NUCLEAR PHYSICS | 2

Why nuclear physics is difficult?

$\boxed{\text{RECAP}}$

→ A general principle:

[Scales are important to understand physics]

$\forall$ check → typical size of a system

→ typical momentum of a system

→ Natural problems

→ natural scale is easy to find

→ easy to estimate the size of an observable on purely dimensional ground (numerical factors $\sim O(1)$)

Example $\rightarrow$ [HYDROGEN ATOM]

$$\left[-\nabla^2 - \frac{2}{a_B r}\right]\Psi(\vec{r}) = -\gamma^2 \Psi(\vec{r})$$

$$\rightarrow \Psi(\vec{r}) = \frac{1}{\sqrt{4\pi}} \frac{2}{a_B^{3/2}} e^{-r/a_B} \quad \text{(SOLUTION)}$$

$\boxed{\text{SCALES}} \longrightarrow$ a_B (length scale)

$Q_B = \frac{1}{a_B}$ (momentum scale)

$$a_B = \frac{\hbar c}{m_e \alpha} \simeq 5.3 \cdot 10^4 \, \mathrm{fm} = 0.53 \, \mathrm{\AA}$$

$$Q_B = m_e \alpha \simeq \frac{0.511 \, \mathrm{MeV}}{137} \simeq 3.7 \, \mathrm{KeV}$$

$$Q_B = \frac{1}{a_B}$$

We can define natural scales

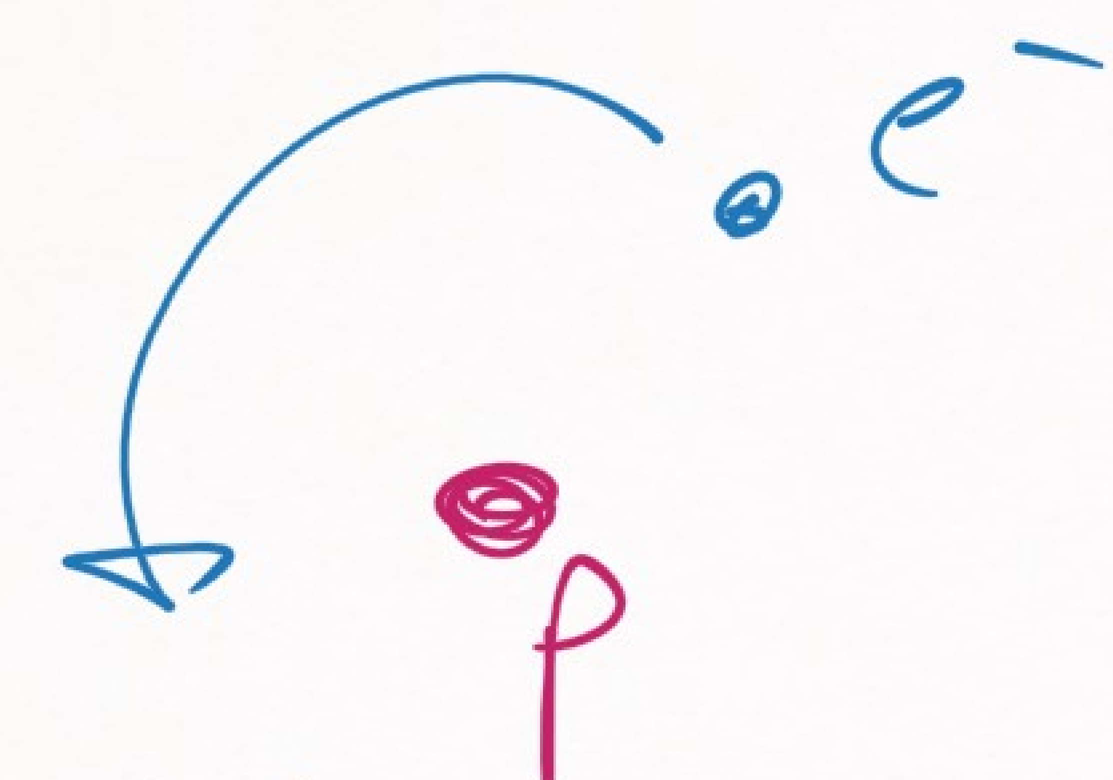
How do we use this knowledge about scales?

$$E_B = -\frac{\gamma^2}{2\mu} \rightsquigarrow \overbrace{\gamma}^{[L]^{-1}} \sim \left(Q_B = \frac{1}{a_B}\right) \times \mathcal{O}(1)$$

$$\sqrt{\langle r^2 \rangle} \sim \mathcal{O}(1) \times a_B$$

$$\frac{\sqrt{\langle p^2 \rangle}}{m_e} \sim \langle v \rangle \sim \mathcal{O}(1) \times \frac{Q_B}{m_e}$$ → Let's see this one

→ Let's see this example of the speed of an electron within the H atom



$$\langle v \rangle \doteq \left(\sqrt{\langle v^2 \rangle}\right) =$$

$$\sqrt{\frac{\langle p^2 \rangle}{m_e^2}} = \sqrt{\langle v^2 \rangle} = O(1) \times \frac{a_B}{m_e} = \circledast$$

$$p = mv$$

$$\circledast = O(1) \times \frac{m_e \alpha}{m_e} = \oplus$$

$$\oplus = O(1) \times \alpha \, , \quad \alpha \simeq \frac{1}{137}$$

$$\sqrt{\langle v^2 \rangle} \sim \frac{1}{137}\, \mathcal{O}(1) \rightarrow \text{what does this mean?}$$

$\rightarrow$ natural units: $\dfrac{v}{c}$ $(c = 1)$

$$\sqrt{\langle v^2 \rangle} \sim \mathcal{O}(1)\, \frac{c}{137} \sim \frac{3 \cdot 10^5\,\mathrm{km/s}}{137} \simeq 2200\ \mathrm{km/s}$$

Expectation (from dimensional argument / from naturalness): $\sqrt{\langle v^2 \rangle} \sim \alpha$

Explicit calculation: $\hat{p}^2 = -\vec{\nabla}^2$

$$\psi(\vec{r}) = \frac{1}{\sqrt{4\pi}} \frac{2}{a_B^{3/2}} e^{-r/a_B}$$

$$\sqrt{\frac{\langle \psi | \hat{p}^2 | \psi \rangle}{m_e^2}} = \alpha \longrightarrow \boxed{\text{spot on}}$$

[Take-home message]

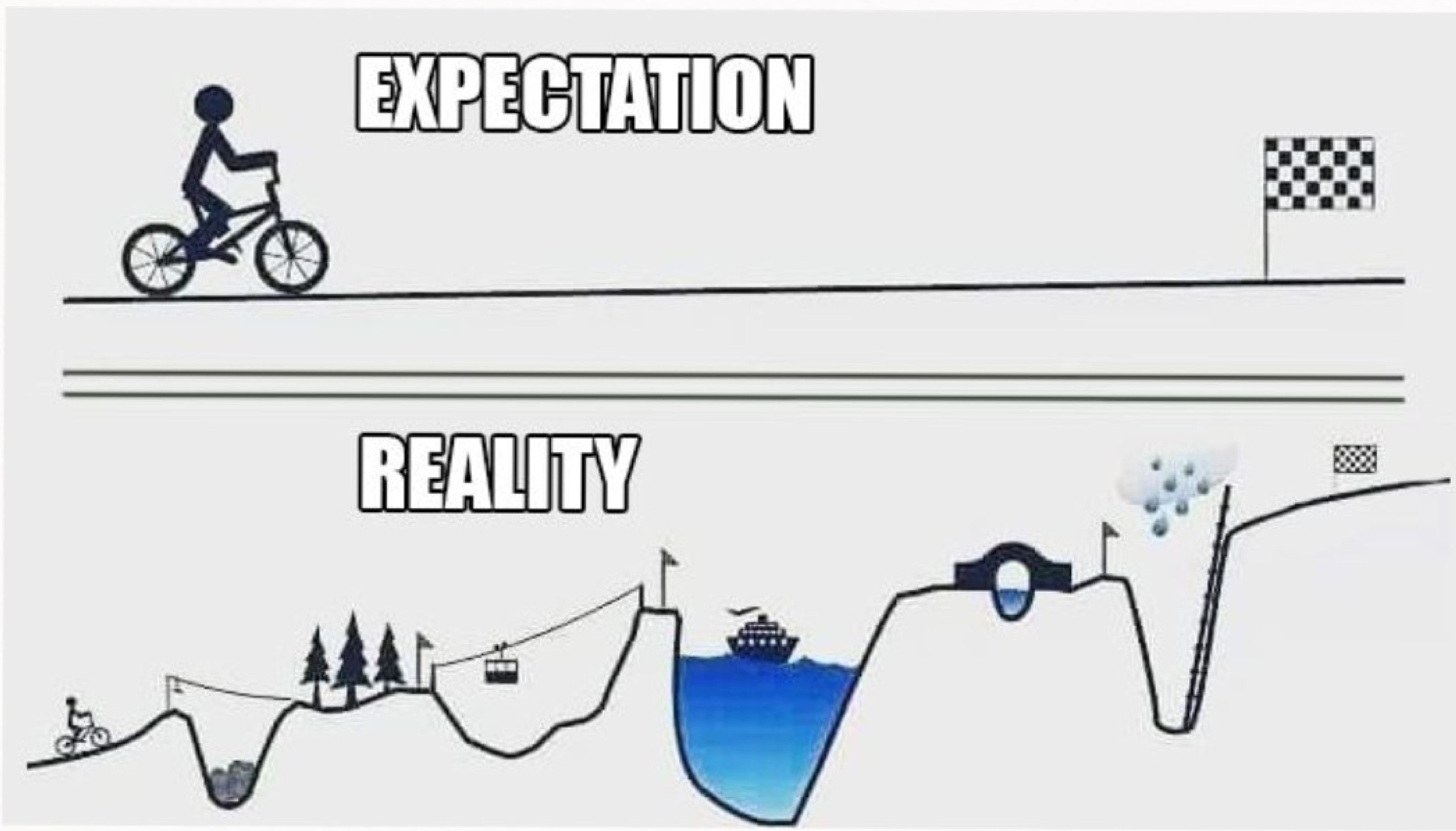
dimensional estimate	actual calculations
$\gamma \sim 1/a_B$	$\gamma = 1/a_B$
$\sqrt{\langle r^2 \rangle} \sim a_B$	$\sqrt{\langle r^2 \rangle} = \sqrt{3}\, a_B$
$\sqrt{\langle v^2 \rangle} \sim \frac{1}{\alpha}$	$\sqrt{\langle v^2 \rangle} = 1/\alpha$

