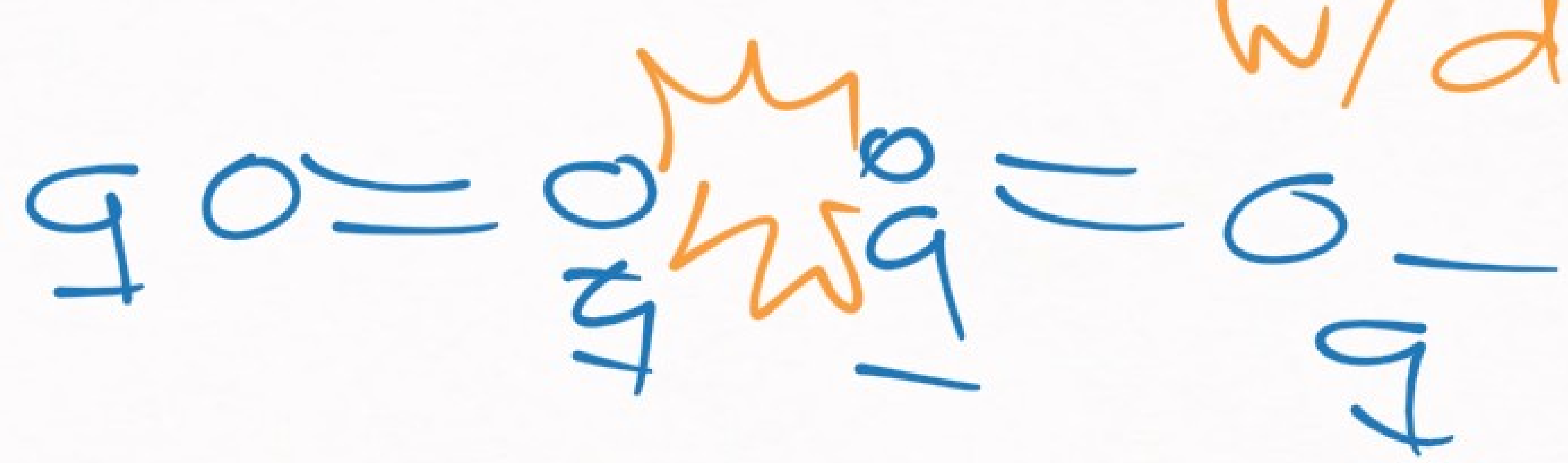
$\alpha_s(r)$ grows with r

$$V_{q\bar{q}}(r) \sim -\frac{\alpha_s(r)}{r}$$

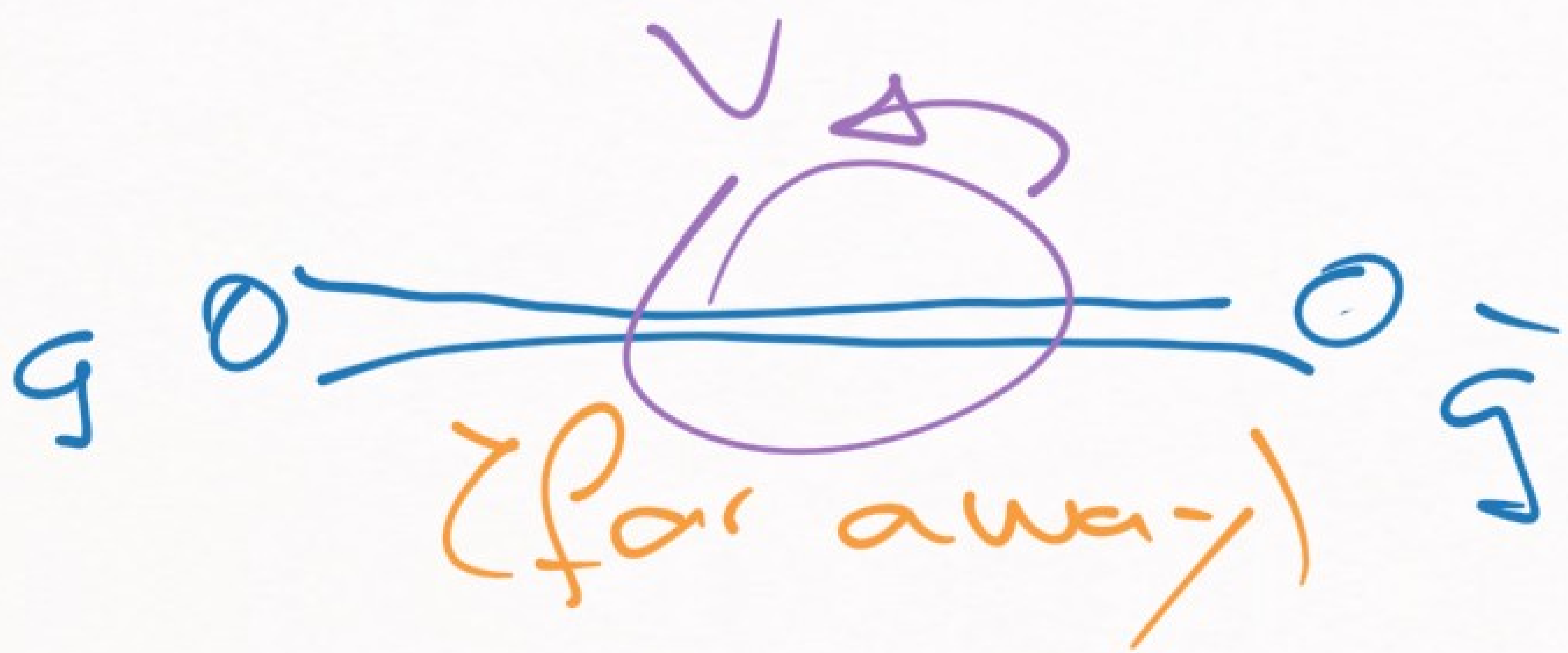
$$\sim -\frac{a}{r} + br$$

grows stronger w/distance



