NUCLEAR PHYSICS (17)

Effective field theory approaches to the nuclear force

RECAP One boson exchange (OBE) model

→ nuclear force comes from exchanges of a series of mesons ($\pi, \sigma, \rho, \omega, \ldots$)

→ $V_{NN} = \sum_{M} V_M = V_\pi + V_\sigma + V_\rho + V_\omega + \ldots$

→ Each meson has a specific job

FUNDAMENTAL PROBLEM OF NUCLEAR PHYSICS

Derivation of nuclear forces from first principles

QCD

[OBE model] → pre QCD

It's not the real solution to the problem of nuclear force

NOWADAYS → [OBE MODEL IS A PHENOMENOLOGICAL MODEL]

Post QCD era

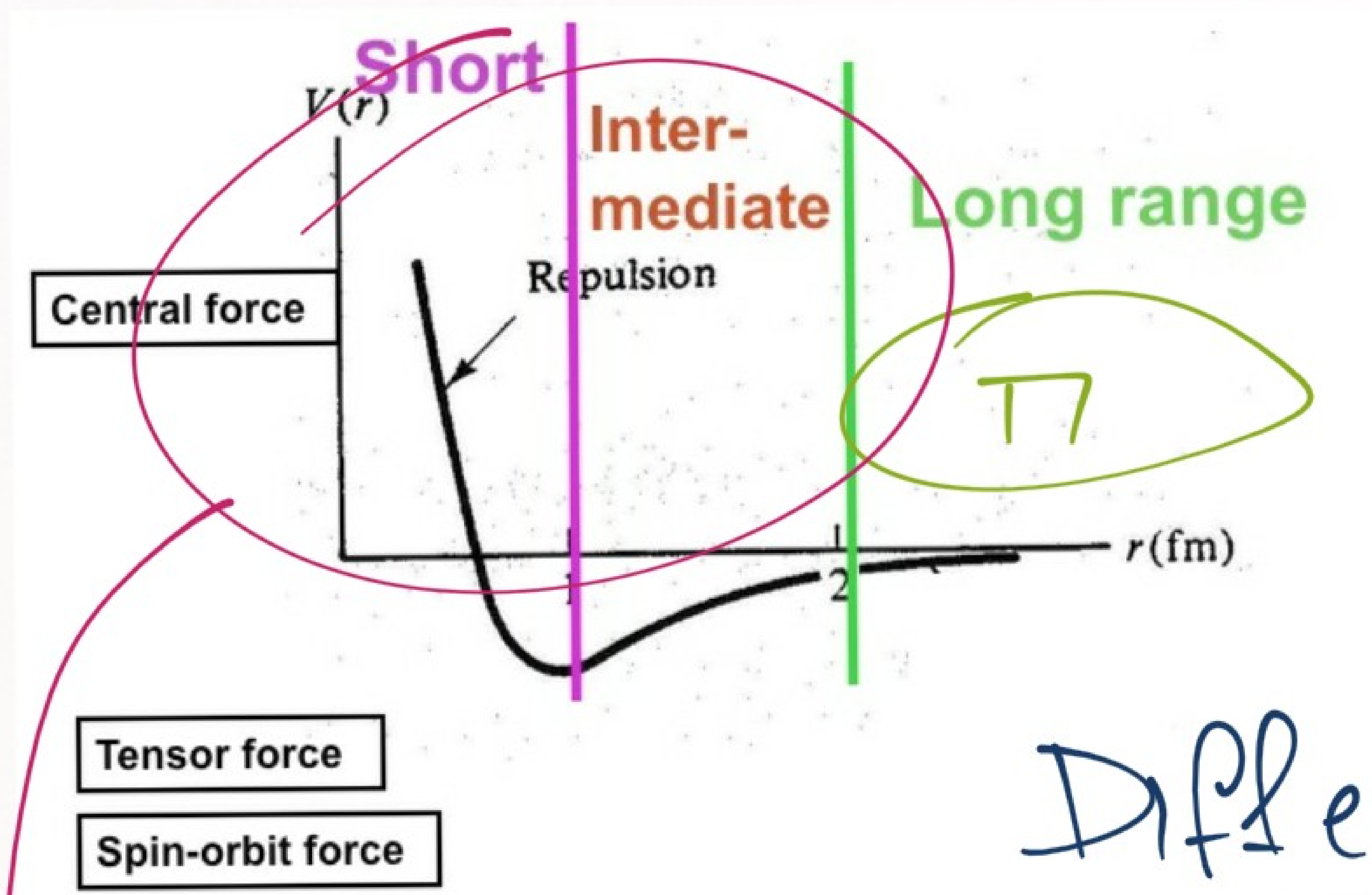
↳ QCD is not solvable at low energies

1) Lattice QCD

↳ need to do really heavy numerical calculations

2) EFT

↳ indirect method



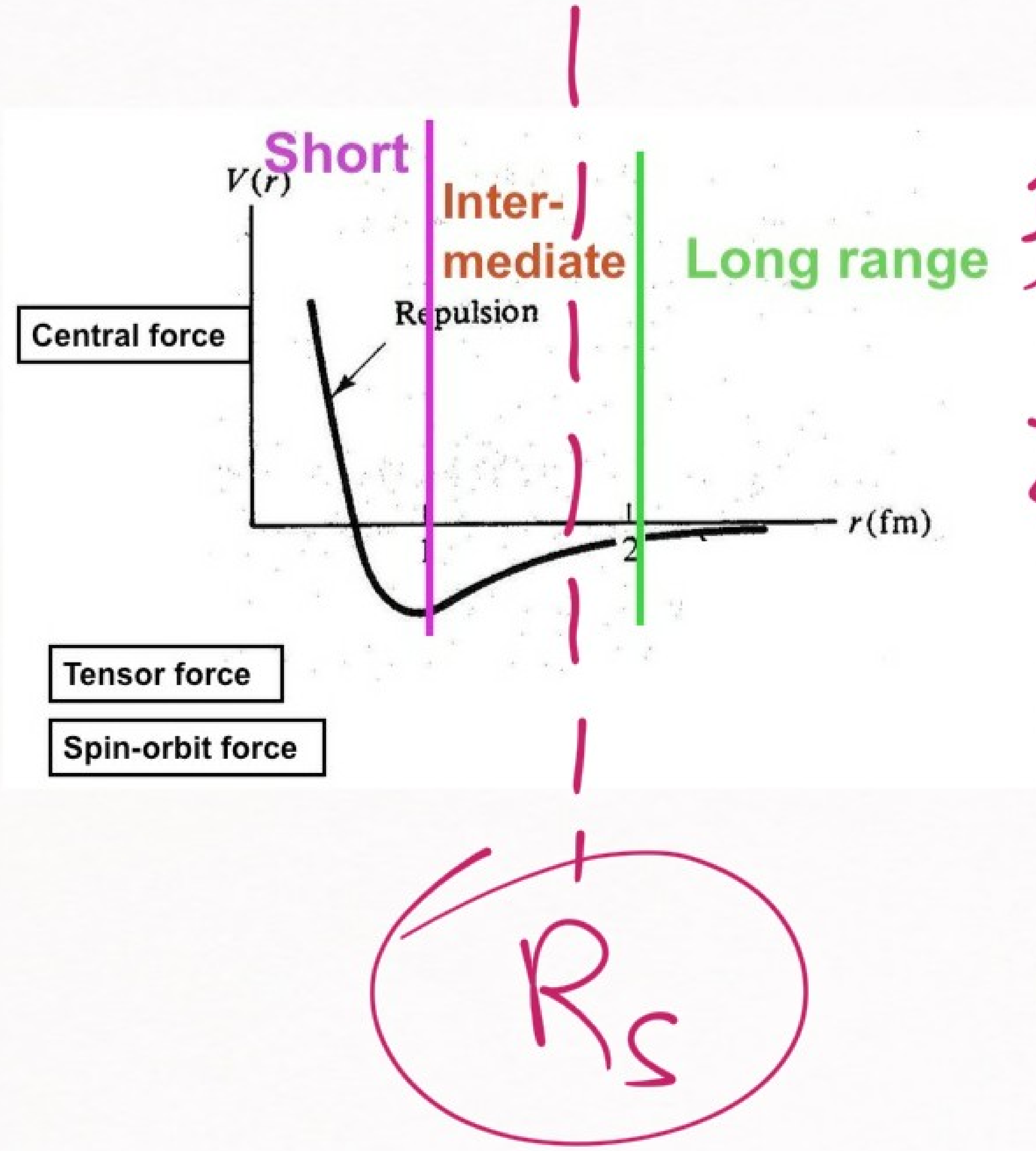
→ Contains this separation (by TNS) of scales (or regions)

Different strategies from OBE & EFT

Long range → we have pions (no discussion)

a) OBE model → meson exchanges

b) EFT → choose a separation scale



1) $r > R_s$ → physics is known

2) $r < R_s$ → physics is unknown

(& I will not assume a specific short-range model)

FIRST STEP:

EFTs → [IDENTIFY A SEPARATION OF SCALES]

FIRST STEP → R_s

→ $m_\pi R_s \sim 1$ (R_s large)

