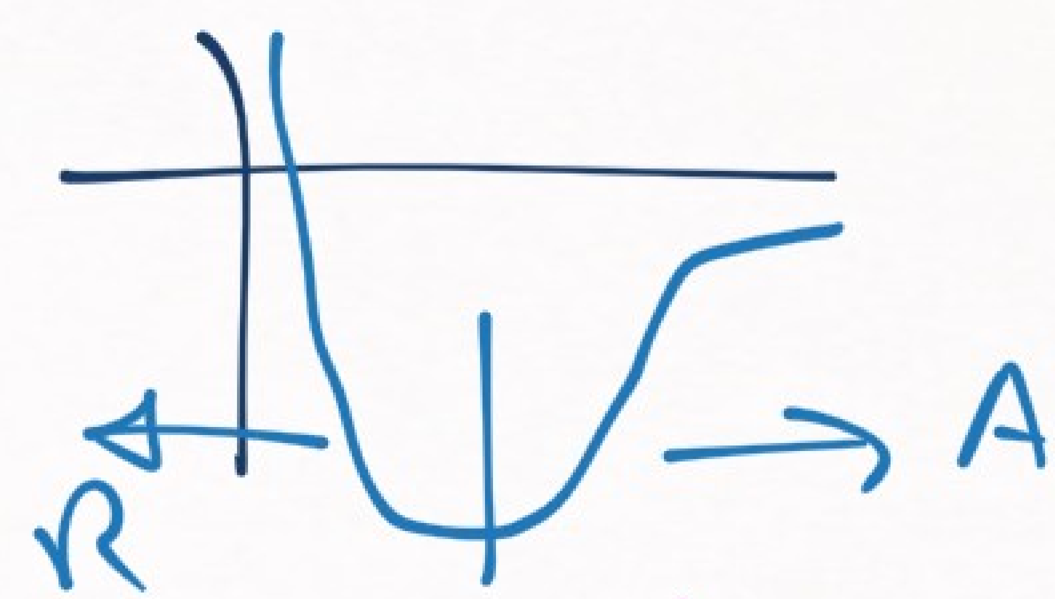
NUCLEAR PHYSICS 5

→ ORIGINS OF THE NUCLEAR FORCE

$\boxed{\text{RECAP}}$ ⟶ Properties of the nuclear force

1) Finite range of about $1/m_\pi \simeq 1.4$ fm
2) Attractive at intermediate r
3) Repulsive at short r



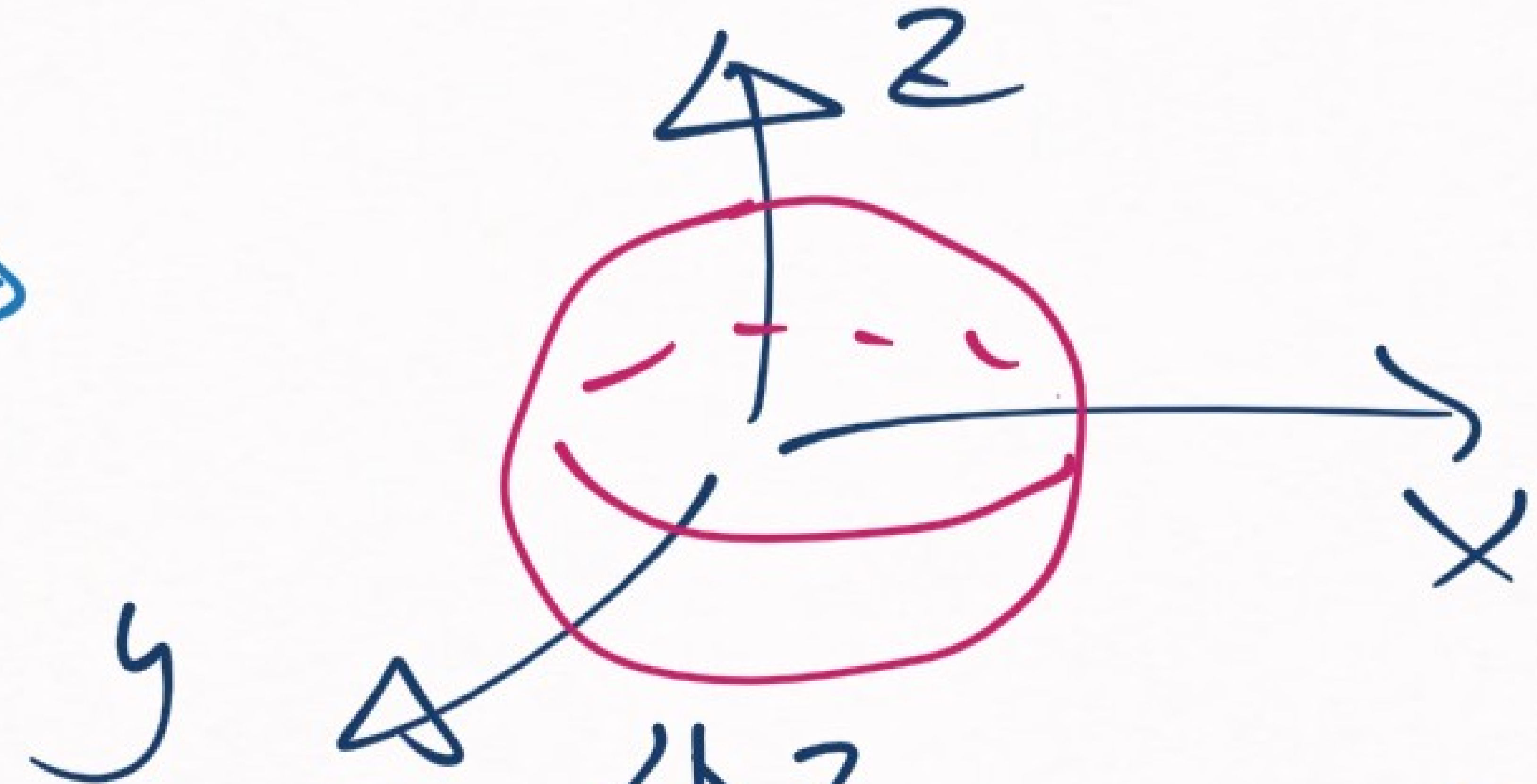
4) Does not distinguish neutrons & protons
5) Not central ⟶ Important for today

$\boxed{\text{Not central}} \longrightarrow \boxed{\text{QUADRUPOLE MOMENT OF THE DEUTERON}}$

deuteron $\rightarrow \rho(\vec{r})$ (charge density)