→ this type of arguments claiming "naturalness" do work in many cases

(not always the case)

(but very often it is the case)

But life is in general not always so easy



→ naturalness assumption

→ real world

[WHEN WILL NATURALNESS WORK?]

NATURAL PROBLEM

(i) CHARACTERISTIC SCALE (SIZE/MOMENTUM)

H atom → (electron mass)

× (strength of Coulomb)

$Q_B = m_e \alpha \simeq 3,7\,\mathrm{keV}$ $\left(\frac{1}{Q_B} = a_B \simeq 0{,}53\,\mathring{A}\right)$

→ It is easy to find a natural scale

(ii) EVERY OBSERVABLE IS $\mathcal{O}(1)$ IN TERMS OF THE CHARACTERISTIC SCALE

$$[\langle \hat{O} \rangle] = [L]^n \rightsquigarrow \langle \hat{O} \rangle \sim \underline{\underline{\mathcal{O}(1)\, a_B^n}}$$

NATURAL PROBLEM

→ characteristic scale

→ $\forall$ natural ($\mathcal{O}(1)$) in this scale

MESSAGE → $\boxed{\text{NATURAL PROBLEMS ARE EASY TO DEAL WITH!}}$

Another example of a natural problem:

BOILING POINT OF CESIUM

liquid/gas transition

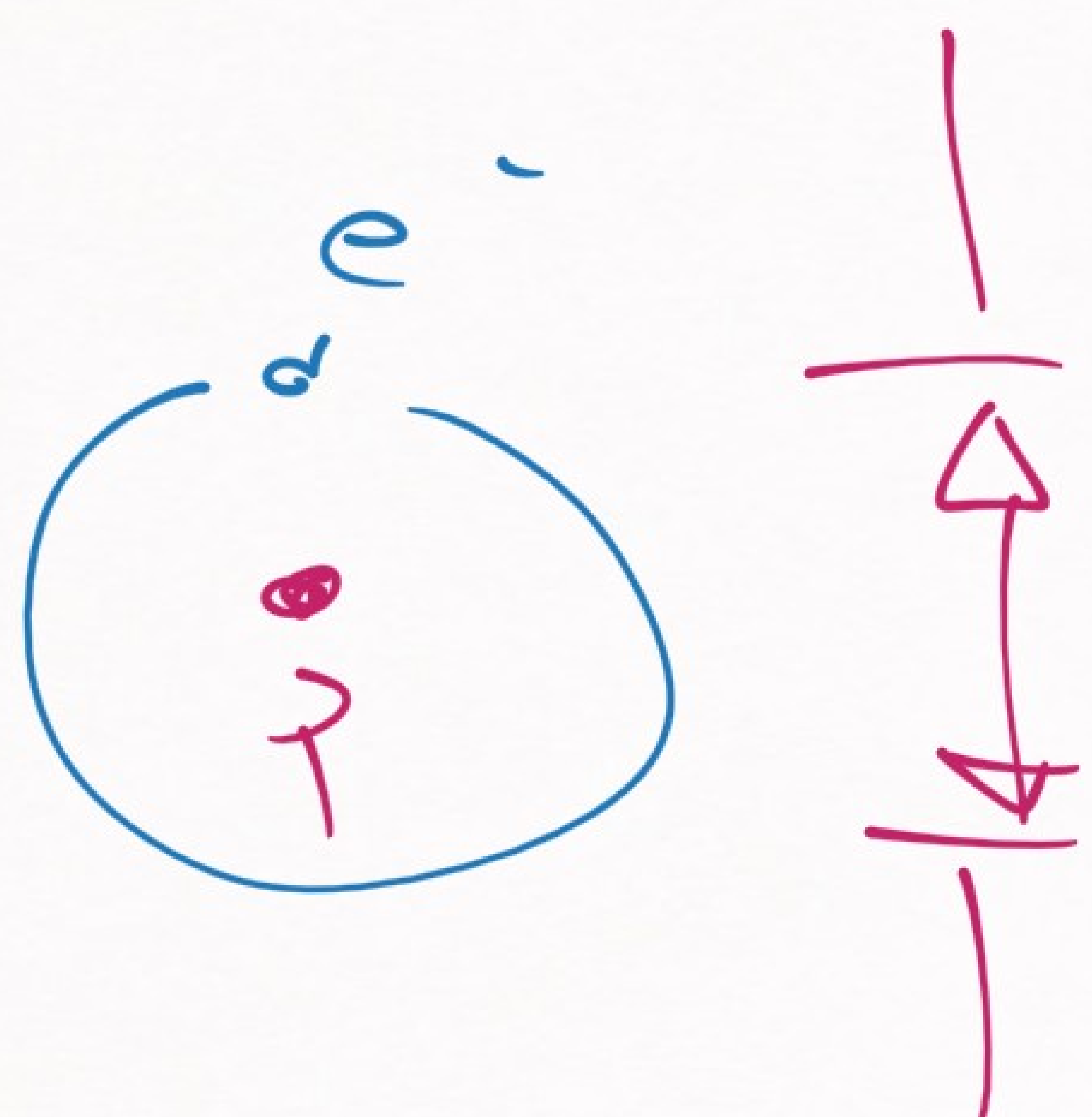
the element

铯

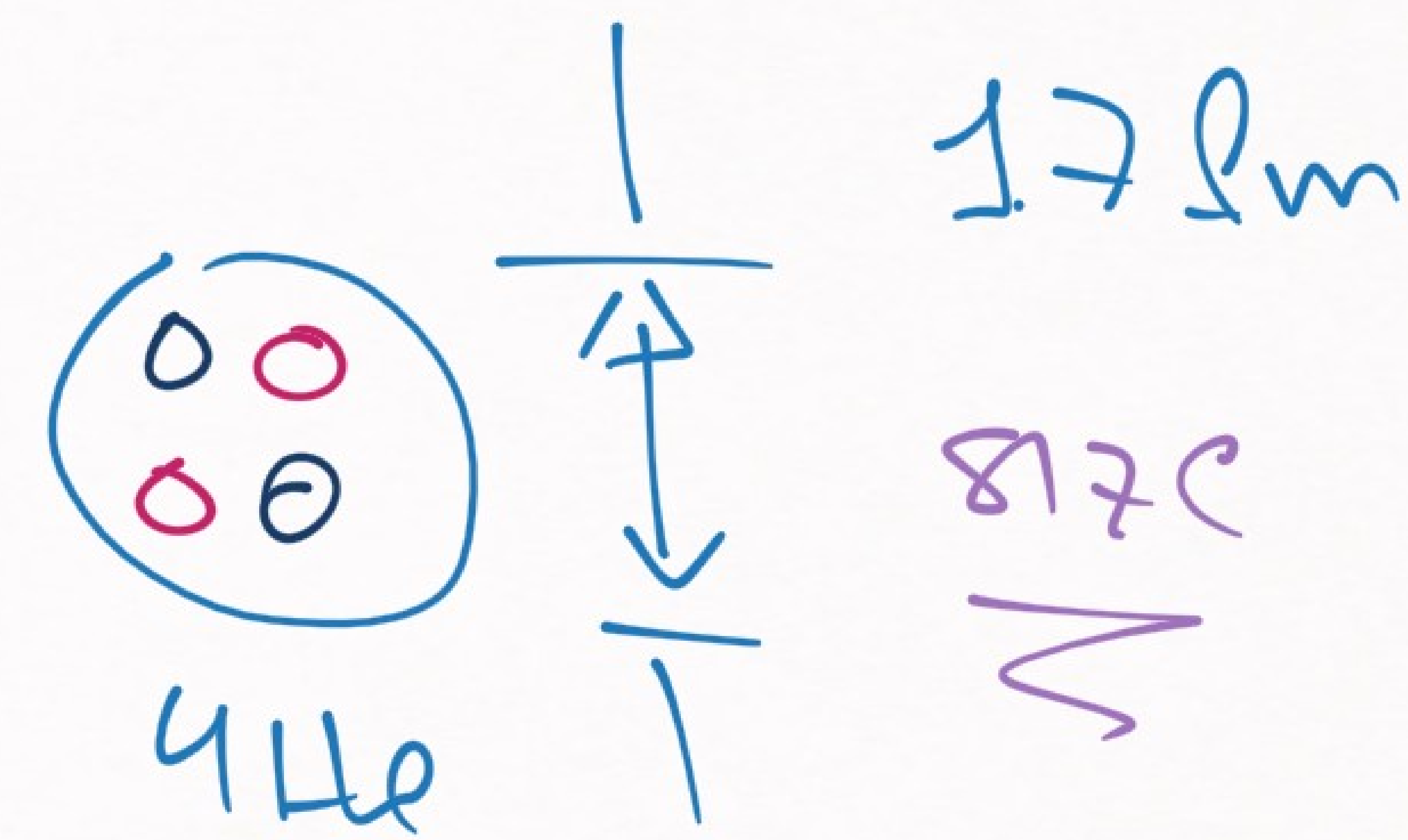
Use scale arguments

→ a part of this calculation is not trivial

↳ identification of the relevant scales



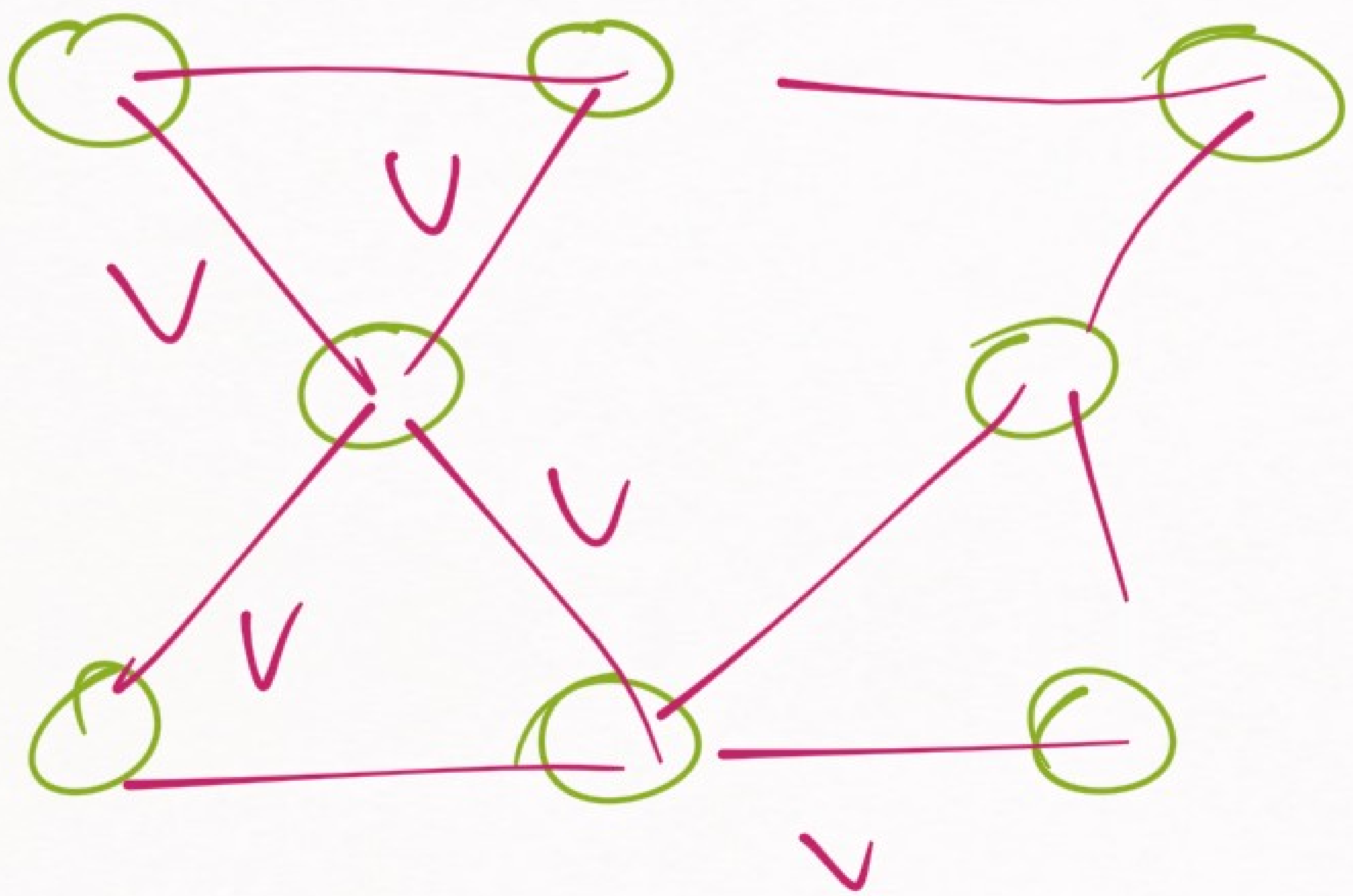
size $\simeq$ relevant scale

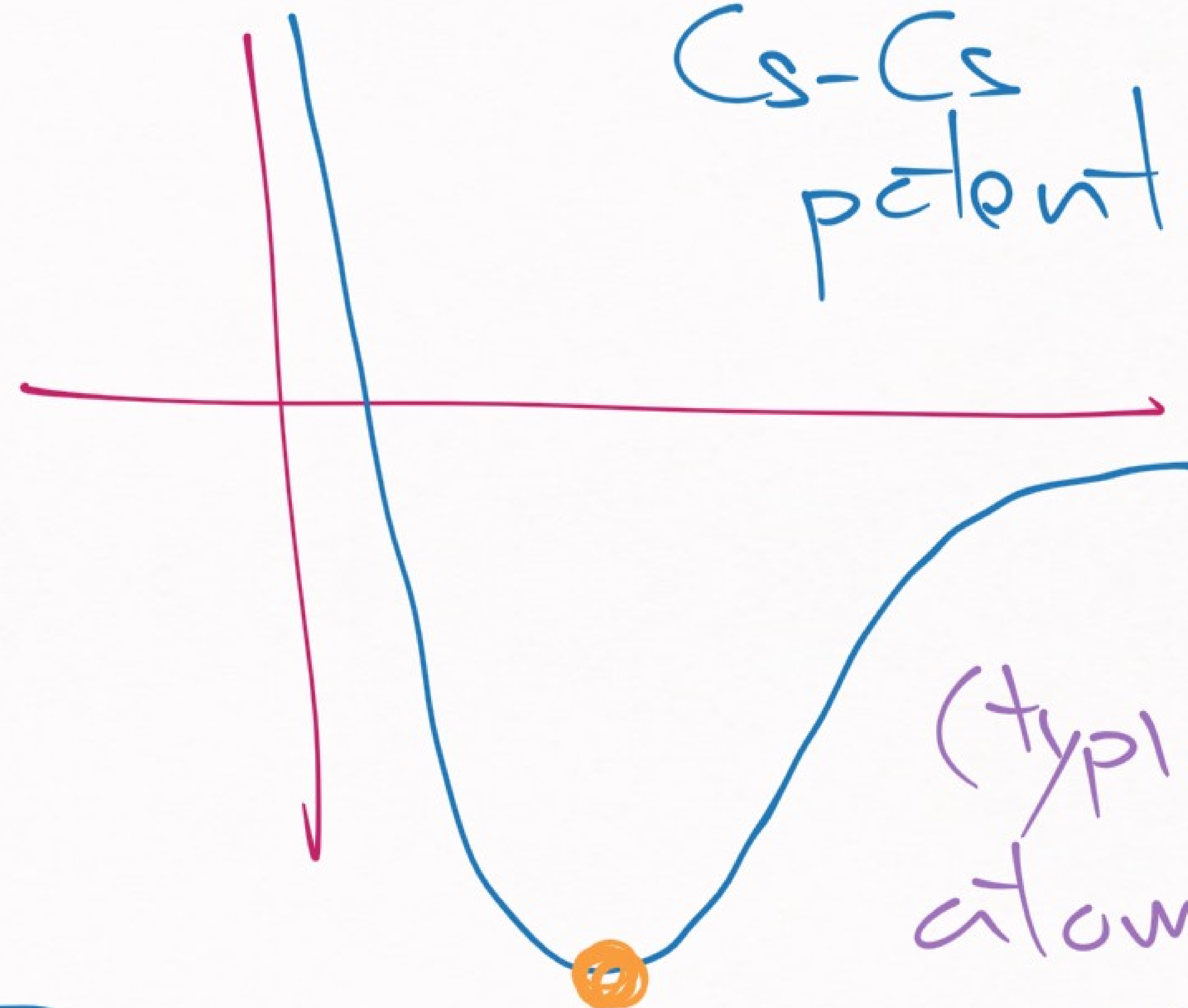


SCHEMATICS OF THE ARGUMENT I WILL USE

→ A group of Cs atom, they will have a natural average speed $\sqrt{\langle v^2 \rangle}$

→ If the thermal speed of atoms is similar to this natural speed ⇒ Cs needs to be a gas





average separations
&
average speed (momentum)

minimum potential energy

→ Estimate the expected momentum of
Cs atoms (when ignoring external
factor such as temperature)

