NUCLEAR PHYSICS ⑫

3

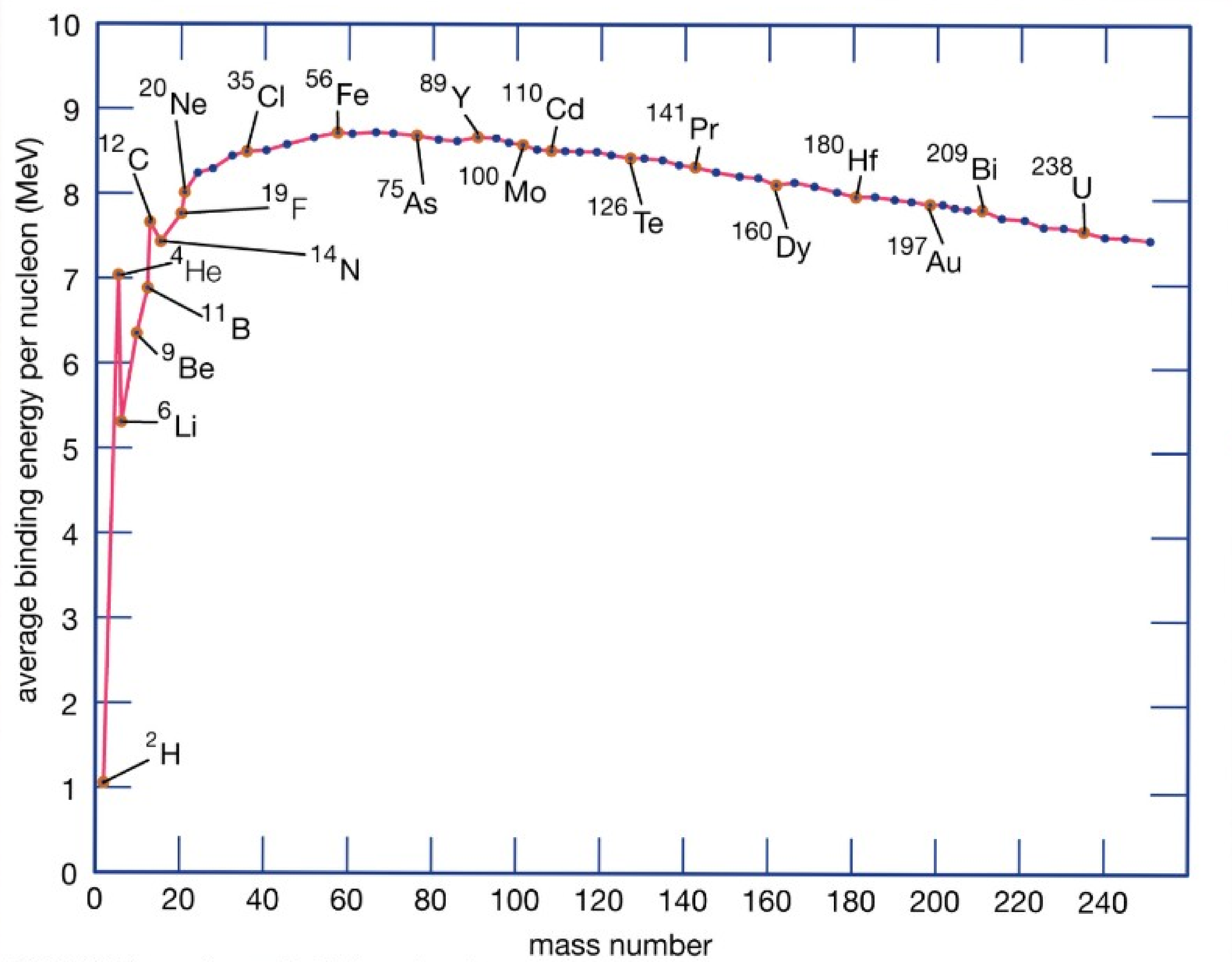
Nuclear properties (cont'd)

Liquid drop model

RECAP → We want to describe nuclei

1) Binding energy
2) Size of the nuclei
3) J^P (angular momentum & parity)
4) Electromagnetic moments
5) Stability & Decays

1) Binding energies → $\frac{B}{A} \sim 8$ MeV/nucleon



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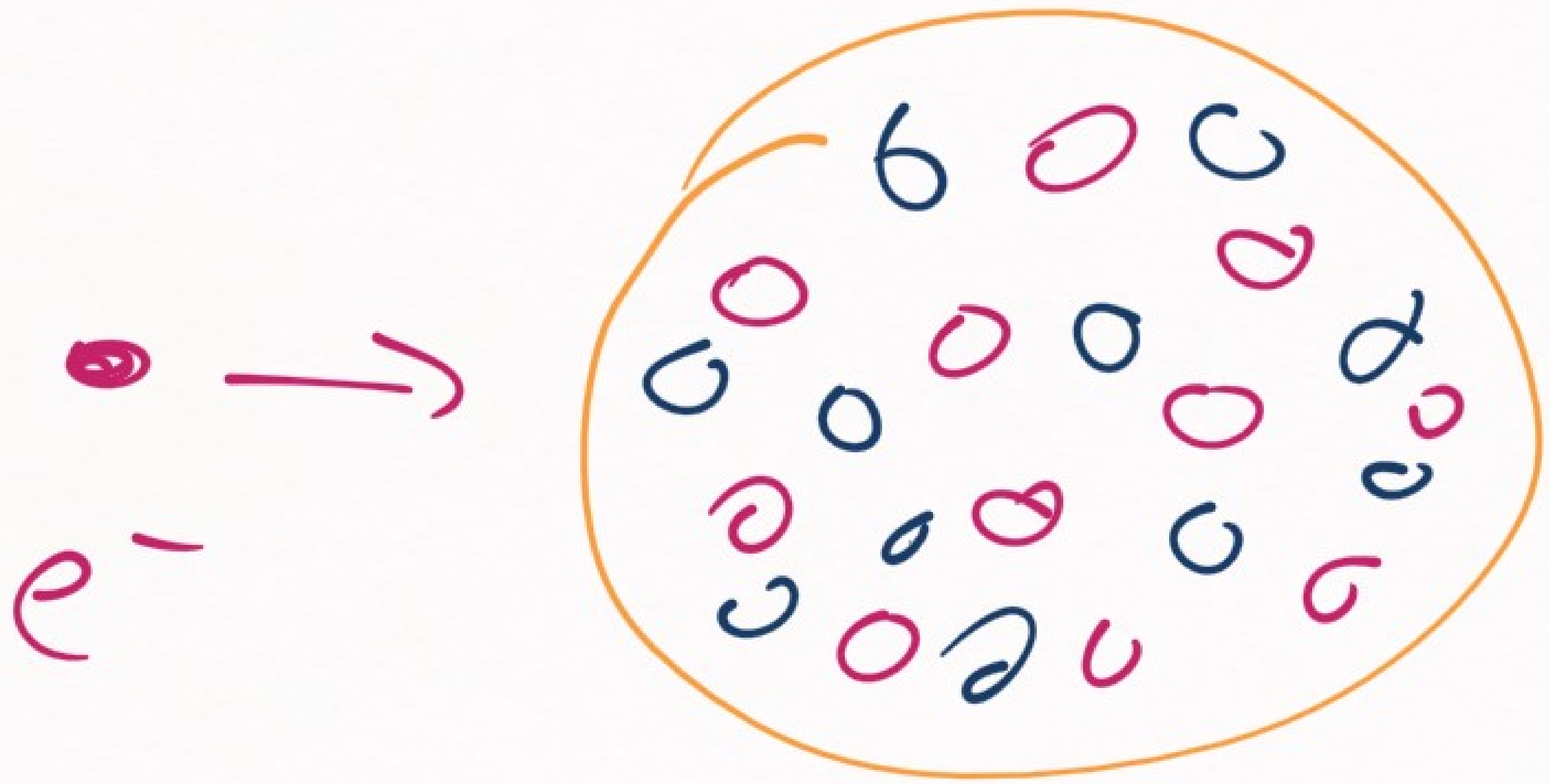
→ Liquid drop model

Σ (we will see)

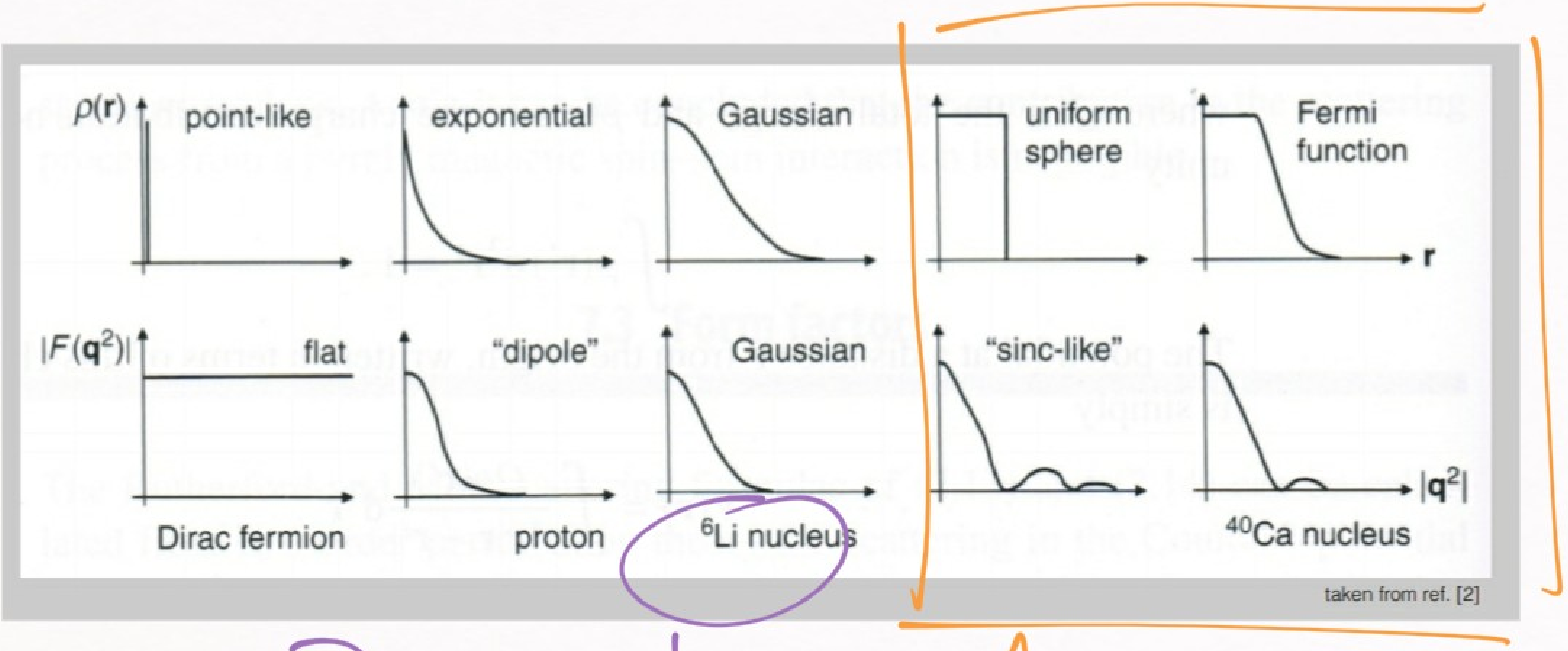
2) Nuclear size & density → Hofstadter experiment



$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{point-like charge}} |F(q^2)|^2$$

$$F(q^2) = \int d^3\vec{r}\, \underbrace{\rho(\vec{r})}_{\sim |\psi(\vec{r})|^2}\, e^{-i\vec{q}\cdot\vec{r}}$$

← Form-factor



taken from ref. [2]