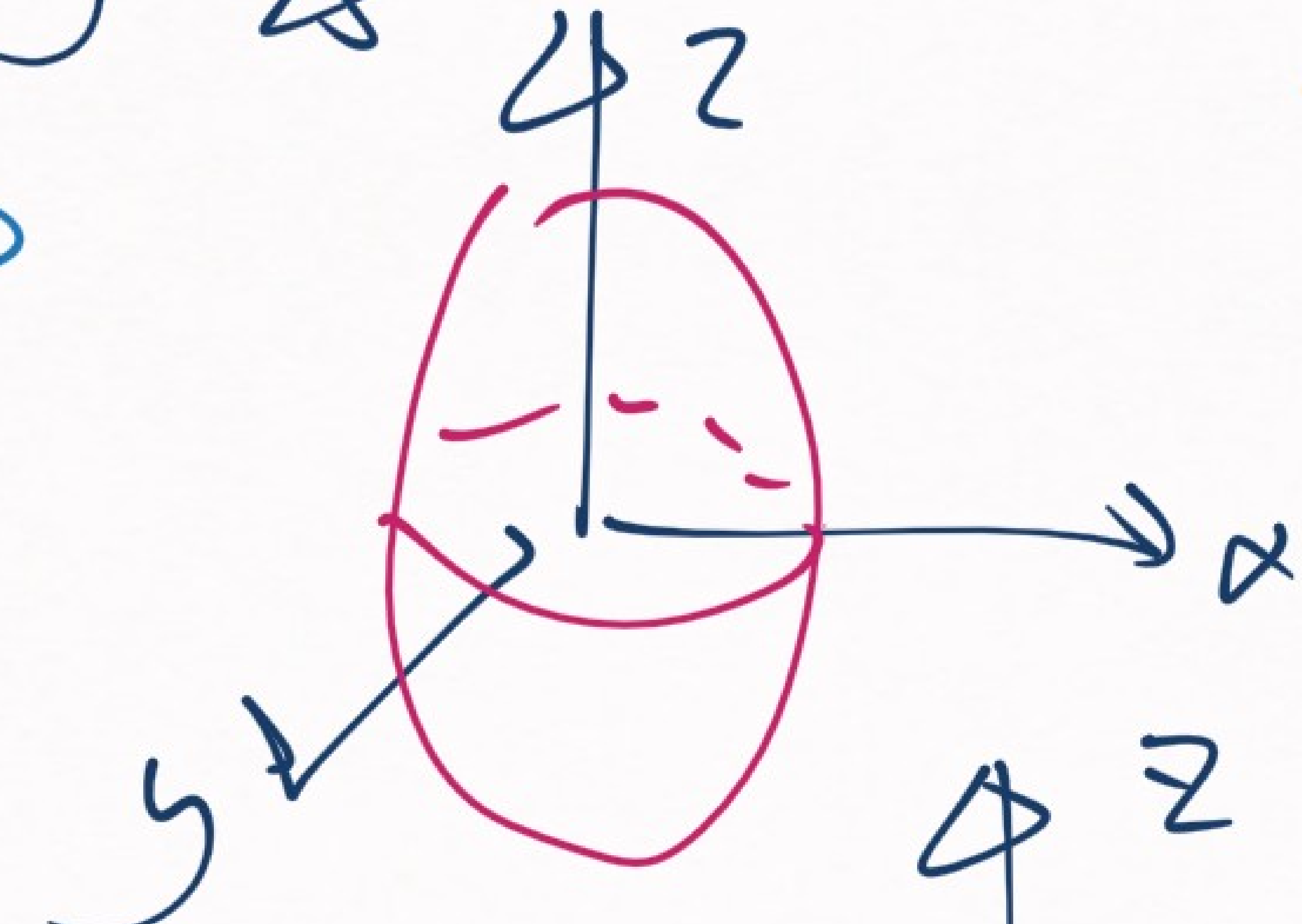
$$\underbrace{Q}_{\text{quadrupole moment}} = \int d^3\vec{r}\, \rho(\vec{r})(3z^2 - r^2)$$

