NUCLEAR PHYSICS

→ Very few : we can change things by agreement (wechat group /private message / here)

→ Online classes suboptimal

→ Recommendations :

1) Take notes (imagine that you were on the classroom & I was writing on the blackboard)

2) Try to make questions

3) Try to check the lessons in advance

→ not possible to make exam

→ $\boxed{\text{exercise}}$

OUTLINE

easy

1) INTRODUCTION (nuclear forces
→ what are their
general features)

2) TWO-NUCLEON SYSTEM (scattering
theory)

dry

→ interesting, but difficult

3) THREE-BODY SYSTEMS (not always)

4) NUCLEAR MODELS

↳ easy

→ class
- → some time for the lessons
- → some short breaks
- → some time for questions

NUCLEAR PHYSICS 1

→ $\boxed{\text{SCALES IN PHYSICS}}$

尺度

→ [Qualitative discussion]

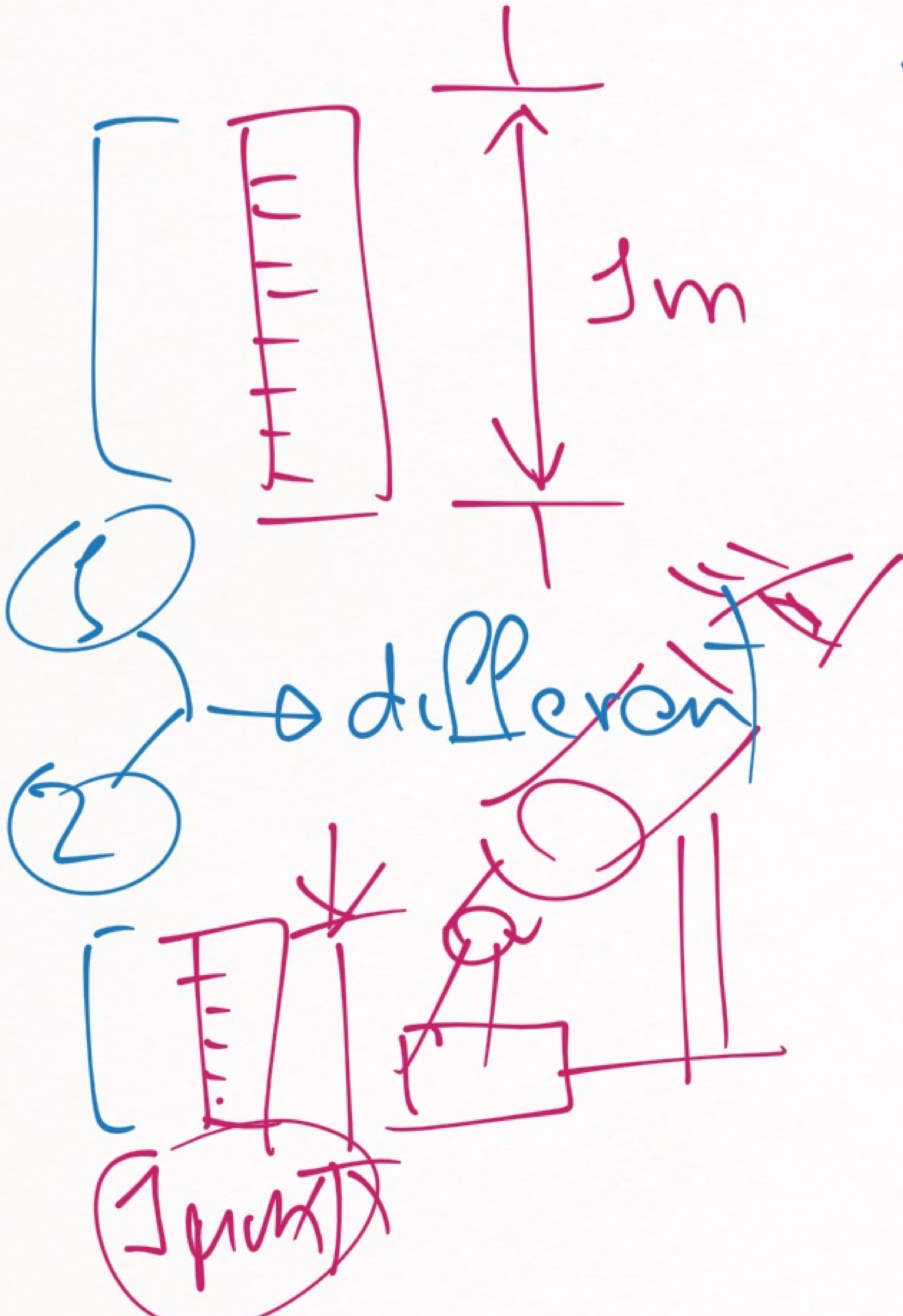
Doing physics → NUCLEAR PHYSICS
PARTICLE PHYSICS
COSMOLOGY
GENERAL RELATIVITY
⋮

