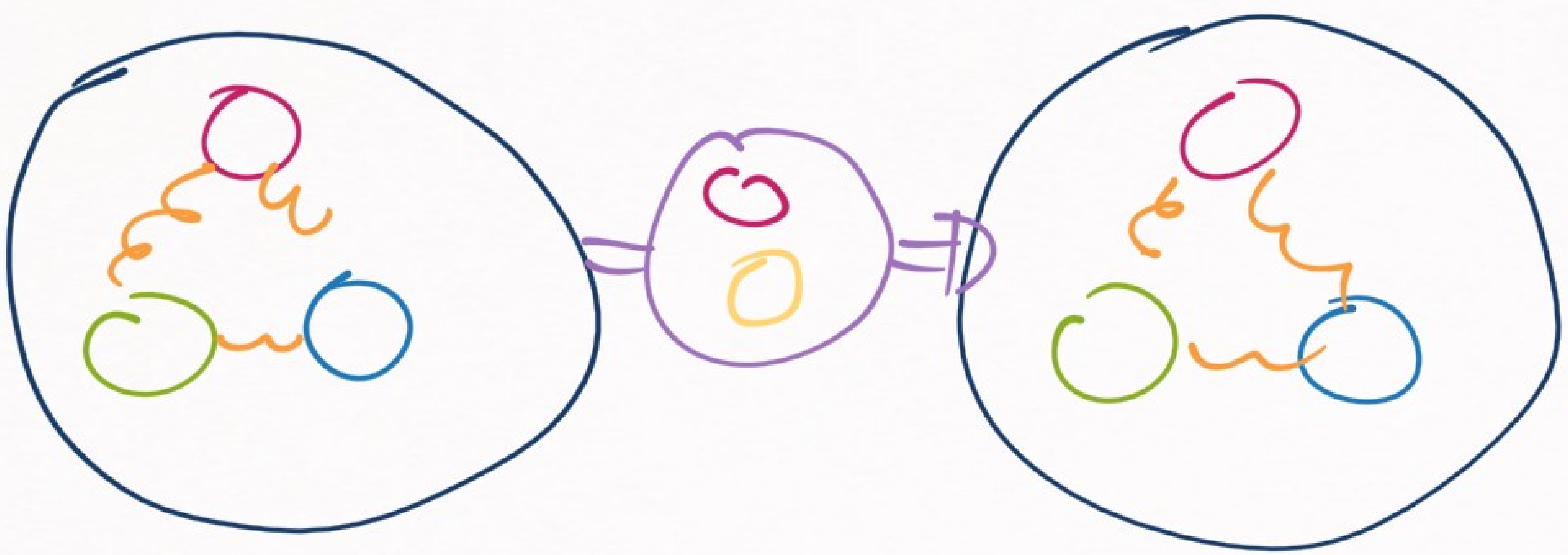
NUCLEAR PHYSICS 6

QUANTUM CHROMODYNAMICS

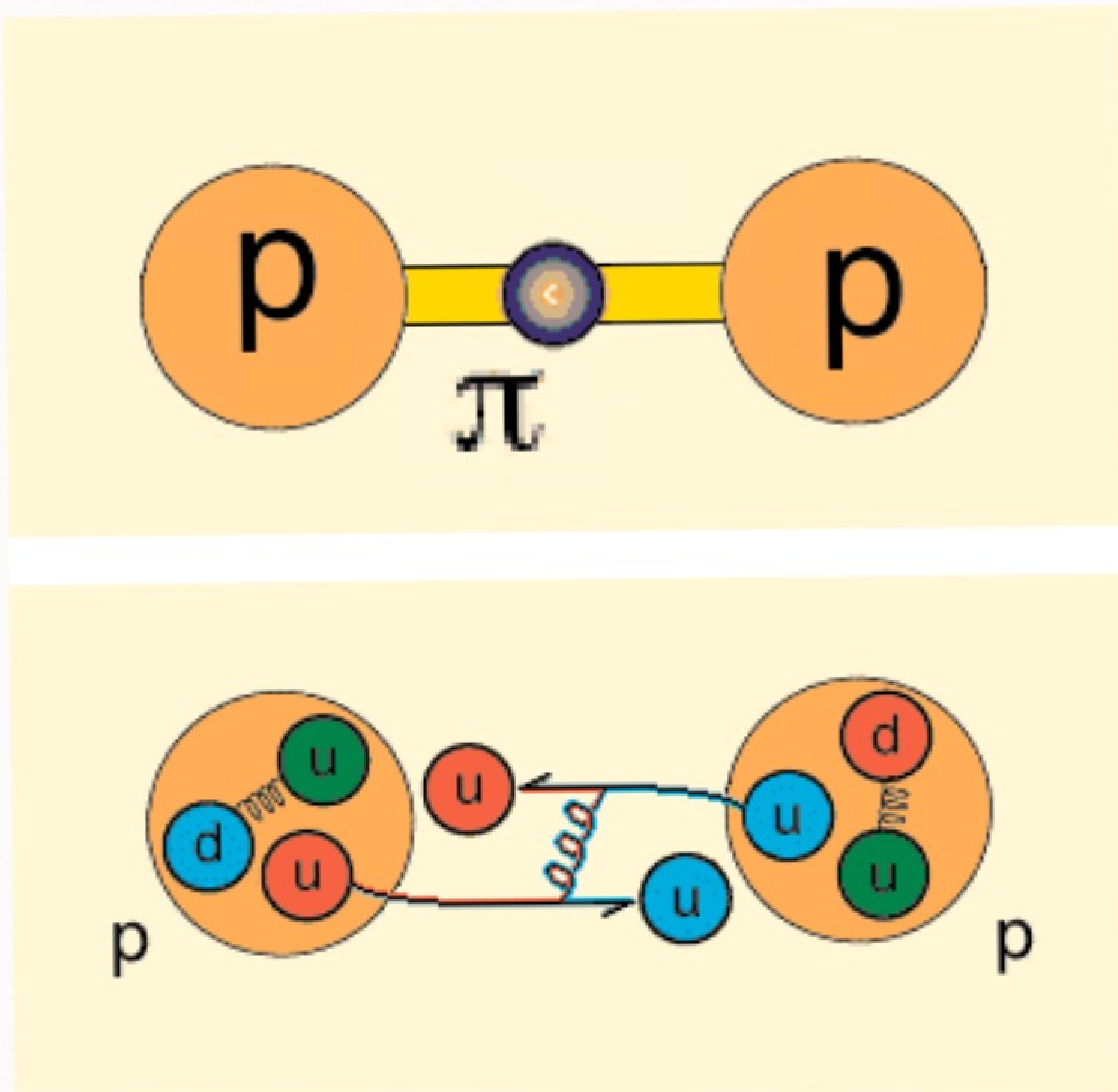
(a qualitative introduction)

