NUCLEAR PHYSICS (21)

THE RESIDUAL INTERACTIONS IN THE SHELL MODEL

RECAP

SHELL MODEL ⇒

* Magic numbers ✓
  N, Z = 2, 8, 20, 28, 50, ...
* Separation energies
  ....

⇓ Assumptions

1) ∃ mean field potential

$$H = \sum_i T_i + \sum_{ij} V_{ij}^{2b} + \sum_{ijk} V_{ijk}^{3b} + \ldots \Rightarrow H = \sum_i (T_i + V_i^{MF}) + \Delta V$$

(Assumptions cont'd)

2) Nucleons are fermions

3) We simply fill the shells → [simple process]

GOOD THINGS

* JP of near closed shell nuclei ($^{16}O \rightarrow^{15/17} O$)

* Excited spectra (near filled shells) ($^{41}Ca, ^{39}Ar$)

* Pairing interaction (simple example ΔV)

[TWO OTEN PROBLEM HERE]

1) How do we find out VMF?

Assumption → $V^{MF} = \frac{1}{2} m\omega^2 r^2 - \xi \vec{e} \cdot \vec{s} - \kappa \vec{e}^{\,2}$ (another addition)

This is phenomenological (no theory behind this)

→ other methods: Hartree-Fock, Skyrme, Gogny, ...

2) How do we deal w/ ΔV?

Pairing interaction → simple example of a phenomenological ΔV

[But in general ΔV will be more complicated

→ [TODAY'S PROBLEM]]

1) MEAN FIELD → NEXT LESSON

TODAY → 2) RESIDUAL INTERACTION

—⊗—

→ [STRATEGY FOR DEALING W/ ΔV]

1) Mean field potential $H = \sum_i h_i + \Delta V$

2) "Monoparticular" wf's

$h_i \phi^{(n)} = \epsilon_n \phi^{(n)}$

3) total wf -> antisymmetrization

$\underline{\Phi}_\alpha = \mathcal{A}\left[\prod_{i=1}^{A} \phi_i\right]$ -> [Reinterpret this]

DETAIL → [$\sum_\alpha$'s actually spans a complete A-body Hilbert space]

→ if we have ∀ possible $\sum_\alpha$'s,
then we have a complete basis
for the A-body system

[DETAIL ON LANGUAGE / FORMALISM]

$$\mathcal{I}_{(2)} = \mathcal{A}\left[\prod_{i=1}^{\hat{A}} \phi_0^{(n_i)}\right]$$

↳ Each α is a different way to fill the levels of the shell model

EXAMPLE OF AN "α" ; ⌐→ infinite examples like this

$\alpha \coloneqq \{$ 2 protons in $1s_{1/2}$, 1 neutron in $1s_{1/2}$, 1 neutron in $1p_{3/2}\}$

"α-space" equivalent to the full Hilbert space

↳ r-space, p-space

$\underline{\psi}(\vec{r})$, $\underline{\psi}(\vec{p})$, $\underline{\psi}_\alpha$

→ understand better w/ 1-body problem

[EXAMPLE]

1) Standard Schrödinger equation

$$\hat{H}|\psi\rangle = E|\psi\rangle \rightarrow -\frac{\hbar^2}{2m}\psi''(x) + V(x)\psi(x) = E\psi(x)$$

2) Oscillator basis (MF → harmonic oscillator)

$$V = V(x) = \underbrace{\frac{1}{2}m\omega^2 r^2}_{V^{MF}} + \underbrace{\left(V(x) - \frac{1}{2}m\omega^2 x^2\right)}_{\Delta V}$$

$H^{HF}|n\rangle = \hbar\omega\left(n+\frac{1}{2}\right)|n\rangle$ → typical 1-dim harmonic oscillator

3) Solve in the oscillator (i.e. HF) basis

$H|\psi\rangle = E|\psi\rangle$, $|\psi\rangle = \sum_n |n\rangle\langle n|\psi\rangle$

$\mathbb{1} = \sum_n |n\rangle\langle n|$ ⟷ $\mathbb{1} = \int \frac{d^3\vec{p}}{(2\pi)^3}|\vec{p}\rangle\langle\vec{p}|$