$Q = 0 \longrightarrow$



Spherical distribution

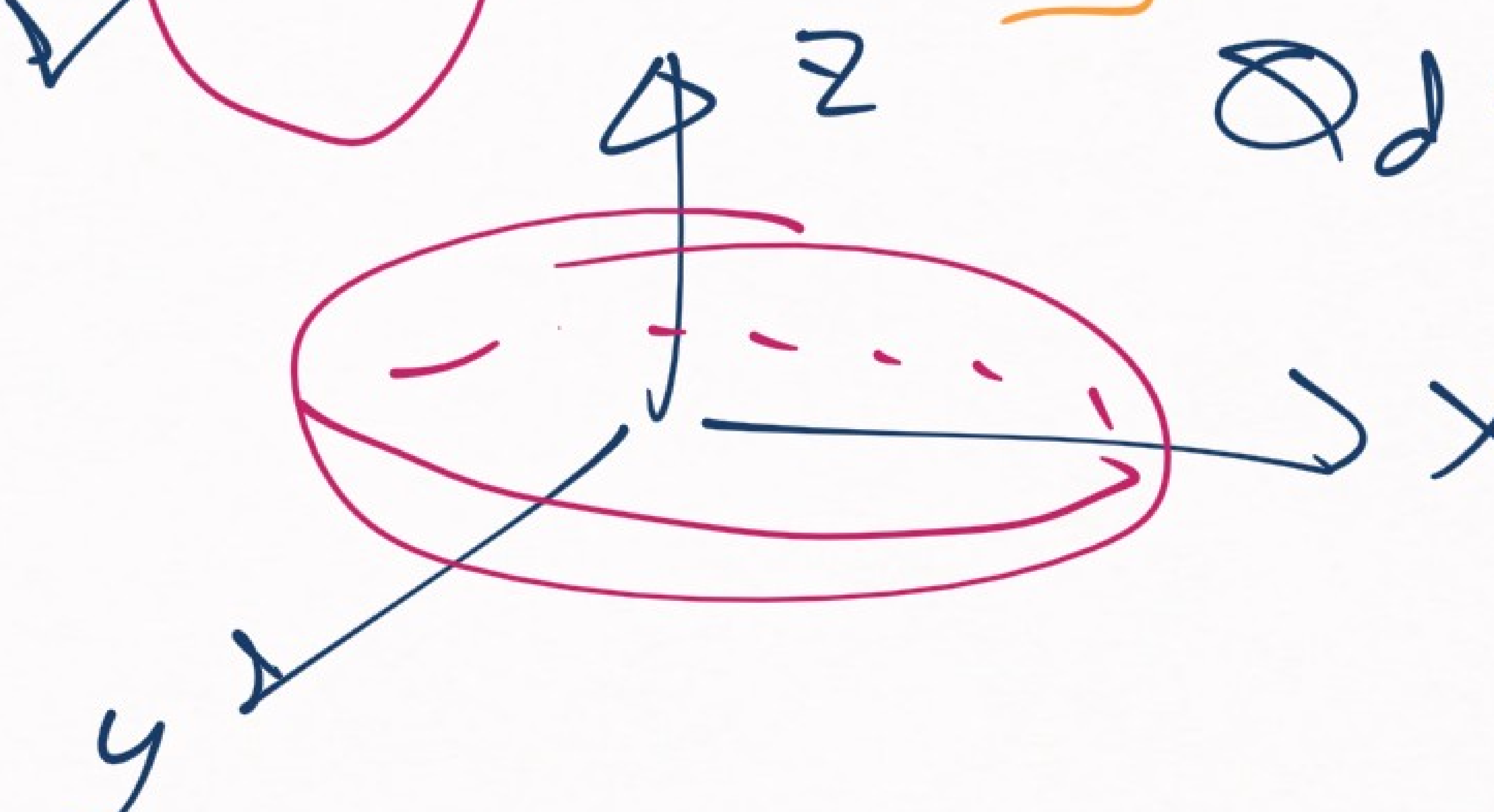
$Q > 0 \longrightarrow$



Deuteron

$Q_d \simeq 0.286 \; e\,fm^2$

$Q < 0 \longrightarrow$

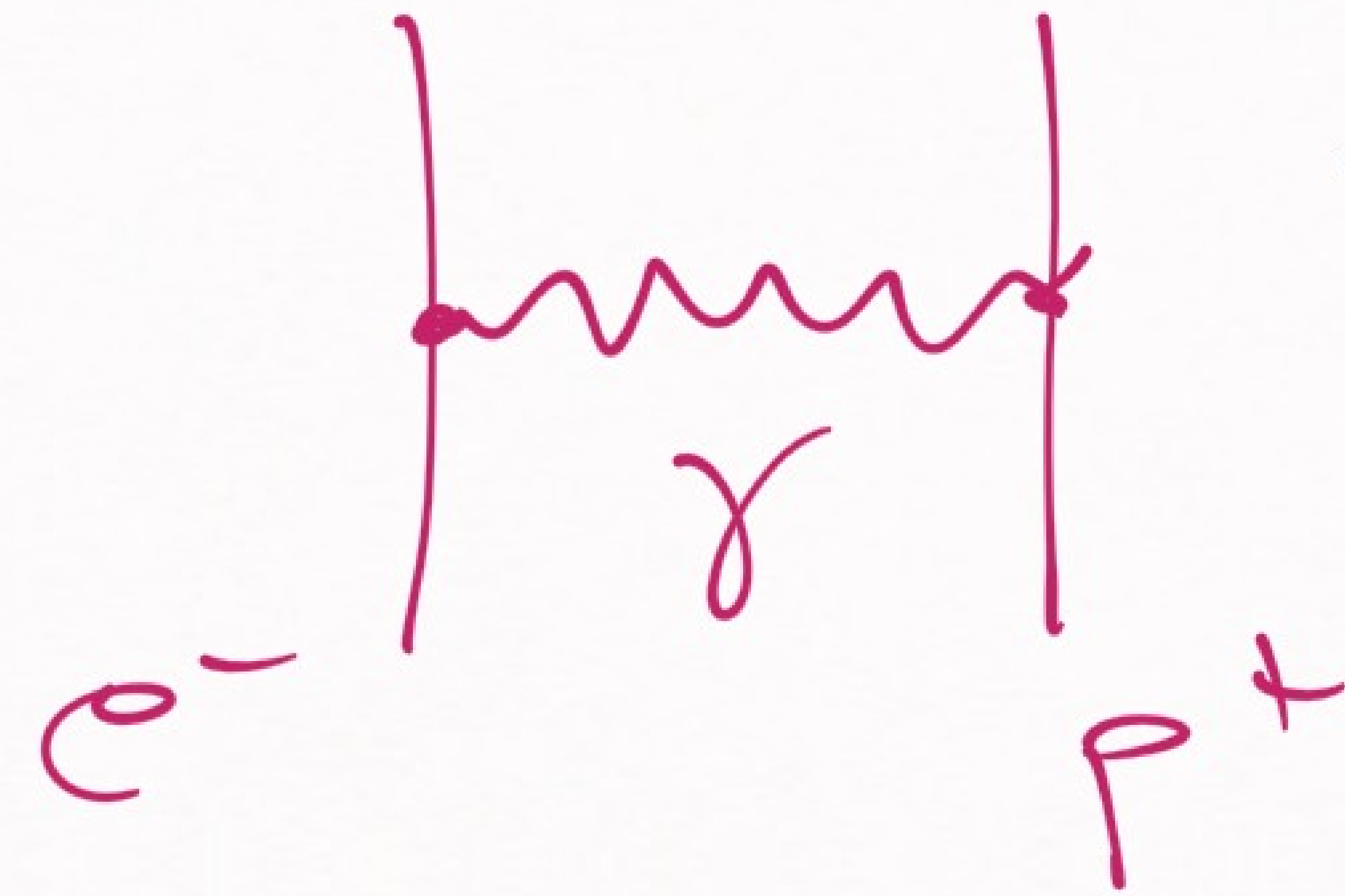


ORIGIN OF NUCLEAR FORCES

→ Where does $V_{NN}(\vec{r})$ comes from?

$\boxed{\text{QFT}}$ → exchange of a virtual particle generates a potential

$\boxed{\text{COULOMB}}$



$\rightarrow$ QFT: $\exists$ rules to calculate this potential

$$V_C(\vec{q}) = -\frac{e^2}{|\vec{q}|^2}$$

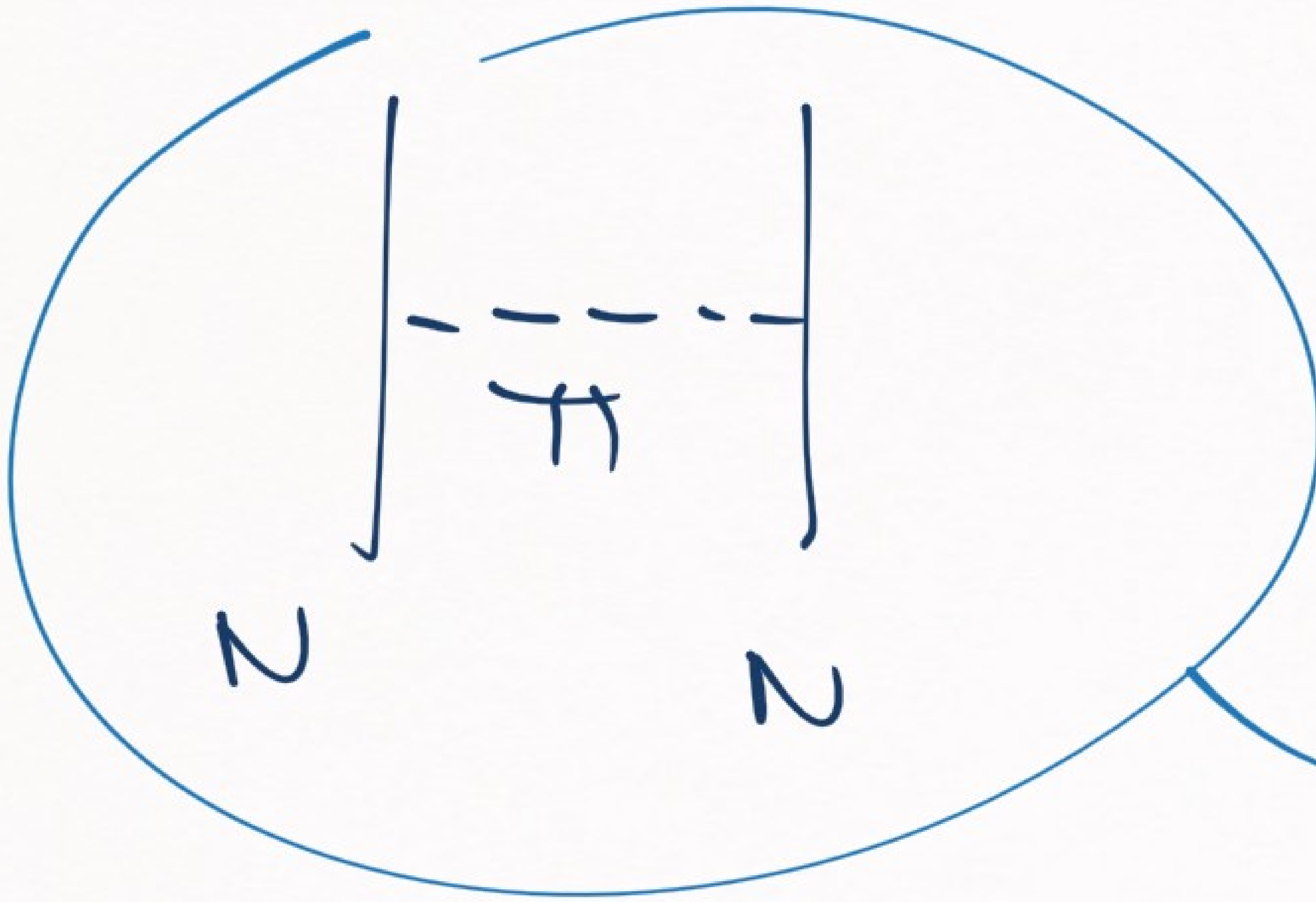
(Peskin & Schröder, Chapter 4.7, Yukawa 4.8, Coulomb)

→ QFT gives us a framework to calculate the potential coming from exchanging a particle (usually a boson)

exchange a photon → Coulomb

exchange a scalar massive boson → Yukawa

YUKAWA'S IDEA → exchange of a scalar meson (which we will call the pion)



Apply QFT rules and get the potential

$$V_Y(\vec{q}) = -\frac{g_Y^2}{\vec{q}^2 + m_\pi^2} \rightsquigarrow V_Y(\vec{r}) = -\frac{g_Y^2}{4\pi}\frac{e^{-m_\pi r}}{r}$$

$\boxed{\text{Yukawa potential}} \to$ central

$V(\vec{r}) = V(|\vec{r}|)$

1) finite-range force ✓
2) attractive at intermediate distances ✓
3) non-central ✗

Why central not acceptable?