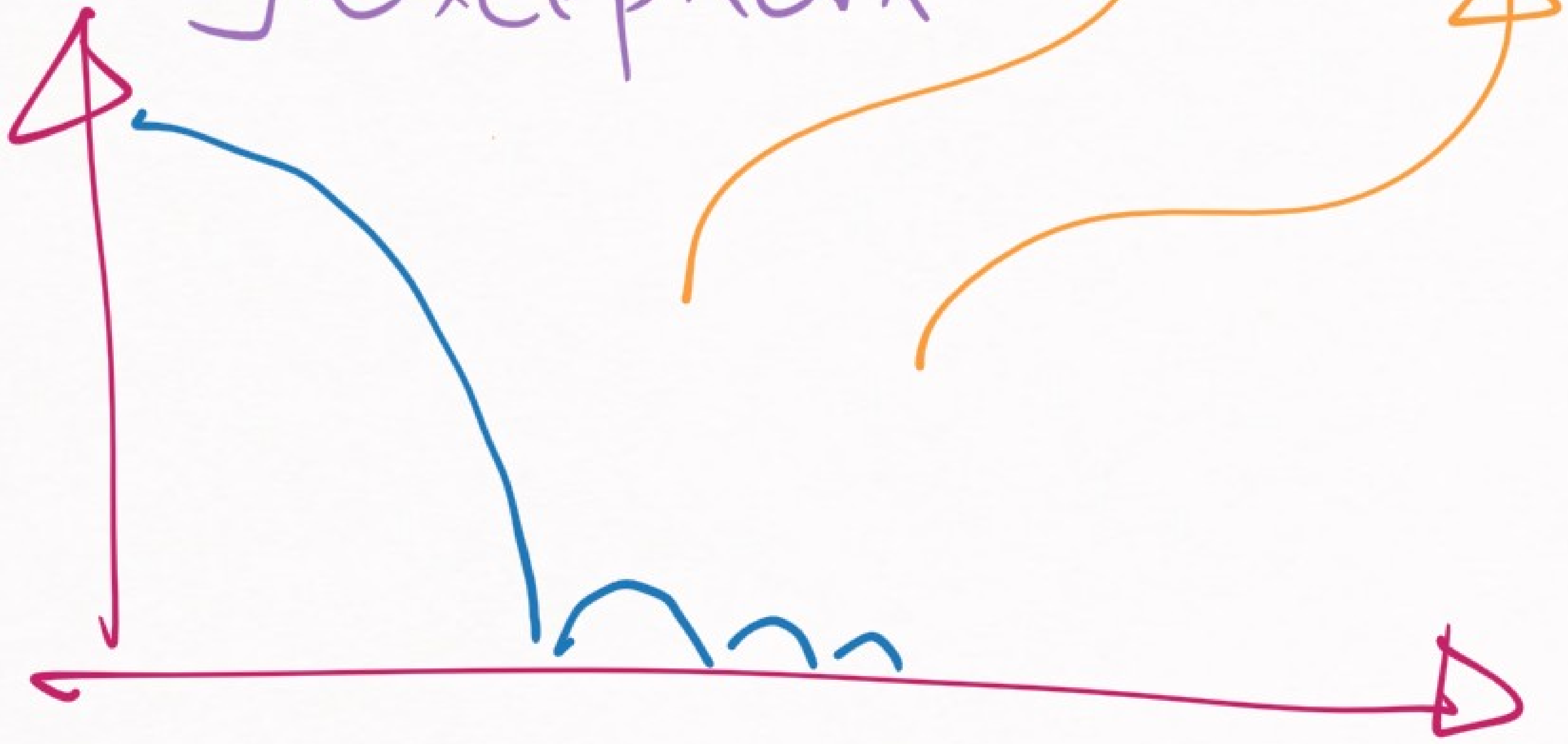
Density profiles

↓

Form factors

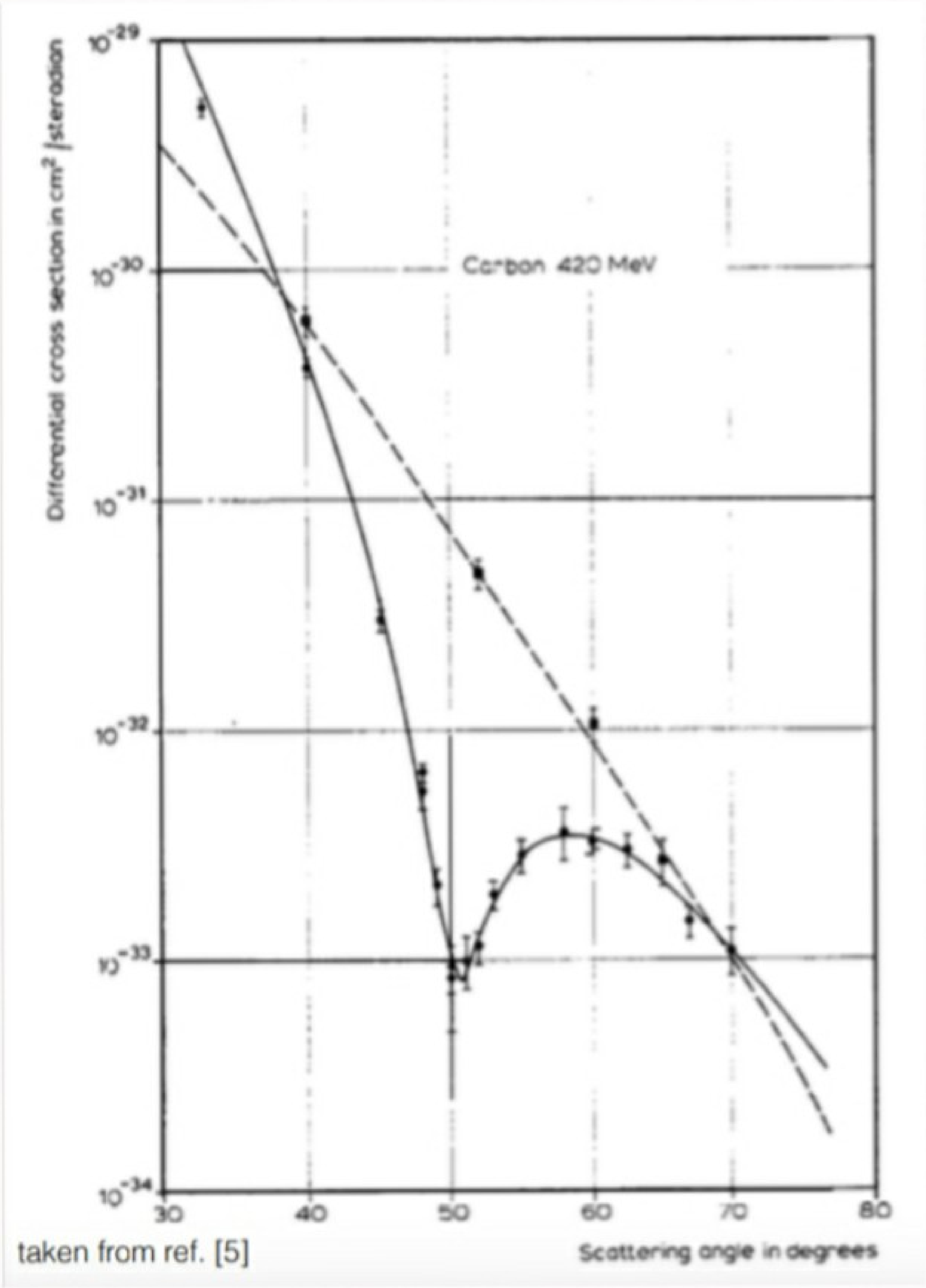
⇒

$\exists$ Exceptions



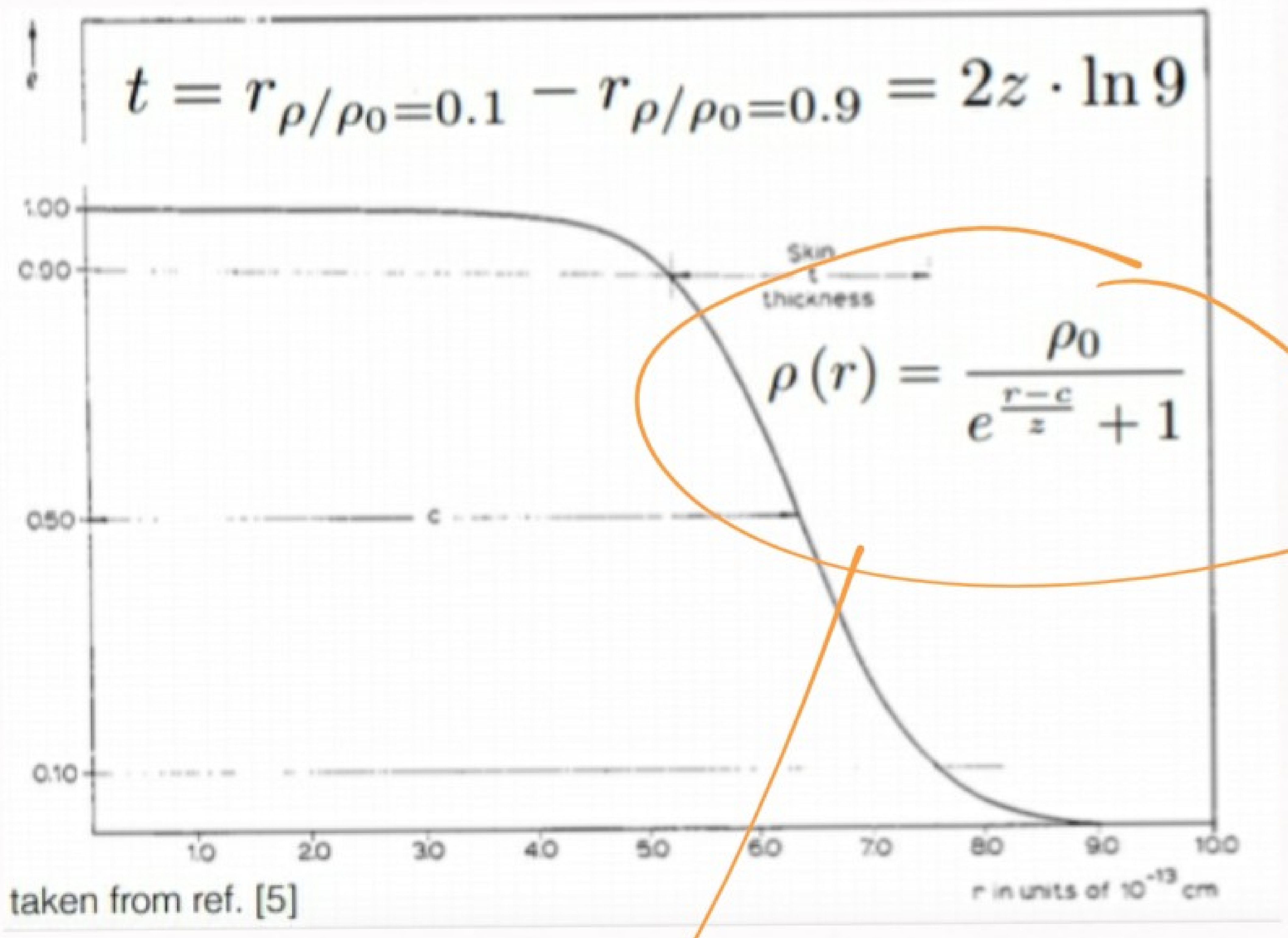
↘ Expected density of nuclei

⇒



taken from ref. [5]

⇒



$$t = r_{\rho/\rho_0=0.1} - r_{\rho/\rho_0=0.9} = 2z \cdot \ln 9$$

$$\rho(r) = \frac{\rho_0}{e^{\frac{r-c}{z}} + 1}$$

taken from ref. [5]

Woods-Saxon distribution

→ Typical mass-distribution : Woods-Saxon
(charge-)

$$\rho(r) = \frac{\rho_0}{1 + e^{(r-R_0)/a_0}}$$

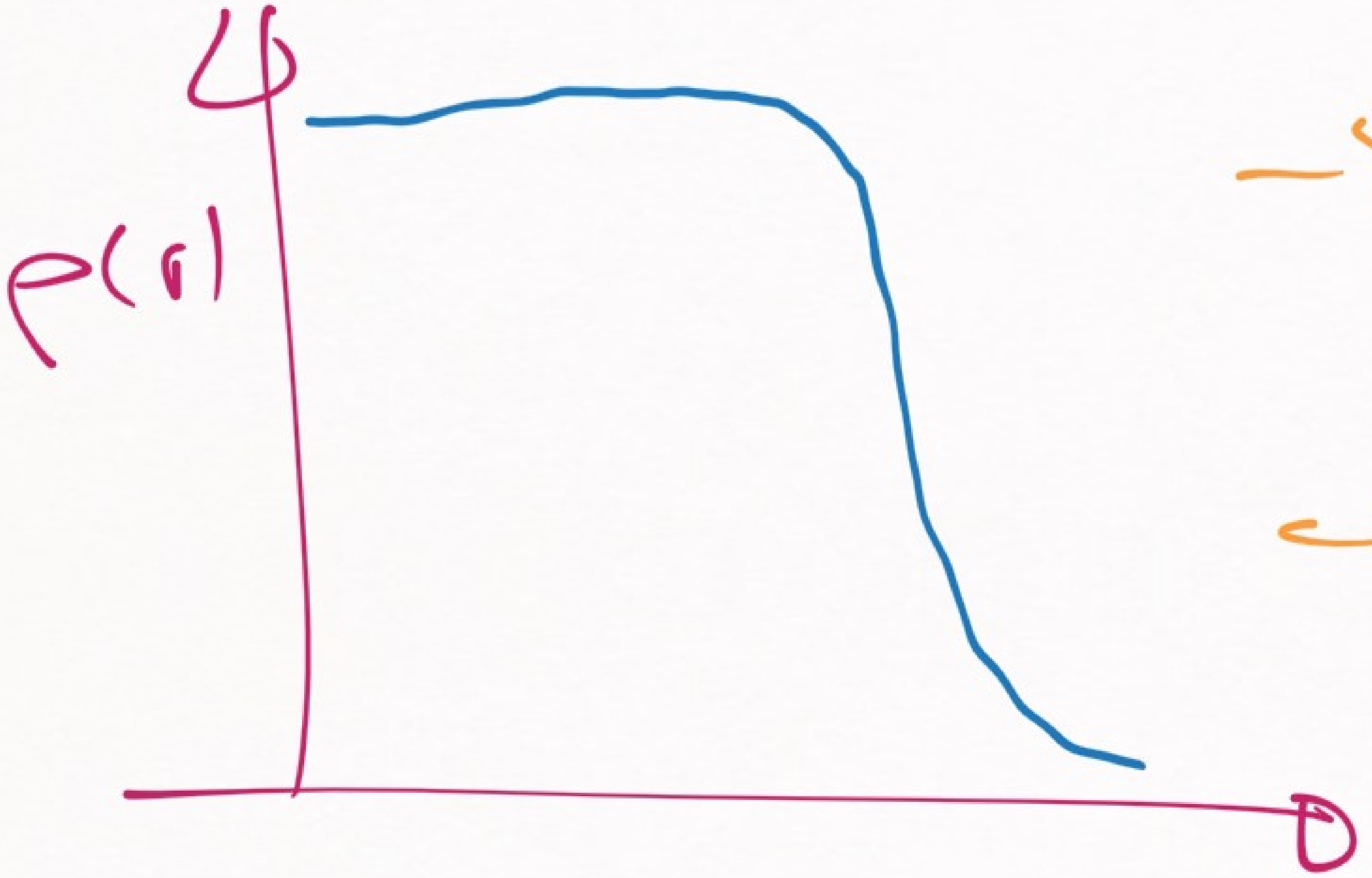
$\rho_0 \simeq 0.17\ \mathrm{fm}^{-3}$

$a_0 \simeq 0.54\ \mathrm{fm}$

$R_0 \simeq \underrightarrow{1.128\, A^{1/3}}\ \mathrm{fm}$

$(1.1 - 1.2)$

Z



→ Saturation ← Binding energy

→ Repulsive core
+
middle-range attraction

3) Angular momentum & Parity (J^P)

