NUCLEAR PHYSICS ⑳

THE COLLECTIVE MODEL

RECAP

Shell-model:

→ Works better close to the magic numbers (N,Z = 2, 8, 20, 28, 50, 82, ...)

(±1 nucleon around N,Z magic)

→ Residual interaction

Pairing interaction → J is large

CURIOUS OBSERVATION

even-even nuclei

⊕

→ 2^+ First excited state

→ 0^+ Ground state

Concrete example → $\boxed{^{38}_{18}\mathrm{Ar}}$

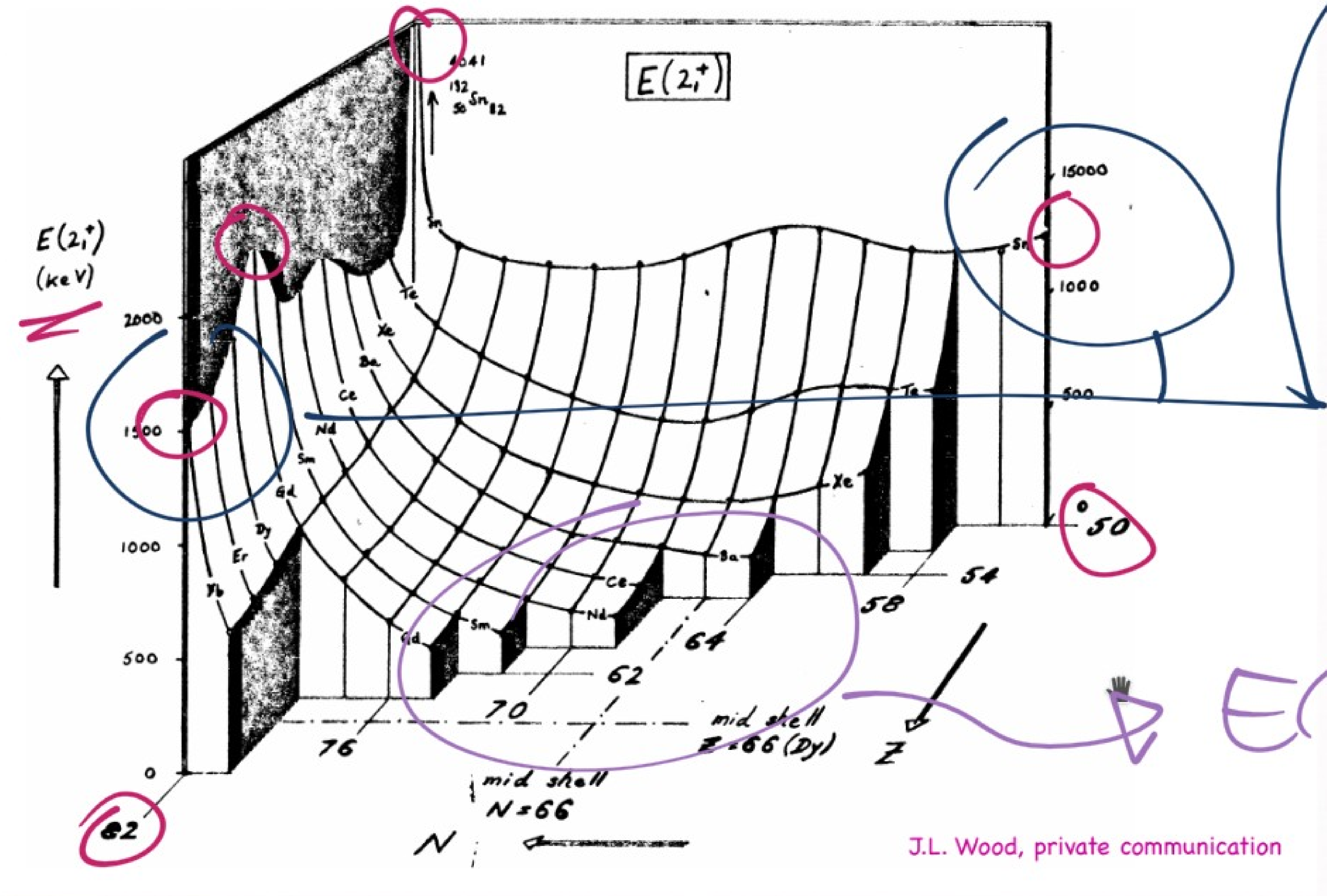
⊕

↓

Almost always happens

—— 2^+ First

—— 0^+ Ground (even-even)



J.L. Wood, private communication

→ high energies

SCALE ARGUMENT

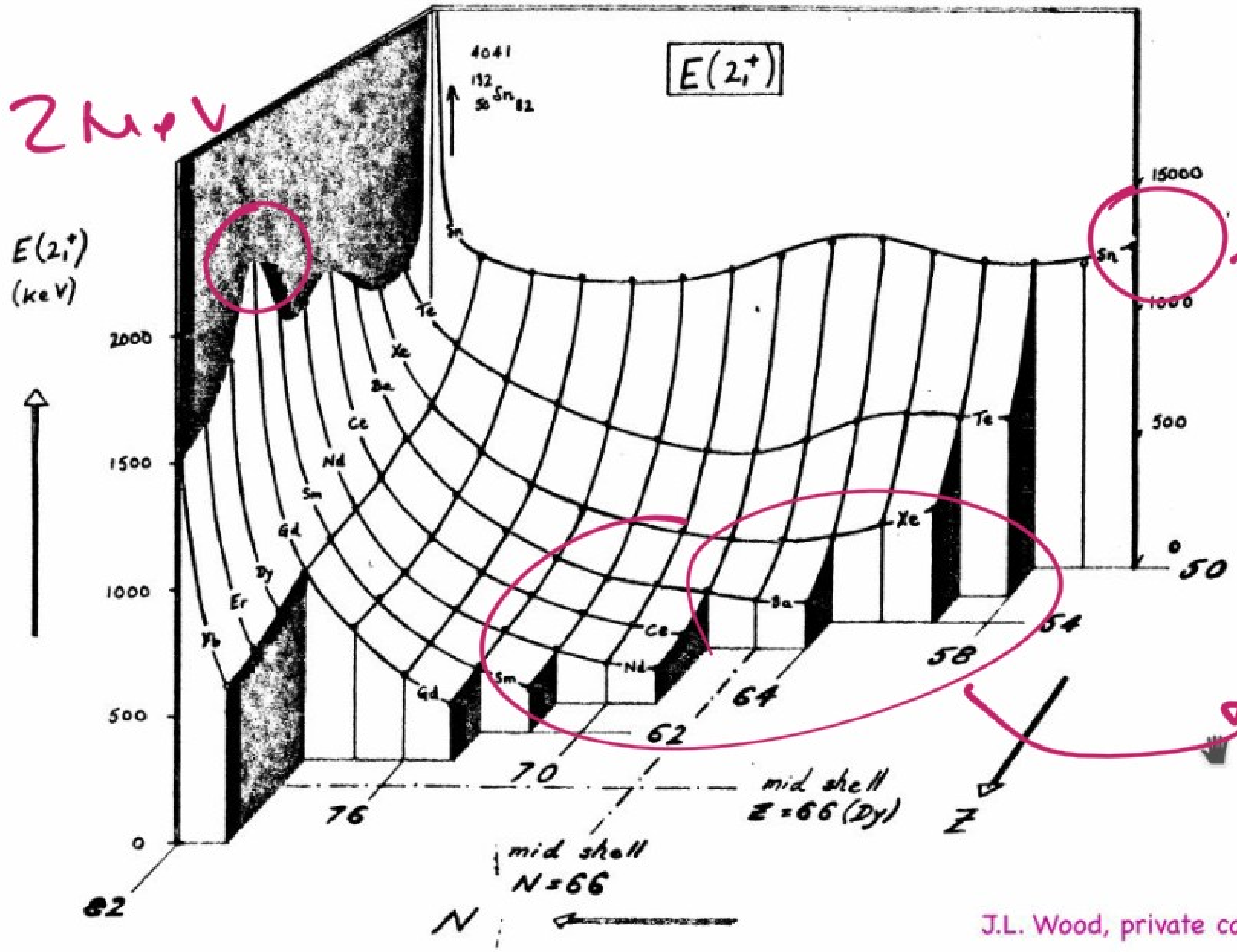
→ near shell closures (~(1-2) MeV)

→ away shell closures

$E(2^+)$ → small energies

↳ (100-200) keV

[CLOSER LOOK] ↳



J.L. Wood, private communication

2MeV

~ 1 MeV

0,1 - 0,7 MeV

SHELL-MODEL :

~ (1 ~ 2) MeV $E(2_1^+)$ =

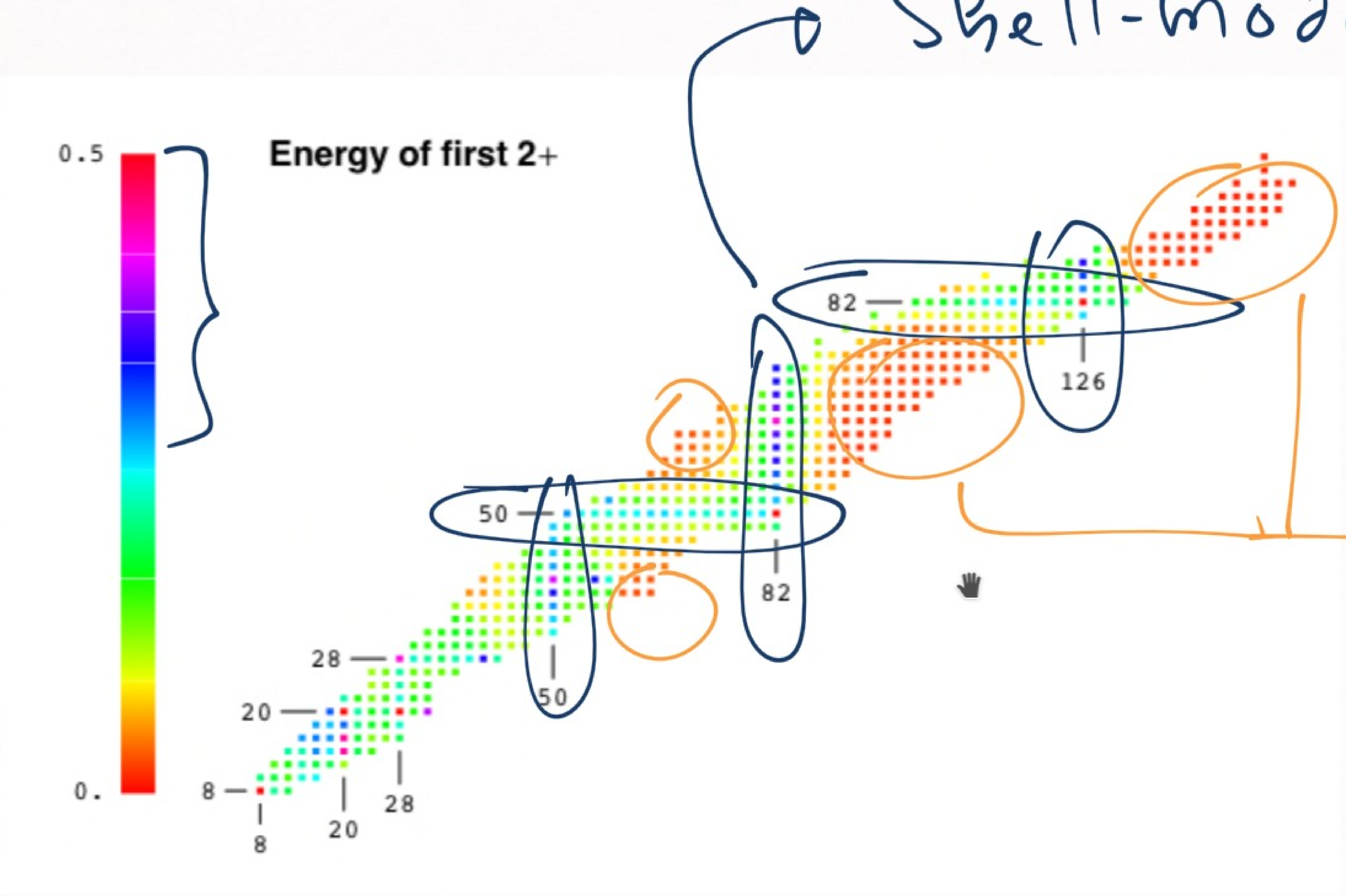
↓

$$V \sim \frac{1}{2} \underline{M_N} \omega r^2 - \xi \vec{l} \cdot \vec{s}$$

↓

$$\hbar\omega \sim \frac{40}{A^{1/3}} \text{MeV} \sim 8 \text{ MeV}$$

$$\hbar^2 \xi \sim \frac{20}{A^{2/3}} \text{MeV} \sim \boxed{1 \text{ MeV}}$$



Shell-model like (expected energy scale)

not shell model like (energy too small)

OBSERVATION → 1) $\overline{\quad\quad}\ 2^+$ $\overline{\quad\quad}\ 0^+$ structure

2) $E(2^+)$ is often too small for the shell-model

⇒ **DIFFERENT NUCLEAR MODEL TO EXPLAIN THIS** ⇒

$\Rightarrow$ $\boxed{\text{COLLECTIVE MODEL}}$

↳ consider the nucleus as a "whole"

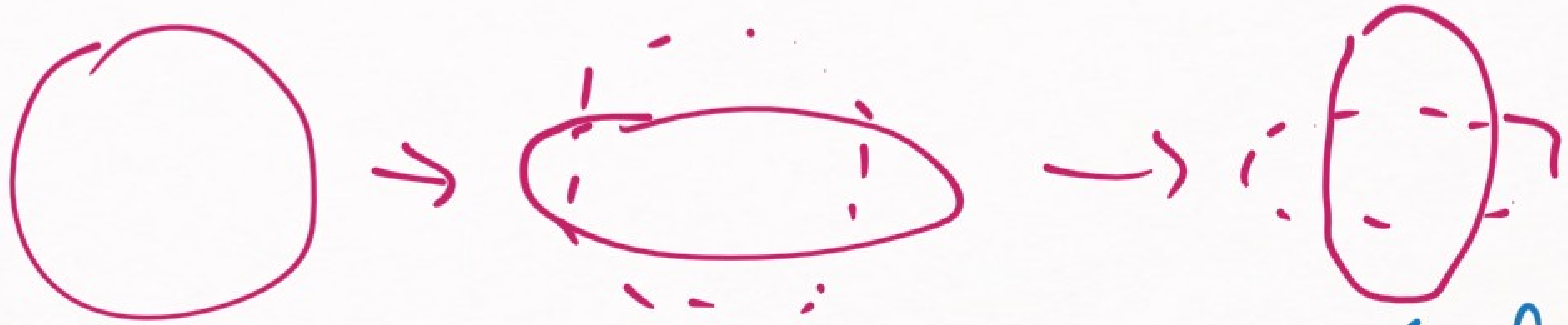
1) Liquid drop model → nucleus as a liquid (collective)

2) Shell-model → individual nucleons in a V_{MF} (not-collective)

3) Collective model → recover a few ideas from 1)