(Oscillator space)

$$|\psi\rangle = \sum_n |n\rangle\langle n|\psi\rangle = \sum_n |n\rangle \psi_n$$

$$\Rightarrow \quad H_{mn} = \langle m|H|n\rangle$$

$$\Rightarrow \boxed{H_{mn}\psi_n = E\,\psi_m}$$

1) $\hat{H}\psi(x) = \left[-\frac{1}{2m}\frac{d^2}{dx^2} + V(x)\right]\psi(x) = E\psi(x)$

↓ (differential equation)

2) $\hat{H}_{mn}\psi_n = E\psi_m$ (matrix equation)

- $\hat{H}_{mn}$ → matrix
- ψ_n → eigenvector
- E → eigenvalue

(infinite-dimensional matrix)

Schrödinger in r-space (or p-space)

$\Leftrightarrow$

Schrödinger in oscillator space

$H|\psi\rangle = E|\psi\rangle$

$H_{mn}\psi_n = E\psi_m$

EQUIVALENT

→ Osmg $V^{HF} = \frac{1}{2} m \omega^2 r^2$ is really

straightforward in 1-dim

⇝

In the shell-model (SM), it is

the same idea (Δ-particles

in 3-dimensions)

BASIC IDEA

→ Write A-body Schrödinger in MF basis

→ It becomes an infinite-dimensional matrix equation

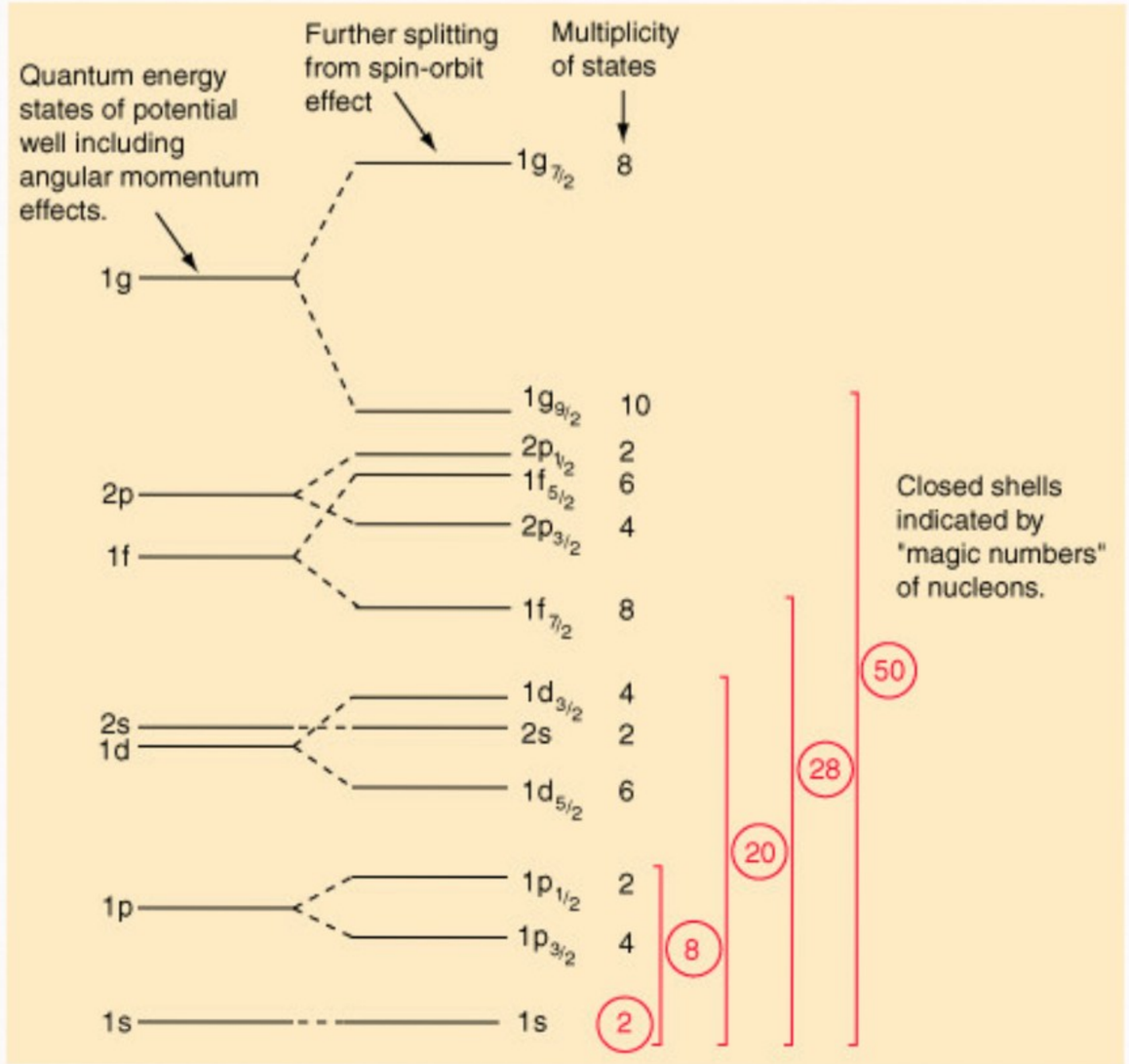
1) $H = \sum_i T_i + \sum_{ij} V_{ij}^{2b} + \sum_{ijk} V_{ijk}^{3b} + \dots \rightarrow \sum_i (T_i + V_i^{MF}) + \Delta V$

