NUCLEAR PHYSICS 4

- NATURAL & UNNATURAL SYSTEMS
- NUCLEAR FORCE & ITS PROPERTIES

RECAP 1

Natural & Unnatural systems

1) Good scale separation ($\exists$ Q / characterizes the system)

2) $\forall$ is going to be $O(1)$ in terms of Q

(H atom)

1) Poor scale separation

2) not $\forall$ will be $O(1)$ in terms of Q

(Square well w/ a shallow b.s.)

[What about nuclear physics?] NEUTRON-PROTON SYSTEM

→ fine-tuning

$E = \langle T \rangle + \langle V \rangle$

$(|E/\langle T \rangle|, |E/\langle V \rangle|)$

⊛ 1/25 for the deuteron (np, $S = 1$)

⊛ 1/1000 for $S = 0$ configuration

np → two spin-½ particles → $\boxed{S = 0, 1}$

Reminder → $S=0$ system, $np \to d\gamma$
$\underbrace{(S=0)}_{\text{finely-tuned}}$

→ poor scale separation $\boxed{\text{similar}}$

→ range of np interaction $\frac{1}{m_\pi} \sim 1.4\,\text{fm}$

→ size of neutron/proton (nucleon) $\sim \boxed{(0.5-1.0)\,\text{fm}}$

neutron-proton system

→ finetuning ($S = 0, 1$ configurations)

→ poor separation of scales

complicated problem, we need to be smart to understand it properly

∃ TWO-TYPES OF FINE-TUNING

1) FORTUITOUS (BY CHANCE)

-> np system -> ∃ of eclipses (relative size of moon & sun the same)

-> solar system (earth within habitable zone)

2) "CONSPIRACY" (UNDERLYING REASON)

-> classical electron radius

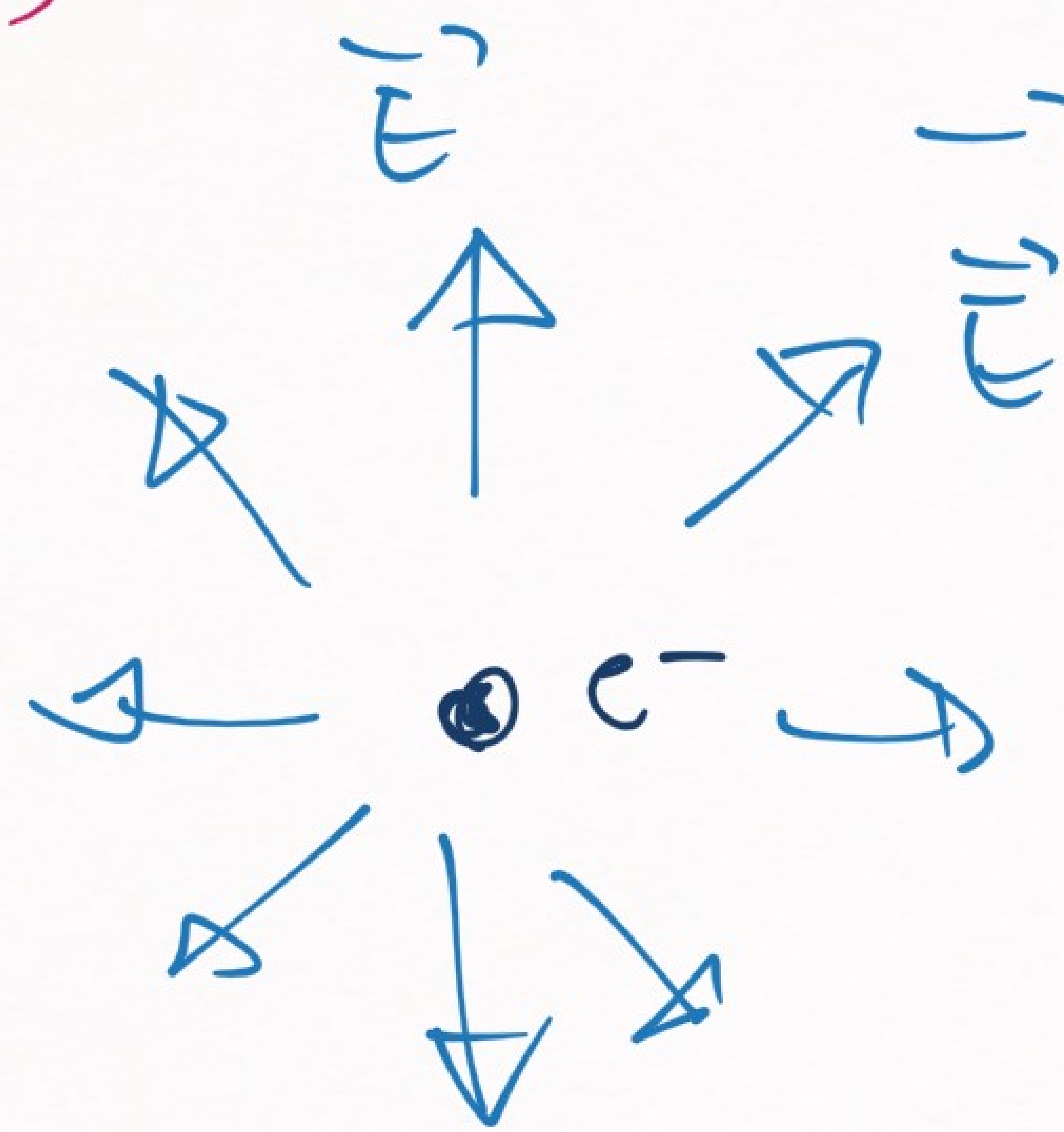
-> Higgs mass? -> cosmological constant?

Second type a bit more interesting

→ Let's see a few examples

1) FINE-TUNING AS "CONSPIRACY"

→ CLASSICAL ELECTRON RADIUS



Electron → generates an electric field

Electric field → It has energy

→ contributes to the mass of the electron

energy of $\vec{E}$

$$m_{phys}(e^-) = \underbrace{m_{bare}(e^-)}_{\text{mass of the electron if } \nexists \text{ electric fields}} + \overbrace{m(\vec{E})}^{\text{energy of } \vec{E}}$$

mass of the electron if $\nexists$ electric fields

Imagine the electron as point-like:

$|\vec{E}| \sim \frac{1}{r^2} \Rightarrow$ if $r \to 0$, $|\vec{E}| \to \infty$

$\Rightarrow m(\vec{E}) \to \infty$

$$\underbrace{m(\vec{E})}_{\to +\infty} + \underbrace{m_{bare}(e^-)}_{\to -\infty} = \underbrace{m_{phys}(e^-)}_{0.5\,MeV}$$

Level of fine-tuning $\frac{1}{\infty}$ $\rightarrow$ infinite fine tuning

$\rightarrow$ $\nexists$ a coincidence that can explain this level of fine-tuning

$\Rightarrow$ [There must be new physics before reaching this point]

⇝ Refine this fine-tuning argument:

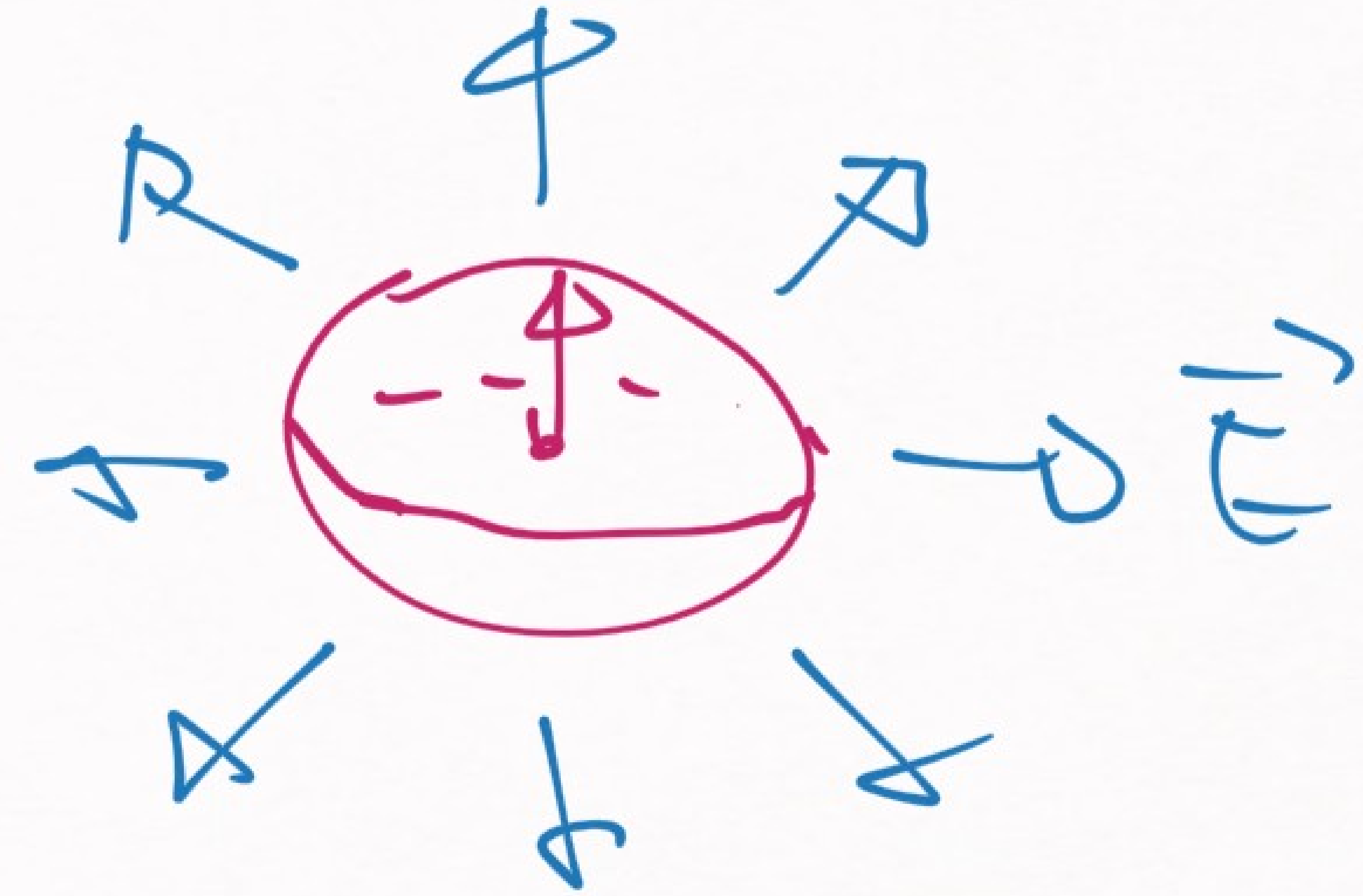
$$\boxed{m_{bare}(e^-) \geq 0} \rightarrow$$ we will reject negative masses