COLLECTIVE MODEL → NUCLEUS AS A WHOLE

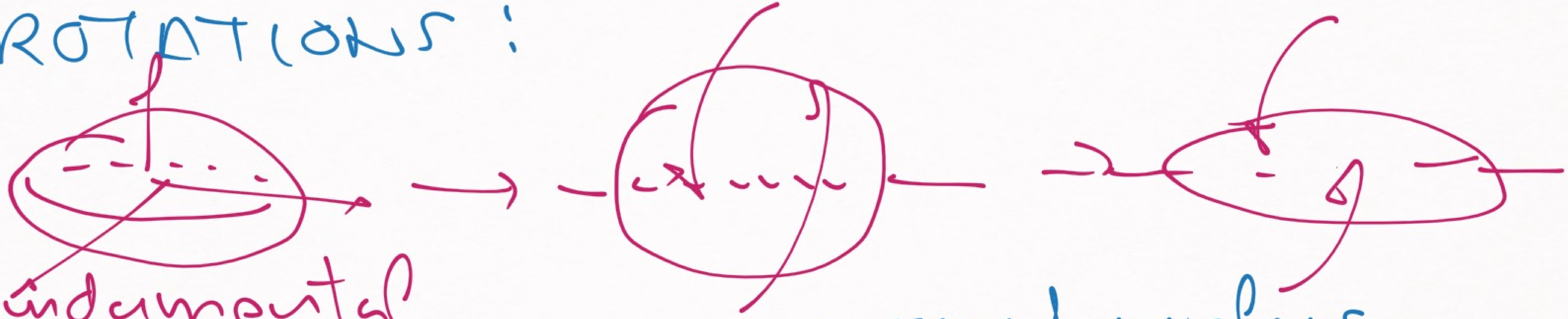
1) VIBRATIONS



fundamental state (spherical)

vibrations around a basic (spherical) form

2) ROTATIONS:



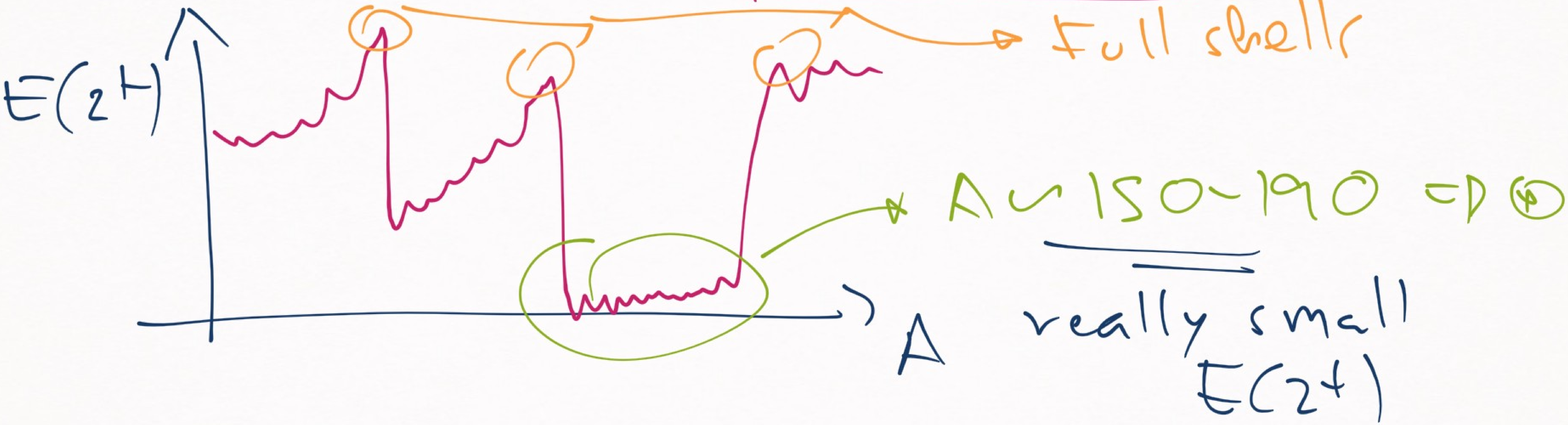
fundamental (deformed)

a rigid nucleus

1) VIBRATIONS → NUCLEUS AS LIQUID

2) **ROTATIONS** → NUCLEUS AS A SOLID

→ We begin w/ **ROTATIONS**

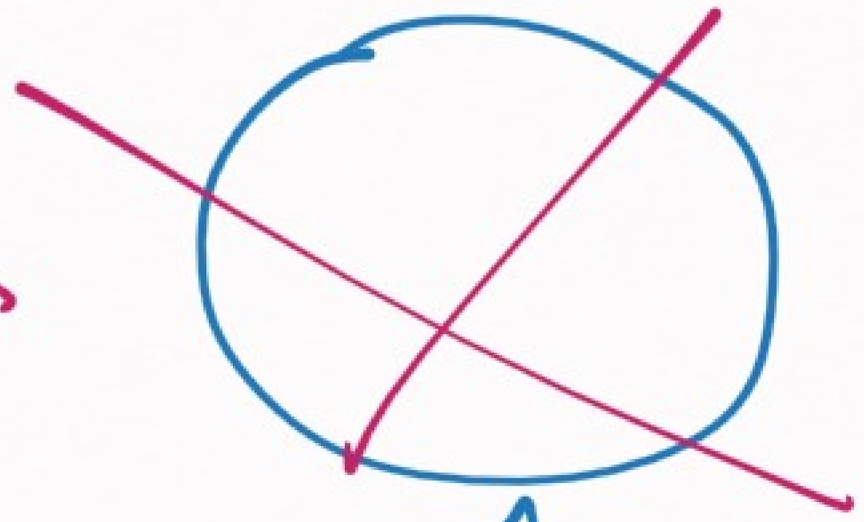


⊕ → $\boxed{\text{NOT USUAL}}$

A ~ (150-190) → really deformed nuclei

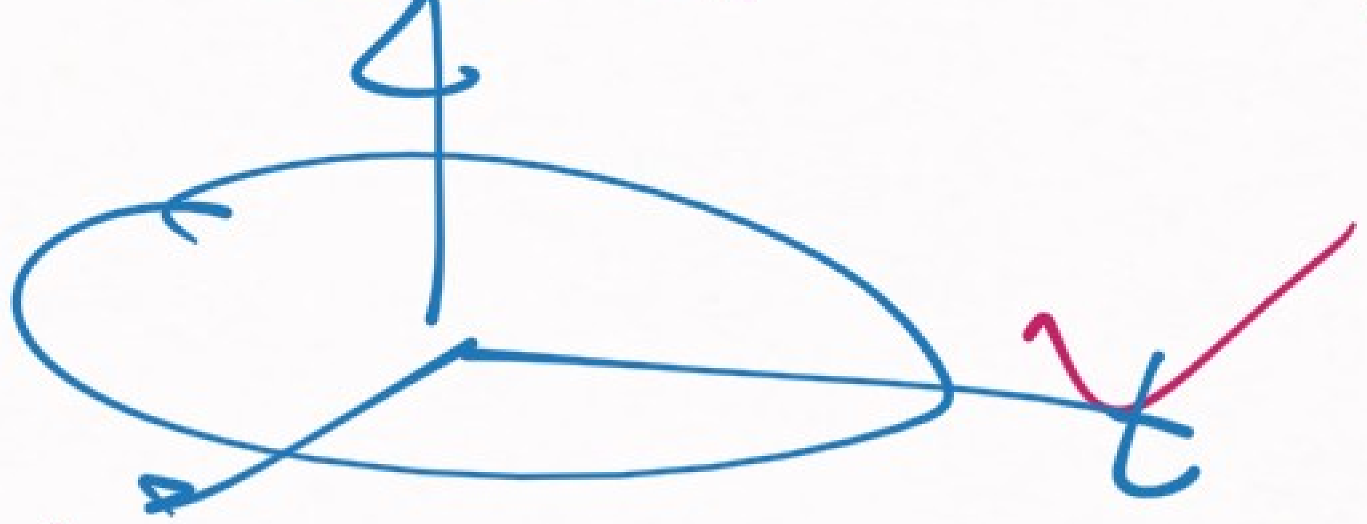
(large quadrupolar moments)

1) Vibrations →



→ no, because not spherical

2) Rotations →



(Q<0)

(because non-spherical)

TO UNDERSTAND ROTATIONS IN NUCLEI

→ QUANTUM ROTATOR

→ it has a few tricks

but we begin w/ obvious features

[QUANTUM ROTATOR]

Solid rigid & moments of inertia (high school physics)

$$H = \frac{1}{2} \underbrace{I}_{\text{moment of inertia}} \underbrace{\vec{\omega}}_{\text{rotation frequency}}^{2}$$

How to quantize it ?

1) $\boxed{\vec{L} = I\vec{\omega}}$ (angular momentum)

→ We know how to quantize $\vec{L}$

2) $\boxed{H = \frac{1}{2} I \vec{\omega}^2} \Longrightarrow \boxed{H = \frac{1}{2I} \vec{L}^2}$

This is easy to quantize

[QUANTUM ROTATOR]

$$H = \frac{1}{2I}\vec{L}^2 \Rightarrow \boxed{\begin{gathered} E(L) = \frac{1}{2I}L(L+1) \\ \text{w/ } L = 0, 1, 2, \dots \end{gathered}}$$

→ SUPER EASY (EXCEPT FOR A SERIES OF COMPLICATIONS)

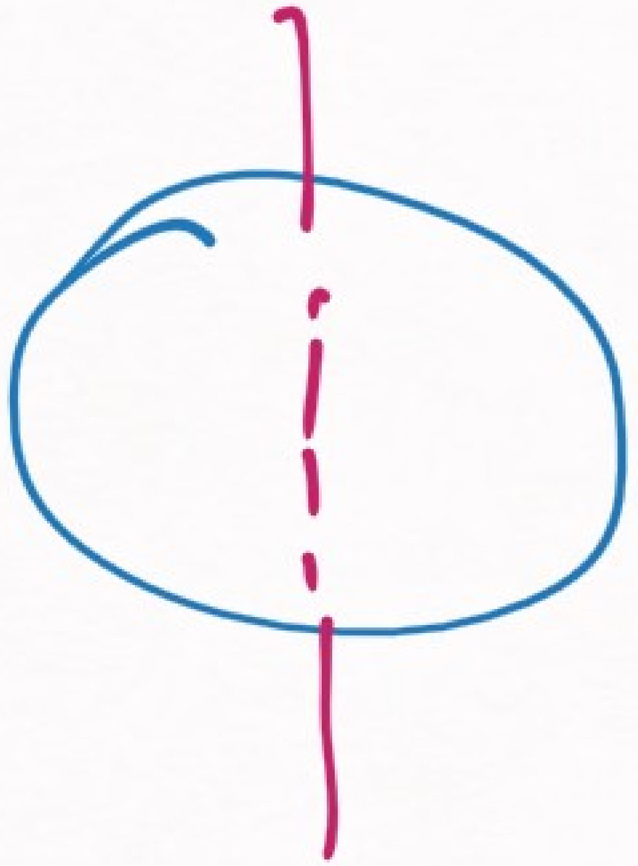
[COMMUTATION] → we should have non-trivial rotations

If a rotation does not change the state of a quantum system

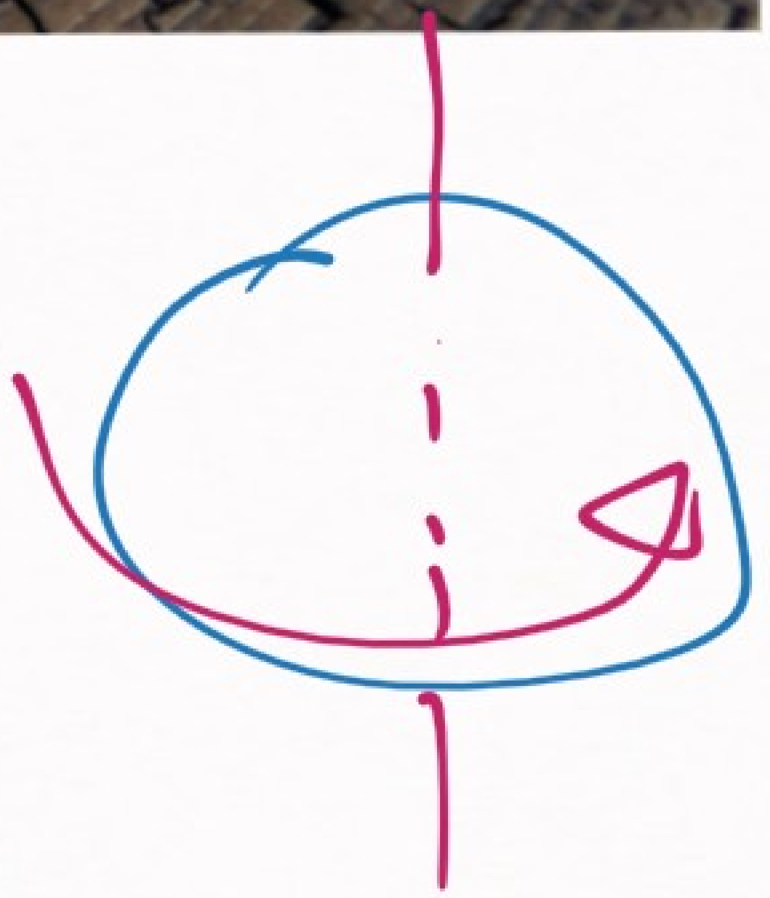
$\Rightarrow$ it should not change the energy state

SIMPLE BUT ABSTRACT

①



②



①, ②

ARE THE SAME QUANTUM STATE

Spherical nucleus $\cong$ Spherical rotating nucleus

$|\psi_1\rangle = |1\rangle$

$|\psi_2\rangle = |2\rangle$

$$\boxed{|1\rangle \cong |2\rangle}$$

→ no difference in the energy levels of $|1\rangle$ and $|2\rangle$

(notice the connection w/ the idea of intrinsic spin)

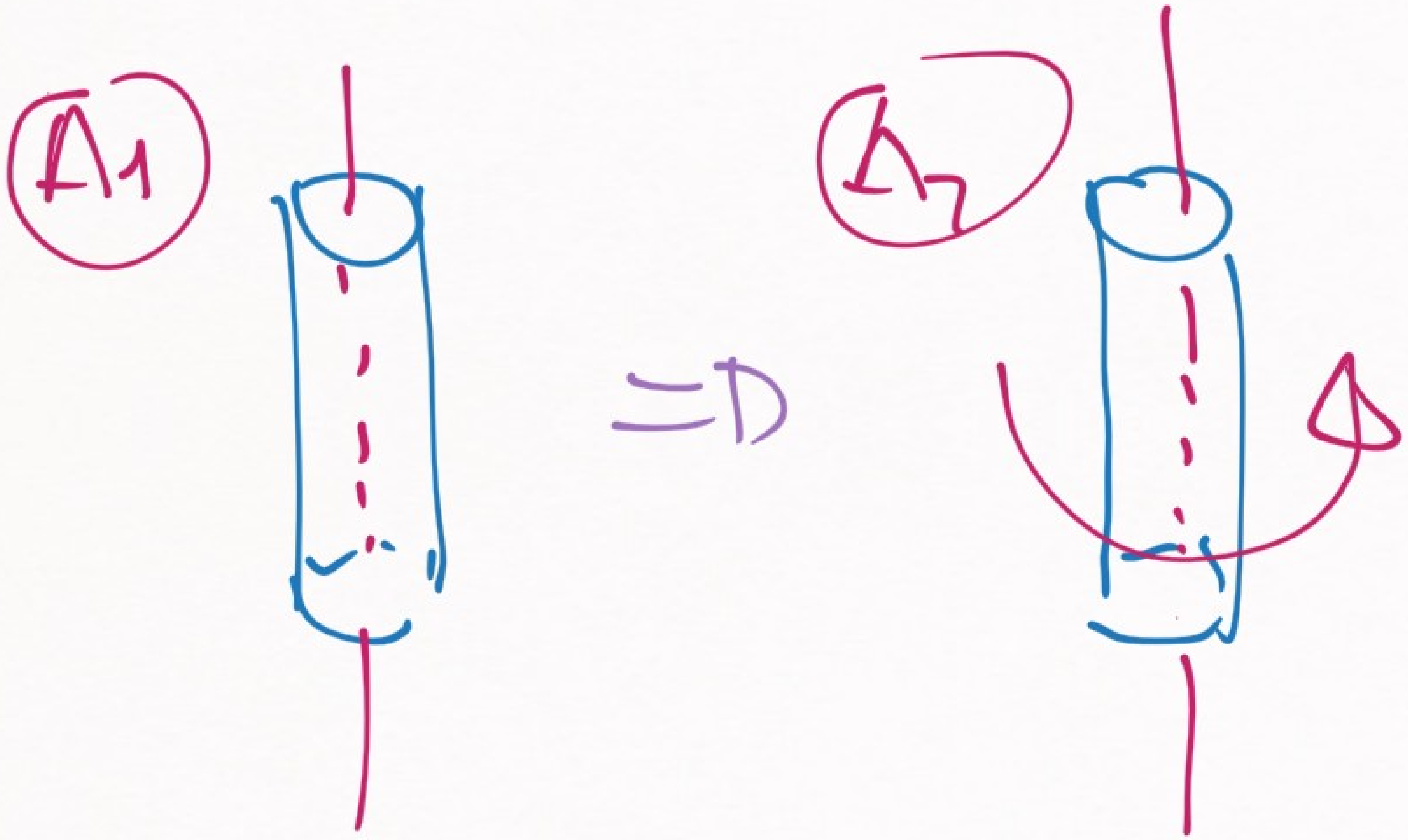
Spherical nucleus $\Longrightarrow$ $\boxed{\nexists \text{ ROTATIONAL ENERGY LEVELS}}$