→ pionless EFT

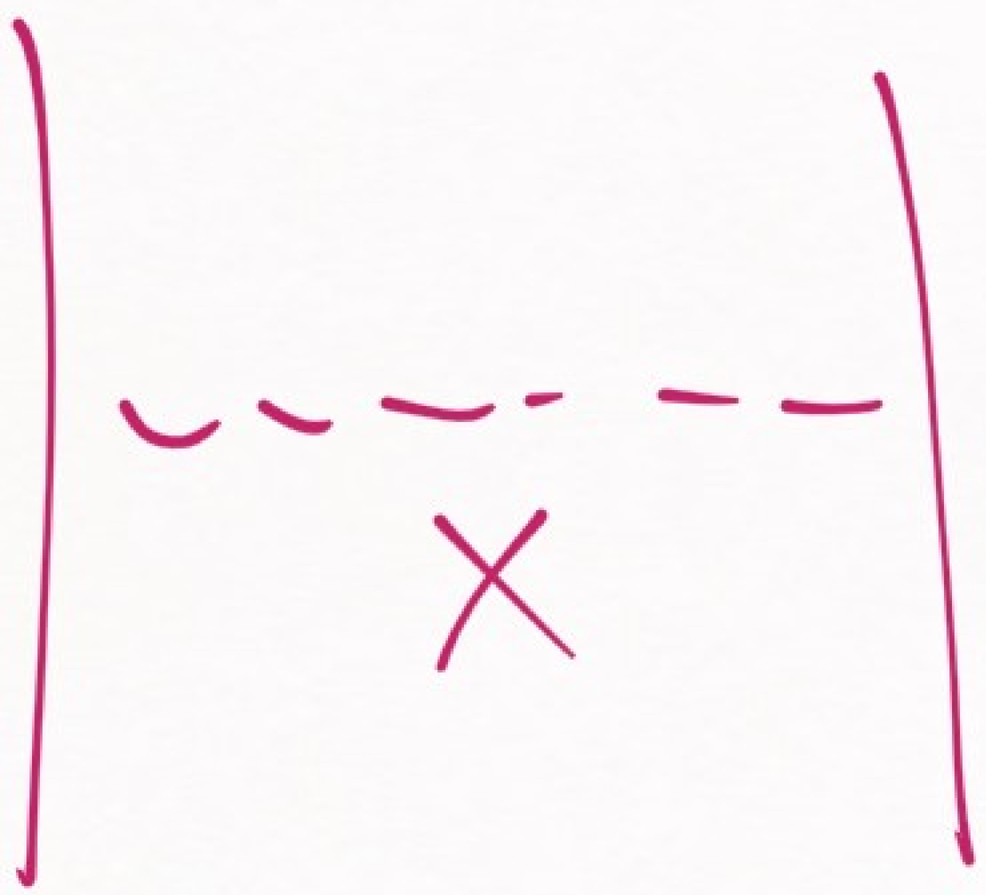
→ $m_\pi R_s \ll 1$ (R_s small)

→ pionful EFT

DECIDING WHICH PHYSICS IS KNOWN AND WHICH PHYSICS IS UNKNOWN

R_ρ → "short-range scale", "separation scale"

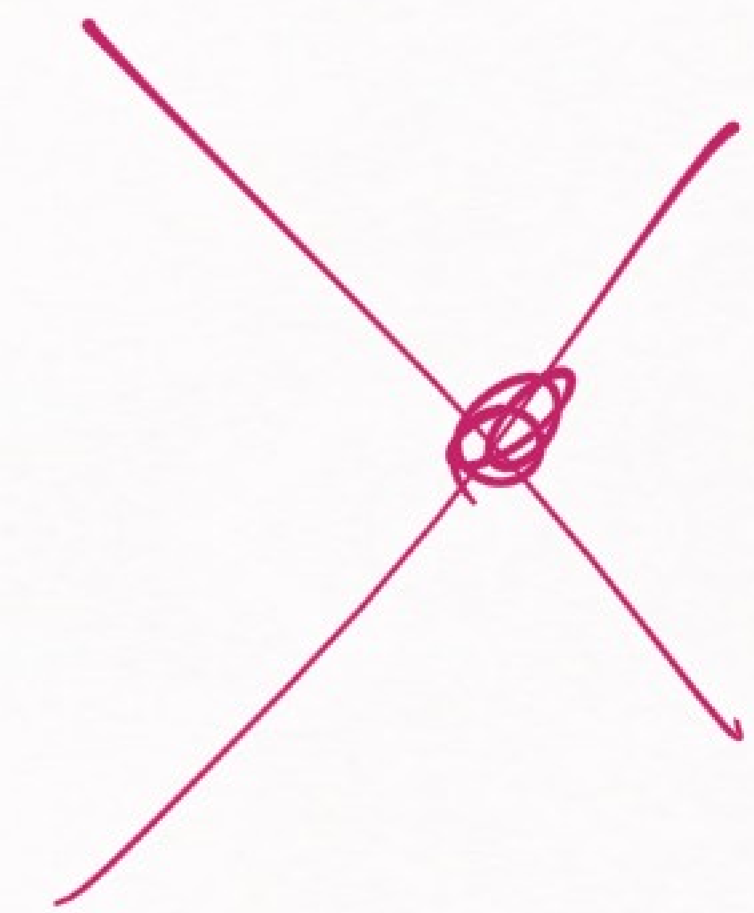
UNKNOWN PART



m_X ("meson X")

$\to \left[m_X R_S \gg 1 \right] \to$

$\sim e^{-m_X R}$ $(\to 0)$



contact-range interaction

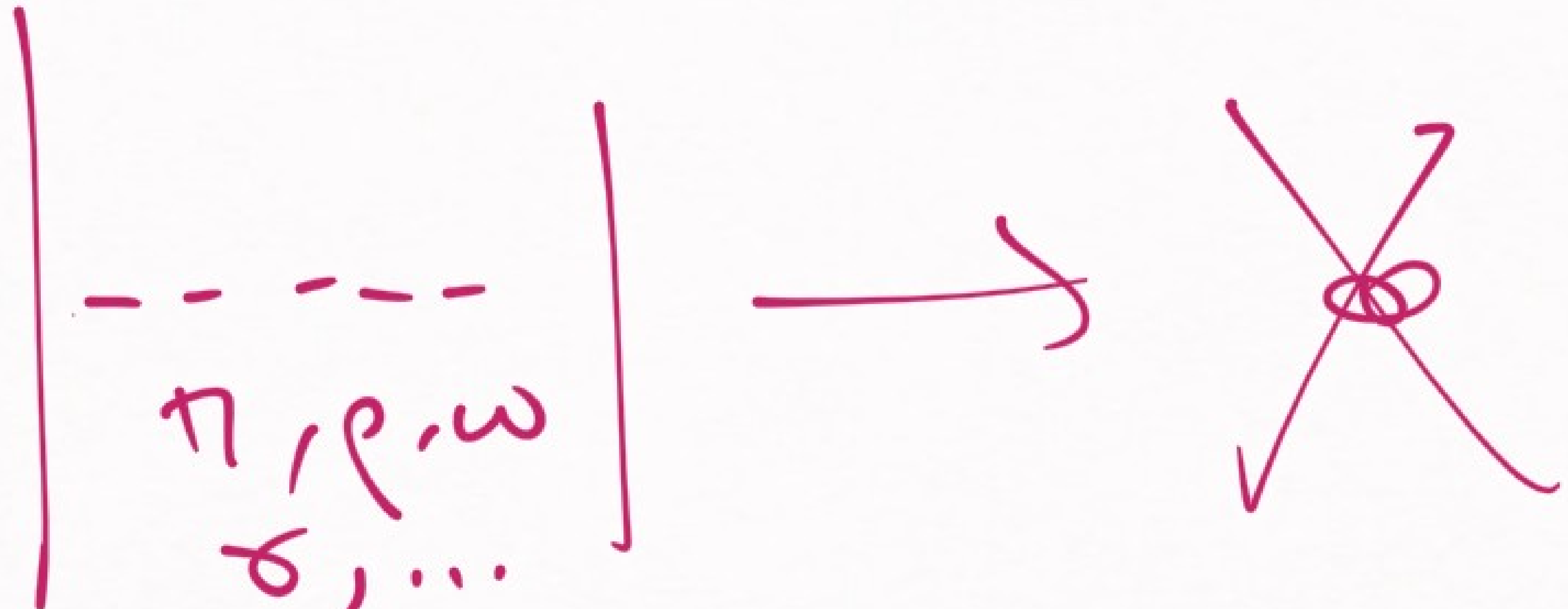
$$\frac{1}{q^2+m^2x} \rightarrow [q^2 \ll m^2 x] \rightarrow \frac{1}{mx^2} - \frac{q^2}{mx^4} + \dots$$

Cont $(2q^2 + \dots$

$$\rightarrow [q^2 \sim m^2 x] \rightarrow$$

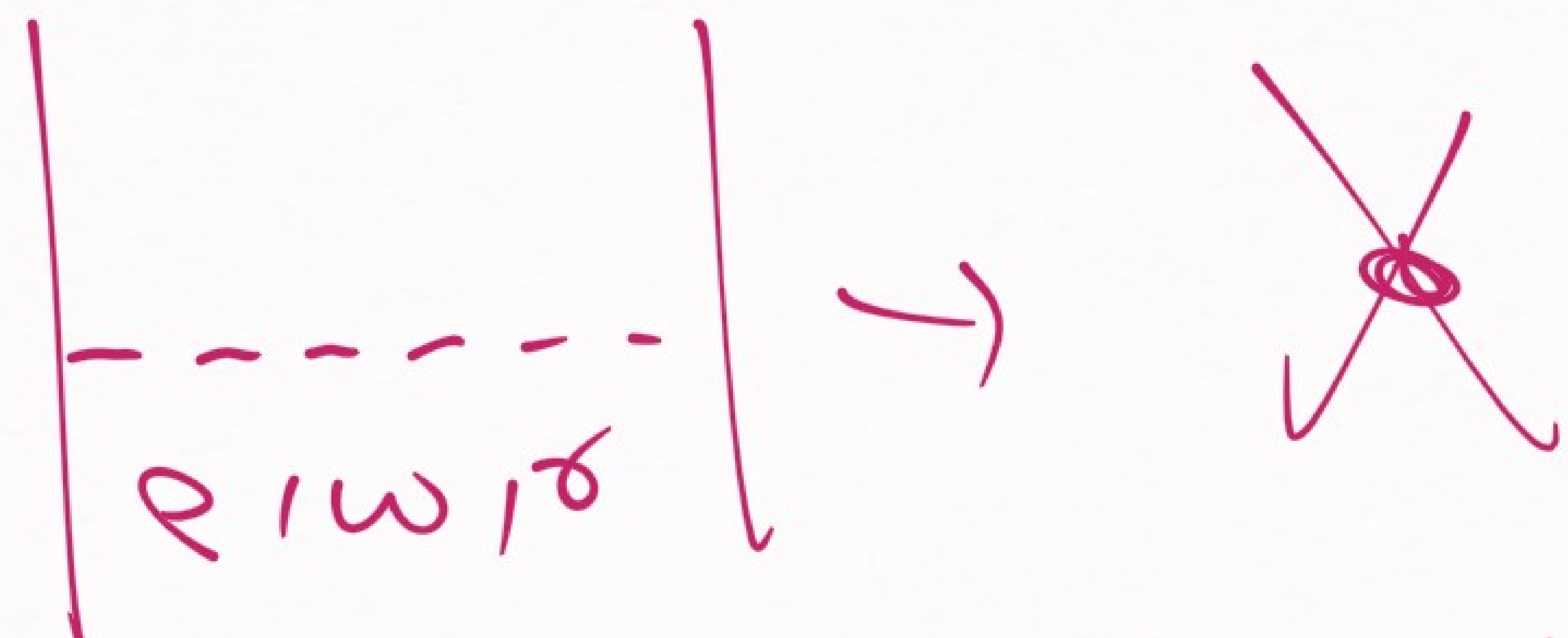
Have to keep the full potentials

Pionless ($m_\pi R_s \sim 1$)



only contacts

Pionful ($m_\pi R_s \ll 1$)



($m_\rho R_s \sim 1$)

contacts + pions

PIONLESS EFT

EFT($\not{\pi}$)

- works for $k < m_\pi$
- really simple & useful
- it only contains contacts

contact

S-waves → $\langle p'|V_c|p\rangle = C_0 + C_2(p^2 + p'^2)$
$+ C_4(p^4 + p'^4) + C_4' p^2 p'^2$
$+ \ldots$

[FUNDAMENTAL TRICK OF EFTS]

→ It does not matter how do you regularize (if you renormalize afterwards)

Ξ

REGULARIZE $\longmapsto$ make it finite (by including a regulator function)

RENORMALIZE $\longmapsto$ fix the free couplings of the EFT to some physical observables

$\geq$

[Once we renormalize

→ does not matter who we regularized the problem]

→ Since all regularizations are equivalent

let's use an easy one for EFT (?)