[CLASSICAL ELECTRON RADIUS]

→ All the mass comes from $\vec{E}$

$$\underbrace{m_{bare}(e^-)}_{0} + m(\vec{E}) = m_{phys}(e^-)$$

$\boxed{\text{In this case}} \rightarrow$



r(empty-shell electron) $= \dfrac{\alpha}{m_e} \rightarrow e^-$ classical radius

$\boxed{r(e^-) \simeq 2.8\ \text{fm}} \rightarrow \boxed{\exists \text{ new physics at } r \geq r(e^-)}$

INDEED, $\exists$ TWO-TYPES OF NEW PHYSICS HAPPENING FOR $r \geq r_{cl}(e^-)$

1) Quantum mechanics: $r \sim a_B \sim 0{,}5\,\text{Å}$

$$\boxed{a_B \gg r_{cl}(e^-)} \checkmark$$

2) Quantum field theory: $r \sim \frac{1}{m_e} \sim 400\,\text{fm}$

$$\boxed{1/m_e \gg r_{cl}(e^-)} \checkmark$$

fine-tuning in this case $\Rightarrow$ new physics

— o —

But even after knowing about QFT/QM

$\rightarrow$ $\exists$ still a new fine-tuning problem

$\boxed{\text{QFT}} \rightarrow e^- \longrightarrow$ $\exists$ a high-energy scale at which the strength of $|\vec{E}| \rightarrow \infty$

$\boxed{\text{QFT}}$ $\Rightarrow$ loop effects $\to$ $|\vec{E}| \to \infty$

$\Rightarrow$ Landau pole

$\Lambda_{UV} \sim 10^{286}$ eV

$\sim 10^{280}$ MeV

$\to$ Before this, we will have a lot of NEW PHYSICS

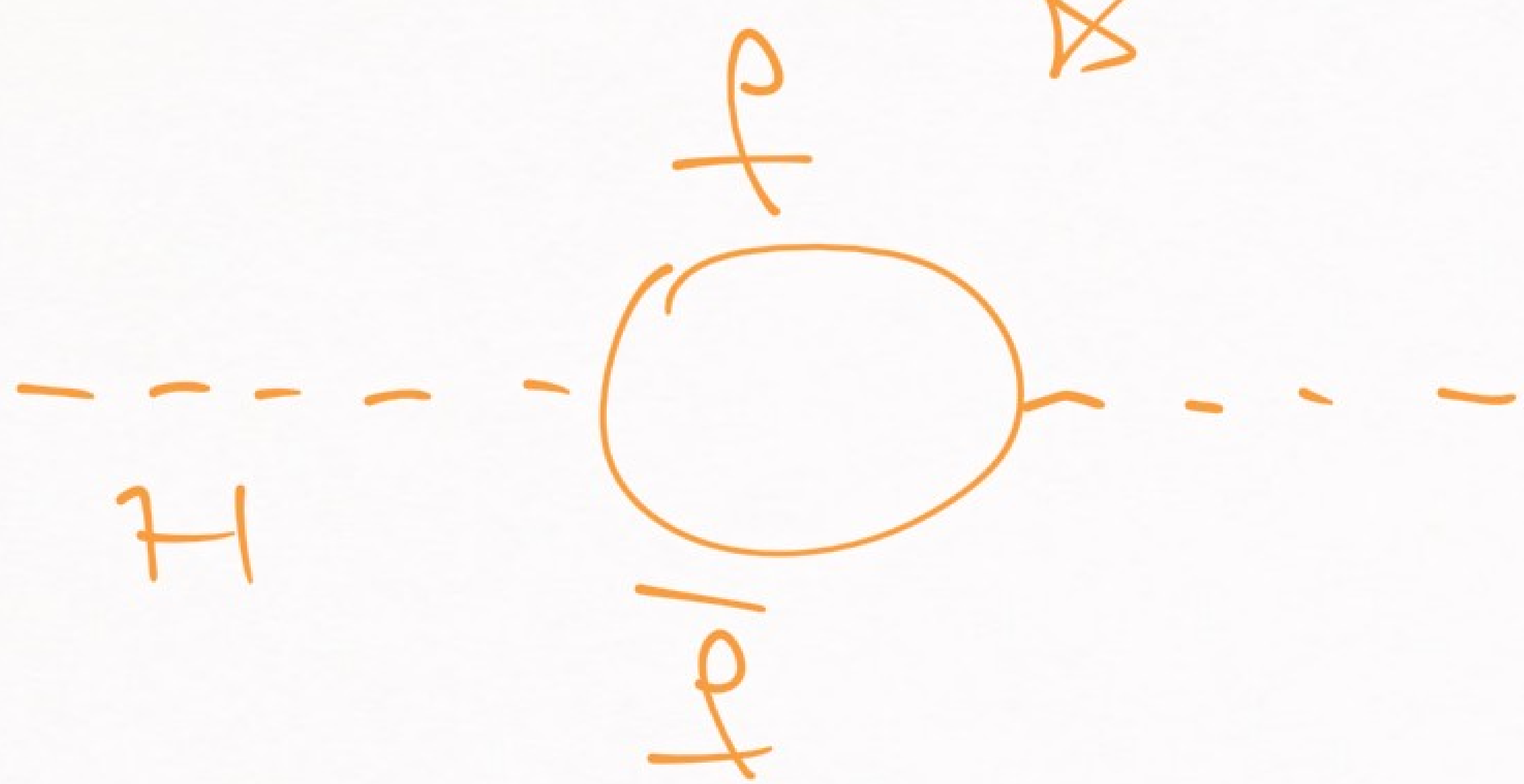
$$\boxed{\Lambda_{UV} \gg M_{Planck}}$$

2) FINE-TUNING "CONSPIRACY": HIGGS MASS

$$m_H^2 = m_{bare}^2 + \Delta m_H^2$$

↳ QFT loops

coupling const.



$$\Delta m_H^2 \propto -\frac{|\lambda_f|^2}{8\pi^2}\left[\Lambda_{UV}^2 + \dots\right]$$

(fermion-antifermion loop)

bottom line → Higgs mass corrections goes as Λ_{UV}^2

(for comparison, electron mass corrections in QED go as $\log\left(\frac{\Lambda_{UV}}{Q}\right)$)

[Higgs supersensitive to short-distance physics]

What is the amount of fine-tuning?

$$\frac{m_H}{\Lambda_{UV}} \rightarrow \text{Higgs mass vs ultraviolet scale}$$

If new physics happen at the Planck scale $\rightarrow$ $\Lambda_{UV} \simeq M_{Planck} \quad (\sim 10^{18}\,\text{GeV})$

$$\Rightarrow \boxed{\frac{m_H}{\Lambda_{UV}} \sim 10^{-16}} \longrightarrow \text{a lot of finetuning}$$

However, if new physics happen before this $\rightarrow$ less fine-tuning

EXAMPLE $\rightarrow$ SUSY (supersymmetry)

$M_{SUSY} \sim (1-10)\,\text{TeV}$ (example)

$M_{susy} \sim (1-10)\,\text{TeV}$

?

$\Rightarrow \boxed{\dfrac{m_H}{M_{susy}} \sim \dfrac{1}{10} - \dfrac{1}{100}}$

Closer to naturalness than 10^{-16}

IMPROVEMENT

3) FINE-TUNING "CONSPIRACY"

→ [COSMOLOGICAL CONSTANT]

(extreme finetuning)

3.a) What is naturalness here?