* * *

Non-spherical nuclei

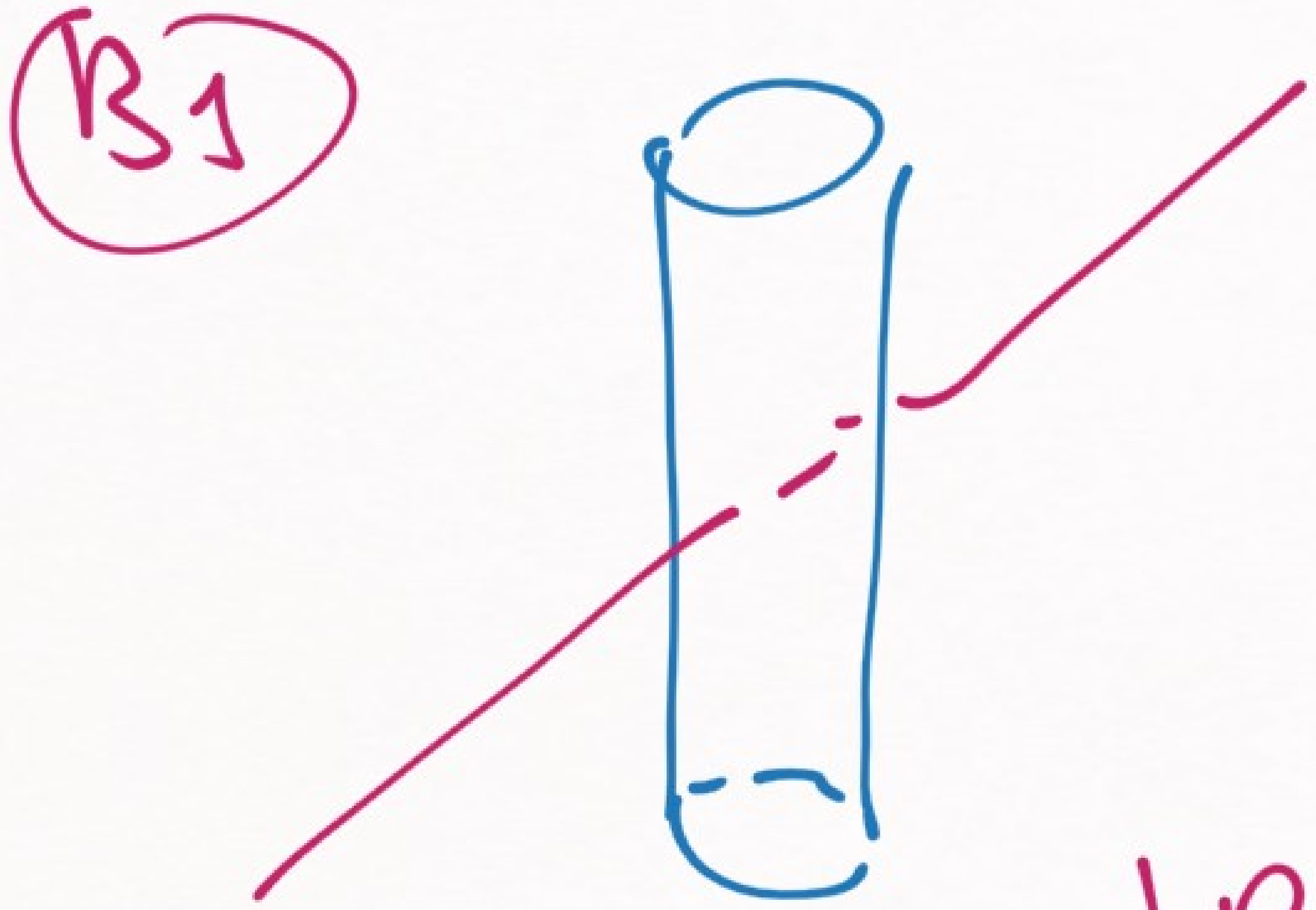
$\Longrightarrow$ [WHAT COUNTS AS A NON-TRIVIAL ROTATION]

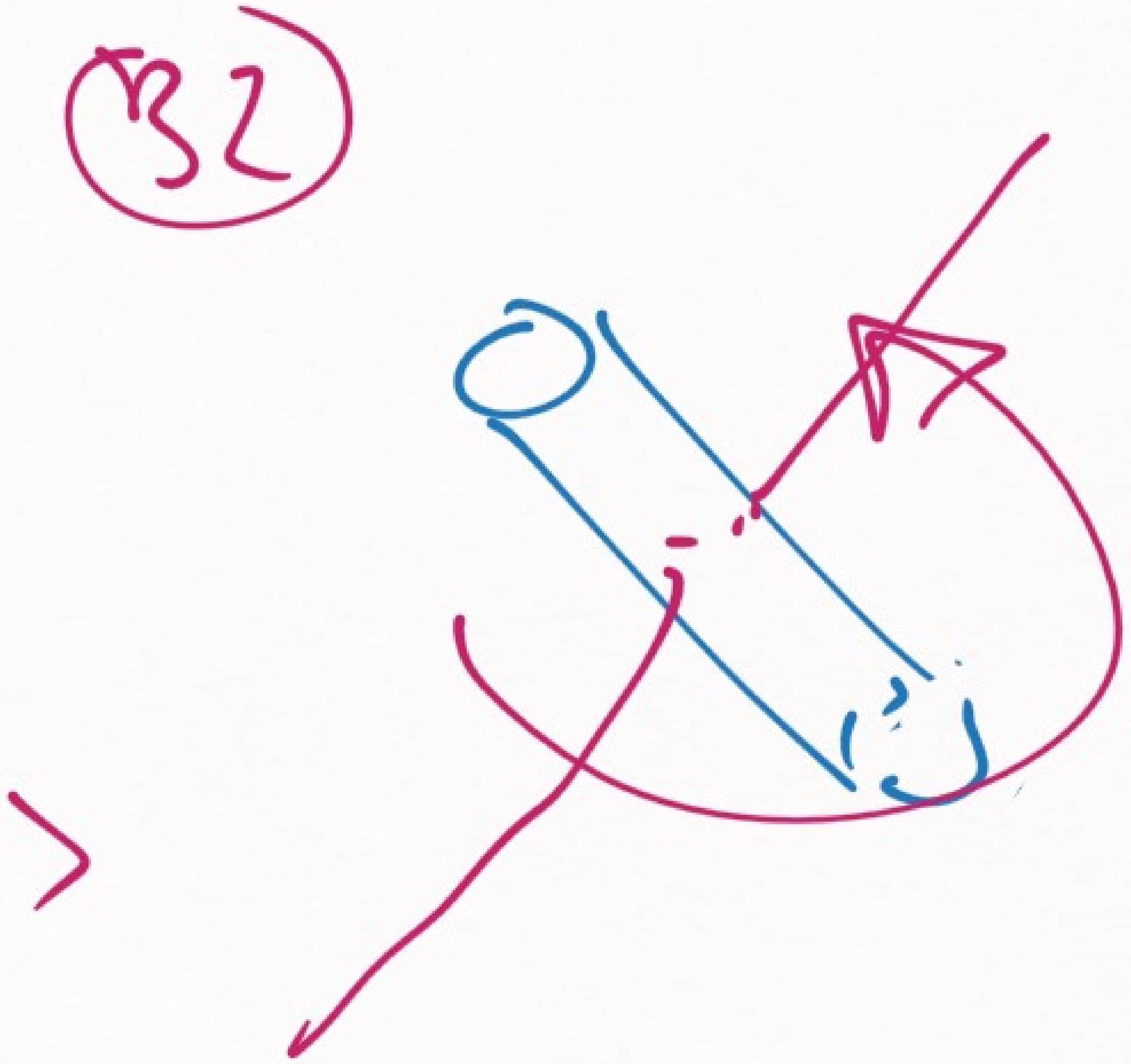
EXAMPLE → QUANTUM ROD



$|A_1\rangle \cong |A_2\rangle$

⇒ This rotation does not count





$$|B_1\rangle \ncong |B_2\rangle$$

→ This rotation is non-trivial

→ ALL DEPENDS ON THE AXIS OF ROTATION

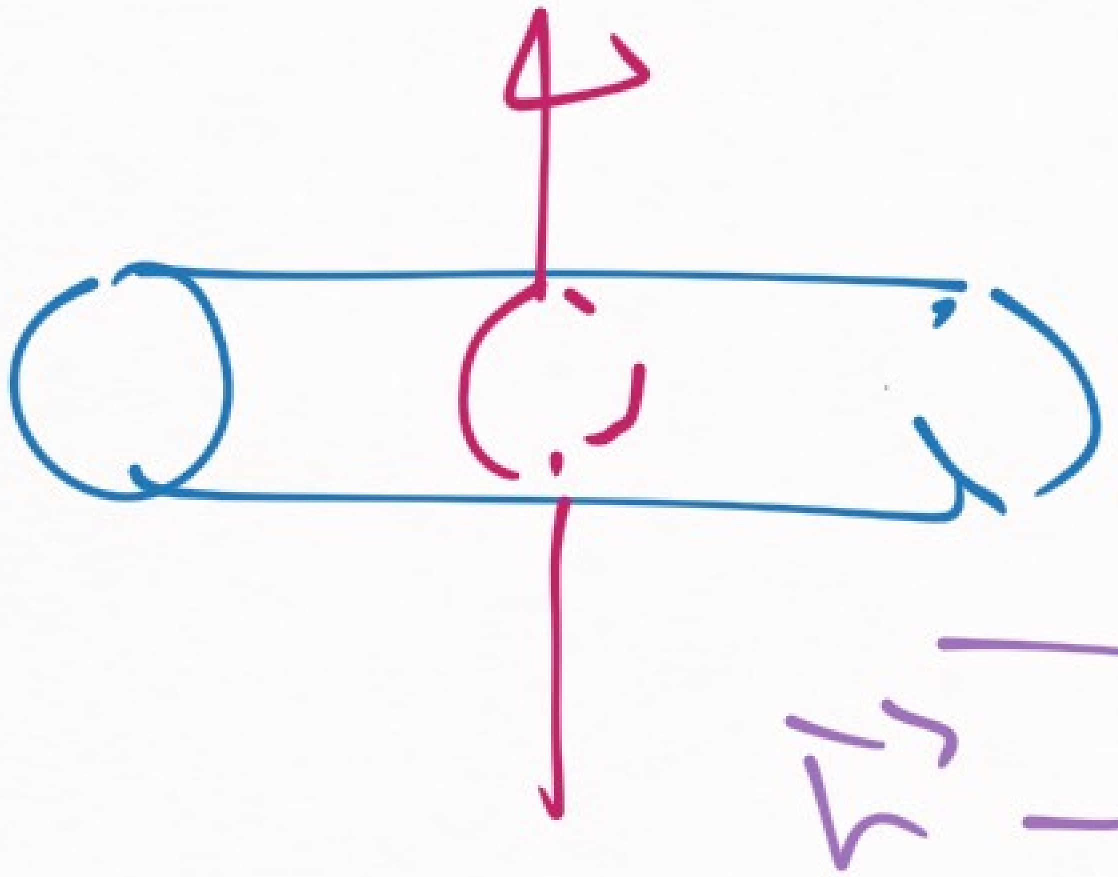
COMPLICATION ②

NON-TRIVIAL

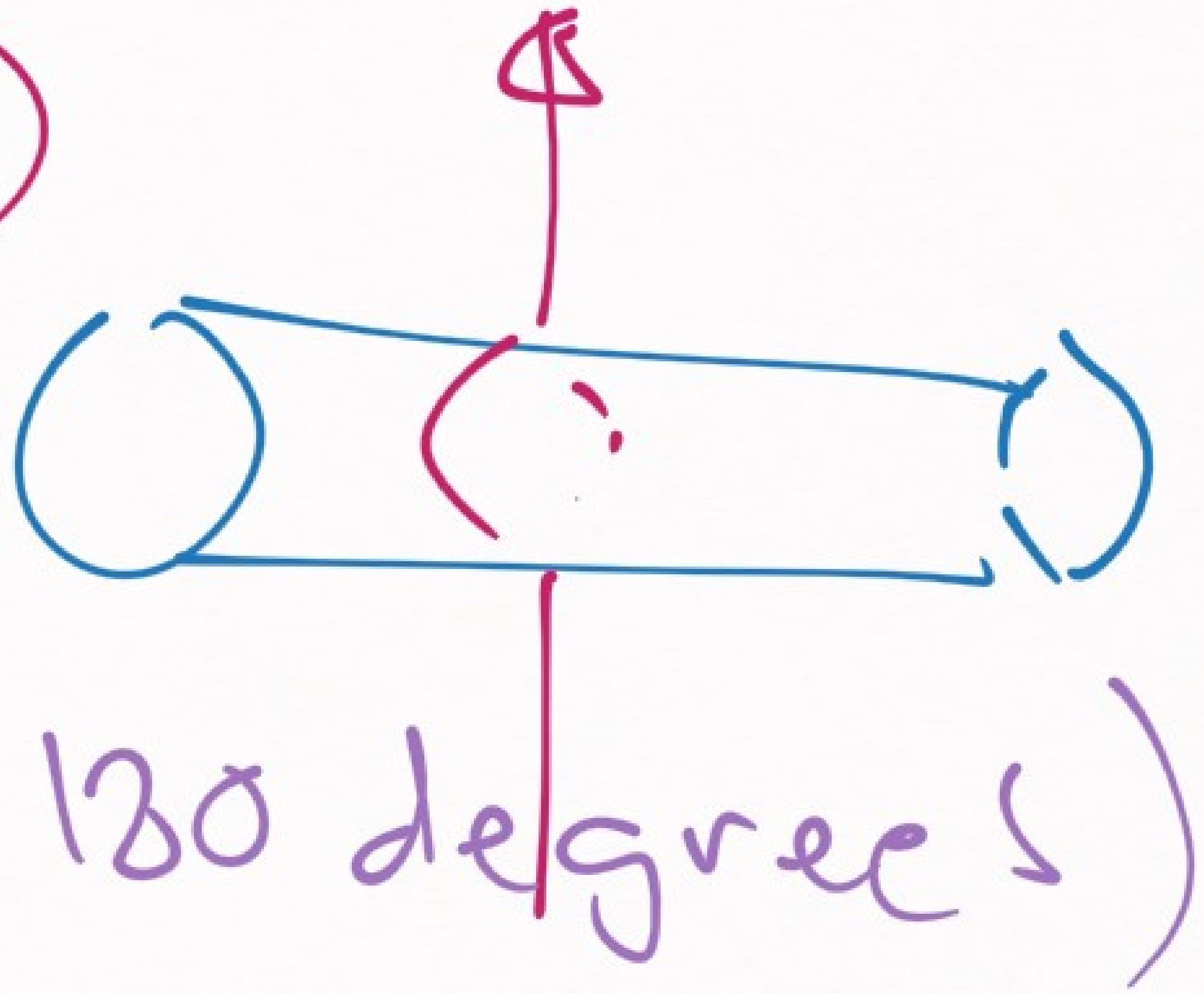
1) COMPLICATION ① → 1.a) SHAPE
1.b) ROTATIONAL AXIS