$\boxed{\text{RECAP}}$ → Nuclear forces are "residual forces"



nucleon & pion

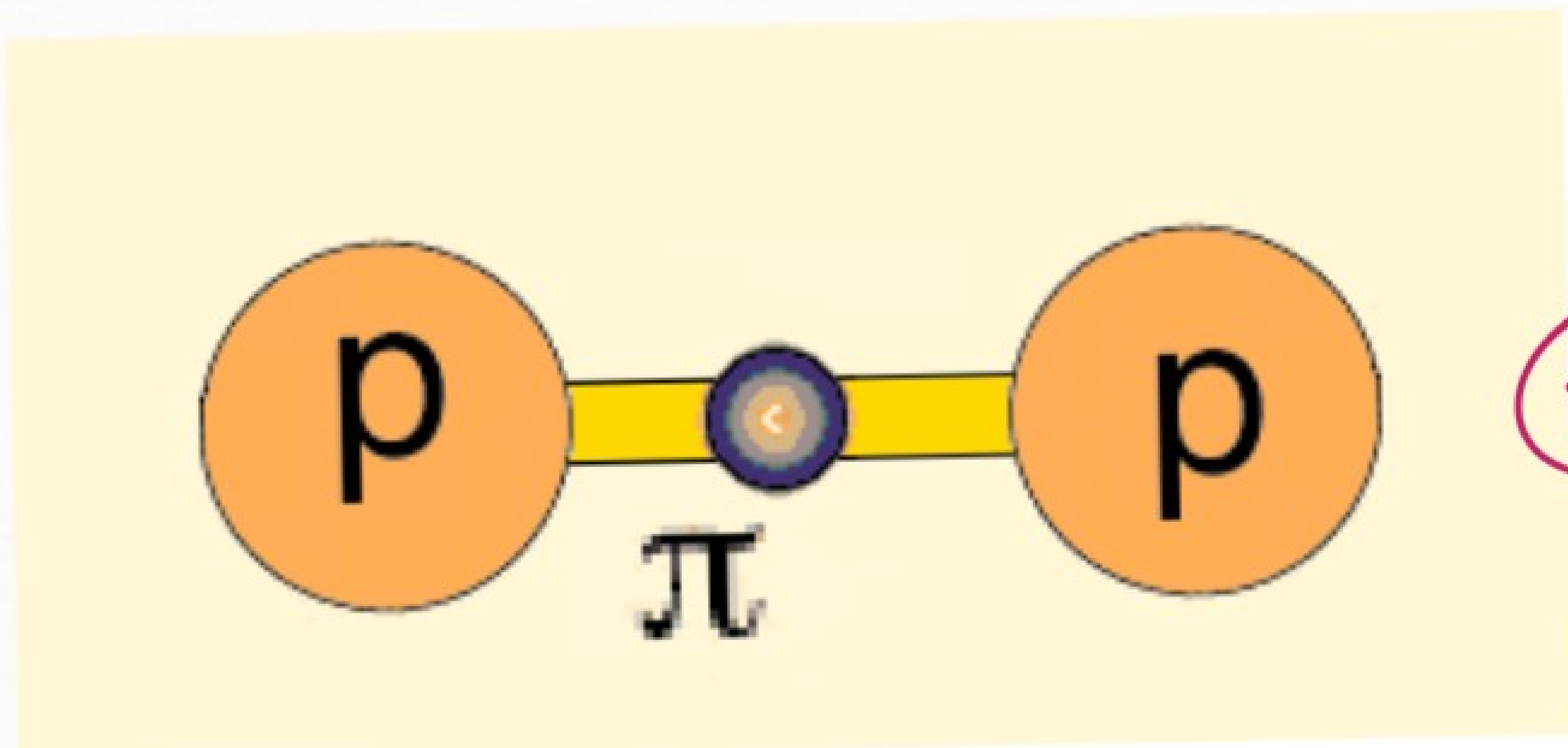
→ gluons & quarks



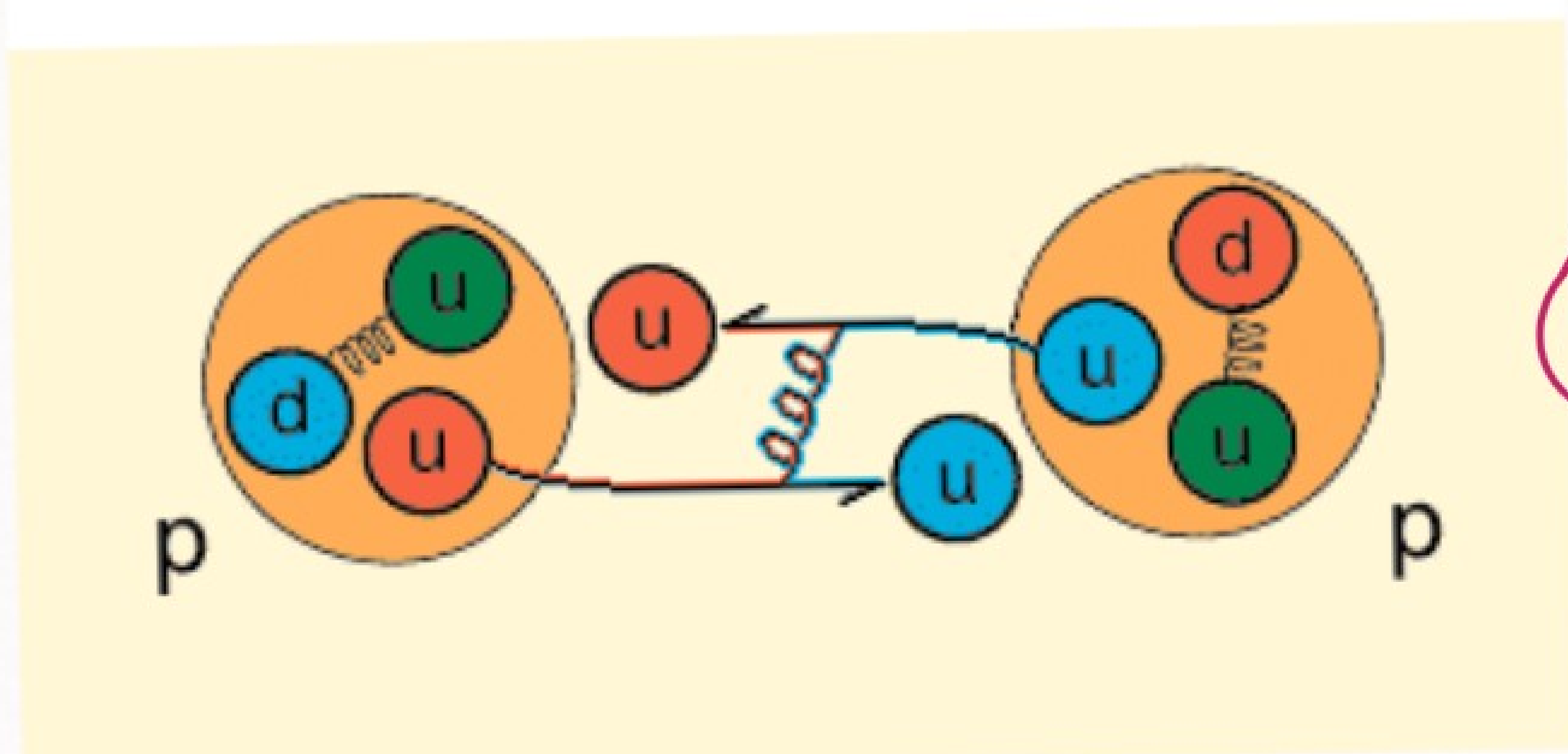
nucleons exchanging pions

quark and gluons being exchange all over the place

(1)



(2)



PROBLEM

How do we explain (1) in terms of (2) ?

$\Downarrow$

Understand QCD

QCD (Quantum chromodynamics)

↳ Fundamental theory that describes quarks & gluons

→ Let's begin with QED

(Quantum electrodynamics)

1) QED is well-known

2) QED is a "simplified version" of QCD

QCD is a theoretical extension of the QED idea

$\boxed{\text{QED}}$ → theory of electrons & photons

↳ QFT textbook → ∀ begins w/ a Lagrangian

interaction

$$\mathcal{L}_{QED} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \rightarrow \otimes$$

Dirac field of the electron

photon field

$\circledast$ → Details not important for now

IMPORTANT IDEA } → QED is a $\boxed{\text{gauge theory}}$

↓

QED & QCD

?

↓

$\circledast$

⊛ → SYMMETRIES

$$\mathcal{L}_{Dirac} = \bar{\psi}(i\not{\partial} - m)\psi$$

$$\psi(x) \to e^{iea}\psi(x)$$

$$\bar{\psi}(x) \to e^{-iea}\psi(x)$$

ψ → Dirac field (spin-1/2 particle)

$$\not{\partial} = \gamma^{\mu}\partial_{\mu}$$

$$\bar{\psi} = \psi^{\dagger}\gamma_0$$

Peskin & Schröder

$$\left.\begin{array}{l}\psi(x) \to e^{ie\alpha}\psi(x) \\ \mathcal{L}_{Dirac} \to \mathcal{L}_{Dirac}\end{array}\right]$$ a symmetry of the Dirac Lagrangian

$\Downarrow$

Global symmetry

$\psi(x) \to \boxed{e^{ie\alpha}}\,\psi(x)$

does not depend on "x"

same everywhere (Global)

$$\psi(x) \to \underbrace{e^{ie\alpha}}\,\psi(x)$$

$U(1)$ symmetry $\longrightarrow$ Group theory

$$U(N) = \{ N \times N \text{ matrices} \,/\, U^{\dagger}U = 1 \}$$

$$U(1) \to z^{\dagger} z = 1 \to \boxed{\begin{aligned} z &= e^{i\varphi} \\ z^{\dagger} &= e^{-i\varphi} \end{aligned}}$$

GLOBAL SYMMETRY $\longrightarrow$ $\psi(x) \to e^{ie\alpha}\psi(x)$

$\Downarrow$

LOCAL SYMMETRY $\longrightarrow$ $\psi(x) \to \underbrace{e^{ie\alpha(x)}}_{\text{now it's local}}\psi(x)$

$\Downarrow$

IS THIS A SYMMETRY OF $\mathcal{L}_{Dirac}$? NO! $\to$ $\oplus$