$J^P(\pi) = 0^-$ $\quad$ $J^P(\rho) = 1^-$ $\quad$ $J^P(\sigma) = 0^+$

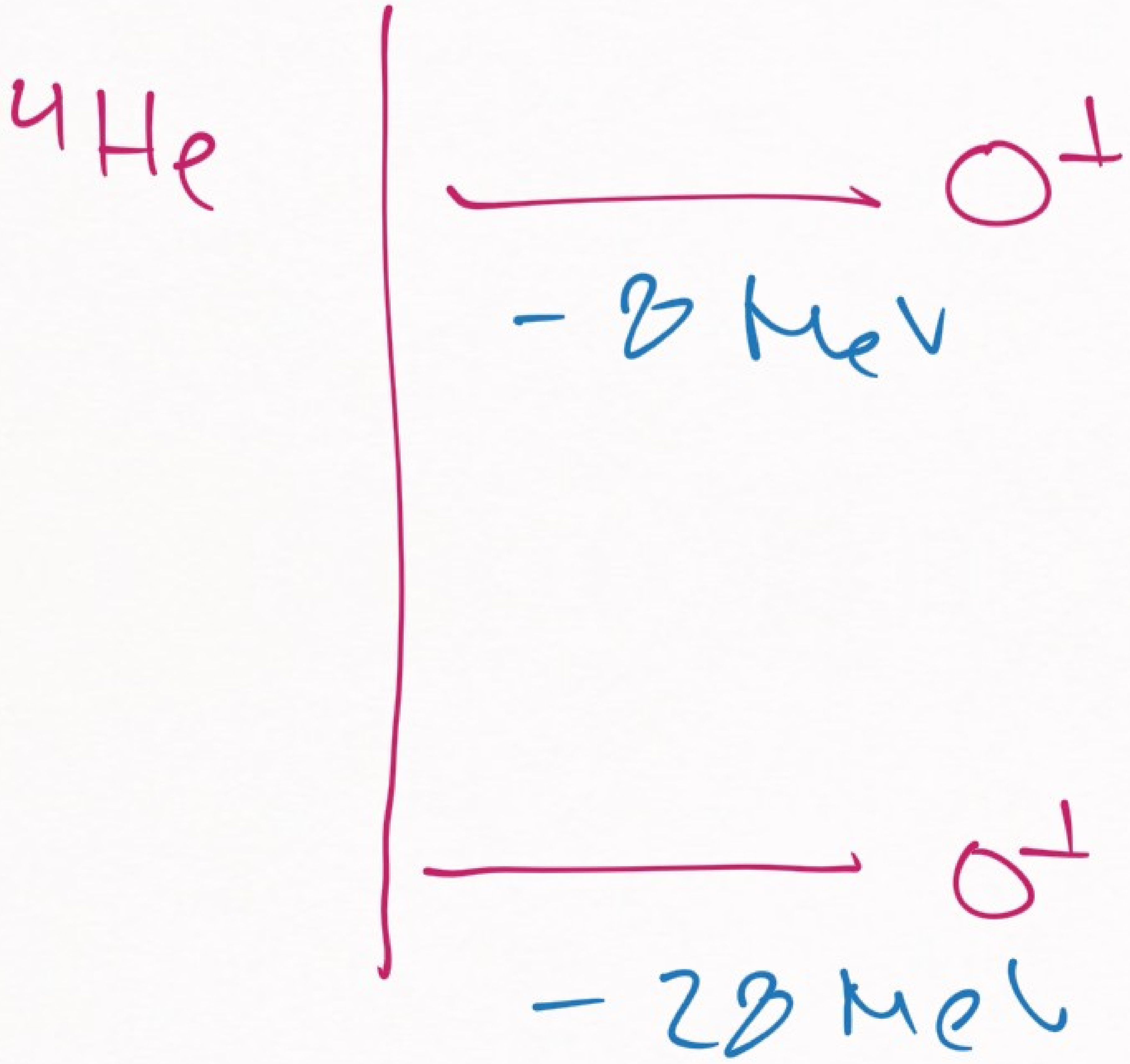
$$\left[\begin{array}{l} J^P(N) = 1/2^+ \ , \ N = n, p \\ J^P(d) = 1^+ \ , \ d = np \ (\text{deuteron}) \\ J^P(^3H/^3He) = 1/2^+ \\ J^P(^4He) = 0^+ \end{array}\right]$$

Excited states → nuclei have excited states

their own J^P

(which we can try to explain)

[nuclear structure]



(first excited state)

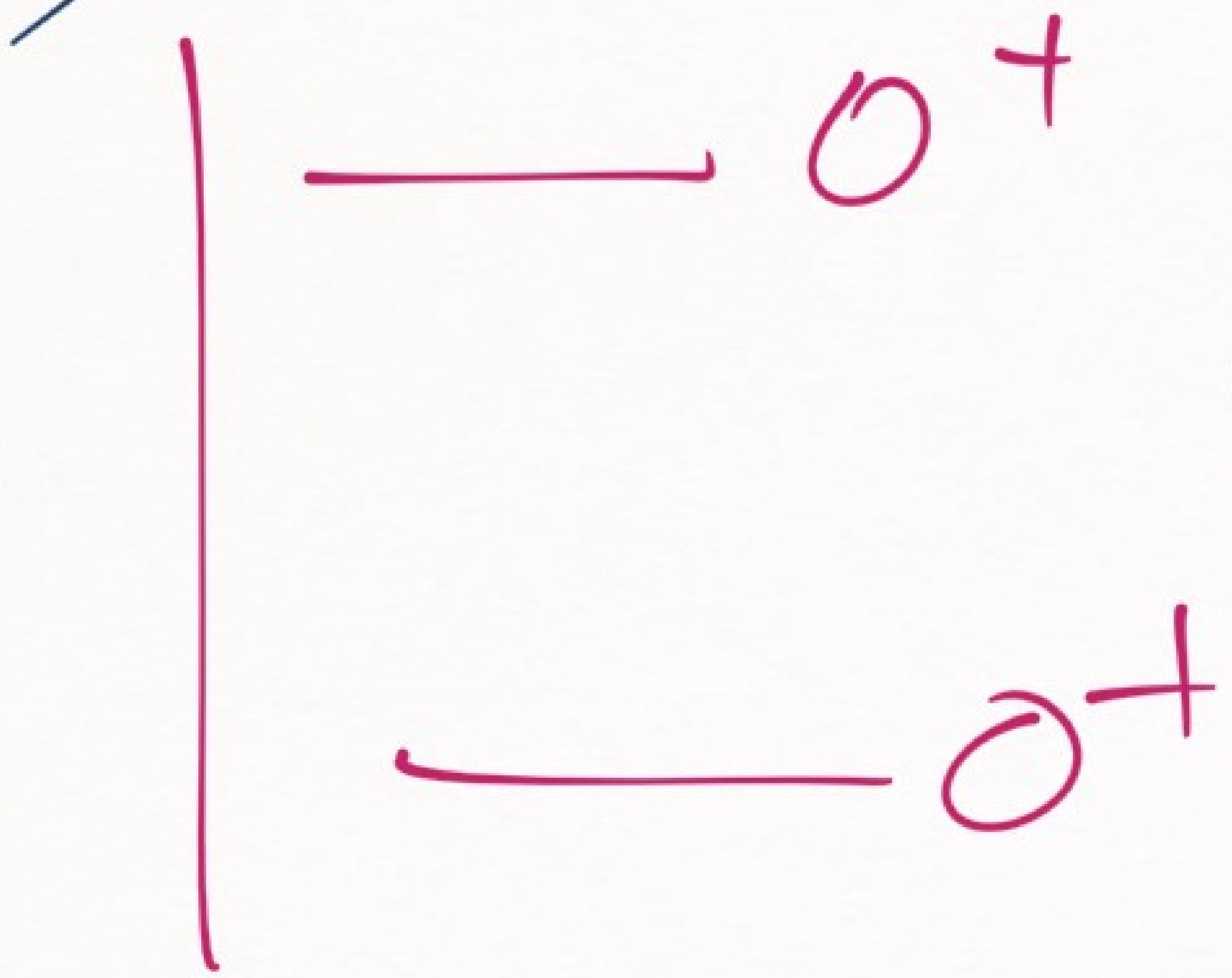
↑ "Breathing mode" of the collective mode

(ground state)

We will check this later on

Different nuclei → different excited state structure → different nuclear structure

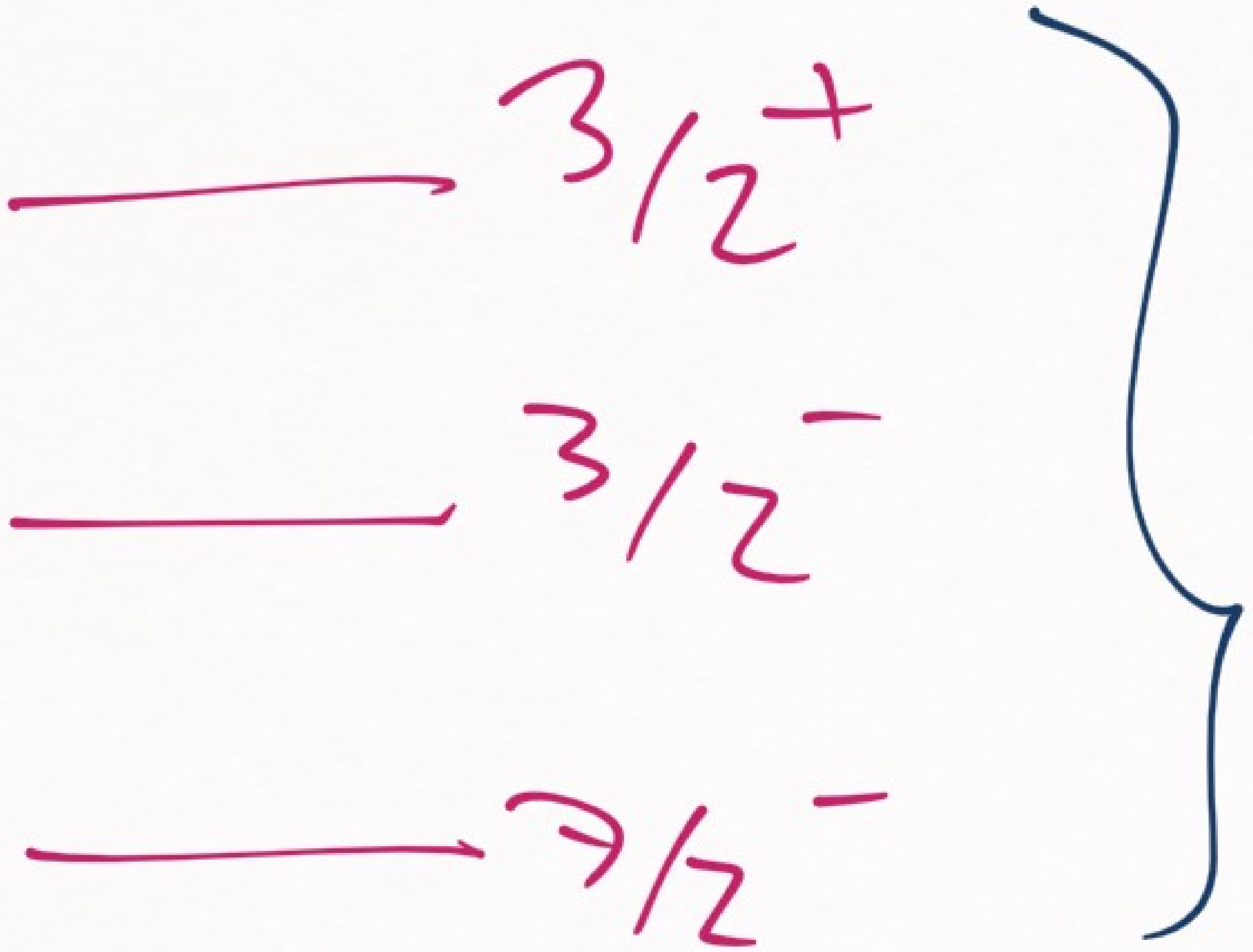
1)