⟶ Delta-shell regulator

$$\langle p'|V_c|p\rangle = C_0 + C_2(p'^2 + p^2) + C_4(p^4 + p'^4) + C_4' p^2 p'^2$$

$$+ \dots$$ (somewhat complex)

↓

$$V_c(r; R_c) = \frac{C_k(R_c)}{4\pi R_c^2}\,\delta(r - R_c)$$ (on-shell equivalent)

$$C_k(R_c) = C_0(R_c) + k^2 C_2(R_c) + k_4 C_4(R_c) + \dots$$

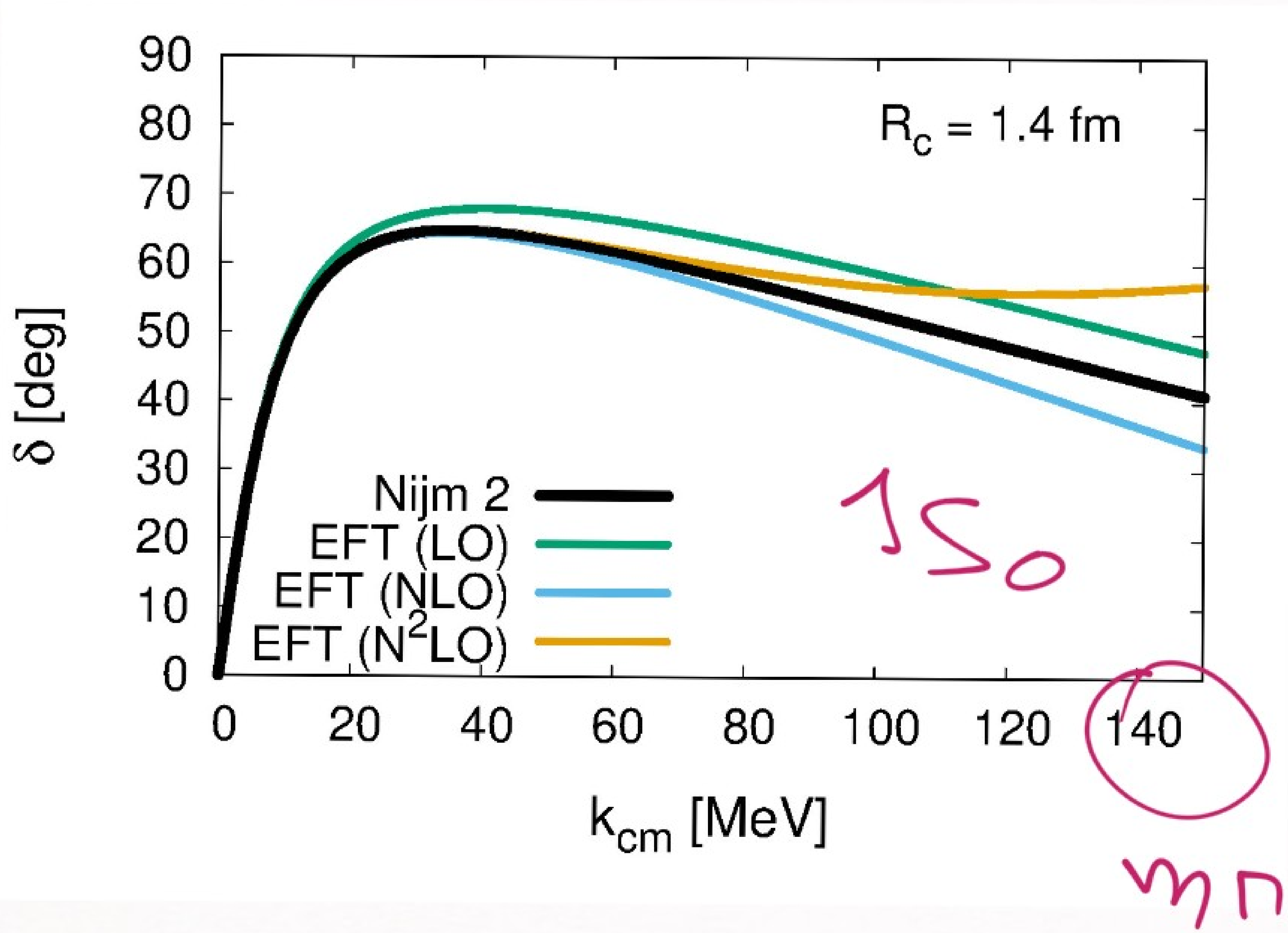
Delta-shell → admits an analytical solution

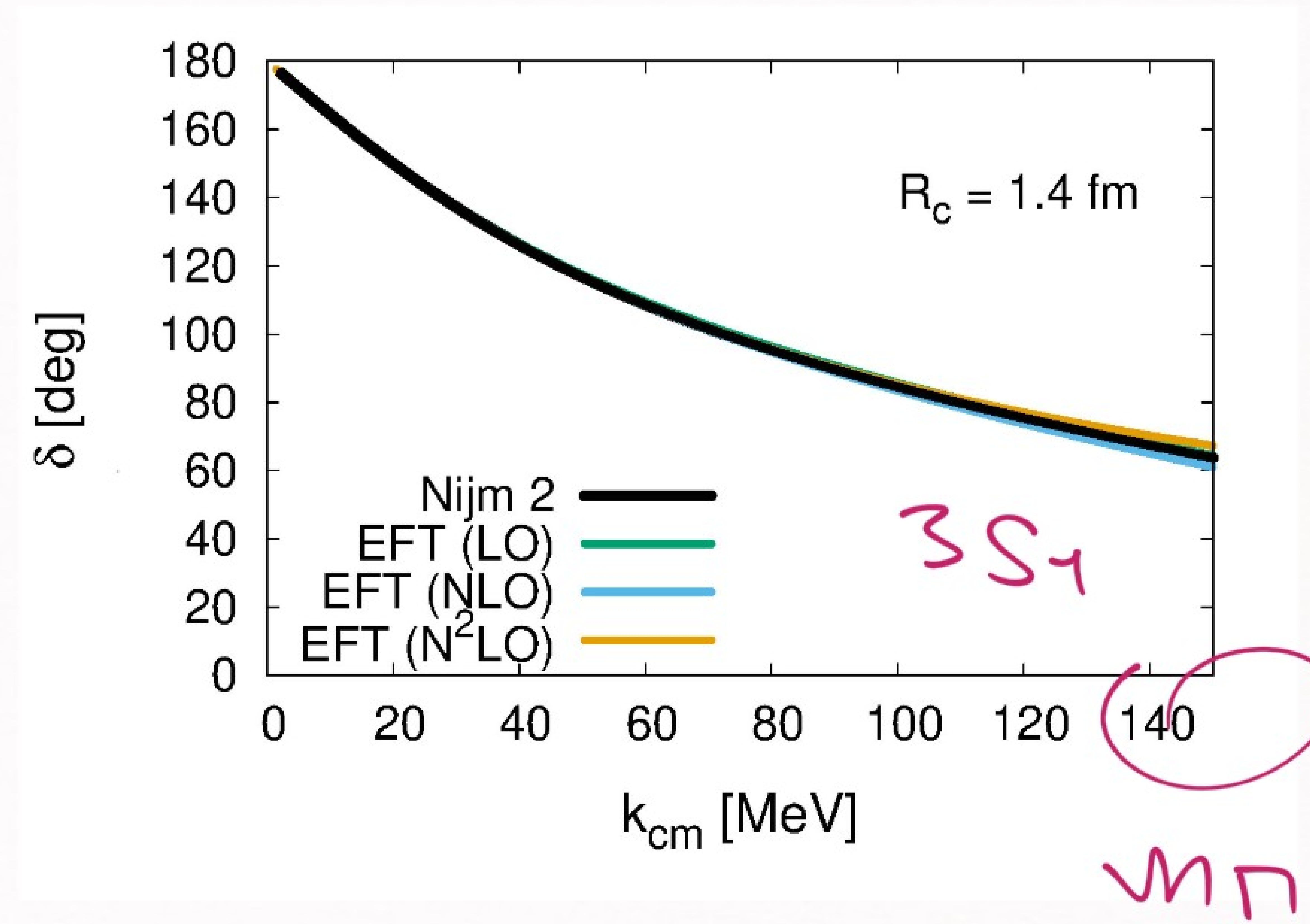
$$k\cot(kR_c+\delta) - k\cot kR_c = 2\mu \frac{c_k(R_c)}{4\pi R_c^2}$$

→ Choose R_c

→ Choose how many $c_{2n}(R_c)$ to keep

→ Fit to the data (or fit to ERE parameters)





pionless EFT works for $k < m_\pi$ ✓

cutoff → $m_\pi \underline{R_c} \sim 1$ (R_c is not R_s!)