$$\mathcal{L}_{Dirac} = \bar{\psi}(x)(i\not\partial - m)\psi(x)$$

$\bar{\psi}(x)\psi(x)$ (1) $\qquad \bar{\psi}(x)\not\partial\psi(x)$ (2)

$$\psi(x) \to e^{ie\alpha(x)}\psi(x)$$

(1) $\bar{\psi}(x)\psi(x) \to \bar{\psi}(x)e^{-ie\alpha(x)}e^{ie\alpha(x)}\psi(x)$

$$= \bar{\psi}(x)\psi(x)$$

(2) $\bar{\psi}(x)\partial_\mu\psi(x) \to \bar{\psi}(x)e^{-ie\alpha(x)}\partial_\mu e^{ie\alpha(x)}\psi(x)$

$$* \longrightarrow \bar{\psi}(x)\partial_\mu \psi(x) \rightarrow \bar{\psi}(x)\partial_\mu \psi(x) + \underbrace{ie\,\partial_\mu \alpha\, \bar{\psi}(x)\psi(x)}_{\text{breaks the local symmetry}}$$

$\mathcal{L}_{Dirac}$ is not local $U(1)$ symmetric:

$$\mathcal{L}_{Dirac} \xrightarrow[\text{local}]{U(1)} \mathcal{L}_{Dirac} - e\,\partial_\mu \alpha\, \bar{\psi}(x)\psi(x)$$

However, we can fix $\mathcal{L}_{Dirac}$ so it will become local $U(1)$-symmetric

→ If we do this, electromagnetism will appear naturally

HOW TO DO IT

1) Introduce a new field:

$$A_\mu \xrightarrow[U(1)]{} A_\mu + \partial_\mu \alpha$$

2) Define a new type of derivative:

$$\cancel{\partial_\mu} \rightsquigarrow \boxed{D_\mu \equiv \partial_\mu - ieA_\mu} \longrightarrow (?)$$

$$\oplus \to \bar\psi(x) D_\mu \psi(x) \to \bar\psi(x)\left(\partial_\mu - \overbrace{ieA_\mu - ie\cancel{\partial_\mu\alpha}}^{5}\right)\psi$$

$$\left(\bar\psi(x)\partial_\mu\psi(x) \to \bar\psi\cancel{(x)\partial_\mu}\psi(x)\right) \qquad + ie\cancel{\partial_\mu\alpha}\,\bar\psi\psi$$

$$\| $$

$$\oplus$$

$$\oplus = \bar\psi(x)(\partial_\mu - ieA_\mu)\psi(x) = \bar\psi(x) D_\mu \psi(x)$$

$$\Rightarrow \boxed{\bar\psi(x) D_\mu \psi(x) \to \bar\psi(x) D_\mu \psi(x)}$$

$$\mathcal{L}_{QED} = \underbrace{\bar{\psi}(i\not{D} - m)\psi}_{\text{local } U(1) \text{ symmetry}} + \underbrace{(\ldots)}_{\text{we need a new term to describe } A_\mu}$$

$$A_\mu \rightarrow A_\mu + \partial_\mu \alpha \quad \times \quad \checkmark$$

(also $U(1)$ local)

$$\boxed{\partial_\mu A_\nu - \partial_\nu A_\mu} \rightarrow \boxed{\partial_\mu A_\nu - \partial_\nu A_\mu} + \cancelto{0}{\partial_\mu \partial_\nu \alpha - \partial_\nu \partial_\mu \alpha}$$