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \underbrace{\boxed{+\Lambda_c g_{\mu\nu}}}_{\text{cosmological term}} = 8\pi G T_{\mu\nu}$$

Dimensions $\rightarrow$ $[\Lambda_c] = [E]^4$

Ansatz $\rightarrow$ $\Lambda_c \sim M_{Planck}^4 \sim (10^{27}\, eV)^4 \sim \underline{\underline{10^{108}\, eV^4}}$

M_{Planck} $\rightarrow$ scale w/ units of mass that can appear naturally in Gravity

$G \sim \frac{1}{M_{Planck}^2} \rightarrow M_{Planck} \sim 10^{18}\, GeV$
$\sim 10^{27}\, eV$

[Natural expectation] $\rightarrow$ $\boxed{\Lambda_c \sim 10^{108}\ eV^4}$

$\rightarrow$ from the rate of expansion of the universe

$\boxed{\Lambda_c \sim 10^{-12}\ eV}$

$$\frac{\Lambda_c}{M_{Planck}^4} \sim \frac{1}{10^{120}}$$ $\rightarrow$ $\boxed{\text{LOTS OF FINE-TUNING}}$

$$\boxed{\frac{\Lambda_c}{M_{pl}^4} \sim \frac{1}{10^{120}}} \longrightarrow$$ What explanations are there for this?

$\rightsquigarrow$ New physics (tricky → $\Lambda_c = 0$ easy, but $\Lambda_c / M_{pl}^4 \sim 10^{-120}$ difficult)

$\rightsquigarrow$ Anthropic principle (we can only make this fine-tuning estimation in fine-tuned universes)

$\rightsquigarrow$ Multiverse (if $\exists$ more than 10^{120} universes $\Rightarrow$ easy to find one that is fine-tuned at this level)

$\rightarrow$ WE DON'T REALLY KNOW

TAKE-HOME MESSAGE

FINE-TUNING

1) Sometimes this is for real

2) Often, there is something else going on

[NUCLEAR PHYSICS & FINE-TUNING]

1) Deuteron binding energy

[-> fine-tuning by chance]

2) Pion mass (very small)

-> new physics type of fine-tuning

→ END OF FINE-TUNING PART

PROPERTIES OF THE NUCLEAR FORCE

$\Rightarrow V(\vec{r})$ nuclear potential

[Properties of $V(\vec{r})$:]

1) Short-ranged
2) Attraction at "intermediate distances"
3) Repulsion at "short distances"
4) Does not distinguish protons & neutrons
5) Not central

Let's explain each one in detail;

1) SHORT-RANGED ($\neq$ SHORT-RANGE)

What does this mean?

1.a) Long-ranged $\rightarrow$ effects of $V(\vec{r})$ extend to infinity

$V(r) \sim 1/r^n$

1.b) Short-ranged $\rightarrow$ effects of $V(\vec{r})$ confined to small region

$V(r) \sim \dfrac{e^{-\lambda r}}{r^n}, \dfrac{\Theta(a-r)}{r^n}$

[long-ranged potential] → Coulomb, gravity,
van der Waals

[Short-ranged potential] → Yukawa,
Square-well, etc.

↓

IN SOME ASPECTS, THEY ARE EASIER
FOR DOING THEORY

DIFFERENT FROM

SHORT-RANGE
LONG-RANGE POTENTIAL

$$V(r) = V_S(r) + V_L(r)$$

$$R_S \ll R_L$$

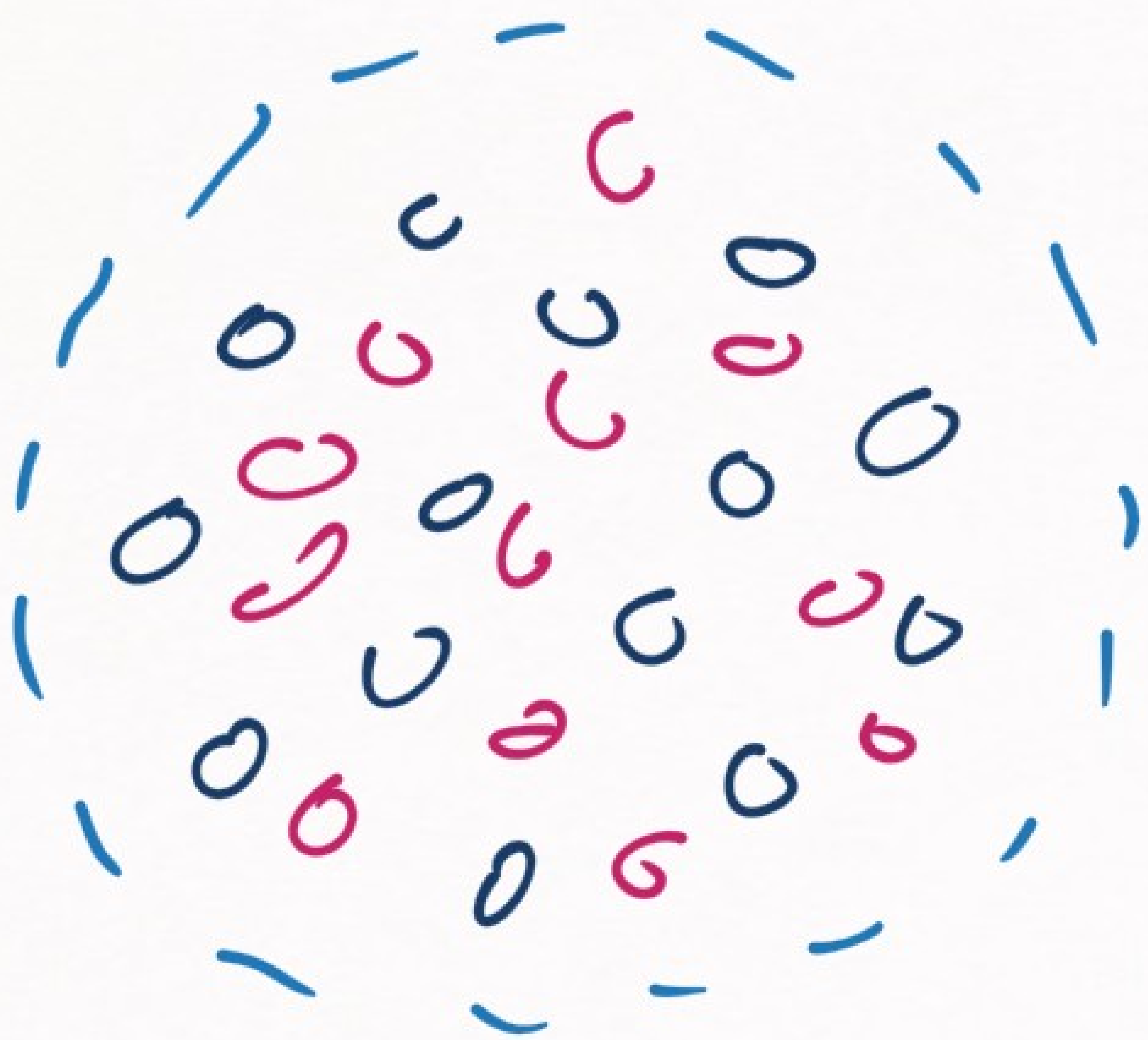
WITHOUT A "D"
AT THE END

How do we know this?