2) $H_{\alpha\beta} \Psi_\beta = E \Psi_\alpha$, $\Psi_\alpha = \mathcal{A}\left[\prod_{i=1}^{A} \phi_i(x_i)\right]$

PROBLEM → INFITE DIMENSIONAL PROBLEM

We have to simplify

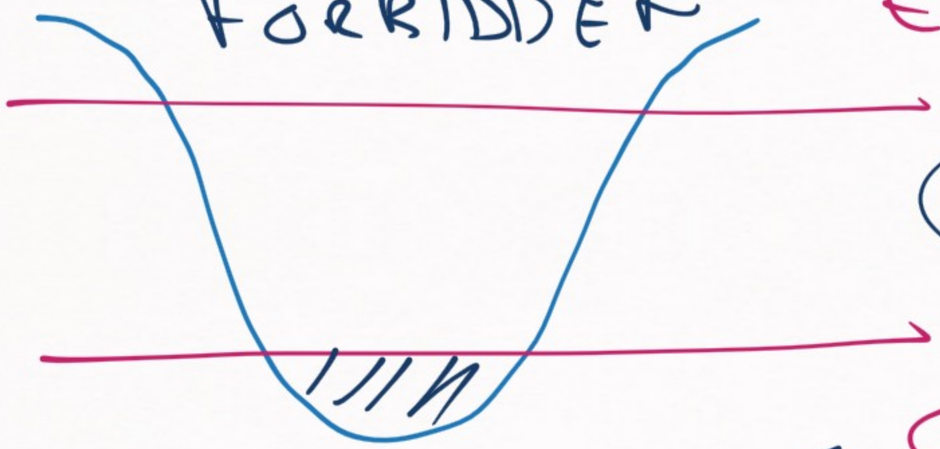
We don't consider all possible shells



→ Not all shells are important for all nuclei

→ We can ignore shells, and have a finite-dim problem

[STANDARD SET-UP]



EXTERNAL → shells that we do not expect to be filled

VALENCE

CORE

shells that are always going to be full

finite_dim

DIVIDE SHELL-SPACE INTO:

1) CORE

2) VALENCE ⟶ this is what matters

3) EXTERNAL

Example →

^{16}O

EXTERNAL

EXTERNAL

2s

1d

1p

1s

1d3/2

2s1/2

1d5/2

1p1/2

1p3/2

1s1/2

neutrons

protons

[16O core]

VALENCE

EXAMPLE (CONT'D)