Objective #1 → $\boxed{\text{We need info about Cs}}$

$\boxed{Cs}$

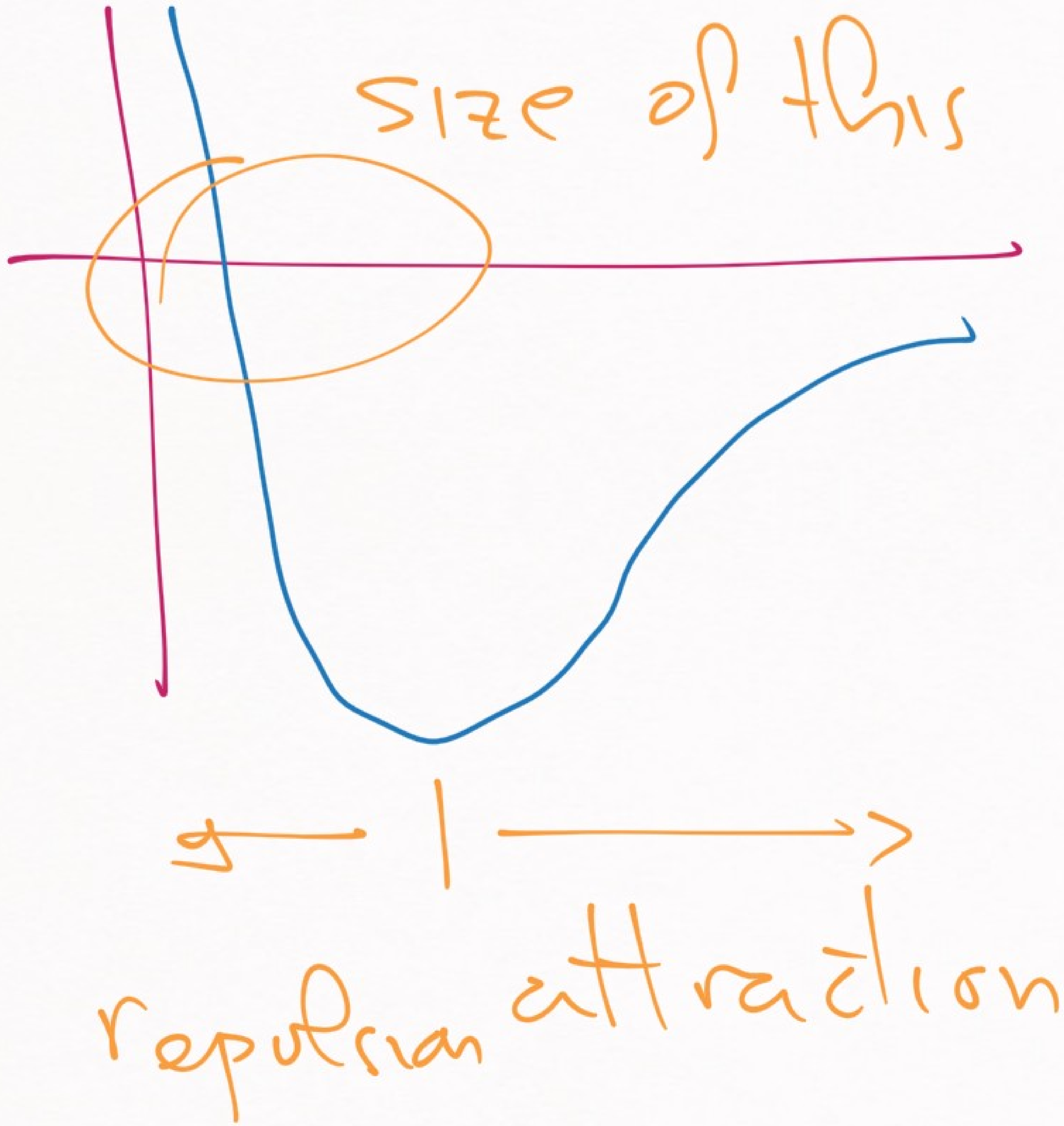
$^{133}Cs \rightarrow$ atomic number : 133

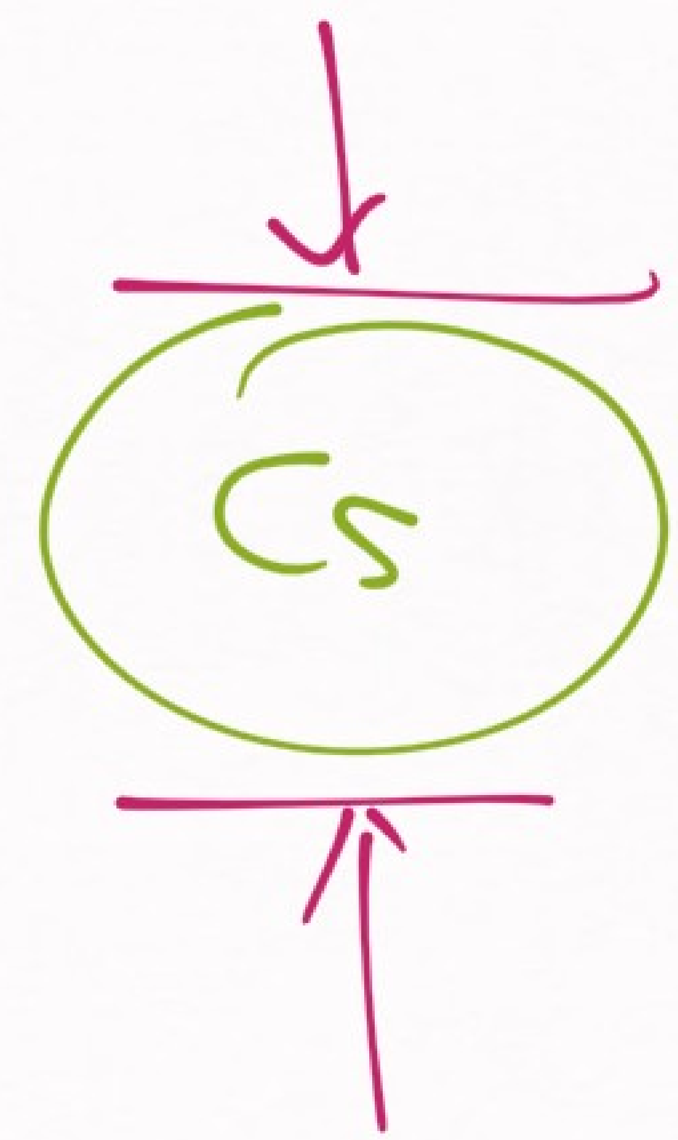
$m(Cs) \simeq 130$ GeV (approximately)

why? $\rightarrow$ 133 nucleons $\quad m_p \simeq m_n \simeq 1$ GeV

(0,94 GeV)

size of this repulsion?





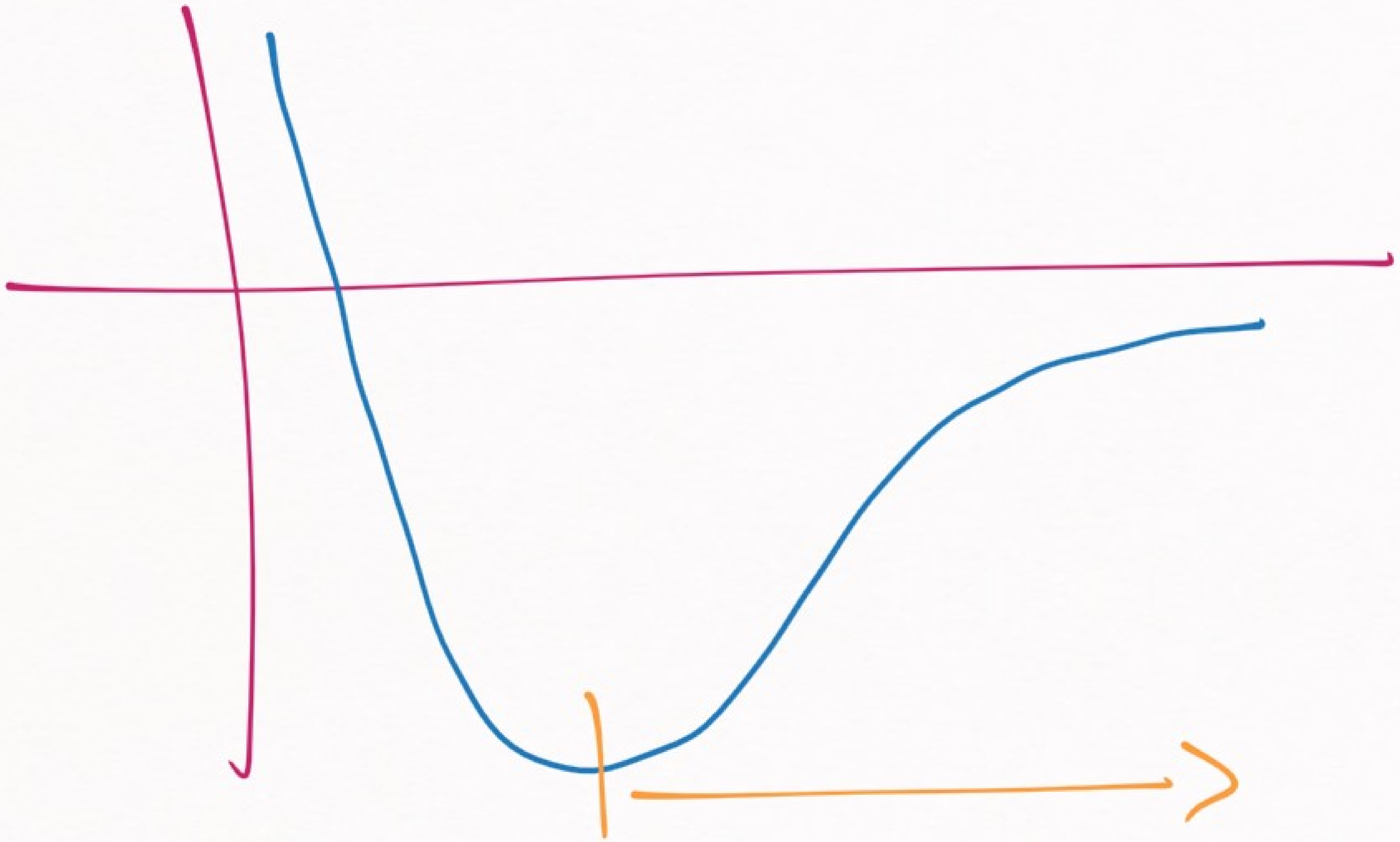
size of Cs

$\simeq 7$ a.u.