↓

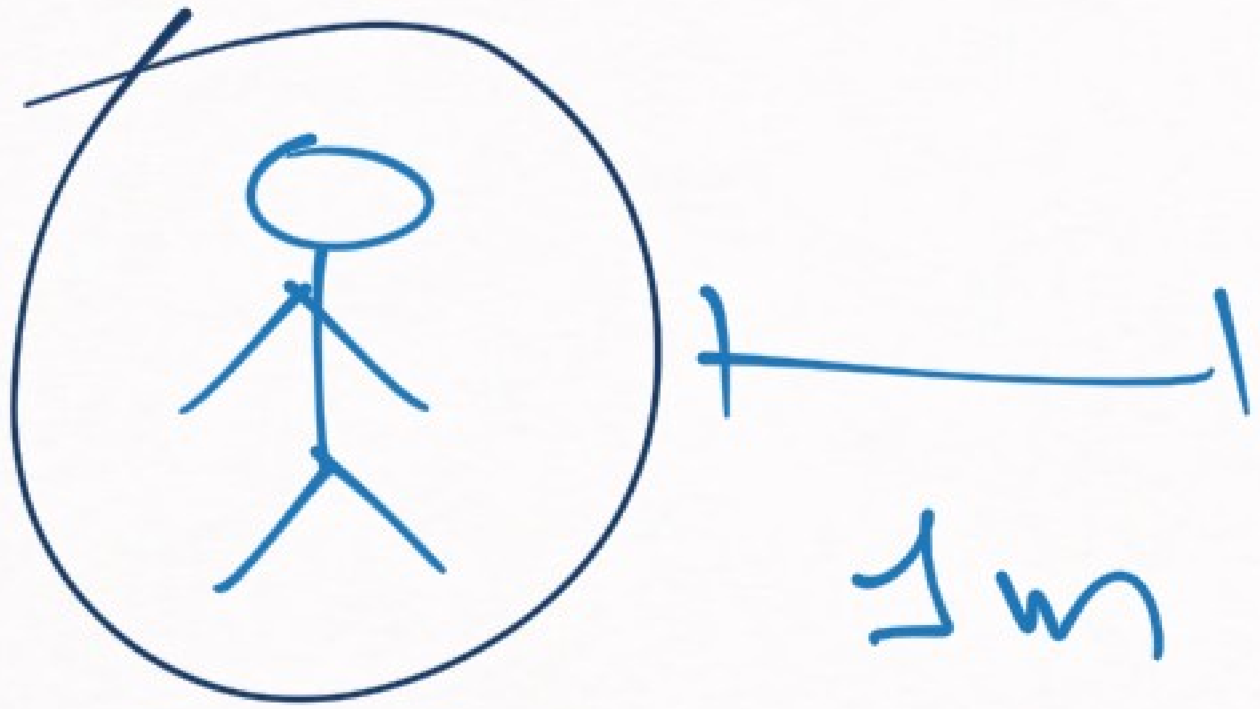
MANY TYPES OF PHYSICS

WHY?

& OTHER SCIENCES



HOW WE SEE THE WORLD DEPENDS ON THE DISTANCE SCALE WE ARE USING TO SEE IT



[PRETTY OBVIOUS OBSERVATION]

humans → [natural scale of the order of meters]

(1-2) km

small

(1-2) mm

$\boxed{\text{OBVIOUS OBSERVATION}}$ ⟶ can be used for improving our understanding of nature

($\forall$ depends on the distance scale we are using to see the world)

$\Rightarrow$

LET'S SEE HOW THIS TRANSLATES
IN THE APPEARANCE OF
DIFFERENT TYPES OF
KNOWLEDGE

CONSIDER A LENGTH SCALE

→ FIELD OF KNOWLEDGE

FEW EXAMPLES

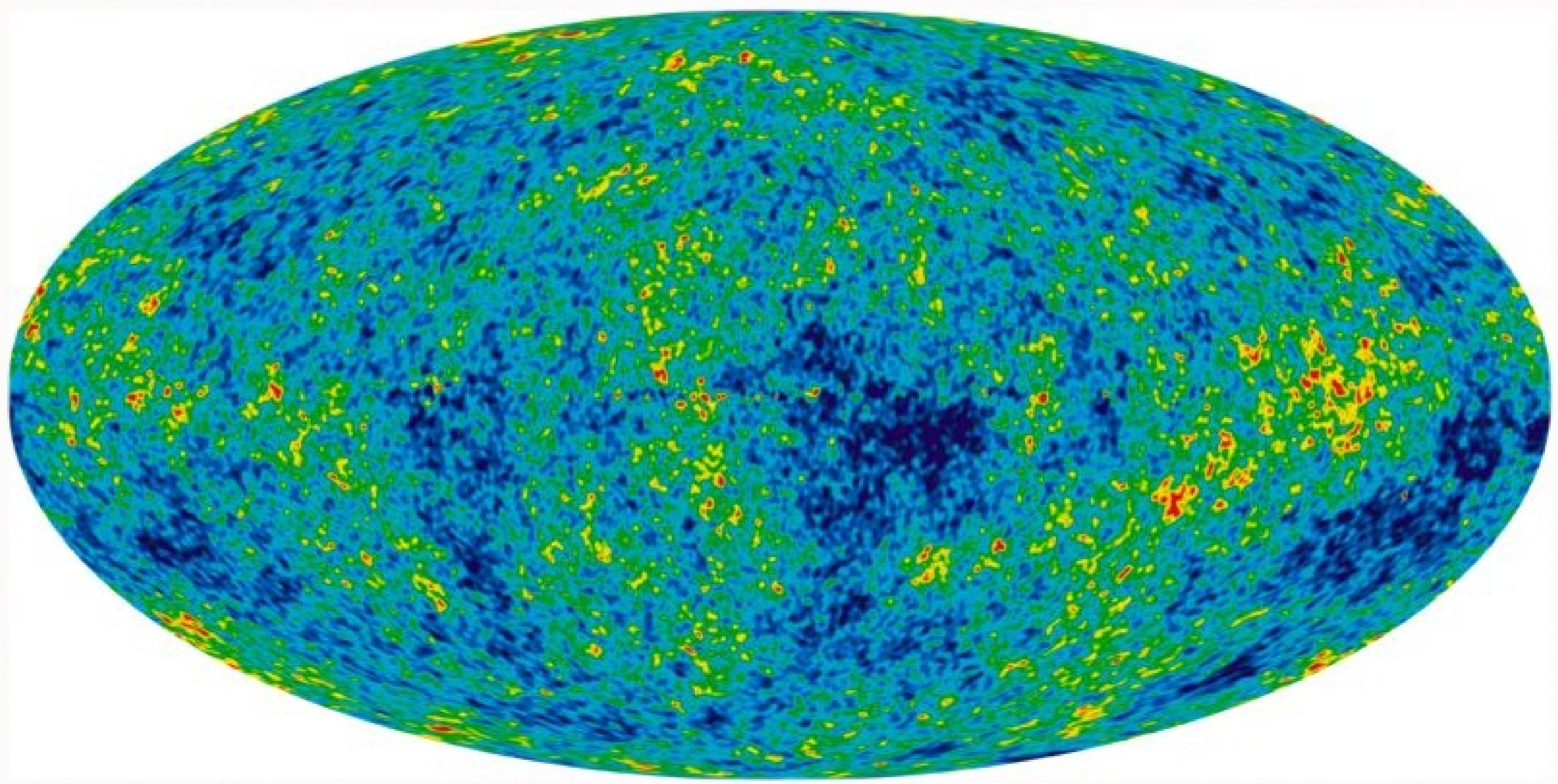
DISTANCE SCALE $\left(\infty \;\; \simeq 0 \right)$