TYPICAL INTERACTING SHELL MODEL

130 →

1) CORE → 1s, 1p orbitals
for neutrons & protons

2) VALENCE → 1d, 2s orbitals
for neutrons

3) EXTERNAL → everything else

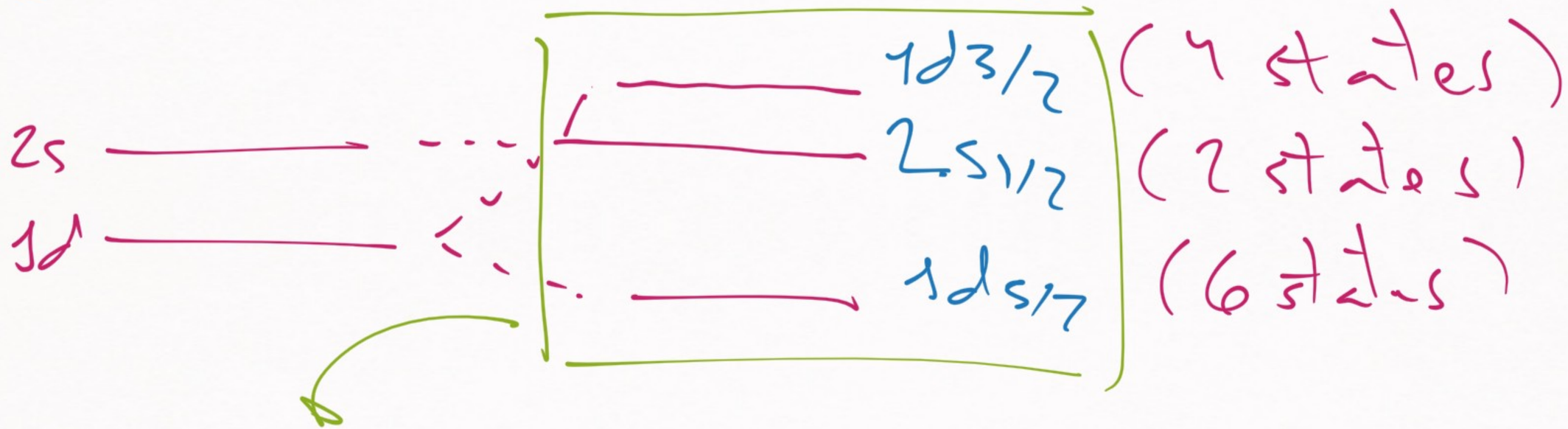
How we solve it?

1) Write all the $\underline{\Psi}_\alpha$ (the basis)

$$\underline{\Psi}_\alpha = \underbrace{\underline{\Psi}_\alpha^{\text{core}}}_{\text{fixed}} \times \underline{\Psi}_\alpha^{\text{valence}} \longrightarrow \text{non trivial}$$

2) We find the dimension of $\underline{\Psi}_\alpha^{\text{valence}}$

[DIMENSION OF $\Psi_\alpha^{valence}$]



VALENCE CONTAINS 12 STATES

→ TWO FERMIONS (B0)

WHAT IS THE DIMENSION OF VALENCE SPACE?

y) ^{17}O → 1 neutron & 12 state

→ $\boxed{D = 12}$

7) ^{18}O → 2 neutrons & 12 state

→ $D = \binom{12}{2} = \frac{12 \cdot 11}{2} = 66$ states

^{130}Te → 66-dimensional basis

BUILD THE BASIS

$\overline{\Psi}_\alpha$, for $\alpha = 1, 2, \ldots, 66$

$\alpha \rightarrow$ $|\, g_{7/2}(+5/2), g_{7/2}(+3/2)\rangle$
$|\, g_{7/2}(+5/2), d_{5/2}(+3/2)\rangle$
$|\, g_{7/2}(+5/2), d_{5/2}(+1/2)\rangle$
$|\, g_{7/2}(+5/2), s_{1/2}(+1/2)\rangle$
$\vdots$

} possible configs

NEXT STEP | → We find the matrix elements of $\hat{H}$

$$\hat{H} \longrightarrow H_{\alpha\beta} = \begin{pmatrix} H_{11} & H_{17} & \cdots \\ H_{21} & H_{77} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

66 × 66 matrix

FINALLY → We diagonalize H to obtain the eigenstates of $\mathcal{H}$

→ GREAT! (REALLY EASY)

(if you have infinite computer power)