1 a.u. $= 1\, a_B$

$\simeq 7\, a_B$

typical atomic potential



attraction

$$V_{AA}(r) = \frac{C_{12}}{r^{12}} - \frac{C_6}{r^6}$$

(Lennard Jones potential)

short-range repulsion

long-range attraction

Long-range attraction → well known

→ given by the van der Waals potential

$$V_{vdW}(r) = -\frac{C_6}{r^6} = -\underbrace{\frac{1}{2\mu}\frac{R_6^4}{r^6}}$$

→ length scale

Why this step? → I want a scale

$$-\frac{C_6}{r^6} = -\frac{1}{2\mu}\frac{R_6^4}{r^6} \rightarrow \boxed{R_6 = (2\mu C_6)^{1/4}}$$

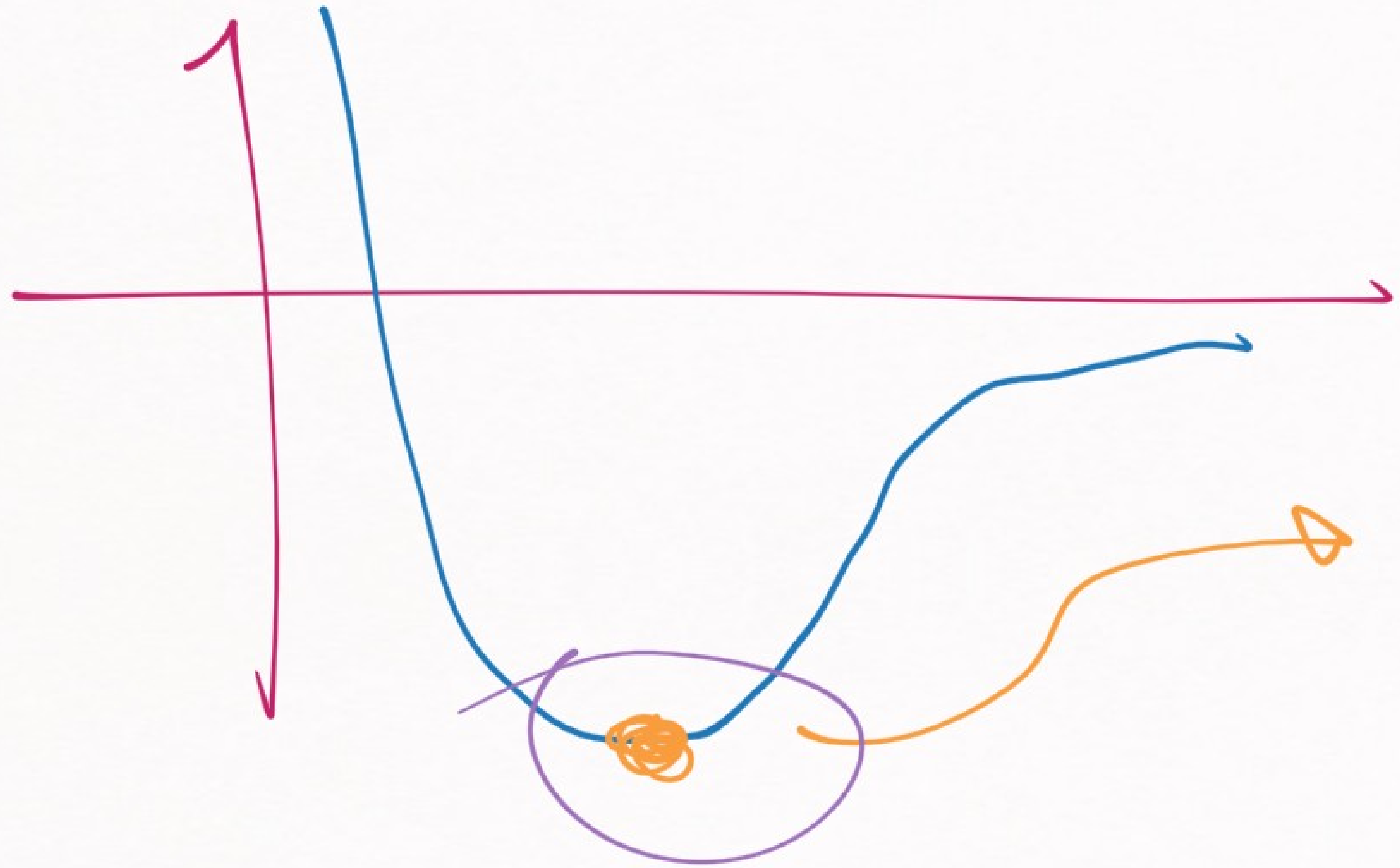
C_6, μ are well known → $\boxed{R_6 \simeq 200 \text{ a.u.}}$

Two possible relevant scales:

$$R(Cs) \equiv R_5 = 7\, a_B$$

$$R(vdW) \equiv R_6 = 203\, a_B$$

→ they are still not the scale we are looking for



Cs atoms will try to get into minimum energy configurations

$E_{kinetic} \simeq E_{potential} \longrightarrow E_{kin} = \frac{1}{2} m \langle v^2 \rangle$

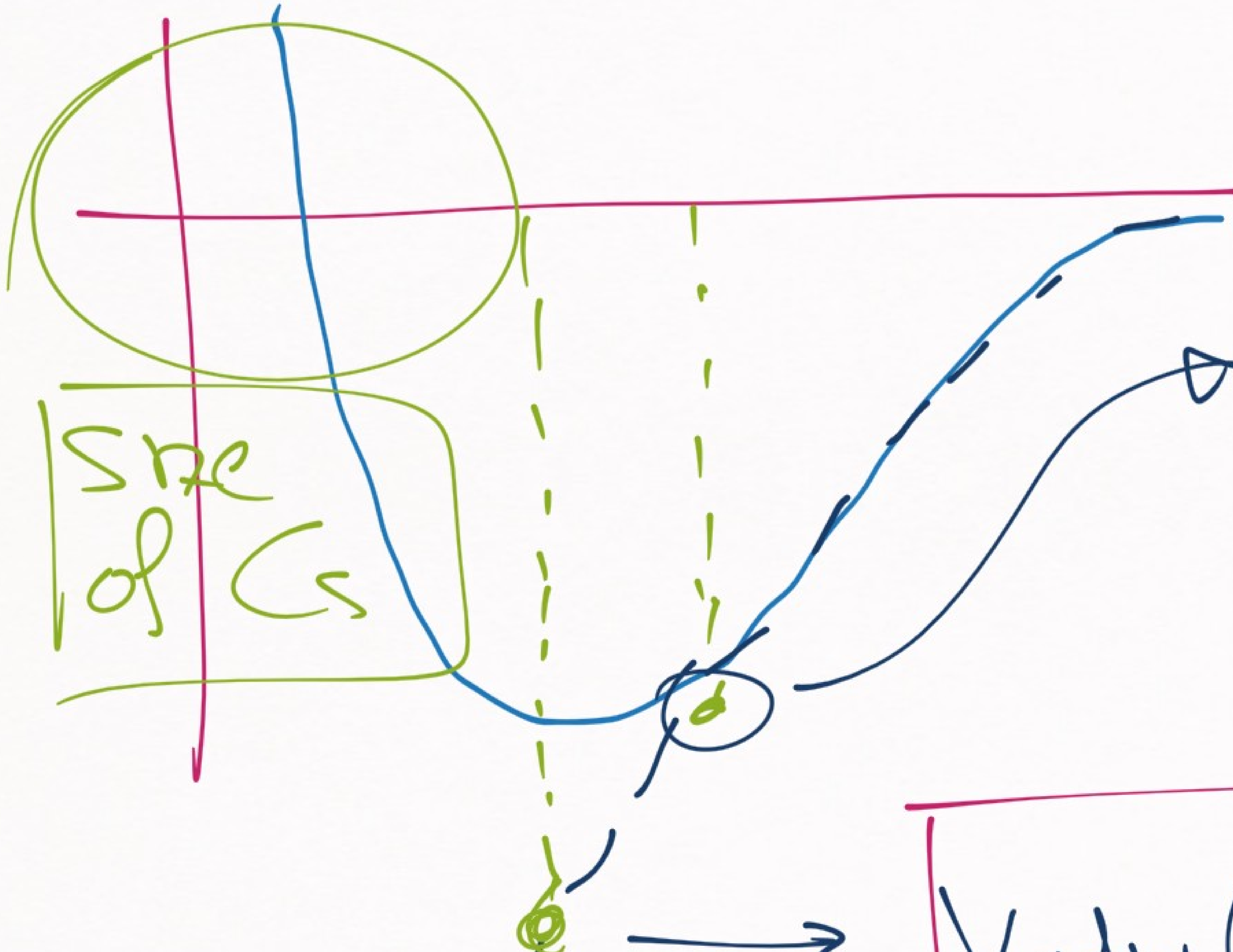
$\langle v^2 \rangle$

AIM $\rightarrow$ get $\langle v^2 \rangle$ for a group of Cs atoms (or $\langle E_{kin} \rangle$)