$V_N(\vec{r}) = V_N(|\vec{r}|) \Rightarrow$ spherically symmetric

$\Downarrow$

Deuteron should also be spherically symmetric

$$\boxed{Q_d = 0} \to \text{incompatible w/ experiment}$$

YUKAWA THEORY → INCOMPLETE ($Q_d = 0$)

↓

[WE HAVE TO MODIFY IT]

($Q_d > 0$)

YUKAWA'S ASSUMPTIONS :

→ exchange a massive boson (a meson)

→ [this meson is a scalar] → modify this scalar meson $\rightarrow$ central potential

[non-scalar meson] $\longrightarrow$ Quantum numbers of the meson

$J = 0, 1, 2, \ldots$
(boson)

J^P $\longrightarrow$ angular momentum & parity

$P = \pm 1$ $\rightsquigarrow$ $\vec{r} \longrightarrow -\vec{r}$
$\vec{p} \longrightarrow -\vec{p}$

$\boxed{\phi \longrightarrow \pm \phi}$

[SCALAR BOSON] $J^P = 0^+$

$$\vec{r} \to -\vec{r}$$
$$\vec{p} \to -\vec{p}$$
$$\boxed{\phi \to +\phi}\ +$$

[VECTOR BOSON] $J^P = 1^-$

$$\vec{r} \to -\vec{r} \quad (1^-)$$
$$\vec{p} \to -\vec{p} \quad (1^-)$$
$$\vec{L} = \vec{r} \wedge \vec{p} \to \vec{L} \quad (1^+)$$
$$\boxed{\vec{\phi} \to -\vec{\phi}}\ -$$

$$\vec{L} = \vec{r} \wedge \vec{p} \rightarrow (-\vec{r}) \wedge (-\vec{p}) = \vec{L}$$

$$\underbrace{\vec{L} \rightarrow \vec{L}}_{\text{pseudovector}} \quad \text{(under parity)}$$

$$\text{pseudovector} \rightarrow \boxed{J^P = 1^+}$$

[PSEUDOVECTOR MESON] $\boxed{\vec{\phi} \rightarrow \vec{\phi}}$ $+$

[PSEUDOSCALAR MESON] $J^P = 0^-$

$$\boxed{\phi \xrightarrow[\mathcal{P}]{} -\phi} \quad -$$

classical analog: $\vec{a}\cdot(\vec{b}\wedge\vec{c})$ 0^-

$\vec{a}, \vec{b}, \vec{c}$ vectors

$(\vec{a} \to -\vec{a}, \vec{b} \to -\vec{b},$
$\vec{c} \to -\vec{c})$

[YUKAWA THEORY] → EXTENSIBLE
(MODIFIABLE)

CHECK WHAT HAPPENS WHEN
THE $J^P = 0^+$ ASSUMPTION CHANGES

↳ try a few J^P choices until
we find one that gives $\underline{Q_d > 0}$

MOST EASY MODIFICATION $\rightarrow$ TRY $J^P = 1^-$

like a heavy photon

QFT $\rightarrow$ Lagrangian $\rightarrow$ Feynman rules / potential

$$\mathcal{L}_{int}(J^P = 0^+) = g_S \bar{\psi}_N \phi \psi_N$$

$$\mathcal{L}_{int}(J^P = 1^-) = g_V \bar{\psi}_N \gamma^\mu \phi_\mu \psi_N + \frac{f_V}{4M_N} \bar{\psi}_N \sigma_{\mu\nu} \psi_N \times (\partial_\mu \phi_\nu - \partial_\nu \phi_\mu)$$

$\rightsquigarrow$ a bit mysterious if you didn't study QFT

$\longrightarrow$ point ; we get a potential

$$V(\vec{q}, J^P = 1^-) = \frac{g_v^2}{\vec{q}^2 + m_v^2} - \frac{(f_v + g_v)^2}{4M_N^2} \frac{(\vec{\sigma}_1 \wedge \vec{q}) \cdot (\vec{\sigma}_2 \wedge \vec{q})}{\vec{q}^2 + m_v^2}$$

$\hookrightarrow$ Fourier transform $\longrightarrow$ ⊕

$\circledast \to \quad V(\vec{r}, 1^-) = \dfrac{g_v^2}{4\pi}\dfrac{e^{-m_v r}}{r}$ similar (except the sign)

spin-spin $\quad + \left(\dfrac{f_v + g_v}{2M}\right)^2 \Big[\dfrac{2}{3} \vec{\sigma}_1 \cdot \vec{\sigma}_2 \dfrac{e^{-m_v r}}{4\pi r}$

$$- \frac{1}{3} \boxed{\left(3 \vec{\sigma}_1 \cdot \hat{\vec{r}}\, \vec{\sigma}_2 \cdot \hat{\vec{r}} - \vec{\sigma}_1 \cdot \vec{\sigma}_2\right)} \frac{e^{-m_v r}}{4\pi r} \times$$

quadrupole like !!

$$\left(1 + \frac{3}{m_v r} + \frac{3}{(m_v r)^2}\right)\Big]$$

$$Q_{ij} = \int d^3\vec{r}\, \rho(\vec{r}) \boxed{(3 r_i r_j - \delta_{ij} r^2)}$$

convention $(Q = Q_{33})$

$$V(\vec{r}, J^P = 1^-) \to S_{12}(\vec{r}) = 3\, \vec{\sigma}_1 \cdot \hat{\vec{r}}\ \vec{\sigma}_2 \cdot \hat{r} - \vec{\sigma}_1 \vec{\sigma}_2$$

$$= \frac{1}{r^2} \left(\underbrace{3\, \vec{\sigma}_1 \cdot \vec{r}\ \vec{\sigma}_2 \cdot \vec{r} - \vec{\sigma}_1 \cdot \vec{\sigma}_2 r^2}_{\sigma_{1i}\, \sigma_{2j} \boxed{(3 r_i r_j - \delta_{ij} r^2)}} \right)$$

Q-like!!

good

$J^P = 1^-$ meson $\rightarrow$ $\boxed{\text{generate } Q \neq 0}$

$\downarrow$

but w/ wrong sign

WHY?

→ We have previously ignore isospin

SPIN-ANALOG TO GROUP THE NEUTRON & PROTON TOGETHER

4) Nuclear forces do not distinguish the neutron & the proton

→ $\exists$ symmetry → ⊗

⊛ → called "isospin"

$$n, p \rightarrow \left.\begin{array}{l} |n\rangle_I = |\tfrac{1}{2}\, {-\tfrac{1}{2}}\rangle \\ |p\rangle_I = |\tfrac{1}{2}\, {+\tfrac{1}{2}}\rangle \end{array}\right\} \quad |I\ M_I\rangle$$

$$\cong$$

$$|e\uparrow\rangle = |\tfrac{1}{2}\, {+\tfrac{1}{2}}\rangle \qquad\qquad |S\ M_S\rangle$$

$$|e\downarrow\rangle = |\tfrac{1}{2}\, {-\tfrac{1}{2}}\rangle$$

Practical implication of isospin

→ If the exchange meson has isospin-1, then we add a $\vec{\tau}_1 \cdot \vec{\tau}_2$ factor in front of the potential

What is this?

$$\vec{\sigma}_1 \cdot \vec{\sigma}_2 = \begin{cases} 1 & S=1 \\ -3 & S=0 \end{cases}$$

$S_1 = 1/2$
$S_2 = 1/2$

spin-spin operator

$$\vec{\tau}_1 \cdot \vec{\tau}_2 = \begin{cases} 1 & T=1 \\ -3 & T=0 \end{cases}$$

$|pp\rangle, |nn\rangle$

$\frac{1}{\sqrt{2}}(|pn\rangle + |np\rangle)$

(deuteron)

$\frac{1}{\sqrt{2}}(|pn\rangle - |np\rangle)$

$\vec{\tau} = \{\tau_x, \tau_y, \tau_z\}$

(Pauli matrices)

$$\left.\begin{array}{l} J^P = 0^+ \\ I = 1 \end{array}\right\} \rightarrow V(\vec{r}) = \left(-\frac{g_\sigma^2}{4\pi}\frac{e^{-m r}}{r}\right) \times (\vec{\tau}_1 \cdot \vec{\tau}_2)$$

$$\left.\begin{array}{l} J^P = 1^- \\ I = 1 \end{array}\right\} \Rightarrow V(\vec{r}) = \vec{\tau}_1 \cdot \vec{\tau}_2 \times \left[+ \frac{g_v^2}{4\pi}\frac{e^{-m_v r}}{r} \right.$$

$$\left. + \left(\frac{f_v + g_v}{2M_N}\right)^2 \left(\frac{2}{3}\vec{\sigma}_1 \cdot \vec{\sigma}_2 \frac{e^{-m_v r}}{4\pi r} + \frac{1}{3} S_{12}(\hat{r}) \frac{e^{-m_v r}}{4\pi r} \times \left(1 + \frac{3}{m_v r} + \frac{3}{(m_v r)^2} \right) \right) \right]$$

Deuteron

$$V(\vec{r}) = -g^2\left[a\,\frac{e^{-m_\pi r}}{4\pi r} - b\,S_{12}(\hat{r})\,\frac{e^{-m_\pi r}}{4\pi r}(\ldots)\right]$$

$b > 0$

~~$J^P = 1^-$~~

$\underline{a > 0}$

$\rightarrow \boxed{Q_d < 0} \longrightarrow \times \quad Q_d^{exp} > 0$

(exercise)

Pion's J^P ?