εν το παν
(en to pan) } all is one

→ METAPHYSICS

(mostly useless speculation)

$\boxed{10^9 \text{ light years}}$



$\rightarrow$ $\boxed{\text{COSMOLOGY}}$

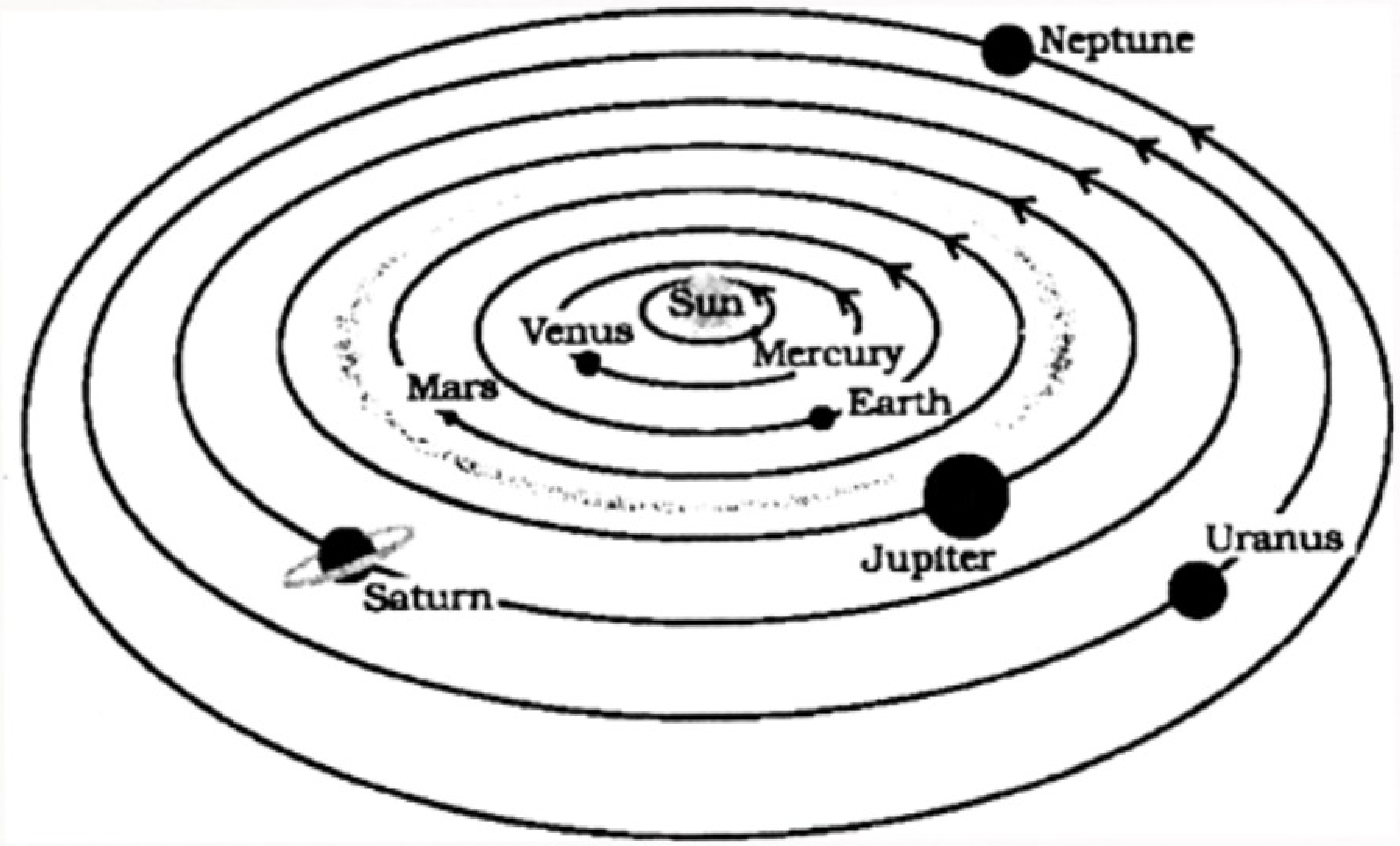
General structure of universe

(GENERAL RELATIVITY, EXPANSION OF UNIVERSE)

$\boxed{10^3 - 10^5 \text{ light years}} \rightarrow \boxed{\text{ASTROPHYSICS}}$

→ we will worry about things like these

$\boxed{\text{100 a.u (astronomical units)}}$ → [size of the solar system]