2) → $\boxed{L = 0, 2, 4, \ldots}$ $L = 1, 3, \ldots \Rightarrow$ TRIVIAL

Δ1



Δ2



$\vec{r} \to -\vec{r}$ (180 degrees)

$|R(180)| \simeq |\text{parity transformation}|$

depends $(\vec{r} \longrightarrow -\vec{r})$

$P|\Delta_1\rangle \simeq |\Delta_2\rangle$ ⟶ does not count as a different state

SUBTLE POINT ⟶

$$P : \vec{r} \longrightarrow -\vec{r}$$

$$R(\theta) = e^{iL\theta} \longrightarrow \underline{\underline{R(\pi) = (-1)^{l}}}$$

$R(\theta) = (-1)^L \Rightarrow$ Equivalent to a parity transformation for $\underline{\underline{\text{odd } L}}$

$L = 1, 3, 5, \ldots$

$\downarrow$

$\underline{\underline{R(\pi) \simeq P}} \longrightarrow \boxed{\text{TRIVIAL}}$

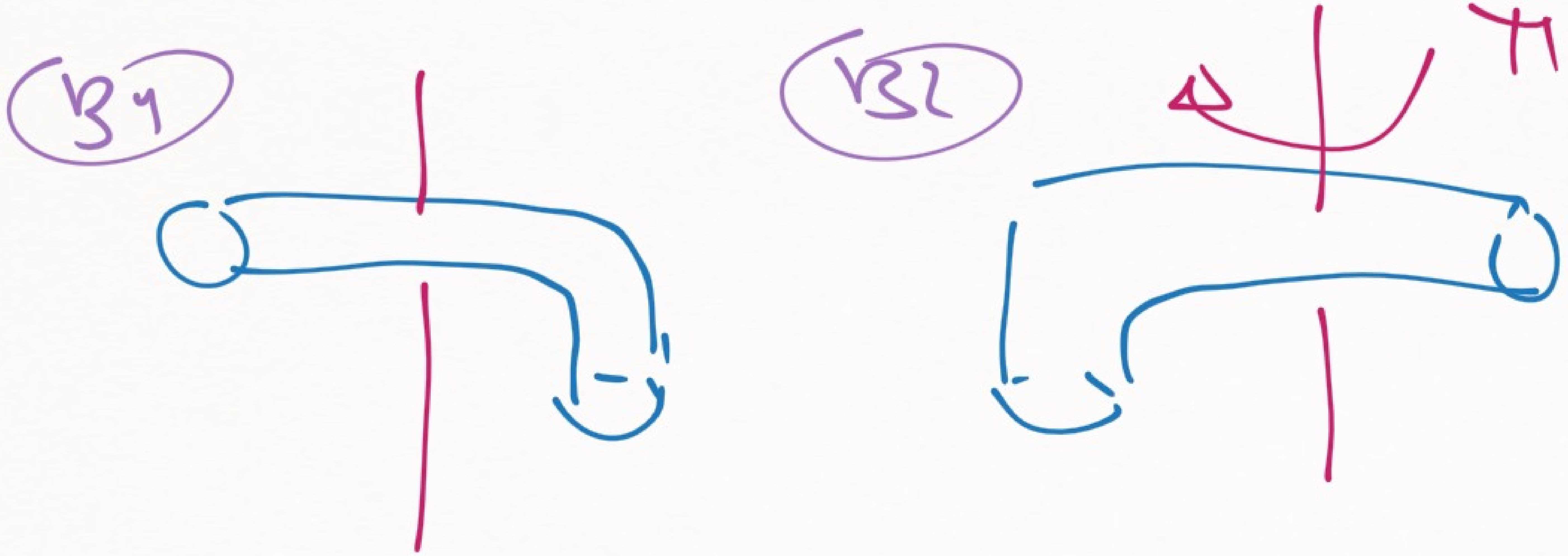
$L = 1, 3, 5, \ldots \Rightarrow$ DON'T COUNT IN GENERAL

$L = 1, 3, 5, \dots$ ROTATIONS $\Rightarrow$ CONTAIN IMPLICIT PARITY TRANSFORMATION

$\Downarrow$

TRIVIAL

$\exists$ EXCEPTIONS:



$$|B_1\rangle \not\simeq |B_2\rangle$$

$$B \rightarrow \boxed{\text{CHIRALITY}}$$

CHIRAL SHAPES →

violate parity

→ they don't happen in nuclei

CONCLUSION) ~~$L = 1, 3, 5, \ldots$~~

NOT ALLOWED

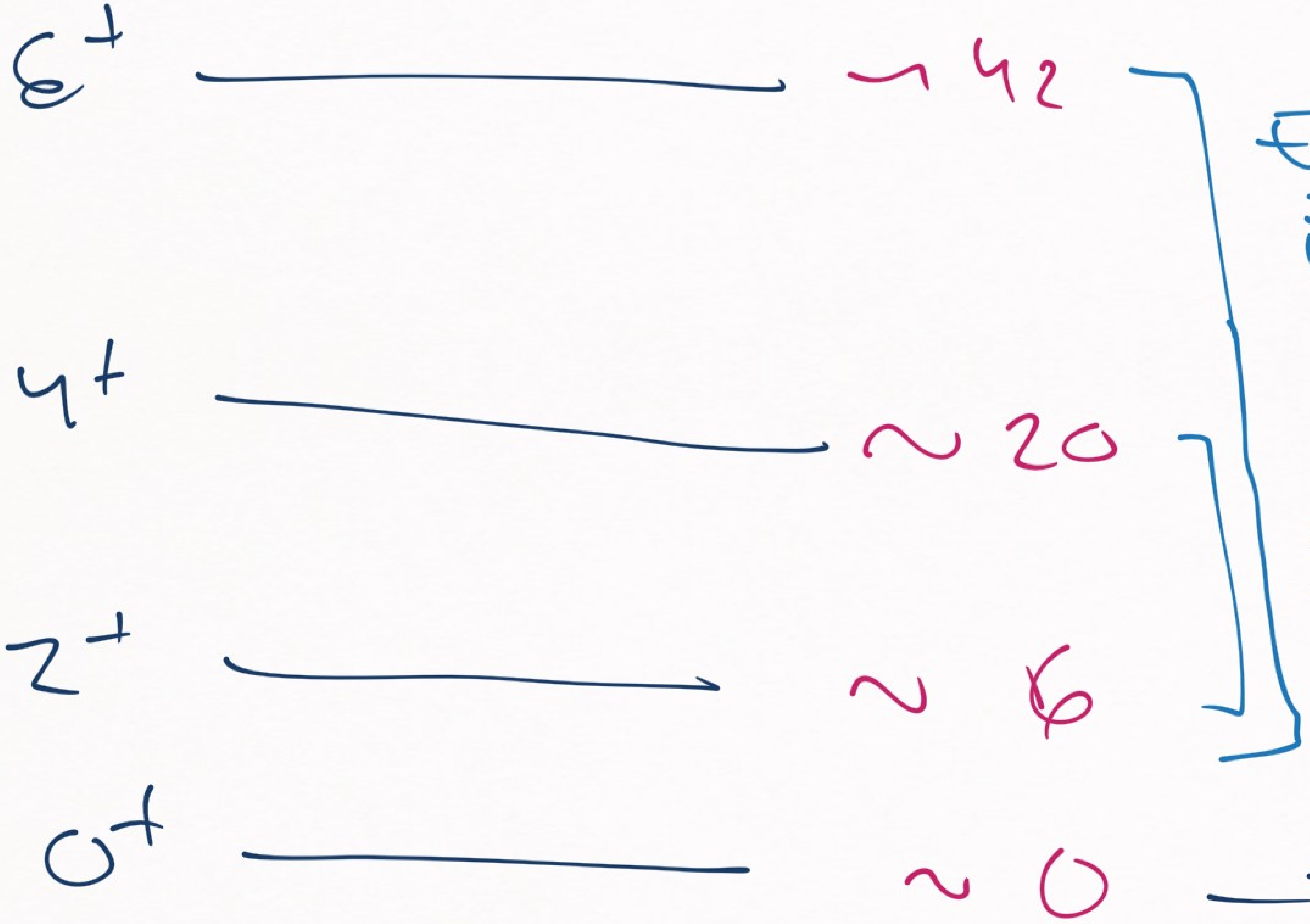
RECAP

ROTATIONAL LEVELS IN NUCLEI:

1) Non-spherical nucleus

2) $L = 0, 2, 4, \dots$ (even L)

ENERGY LEVELS: $E(L) = \frac{1}{2I} L(L+1)$

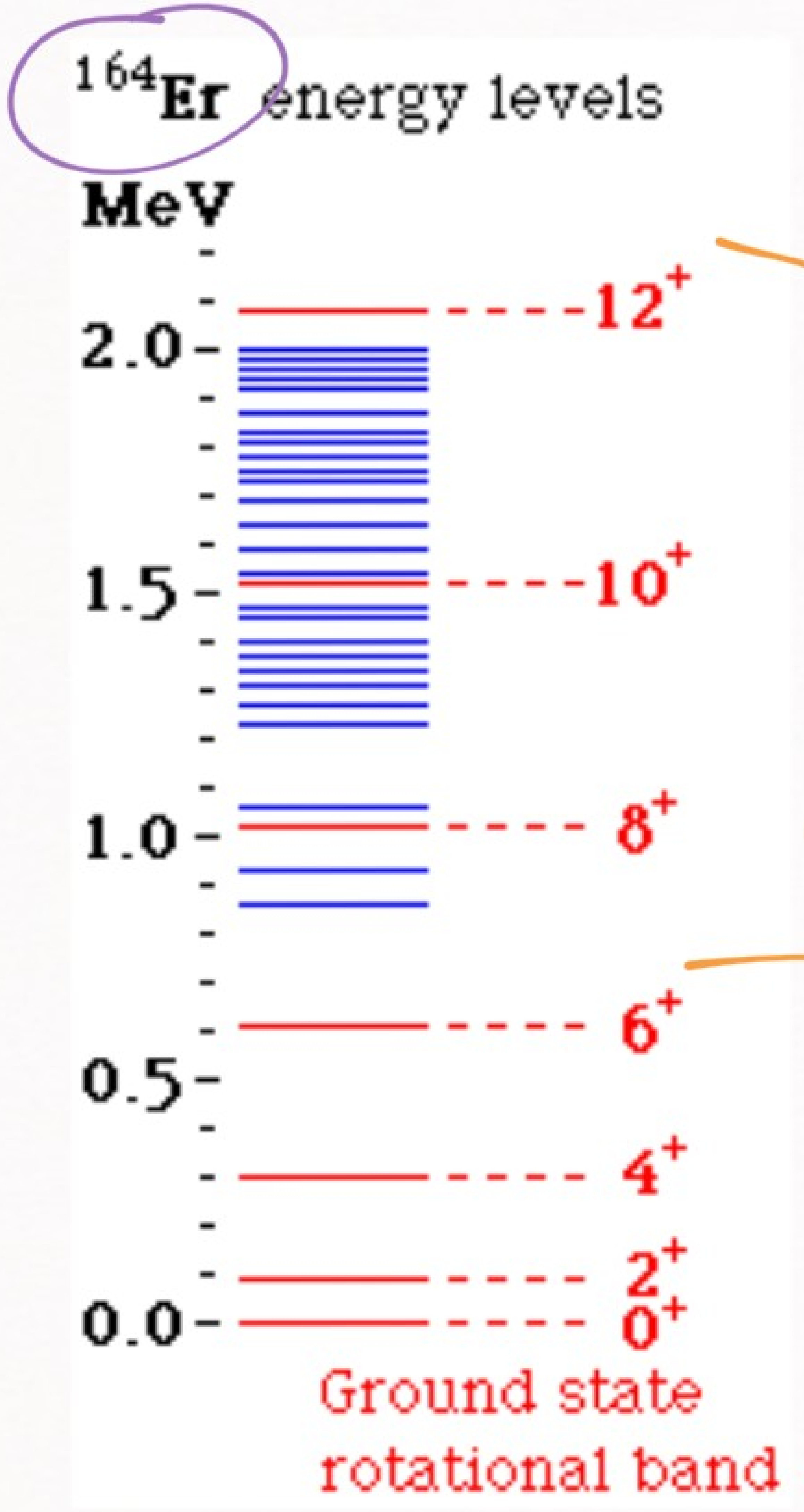


$$\frac{E(6^+)}{E(2^+)} = 7$$

$$\frac{E(4^+)}{E(2^+)} = \frac{20}{6} \simeq 3{,}33\ldots$$

→ TYPICAL ROTATIONAL SPECTRUM

LET'S MAKE A FEW COMPARISONS:



$A \simeq 164$

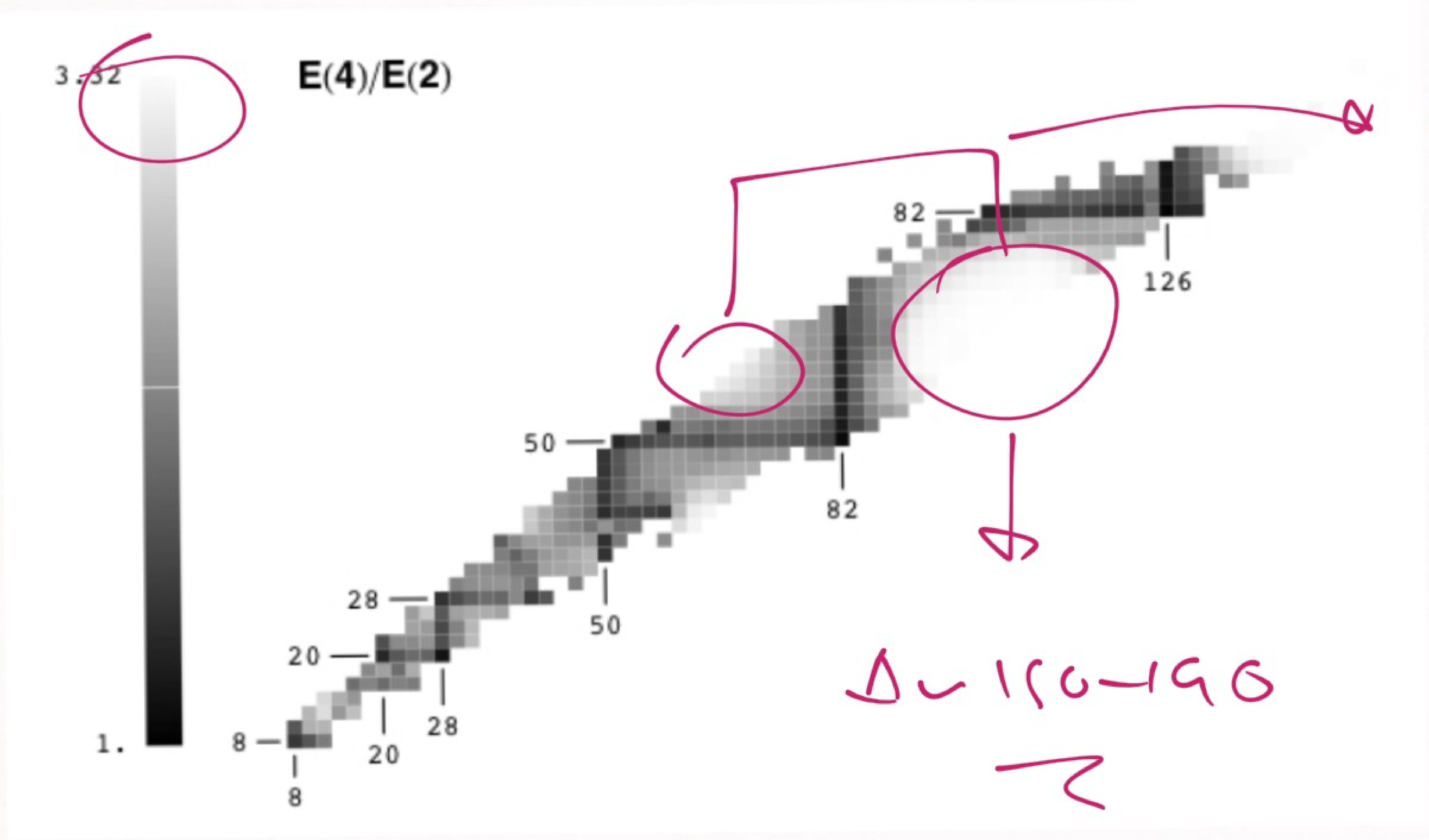
$A \sim 150 - 190$

→ MIXTURE OF ROTATIONAL & NON-ROTATIONAL

✓

→ ∀ rotational → $\boxed{E(I) \propto I(I+1)}$

PLOT OF E(4+)/E(2+) (→ 3.33 IF ROTATIONAL)