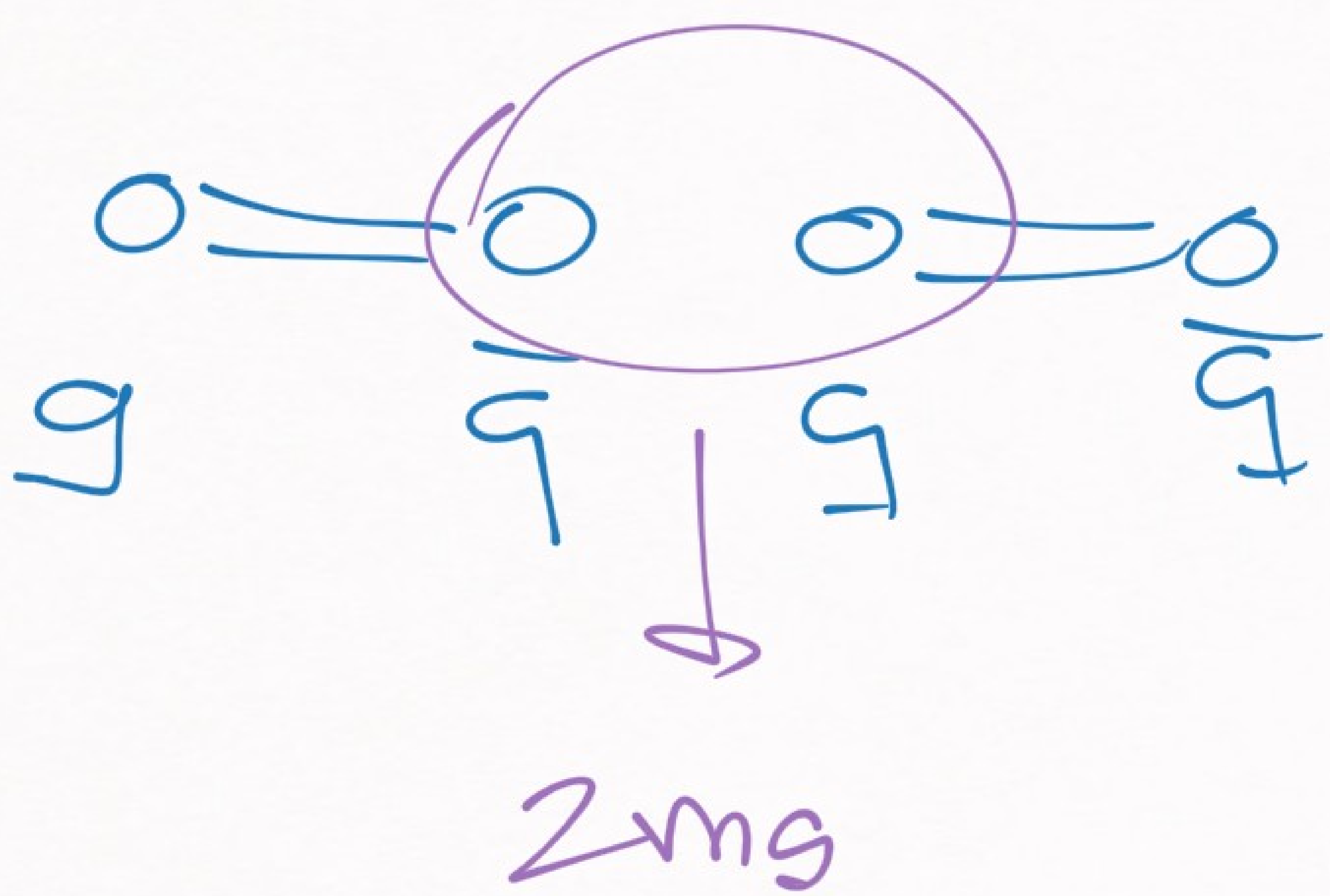


Minimum energy configuration:



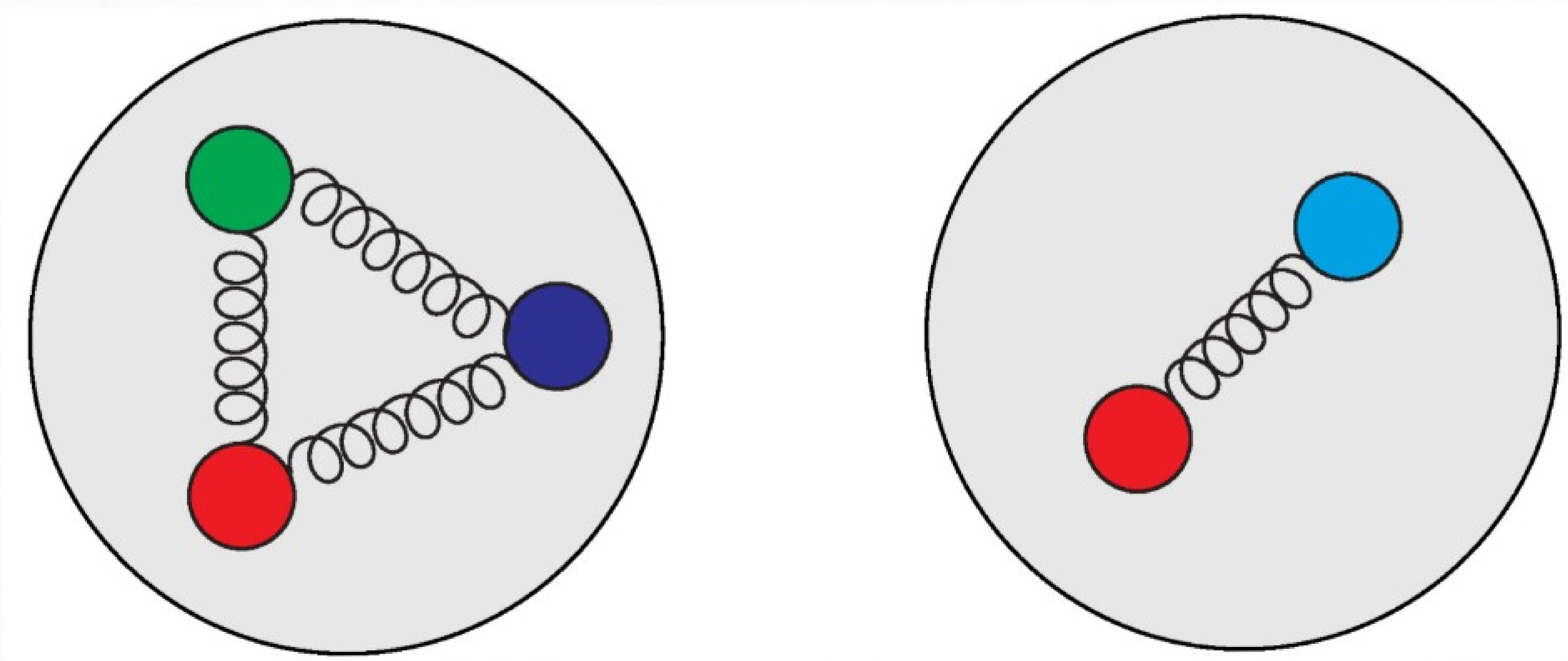
(1) E_1

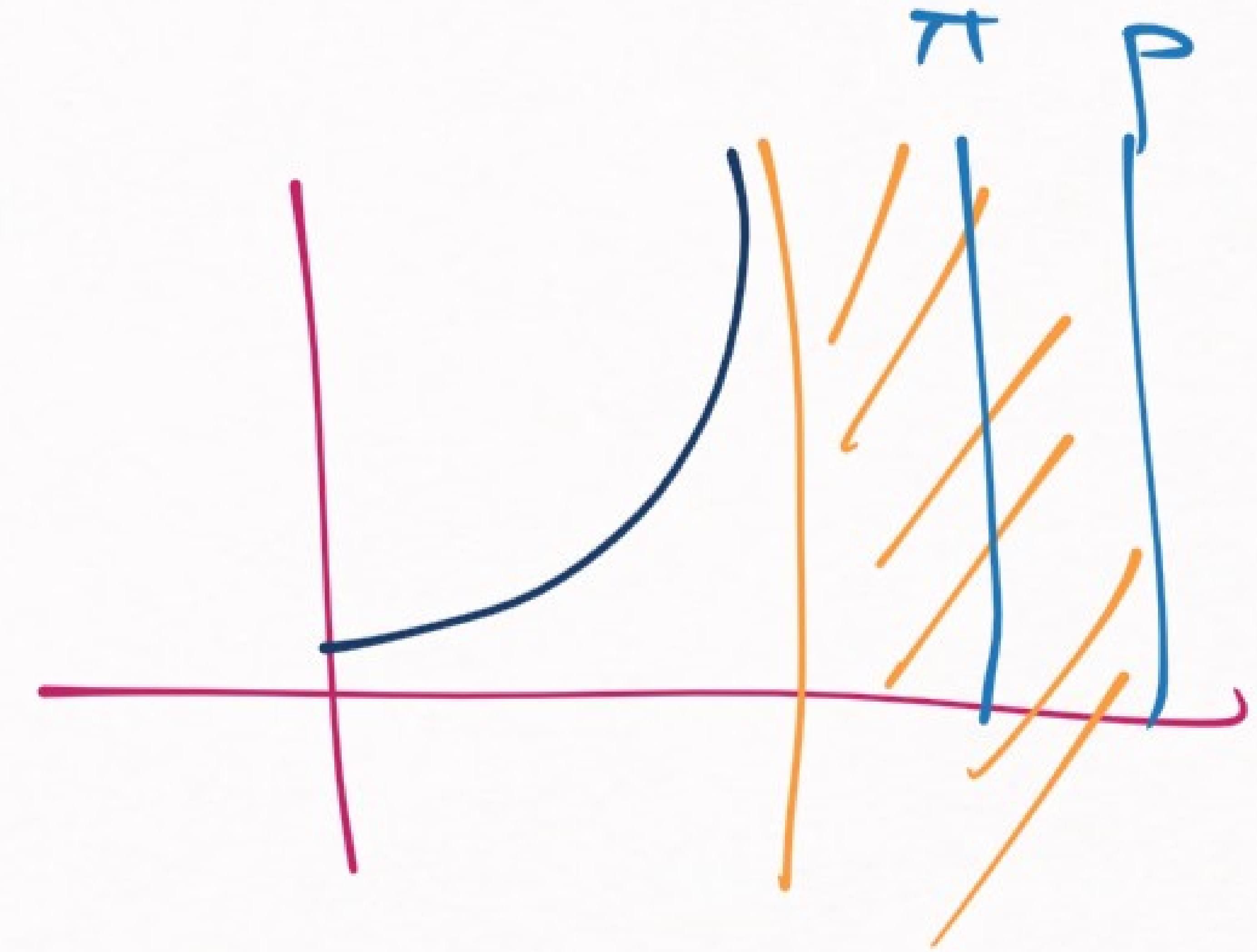
$\vee$



(2) E_2 $\rightarrow$ nature's choice

$|V| > 2mq$





0.5 fm

Baryon (qqq)