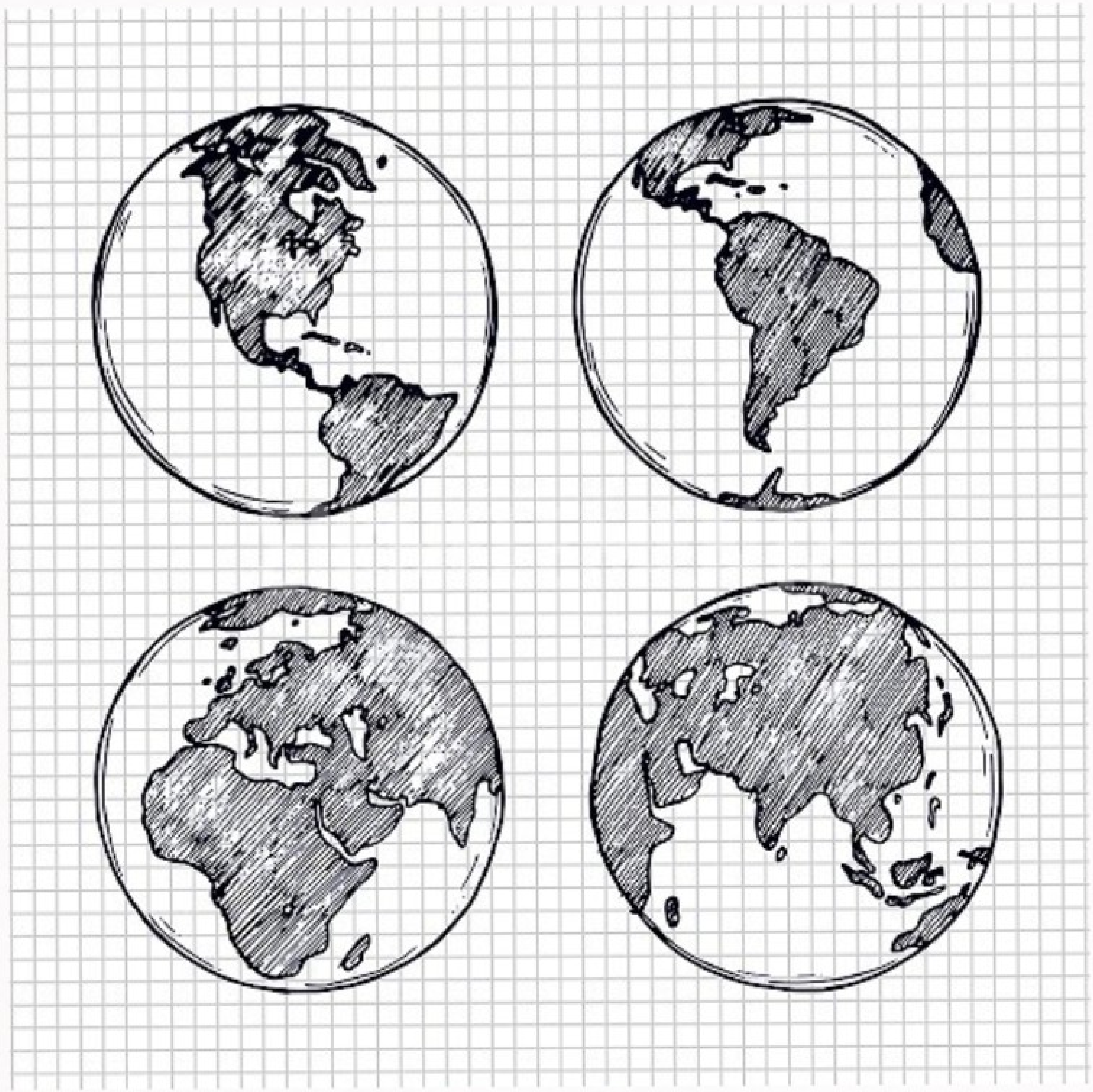


WE WILL WORRY ABOUT

→ CLASSICAL MECHANICS

→ NEWTONIAN GRAVITY

$\boxed{10^4 \text{ km}}$ → size of the earth



→ GEOGRAPHY

→ ECOLOGY

→ CLIMATE SCIENCE

$\boxed{10^3\ km}$ → countries, ecosystems

→ OUR UNDERSTANDING OF THINGS CHANGES AGAIN

$\boxed{1\,\text{km}} \rightarrow$ our normal, ordinary lives

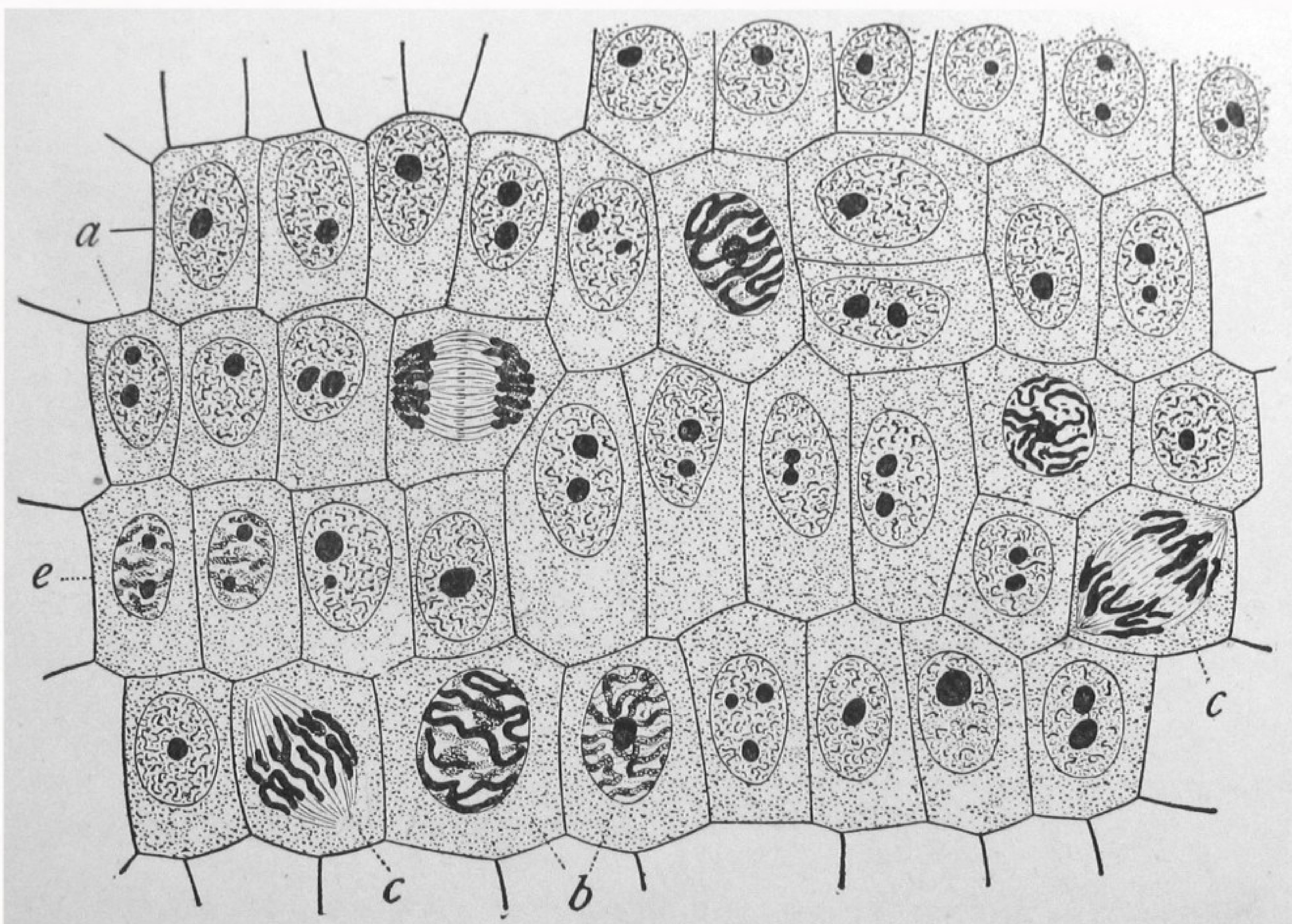
$\rightarrow$ SOCIAL INTERACTIONS

URBAN PLANNING

$\boxed{1\,\text{m}} \rightarrow$ people, throwing things

(CLASSICAL MECHANICS)