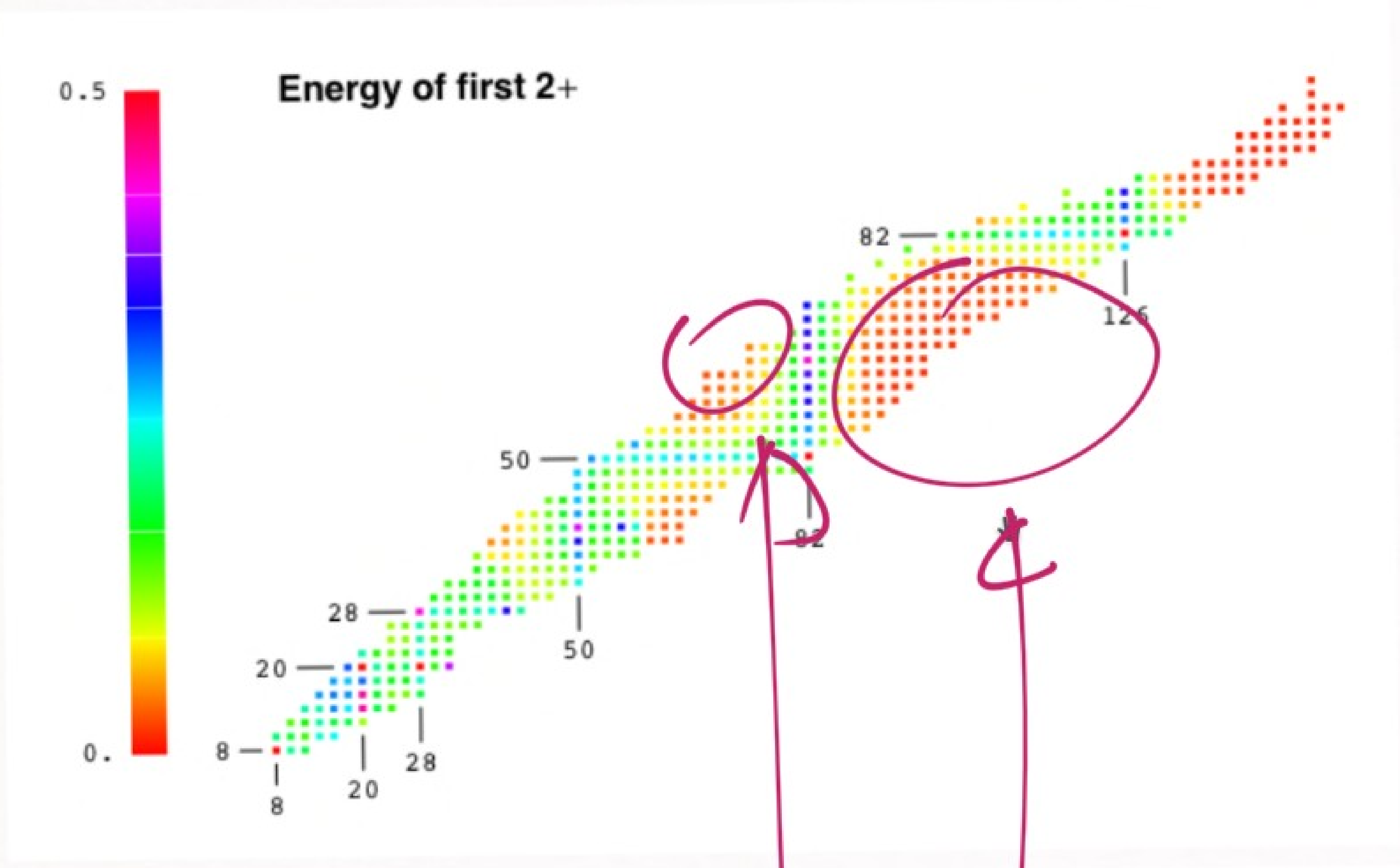


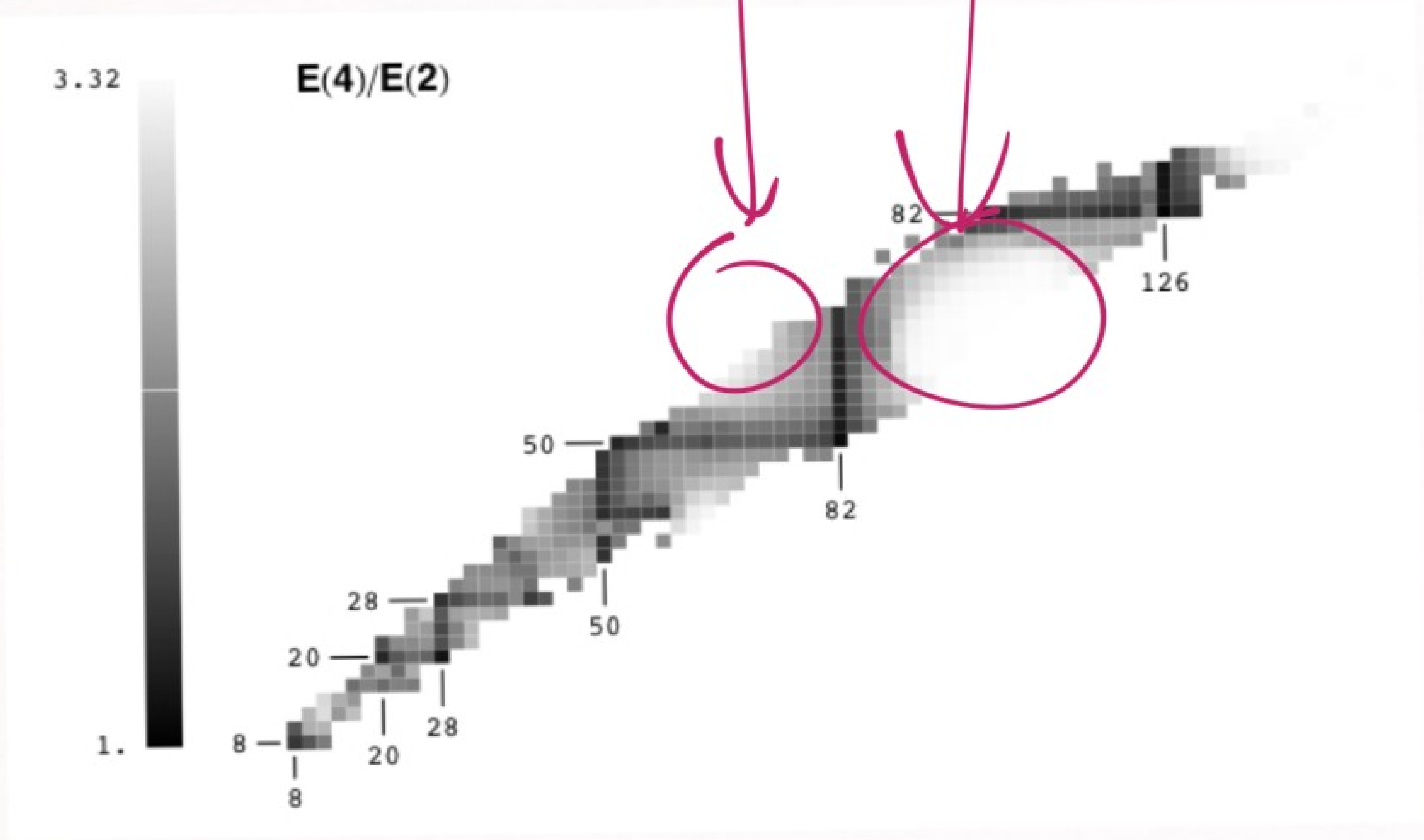
ROTATIONAL SECTOR

3

A ~ 150-190

2





ROTATIONAL EXCITATION ENERGIES

↓

VERY SMALL

[RECAP THE SCALES]

1) SHELL-MODEL → ~ (1-2) MeV

2) ROTATIONAL LEVELS → ~ 0.1 MeV (or less)

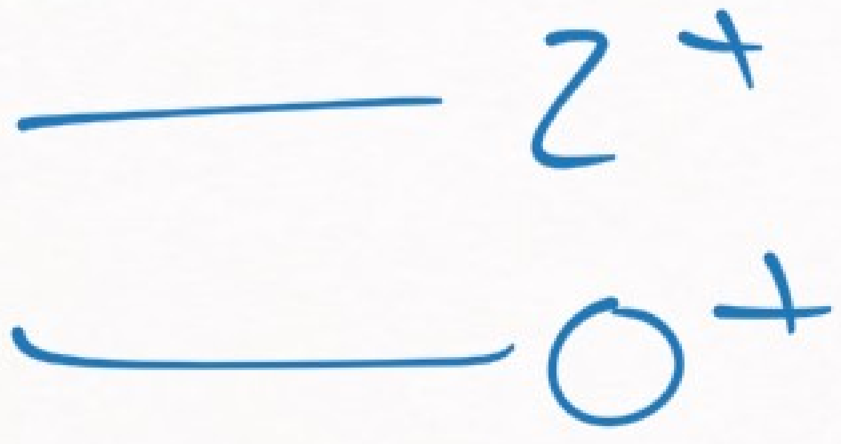
$E(4^+)/E(2^+) \sim 3.33$

3) VIBRATIONAL LEVELS (Now)

~ 0.5 MeV

$$\frac{E(4^+)}{E(2^+)} \sim 2$$

→ VIBRATIONS IN THE COLLECTIVE MODEL

1) 

2) $E(2^+)$

3) $\dfrac{E(4^+)}{E(2^+)}$

1+2+3) → they require some explanation

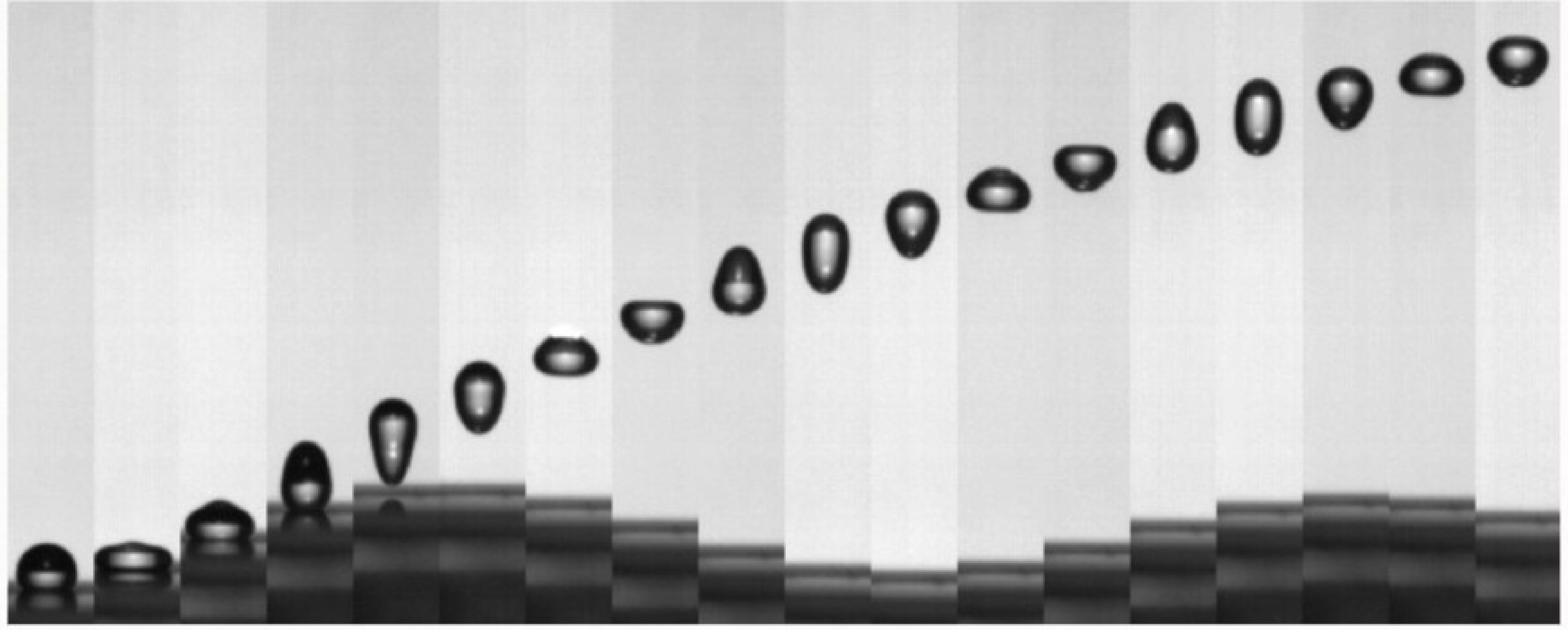
(explanation not unique
→ depends on nucleus)