^4He → Breathing mode

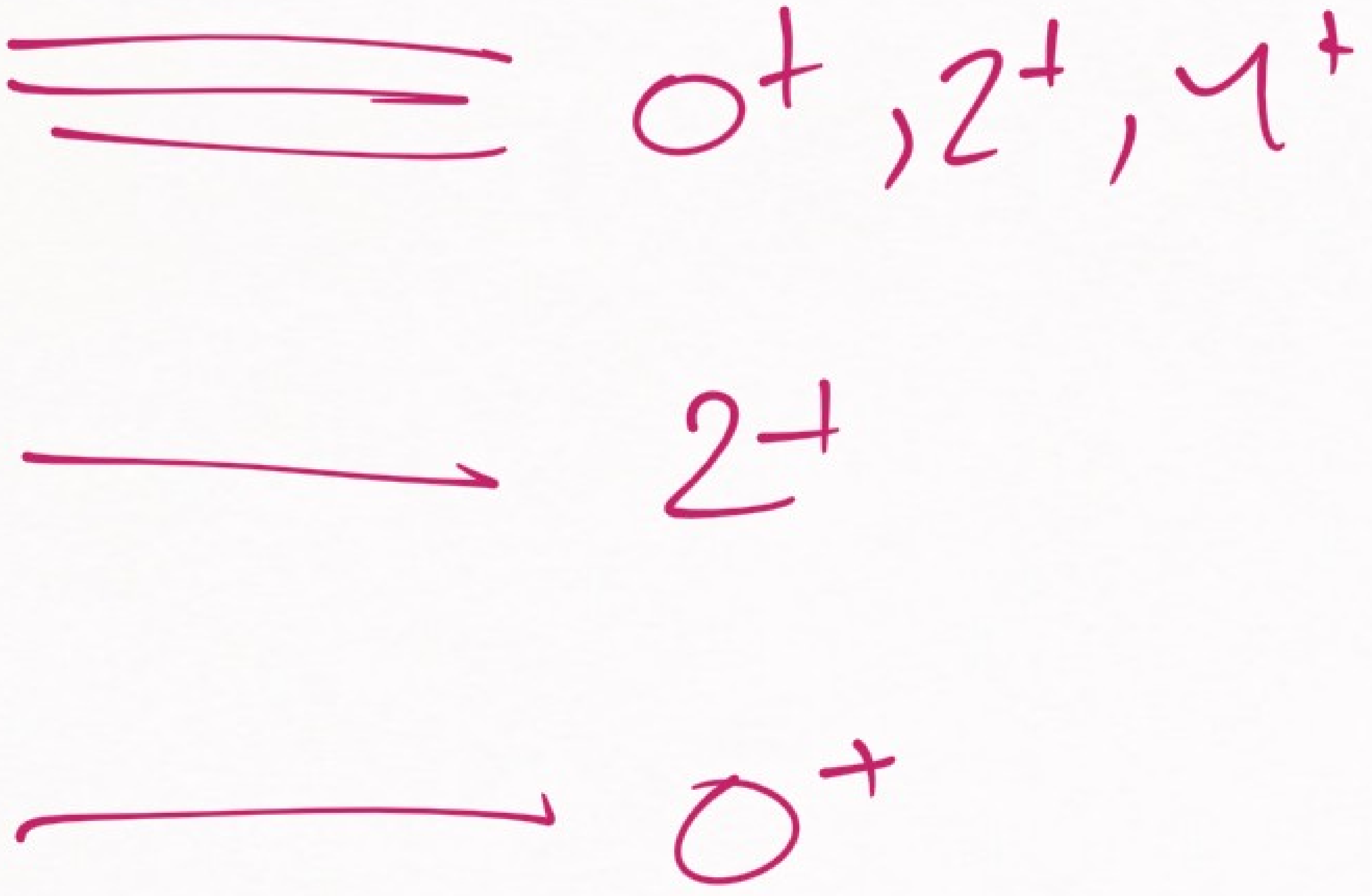
2) $^{41}\mathrm{Ca}$

We will also study this later



Typical shell-model spectrum

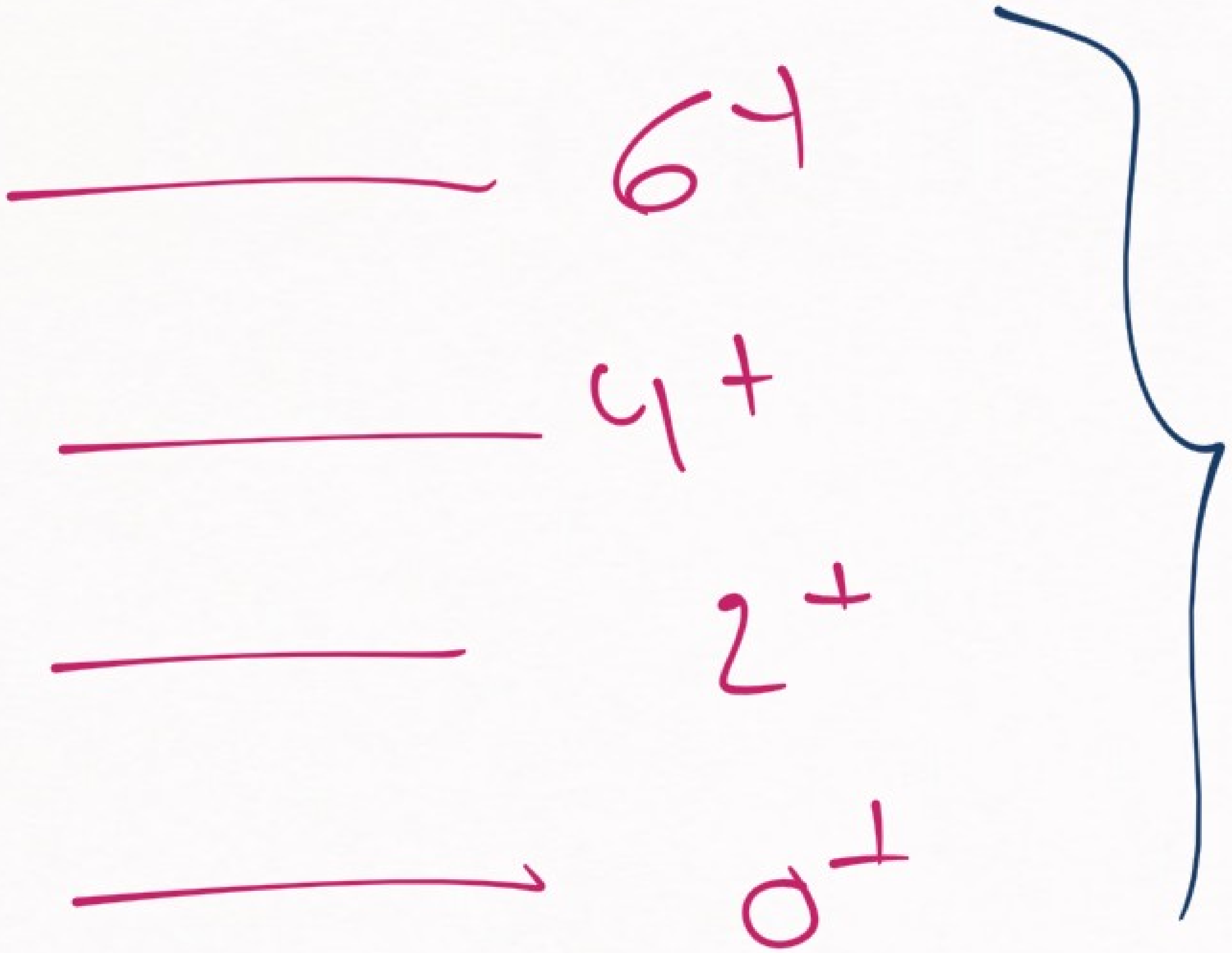
3) $^{122}\mathrm{Te}$



typical vibrational spectrum

another type of structure

4) ^{166}Er



typical rotational spectrum

1), 2), 3), 4) → different J^P spectra (indicating different nuclear structures)

4) Electromagnetic properties of nuclei

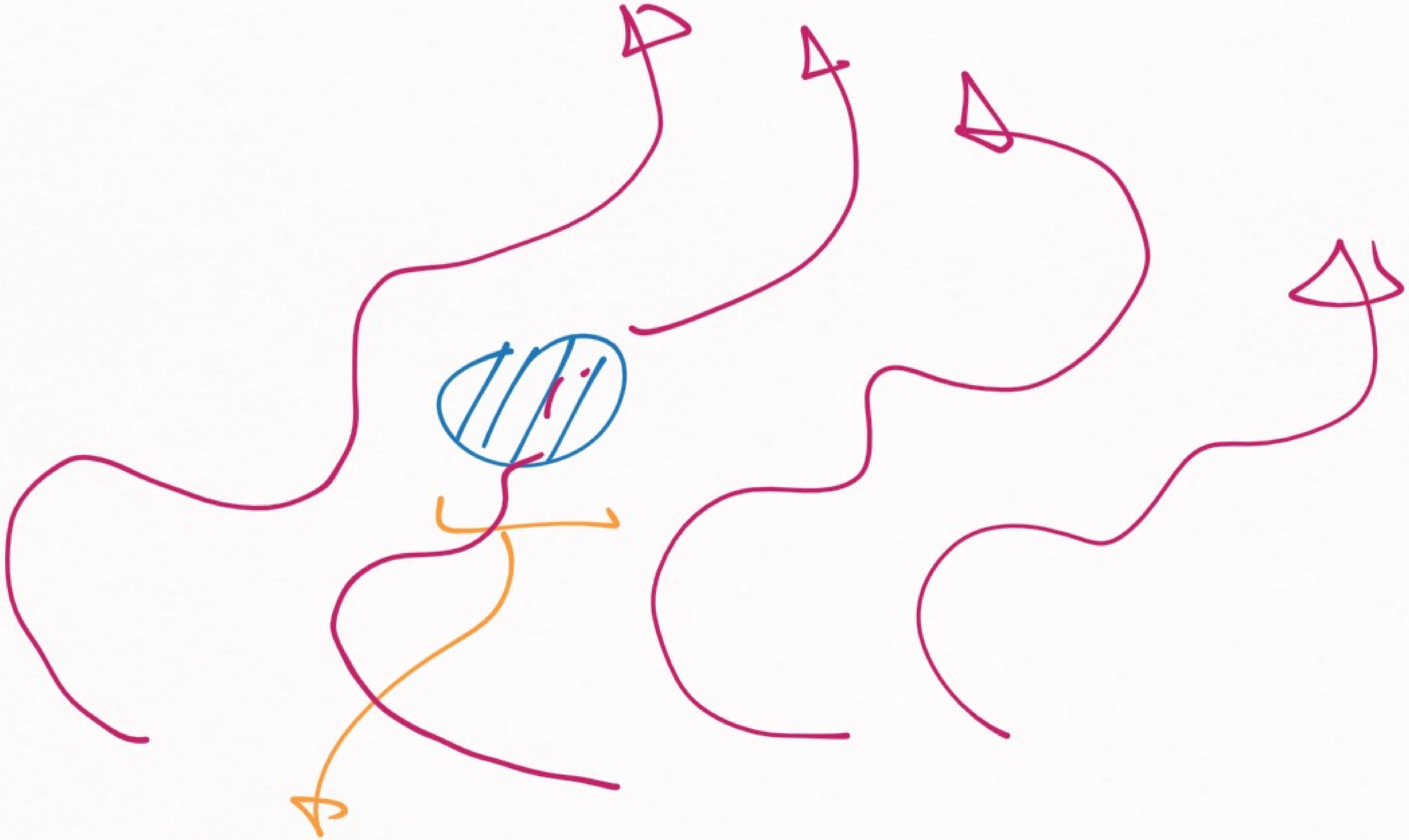
(Magnetic dipole moment &
Electric quadrupole moment)

↓

What are these things?

"MOMENTS"

$\vec{E}$ $(\vec{B})$



DESCRIBE
THE POTENTIAL
ENERGY
OF $\rho(\vec{r})$
IN THIS
FIELD

Charge distribution $(\rho(\vec{r}))$

ELECTRIC POTENTIAL:

$$V = \underbrace{\int d^3\vec{r}\, \underset{\circledast}{\rho(\vec{r})}\, \Phi(\vec{r})}_{\downarrow} \rightarrow \text{scalar potential}$$

$$\vec{E} = +\vec{\nabla}\Phi$$

TAYLOR-EXPAND THIS QUANTITY

$\circledast$ → We don't know the exact $\rho(\vec{r})$

$$\Phi(\vec{r}) = \Phi(0) + r_i \partial_i \Phi\big|_{\vec{r}=0} + \frac{1}{2} r_i r_j \partial_i \partial_j \Phi\big|_{\vec{r}=0}$$

$+ \ldots$ (Taylor expansion of $\Phi(\vec{r})$ around $\bar{r}=0$)

this assumes $\rho(\vec{r}')$ centered around $\vec{r}'=0$

$$V = \Phi\big|_{\vec r=0} \int d^3\vec r \, \underline{\underline{\rho(\vec r)}} + \partial_i \Phi\big|_{''} \int d^3\vec r \, \underline{\underline{r_i \rho(\vec r)}}$$

$$+ \frac{1}{2} \partial_i \partial_j \Phi\big|_{''} \int d^3\vec r \, \underline{\underline{r_i r_j}} \, \rho(\vec r) + \ldots$$

→ We rearrang this a bit (& define "moments")

$q = \int d^3\vec{r}\, \rho(\vec{r})$ → charge

$\vec{d} = \int d^3\vec{r}\, \rho(\vec{r})\, \vec{r}$ → electric dipole

$Q_{ij} = \int d^3\vec{r}\, (3 r_i r_j - \delta_{ij} r^2)\, \rho(\vec{r})$

↓

electric quadupole

$\ldots = \ldots$

$$V = q\,\Phi + \vec{d}\cdot\vec{\nabla}\Phi + \frac{1}{6} Q_{ij}\,\partial_i\partial_j\Phi + \dots$$

$$= q\cdot\Phi - \vec{d}\cdot\vec{E} - \frac{1}{6} Q_{ij}\,\partial_i E_j + \dots$$

→ Potential energy of a charge distribution within an electric field ↲

→ Magnetic field $\vec{\mu}(\vec{r})$ "magnetic moment distribution"

$$V = \underbrace{-}_{}\int d^3\vec{r}\;\vec{\mu}(\vec{r})\cdot\vec{B}(\vec{r})$$

↳ check the signs in the previous pages

$$\vec{B}(\vec{r}) = \vec{B}(0) + \vec{r}\cdot\vec{\nabla}B\big|_{\vec{r}=0} + \dots$$

$$V = -\vec{\mu}\cdot\underbrace{\vec{B}\big|_{\vec{r}=0}} - \frac{1}{6}\, Q_{ij}\,\underbrace{\partial_i B_j} + \dots$$

magnetic field (Taylor expansion)

$$\vec{\mu} = \int d^3\vec{r}\;\vec{\mu}(\vec{r}) \qquad Q_{ij} = \int d^3\vec{r}\,\Big[\frac{3}{2} r_i \mu_j(\vec{r})$$

$$+ \frac{3}{2} r_j \mu_i(\vec{r}) - \vec{r}\cdot\vec{\mu}(\vec{r})\,\delta_{ij}\Big]$$

→ We have defined moments

COMMENT → For ∀ nuclei:

$$\vec{d}_E = \vec{0} \quad , \quad Q_{MJ} = 0$$ → **WHY?**

electric dipole

magnetic quadrupole

$\oplus \rightarrow \quad \rho(\vec{r}) = \rho(-\vec{r}) \quad \rightarrow \quad 0^{+}$ (scalar)

$\vec{\mu}(\vec{r}) = \vec{\mu}(-\vec{r}) \quad \rightarrow \quad 1^{+}$ (pseudo-vector)

$\Downarrow$

$$\int d^3\vec{r}\, \rho(\vec{r})\,\vec{r} = \vec{0} \Leftarrow (\vec{d} = -\vec{d})$$

$$\left(= \int d^3\vec{r}\, \rho(-\vec{r})(-\vec{r}) = -\int d^3\vec{r}\, \rho(\vec{r})\,\vec{r} \right)$$