PROBLEM → Dimensionality grows quickly

$^{60}_{30}Zn_{30}$ → $^{40}_{20}Ca_{10}$ + $^{10}_{10}n$ in p/f shells

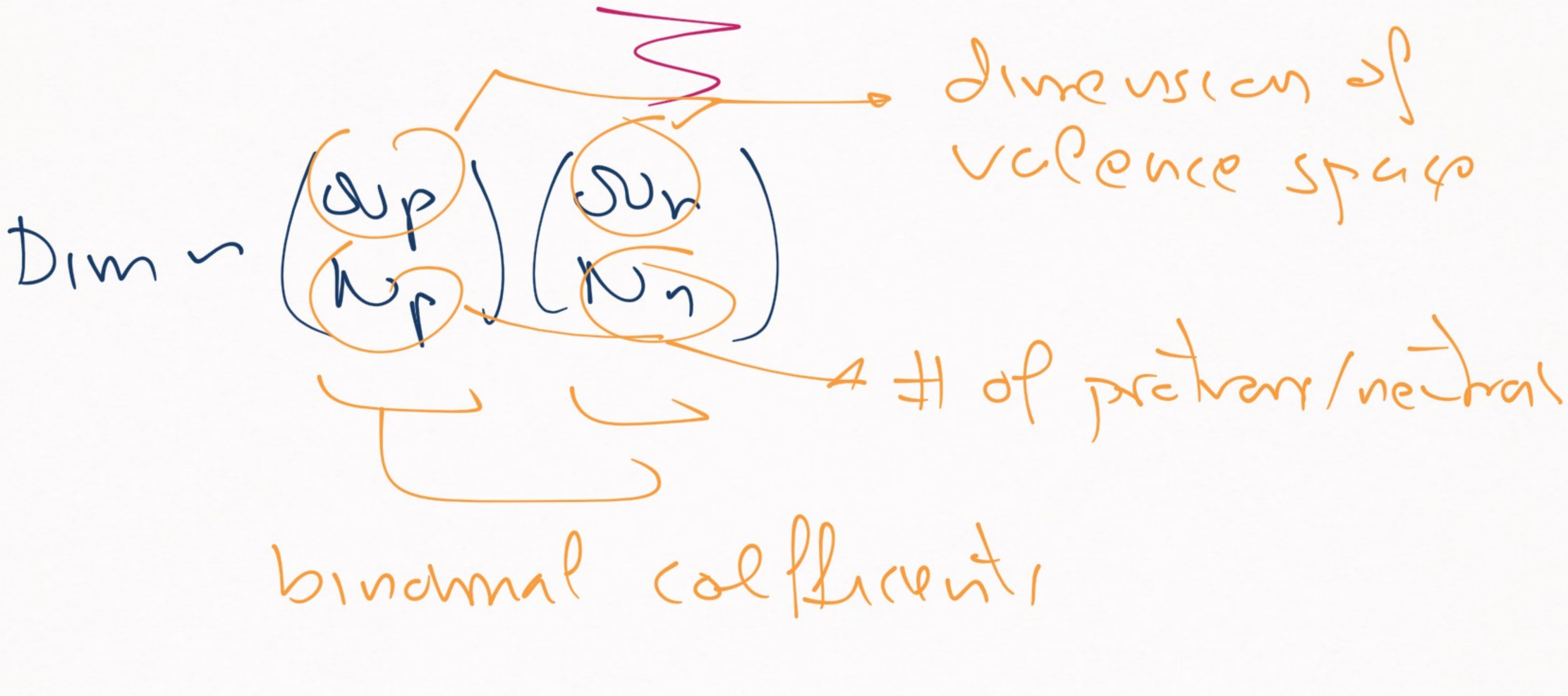
CORE

$D(^{60}_{30}Zn) = (20)(20)(10)(10) \sim 3.4 \cdot 10^{10}$

p/f shells neutrons protons

VALENCE (20 states)

INTERACTING SHELL MODEL CAN BECOME SOMEWHAT COMPLEX VERY EASILY

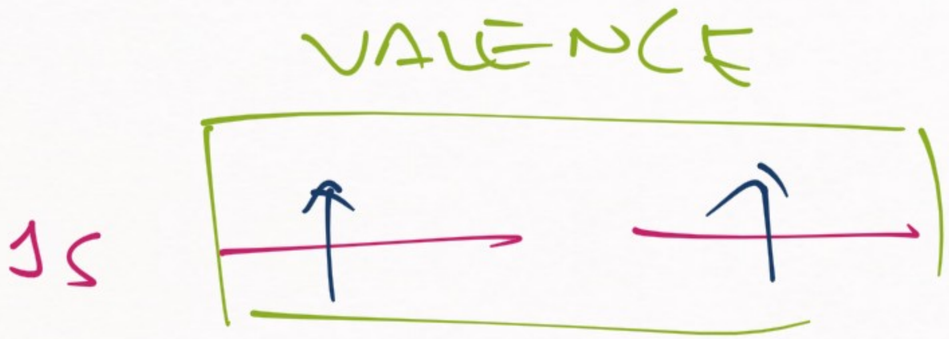


GOOD NEWS → We don't need such large matrices if we concentrate in nuclei w/ specific JP

$$Dim(JP) << (N_p)(N_n)$$

Concentrate on orbital combos that give you JP

[Trivial example] -> Deuteron ($J^P = 1^+$)