COLLECTIVE MODEL

1) ROTATIONAL MODES → non-spherical

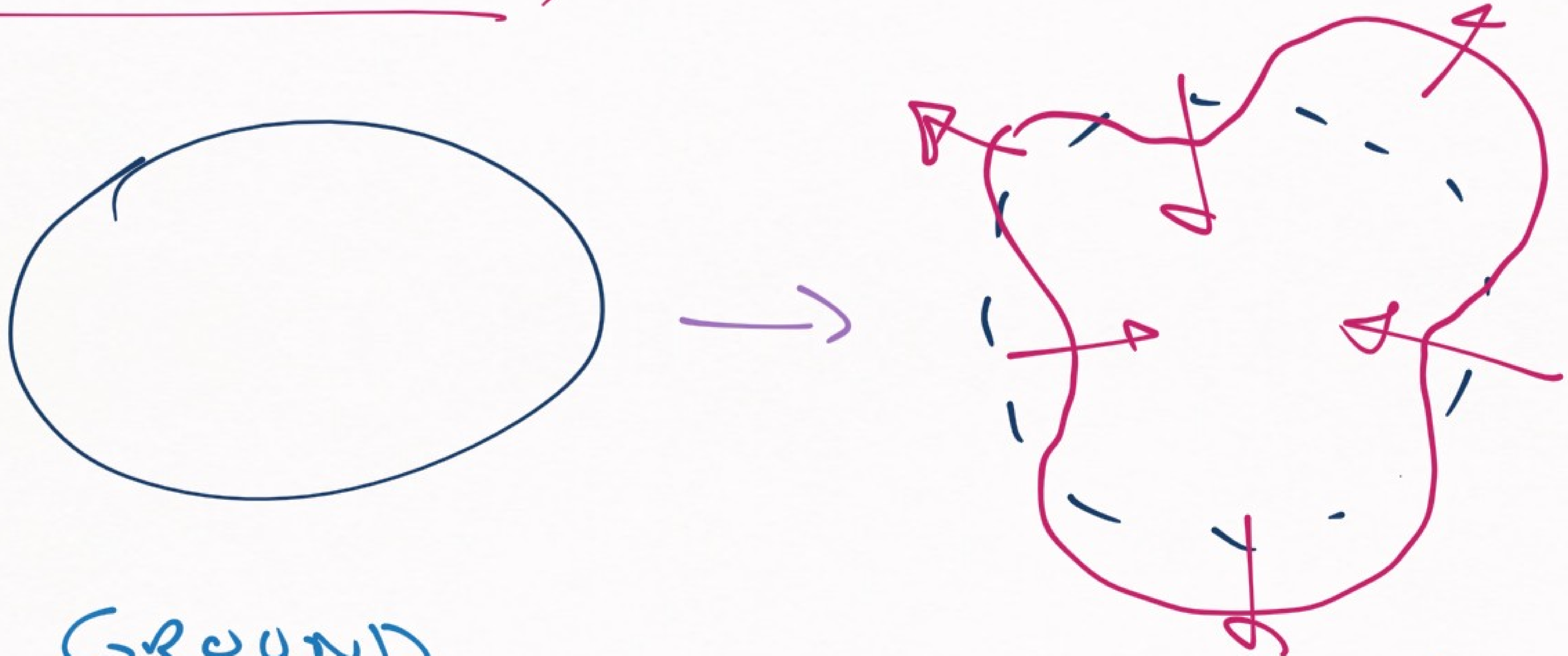
2) VIBRATIONAL MODES → spherical

(→ liquid)



→ [QM version of this]

BASIC IDEA



GROUND STATE

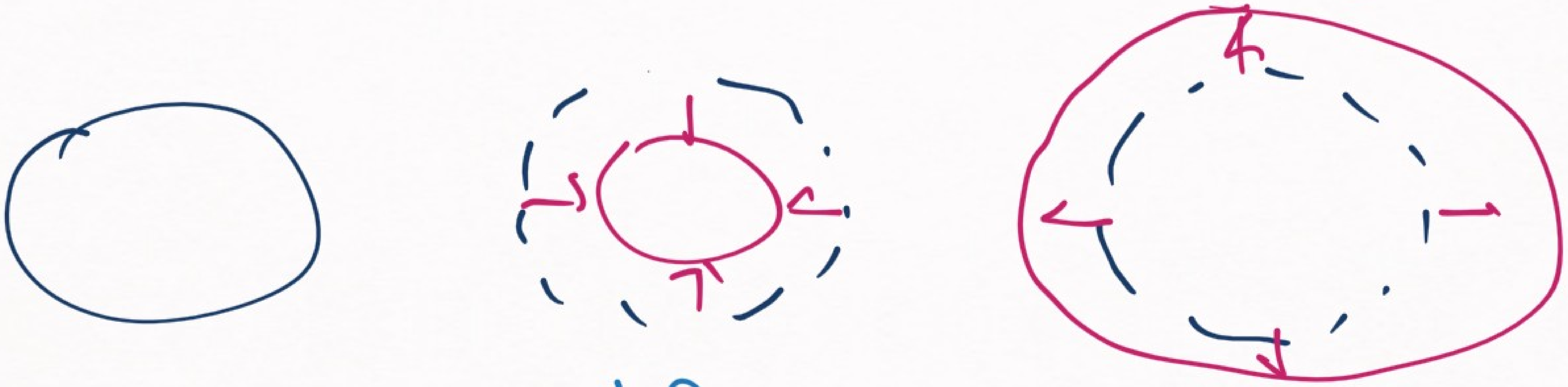
EXCITED STATE:

[QUANTIZED VIBRATION]

(→ maybe seen is statistical mechanics)

[POSSIBLE TYPES OF VIBRATION]

0)

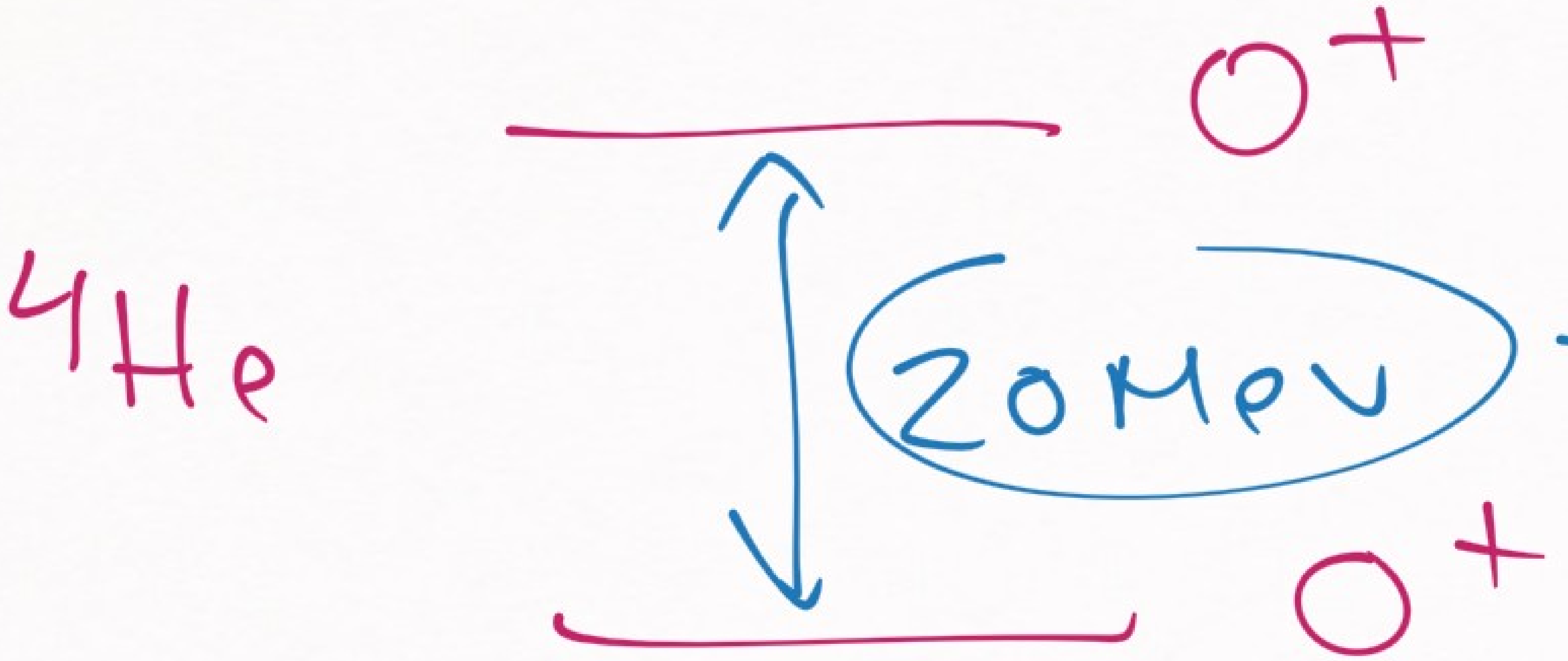


$\boxed{L=0}$ → "breathing mode"

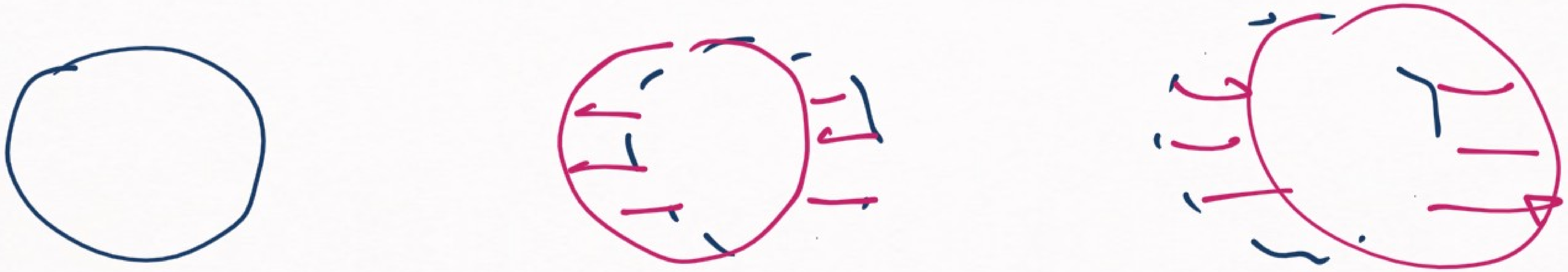
→ high-energy modes (liquids are not particularly compressible)

BREATHING MODES → HIGH-ENERGY

$(L=0) \rightarrow 0^+ \quad (L^{(-1)^L})$



1) DIPOLAR (L=1)

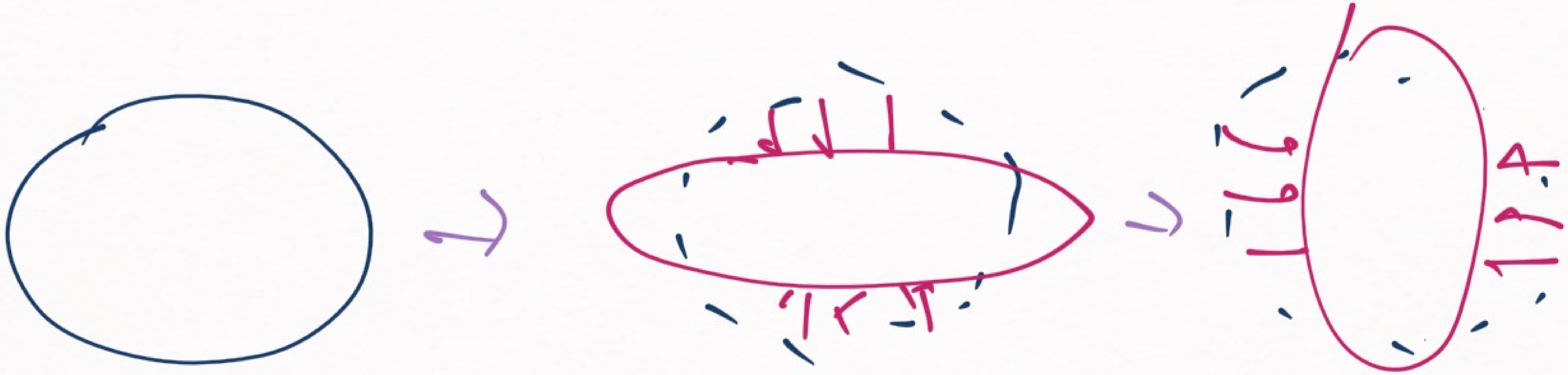


SOMETHING FISHY → THIS IS ACTUALLY A TRANSLATION

NOT A REAL, INTERNAL VIBRATION

→ IGNORE IT

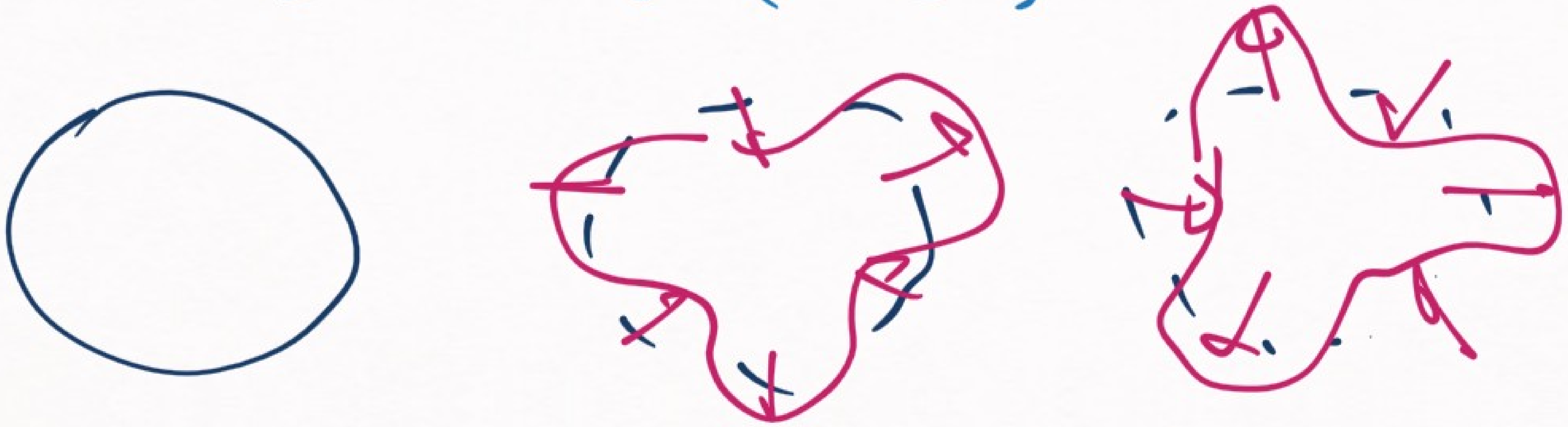
2) QUADRUPOLAR ($l = 2$)



→ FIRST NON-TRIVIAL VIBRATION
(w/ low energy)

→ MOST IMPORTANT TYPE OF VIBRATION

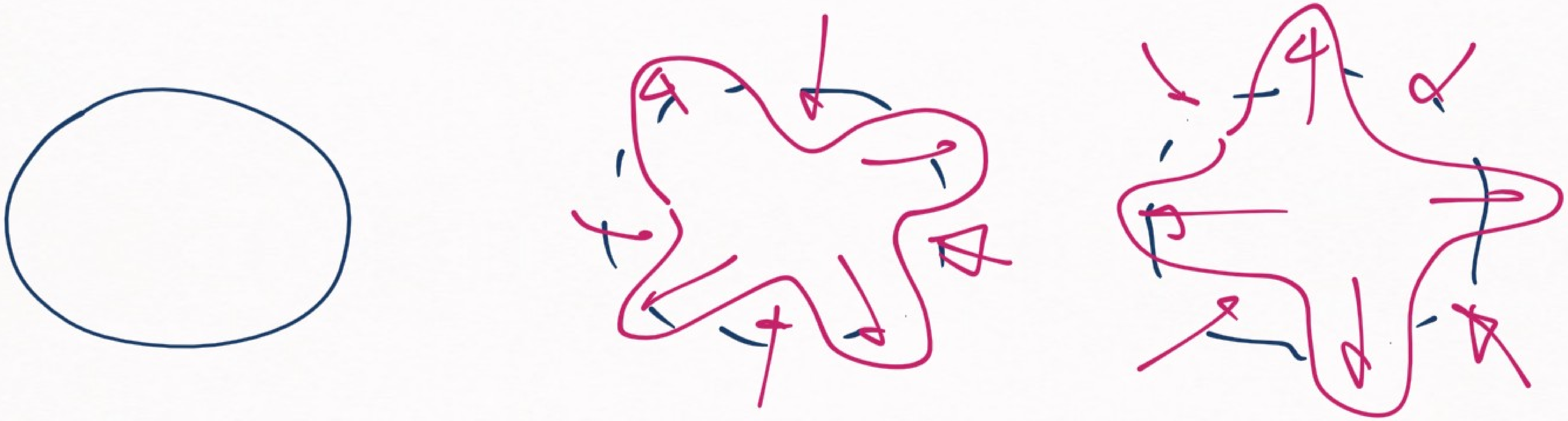
3) OCTUPOLAR ($L=3$)



$\rightarrow \omega_3 \simeq (2-3)\,\omega_2$ → QUADRUPOLAR VIBRATION ENERGY

OCTUPOLAR VIBRATION ENERGY

4) HEXADECUPOLAR (l=4)