(proton, $r_p \sim 0.85\,\text{fm}$)

Meson ($q\bar{q}$)

(pion, $r_\pi \sim 0.65\,\text{fm}$)

their size is simply too big to apply QCD (directly)

→ I can't apply QCD analytically

→ I want a derivation of nuclear forces based on QCD

→ There are ways around this

$\boxed{\text{TWO POSSIBILITIES}}$

1) Lattice QCD

2) Effective field theories

1) LATTICE QCD :

Use a (super) computer to solve QCD numerically

NUCLEAR PHYSICS

$p, n \to \boxed{uud, udd}$

$\pi \to \boxed{u\bar{d}}$

$m_u, m_d \to 0$

[PROBLEM : $m_q \to 0$

$\Rightarrow$ calculations are super difficult]

2) EFFECTIVE FIELD THEORY (EFT) ← our choice

More abstract than lattice QCD

→ "Renormalizion group analysis" (RGA)
to solve QCD indirectly

(construct a theory for large distances
that is equivalent to QCD)

How to do EFTs?

→ We use renormalization → history

[What is renormalization?]

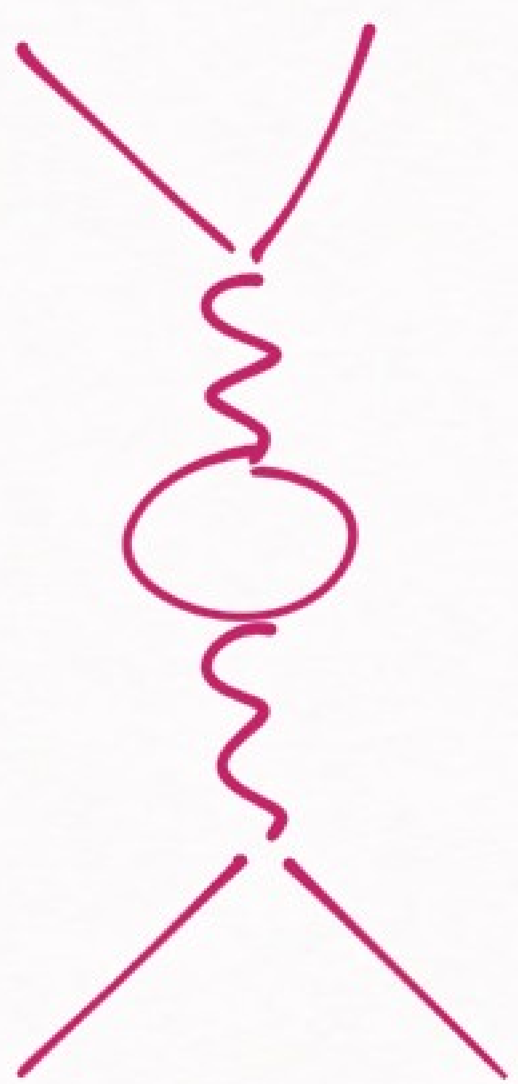
1) Old ideas of renormalization

2) Modern understanding of renormalization

1) OLD SCHOOL RENORMALIZATION

- → POCONO CONFERENCE 1948
- → Lectures by Feynman & Schwinger

(issue was the infinities in QED)



Feynman & Schwinger → find "weird methods" to deal w/ this problem

(Tomonaga too)

(Dyson → equivalence between their methods)

→ this is what you find in older books

INTERESTING OBSERVATION BY FEYNMAN:-

我们为求出n和j所玩的壳层游戏，在专业上叫做“重正化”（renormalization）。但是，不管这个词听来多聪明，我却说这个过程是蠢笨的！求助于这类戏法妨碍了我们去证明量子电动力学在数学上的自洽性（self-consistent）。令人不解的是，尽管人们用了各种办法，这个理论至今仍未被证实是自洽的；我猜想，重正化在数学上是不合法的。我们还没有一种好的数学方法描述量子电动力学，这是肯定的——像这样描述n、j同m、e之间关系的语言不是好的数学。[23]

→ "arcane" thing you do to obtain finite results

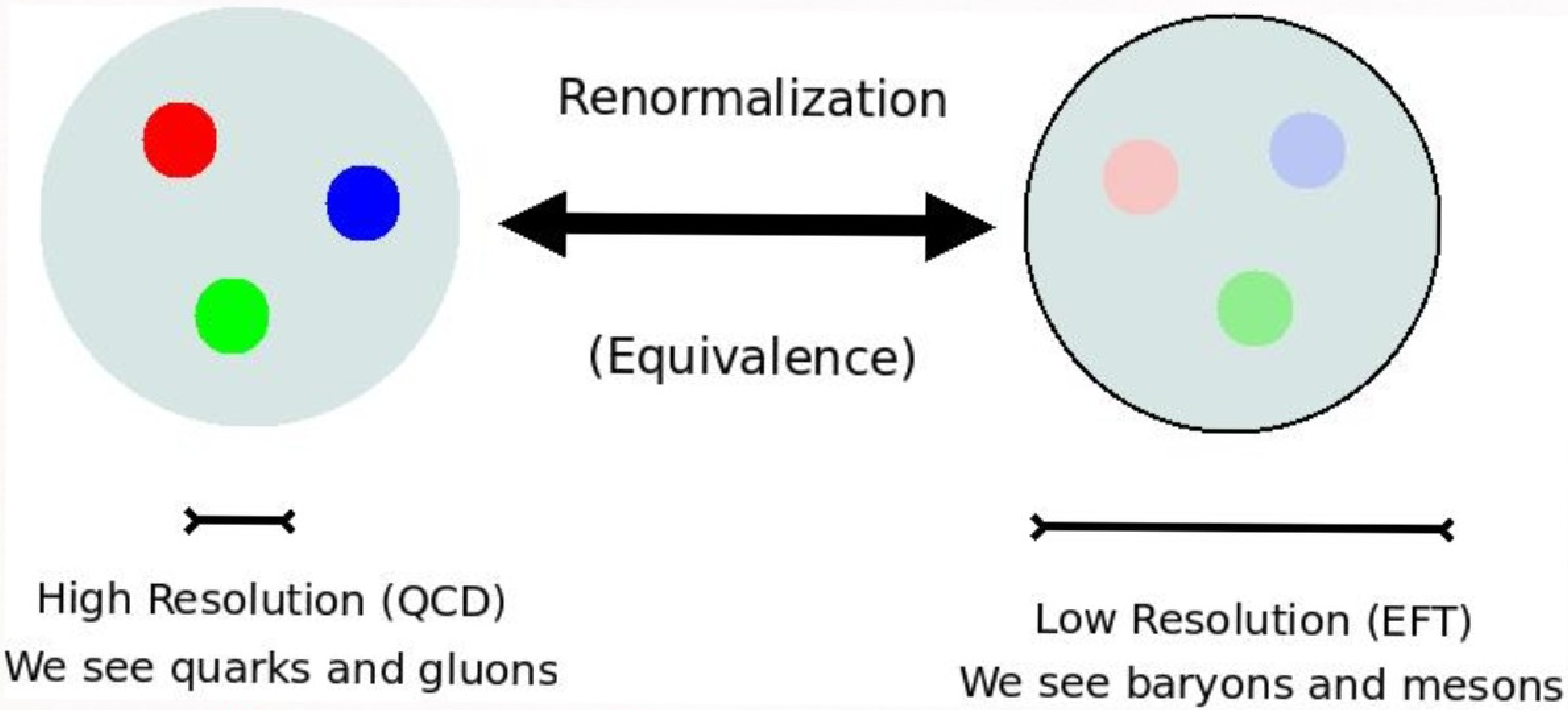
- So it appears that the only things that depend on the small distances between coupling points are the values for n and j-theoretical numbers that are not directly obseroable any- way; everything else, which can be observed, seems not to be affected. The shell game that we play to find n and j is technically called "renormalization." But no matter how clever the word, it is what I would call a dippy process! Having to resort to such hocus-pocus has prevented us from proving that the theory of quantum electrodynamics is mathematically self-consistent. It's surprising that the theory still hasn't been proved self-consistent one way or the other by now; I suspect that renormalization is not mathematically legitimate. What is certain is that we do not have a good mathematical way to describe the theory of quantum electrodynamics: such a bunch of words to describe the connection between n and j and m and e is not good mathematics.
  - Richard Feynman, *QED: The Strange Theory of Light and Matter* (1985), Chap. 4. Loose Ends