(limitation -> only dimensional arguments)

$\leftarrow$ Looking at previous page

$\langle E_{kin} \rangle = \langle V_{min} \rangle \rightarrow$ we want to estimate this



$V_{min} \simeq V_{vdW}(R \gtrsim R_s)$

$\boxed{V_{vdw}(R_{Cs}) < V_{min}}$

van der Waals → we know

$R_s \simeq 7a_B$ $\Rightarrow$ (50% more maybe)

(approximations)

$\Rightarrow$ 10 a_B

$$\boxed{V_{min} \simeq V_{vdW}(R = 10\, a_B)}$$

$$V_{min} = -\frac{1}{2\mu}\frac{R_6^4}{R_{min}^6}$$

$R_{min} \simeq 10\, a_0$

from here I will get the expeted size of the speed/energy of Cs atom

$$|\langle E_{kin}\rangle| \simeq |V_{min}| \Rightarrow \underbrace{\frac{1}{2\mu} Q_{cs}^2}_{\text{kinetic energy}} = \frac{1}{2\mu} \frac{R_6^4}{R_{min}^6}$$

→ doing the calculation

$$\boxed{Q_{cs} \sim 20\, \underbrace{Q_B}_{3.7\,\text{keV}} \sim \frac{20}{a_B}}$$

$$\langle R \rangle \sim R_{min}$$

$$\langle E_{kin} \rangle \sim \frac{1}{2M} Q_{cs}^2$$

$$\boxed{\langle v \rangle \sim \frac{Q_{cs}}{m_{cs}}}$$

$$\boxed{Q_{cs} \sim 70\,\text{keV}} \longrightarrow \boxed{\langle v \rangle \sim 350\,\text{m/s}}$$

Compare this $\langle v \rangle_{cs}$ w/ $\langle v \rangle_{thermal}$

$\Rightarrow$ when $\langle v \rangle_{thermal} \geq \langle v \rangle_{cs}$

$\Rightarrow$ only gas state is possible

$\Rightarrow$ Boiling point

$$\boxed{E_T = k_B T}$$

$k_B \rightarrow$ Boltzmann costant

$$k_B = \frac{1\,eV}{11\,000\,°K}$$

Room temperature

$T \sim 300\,°K$

$$\boxed{E_T \sim 50\ meV}$$

$$\frac{1}{2} m \langle v \rangle^2 = k_B T \longrightarrow T = \frac{1}{2} m \langle v^2 \rangle \frac{1}{k_B}$$

(kinetic = thermal)

$$\langle v \rangle \simeq \frac{Q}{m_{Cs}} \simeq 350\,m/s$$

$$\boxed{T(Cs) \simeq 1000\,^{\circ}K}$$

$$\langle v_T \rangle \simeq 350\,m/s$$

Actual boiling point

$$\boxed{T(Cs)_{exp} \simeq 944\,^{\circ}K}$$

→ Cs boiling point (natural problem)

Dimensional argument → $T(Cs) \simeq 1000°K$

Experiment → $T(Cs) \simeq 944°K$

→ $\boxed{\text{WORKS REALLY WELL}}$

→ FOR NATURAL PROBLEMS YOU CAN USE REALLY DUMB APPROX.

(AND THEY WILL WORK)

$$T_{dim}(Cs) \simeq 1000^\circ K \qquad T_{real}(Cs) \simeq 944^\circ K$$

NATURAL PROBLEMS $\rightarrow$ EASY

(usually $\exists$ one relevant scale)

most problems in physics involve several scale (multiscale problems)

corrections

[HOW TO DEAL W/ THESE CORRECTIONS]

→ [H ATOM]

1) most important scale $a_B = \frac{1}{m_e \alpha}$ $\left(\frac{1}{a_B} = Q_B\right)$

2) but ∃ other scales

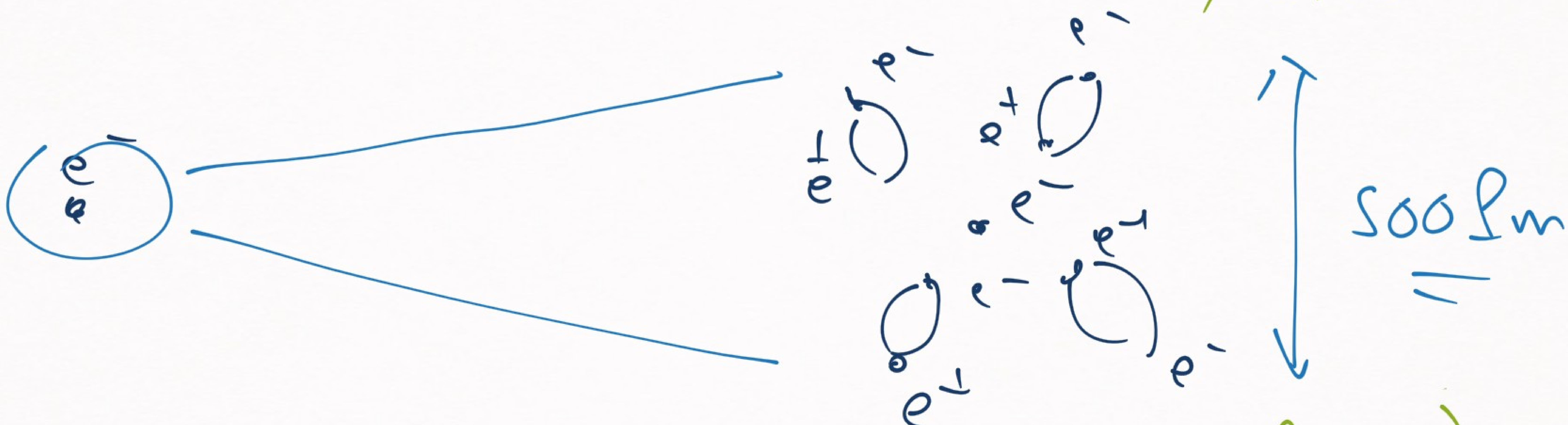
r_e, r_p → electron & proton size

?

Other scales → $\boxed{\text{ARE THEY BIG OR SMALL?}}$

→ electron size (isn't the electron point-like?)

yes, but...

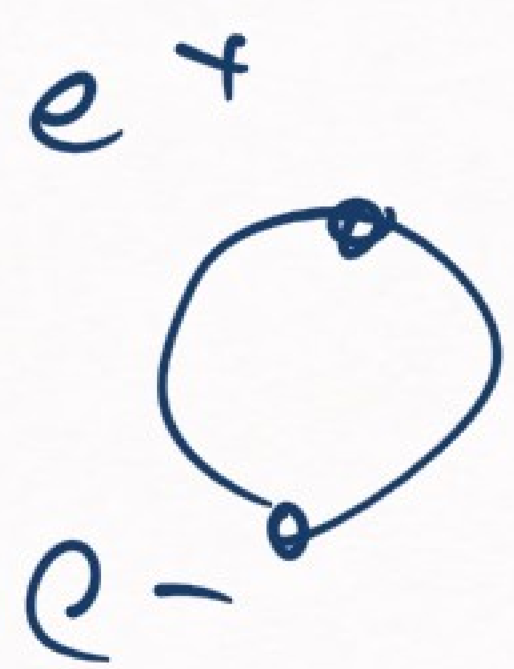


... vacuum polarization.

→ What is vacuum polarization?

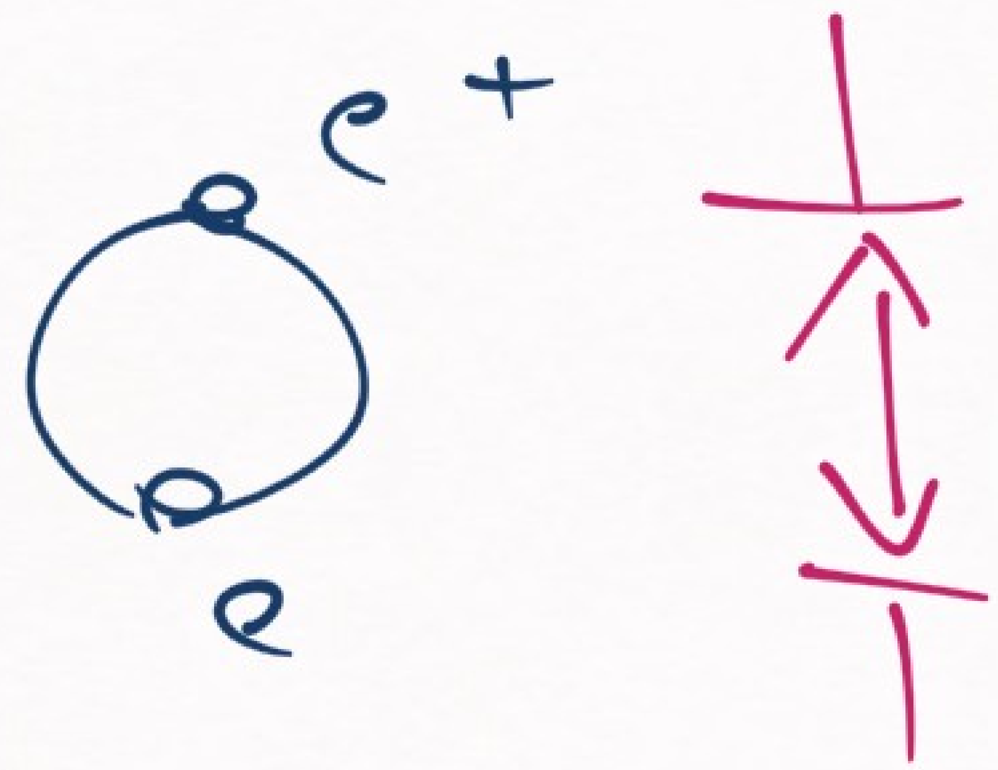
-> creation of electron-positron pairs that change the electromagnetic properties of vacuum