$Dim = \binom{?}{1}\binom{?}{1} = 4$

$Dim(1^+) = 3$

$Dim(0^+) = 1$

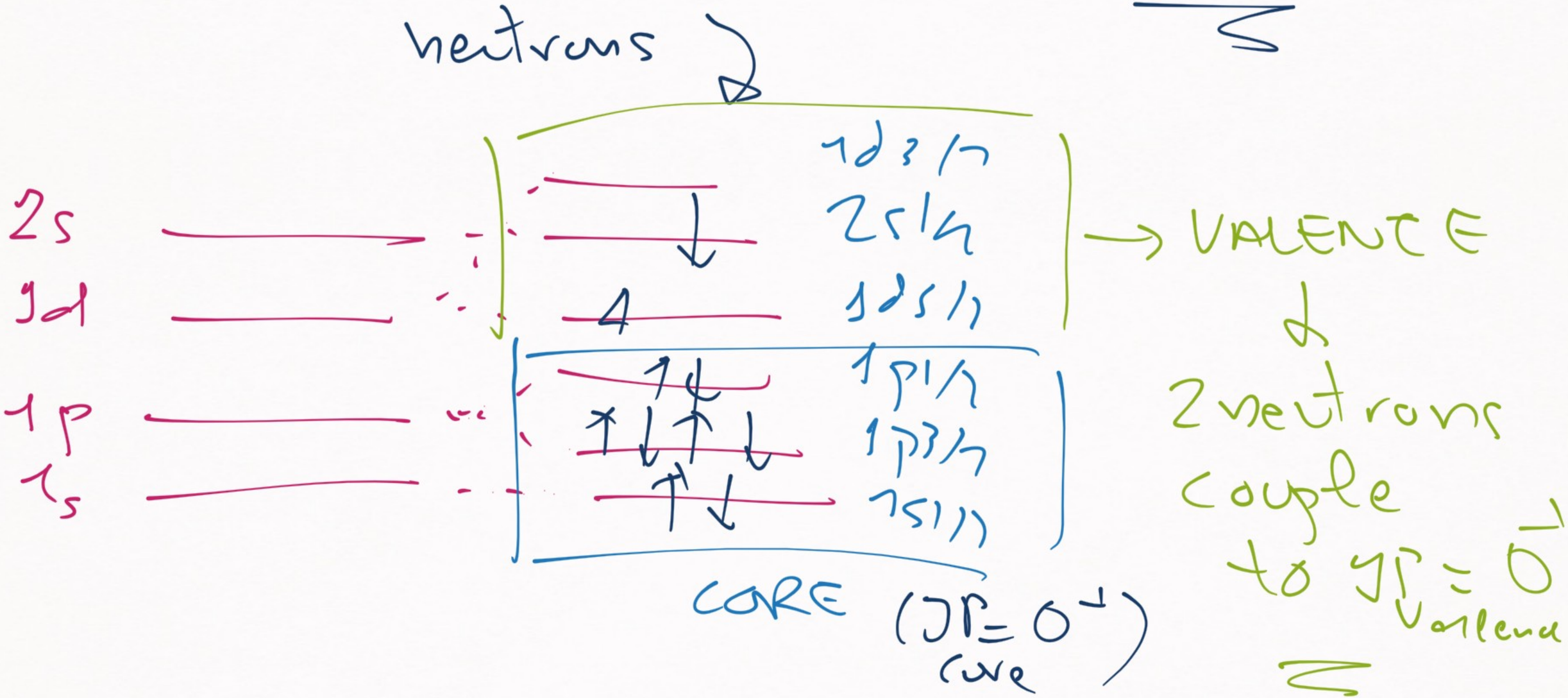
$Dim(J^P) \ll Dim$

Of course, we don't use the shell-model w/ the deuteron (bc/ it's a shallow state)