B → binding energy

A → # of nucleons

→ Binding energy of nuclei



$$\frac{B}{A} \to (8-9)\ \text{MeV}$$

for A large

→ $\boxed{\text{SATURATION}}$

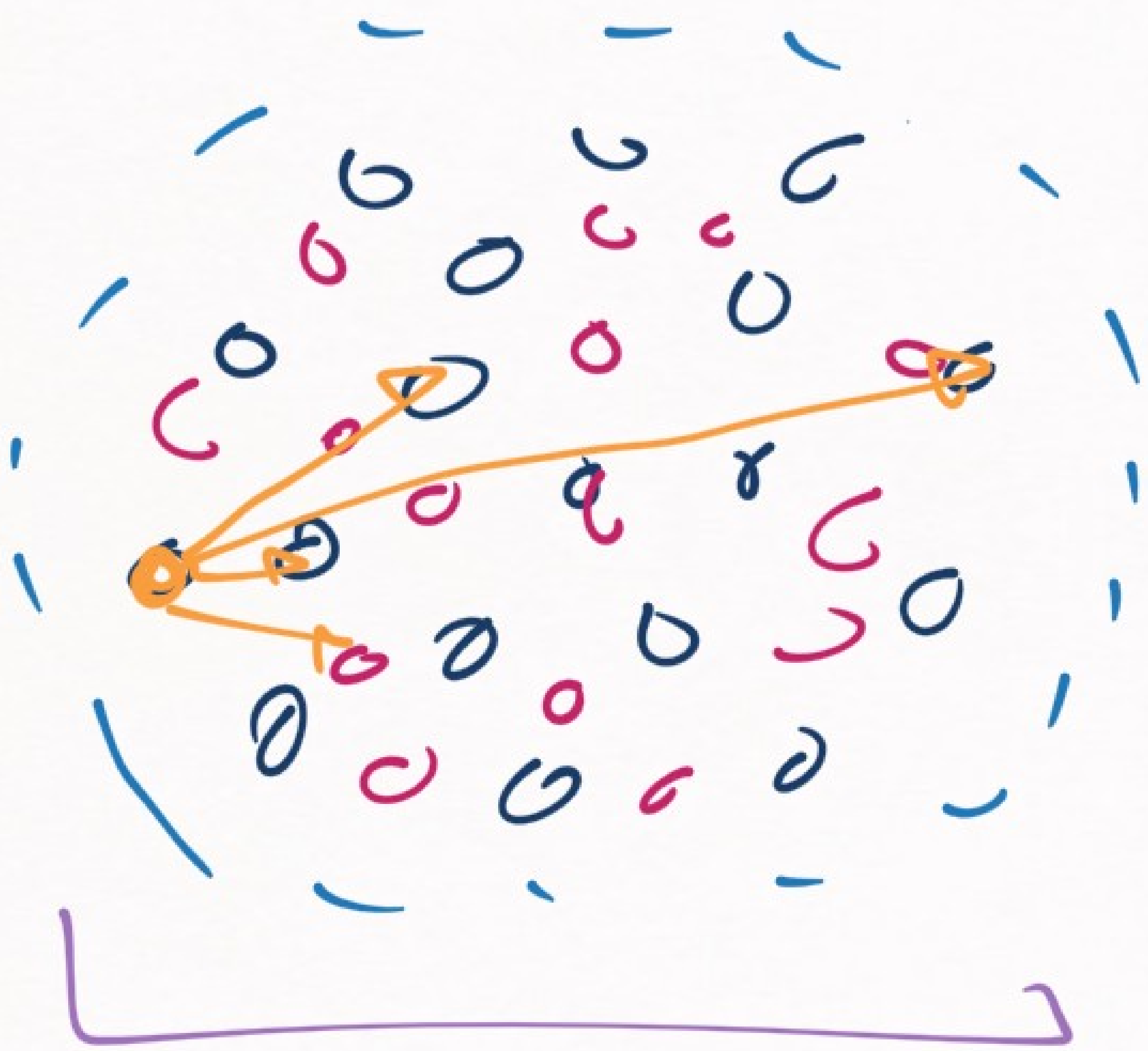
SATURATION

→ $A = 2$, $B \sim 2\ MeV$ $\Rightarrow$ $\frac{B}{A} \sim 1\ MeV$
(Deuteron)

→ $A = 3$, $B \sim 8\ MeV$ $\Rightarrow$ $\frac{B}{A} \sim (2-3)\ MeV$
(Triton, 3He)

→ $A = 4$, $B \sim 28\ MeV$ $\Rightarrow$ $\frac{B}{A} \sim 7\ MeV$
(Alpha particle: 4He)

IMAGINE A LONG-RANGED FORCE



A nucleons

→ each nucleon will interact with each other

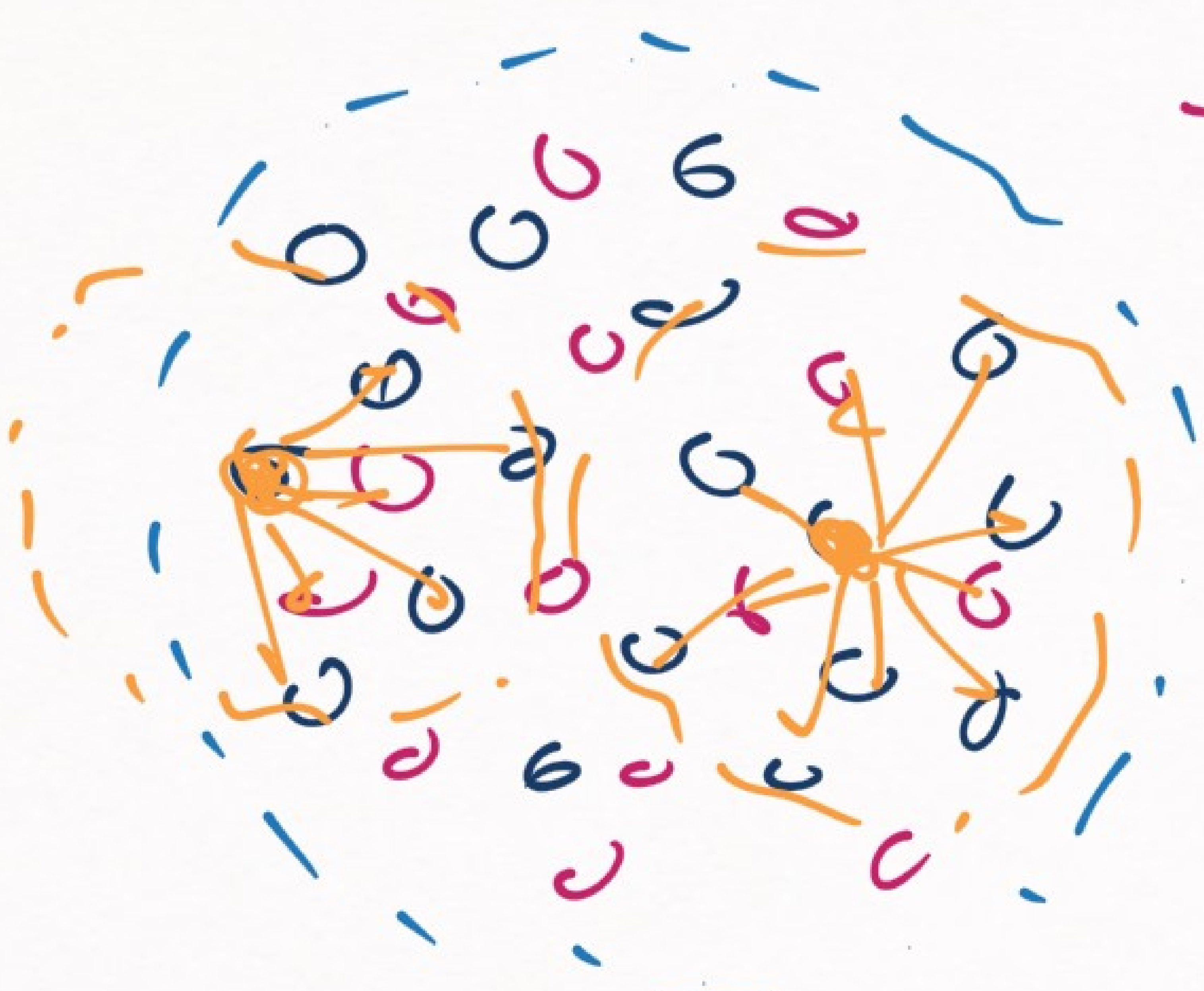
→ $\frac{A(A-1)}{2}$ interaction pairs

$$B \propto \frac{1}{2} A(A-1)\langle \bar{V} \rangle \propto A^2$$

$\boxed{\text{But it's not like this}}$ ← $\cancel{\frac{B}{A^2} \to \text{constant}}$

But we see this : $\boxed{\dfrac{B}{A} \to \text{const}}$

Short-ranged force (w/ finite range)



→ potential range smaller than size of nucleus

$\Rightarrow \forall$ nucleon only see a few neighbors

$B \sim A \times$ (# of neigborr) $\times \langle \bar{v} \rangle$

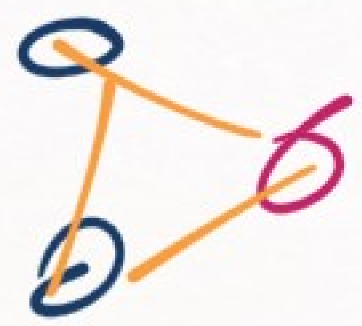
$\propto A$

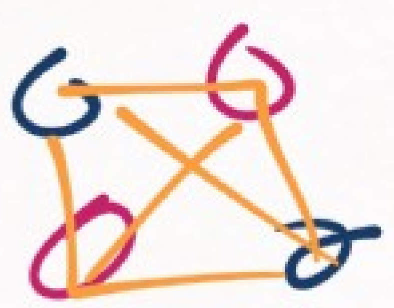
$\Rightarrow \boxed{\frac{B}{A} \to \text{const}}$ → What is the range?

↓

Argument from Wigner

1  $B/A \sim 1$

3  $B/A \sim 3$

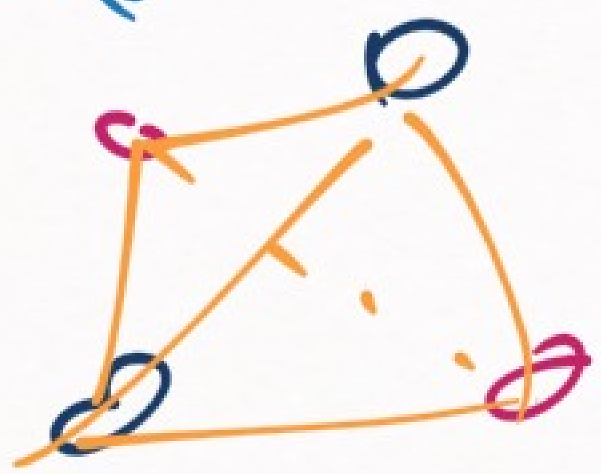
6  $B/A \sim 7$

→ Grows quickly

↳ the size $\simeq$ range of nuclear forces

After $A = 4$, grows slowly

^4He



$\sim \boxed{1.7\ \text{fm}}$ → the range of the nuclear force should be around this value

PION EXCHANGES

$$\boxed{\frac{1}{m_\pi} \sim 1.4\ \text{fm}}$$

BOTH ESTIMATIONS COMPATIBLE

We continue:

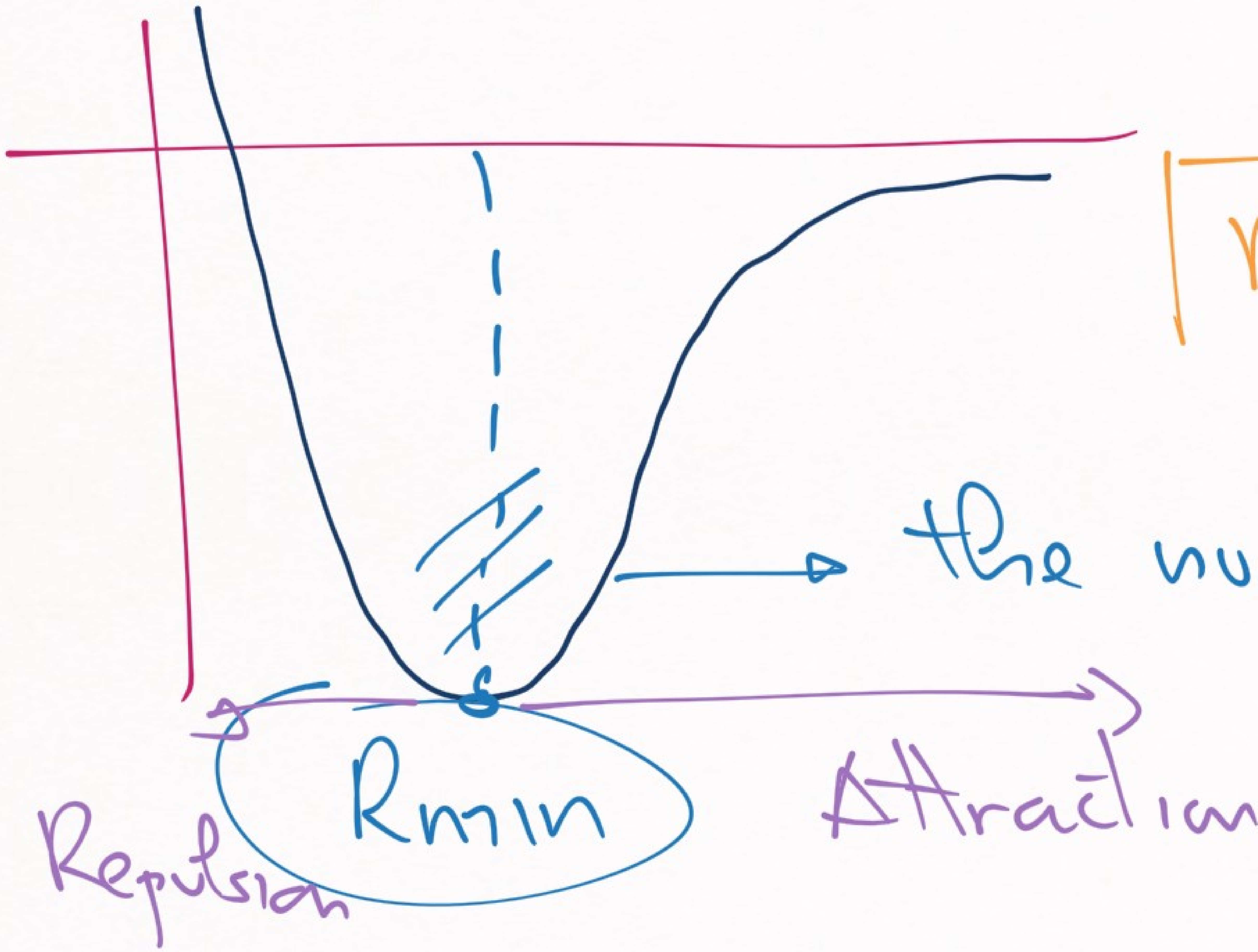
2) ATTRACTIVE AT INTERMEDIATE DISTANCES

$\rightsquigarrow$ density of heavy nuclei is constant

how do we explain this?

1) Nuclei bound
$\Rightarrow$ attraction

2) Density constan
$\Rightarrow$ repulsion at short distances



$\boxed{R_{min} \sim 1.8\ fm}$

the nucleons will love this spot

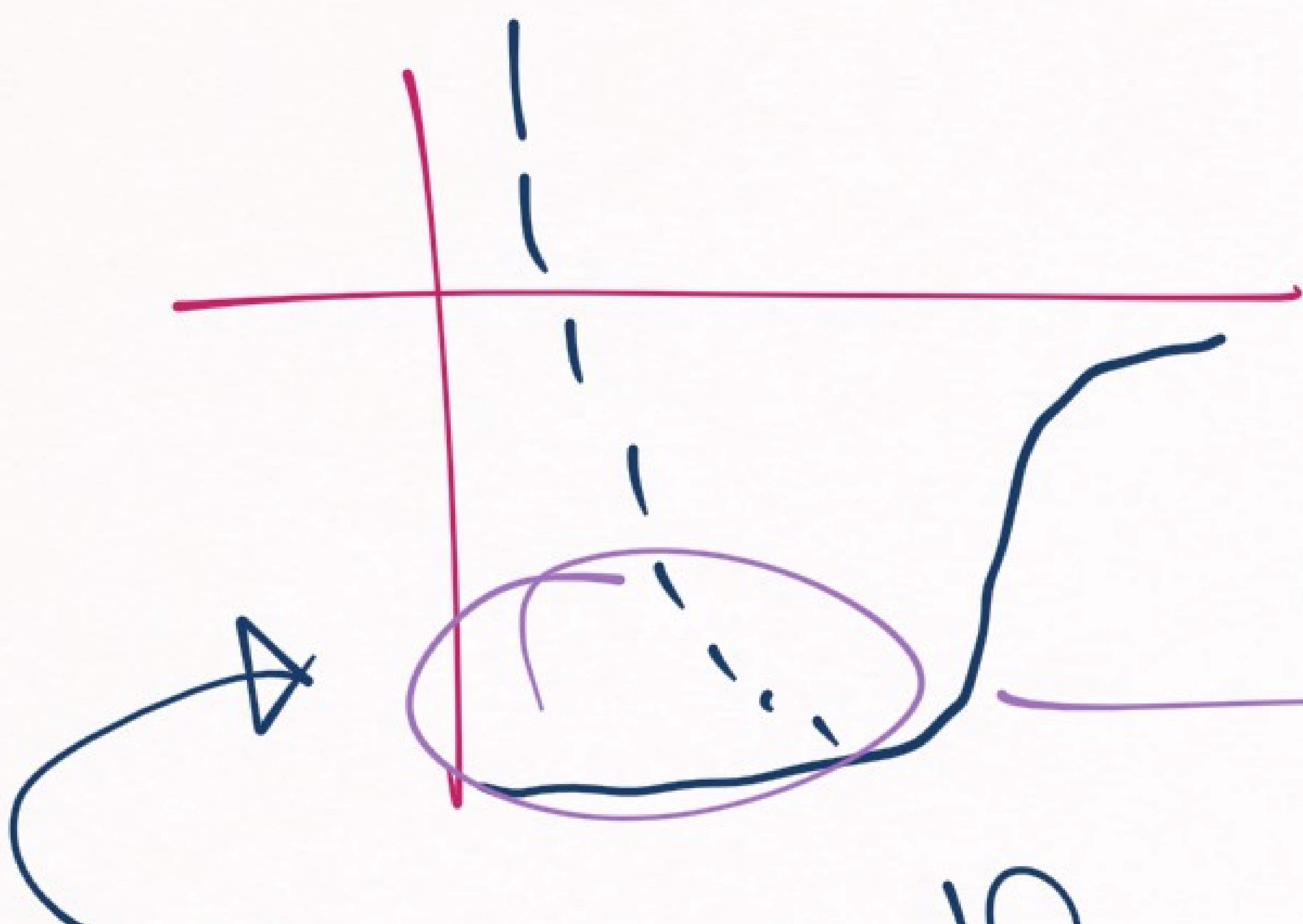
Density of heavy nuclei $\rightarrow$ 0.17 nucleon per fm^3

$$\rho_0 \sim 0.17\, fm^{-3} \sim \left(\frac{1}{R_{min}}\right)^3$$ (more or less)

$$\Rightarrow \boxed{R_{min} \sim 1.8\, fm}$$

Next one : 3) REPULSIVE AT SHORT-DISTANCES

3.a) We go back to the constant density



→ more nucleons ⇒ compress them more

→ no repulsion

$$\boxed{\cancel{\begin{array}{c}\rho(A) > \rho(A_0) \\ A > A_0\end{array}}}$$

not what happens

Imagine this potential

what happens is $\boxed{\rho(A) \wedge \rho(D_0)}$

$\to$ probably $\exists$ repulsive one

3.b) Phase shifts of ${}^1S_0$ channel

$\to$ scattering of np in S-wave, $S=0$

What is a phase shift?