→ What is the size of this effect?



$$\Delta E \Delta t \simeq \frac{\hbar}{2}$$ (uncertainty relations)

$\Delta x \, \Delta p \sim \hbar$



$$\Delta x \simeq \frac{\hbar c}{\Delta p}$$

m_e

effect from QFT

"size" of electron

$\geq$

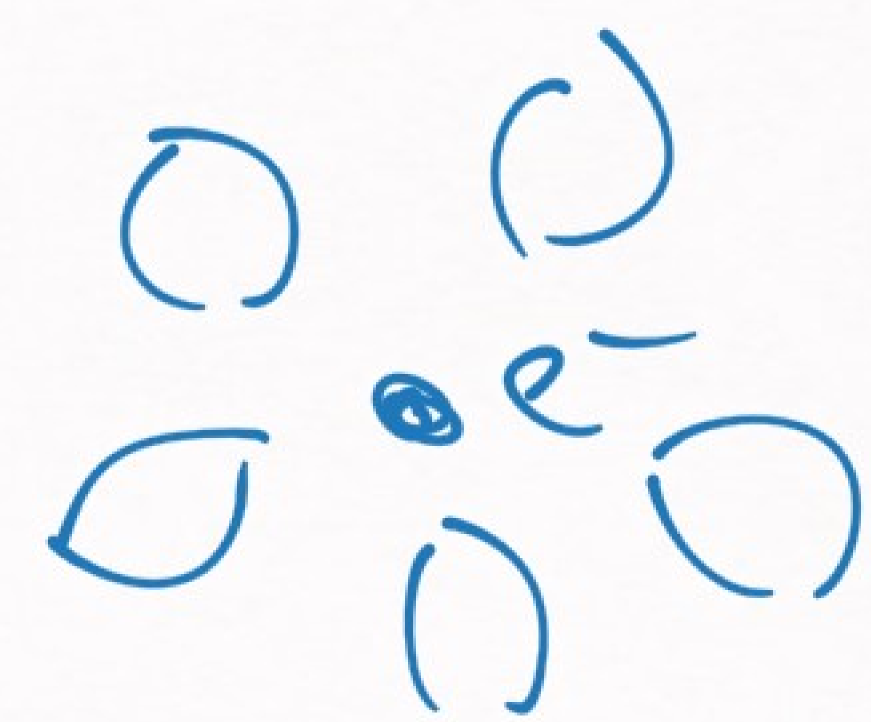
$$\rightarrow \left[\frac{1}{m_e} = \frac{\hbar c}{m_e} \simeq 400 \text{ fm} \right]$$

$a_B \simeq 53000 \text{ fm}$ $\qquad\qquad$ $\frac{1}{m_e} \simeq 400 \text{ fm}$

e^-

(point-like)

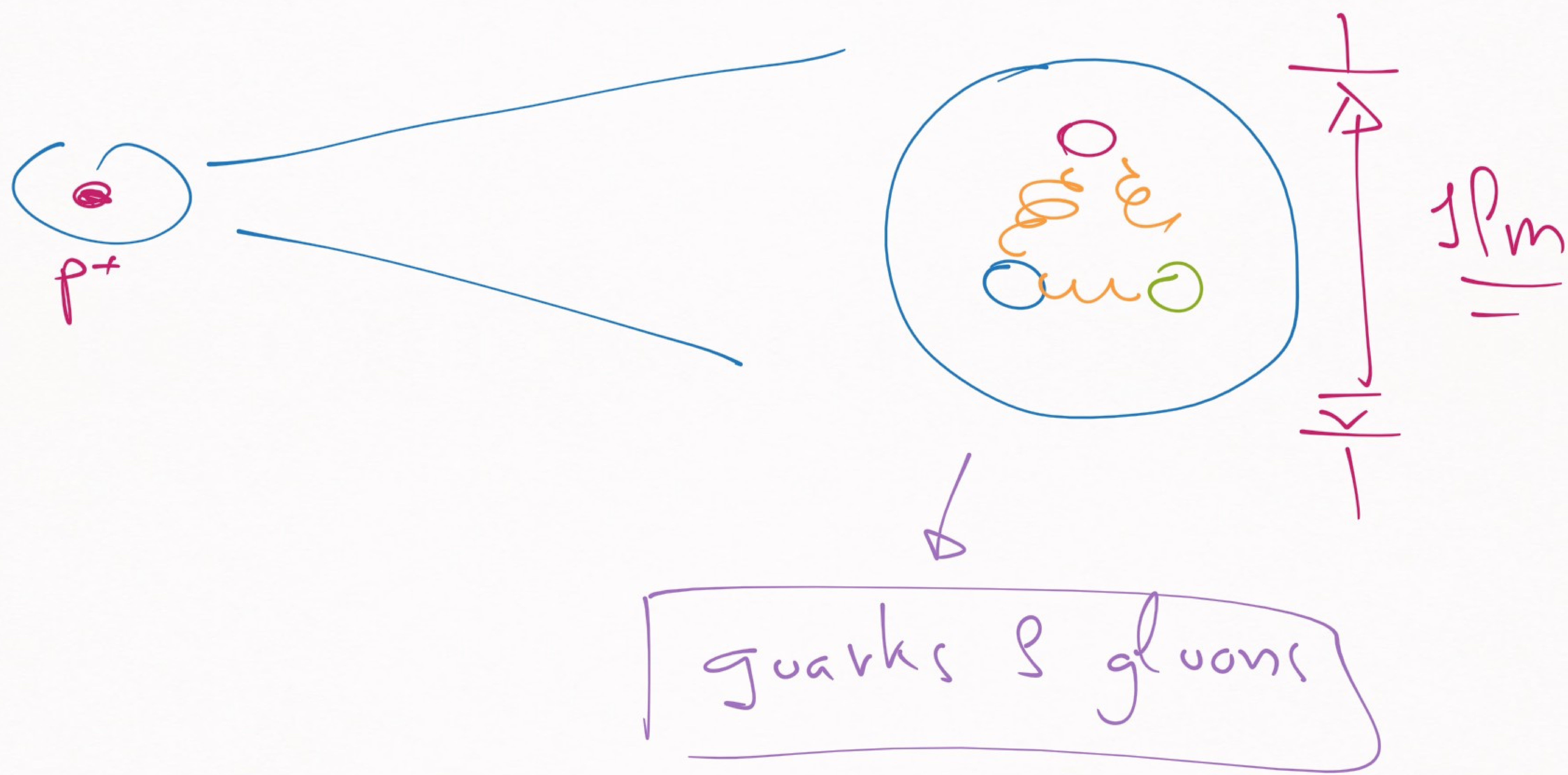
$\longrightarrow$



structure

long distances $\rightarrow$ no structure

short distances $\rightarrow$ structure


p+
1fm
quarks & gluons

HYDROGEN ATOM $\longrightarrow$ MULTISCALE PROBLEM

$a_B \sim 53000\,\mathrm{fm}$ (very big)

[$\exists$ a separation of scales]

$1/m_e \sim 400\,\mathrm{fm}$ $\left(\simeq \frac{a_B}{137}\right)$

$r_p \sim 1\,\mathrm{fm}$ $\left(\simeq \frac{a_B}{53000}\right)$ (very small)

Bigger scales, smaller scales

→ good scale separation

1) Most important properties → bigger scale

2) Smaller scales → $\boxed{\text{corrections}}$

2) Smaller scales:

2.a) $r_e \sim \frac{1}{m_e} \Rightarrow \frac{r_e}{a_B} \simeq \frac{1}{137}$

(even smaller because dynamics forbid r_e/a_B corrections)

2.b) $\frac{r_p}{a_B} \sim 2 \cdot 10^{-5}$

$\boxed{\text{EXTRA SCALES}} \longrightarrow \boxed{\exists \text{ CORRECTIONS}}$

CORRECTION 1) electron size

relativistic in nature

$$\left(\frac{r_e}{a_B}\right)^2 \sim \left(\frac{1}{137}\right)^2 \sim \underline{\underline{5 \cdot 10^{-5}}}$$