$\boxed{10^{-5}\,m}$ → cells (BIOLOGY)

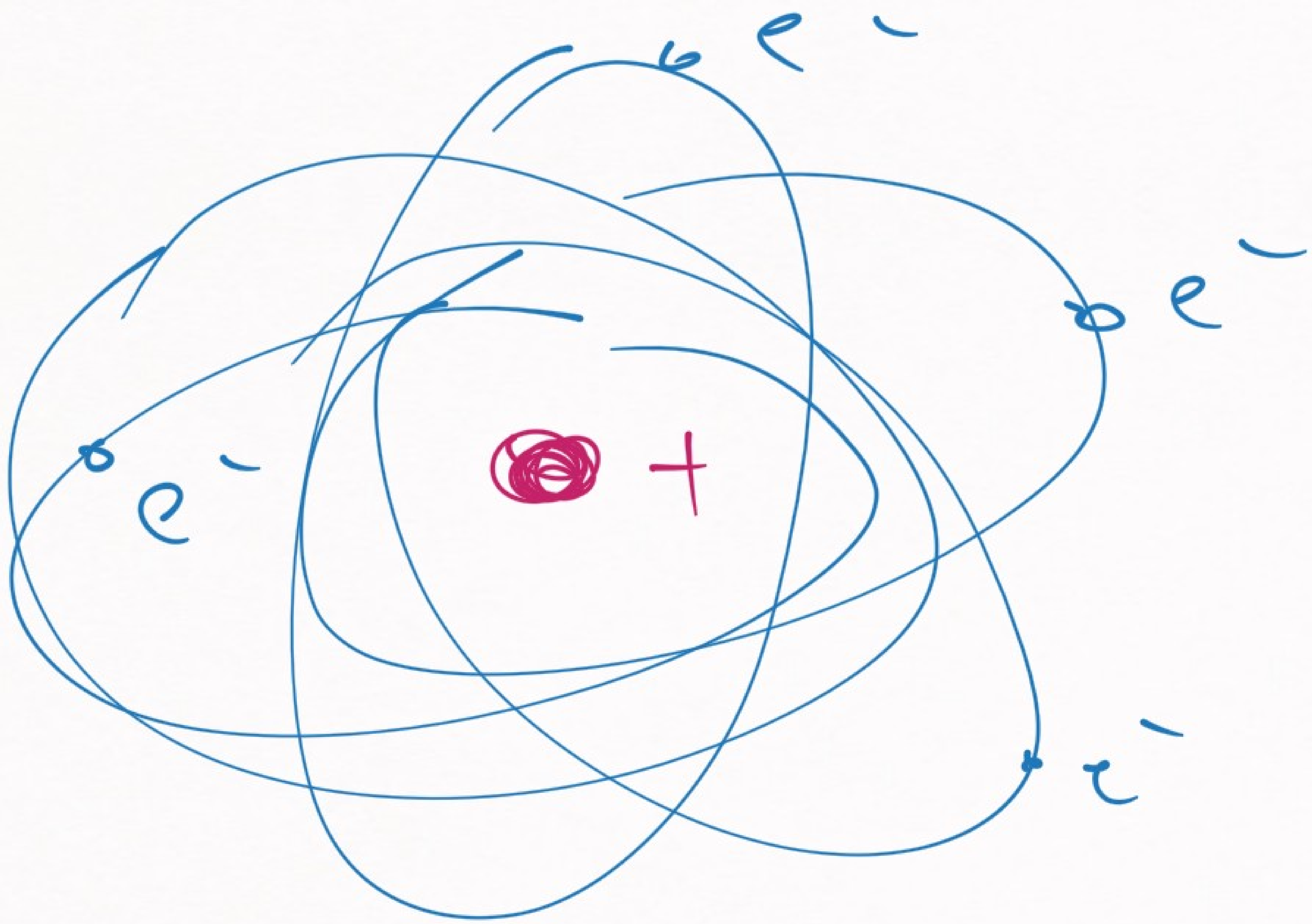


$\boxed{10^{-9}\,m}$

→ molecoles

(CHEMISTRY)