$\mathcal{L}_{QCD} = \bar{\psi}(i\cancel{D} - m)\psi + (\ldots)$

for $A_\mu \rightarrow F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$\downarrow$

$(\ldots)$ is written in term of $F_{\mu\nu}$

$(F_{\mu\nu})^2 \rightsquigarrow \lambda F_{\mu\nu} F^{\mu\nu}$

$\downarrow$

we remove the Lorentz indices

$$\mathcal{L}_{QED} = \bar{\psi}(i\not{\partial} - m)\psi + \lambda F_{\mu\nu}F^{\mu\nu}$$

We have to determine this

$\exists$ a kinetic term for the A_μ field

$A_\mu \to$ spin - 1 field

Let's compare this with spin-0 field $\bigg\} \to$ $\mathcal{L}_{KG} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}m^2\phi^2$

Kinetic term for Klein-Gordon:

$$\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi \longrightarrow -\frac{1}{2}\phi\,\Box^2\phi$$ (integration by part;

$$\left(S = \int d^4x\,\mathcal{L}\right)$$

$$\int d^4x\,\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi =$$

$$\int d^4x\left(-\frac{1}{2}\phi\,\Box^2\phi\right)\Big)$$

equivalent $\overline{\overline{S}}$

fundamental quantity

Spin-0: $-\frac{1}{2}\phi\,\Box^2\phi \longrightarrow$ spin-1 ?

$$A_\mu = (A_0, \vec{A}) \rightsquigarrow -\frac{1}{2}\underbrace{\vec{A}\,\Box^2\vec{A}}_{\text{for } \vec{A}}$$

$\vec{A}$ → physical photon

$\not\!\!\!\Delta\ A_0, \vec{A}$

$$A_\mu(x) = \underset{\text{polarization}}{\epsilon_\mu}\, e^{-i q\cdot x}, \quad q_\mu \epsilon^\mu = 0$$

$\rightsquigarrow$ the metric: $g_{\mu\nu} = \begin{pmatrix} 1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix}$

$$-\frac{1}{2}\left(\vec{A}\,\Box^2\right)\vec{A}$$

$$\longrightarrow \frac{1}{2} A_\mu \Box^2 A^\mu$$

$$\left(-\frac{1}{2}\phi\,\Box^2\phi\right)$$

sign difference

$$\mathcal{L}_{kin}(A_\mu) = +\frac{1}{2} A_\mu \Box^2 A^\mu \Rightarrow \boxed{\lambda = -\frac{1}{4}}$$

$$\boxed{\mathcal{L}_{QED} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}}$$

$$\left\{ \begin{array}{c} \text{A CONSEQUENCE OF APPLYING LOCAL} \\ \text{U(1) SYMMETRY TO } \mathcal{L}_{Dirac} \end{array} \right\}$$

$\boxed{\text{QCD}}$ $\longrightarrow$ Just like QED, but w/ a different local symmetry group

QED $\to$ local $U(1)$

QCD $\to$ local $SU(3)$

$SU(N) = \{ N \times N \text{ matrices} \,/\, U^{\dagger}U = 1,\ \det U = 1 \}$

Why SU(3)?

→ $\boxed{\text{QED}}$ → electrons & photons

1 type of charge

→ $\boxed{\text{QCD}}$ → quark & gluons

3 types of charges

→ colors $\boxed{\text{red green blue}}$

$$\psi(x) \longrightarrow \psi_a(x)\,, \quad a = R, G, B$$
$$(a = 1, 2, 3)$$

SYMMETRY: $\sum_a \bar{\psi}_a(x)\,\psi_a(x) \longrightarrow \sum_a \bar{\psi}_a(x)\,\psi_a(x)$

$\rightarrow$ complex field : $U(3)$ or $SU(3) \longrightarrow$

(real field $\rightarrow$ $O(3)$ or $SO(2)$)

⊕ → SU(3) because experimentally hadrons don't have a strong charge

gluon

never happens → $\cancel{U(3)}$

[QCD Lagrangian] → for completeness

→ Quarks come in different types (flavors)

6 flavors : $\boxed{u, d, s, c, b, t}$

$$\mathcal{L}_{\text{quarks}} = \sum_{j=1}^{n_f} \bar{\psi}_j \left(i\not{\partial} - m_j \right) \psi_j$$

→ add local SU(3) symmetry

$(n_f = 6)$

⊛ → define D_μ , add a new term for the gluon fields

$$\mathcal{L}_{QCD} = \sum_{j=1}^{n_f} \bar{q}_j (i\not{D} - m_j) q_j - \frac{1}{4} G_{\mu\nu}^{a} G^{a\mu\nu}$$

$$\wr \qquad\qquad\qquad\qquad \wr$$

$$\left(\bar{\psi}(i\not{D} - m)\psi \qquad - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right)$$

→ analogous to QED

but more complex → more types of gluons

1 type of gluon for each "generator" of the local symmetry group

$\boxed{\text{REVIEW GROUP THEORY}}$

$SU(N)$ has N^2-1 gens

$U(N) \to N^2$

$SO(N) \to \frac{N(N-1)}{2}$

SU(3) → 8 generators → 8 types of gluons

$$D_\mu = \partial_\mu - ig \sum_a \frac{\lambda^a}{2} A^a_\mu \quad , \quad \underline{a = 1, \dots, 8}$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \boxed{g f_{abc} A^b_\mu A^c_\nu}$$

non abelian SU(3)

$$\underbrace{\left[\frac{\lambda^b}{2}, \frac{\lambda^c}{2} \right]}_{\text{non commutative (non abelian)}} = i f^{abc} \frac{\lambda^a}{2} \longrightarrow \text{3×3 matrices}$$

generators of SU(3)

Gell-mann matrices

SU(2) $\rightarrow$ generators are σ_i, $i=1,2,3$

$[\sigma_i, \sigma_j] = i\epsilon_{ijk}\sigma_k$

Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

non-abelian

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

(SU(3) analogous, but more complex)

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

BASIC IDEA → QCD is a complicated version of QED

DIFFERENCES → $N_c = 3$ & $N_f = 6$

colors

flavors

↓

non-trivial effects

most important QCD surprise

→ $\boxed{\text{ASYMPTOTIC FREEDOM}}$

The strength of the strong interaction goes to zero at high energies

(or short distances)

→ Why does this happen?