Comments about EFT($\not\pi$):

→ EFT($\not\pi$) is equivalent to the ERE (in the two-body case)

Nucleon-nucleon effective field theory without pions

Jiunn-Wei Chen (Washington U., Seattle), Gautam Rupak (Washington U., Seattle), Martin J. Savage (Washington U., Seattle and Jefferson Lab)

Feb 24, 1999

27 pages

Published in: *Nucl.Phys.A* 653 (1999) 386-412

e-Print: nucl-th/9902056 [nucl-th]

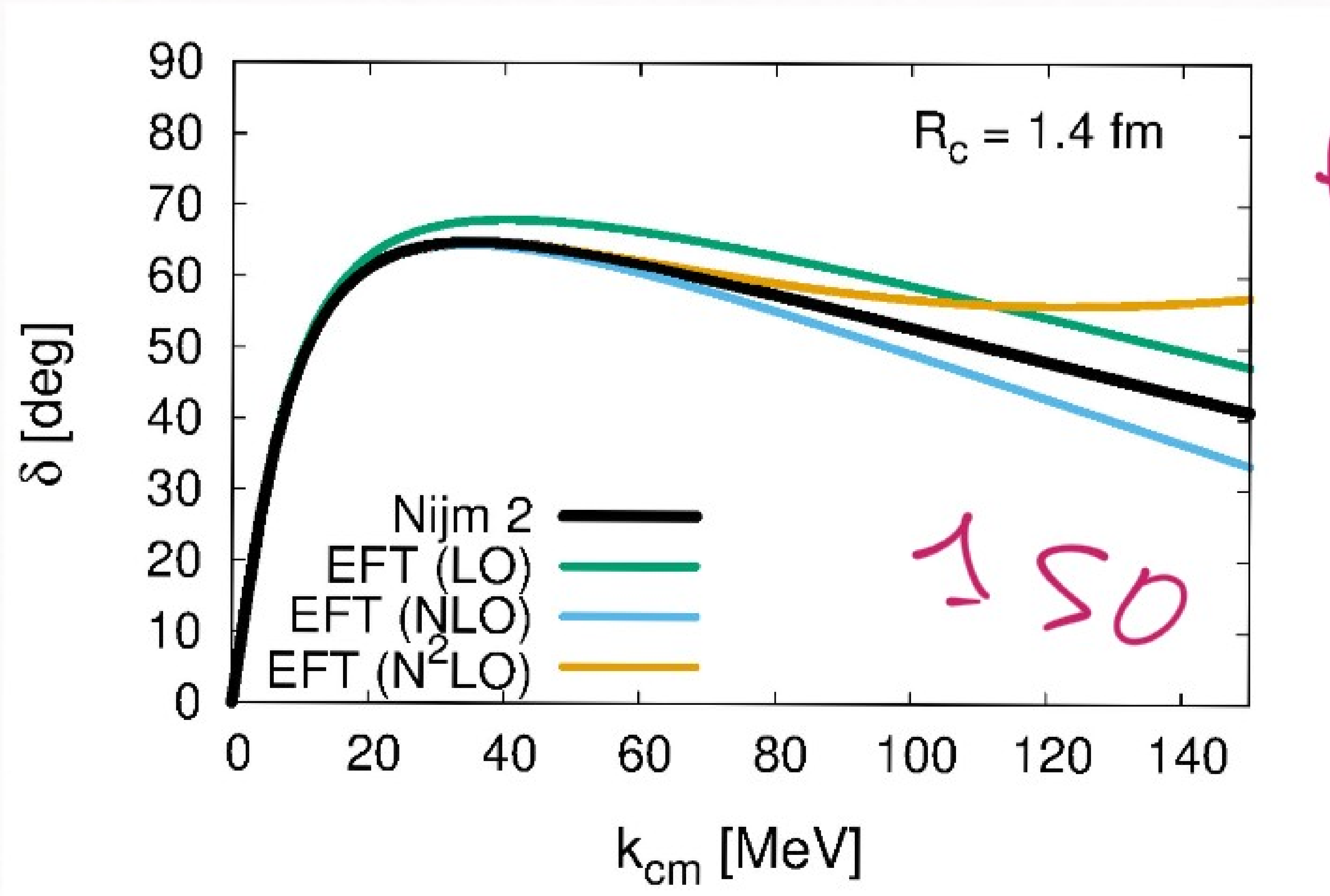
DOI: 10.1016/S0375-9474(99)00298-5

Report number: NT-UW-99-14, JLAB-THY-99-50

View in: OSTI Information Bridge Server, ADS Abstract Service

pdf links cite

→ You should be able to understand this one

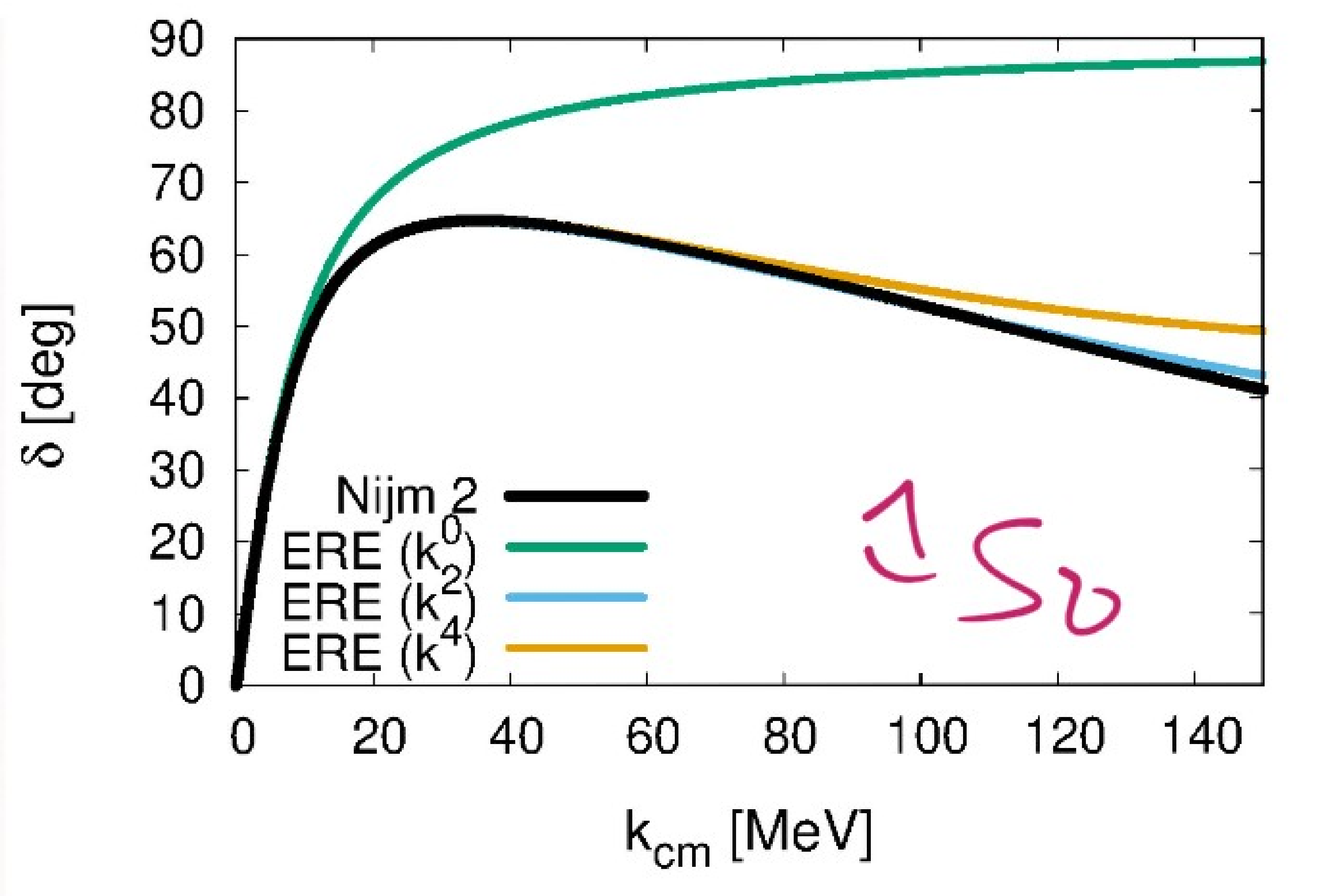


EFT(π̸)

→ SimPar

3

ERE



→ **CUTOFF DEPENDENCE**

not exact $\frac{d}{d R_c} \langle \psi | \hat{O} | \psi \rangle \neq 0$

but exact modulo higher order corrections

$$\frac{d}{d R_c} \langle \psi | \hat{O} | \psi \rangle = 0 \ \underline{\underline{\text{mod}}} \text{ "higher orders"}$$