→ even higher energy

→ not particularly important

RECAP → Basic idea: vibrations around a spherical shape

$$R(\theta,\varphi) = \underline{\underline{R_0}}\left[1 + \sum_{\lambda\mu} \alpha^*_{\lambda\mu} Y_{\lambda\mu}(\theta,\varphi)\right]$$

↓

radius of fundamental state

$\lambda = 0$ → compression (high energy → ^{4}He)

ignore

$\lambda = \cancel{1}$ → translation (not a deformation)

$\lambda = 2$ → quadrupolar (most common one)

$\lambda = 3$ → octupolar

$\lambda = 4, 5, 6 \ldots$ → all is possible here

QM DESCRIPTION

→ Second quantization

(this operator language we often use when quantizing the harmonic oscillator)

→ the quantized vibrational excitations are called "phonons"

$$|\text{vibrational excitation}\rangle$$

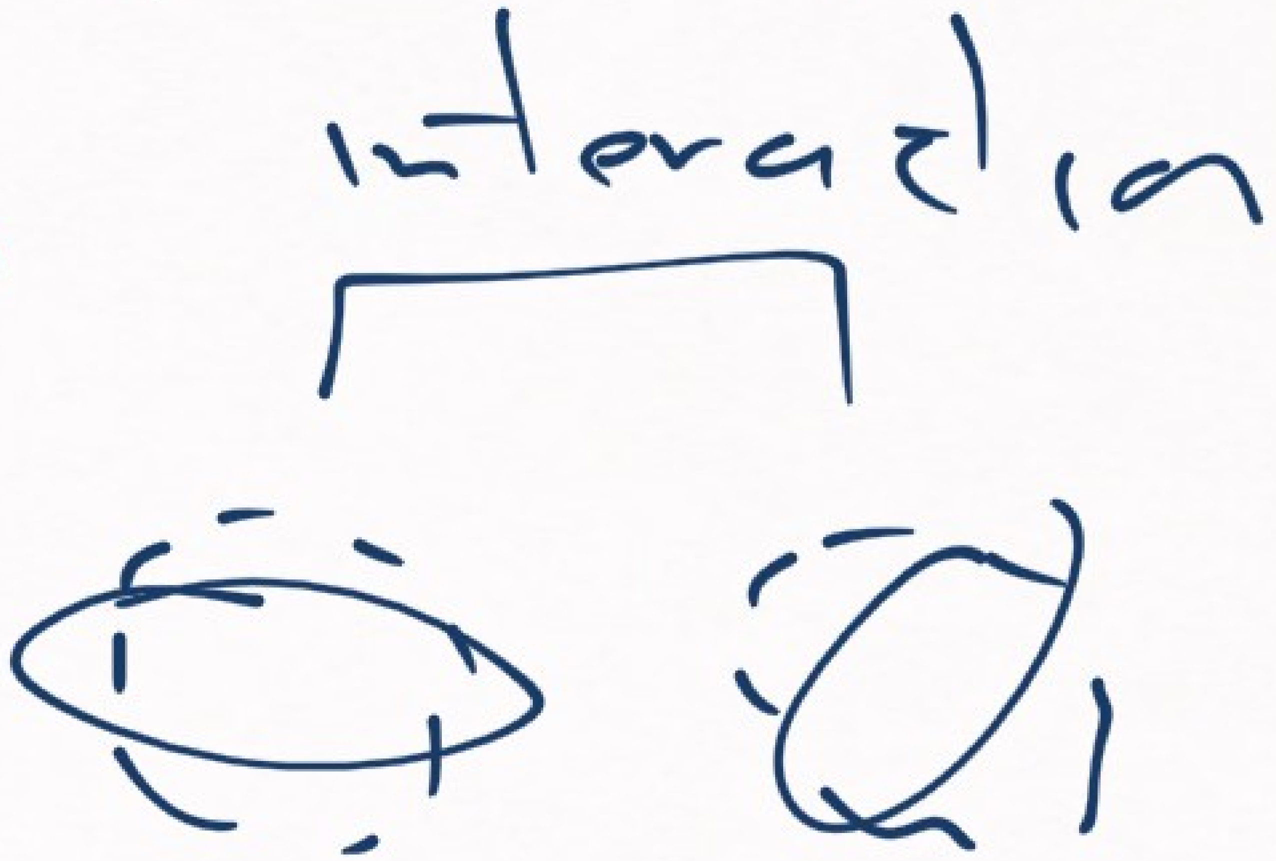
$$= |\underbrace{n_0}_{\text{\# of breathing excitations}}, \underbrace{n_2}_{\text{\# of quadrupolar excitations}}, \underbrace{n_3}_{\text{\# of octupolar excitations}}, n_4, \dots\rangle$$

$$H\,|n_0, n_1, n_3, \dots\rangle =$$

$$\hbar\,(n_0\omega_0 + n_1\omega_1 + n_3\omega_3 + \dots)\,|n_0, n_1, n_3, \dots\rangle$$

small

$$\Big[\,+\,(\text{interactions})\Big] \rightarrow$$



phonons can interact w/ each other.

FIRST APPROXIMATION :

$$H|n_0, n_2, n_3, \ldots\rangle = \hbar(n_0\omega_0 + n_2\omega_2 + n_3\omega_3 + \ldots)|n_0, n_2, n_3, \ldots\rangle$$

→ count the number of each type of excitation and multiply by their energy

$\boxed{J^P}$ → monopole : 0^+

quadrupole : 2^+

octupole : 3^-

$\Rightarrow$ more than one phonon

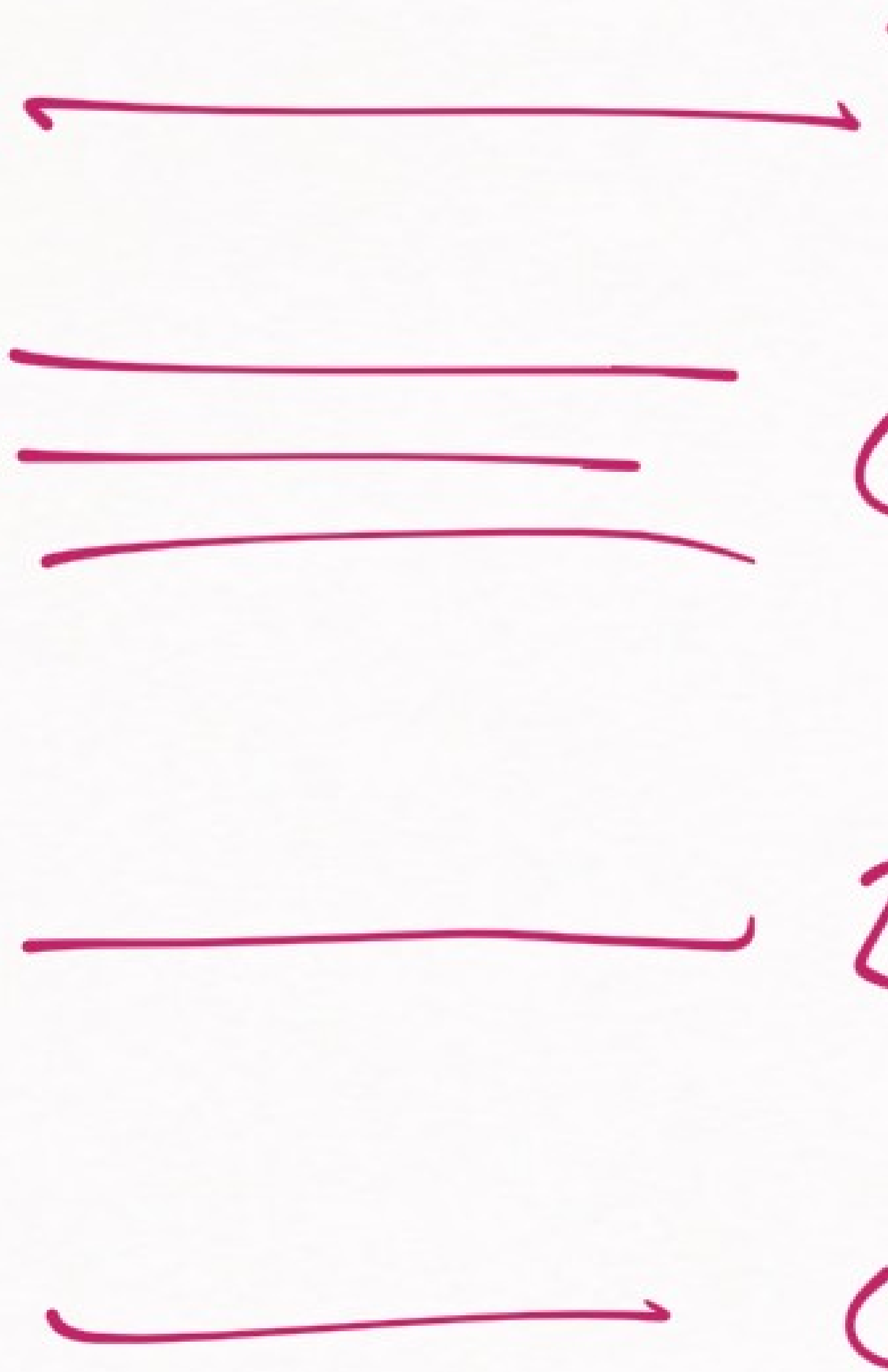
→ just couple their angular momentum

2. quadrupolar phonons:

$$2^+ \otimes 2^+ = \underbrace{0^+ \oplus 2^+ \oplus 4^+}$$

no 1^+ or 3^+ → $\boxed{\text{Exercise}}$

TYPE OF SPECTRUM WE GET FROM VIBRATIONS:

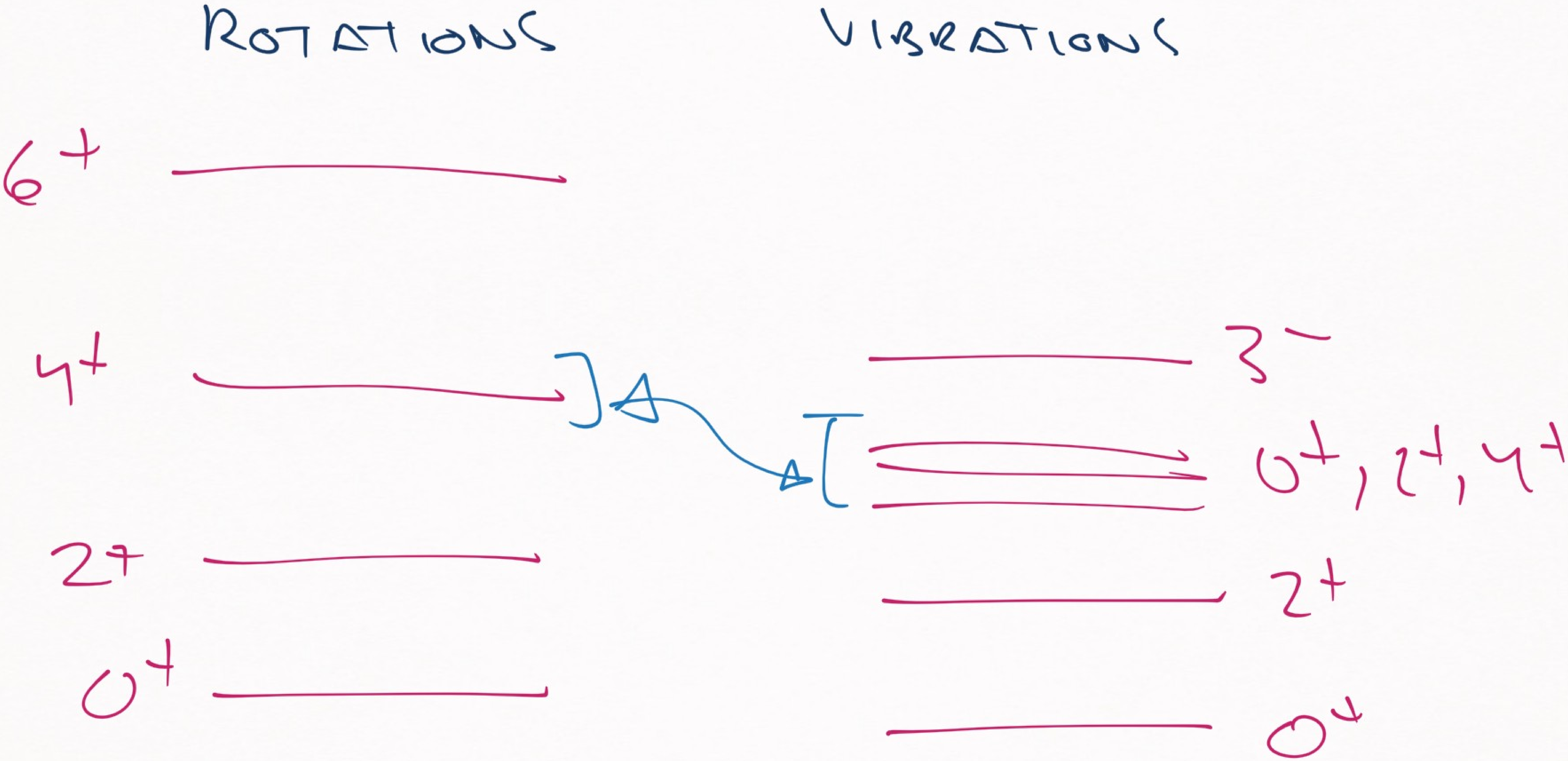


3^- (one $l=3$ phonon) ω_3 $(\sim 2\cdot 3\ \omega_2)$

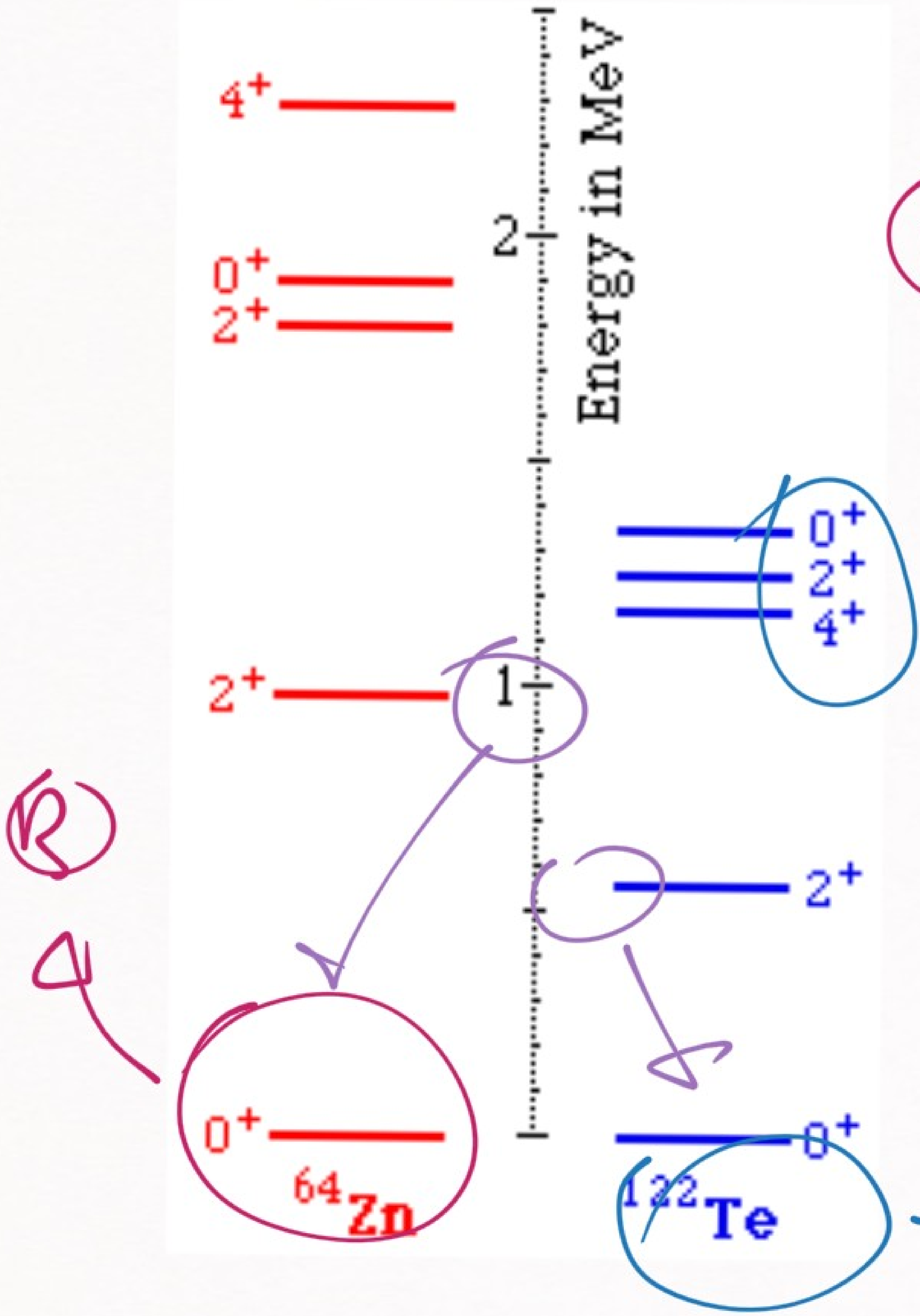
$0^+, 2^+, 4^+$ (two $l=2$ phonons) $2\omega_2$