[BECAUSE GLUONS CARRY COLOR]

≠ QED, WHERE PHOTONS ARE NEUTRAL

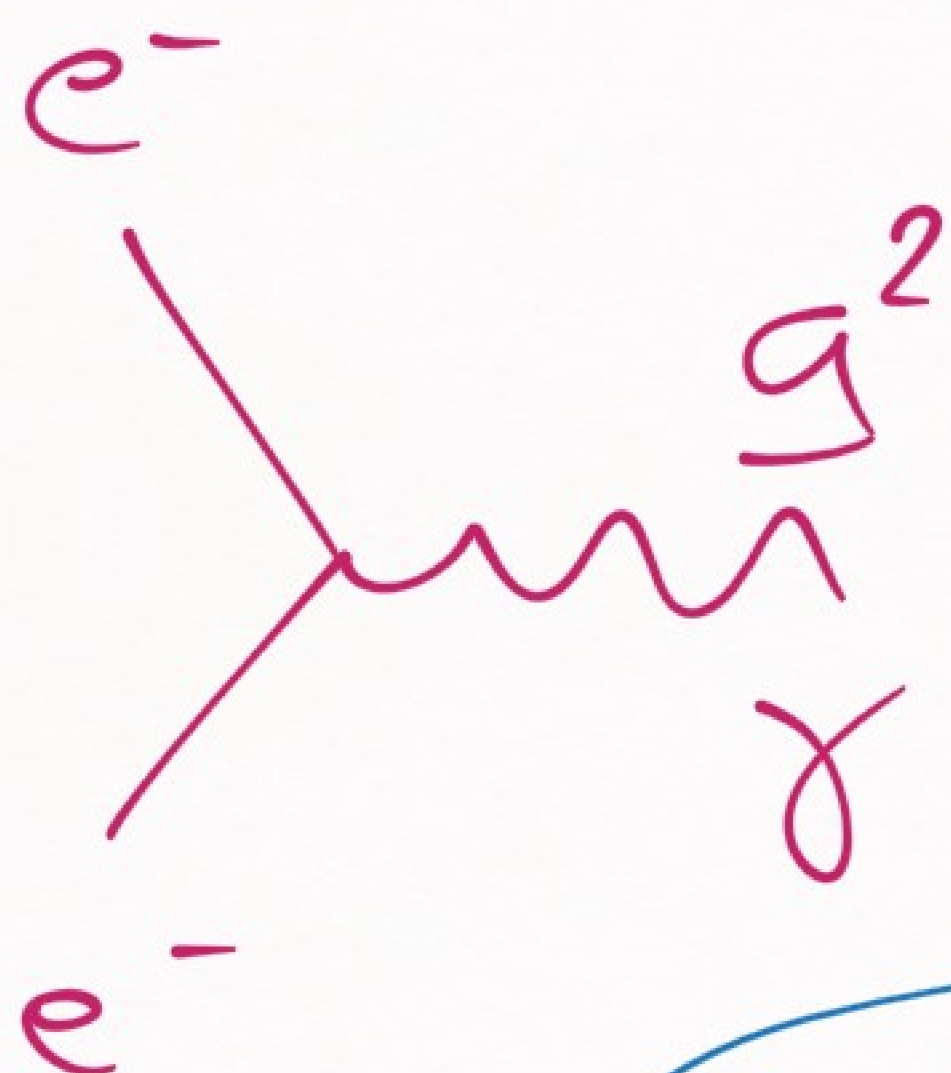
RED-ANTIGREEN

(OR ANTIRED-GREEN)

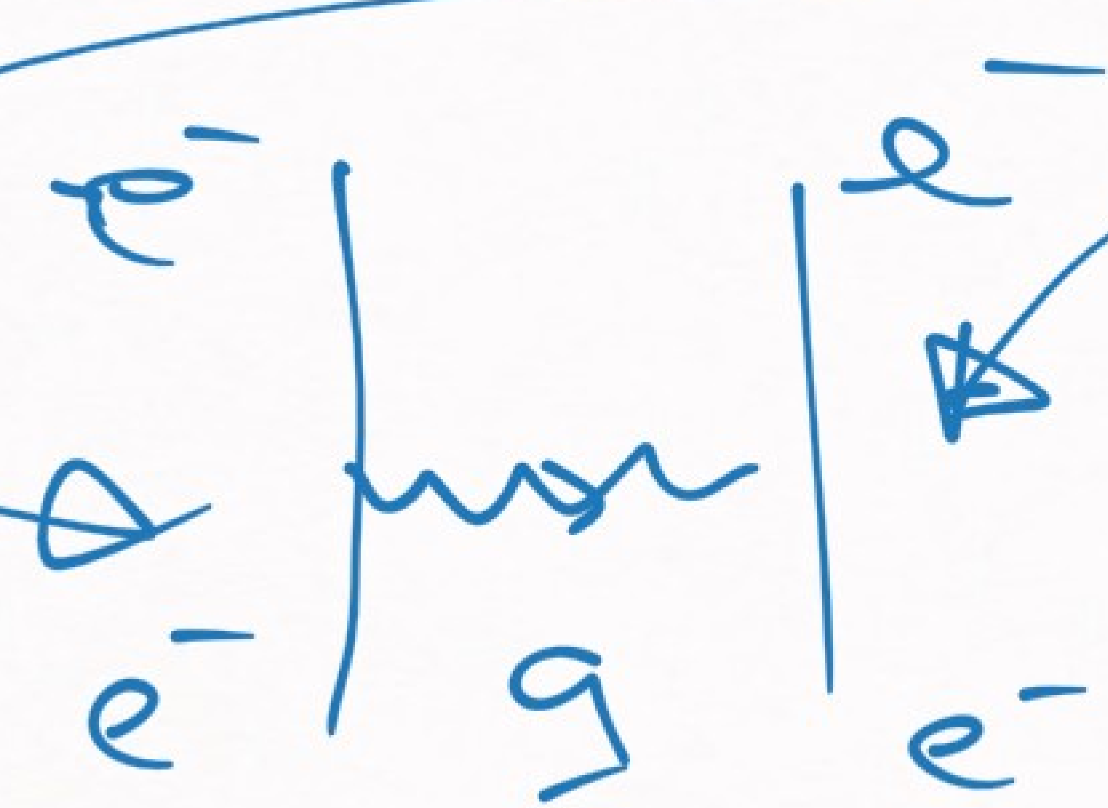
RED QUARK

GREEN QUARK

QED $\rightarrow$ strength of electromagnetism



$$= ie\gamma^\mu \cong i\sqrt{4\pi\alpha}$$



$$V(\bar{q}) = \frac{4\pi\alpha^2}{q^2}$$

QUANTUM CORRECTIONS TO THE STRENGTH OF α

$$\propto i\sqrt{4\pi\alpha(q^2)}$$

→ effective strength of electromagnetism changes w/ energy

$$V(q) = 4\pi \frac{\alpha}{q^2} \qquad \alpha \simeq 1/137$$

$$\xrightarrow{\text{quantum corrections}} \quad 4\pi \frac{\alpha(q^2)}{q^2} + \dots$$

coupling depends on q^2

$$\alpha(q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi} \log\left(\frac{q^2}{\mu^2}\right)}$$