$\boxed{10^{-10}\,m}$ → atoms → ATOMIC PHYSICS

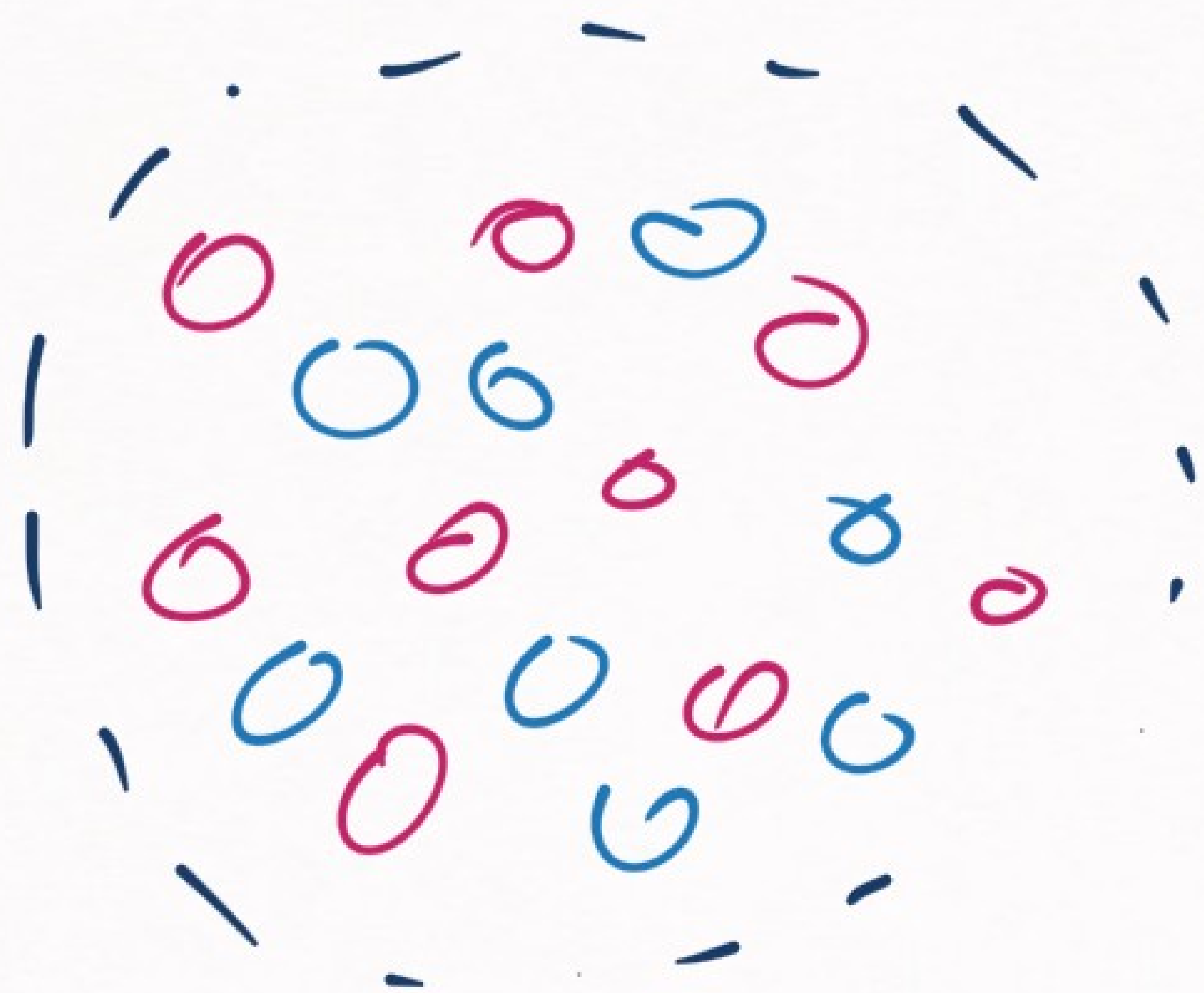


~~CLASSICAL MECHANICS~~

NO LONGER VALID

QUANTUM MECHANICS

$\boxed{10^{-15}\,m}$ → nuclei, protons, neutrons



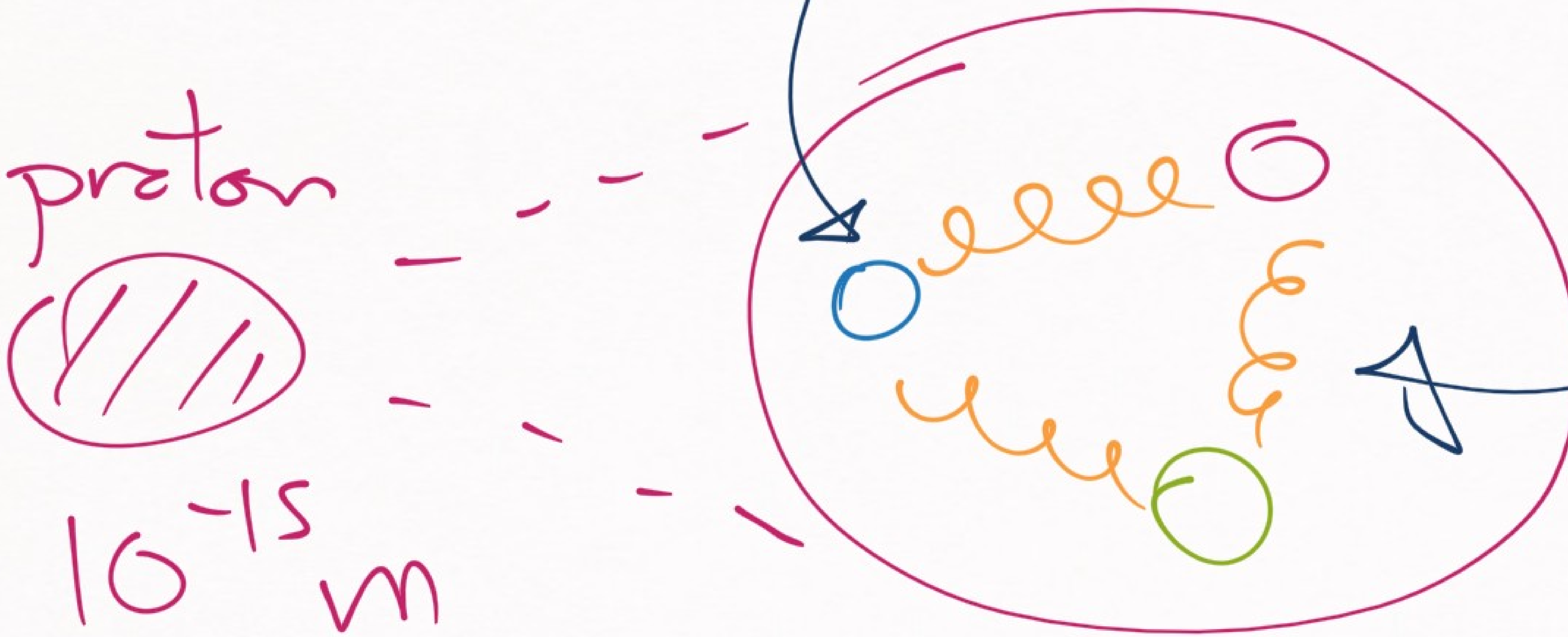
→ $\boxed{\text{NUCLEAR PHYSICS}}$

here

$10^{-15}\,m = \underline{1\ fm}$ (fermi)