$\Rightarrow$ Same argument applies for $\int d^3\vec{r}\, r_i \mu_j(\vec{r})$

$$\boxed{\begin{aligned}\vec{d}_E &= \vec{0}\\ Q_{\mu ij} &= 0\end{aligned}}$$

$$\downarrow$$

($\neq 0$ for transitions)

WE MOSTLY NEED TO WORRY ABOUT:

1) MAGNETIC DIPOLE MOMENT

2) ELECTRIC QUADRUPOLE MOMENT

[3) CHARGE] → TRIVIAL

→ Higher orders: electric hexadecupole
(16-pole) moment

& magnetic octopole moment
(8-pole)

(But they are small & less important)

ELECTRIC CHARGE → eZ (TRIVIAL)

(does not give us much info
about ^ nuclear structure)
detailed

QUADRUPOLE MOMENT → DEUTERON
(ELECTRIC)

(D-WAVE,
TENSOR FORCE)

$$\rho(\vec{r}) = \langle \Psi_A | \sum_{k=1}^{A} \rho_k | \Psi_A \rangle$$

$$= \langle \Psi_A | \sum_{k=1}^{A} e_k \, \delta(\vec{r} - \vec{r}_k) | \Psi_A \rangle$$

$$Q_{ij} = \int d^3\vec{r} \, (3 r_i r_j - r^2 \delta_{ij}) \rho(\vec{r})$$

1) Deuteron : $Q_d = 0 \cdot 286 \; e\,\text{fm}^2$

↳ sometimes we don't indicate "e"

2) Triton, 3He ; $\boxed{Q_d = 0}$

3) 4He : $\boxed{Q_d = 0}$

Trick ; $\exists \, (2J+1)$ moments for any compound thing

prolate shape

$(2J+1)$ moments :

$\boxed{J=0}$ → Charge (magnetic & quadrupole moments $= 0$)

$\boxed{J=1/2}$ → Charge, magnetic moment (e^-, p, etc)

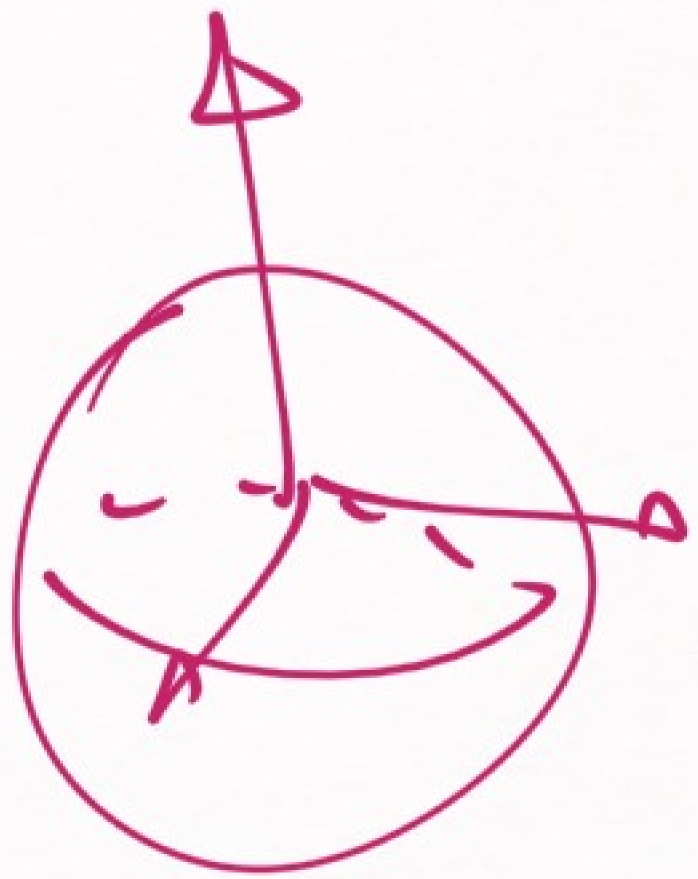
$\boxed{J=1}$ → Charge, magnetic, quadrupole (higher ones $= 0$)

$J = 3/2$ → $\exists$ magnetic octupole moment

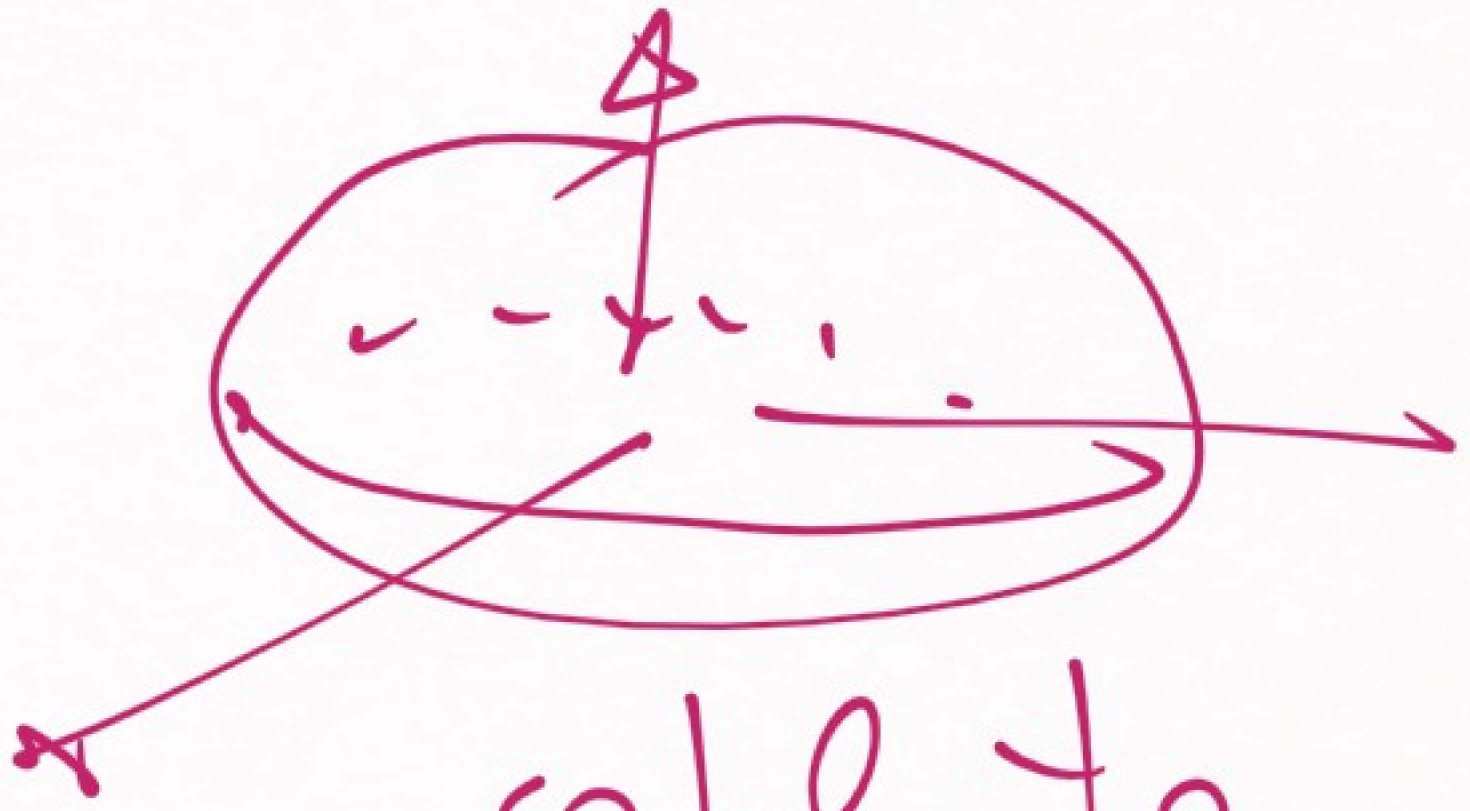
ETC. → pattern is easy to understand

QUADRUPOLE MOMENT :

DEUTERON → $$\underbrace{Q_d > 0}_{\text{not the standard nuclei}}$$

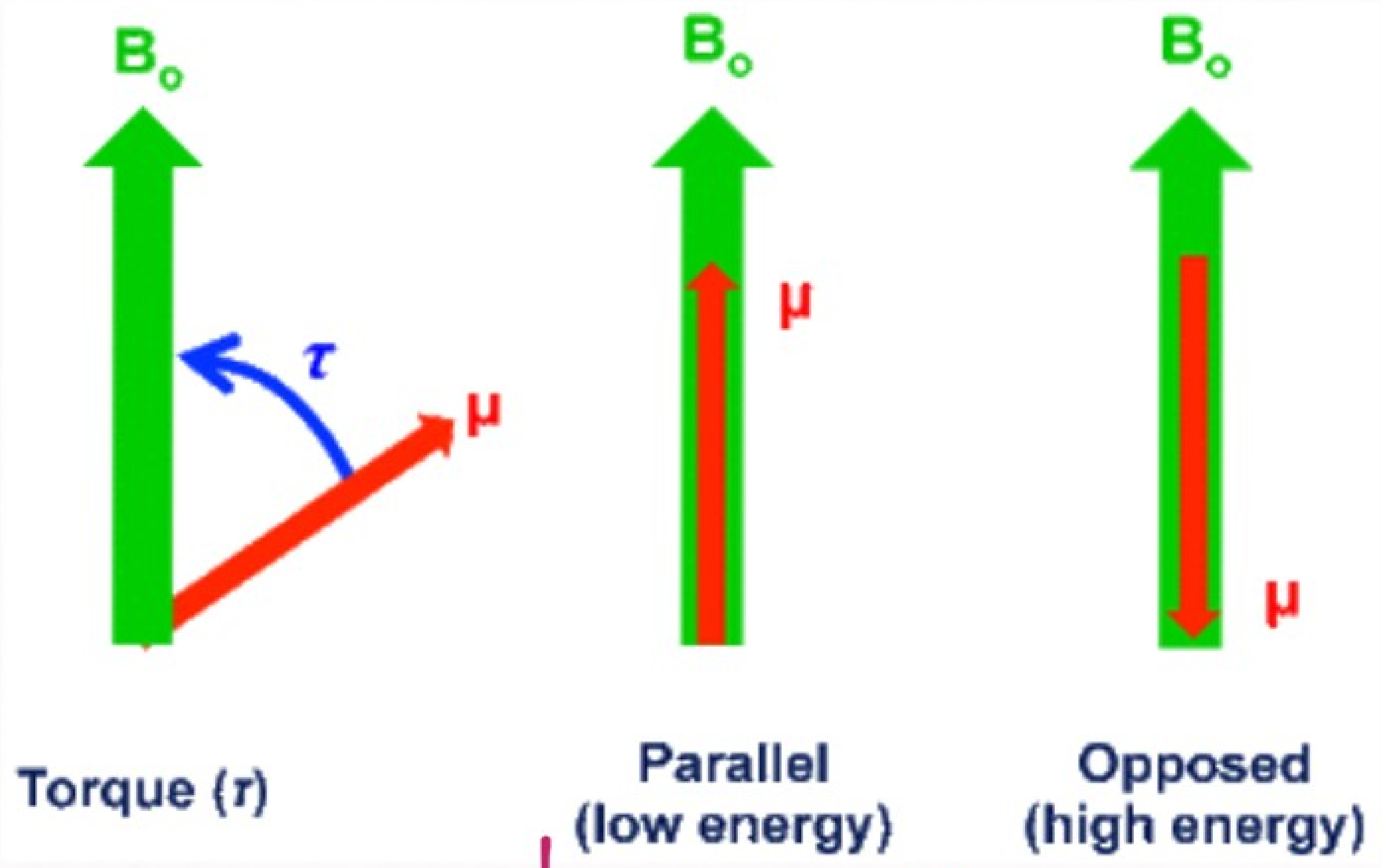


MOST HEAVY NUCLEI → $\underline{Q_d < 0}$



Magnetic moment

$H = -\vec{\mu} \cdot \vec{B}$



→ $\vec{\mu} \parallel \vec{B}$ is the minimal energy state

$\Downarrow$

$\vec{\mu}$ → pseudovector

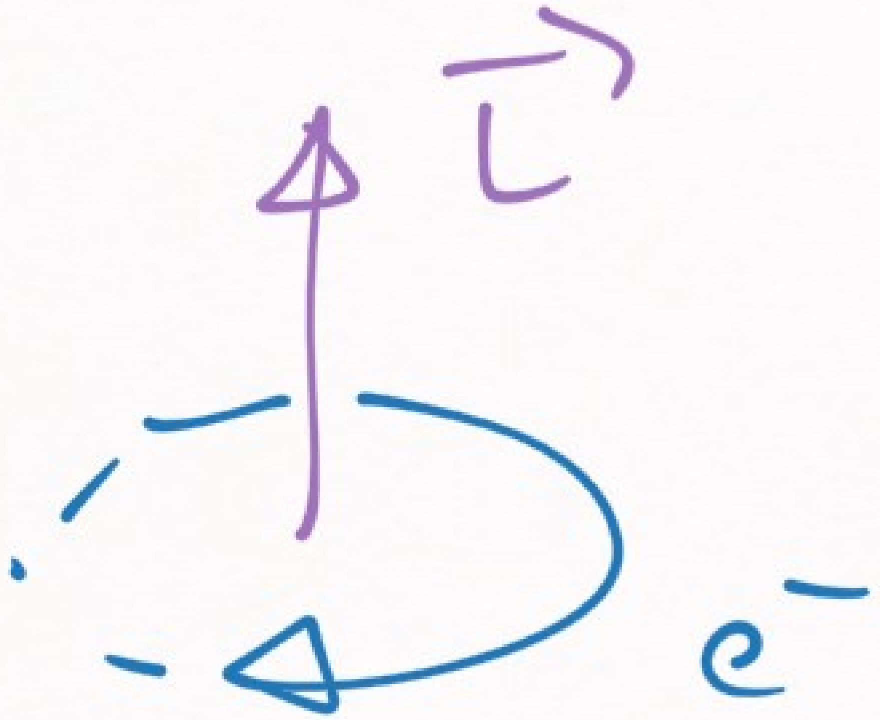
(compass points north also in southern hemisphere)

Ground state

$\Downarrow$

[CLASSICAL MECHANICS] →

$$\boxed{\vec{\mu}_L = \frac{e}{2m}\vec{L}}$$



[QUANTUM MECHANICS] → ∃ spin (intrinsic spin)

CHANGES ←

$$\vec{J} = \vec{L} + \vec{S} \Rightarrow \vec{\mu} = \vec{\mu}_L + \boxed{\vec{\mu}_s}$$

?