$\rightarrow$ $\boxed{\text{If } q^2 > \mu^2 \Rightarrow \alpha(q^2) > \alpha(\mu^2)}$

Q^2 for which $\underbrace{\alpha(Q^2) \rightarrow \infty}_{\text{LANDAU POLE}}$

$=$

Landau pole $\Rightarrow$ $$\alpha(q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi}\log\left(\frac{q^2}{\mu^2}\right)}$$

$$\alpha(m_e^2) \simeq 1/137$$

$$1 - \frac{\alpha(m_e^2)}{3\pi}\log\left(\frac{Q^2}{m_e^2}\right) = 0$$

$$\Rightarrow \quad Q^2 = m_e^2 \exp\left(\frac{3\pi}{\alpha(m_e^2)}\right)$$

$$\boxed{Q_{\text{Landau}} \simeq 10^{280}\ \text{MeV}}$$

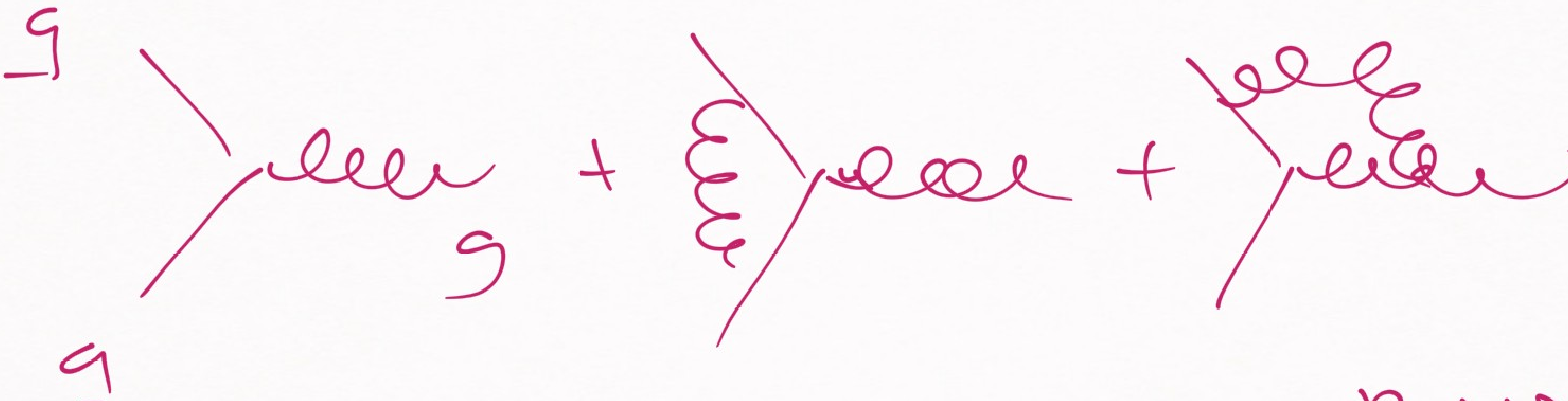
$$\rightarrow \boxed{Q_{\text{Landau}} \gg M_{\text{Planck}}}$$

↳ it's mostly a theoretical problem

(we will not ever notice
at normal energies)

→ [WHAT'S THE EQUIVALENT OF Landau IN QCD?]

↳ very different from QED



+ ... → RUNNING →

$$\rightarrow \quad \alpha_s(q^2) = \frac{\alpha_s(\mu^2)}{1 + \frac{\alpha_s(\mu^2)}{12\pi}(33 - 2n_f)\log\frac{q^2}{\mu^2}}$$

!

→ very important

(cause: gluons carry color)

$$\alpha(q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi}\log\left(\frac{q^2}{\mu^2}\right)}$$