$\boxed{10^{-16}\,m}$ → quarks & gluons

proton



$10^{-15}\,m$

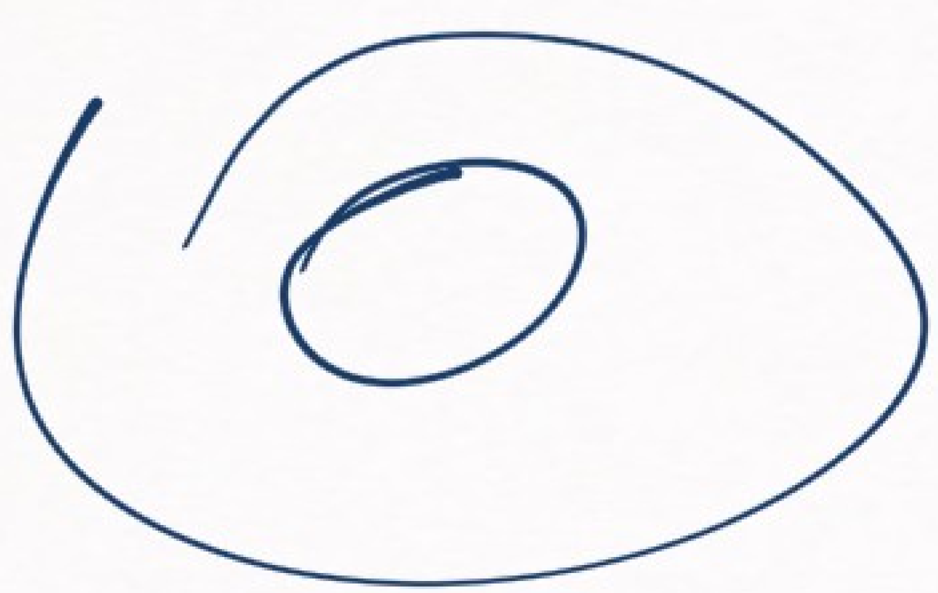
$10^{-16}\,m$

QUANTUM
CHROMO
-DYNAMICS

(QCD)

$\boxed{10^{-35}\,\mathrm{m}} \rightarrow$ Planck scale

$\rightarrow \boxed{\text{QUANTUM GRAVITY}}$

 $\rightarrow$ METAPHYSICS AGAIN

CHOOSE DISTANCE SCALE ⇒ EACH DISTANCE SCALE REQUIRES A DIFFERENT EXPLANATION

→ SOME INDEPENDENCE BETWEEN EXPLANATIONS @ DIFFERENT SCALES

→ Meaning :

* We don't need to know QUANTUM MECHANICS to understand CLASSICAL MECHANICS (but it helps)

* → similar examples

→ [INDEPENDENCE OF EXPLANATIONS]

↳ (RENORMALIZATION)

↓

[this isn't only about removing <u>infinities</u>]

→ WE ARE BACK

→ IF I CONTINUE BEYOND THE TIME FOR THIS CLASS

TELL ME !!

PRACTICAL EXAMPLE
(SCALES IN PHYSICS)

↪ **ATOMIC PHYSICS**

Hydrogen atom (probably seen this when undergraduate)

ATOMIC PHYSICS

hydrogen atom

1) Standard approach

(ground state & its properties)

2) Scales-based approach

(exploit the $\exists$ natural scale)

1) STANDARD APPROACH

1.a) Two-body problem (evident)

1.b) Potential (Coulomb)

1.c) Solve Schrödinger equation

1.b) $V(r) = -\frac{\alpha}{r}$

$V(r) = -\frac{\alpha}{r}$ → fine structure constant

distance

$$\alpha = \frac{e^2}{4\pi} \simeq \frac{1}{137}$$

units → $[E]$ $[L]$ } interchangeably

conversion factor → $\hbar c = 197.3$ MeV fm

$$\underbrace{V(r)}_{\text{MeV}} = -\frac{\alpha}{\underset{\text{fm}}{r}}$$

$\left.\begin{array}{l}\text{MeV}\\ \text{fm}\end{array}\right] \to$ natural for nuclear physics

$r = 1\,\text{fm} \Rightarrow V(r) = -\alpha(1\,\text{fm})^{-1}$

$\left.\begin{array}{l}\text{eV}\\ \text{Å}\end{array}\right] \to$ also possible

$\Rightarrow V(r) = -\alpha(1\,\text{fm})^{-1}\hbar c$

$= -\alpha(200\,\text{MeV})$

$\hbar c = 1973\,\text{Å eV}$ (check)

$\boxed{\text{Units}} \longrightarrow$ fm , MeV

use $\hbar c$ as a conversion factor

$$\boxed{V(r) = -\frac{\alpha}{r}} \longrightarrow \text{Coulomb potential}$$

1.c) Schrödinger equation

$$\left[-\frac{1}{2\mu}\vec{\nabla}^2 + V(\vec{r})\right]\Psi(\vec{r}) = E\,\Psi(\vec{r})$$

reduced mass $\rightarrow \dfrac{1}{\mu} = \dfrac{1}{m_e} + \dfrac{1}{m_p} \simeq \dfrac{1}{m_e}$

Standard approach → solve Schrödinger

$$\left[-\frac{1}{2\mu}\vec{\nabla} + V(\vec{r})\right]\Psi(\vec{r}) = E\,\Psi(\vec{r})$$

$$\left[-\vec{\nabla} + \underbrace{2\mu V(\vec{r})}\right]\Psi(\vec{r}) = \underbrace{2\mu E}\,\Psi(\vec{r})$$

$$U(\vec{r}) = 2\mu V(\vec{r})$$

"reduced potential"

$$\underline{\underline{2\mu E}} = -\gamma^2$$

wave number

$$\left[-\vec{\nabla}^2 + 2\mu V(\vec{r})\right]\Psi(\vec{r}) = -\gamma^2 \Psi(\vec{r}) \Rightarrow (*)$$

$$2\mu V(\vec{r}) = -2\mu \frac{\alpha}{r} = -\frac{2}{a_B r}$$

$$a_B = \frac{1}{\mu \alpha} \simeq \frac{1}{m_e \alpha}$$

length scale

(Bohr radius)

$$a_B \simeq \frac{137}{0.5\,\mathrm{MeV}} \simeq 270\,\mathrm{MeV}^{-1} \simeq 54000\,\mathrm{fm}$$

$$\hbar c \simeq 200\,\mathrm{MeV\,fm}$$

$$(*) \Rightarrow \left[-\nabla^2 - \frac{2}{a_B r} \right] \Psi(\vec{r}) = -\gamma^2 \Psi(\vec{r})$$

$$a_B = \frac{1}{\mu \alpha} \simeq 5.4 \cdot 10^{4} \text{ fm}$$

$$\left[\nabla^2 + \frac{2}{a_B r} \right] \Psi(\vec{r}) = \gamma^2 \Psi(\vec{r})$$

$\hookrightarrow$ we can solve it

$$\Psi(\vec{r}) = \frac{1}{\sqrt{4\pi}} \frac{2}{a_B^{3/2}} e^{-r/a_B} \quad , \quad \gamma = \frac{1}{a_B}$$