→ 75 years has passed since Pocono

[NOWADAYS WE UNDERSTAND RENORMALIZATION MUCH, MUCH BETTER]

Basic idea → (Renormalization)

Physics at long-distances does not depend on short-distance details



1 $\simeq$ 2

(same)

$\downarrow$

Renormalization

1 short-distance understanding (quarks & gluons

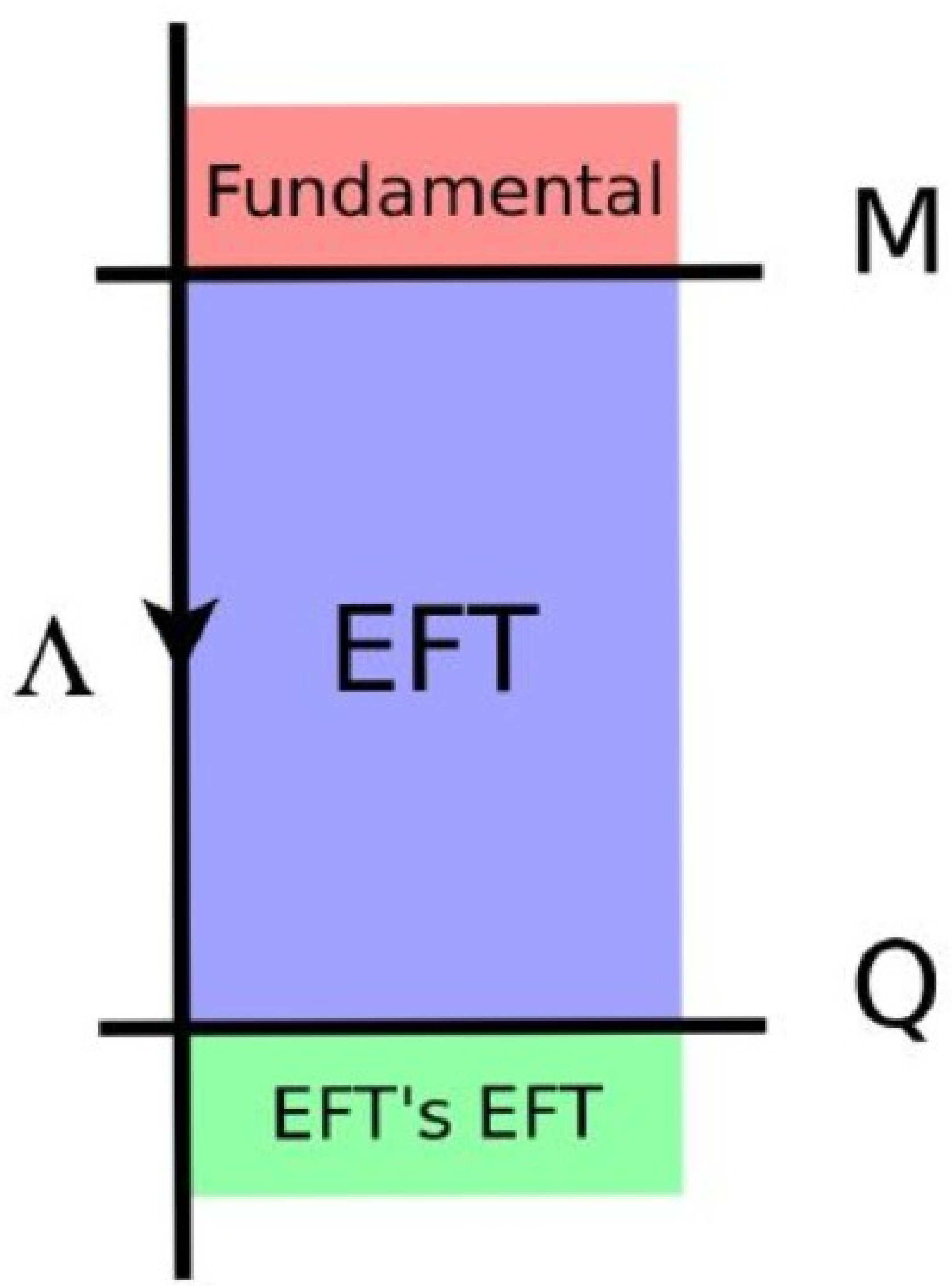
2 long-distance understanding (baryons & mesons)

⊕ → RENORMALIZATION is just the set of theoretical tools to prove this equivalence

(or the idea that long-range physics is independent of the (often unknown) short-range details)

[Caveat → this will be very abstract]

RENORMALIZATION



Physics is unique, but choice of theory depends on resolution Λ:

- $\Lambda \geq M$: Fundamental
- $M \geq \Lambda \geq Q$: EFT

For equivalent descriptions:

$$\boxed{\frac{d}{d\Lambda}\langle \Psi | \mathcal{O} | \Psi \rangle = 0}$$

Renormalization group invariance

HIGH ENERGY

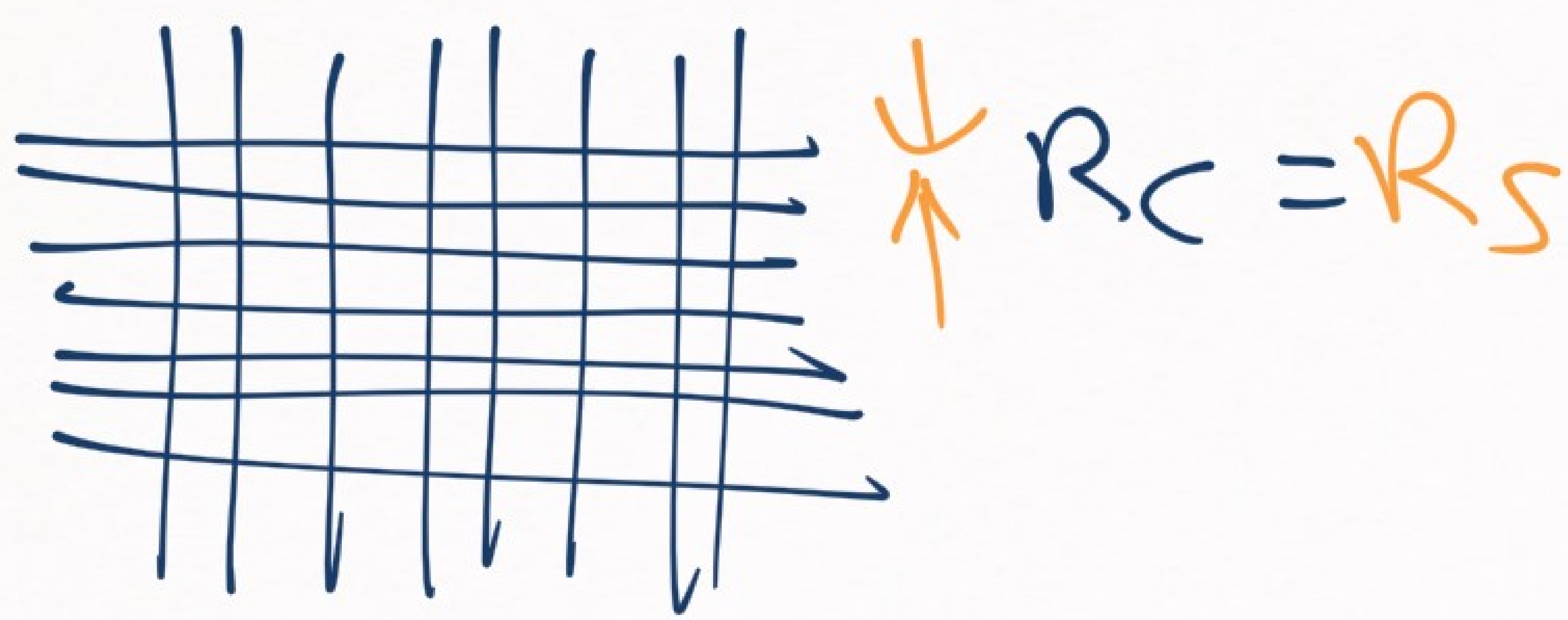
→ FUNDAMENTAL THEORY

→ sometimes unkown

→ sometimes unsolvable (QCD)

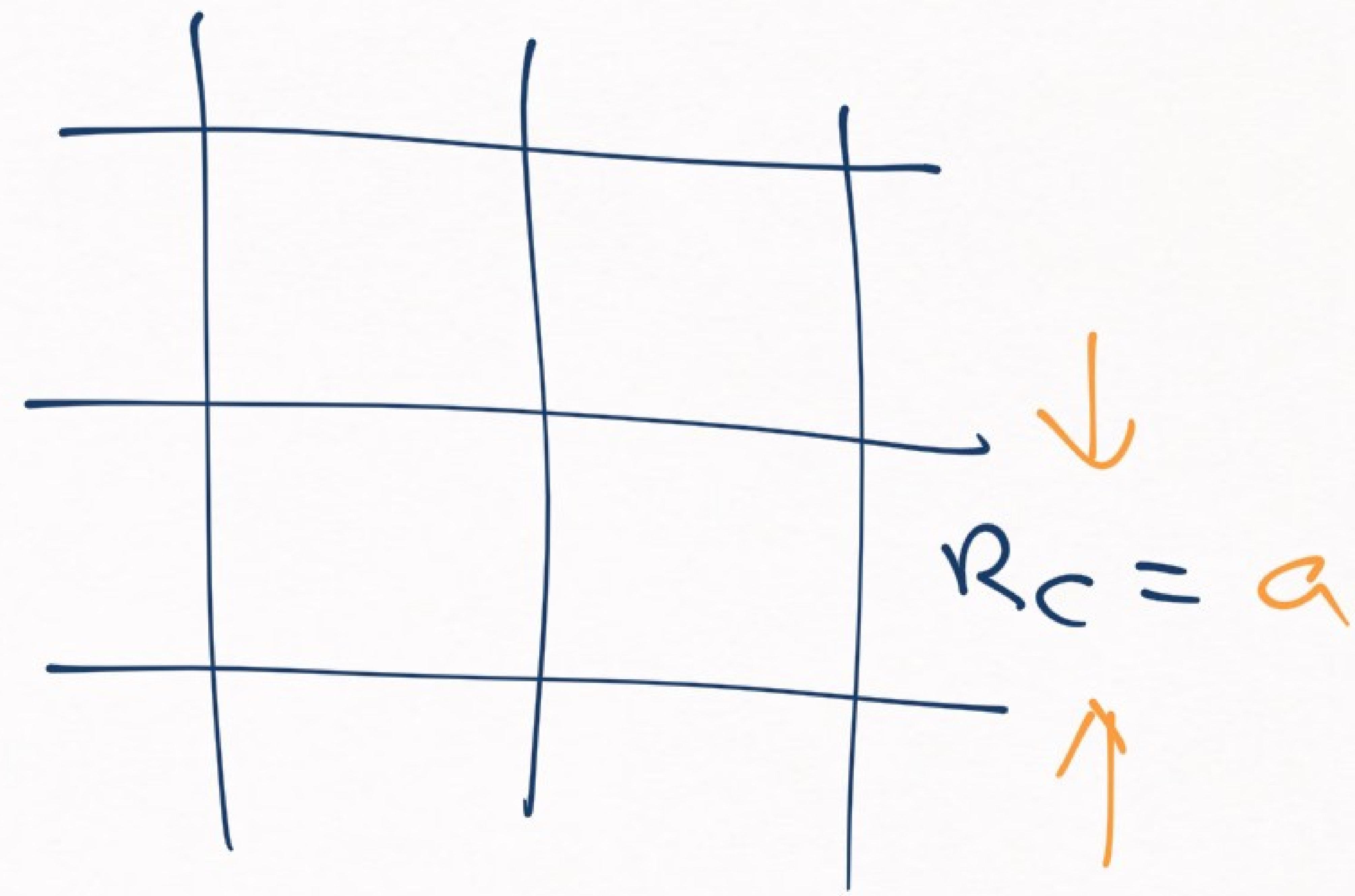
How we see the world depends on the resolution:

HIGH RESOLUTION



(fundamental)

LOW RESOLUTION



(effective or long-distance description)

Specific example : NUCLEAR PHYSICS

1) HIGH RESOLUTION $\rightarrow$ $R_s \sim (0.1 - 0.2)\,fm$

$\rightarrow$ quarks & gluons

2) LOW RESOLUTION $\rightarrow$ $a \sim (1.0 - 2.0)\,fm$

$\rightarrow$ hadrons (nucleons & pions)

$R_c \longrightarrow \boxed{\text{cutoff}} \longrightarrow$ choice of resolution

$R_c \to 0 \Rightarrow$ HIGH RESOLUTION

$R_c \to \infty \Rightarrow$ LOW RESOLUTION

Idealizations

We want HIGH RESOLUTION & LOW RESOLUTION views to be EQUIVALENT

How to do this?

RENORMALIZATION GROUP EQUATION

→ EQUIVALENCE : observable quantities should be the same independently of the resolution