QED : $q^2 > \mu^2 \Rightarrow \alpha(q^2) > \alpha(\mu^2)$

strength grows w/ energy

QCD : $q^2 > \mu^2 \Rightarrow \alpha_s(q^2) < \alpha_s(\mu^2)$

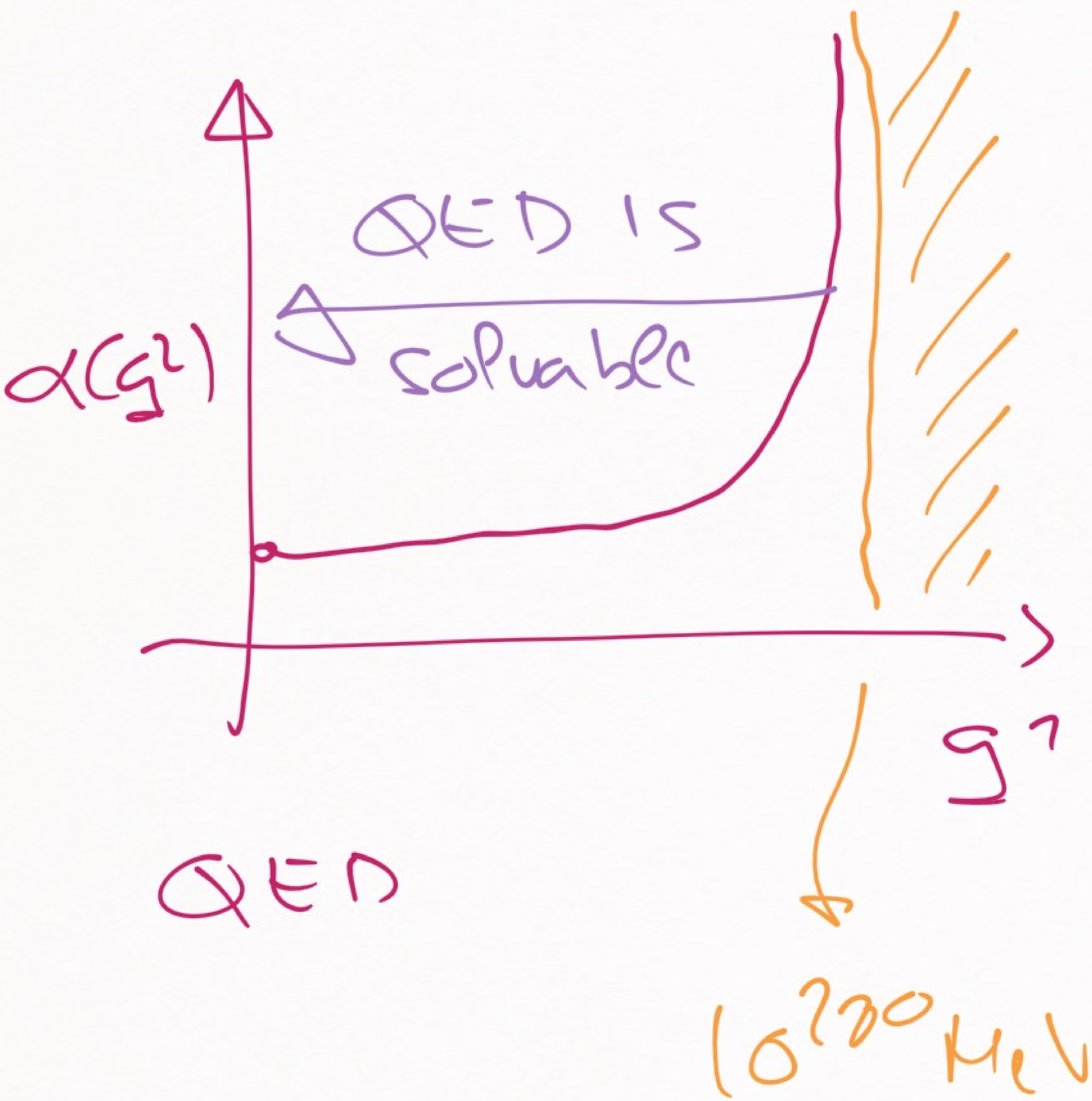
strength decreases w/ energy

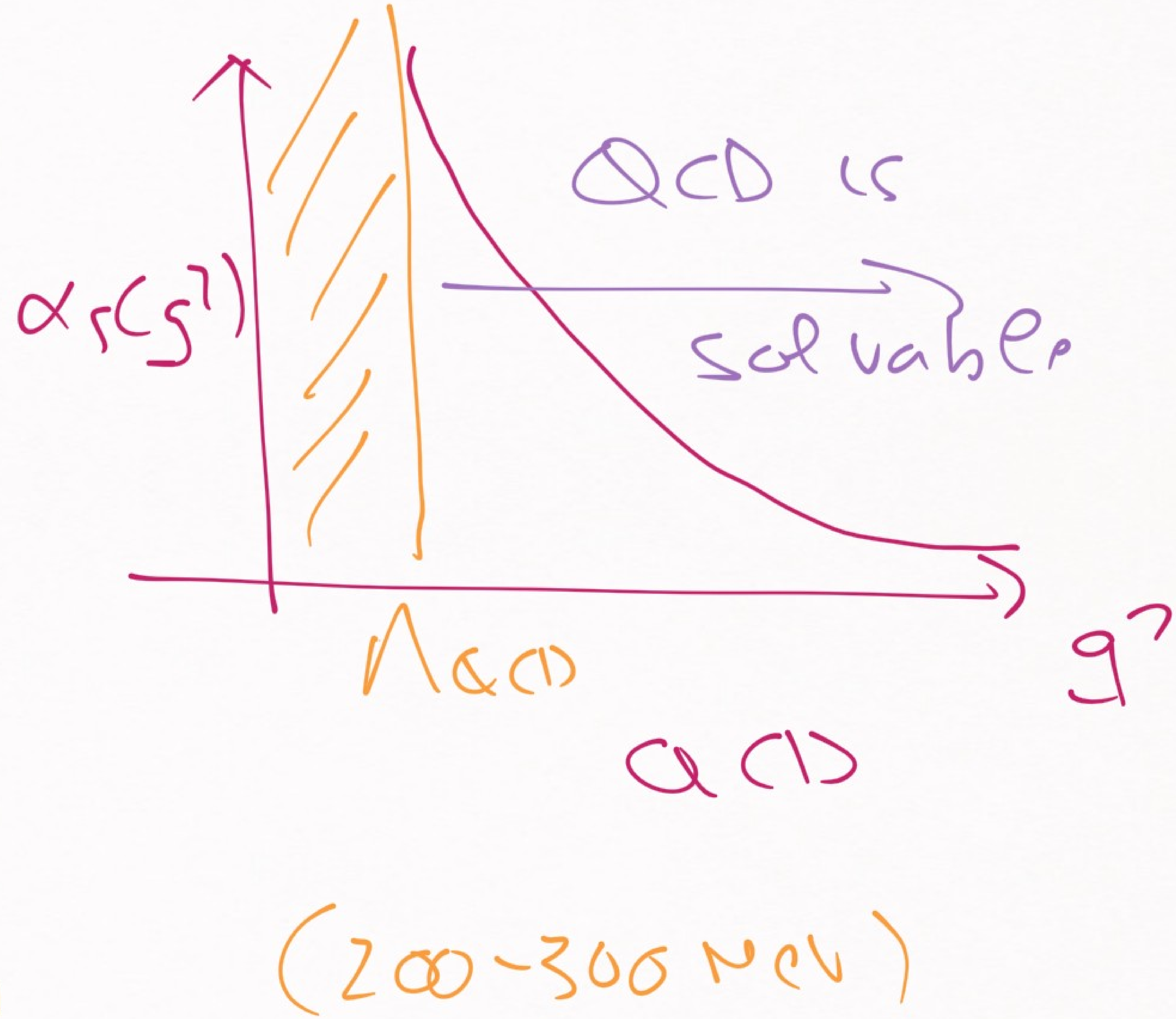
$\exists Q \,/\, \alpha_s(Q^2) \to \infty$

$$\Rightarrow \Lambda_{QCD} = \mu \exp\left[-\frac{12\pi}{(33-2n_F)\alpha_s(\mu^2)}\right]$$

$$\alpha_s(q^2) = \frac{12\pi}{(33-2n_F)\log\left(\frac{q^2}{\Lambda_{QCD}^2}\right)}$$

$$\boxed{\Lambda_{QCD} \sim (200-360)\,\mathrm{MeV}}$$


QED is solvable
α(q²)
q²
QED
10^280 MeV


QCD is solvable
α_s(q²)
Λ_QCD
q²
QCD
(200-300 MeV)

Low energy → nucleons → can't use QCD

(at least perturbative QCD)

not trivial to connect them to QCD

$\boxed{\Lambda_{QCD}}$ → natural/characteristic scale for strong interactions

u, d, s, c, b, t quarks

chiral symmetry

heavy quark symmetry

$$\boxed{m_u, m_d, m_s} \ll \Lambda_{QCD} \ll \boxed{m_c, m_b, m_t}$$

$\Lambda_{QCD} \longrightarrow$ nucleon, e, etc.