$\rightarrow$ ground state solution of the hydrogen atom

$$E_B = -\frac{\gamma^2}{2\mu} = -\frac{1}{2\mu}\left(\frac{1}{a_B}\right)^2 \simeq \underline{\underline{-13.6\ eV}}$$

$\sqrt{\langle r^2 \rangle}$ → mean square radius of the hydrogen atom

$$\langle r^2 \rangle = \langle \Psi | r^2 | \Psi \rangle = \int d^3\vec{r}\; r^2 |\Psi(\vec{r})|^2$$

$$= \underline{\underline{3a_0^2}} \Rightarrow \underline{\underline{\sqrt{\langle r^2 \rangle} = \sqrt{3}\, a_0}}$$

[STANDARD APPROACH]

Potential → Schrödinger equation → Solution

$$E_B = -\frac{1}{2\mu}\gamma^2 \quad , \quad \boxed{\gamma = \frac{1}{a_B} \quad , \quad \sqrt{\langle r^2 \rangle} = \sqrt{3}\, a_B}$$

non-relativistic theory

powers of a_B

[SCALES-BASED APPROACH]

→ method for lazy people

↓

this can be a good thing

$\hat{O} \rightarrow$ observable

$\langle \hat{O} \rangle = \langle \psi | \hat{O} | \psi \rangle$

$|\psi\rangle \rightarrow$ hydrogen atom wave function

$\rightarrow$ dimensions of $[L]^n$

$$\left[\nabla^2 + \frac{2}{a_B r}\right]\psi(\vec{r}) = k^2 \psi(\vec{r})$$

$\rightarrow$ input $\leftarrow$

$\Rightarrow$ THE INPUT ONLY CONTAINS ONE PARAMETER

$\boxed{a_B}$

$$[\langle \hat{O} \rangle] = [L]^n \Rightarrow \boxed{\langle \hat{O} \rangle \sim \text{const} \times a_B^n}$$

$[\ldots] \longrightarrow$ dimensions (length, energy)

1) $[\langle \hat{O} \rangle] = [L]^n \Rightarrow \boxed{\langle \hat{O} \rangle = C_{\hat{O}}\, a_B^n}$

$\Uparrow$

2) a_B is the only input parameter in the Schrödinger equation

$\boxed{1) + 2)}$

?

? $\rightarrow$ what is the size of $\underline{\underline{C_{\hat{O}}}}$

$\boxed{\text{SCALES}}$ $\rightarrow$ $\langle \hat{O} \rangle = c_{\hat{O}}\, a_g^n$

$\boxed{\text{NATURALNESS}}$ $\leftarrow$ [size of this]

(assumption) $\rightarrow$ physical problems are not always natural

$$\boxed{c_{\hat{O}} \sim O(1)}$$

→ back to the STANDARD APPROACH

$$\boxed{\begin{aligned} \gamma &= \frac{1}{a_B} \\ \sqrt{\langle r^2 \rangle} &= \sqrt{3}\, a_B \end{aligned}}$$

EXPLICIT CALCULATION

$\Longleftrightarrow$

SCALE APPROACH

$$\boxed{\begin{aligned} \gamma &= \frac{c}{a_B}, \quad c \sim O(1) \\ \sqrt{\langle r^2 \rangle} &= d\, a_B, \quad d \sim O(1) \end{aligned}}$$

$$\gamma = \frac{c}{a_B} \longrightarrow c = 1 \sim O(1)$$ ✓

$$\sqrt{\langle r^2 \rangle} = d\, a_B \longrightarrow d \simeq \sqrt{3} \simeq 1.7 \sim O(1)$$ ✓

$$\longleftarrow \otimes \longrightarrow$$

$$\boxed{\text{SCALES + NATURALNESS}} \longrightarrow \boxed{\text{VERY GOOD APPROX. TO ANY PROBLEM}}$$

1) Physics depends on a characteristic scale

+

2) Physics is natural ($\sim O(1)$)

$\boxed{1) + 2)}$ $\longrightarrow$ $\boxed{\text{good estimations to a lot of problems}}$

→ PROBLEM → NUCLEAR PHYSICS

GOOD: WORK FOR PHYSICISTS

ASSUMPTION OF NATURALNESS WILL FAIL

$\boxed{\text{EXAMPLE}}$

$\underbrace{n}_{} + \underbrace{p}_{}$ system

$s = 1/2$

np $\rightarrow$ $\begin{cases} s = 0 \rightarrow \text{virtual state (quasi bound)} \\ s = 1 \rightarrow \text{deuteron (bound state)} \end{cases}$

$\boxed{\gamma_0, \gamma_1}$ (γ_s)

1) What does natural means in nuclear physics?

ATOM

$$V(r) = -\frac{\alpha}{r}$$

$$\alpha = \frac{e^2}{4\pi}$$

$$\longrightarrow \quad V_{np}(r) = -\frac{g^2}{4\pi}\frac{e^{-mr}}{r}$$

β

$$a_y = \frac{1}{\mu p}$$

$m \longrightarrow$ this one

→ usual solution is to use m

(solution 2 → $a_\chi \simeq \frac{1}{\Lambda_\chi}$, $\Lambda_\chi \simeq 300\,\mathrm{MeV}$)

$$\boxed{m \simeq 140\ \mathrm{MeV}}$$

→ convention

$\gamma_0 \sim \mathcal{O}(1) \times m_\pi$

$\gamma_1 \sim \mathcal{O}(1) \times m_\pi$

natural solution

$$\boxed{\begin{matrix} \gamma_0 \sim 8\,\mathrm{MeV} \\ \gamma_1 \sim 45\,\mathrm{MeV} \end{matrix}}$$

$\gamma_0 \sim 8\,\mathrm{MeV} \rightarrow \frac{m_\pi}{17} \rightarrow \frac{1}{17} \ll 1$

$\gamma_1 \sim 45\,\mathrm{MeV} \rightarrow \frac{m_\pi}{3} \rightarrow \frac{1}{3} \lesssim 1$

$\boxed{\text{FOR LATER}}$

$\rightarrow$ ATOMIC PHYSICS : NATURAL $\rightarrow$ EASY

$\rightarrow$ NUCLEAR PHYSICS : NON NATURAL

(FINE TUNED)

$\downarrow$

DIFFICULT

END FOR TODAY

→ FRIDAY 15:50