1) $J^P = 0^+ \longrightarrow Q_d = 0$ no!

2) $J^P = 1^- \longrightarrow Q_d < 0$ no!

3) $J^P = 0^- \longrightarrow \boxed{Q_d > 0}$ will work

our chap

4) $J^P = 1^+$

?

Let's see how it works:

$$\mathcal{L}_{int} = i\, g_{PS}\, \bar{\Psi}_N \gamma_5 \phi\, \Psi_\alpha$$

$$V(\vec{q}) = -\left(\frac{g_{PS}}{2M_N}\right)^2 \frac{\vec{\sigma}_1 \cdot \vec{q}\ \vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^2 + m^2}$$

$\rightarrow$ Fourier transform $\rightarrow$

$$\rightarrow \quad V_{\pi s}(\vec{r}) = \left(\frac{g_{\pi}}{2M_N}\right)^2 \left[\vec{\sigma}_1 \cdot \vec{\sigma}_1 \frac{e^{-mr}}{4\pi r} + S_{12}(\vec{r}) \frac{e^{-mr}}{4\pi r} (\ldots) \right] \times (\vec{\tau}_1 \cdot \vec{\tau}_2)$$

For deuteron $(S=1, I=0)$

$$V(\vec{r}) = -g^2 \left[a \frac{e^{-mr}}{4\pi r} + b\, S_{12}(\hat{r}) \frac{e^{-mr}}{4\pi r} (1 + \ldots) \right]$$

$a > 0 \qquad b > 0$

→ $\boxed{Q_d > 0}$ yes!! (correct one)

→ $J^P = 0^-$ works as a possible quantum number of the pion

→ [ORIGIN OF THE NUCLEAR FORCES]

↳ IDEA BY YUKAWA : EXCHANGE OF A MESON

We tried :

1) $J^P = 0^+ \longrightarrow$ ~~$Q_d = 0$~~

2) $J^P = 1^- \longrightarrow$ ~~$Q_d < 0$~~

3) $J^P = 0^- \longrightarrow \boxed{Q_d > 0}$ → Best option

4) $J^P = 1^+ \longrightarrow$ ~~repulsive for the deuteron~~

WE HAVE LEARNED THAT

THE EXCHANGE OF A $J^P = 0^-$, $I = 1$ MESON EXPLAINS MOST OF THE QUALITATIVE FEATURES OF THE NUCLEAR FORCE

→ PION IS A $I = 1$, $J^P = 0^-$ MESON

PION → EXPLAINS NUCLEAR FORCES

FOR $r > 1/m_\pi$

→ [WHAT IS THERE BEYOND THE PION?]

→ WHAT IMPROVEMENTS ARE POSSIBLE?

IMPROVEMENTS

1) More pions

2) More mesons

→ two most obvious extensions of Yukawa's idea

History of nuclear physics

1) MORE PIONS

→ one-pion exchange potential (OPE potential)

Two-pion exchange (TPE) potential

(a) planar box
(b) crossed box
(c) football
(d) triangle

→ first time TPE calculated : 50's

1950's multipion theories :

→ full of trouble

(infinities everywhere) → why → ⊕

$\circledast \to$ why?

1) Wrong pion dynamics

2) Renormalization
(not well understood in the 1950's)

$\Longrightarrow$

$\boxed{\text{WRONG PION DYNAMICS}}$ → How is this possible?

↓

"Inverse scattering problem"

↓

$V(\vec{r}),\ V'(\vec{r}),\ V''(\vec{r})$

$\Rightarrow$ Same observable consequences

Cross sections / observables

$\Rightarrow$ does not uniquely determine the potential

$\Rightarrow$ not only a QM problem, it is also a QFT problem pion dynamics (1950 version)

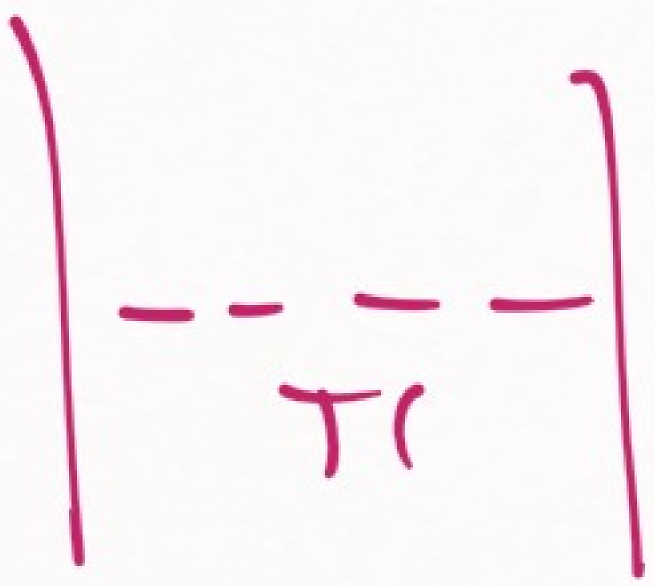
$$\mathcal{L}_{int} = i g \bar{\psi}_N \gamma_5 \vec{\tau} \cdot \vec{\pi} \psi_N$$

non-derivative term

pion dynamics (post chiral symmetry)

$$\mathcal{L}'_{int} = \frac{g'}{2 f_\pi} \bar{\psi}_N \gamma^5 \gamma^\mu \vec{\tau} \cdot \partial_\mu \vec{\pi} \psi_N$$

derivative term



$\Rightarrow \mathcal{L}_{int}, \mathcal{L}'_{int}$ give the same result

$\Rightarrow \mathcal{L}_{int}, \mathcal{L}'_{int}$ give different results

1950 → $\mathcal{L}_{int}$ (non-derivative)

→ Wrong results for TPE

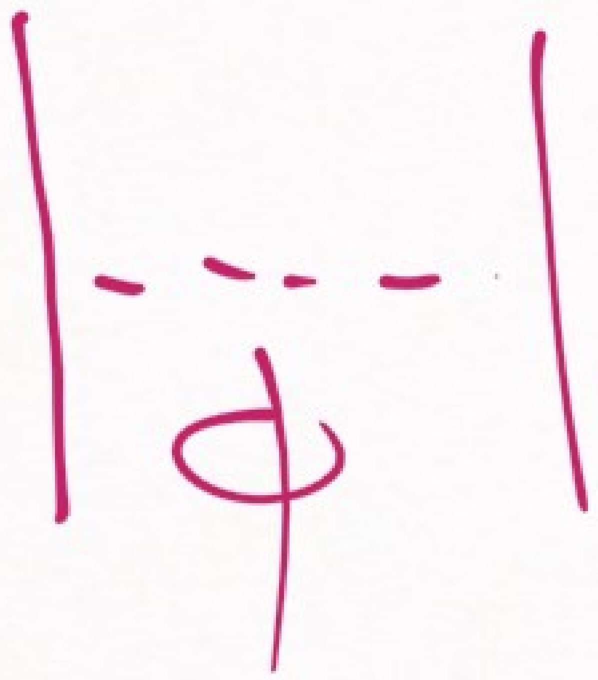
→ PEOPLE DECIDED TO GIVE UP ON MULTI-PION EXCHANGES

2) MORE MESONS → One boson exchange (OBE) model

multipion exchange fail

→ simply try other mesons

why? → $\boxed{\exists \text{ a lot of mesons besides the pion}}$



$\phi = \pi$, $J^P = 0^-$, $m_\pi = 140\,\text{MeV}$

$I=1$ $\quad$ $I=0$

$\phi \to \rho, \omega \longrightarrow J^P = 1^-$, $m_\rho \simeq 770\,\text{MeV}$

$m_\omega \simeq 780\,\text{MeV}$

(vector mesons)

$\phi \to \sigma \longrightarrow J^P = 0^+$, $m_\sigma \simeq 550\,\text{MeV}$

$I=0$, $J^P = 0^+$

(scalar meson)

$$V_{NN}(\vec{r}) = \underbrace{V_{\pi}(\vec{r})}_{Q_d > 0} + V_{\rho}(\vec{r}) + V_{\omega}(\vec{r}) + V_{\sigma}(\vec{r}) + \dots$$

very easy potential

gives you short-range repulsion

yes!!

strong medium range attraction

yes!!