u, d quarks

$m_u, m_d \ll \Lambda_{QCD}$

1) mass of ρ-meson ($u\bar{d}$) $m_\rho = 770$ MeV

$m_\rho \neq m_u + m_d \to \Lambda_{QCD}$

$m_u \sim 2$ MeV

$m_d \sim 5$ MeV

$$\boxed{m_\rho \simeq 2\Lambda_{QCD}}$$

$m_\rho \simeq 0.6$ GeV

2) mass of nucleon (udd, uud) $m_N = 940$ MeV

$$m_N \simeq 3\Lambda_{QCD} \simeq 0.9\ \text{GeV}$$

3) D-meson $(c\bar{c})$

$m(D) \simeq 1.8\ \text{GeV}$

$m(c) \simeq 1.2\ \text{GeV}$

$$m(D) \simeq m(c) + 2\Lambda_{QCD} \simeq \underline{\underline{1.8\ \text{GeV}}}$$

$\rightarrow$ [Λ_{QCD} gives non-trivial information about QCD dynamics]

$\rightarrow$ $\boxed{\text{PROBLEM : PION}}$

4) $\pi\ (u\bar{d})$ $\quad m(\pi) \simeq 0.14\ \text{GeV}$

$$m(\pi) \simeq 2\Lambda_{QCD} \simeq 0.6\ \text{GeV}$$

$$\frac{m_\pi}{2\Lambda_{QCD}} \sim \frac{1}{4} \qquad \frac{m_\pi}{m_p} \sim \frac{1}{6}$$

small $\rightarrow$ ⊕

⊛ → ∃ fine-tuning for the pion

Pion mass fine-tuned →
- → coincidence ✗
- → conspiracy ✓ (symmetry)

$$\boxed{\frac{m_u}{\Lambda_{QCD}},\ \frac{m_d}{\Lambda_{QCD}} \ll 1}$$

$\boxed{\text{CHIRAL SYMMETRY}}$

RECAP

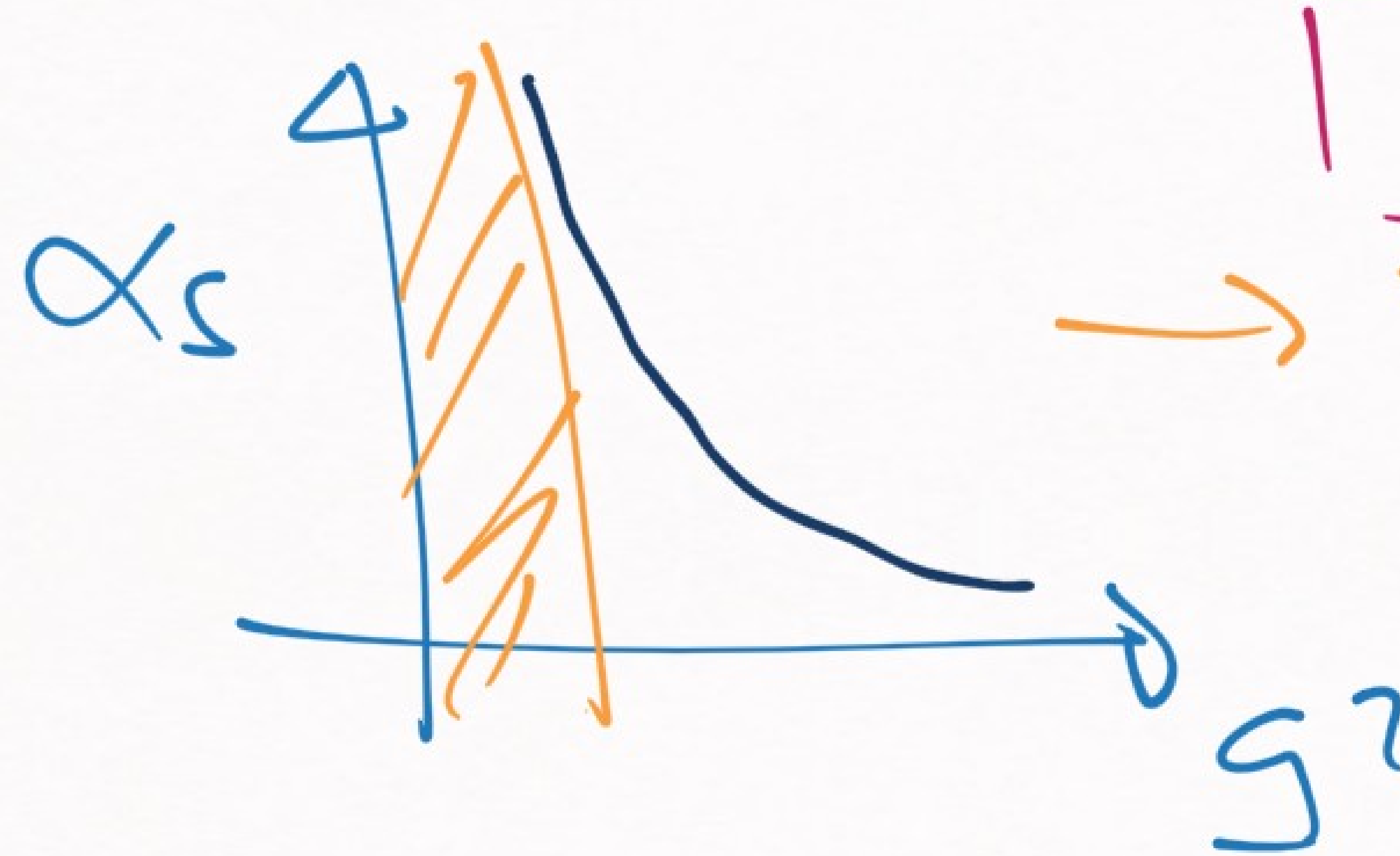
1) HADRONS → QUARKS & GLUONS

It would be nice to explain hadrons in terms of quarks & gluons

2) QUARK & GLUONS → QCD

We can try to apply perturbation theory to quarks & gluons

3) QCD → ASYMPTOTIC FREEDOM



$\boxed{\Lambda_{QCD}}$

→ we cannot use perturbation theory to describe hadrons in QCD

1) → FIND A DIFFERENT WAY

4) PION IS SPECIAL → $\boxed{\dfrac{m_\pi}{2\Lambda_{QCD}} \ll 1}$

→ check

$\exists$ PROBLEM IN EXPLAINING NUCLEAR PHYSICS FROM QCD

THEORETICIANS → WHAT WE WANT IS TO EXPLAIN NUCLEAR PHYSICS FROM QCD

FUNDAMENTAL ← PROBLEM OF NUCLEAR PHYSICS