R_c

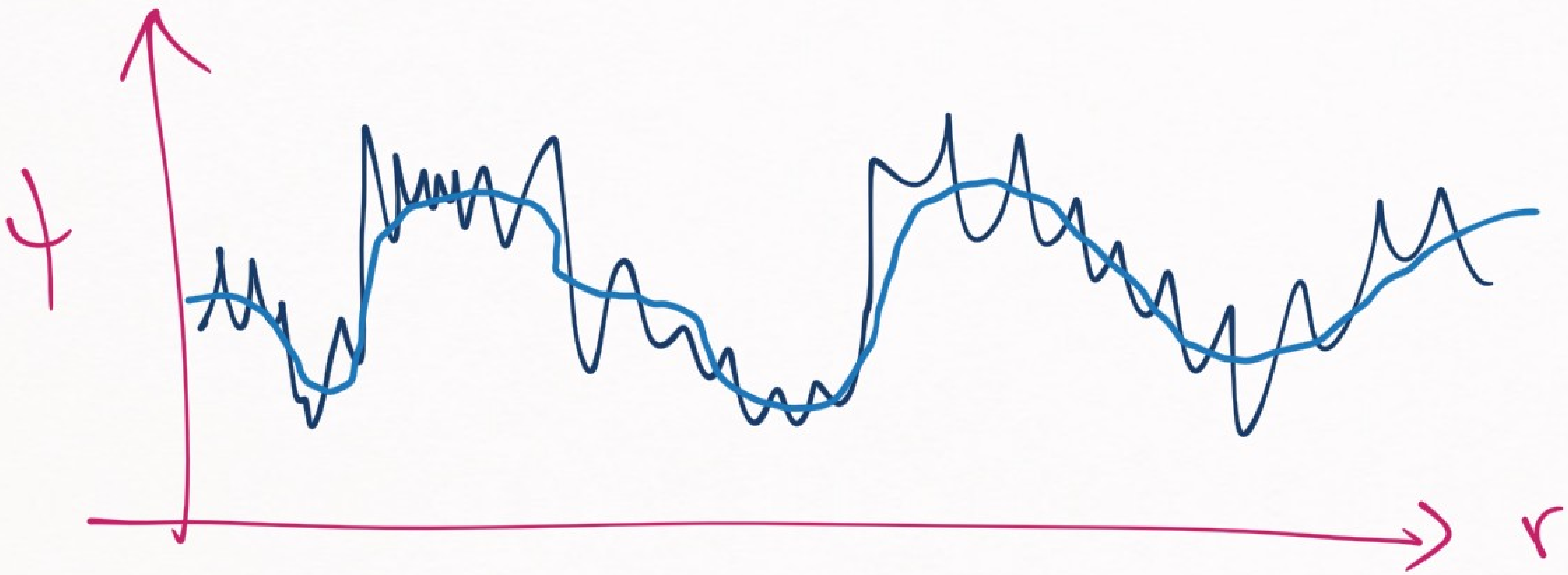
$\langle \hat{O} \rangle = \langle \psi | \hat{O} | \psi \rangle$

this is how you calculate an observable

EQUIVALENCE

$$\boxed{\frac{d}{dR_c}\langle \hat{O} \rangle = 0}$$ → Observables independent of resolution scale

$$\underbrace{\langle \hat{O} \rangle}_{\text{observable}} = \langle \underbrace{\psi}_{\text{wave function; not observable}} | \hat{O} | \psi \rangle$$



High resolution ψ

Low resolution ψ

$$\langle \psi | \hat{O} | \psi \rangle = \langle \psi | \hat{O} | \psi \rangle$$

→ which $|\psi\rangle$ I use is not important

$$\boxed{\frac{d}{dR_c}\langle\psi|\hat{O}|\psi\rangle = 0}$$

→ this is how we formulate renormalization problems

SPECIFIC EXAMPLES IN THE FOLLOWING LESSONS

QUALITATIVE EXAMPLE →

$\boxed{\text{TEAPOTS \& TEACUPS}}$ $\rightarrow$ Which one cools faster?

1 or 2 ?

1 2

o o o o o $\Rightarrow$

ANSWER IS 2 → If you put tea in a cup, it cools faster than in the teapot

looks trivial,

but you know that before considering the physics of cooling →

$\textcircled{3}$ → [YOU KNOW THE ANSWER W/O SOLVING ANY EQUATION]

(you have unwittingly renormalized this problem)

I WILL EXPLAIN ...

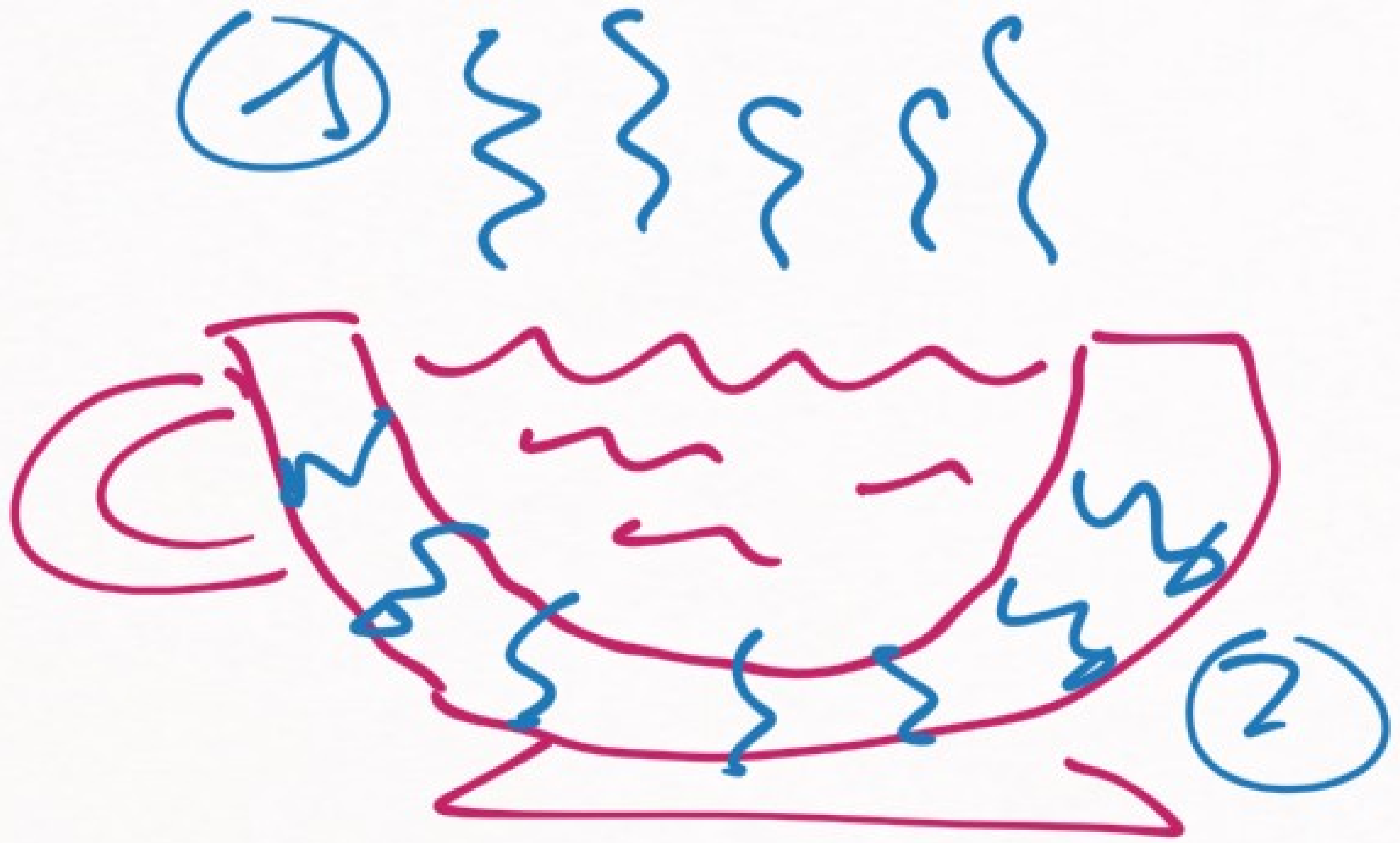
1

2

2 cools down faster

WHICH PHYSICS DO WE HAVE HERE?

FUNDAMENTAL THEORY?



How does this cool down?