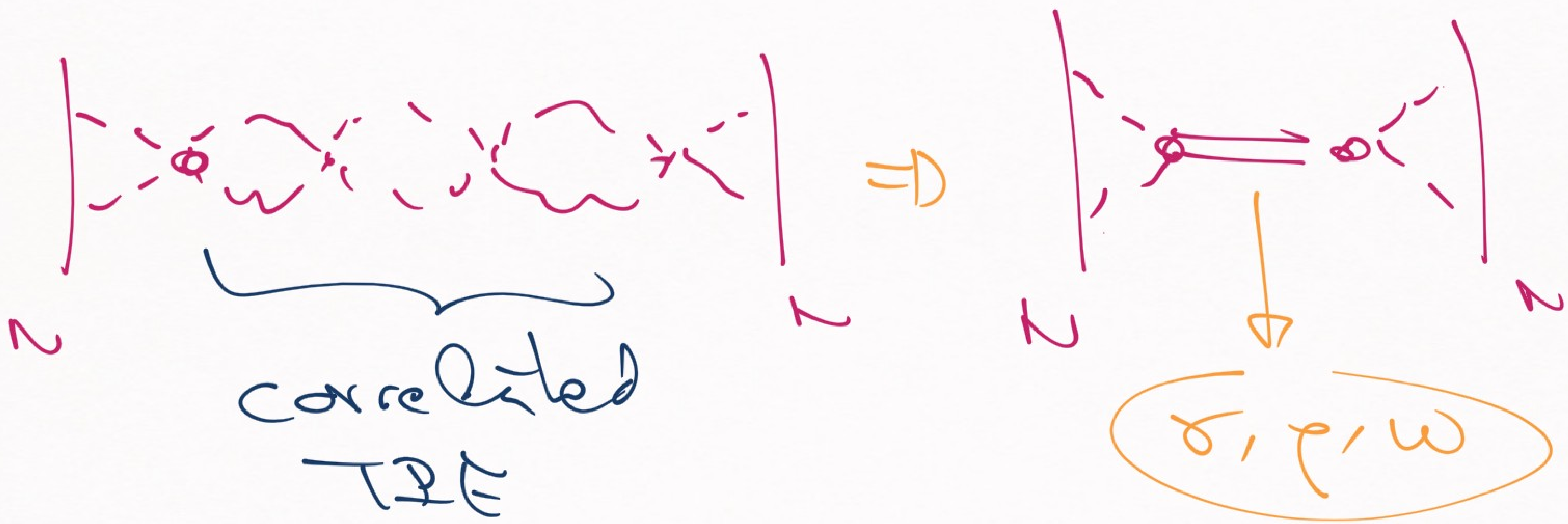
OBE → good features (first quantitative successful explanation of VNN)

∃ theoretical motivation: relation to TDE

1970 → 1) uncorrelated TDE (boxes, football, triangle, ...)
→ not important (then)

2) correlated TDE
→ important part

Correlated TPE :



(rescattering → complicated)

hypothesis → exchange of ρ, ω, σ

(1950's) $\simeq$ exchange of multiple pions

Now → this is not exactly correct

(or might be an exception)

→ OBE → PHENOMENOLOGICAL MODEL

PHENOMENOLOGICAL MEANS:

IT WORKS, BUT THE THEORETICAL BASIS IS NOT GOOD

→ Convenient, but not necessarily correct approach

from a theory point of view

$\boxed{\text{OBE MODEL}}$ $\longrightarrow$ THEORETICAL PROBLEMS

1) ρ, ω, σ $\to$ too short-ranged (maybe)

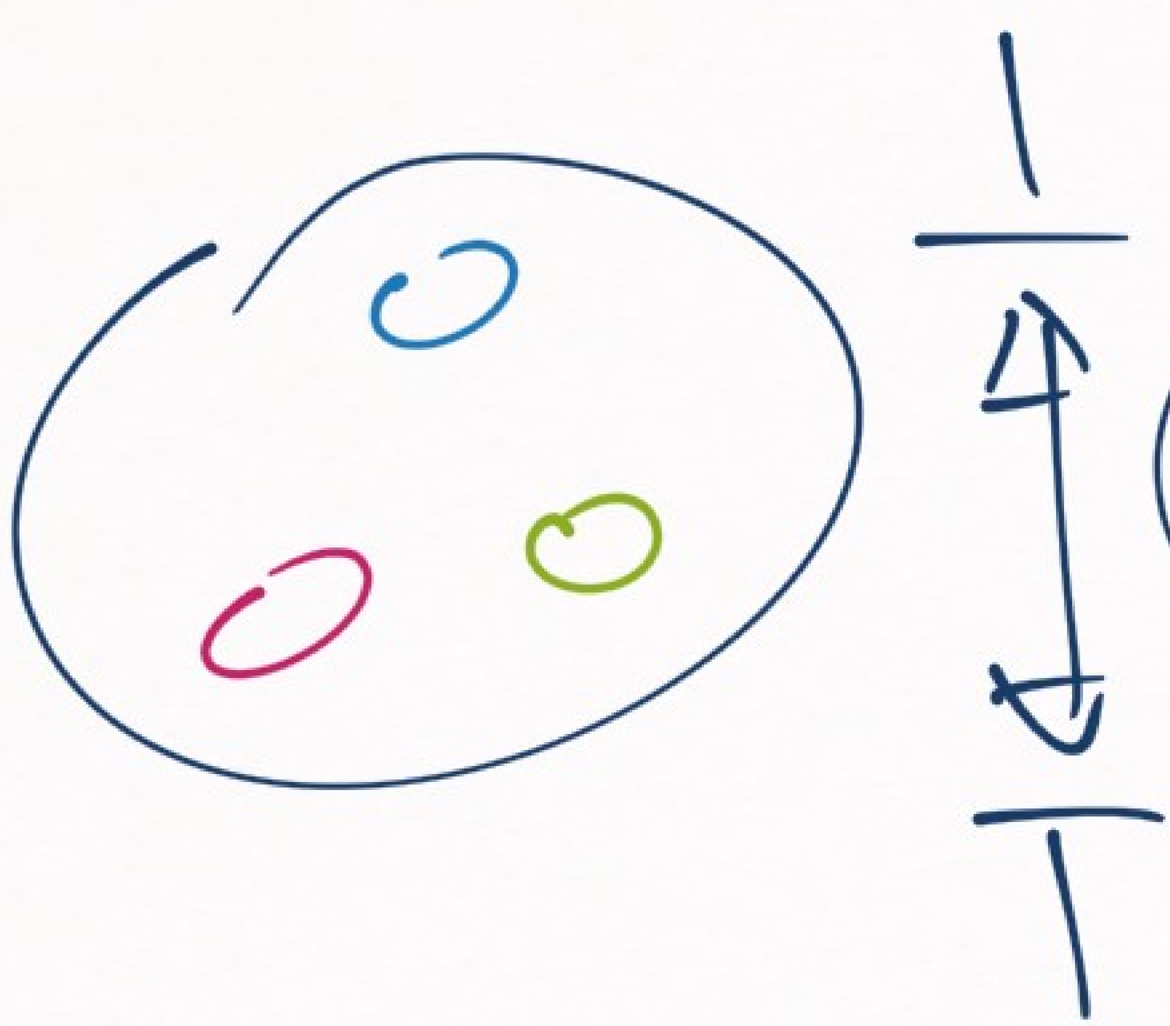
2) singular potentials $\left(\sim \frac{1}{r^3}\right)$ $\leftarrow$ form factors

$\longrightarrow$ no ground state (collapse)

3) σ meson was not found experimentally

1) Range of $(\omega, \rho, \sigma$

$$|\text{---}| \sim \frac{e^{-mr}}{r} \rightsquigarrow \frac{1}{m} \sim (0.2 - 0.3)\,\text{fm}$$



$0.5 - 1.0$ fm

$$\frac{1}{m} \lesssim \sqrt{\langle r^2 \rangle}_{\text{nucleon}}$$

$\boxed{\text{THEORETICAL PROBLEM}}$

Partial solution → range is actually larger than $1/m$

→ we could use a different definition

EXAMPLE → $\frac{\pi}{m}$

1) Singular potentials

$$|\ldots| \sim S_{12}(\hat{r}) \frac{e^{-mr}}{4\pi r}\left(1 + \frac{3}{mr} + \frac{3}{(mr)^2}\right)$$