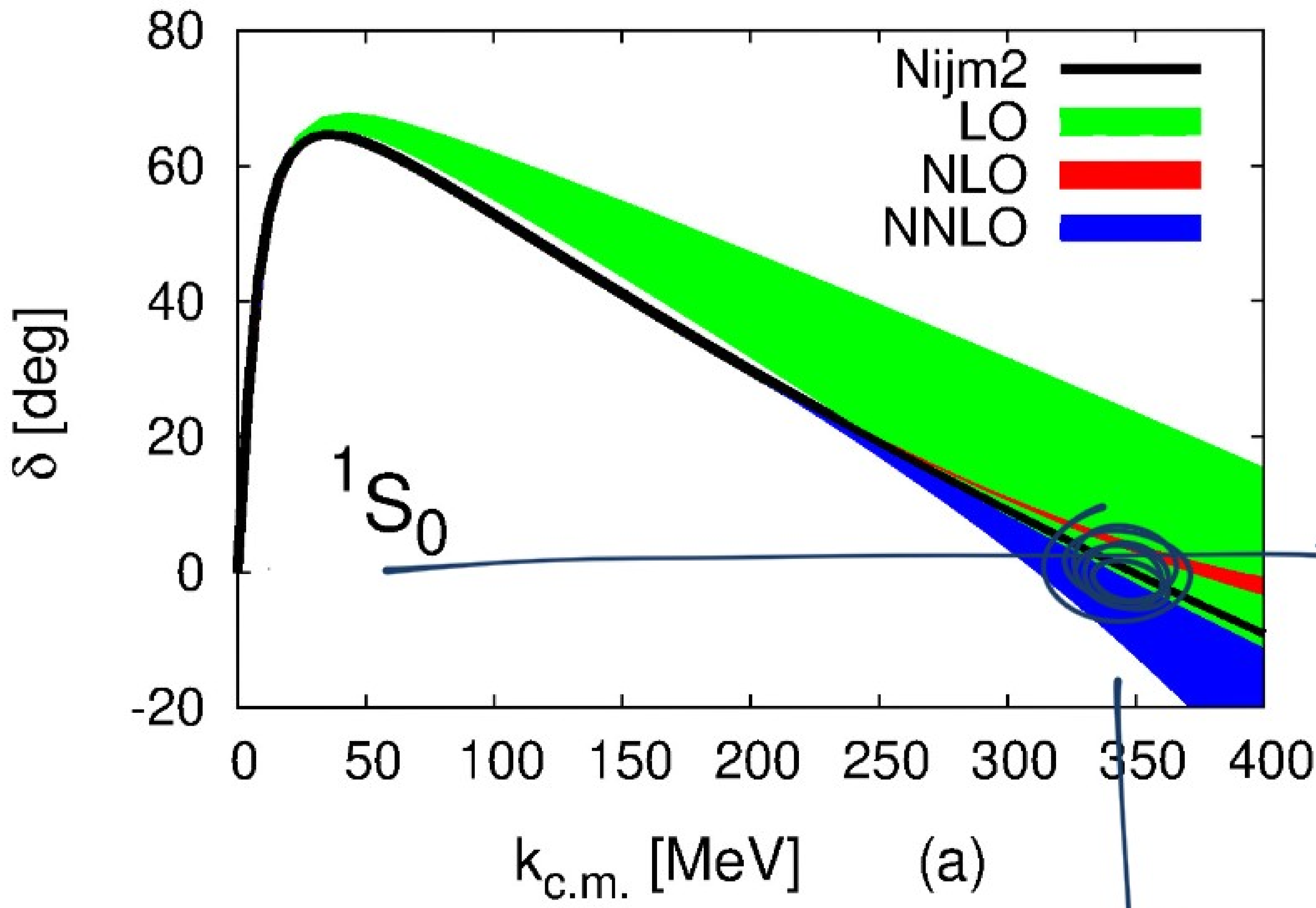
$$\left[\nabla^2 + U(\vec{r})\right]\Psi(\vec{r}) = k^2\,\Psi(\vec{r})$$

$$\Psi(\vec{r}) \propto \frac{u(r)}{r} \quad \text{(S-wave)}$$

If $U(\vec{r})$ is short-ranged $\Rightarrow$

$$u(r) \to \sin(kr + \delta)$$

$$\boxed{\text{PHASE SHIFT}}$$



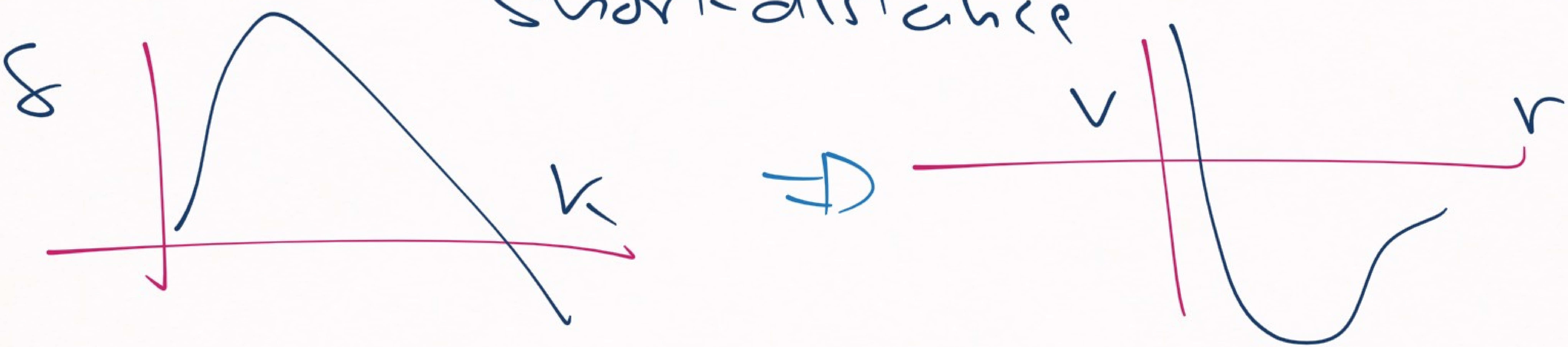
Black line is "experiment"

has a zero

curious ⇒ ⊕

⊛ ⟹ If you make lots of calculations w/ different potentials

⟹ This is easy to explain if the potential is repulsive at short-distance



Next one: 4) NUCLEAR FORCE DOES NOT DISTINGUISH NEUTRONS FROM PROTONS

How do we know?

$$\underbrace{B(^3He)}_{7{,}72\,\text{MeV}} \sim \underbrace{B(^3H)}_{8{,}48\,\text{MeV}}$$

^3He (ppn)

^3H (pnn)

Almost the same

(ΔB comes from Coulomb)

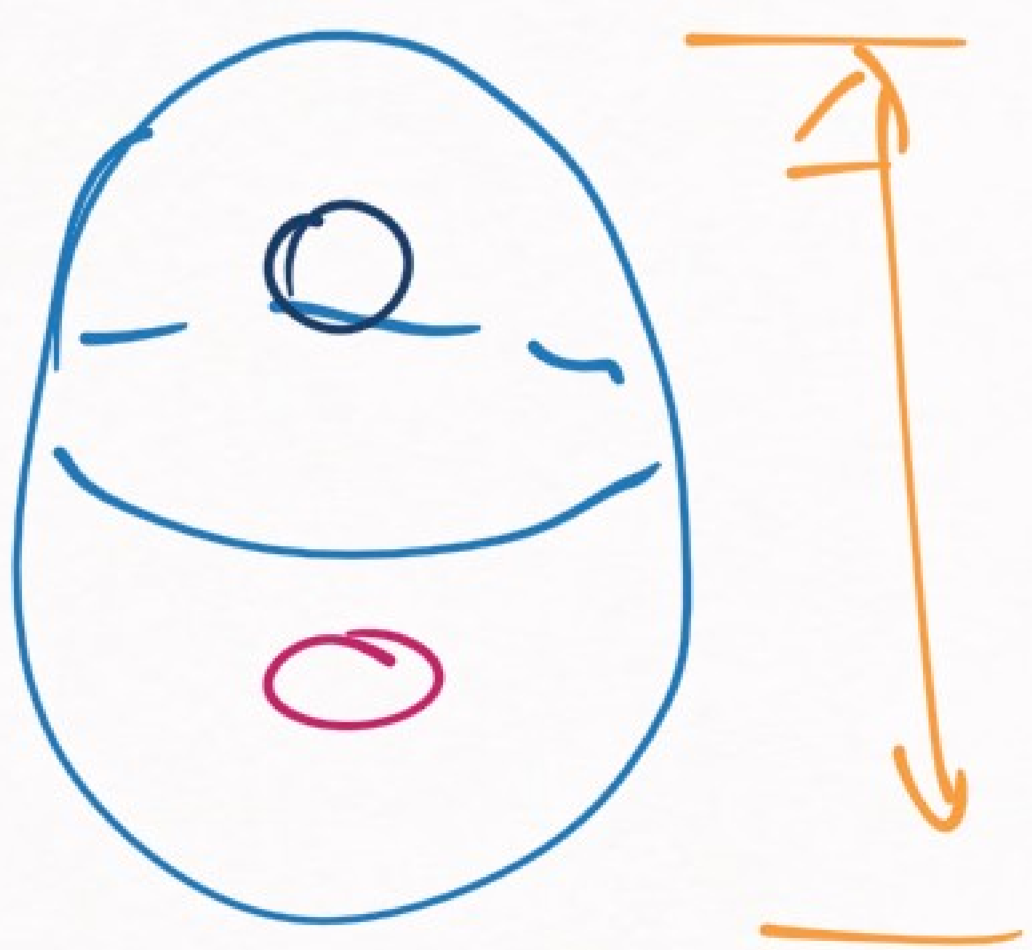
Last one: 5) NUCLEAR FORCE IS NOT CENTRAL

Reminder → Central force is $V(\vec{r}) = V(|\vec{r}|)$

Why not central? → Deuteron

↓

not spherical



Deuteron is a bit like a Youtiao (food)