1) First guess: $\vec{\mu}_L = \frac{e}{2m}\vec{L}$, $\cancel{\vec{\mu}_s = \frac{e}{2m}\vec{S}}$

$\boxed{\text{WRONG}}$

2) Correct answer:

$$\vec{\mu}_s = \boxed{g_s}\,\frac{|e|}{2m}\vec{S}$$

$\longrightarrow$ gyromagnetic factor

GYROMAGNETIC FACTOR -> ELECTRON (POINT-LIKE)

1) DIRAC EQUATION $g_s(e^-) = -2$

(QM + Special relativity)

2) QED CORRECTIONS (this is why "g-2" is important)

$$g_s(e^-) = -2\left[1 + \boxed{\frac{\alpha}{2\pi}} + O(\alpha^2)\right]$$

$\vec{\mu}_s = g_s \frac{|e|}{2m} \vec{S}$ → If protons & neutrons were point-like

$$\boxed{\begin{array}{l} g_p = 5.586 \\ g_n = -3.826 \end{array}} \quad \Leftarrow \quad \xcancel{\begin{array}{l} g_p \simeq 2 \\ g_n \simeq 0 \end{array}}$$

↳ neutron & proton are not point-like (but composed of quarks)

SUMMARIZE → $\vec{\mu} = \mu_N(\vec{l} + g_s\vec{s})$

w/ $\mu_N = \frac{e}{2M_n}$ (nuclear magneton) (a unit for magnetic moments)

∃ more than one way to write this :

$$\vec{\mu} = \mu_N(\vec{l} + \mu_s\vec{\sigma})$$

$$\mu_s = \frac{g_s}{2}$$

$\mu_s(p) = +2{,}793$ $\quad (\mu_p = +2.793\,\mu_N)$

$\mu_s(n) = -1{,}913$ $\quad (\mu_n = -1.913\,\mu_N)$

→ more usual in certain contexts

$$\mu = \langle JJ | \underbrace{\hat{\mu}_3}_{\boxed{\text{operator}}} | JJ \rangle$$

$$\mu_p = \frac{g_p}{2}\,\underline{\underline{\mu_N}}$$

$$\mu_n = \frac{g_n}{2}\,\underline{\underline{\mu_N}}$$

[MAGNETIC MOMENT] → STRUCTURE OF A NUCLEUS

EXAMPLE → $\boxed{\text{DEUTERON}}$

↓

$$\boxed{\mu_d = 0.8573\ \mu_N}$$ → Experiment

→ let's connect this number w/ properties of the deuteron

$$\mu({}^{A}_{Z}X_{N}) = \langle JJ | \hat{\mu}_3 | JJ \rangle$$

$$\vec{\hat{\mu}} = \sum_{i=1}^{A} \vec{\hat{\mu}}_i \quad , \vec{\hat{\mu}}_i = \mu_N (\vec{L} + \mu_s(i) \vec{\sigma})$$

[DEUTERON] → $\hat{\vec{\mu}}_d = \hat{\vec{\mu}}_p + \hat{\vec{\mu}}_n$

$$= \mu_N \vec{L}_p + \mu_p \vec{\sigma}_p + \mu_n \vec{\sigma}_n$$

↓

$$\mu_d = \langle 11 | \hat{\mu}_{3d} | 11 \rangle \quad (J=1)$$

If the deuteron was a pure S-wave state:

$$\mu_d = \langle 1\,1\,1 |\, \cancelto{0}{\mu_N L_{p3}} + \mu_p \cancelto{1}{\sigma_{p3}} + \mu_n \cancelto{1}{\sigma_{n3}} \,| 1\,1\,1 \rangle$$

$$= \mu_p + \mu_n \simeq 0.88\, \mu_N$$

$$\left(\neq \mu_d^{exp} = 0.8573\, \mu_N \right)$$

$$0.86\, \mu_N$$

difference

$$|1\,1\,1\rangle = |\tfrac{1}{2}\,\tfrac{1}{2}\rangle_p\, |\tfrac{1}{2}\,\tfrac{1}{2}\rangle_n$$

$$\left.\begin{array}{l}\mu_d^{S\text{-wave}} \simeq 0.88\,\mu_N \\ \mu_d^{exp} \simeq 0.86\,\mu_N\end{array}\right\} \rightarrow$$

Difference is because the deuteron is not a pure S-wave

(Qd) Deuteron has a D-wave component

→ change to $\underline{\underline{\mu_d}}$

Solution → $|\psi_d\rangle = a_s|3S_1\rangle + a_d|3D_1\rangle$

$$|a_s|^2 + |a_d|^2 = 1$$

$$\left.\begin{array}{l}\mu(3S_1) = 0.88\,\mu_N \\ \mu(3D_1) = 0.31\,\mu_N \checkmark\end{array}\right] \longrightarrow \oplus$$

$$\oplus \rightarrow \mu_d = |a_s|^2\mu(3S_1) + |a_d|^2\mu(3D_1)$$

$$= P_S\,\mu(3S_1) + P_D\,\mu(3D_1)$$

$$\Rightarrow \boxed{\begin{array}{l} P_S \simeq 0,96 \\ P_D \simeq 0,04 \end{array}} \Rightarrow \boxed{\mu_d = \mu_d^{exp}?}$$

→ [the D-wave probability of the deuteron is about 3-4%.]

If any of you is a QM purist

→ "The wave function is not an observable quantity"

↙

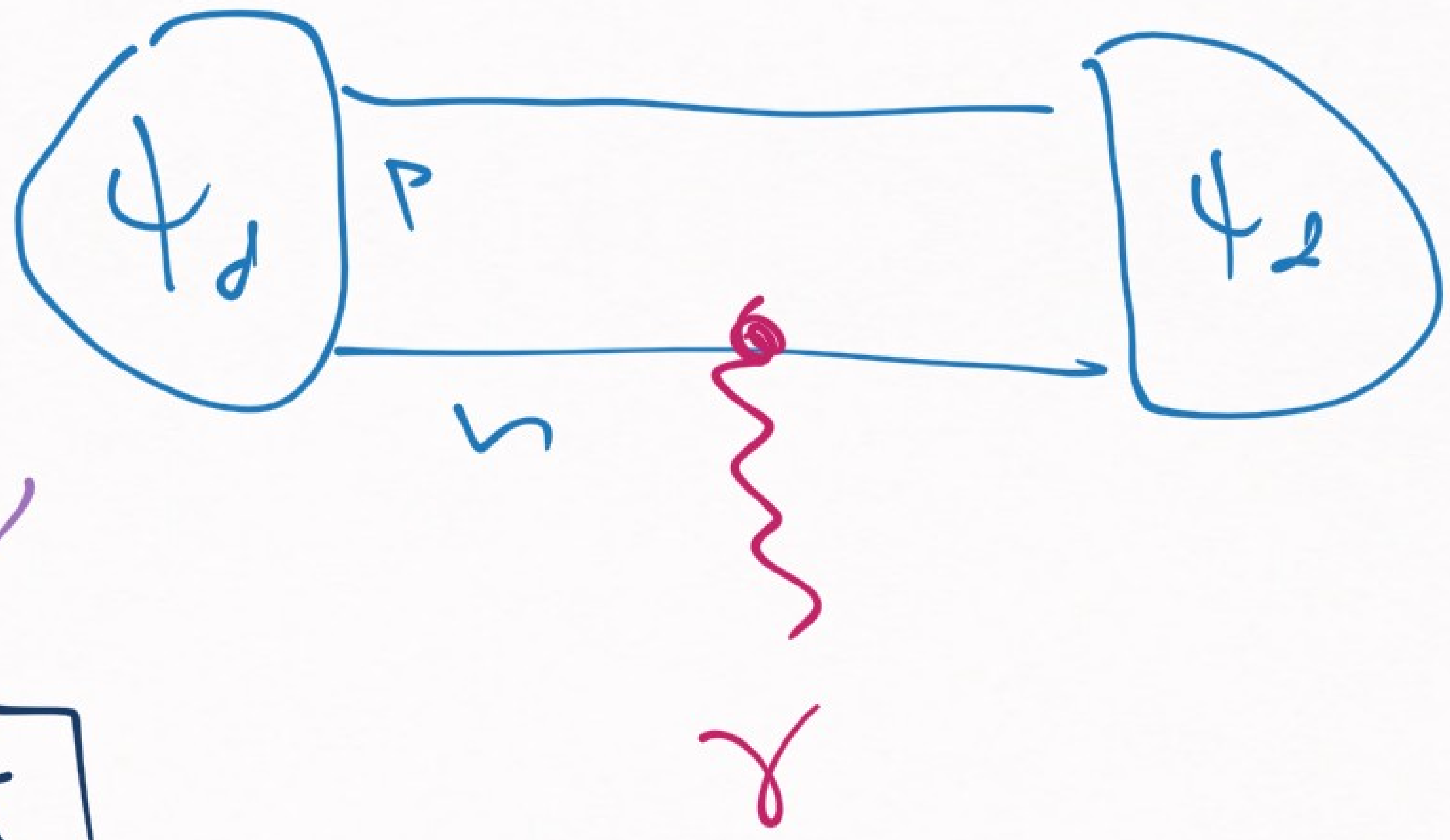
"P_S & P_D cannot be determined"

↳ $\exists$ missing in our previous calculation

What is missing → TWO-BODY CURRENTS

$P_S \simeq 0.96$

$P_D \simeq 0,04$ →



PHOTON ONLY HITS ONE NUCLEON → ONE-BODY CURRENT

ONE BODY CURRENTS → E.M. PROPERTIES OF A NUCLEUS ARE SIMPLY THE SUM OF THE E.M PROPERTIES OF NUCLEONS

↓

THIS IS WHAT WE USUALLY EXPECT

NUCLEUS → NUCLEONS + "POTENTIAL ENERGY"