$(\neq 0)$ → residual cutoff dependence

EXAMPLE: $u_k(R_c) = u_0(R_c)$ → fix it to the scattering length

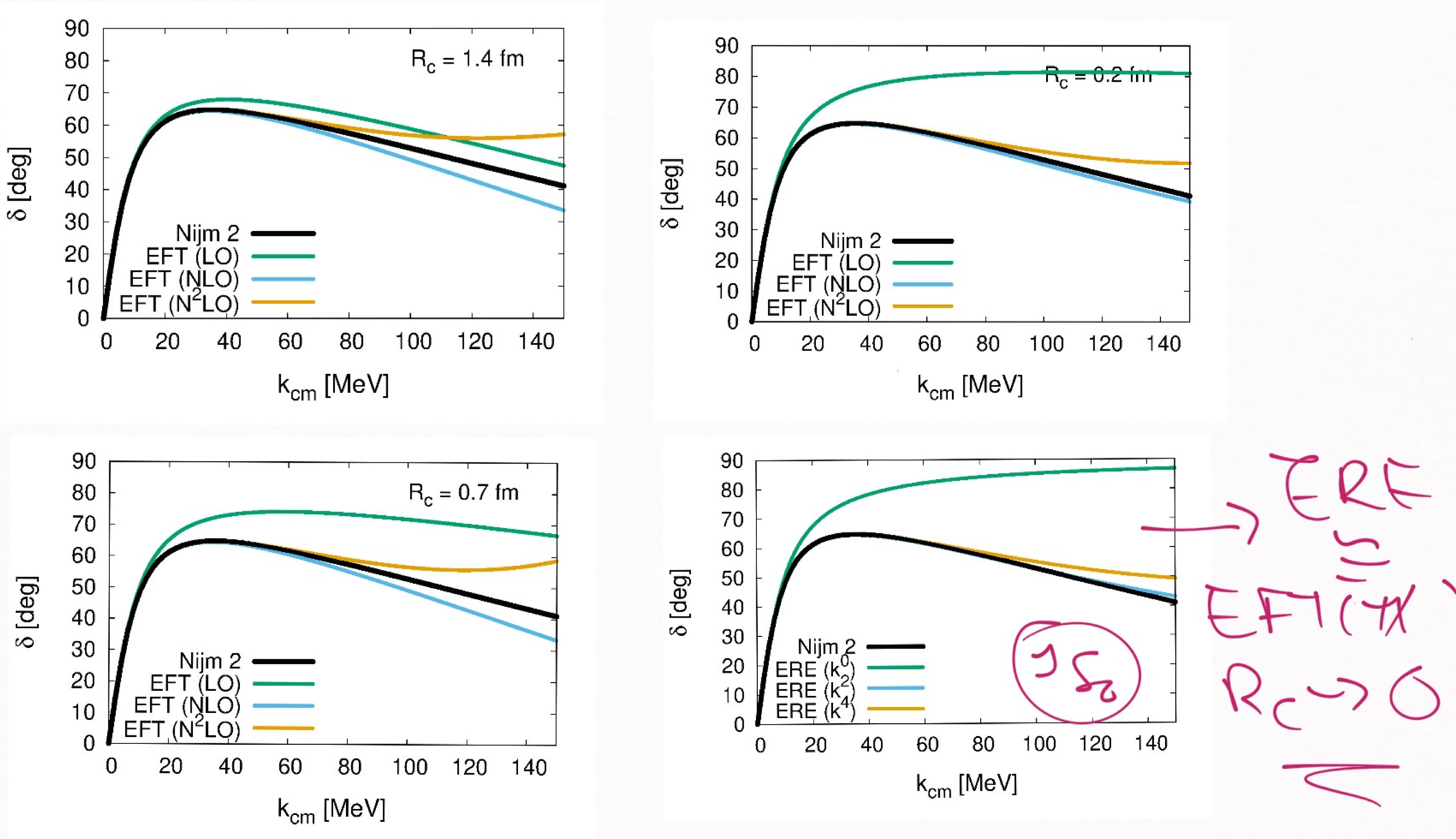
$$k\cot(kR_c + \delta) - k\cot kR_c = \text{constant}$$

$$k\cot\delta \to -\frac{1}{a_0} + \lambda_0 R_c k^2 + \ldots$$

$\frac{1}{2} r_0 k^2$ → $(\neq 0)$

In this example $\rightarrow$ $r_0 \sim \frac{1}{m_\pi}$

If $m_\pi R_c \gtrsim 1 \Rightarrow$ the residual r_0 we accidentally generate will be smaller than the physical one



R_c = 1.4 fm
δ [deg]
k_cm [MeV]
Nijm 2
EFT (LO)
EFT (NLO)
EFT (N2LO)
R_c = 0.2 fm
δ [deg]
k_cm [MeV]
Nijm 2
EFT (LO)
EFT (NLO)
EFT (N2LO)
R_c = 0.7 fm
δ [deg]
k_cm [MeV]
Nijm 2
EFT (LO)
EFT (NLO)
EFT (N2LO)
δ [deg]
k_cm [MeV]
Nijm 2
ERE (k0)
ERE (k2)
ERE (k4)
1S0
ERE
≃
EFT(#)
R_c → 0

→ CUTOFF INDEPENDENCE

often understood simply as the existence of $R_c \to 0$ limit

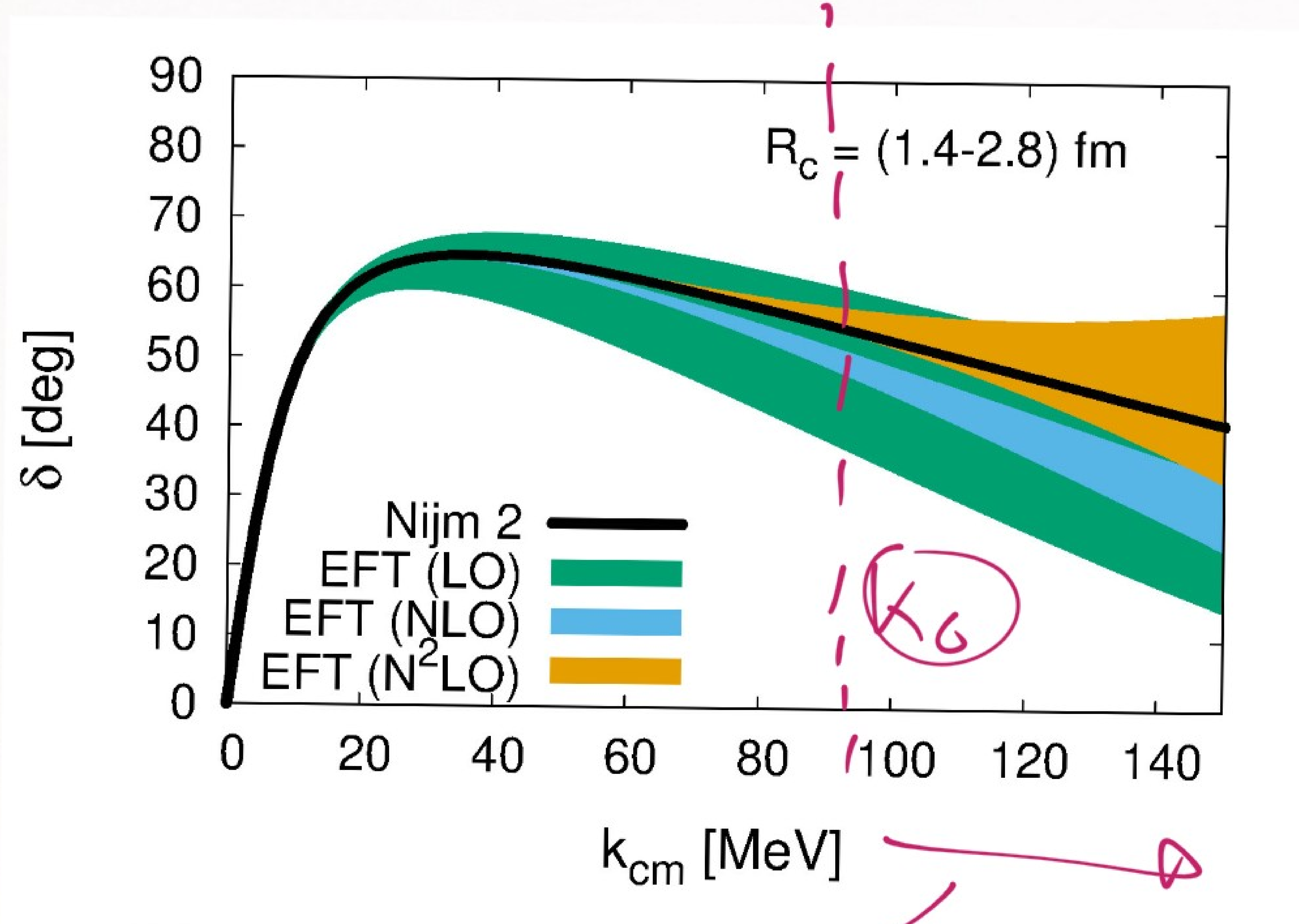
$$\text{EFT}(\not\pi) \xrightarrow[R_c \to 0]{} \text{ERE}$$

Residual cutoff dependence

$\rightarrow$ higher order effect] WHY?

$$\frac{d}{dR_c}\langle\psi|\tilde{O}|\psi\rangle = 0$$ modulo higher order correction

$\Rightarrow$ $(\neq 0)$ is a higher order effect (for $R \lesssim R_s$)



→ Used [residual cutoff dependence] to generate error bands ⊕

⇒

error bands grow (instead of shrinking)

⊕ → it can easily underestimate errors

$\Rightarrow$ $k > k_0$ ($k_0 \sim 80$-100 MeV)