2^+ (one $l=2$ phonon) ω_2

0^+ (zero phonons) 0


ROTATIONS
VIBRATIONS
6+
4+
2+
0+
Δ
Δ
3−
0+, 2+, 4+
2+
0+

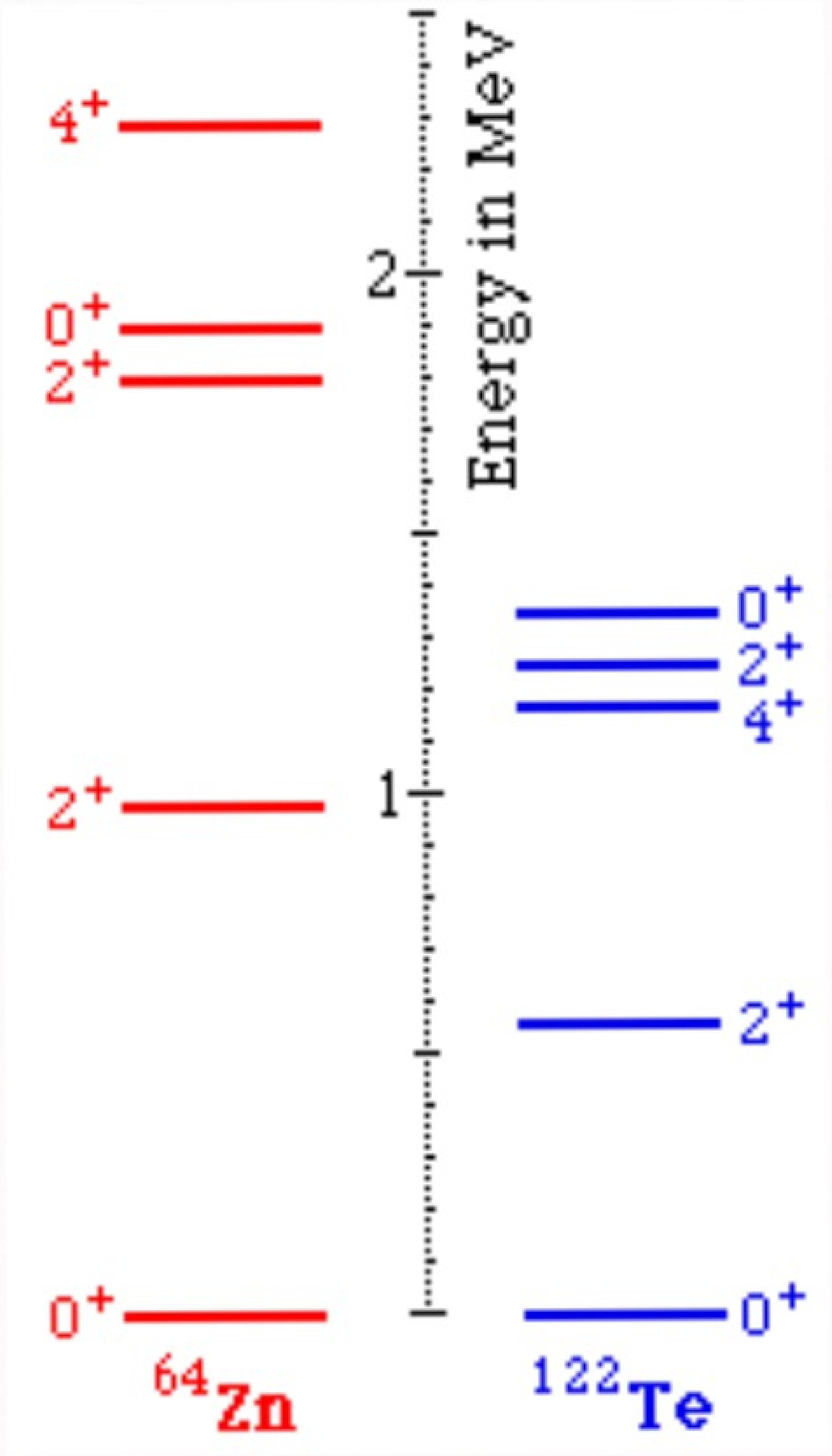
EXAMPLE



(A) → typical vibrational spectrum

(B) → transition between shell-model & vibrational spectrum

(A) & (B) → Different energy scales



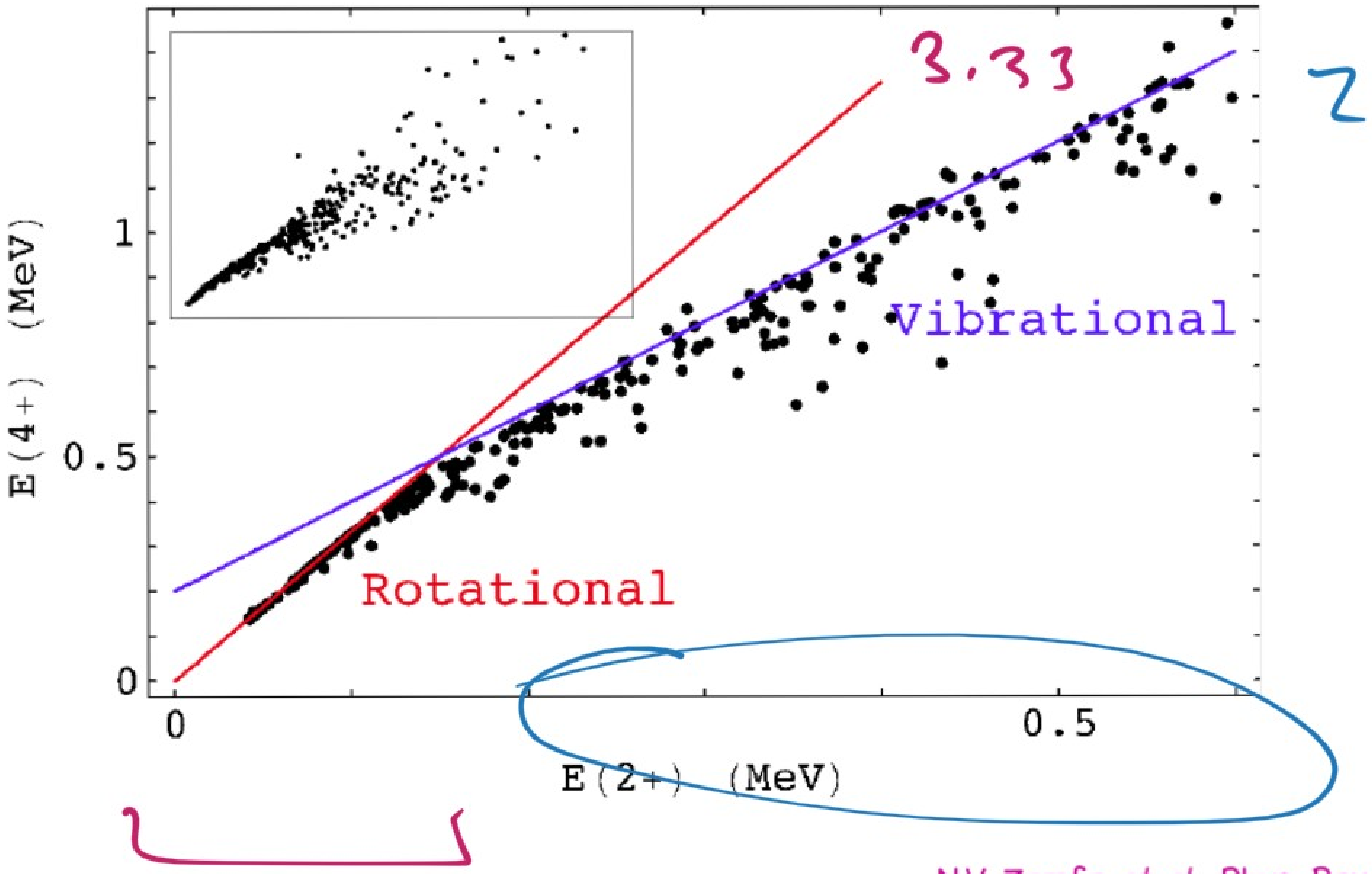
Ⓐ → classical vibrational spectrum

$E(2^+) \sim 0{,}5$ MeV

Ⓑ → in-between SM & vibrations

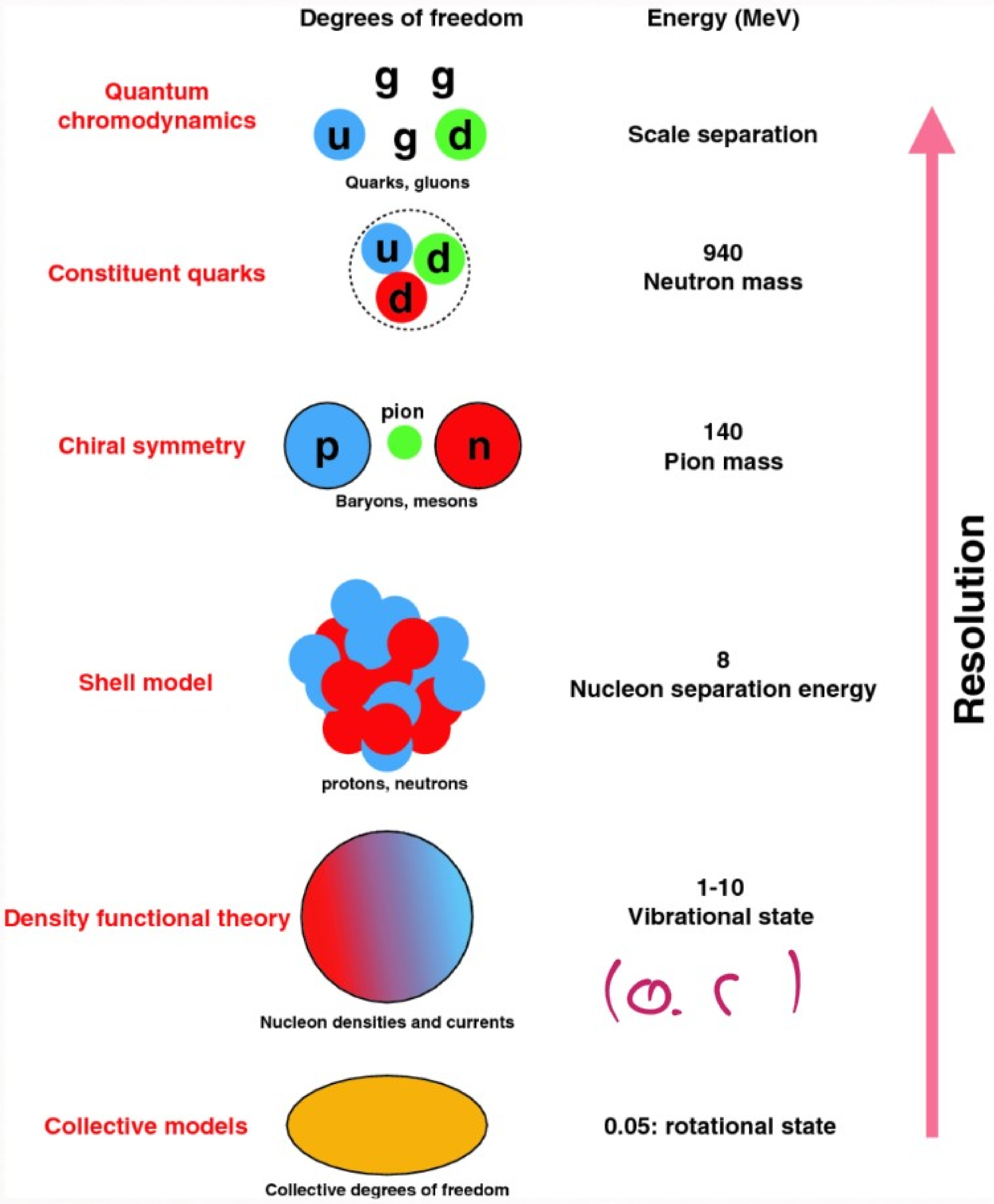
$E(2^+) \sim 1$ MeV

$$\frac{E(4^+)}{E(2^+)} \sim 2 \quad \checkmark$$



N.V. Zamfir *et al.*, Phys. Rev. Lett. **72** (1994) 3480

$E(2^+)$ very small



	Degrees of freedom	Energy (MeV)
Quantum chromodynamics	Quarks, gluons	Scale separation
Constituent quarks		940 Neutron mass
Chiral symmetry	Baryons, mesons	140 Pion mass
Shell model	protons, neutrons	8 Nucleon separation energy
Density functional theory	Nucleon densities and currents	1-10 Vibrational state (0. ( )
Collective models	Collective degrees of freedom	0.05: rotational state

Resolution

→ **RESIDUAL INTERACTIONS IN THE SHELL MODEL**

(beyond pairing interaction)

SHELL-MODEL → STRONG POINTS:

1) magic numbers ↳ $N, Z = 2, 8, 20, 28, 50 \ldots$

2) separation energies right

Basic assumptions :

1) MEAN FIELD POTENTIAL

$$H = \sum_i T_i + \sum_{ij} V^{2B}_{ij} + \sum_{ijk} V^{3B}_{ijk} + \dots$$

$$\longrightarrow \quad H = \sum_i (T_i + V_i) + \Delta V$$

2) NUCLEONS ARE FERMIONS

3) WE FILL SHELLS

SHELL MODEL →

1) JP near closed shell nuclei ($^{16}O \to {}^{15}O/{}^{17}O$)

2) Excited spectra near a closed shell (^{41}Ca, ^{38}Ar)

3) Pairing interaction (example of ΔV) (^{203}Tl, ^{205}Tl, ^{209}Pb)

→ This are just the basic features

[TWO BIG PROBLEMS LEFT]

1) HOW TO DEFINE V^{MF} ?

2) HOW TO DEAL w/ ΔV ?

→ before we just propose a simplified answer to this ↳

1) $V^{MF} = \frac{1}{2} m\omega r^2 - \xi \vec{\ell}\cdot\vec{s} - k\vec{\ell}^2$

→ REALLY SIMPLE & EFFECTIVE

Advanced options → Hartree-Fock, Skyrme, Gogny, etc

2) Pairing (SIMPLEST ONE)

→ Advance methods for AV

[NEXT TWO LESSONS]

1) Residual interactions

2) Mean field

→ WE CONTINUE ON MONDAY