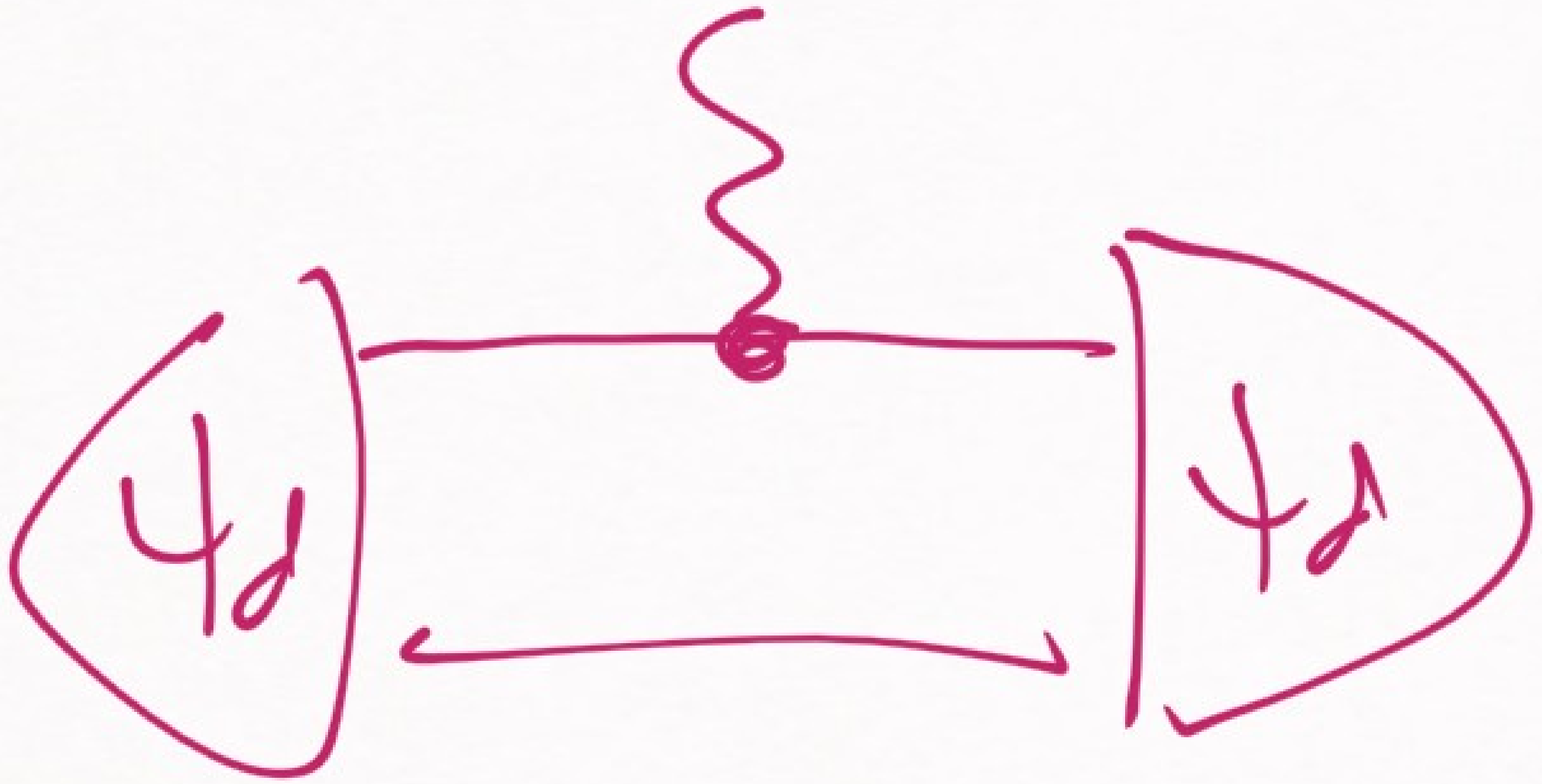
↓

PIONS IN-FLIGHT

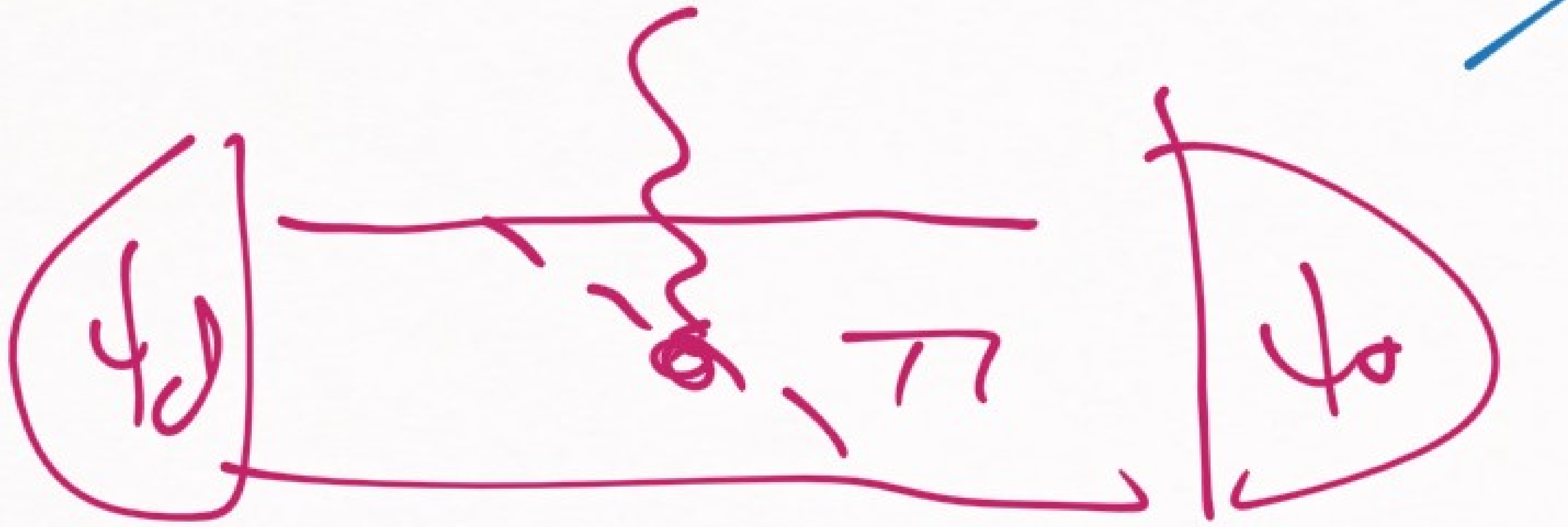
+ OTHER THINGS

↙

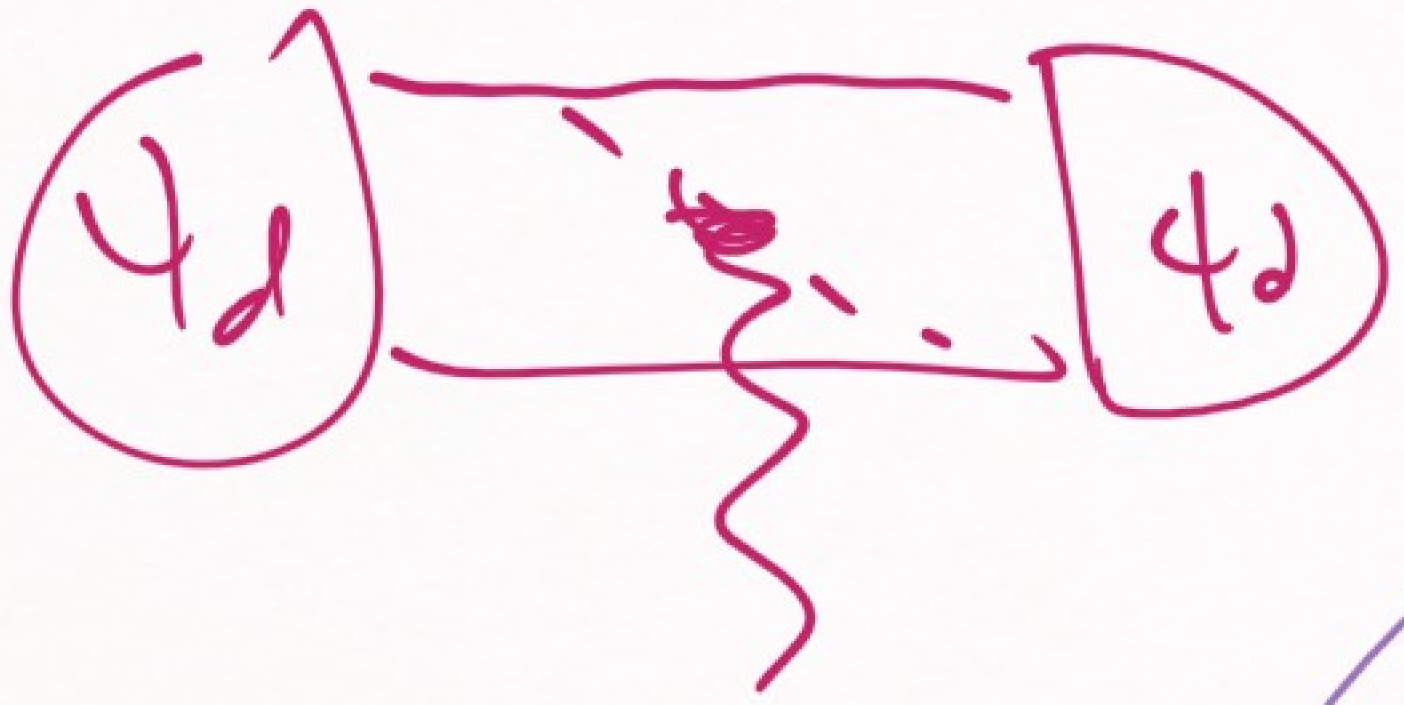
Have an impact on the e.m. properties of nuclei



corrections to μ_d, Q_d



→ photon hitting a pion in-flight



→ TWO-BODY CURRENTS

IT REQUIRES TWO NUCLEONS INTERACTING

THEY ARE SOMEWHAT DIFFICULT TO COMPUTE

LUCKILY THEIR CONTRIBUTION IS SMALL

[Two-body currents] → small corrections

EXAMPLE : DEUTERON QUADRUPOLE MOMENT

$Q_d^{exp} = 0.2859(3)\,e\mathrm{fm}^2$

$$\left.\begin{aligned} Q_d^{1B} &\simeq 0.276\,e\mathrm{fm}^2 \\ Q_d^{2B} &\simeq 0.010\,e\mathrm{fm}^2 \end{aligned}\right] \quad |Q_d^{1B}| \gg |Q_d^{2B}|$$

$$\rightarrow \quad \mu_d = \mu_d^{1B} + \boxed{\mu_d^{2B}} \rightarrow$$

this is why is
somehow difficult
to pinpoint
the D-wave
probability

3

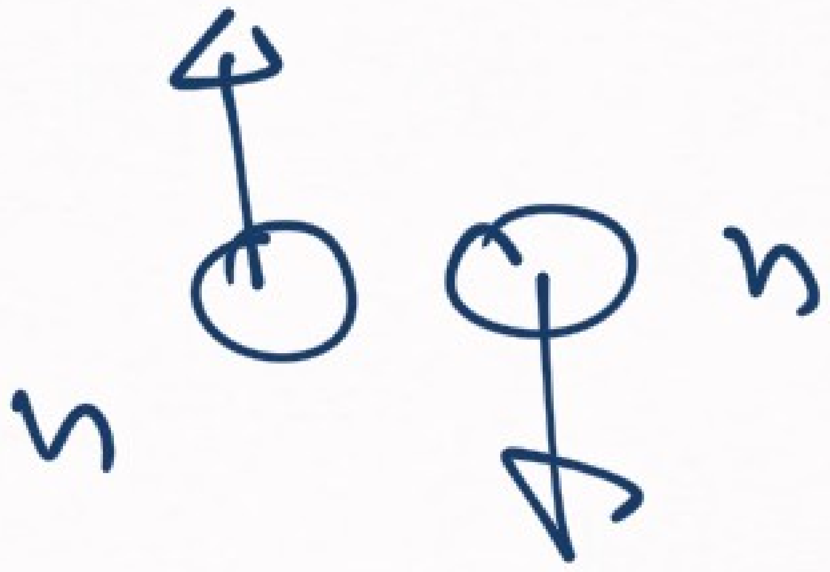
[MAGNETIC MOMENT OF NUCLEI]

even-even nuclei → **PAIRING**

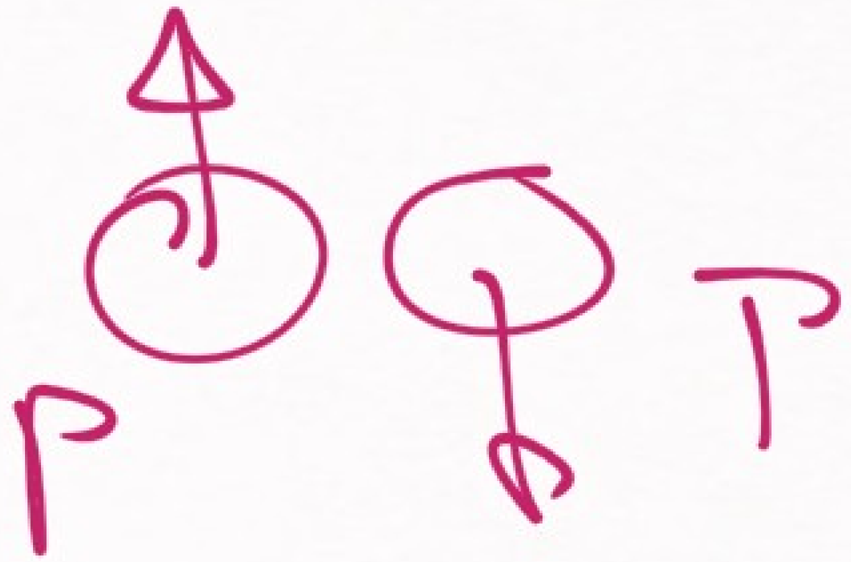
↓ ↓

even Z even N

↓

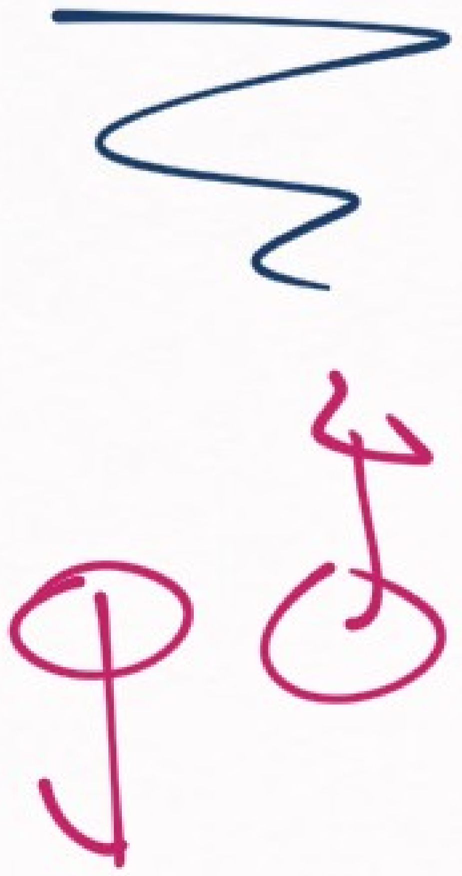


$\underline{\vec{S} = \vec{0}}$



$\vec{S} = \vec{0}$

Every time we have a pair of neutrons or protons their spin will couple to zero



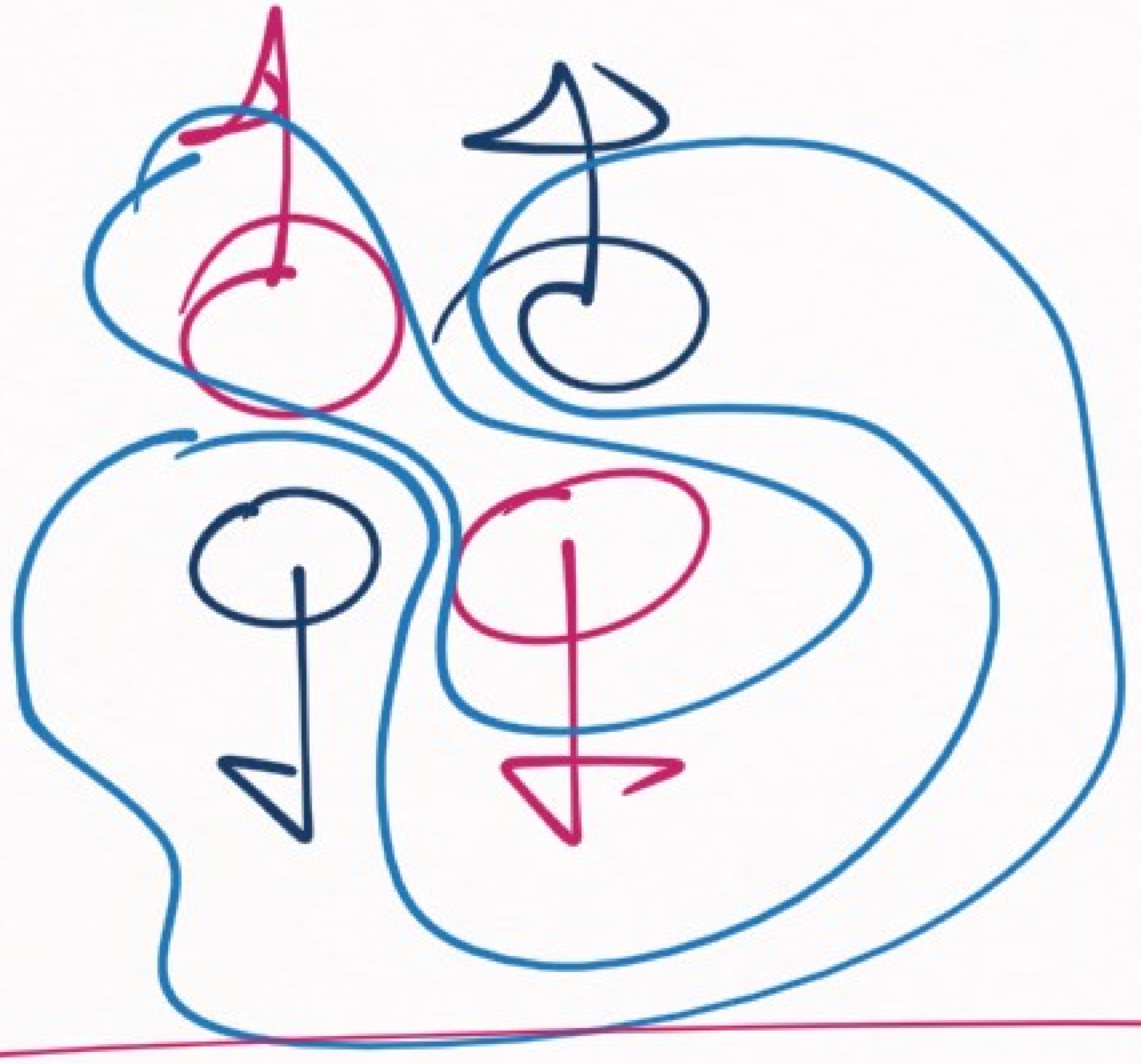
Even-even nuclei] →

$$\mu_{pair} = \vec{\mu}_1 + \bar{\mu}_2$$

$$= \mu_{n/p}(\vec{\gamma}_1 + \vec{\gamma}_2)$$

$$= 0$$

${}^4He$



$\rightarrow \mu({}^4He) = 0$

(alternavely $J^P({}^4He) = 0^+$)