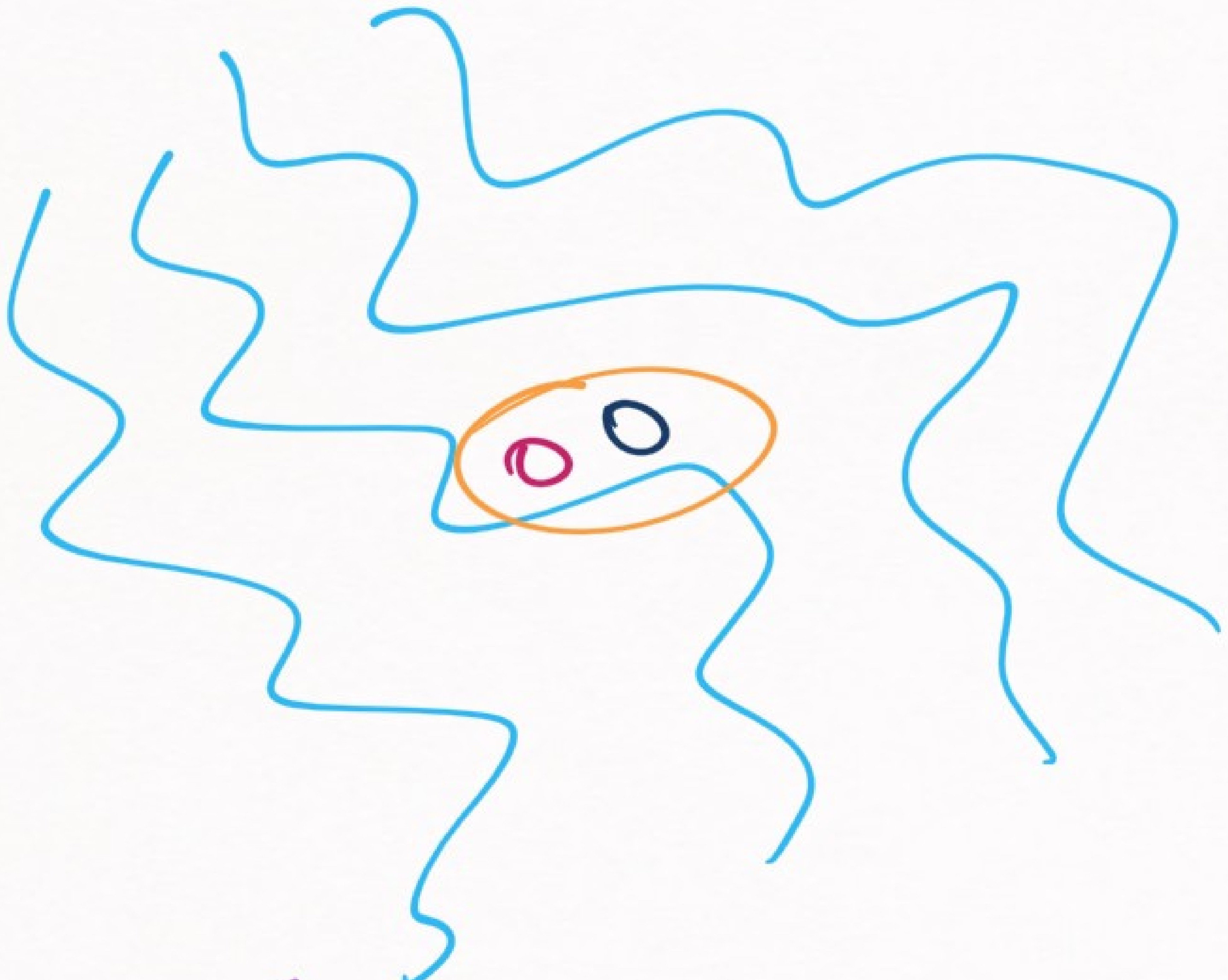
→ not completely central

How do we know that the deuteron is not spherical?

DEUTERON → QUADRUPOLE MOMENT

electromagnetic property → charge distribution

$\boxed{\text{REMINDER}}$ → WHAT IS A QUADRUPOLE MOMENT?



How does the deuteron experience this $\Phi(\vec{r})$?

$$V = q\,\Phi(\vec{r}) + \vec{d}\cdot\vec{\nabla}\Phi(\vec{r}) + \frac{1}{6} Q_{ij}\,\partial_i \partial_j \Phi(\vec{r}) + \dots$$

$\Phi(\vec{r})$ → e.m. potential

$$\vec{E} = \vec{\nabla}\Phi(\vec{r})$$

⊕ → "multipolar expansion of the potential"

$$V = \int d^3\vec{r}\ \underbrace{\rho(\vec{r})}_{\text{charge distribution}}\ \underline{\Phi}(\vec{r})$$

$$\int d^3\vec{r}\ \rho(\vec{r}) = e$$

If $\Phi(\vec{r})$ changes at distances larger than the charge distribution of the object inside it

$$\Rightarrow \quad \Phi(\vec{r}) = \Phi(\vec{0}) + \vec{r} \cdot \vec{\nabla} \Phi(\vec{r})\Big|_{\vec{r}=0}$$

$$+ \frac{1}{2} r_i r_j \partial_i \partial_j \Phi(\vec{r})\Big|_{\vec{r}=0} + \ldots$$

$$V = \int d^3\vec{r}\; \rho(\vec{r})\, \Phi(\vec{r})$$

$$= \underbrace{q}_{\text{charge}}\, \Phi + \underbrace{\vec{d}}_{\substack{\text{dipolar}\\ \text{moment}}} \cdot \vec{\nabla}\Phi + \frac{1}{6} \underbrace{Q_{ij}}_{\substack{\text{quadrupolar}\\ \text{moment}}}\, \partial_i \partial_j \Phi + \ldots$$

Charge $\rightarrow$ $q = \int d^3\vec{r}\ \boxed{\rho(\vec{r})}$ $\rightarrow$ charge distro

Dipole moment $\rightarrow$ $\vec{d} = \int d^3\vec{r}\ \vec{r}\ \rho(\vec{r})$

Quadrupolar moment $\rightarrow$ $Q_{ij} = \int d^3\vec{r}\ \rho(\vec{r})\left(3r_ir_j - r^2\delta_{ij}\right)$

9 moments $\rightarrow$ but $Q_{ij} = Q_{ji}$ & $\sum Q_{ii} = 0$ $\rightarrow$

→ S moments

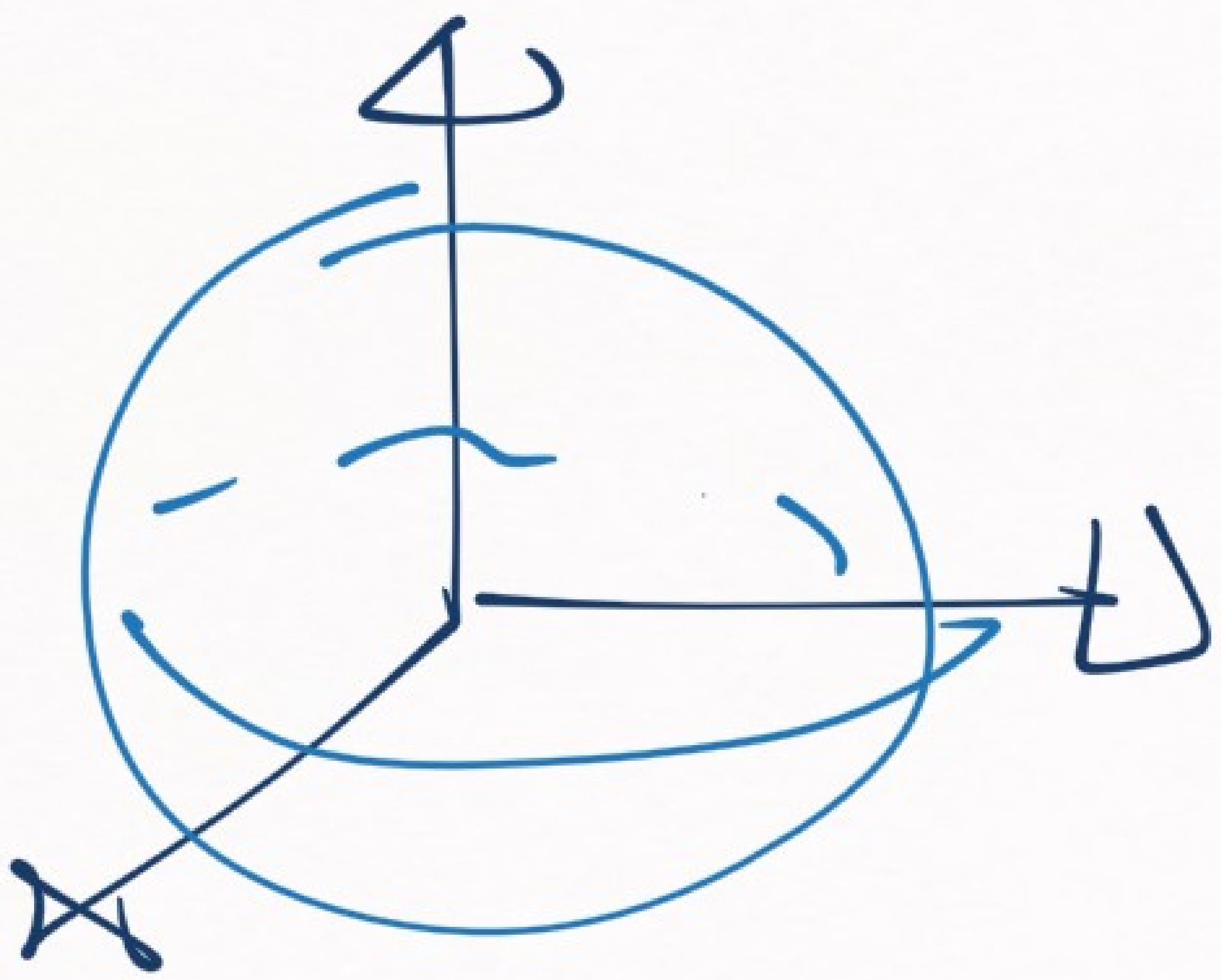
Quadrupole moment of a charge distribution