EXPECTATION → r_e corrections of a relative size of $\boxed{5 \cdot 10^{-5}}$

REALITY → fine-structure corrections

2P

$\downarrow$ 10.2 eV

1S

②

$<$

$2P_{3/2}$

$4.5 \cdot 10^{-5}$ eV ①

$2P_{1/2}$

$\downarrow$

r_e

$\frac{①}{②} \sim \boxed{5 \cdot 10^{-6}}$

not perfect (factor of 10 difference) $\underline{\underline{10}}$ ⊕

→ but still pretty good

⊕ → ∃ - waves, $\left(\frac{1}{2\ell+1}\right)^2 \sim \left(\frac{1}{3}\right)^2 \sim \frac{1}{9}$

(remember: ∃ numerical factors)

CORRECTION 2) proton size correction

(expectation $\rightarrow \left(\frac{r_p}{a_B}\right) \sim 2 \cdot 10^{-5}$)

Hyperfine corrections

$\frac{(1)}{(2)} \simeq 5 \cdot 10^{-7}$ → magnetic effect

1 S

(−13.6 eV)

(2)

↑↑

↑↓

$5.9 \cdot 10^{-6}$ eV

(1)

LESSONS

Systems that are

1) natural
2) good separation of scales

NATURAL SYSTEMS

→ [very easy to deal with]

→ good appox. w/ little effort

$\boxed{\text{COROLARY}}$ Systems that

1) are not natural
2) have a poor scale separation → UNNATURAL SYSTEMS

→ $\boxed{\text{DIFFICULT}}$

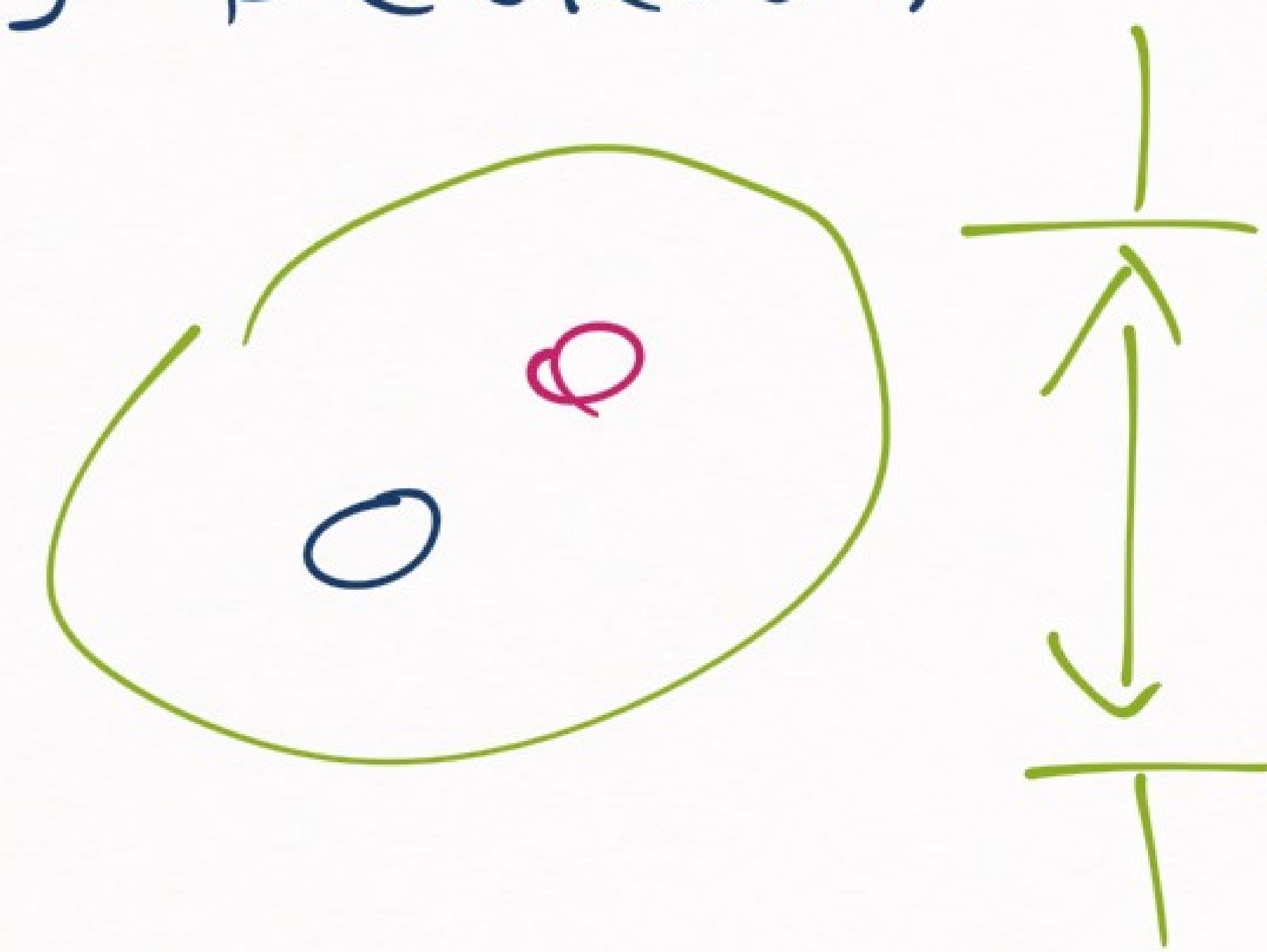
↳ [good for research]

$\boxed{\text{NUCLEAR PHYSICS}}$

NATURAL
or
UNNATURAL

Let's check info about these nuclear systems:

1) Deuteron → np bound state $S = 1$, $J = 0$



$\boxed{r_d \sim 5\,\text{fm}}$ is this big or small?

2) Range of nuclear forces

$$V_Y(r) = -\frac{g^2}{4\pi}\frac{e^{-mr}}{r}$$

pion mass

$m \simeq 140\,\text{MeV}$

$\frac{1}{m} \simeq 1.4\,\text{fm}$

$$V_C(r) = -\frac{e^2}{4\pi}\frac{1}{r}$$

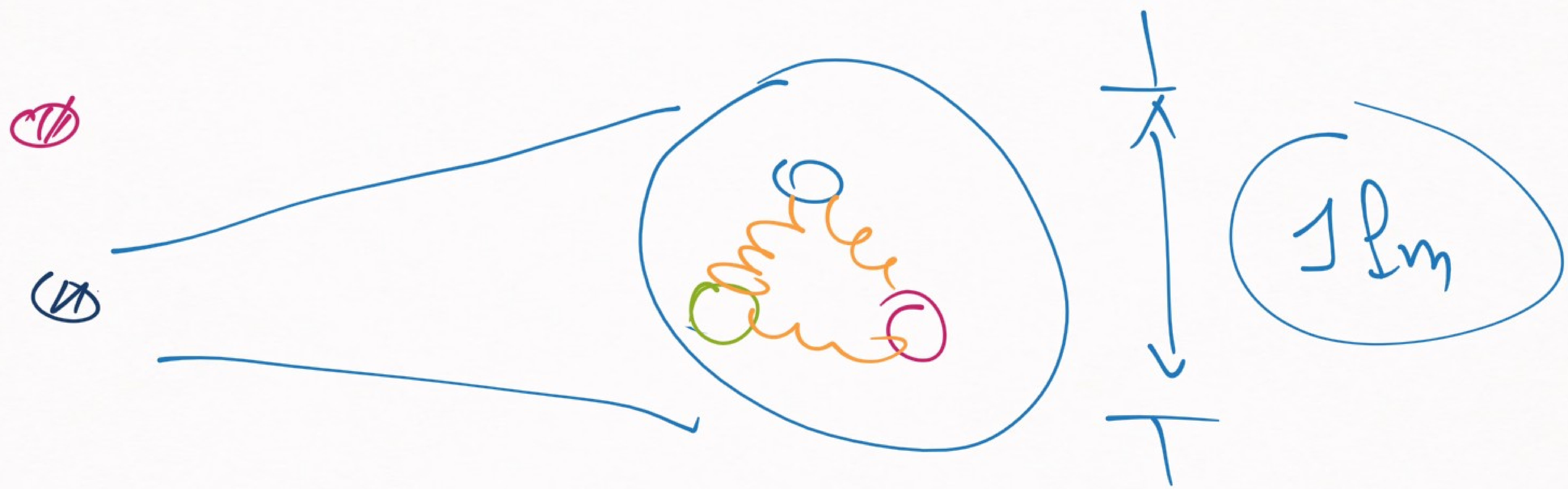
$\alpha \simeq \frac{1}{137}$

$\boxed{\alpha_Y \simeq 15}$!! $\rightarrow$ wow!! $\rightarrow$ more complex

1 + 2) $\rightarrow$ $\boxed{r_d \simeq 3/m_\pi}$ $\rightarrow$ 3 $\sim \mathcal{O}(1)$

"sort of"

3) Other scales $\rightarrow$ size of nucleons



$r_d \sim 5$ fm

(1) 4

$1/m_\pi \sim 1.4$ fm

$r_p, r_n \sim 1$ fm (2)

different from atomic physics

(1) not particularly natural

(2) poor separation of scales

1 + 2 $\rightarrow$ $\boxed{\text{NUCLEAR PHYSICS WILL PROBABLY BE MORE DIFFICULT THAN ATOMIC PHYSICS}}$

how much more difficult $\longrightarrow$

→ we have been studying nuclear forces for 70-80 years

→ yet, we still do not understand nuclear forces well

→ $\boxed{\text{PRETTY DAMN DIFFICULT}}$