$$\boxed{\mu(\text{even-even nuclei}) = 0}$$

$$\boxed{J^P(\text{even-even nuclei}) = 0}$$

NEXT CASE : even-odd nuclei

↓ $\cong$ even-even core + single nucleon

$$J(A) = \underset{0}{\cancel{J_{core}}} + J_N = \boxed{J_N}$$

$$\mu(A) = \underset{0}{\cancel{\mu_{core}}} + \mu_N = \boxed{\mu_N}$$

$$\boxed{\vec{J}, \vec{L}, \vec{S}}$$

1) $\hat{\vec{\mu}}_n = \mu_n(\vec{L} + \mu_s \vec{\sigma})$

$$\left[\begin{array}{l} g_L(p) = 1 \\ g_L(n) = 0 \end{array}\right]$$

2) $S = 1/2$, $J = L \pm 1/2$, L is given

3) $$\mu_n(J = L + 1/2) = \boxed{g_L}(J - 1/2) + \frac{1}{2} g_s$$

$$\mu_n(J = L - 1/2) = g_L \frac{J(J + 3/2)}{J + 1} - g_s \frac{J}{2(J+1)}$$

→ **SCHMIDT VALUES**

Recap (magnetic moments)

1) Deuteron contains a D-wave $\mu_d(^3S_1) = 0{,}88\,\mu_N$ (exp: $0{,}86\,\mu_N$)

2) Nuclei look complicated, but ∃ simplifications

2.a) even-even nuclei $\mu(\text{even-even}) = 0$

2.b) even-odd nuclei   Schmidt values

$$\mu(A) = \underset{0}{\cancel{\mu_{core}}} + \mu_N = \boxed{\mu_N}$$

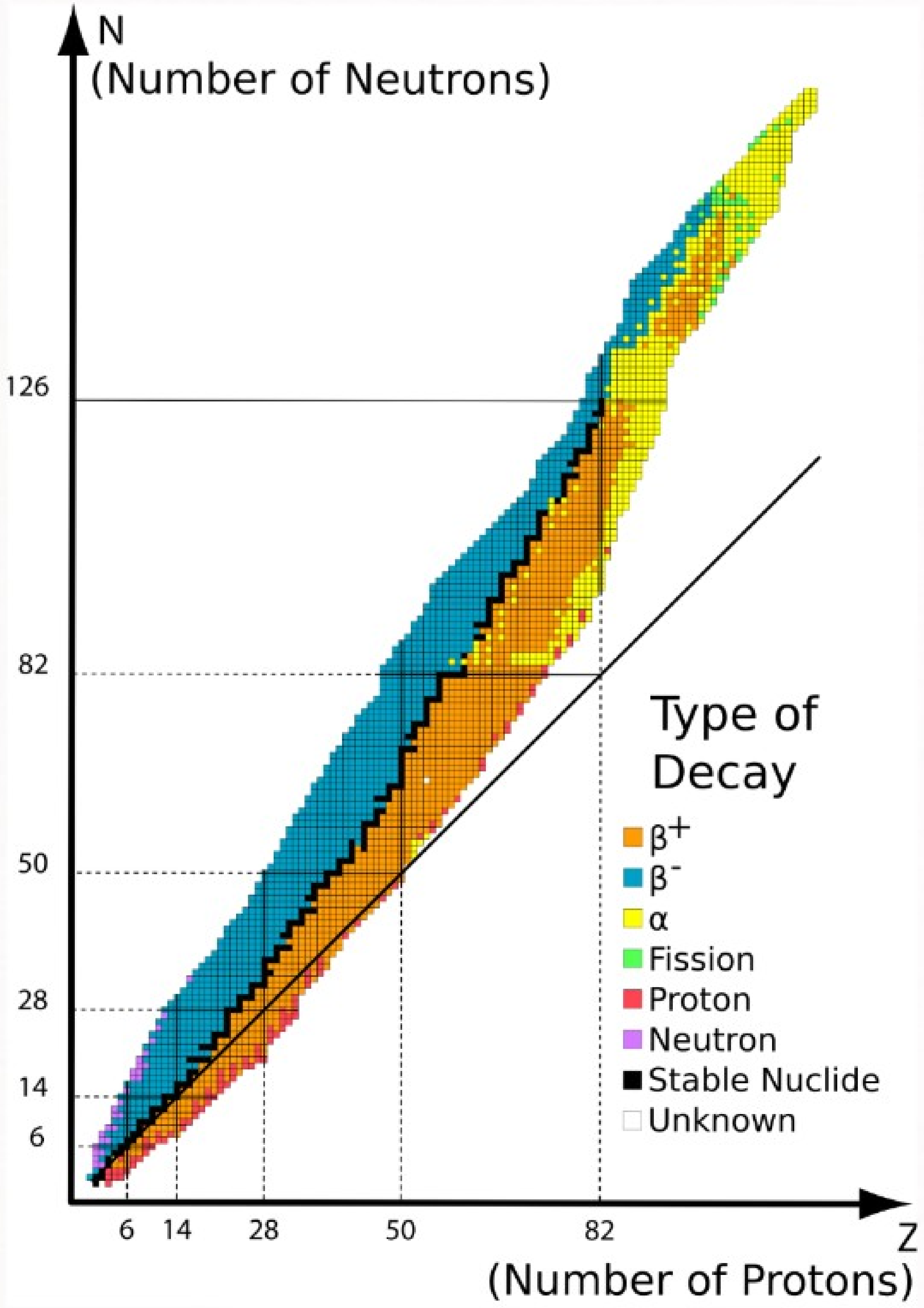
2.c) odd-odd nuclei (5 stable cases)

→ case-by-case bases

↓

deuteron is one example

$\boxed{5}$ NUCLEAR STABILITY & DECAYS



(α, β, γ decays)

which nuclei will decay?

→ any nuclei for which decaying is energetically allowed

[NUCLEAR STABILITY]

$$\left[E(\text{initial nucleus}) > E(\text{final nucleus} + \text{decay product}) \right]$$

$$\Downarrow$$

$$\boxed{\text{DECAY}}$$

I) α-decay $\;{}^{A}_{Z}X_{N} \rightarrow {}^{A-4}_{Z-2}X_{N-2} + \alpha$

(4He nucleus)

Energetically possible:

$$S_\alpha(Z,N) = B(Z,N) - B(Z-2,N-2) - B(2,2)$$

$\boxed{S_\alpha > 0} \Rightarrow$ We have alpha decay

2) β^+ & β^- decays:

$$\beta^- : \; {}^{A}_{Z}X_{N} \rightarrow {}^{A}_{Z+1}X_{N-1} + e^- + \bar{\nu}_e$$

$$\beta^+ : \; {}^{A}_{Z}X_{N} \rightarrow {}^{A}_{Z-1}X_{N+1} + e^+ + \nu_e$$

Simplest case → neutron decay

$$n \rightarrow p + e^- + \bar{\nu}_e \, , \quad Q = -(m_p + m_e + \overset{\simeq 0}{\cancel{m_\nu}} - m_n)$$

$$\simeq 1.29\,\text{MeV}$$

$$\Gamma(n \to p e \bar{\nu}) = 2\pi G_V^2 (1 + 3 g_A^2)\, \pi(Q)$$

$$\pi(Q) = \frac{m_e^5}{4\pi^4} f\left(\frac{Q}{m_e}\right), \quad f(x) = \int_0^\infty (\ldots\ldots)\, dx$$

→ Details can be consulted later

$$\tau = \frac{\hbar c}{\Gamma} \frac{1}{c} \simeq 930 \text{ seconds}$$

Next example: $\underline{\underline{\text{TRITON}}}$ (^3H nucleus)

$$^3H \to {}^3He + e^- + \bar{\nu}_e$$

$$Q = +B(^3H) + (m_n - m_p) - B(^3He)$$

$$- m_e \geq 0$$

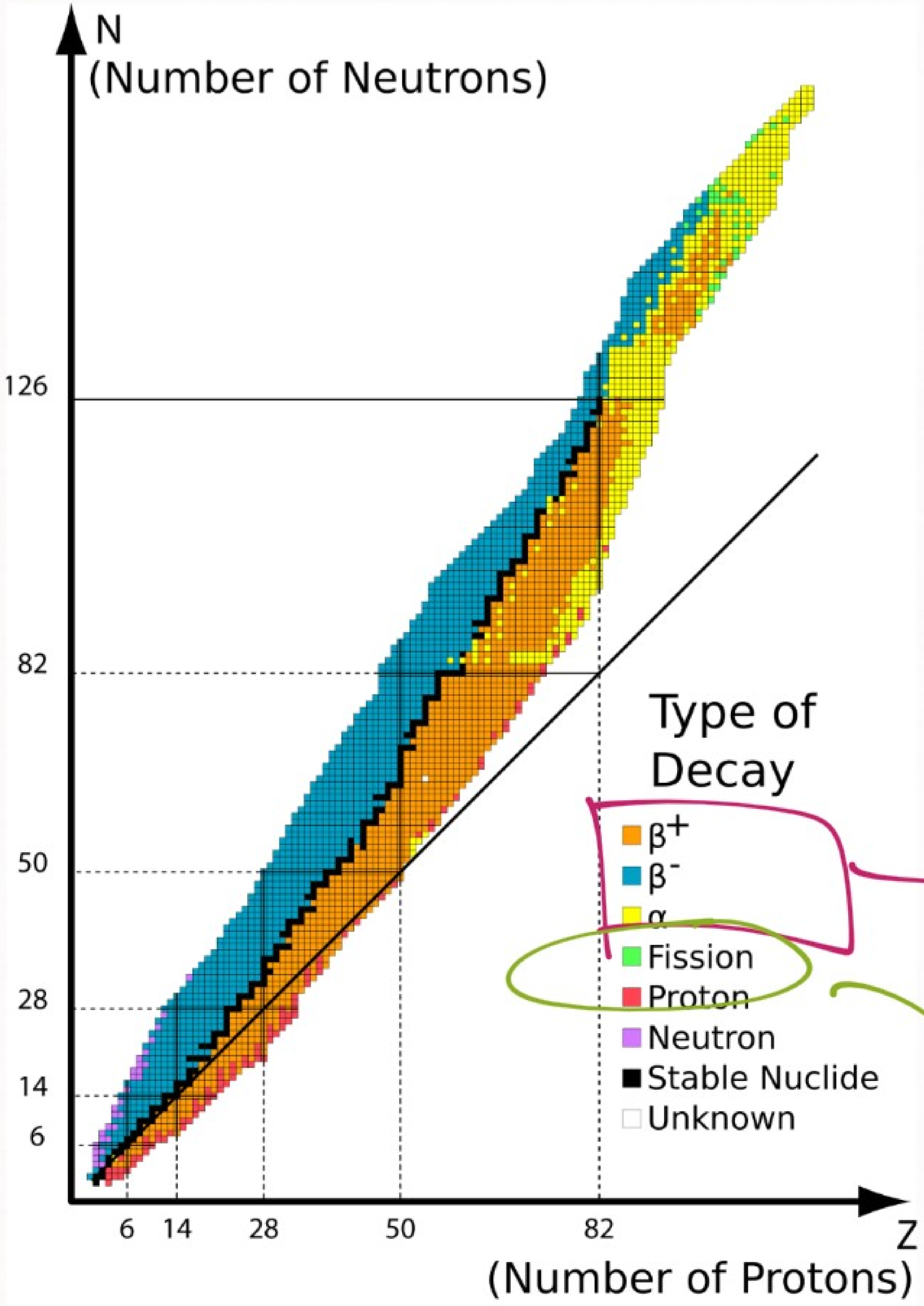
$$Q \simeq 0.0186\,\text{MeV} \to \boxed{\tau \sim 10\,\text{years}}$$

3) γ-decay

$${}^{A}_{Z}\underbrace{X^{*}_{n}}_{\text{excited state of a nuclel}} \rightarrow {}^{A}_{Z}\underbrace{X_{n}}_{\text{less-excited state}} + \gamma$$

→ happens super fast

→ between states of same nucleus



→ most of the decays

→ liquid drop model

NEXT LESSON

→ NUCLEAR MODELS