Better example → H2O

not good for SM

EXAMPLE → ^{18}O, only $J^P = 0^+$ states

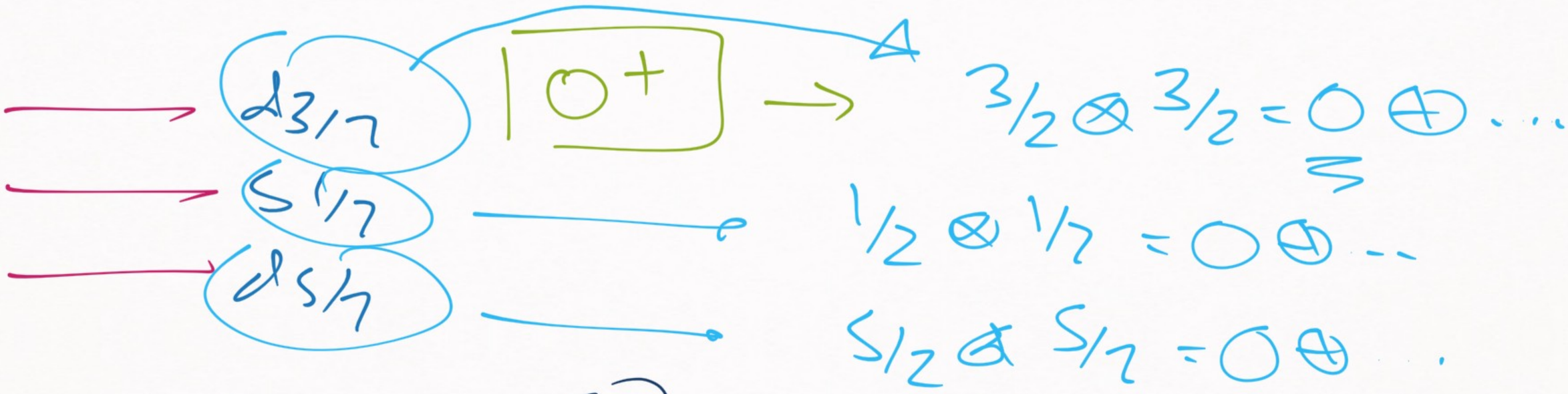


$|^{12}O(0^+)| \rightarrow |^{16}O(J^P_{core}=0^+)| + |$ 2 neutrons in s1d shell coupled to $J^P = 0^+$ $|$

How many configurations give you $J^P_{valence} = 6^+$?

↓

Easy problem


d3/2
s1/2
d5/2
0+
3/2 ⊗ 3/2 = 0 ⊕ ...
1/2 ⊗ 1/2 = 0 ⊕ ...
5/2 ⊗ 5/2 = 0 ⊕ ...
1
2
3
otherwise
J ≠ 0
[only 3 possibilities]

We also have to check symmetry

$\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$ ✗ forbidden (fermions)

$\frac{3}{2} \otimes \frac{3}{2} = 0 \oplus 1 \oplus 2 \oplus 3$ ✗

$S_{1/2} \otimes S_{1/2} = 0 \oplus 1 \oplus 2 \oplus 3 \oplus 4$ ✗

→ allowed

180 ($J^P = 0^+$)

↓

16O core

+ 2n in s/d

↓

Dim = 3 (<< 66)

Calculation is doable

↓

How to do it?

$h_{sp}\,\phi_\alpha = \epsilon_\alpha \phi_\alpha \quad , \; \alpha = n, p$

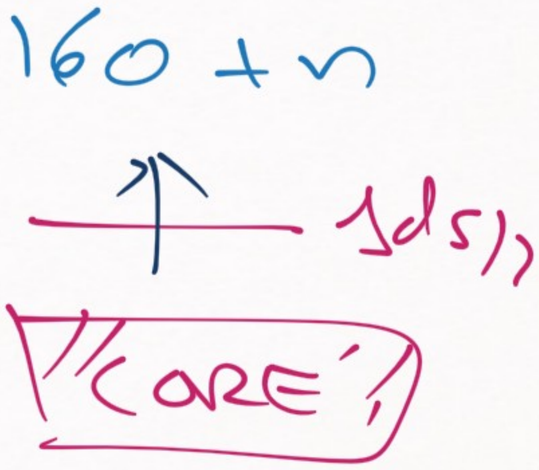
ϵ_α → monoparticular energies

1) FIND THE SINGLE PARTICLE LEVELS

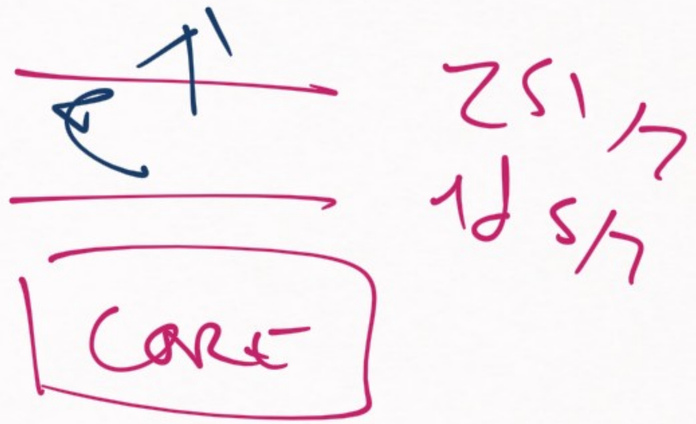
$h^{MF}\,\phi_\alpha = \epsilon_\alpha \phi_\alpha \rightarrow$ [$p_{1/2}$, $1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$]