1 convection → a

2 conduction → b

a → hot water/tea exposed to the air

b → hot water/tea in contact w/ the ceramic material used to make the cup

① & ② → $\boxed{\text{DESCRIBED BY A SERIES OF EQUATIONS}}$

∃ CONVECTION EQUATION

∃ HEAT TRANSFER EQUATION

(FOR CONDUCTION) FOURIER

FUNDAMENTAL THEORY

$\boxed{\text{COMMENT :}}$ this FUNDAMENTAL THEORY is not easy to solve

By solving this directly, it will be difficult to know whether the teapot or the teacup cools <u>faster</u>

→ But we don't solve this problem like that in the real world

WE KNOW THIS IS TRUE

ONE THAT

COOLS FASTER

$\boxed{\text{EFT mindset}}$

EFT's are the lazy person's way to solve a physical problem

only involves a particular way of thinking

[EFT way of thinking for teacups & teapots]

→ Step-by-step guide

1) Some dynamics (as little as possible)

Newton's law of cooling:

$$\left[\; T(t) = T_0(t_0)\, e^{-\lambda(t-t_0)} \;\right]$$ $\lambda \to$ coefficient

$T, T_0 \to$ final & initial temperature

$t, t_0 \to$ final & initial time

2) Find the relevant degrees of freedom

(find what is relevant at low energies)

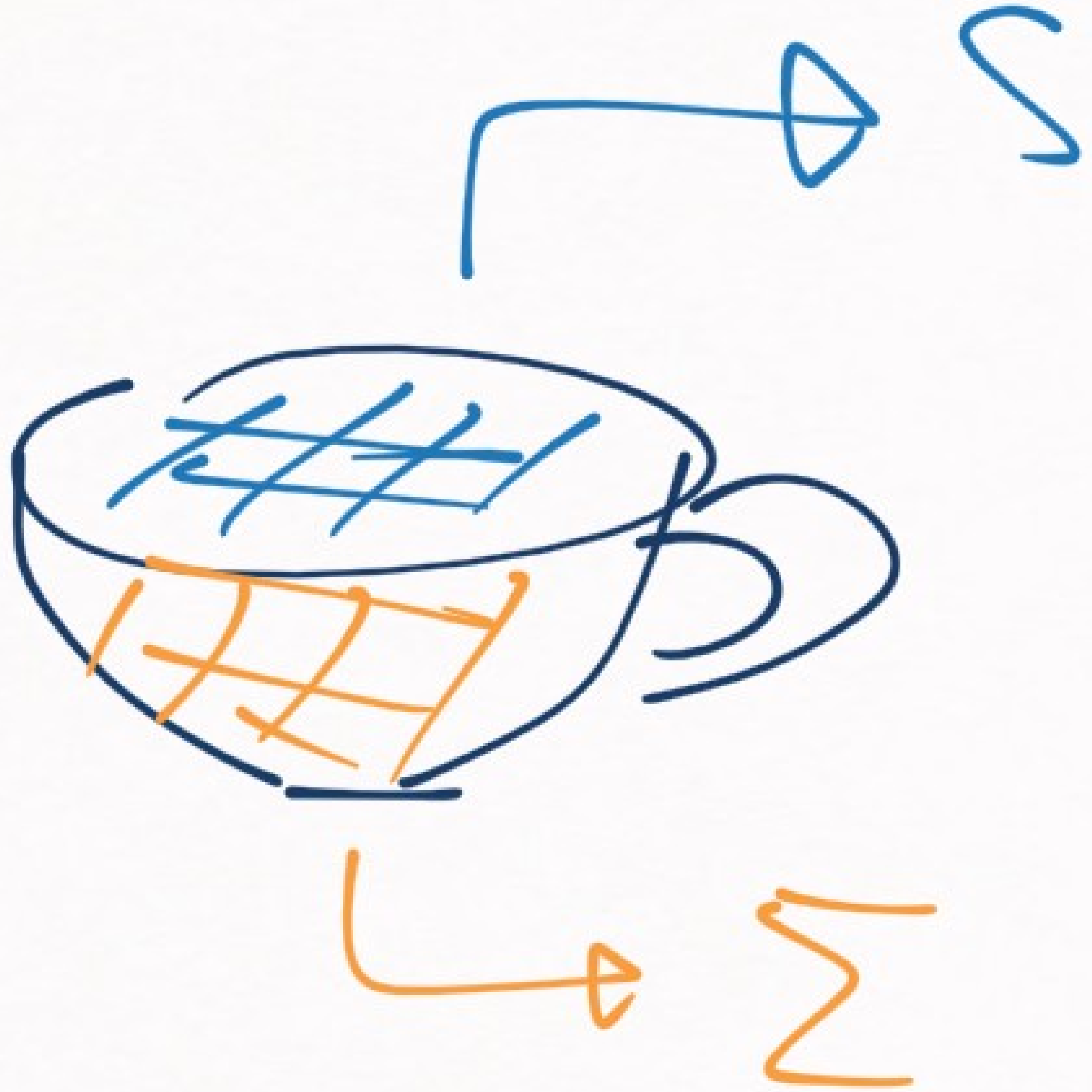
→ surface exposed to the air (S)

→ ceramic surface (Σ)

(S, Σ) → how quickly this cools down

3) Propose a "power counting"

(propose a way to identify which degrees of freedom are more important).



3.a) I know that S gives more cooling than Σ

3.b) So I can propose the following:

$T = T_0 e^{-\lambda(t-t_0)} \rightsquigarrow \underline{\underline{\lambda \propto S + \text{corrections}}}$

$\lambda = \underline{\underline{S}}(c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \dots)$

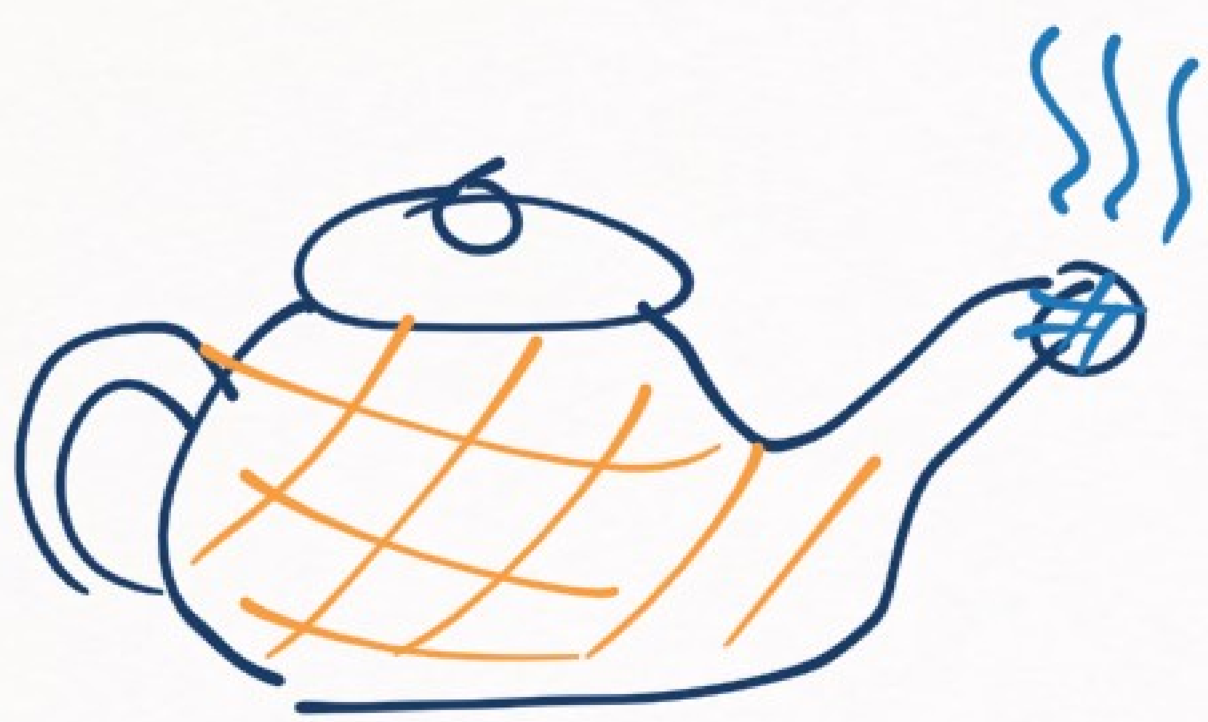
x ?