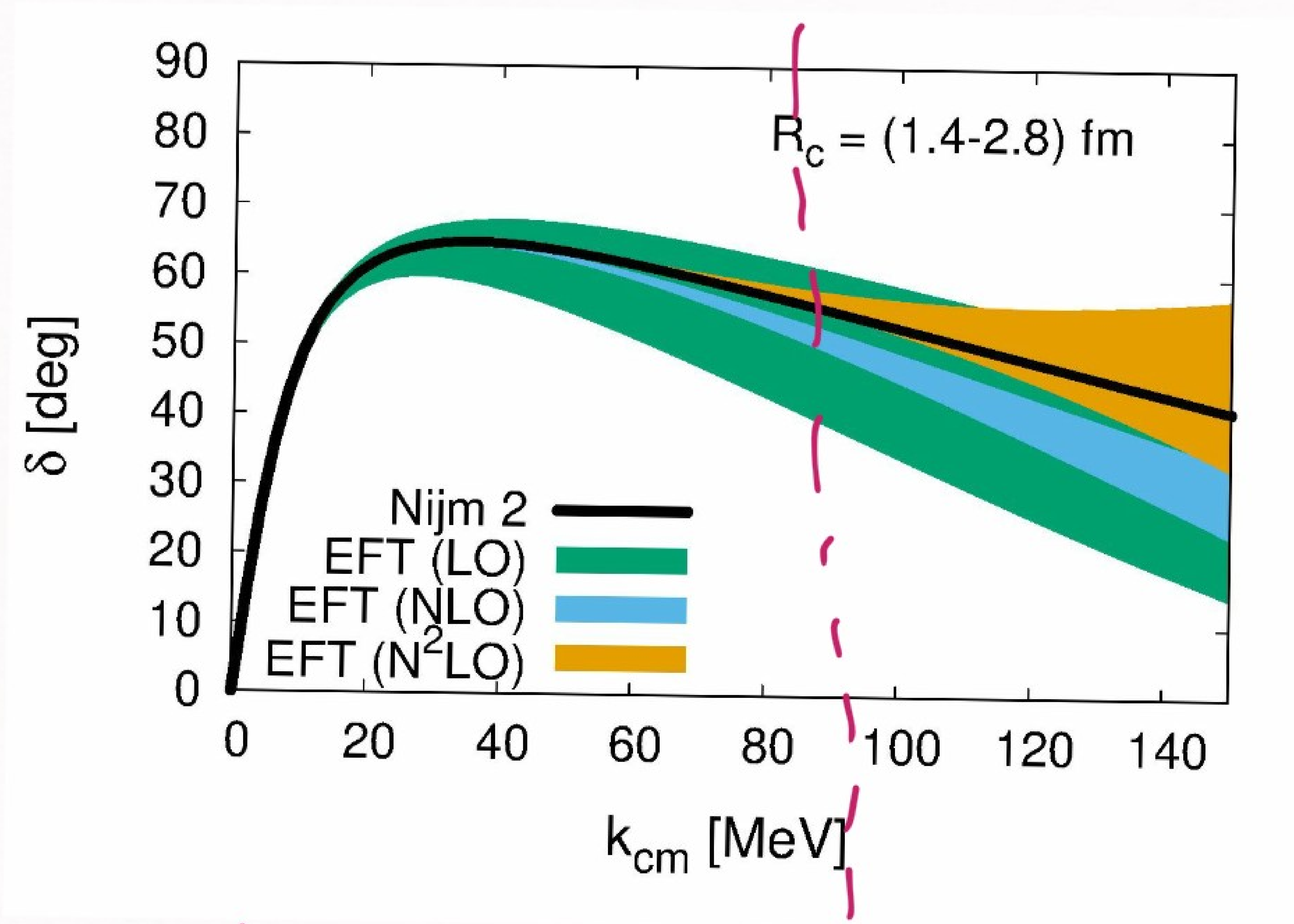
↪ error bands grow

↪ each order seems to be a bit worse than the previous one

$\Rightarrow$ EFT($\not\pi$) only converges for $k < m_\pi/2$

EFT's always have a radius of convergence

(EFT-s can only be used at low energies)



We can use pionful EFT here

EFT($\not\pi$) converges (useful)

EFT diverges ($\not\pi$) (not useful)

EFT (π)

1) $\simeq$ ERE

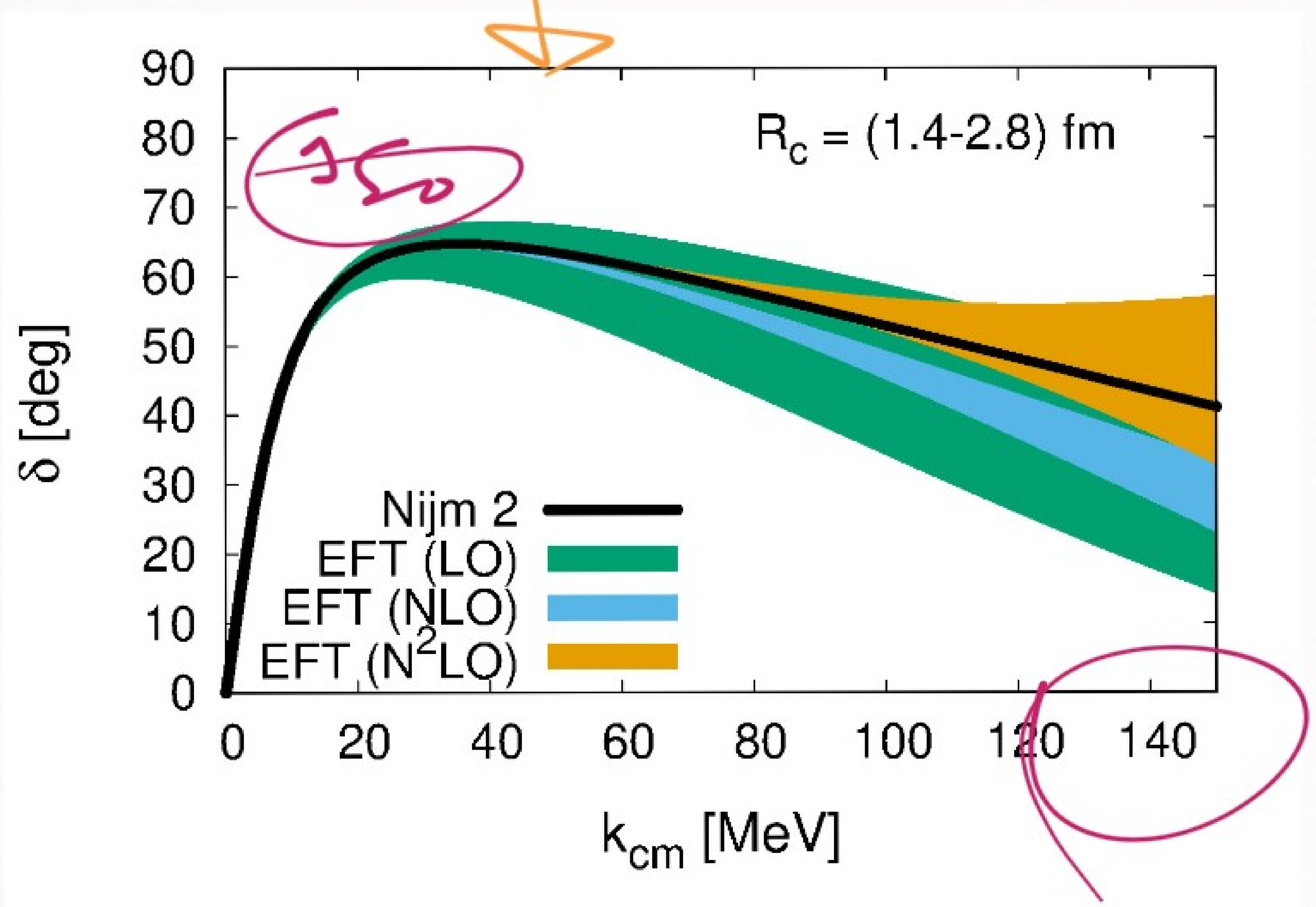
2) $\exists$ $R_c \to 0$ limit (but $\exists$ residual cutoff dependence)

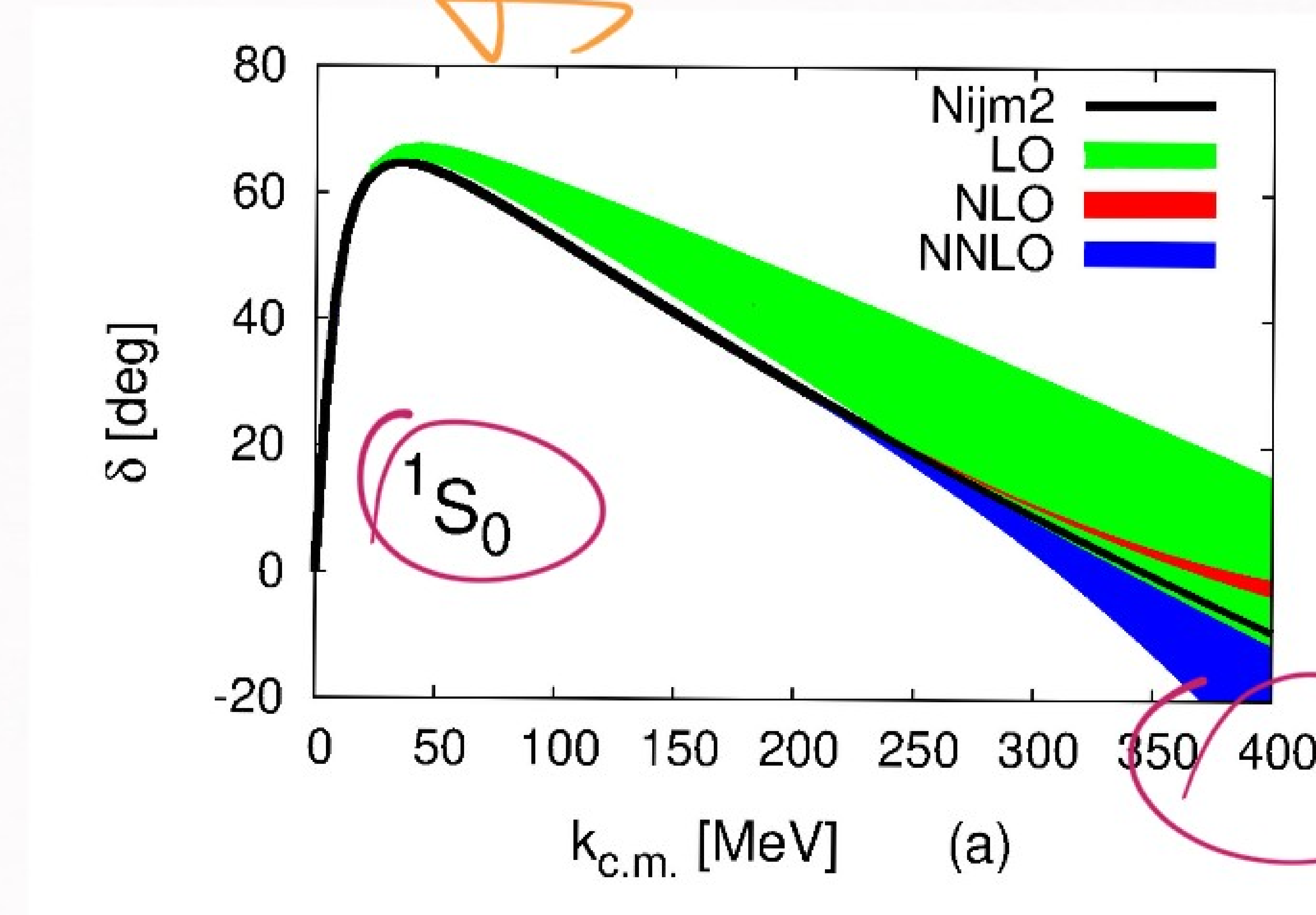
↳ smaller than EFT's truncation error

3) EFT's only work for $k < k_0$

↳ depends on the EFT used

EFT (π̸) → EFT (π)





m_π

$\frac{m_\pi}{2}$

Effective field theory of short range forces

U. van Kolck (Caltech, Kellogg Lab and Washington U., Seattle)

Aug 4, 1998

38 pages
Published in: *Nucl.Phys.A* 645 (1999) 273-302
e-Print: nucl-th/9808007 [nucl-th]
DOI: 10.1016/S0375-9474(98)00612-5
Report number: KRL-MAP-230, NT-UW-98-01
View in: ADS Abstract Service

pdf   cite

Nucleon-nucleon effective field theory without pions

Jiunn-Wei Chen (Washington U., Seattle), Gautam Rupak (Washington U., Seattle), Martin J. Savage (Washington U., Seattle and Jefferson Lab)