Usual definition → $\boxed{Q = Q_{33}}$

Deuteron : $\boxed{Q_d = 0.286 \ \text{fm}^2}$

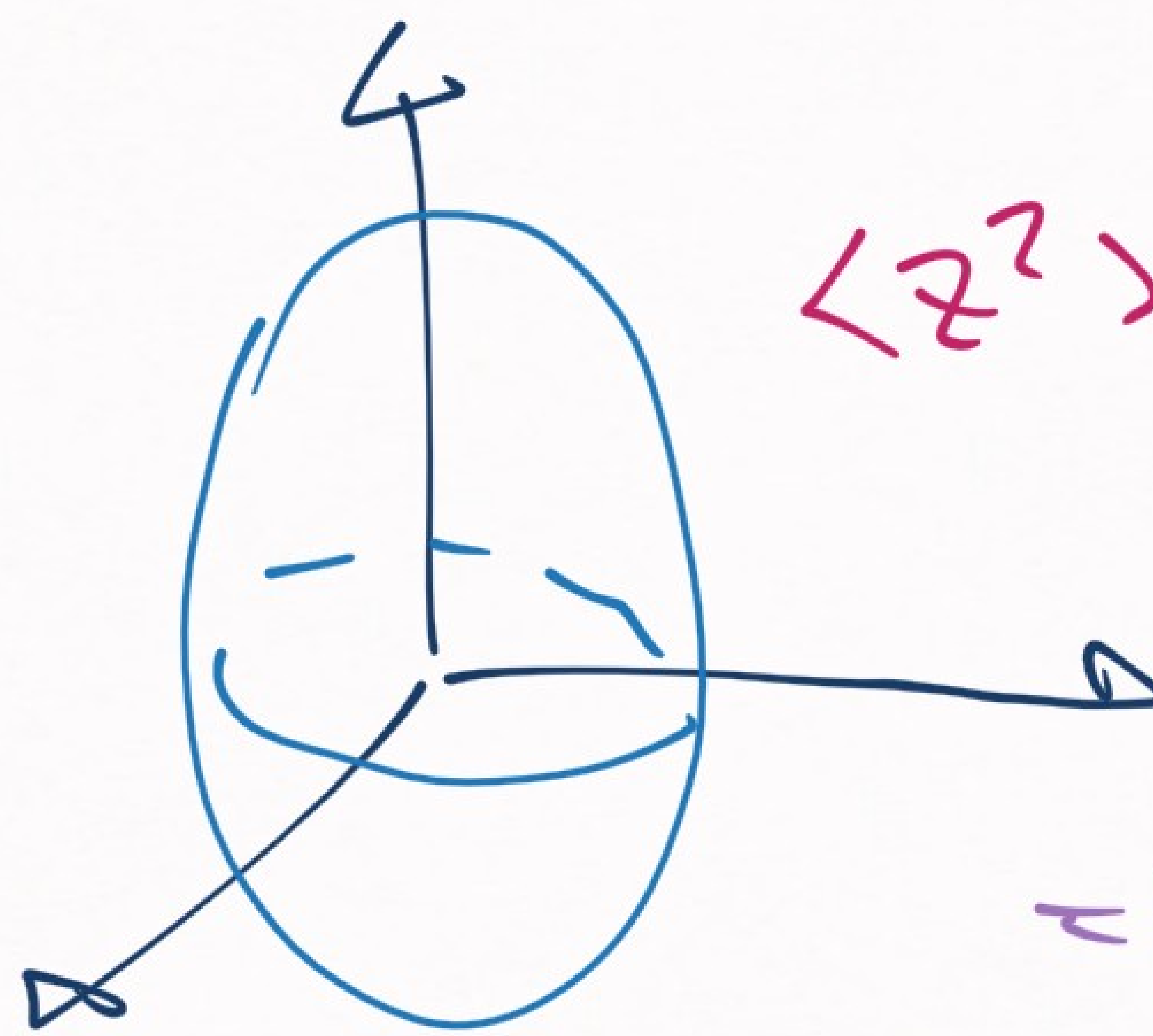
$\boxed{Q_d > 0}$ → $Q_{33} = \int d^3\vec{r}\, \rho(\vec{r}) \left(3z^2 - r^2\right)$

$$Q_{33} > 0 \Rightarrow \int d^3\vec{r}\, z^2 \rho(\vec{r}) > \int d^3\vec{r} \left(\frac{r^2}{3}\right) \rho(\vec{r})$$



spherical

$$\to \langle x^2 \rangle = \langle y^2 \rangle = \langle z^2 \rangle$$



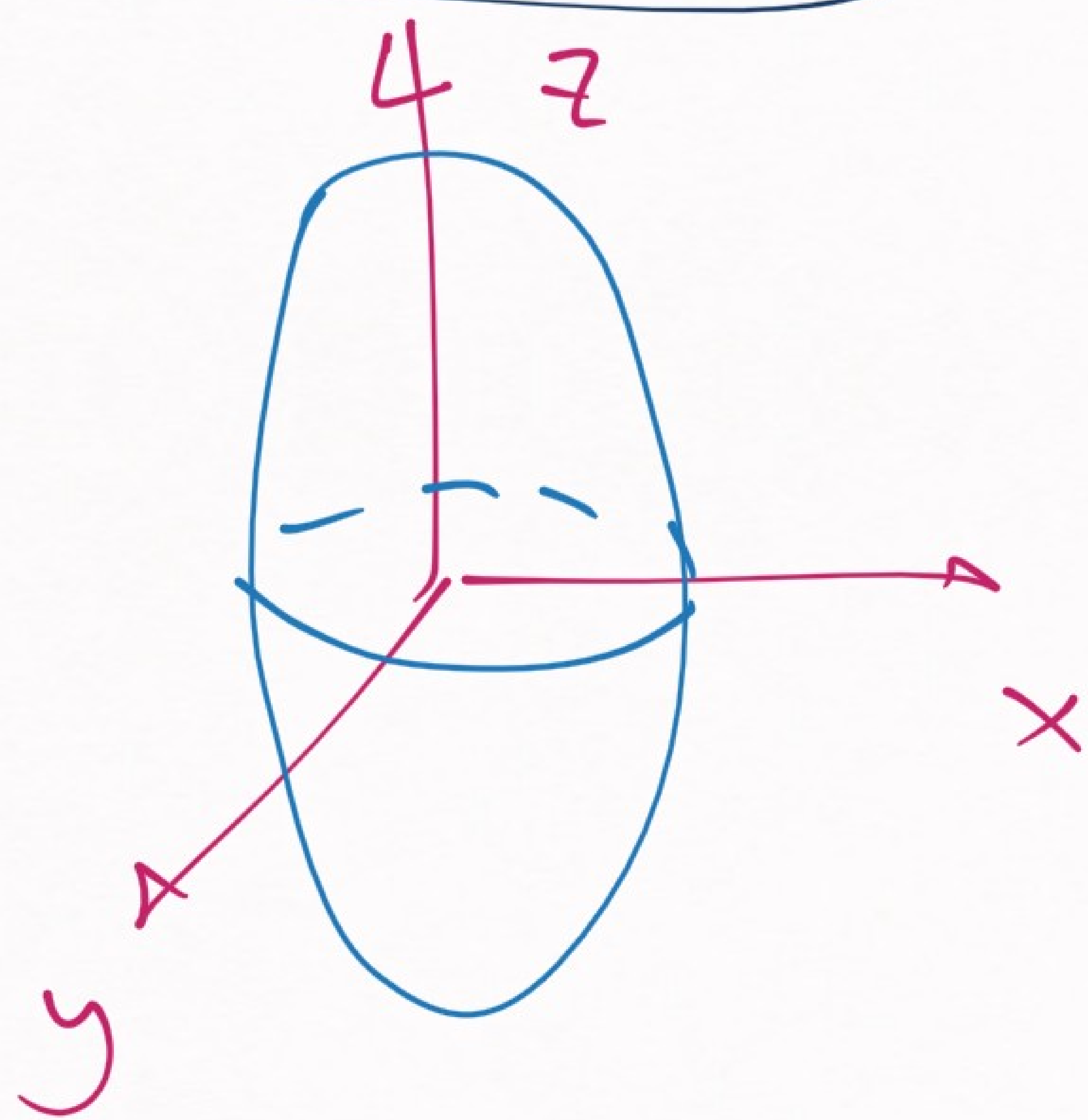
$$\langle z^2 \rangle \geq \langle x^2 \rangle = \langle y^2 \rangle$$

$$\langle r^2 \rangle = \langle x^2 + y^2 + z^2 \rangle$$

$$\Rightarrow \boxed{\langle z^2 \rangle \geq \left\langle \frac{r^2}{3} \right\rangle}$$

$$\boxed{Q_d = 0.286\ \mathrm{fm}^2}$$

$\Rightarrow$



$\rightarrow$ Why is this the case?

$\downarrow$

ORIGIN OF NUCLEAR FORCES

→ FOR THE NEXT LESSON