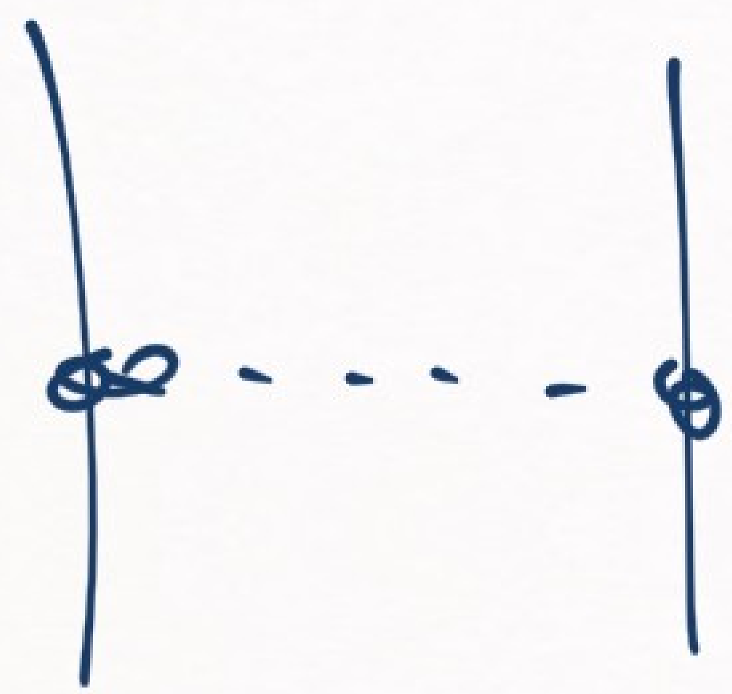
$$\sim \frac{S_{12}(\hat{r})}{r^3} \rightarrow \boxed{\text{OUPS !!}}$$

↳ it can be shown that this has not ground state → ☹

- → neutron-proton system should collapse to a bound state w/ infinite energy

$\boxed{B_d \to \infty}$ → theoretical problem

Solution → form factors

 $\longrightarrow \mathcal{L}_{int} \longrightarrow \dfrac{\vec{\sigma}\cdot\vec{q}}{2M}$ → point-like

→ but the nucleons have a size

$$\Rightarrow \frac{\vec{\sigma}\cdot\vec{q}}{2M} \to \frac{\vec{\sigma}\cdot\vec{q}}{2M} f(\vec{q}) \quad / \quad f(|\vec{q}|\to\infty) \longrightarrow 0$$

By including these form factors $f(\vec{q})$

$\Rightarrow$ Finite potential

$\Rightarrow \boxed{B_d < \infty}$ ✓

JUSTIFICATION $\longrightarrow$ finite size of the nucleus

$$\underbrace{V_Y(\vec{r})}\ = -\frac{g_Y^2}{4\pi}\frac{e^{-mr}}{r}$$

$\hookrightarrow$ only valid for $|\vec{r}| > \sqrt{\langle r^2 \rangle_{\text{nucleon}}}$

$$V_Y(\vec{r}) = -\frac{g_Y^2}{4\pi}\frac{e^{-mr}}{r}\, f\!\left(\frac{r}{r_N}\right) \quad / \quad f\!\left(\frac{r}{r_N}\right) \to 0$$

for $\frac{r}{r_N} \to 0$

3) σ meson was never found in experiments

π, ρ, ω → known experimentally

σ → missing / but really important (because its attraction)

→ 2000's when it was derived as a $\pi\pi$ resonance in $\pi\pi$ scattering

1) mesons too short-ranged
2) singular potential → form factors
3) sigma not found

→ OBE model ∨ phenomenological

[open theoretical problems]

History $\longrightarrow$ old pion theories

(infinities)

$\downarrow$

OBE model

(where is the sigma?)

$\downarrow$

Later on $\longrightarrow$ modern multipion theories

(includes lot of theory not known in 1950's)

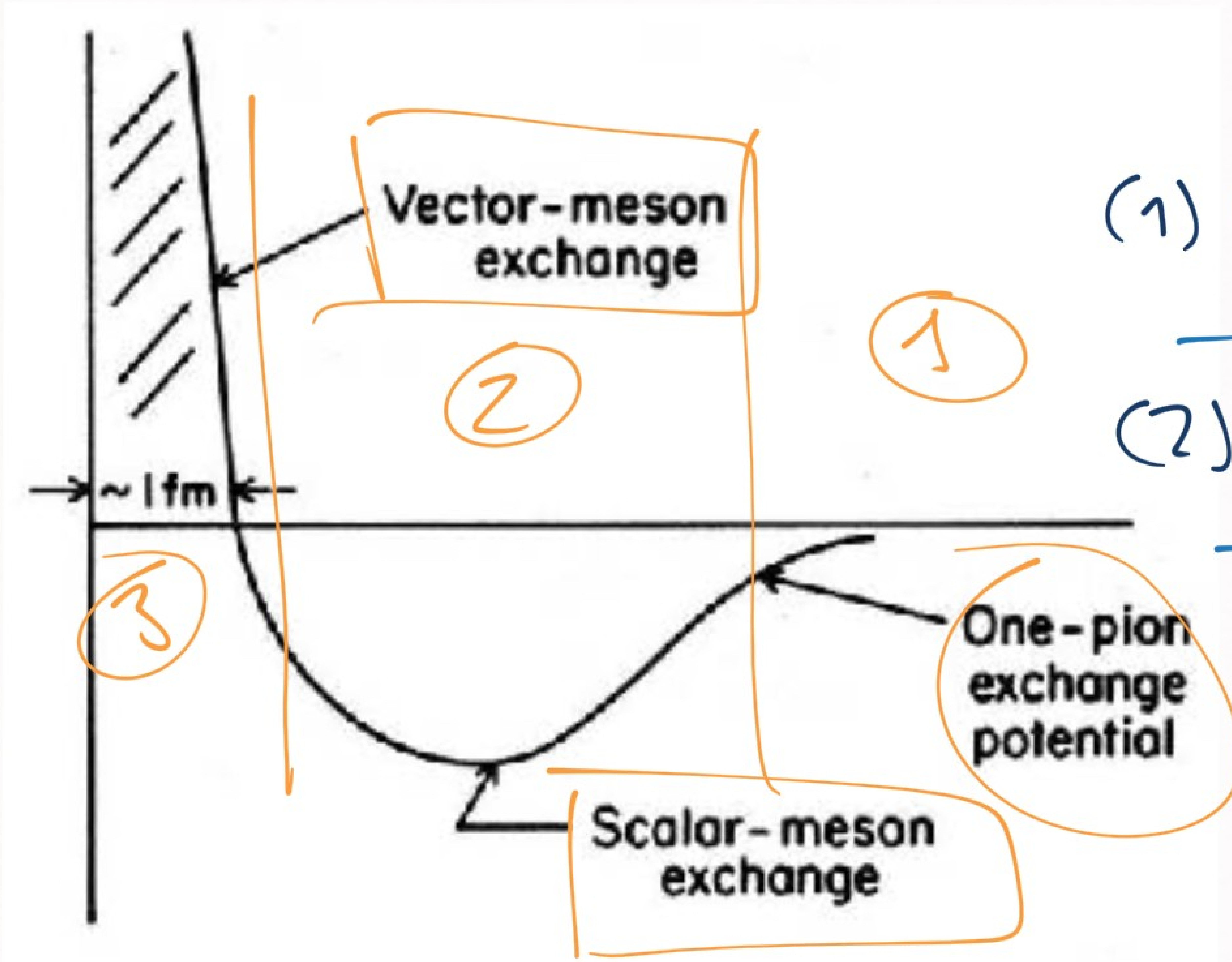
PAIR OF INTERESTING IDEAS
ABOUT THE NUCLEAR FORCE

→ TNS (TAKETANI - NAKAMURA - SASAKI)
SEPARATION OF SCALE

→ NUCLEAR FORCES AS RESIDUAL FORCES

Useful for understanding V_{NN}

TNS CLASSIFICATION → distinguish several regions



(1) CLASSICAL ZONE

→ OPE, well-known

(2) DYNAMICAL ZONE

→ TPE, σ (we can cope)