$x \propto \frac{\Sigma}{S} \rightsquigarrow x = \frac{\Sigma}{\rho S}, \underline{\underline{\rho \gg 1}}$

3.c) $\exists$ several possibilities for this expansion

$$\lambda = S\left(c_0 + c_1 x + c_2 x^2 + \dots\right), \quad x = \frac{\Sigma}{\rho S}$$

$(x < 1)$ $(\rho > 1)$



$$\lambda = \Sigma\left(d_0 + \frac{d_1}{x} + \frac{d_2}{x^2} + \dots\right)$$

$(x > 1)$

(two different power countings)

4) Write down the theory:

$$T = T_0 e^{-\lambda(t-t_0)}$$

power counting a) $\lambda = \Sigma\left(c_0 + c_1 x + c_2 x^2 + \dots\right)$

power counting b) $\lambda = \Sigma\left(d_0 + \frac{d_1}{x} + \frac{d_2}{x^2} + \dots\right)$

$\rightarrow$ LECs (low energy constants)

5) Choose the accuracy we want for our theoretical calculations

$$\lambda = S \sum_{n=0}^{\infty} c_n x^n \longrightarrow$$ PROBLEM:

infinite number of c_n's

no predictive power $\longrightarrow$ SOLUTION $\Rightarrow$

SOLUTION → choose a level of accuracy

→ $\mathcal{O}(x^1)$ $\qquad \lambda = S c_0 (1 + \mathcal{O}(x))$ LO

→ $\mathcal{O}(x^2)$ $\qquad \lambda = S(c_0 + c_1 x + \mathcal{O}(x^2))$ NLO

→ $\mathcal{O}(x^3)$ $\qquad \lambda = S(c_0 + c_1 x + c_2 x^2 + \mathcal{O}(x^3))$ NNLO

LO → leading order

NLO → Next-to-leading order

NNLO → Next-to-next-to-leading order

More terms → more accuracy

(but less predictive power / more parameters)

$x \simeq 0.1$ $\Rightarrow$ $\boxed{\text{Errors}}$

LO $\rightarrow$ 10% error

NLO $\rightarrow$ 1% error

NNLO $\rightarrow$ 0.1% error

(Example for $x \simeq 0.1$)

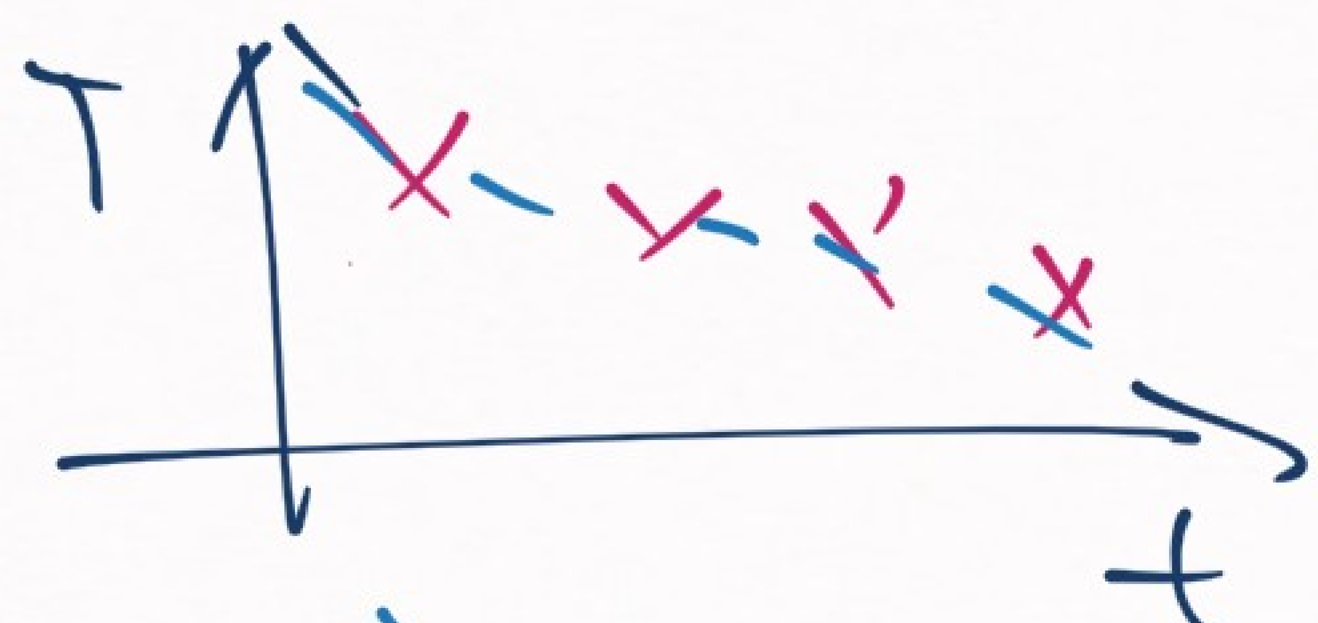
$\rightarrow$ for $\forall$ EFT, x will be different

6) Fit the LECs to experiment

c_n's / d_n's

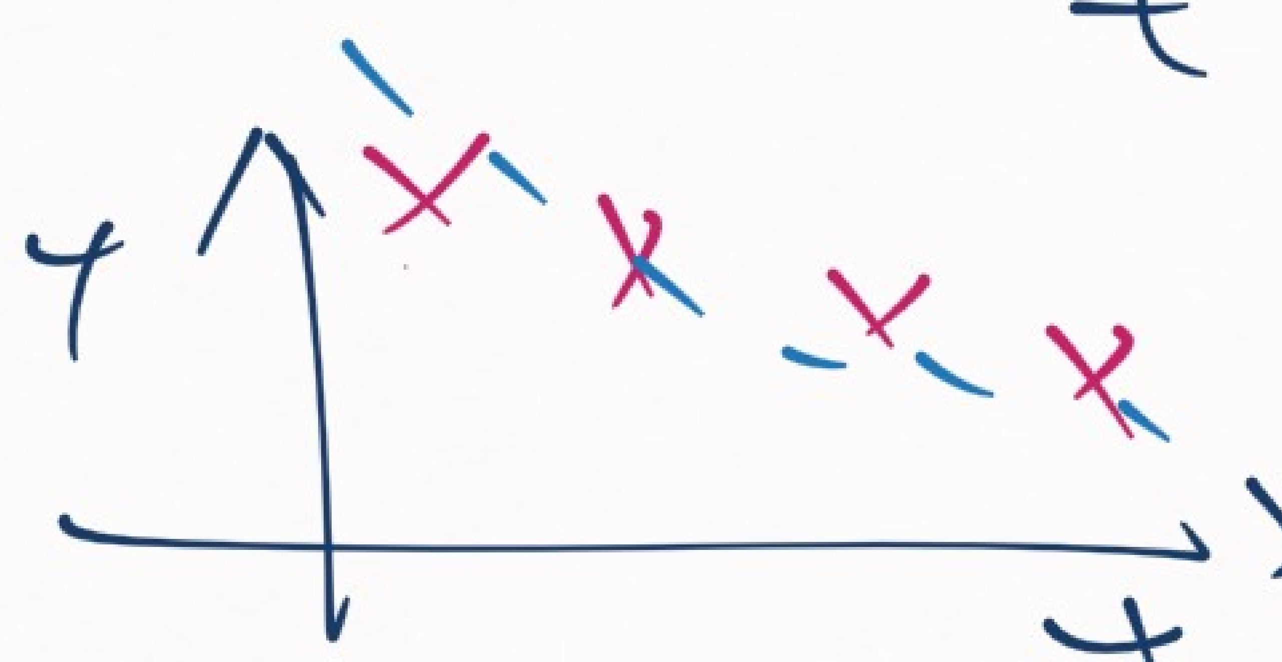
→ Instruments

① 



→ λ $\quad$ S, Σ

② 



→ λ' $\quad$ S', Σ'

c_0, c_1

[ALGORITH FOR FORMULATING AN EFT]

1) Some dynamics (cooling/QM/QFT)
2) Some degrees of freedom (types of surfaces/ /types of particles/...)
3) Power counting ($x < 1$, $\sum c_n x^n$)
4) Write down the theory ($\rightarrow$ LECs)
5) Choose the accuracy (LO, NLO, N^2LO, ...)
6) Fit the LECs / make predictions

MORE TYPICAL EXAMPLES

IN THE NEXT LESSONS