Comparing energies of different nuclei

$\epsilon(1d_{5/2}) = E_B(^{17}O) - E_B(^{16}O)$



$\epsilon(2s_{1/2}) = \epsilon(1d_{5/2}) + (E_x^*(^{17}O, 1/2^+) - E_B(^{17}O))$



$\epsilon(2s_{1/2}) - \epsilon(1d_{5/2}) = E^*(^{17}O, 1/2^+) - E(^{17}O, 5/2^+)$

$$\epsilon(1d_{3/2}) - \epsilon(1d_{5/7}) = E_B(17O, 3/2^+) - E_B(17O, 5/2^+)$$

→ it's just an elaborate process of comparing energies of nuclei that have an easy explanation in terms of SM orbital

$\epsilon(1d_{5/2}) = -4.1\,MeV$

$\epsilon(2s_{1/2}) = -3.3\,MeV$

$\epsilon(1d_{3/2}) = +0.9\,MeV$

} → Pairing of energy levels

^{18}O → $H_{\alpha\beta} = 2\delta_{\alpha\beta}\epsilon_\alpha + V_{\alpha\beta}$

$\alpha, \beta = 1, 2, 3$

1 → 2n in $1d_{5/2}$

2 → 2n in $2s_{1/2}$

3 → 2n in $1d_{3/2}$

$V_{\alpha\beta}$ → Depends on your potential model

Let's imagine we obtain this:

$$V = \begin{pmatrix} -2.2 & -1.3 & -3.7 \\ -1.3 & -2.1 & -1.1 \\ -3.7 & -1.1 & -2.2 \end{pmatrix}$$

(u / same potential)

H + V → 3×3 matrix → find eigenvalues

→ −12.6, −8.1, +0.6

Ground state
1S0 (0+)

two excited state

-> Normally we use energy differences →

$\Delta E(0_1^+) = 4.4\ MeV$

$\Delta E(0_2^+) = 13.1\ MeV$

} this is how you will do the calculation (= still the problem of calculation $V \propto \rho$)

If you want to get the full spectrum

⇒ Repeat this proces for:

$J^P = 1^+, 2^+, 3^+, 4^+$

(5^+ not possible because it is symmetric)

EXTRA EXAMPLE ↪ $^{130}_{}$, $J = 3^+$

AGAIN → 2n in d/s shells

(must couple to 3^+)

$1d_{3/2}$

$2s_{1/2}$

$1d_{5/2}$

$\frac{3}{2} \otimes \frac{3}{2} = 0 \oplus 1 \oplus 2 \oplus 3$ ~~Sym~~

$\frac{1}{2} \otimes \frac{5}{2}$

$\frac{3}{2} \otimes \frac{5}{2}$

} two options here

Dim = 2

1) Possible states w/ $J^P = 3^+$

$$\alpha = \{(1d_{5/2})(2s_{1/2}), (1d_{5/2})(1d_{3/2})\}$$

$$= \{1, 2\} \text{ (ignoring } m_J\text{)}$$

2) Hamiltonian

$$H_{11} = \epsilon(1d_{5/2}) + \epsilon(2s_{1/2}) + V_{11}$$

$$H_{12} = H_{21} = V_{21} = b_n$$

$$H_{22} = \epsilon(1d_{5/2}) - \epsilon(1d_{3/2}) + V_{22}$$

Imagine that $V = \begin{pmatrix} 0.2 & 0.7 \\ 0.7 & 0.6 \end{pmatrix}$ MeV

Diagonalize H $\Longrightarrow$

$\rightarrow$ -6.6 , -7.5 MeV

$\rightarrow$ Just like in 0^+

→ [General pattern]

(Repeat this for any nuclei of interest)

CLASSICAL SM CALCULATIONS

NOWADAYS → NO-CORE SHELL MODEL

(much bigger matrices)

→ SEE YOU ON THURSDAY