(3) PHENO ZONE

→ ρ, ω, nuclear size (not well-known)

TNS → known / unknown distinction

4

→ long / medium / short-range physics

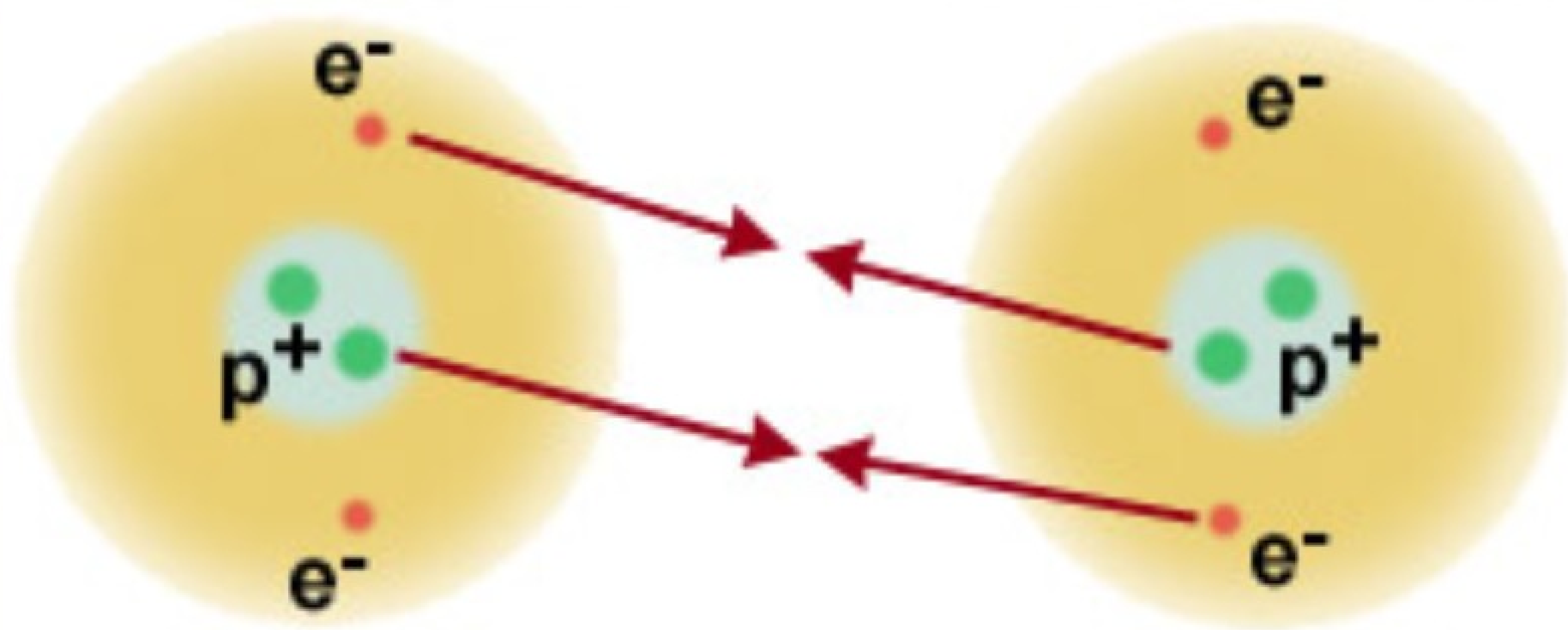
NUCLEAR FORCES
AS RESIDUAL FORCES

atoms → electrically neutral (Coulomb = 0)

→ but ∃ a potential

↓

VdW



Residual E-M force in action: the atoms are electrically neutral, but the electrons in one are attracted to the protons in another, and vice versa!

→ even though atoms are neutral,

→ they are still compose of non-neutral part

after doing calculations, $V(\vec{r}) \neq 0$

$$V(r) = -\frac{C_6}{r^6} - \frac{C_{10}}{r^{10}} - \frac{C_{12}}{r^{12}} - \ldots \Big\} \rightarrow \boxed{\text{residual force}}$$

van der Waals

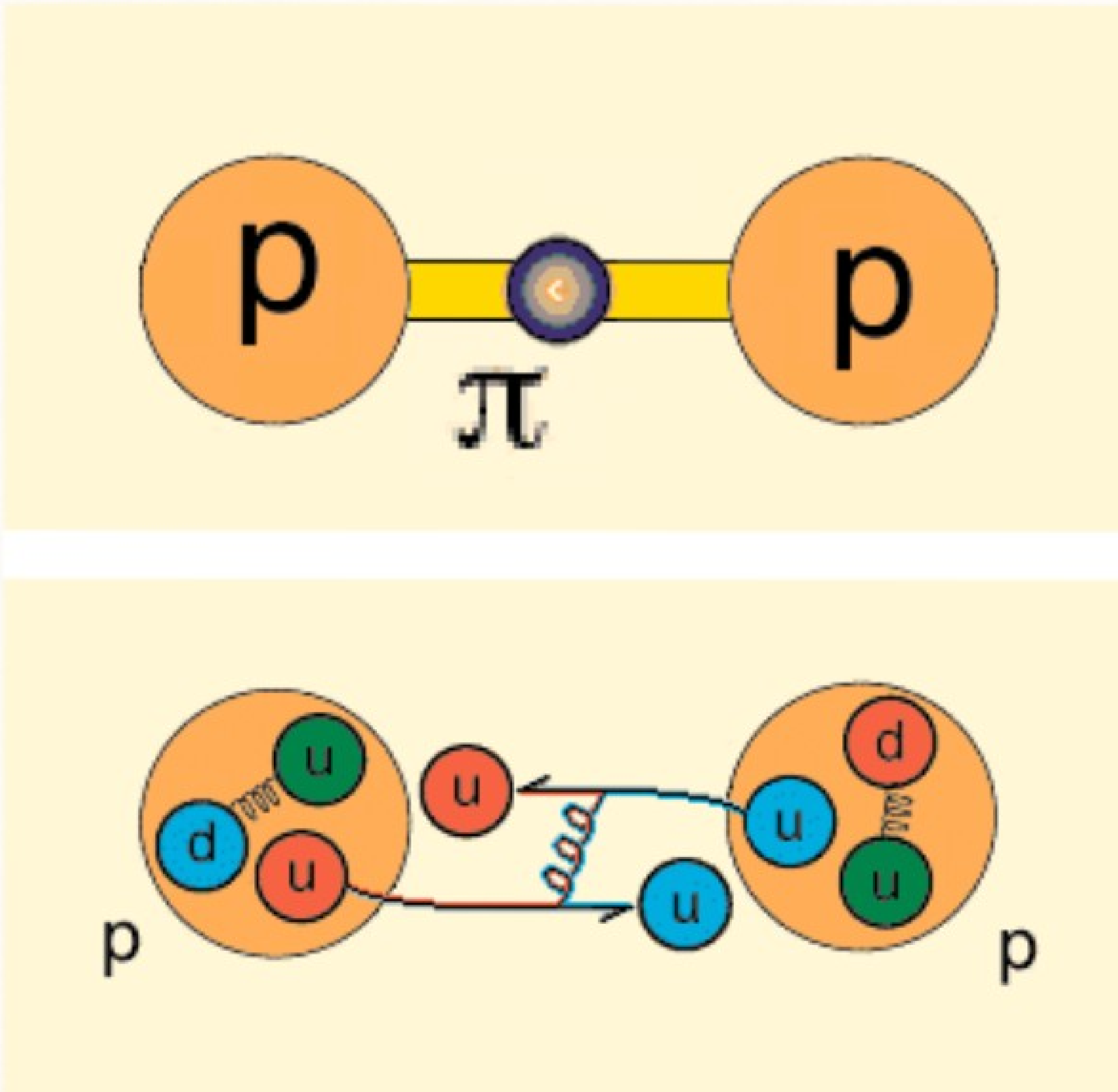
corrections

Fondamental force → Coulomb = 0

van der Waals is electromagnetic in nature

→ but is residual

(what survives after a lot of
calculations of forces that
cancel each other,
but not perfectly)



→ Yukawa or OPE

→ what really happen

nucleons → quarks & gluons

pions →

NUCLEAR FORCES (not fundamental)

→ RESIDUAL STRONG FORCE
INTERACTION

fundamental

THE END