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→ TWO IMPORTANT PAPERS ABOUT EFT($\not\pi$)

PIONFUL EFT | EFT(π)

EFT($\not\pi$) $\rightarrow R_S \sim \frac{1}{m_\pi} \sim 1.4\ \mathrm{fm}$

$R < R_S$ $\rightarrow$ contacts

EFT(π) $\rightarrow R_S \sim \frac{1}{m_c}, \sqrt{\langle r^2 \rangle_{p,n}} \sim$ 0.5-1.0 fm

Including pions

→ Including chiral symmetry

(go back to first part of the course & check it)

→ [MAIN DIFFERENCE W/ EFT ($\not\pi$)]

EFT(TI) PROBLEMS

→ Regularization & Renormalization

→ Power counting

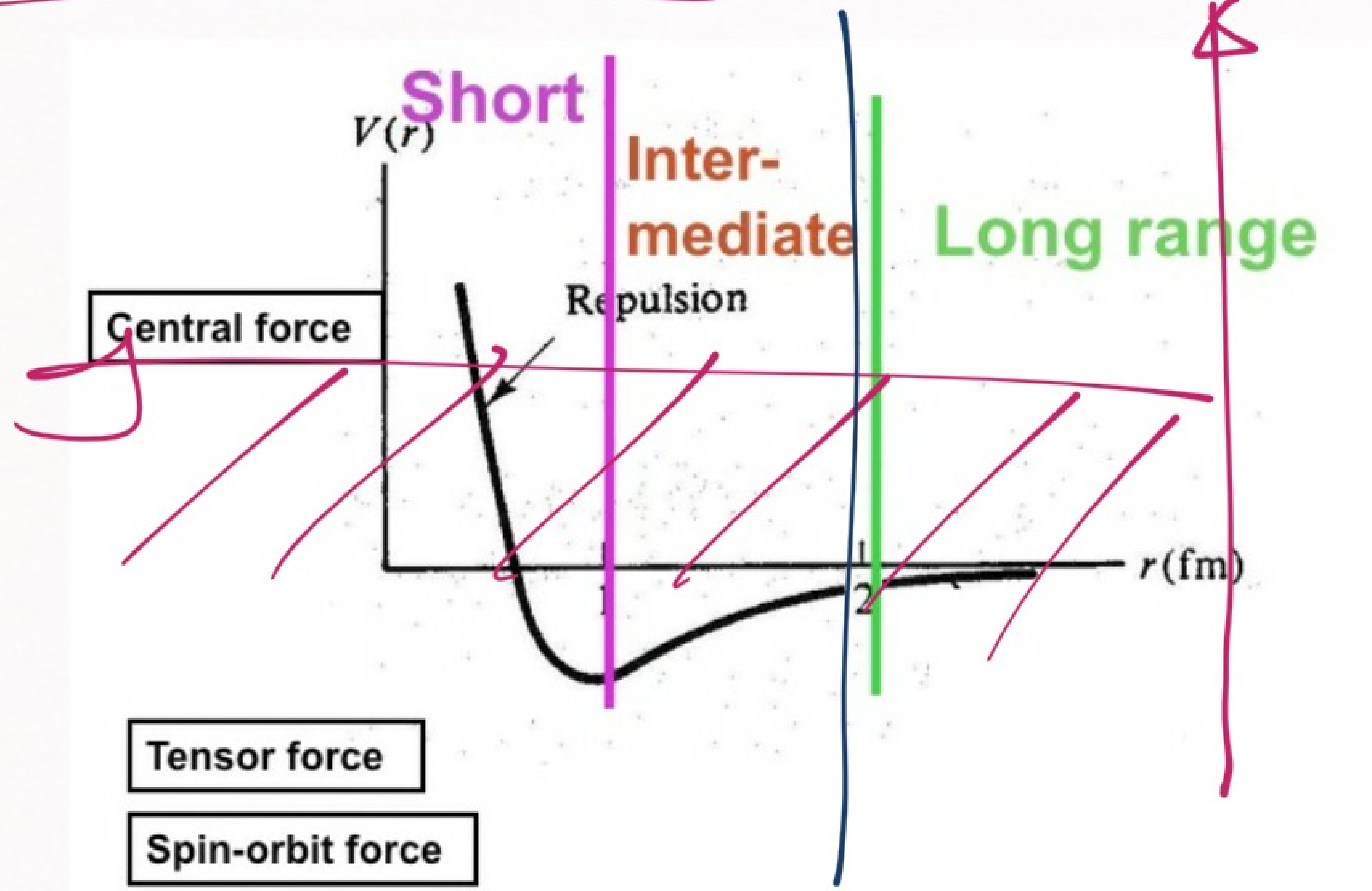
→ **OPEN PROBLEMS**

(good research problem)

KEY POINT

R_s for EFT($\not\pi$)

(no pions)



R_s for EFT(π) → pions

A little tour around EFT (TH):

1) (LO) → (V_{EFT}) → expand this in powers of $\frac{Q}{M}$

Q → low energy physics that we explicitly consider

M → high energy (which we ignore)

LO $\rightarrow$ 

(σ, ρ, ω, nucleon size, ...)

$$V_{LO} = V_C^{(0)} + V_\pi^{(0)}$$

$$V_C^{(0)} = C_S + C_T \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

$$V_\pi^{(0)} = -\frac{g_A^2}{4 f_\pi^2} \vec{\tau}_1 \cdot \vec{\tau}_2 \frac{\vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q}}{q^2 + m_\pi^2}$$

→ Regularize & Renormalize

p-space → $V^{(0)}_{LO}$ → $\langle \vec{p}' | V^{(0)}_{LO,Reg} | \vec{p} \rangle$

$$= \langle p' | V^{(0)}_{LO} | p \rangle \, f\left(\frac{|\vec{p}'|}{\Lambda}\right) f\left(\frac{|\vec{p}|}{\Lambda}\right)$$

(unreg)

(Example)

r-space

$V_{co}(\vec{r})$

$$V_{EFT}^{(reg)} = C_0(R_c)\,\frac{\delta(r-R_c)}{4\pi R_c^2} + V_\pi(\vec{r})\,\theta(r-R_c)$$

→ just a possible example of regularization ←

Problem w/ EFT(π) & Renormalization:

→ Appearance of singular interactions

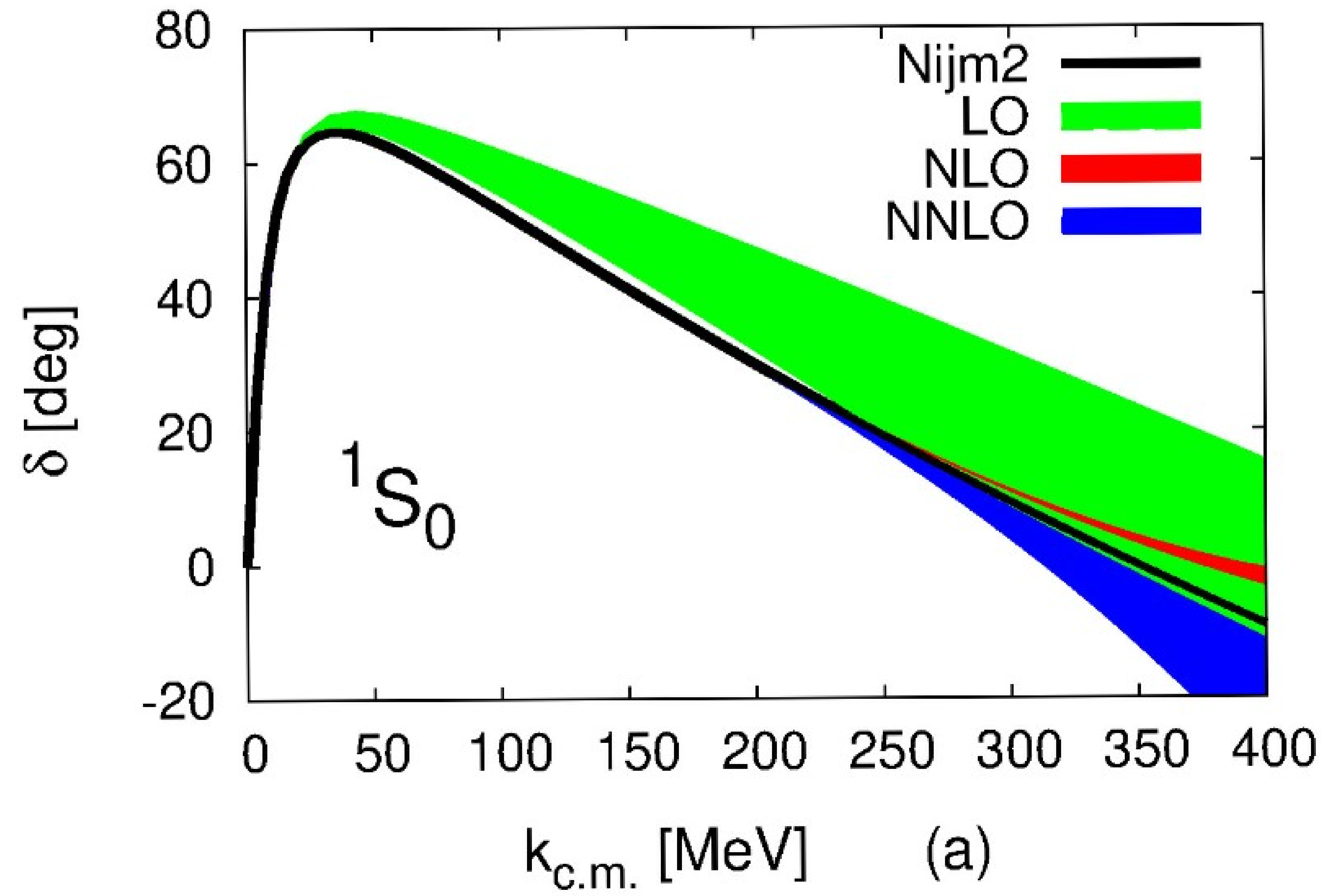
$V_{EFT} = V_C + V_F$

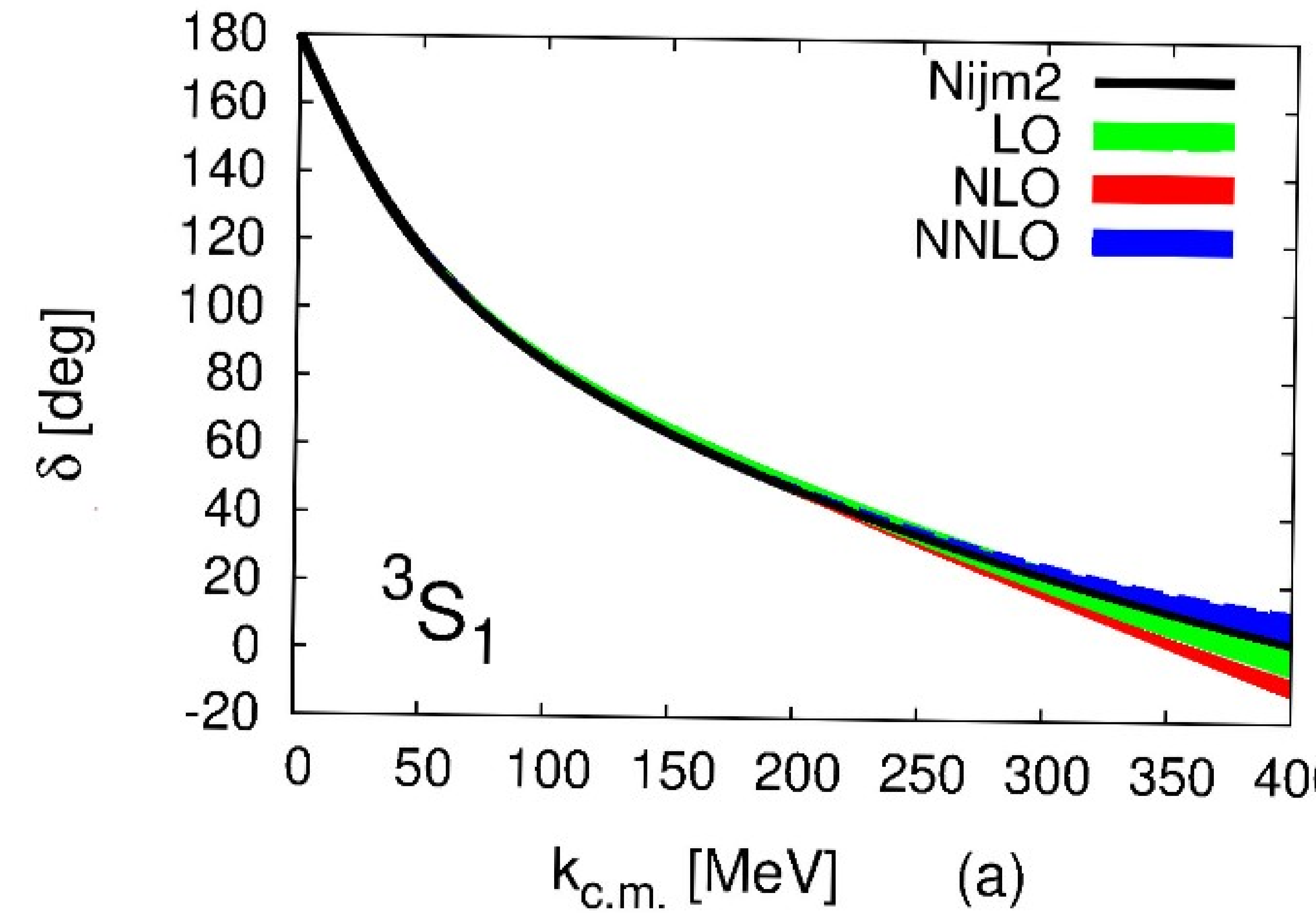
V_C → singular $\delta^{(3)}$-type → under control

V_F → singular $\frac{1}{r^n}$-type

finite-range potential → accepts a long-range expansion

$$V_F(\vec{r}) = \sum_n \frac{e^{-\lambda_n r}}{r^n} \sim \left(\frac{l}{r}\right)^n$$

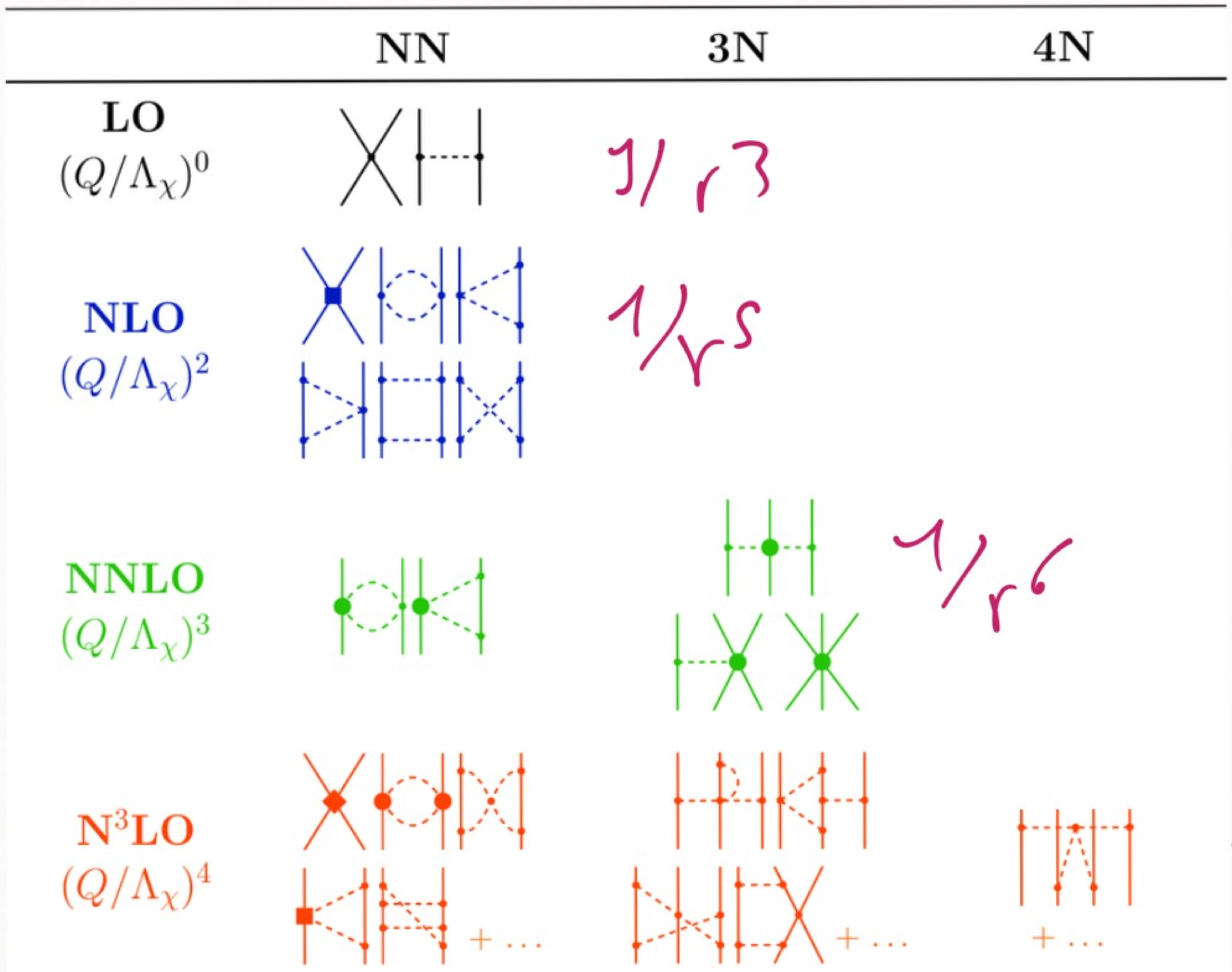




→ It can be renormalized
(but it's tricky)

Renormalized EFT w/ pions

→ tricky (subleading corrections perturbative, modify the power counting, etc)



$1/r^3$

$1/r^5$

$1/r^6$

$1/r^7$

EFT (χ) contributions

(NDA → Naive dimensional analysis)

IF ONE SIMPLE DOES THIS :

1) $V_{EFT} = \sum_{\nu} \left(V_C^{(\nu)} + V_F^{(\nu)} \right)$

2) $V_{EFT} \longrightarrow V_{EFT} \times \text{regulator}$

3) Include it in Schrödinger.

$\Rightarrow$ This doesn't work well

Renormalization of one-pion exchange and power counting

A. Nogga (Julich, Forschungszentrum), R.G.E. Timmermans (Groningen, KVI), U. van Kolck (Arizona U.)

Jun 2, 2005

19 pages

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DOI: 10.1103/PhysRevC.72.054006

Report number: FZJ-IKP-TH-2005-19

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↳ this paper explains some of the problems w/ Reno in EFT(π)

Message to take:

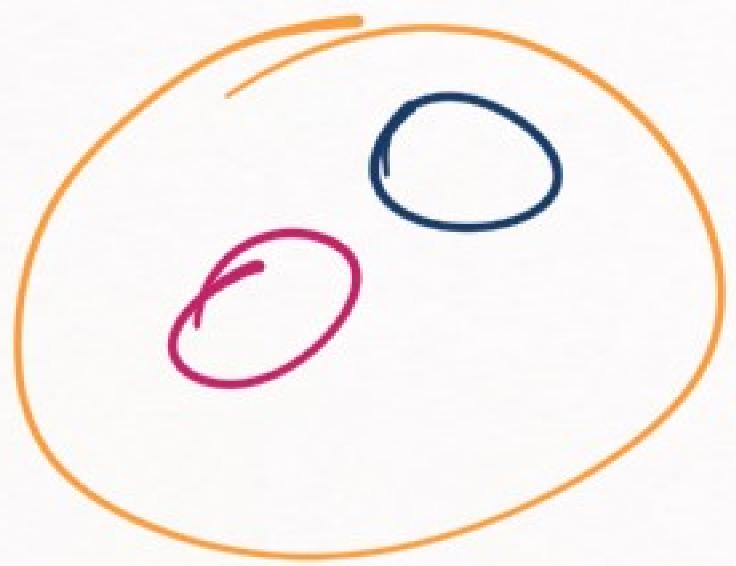
1) EFT (H) more complicated
2) Simple approaches → $\nexists$ $R_c \to 0$ limit
3) We know how to solve this
4) But it is complicated to do
5) Most people keep doing (2)

→ It's a good research topic (w/ looks of polemics)

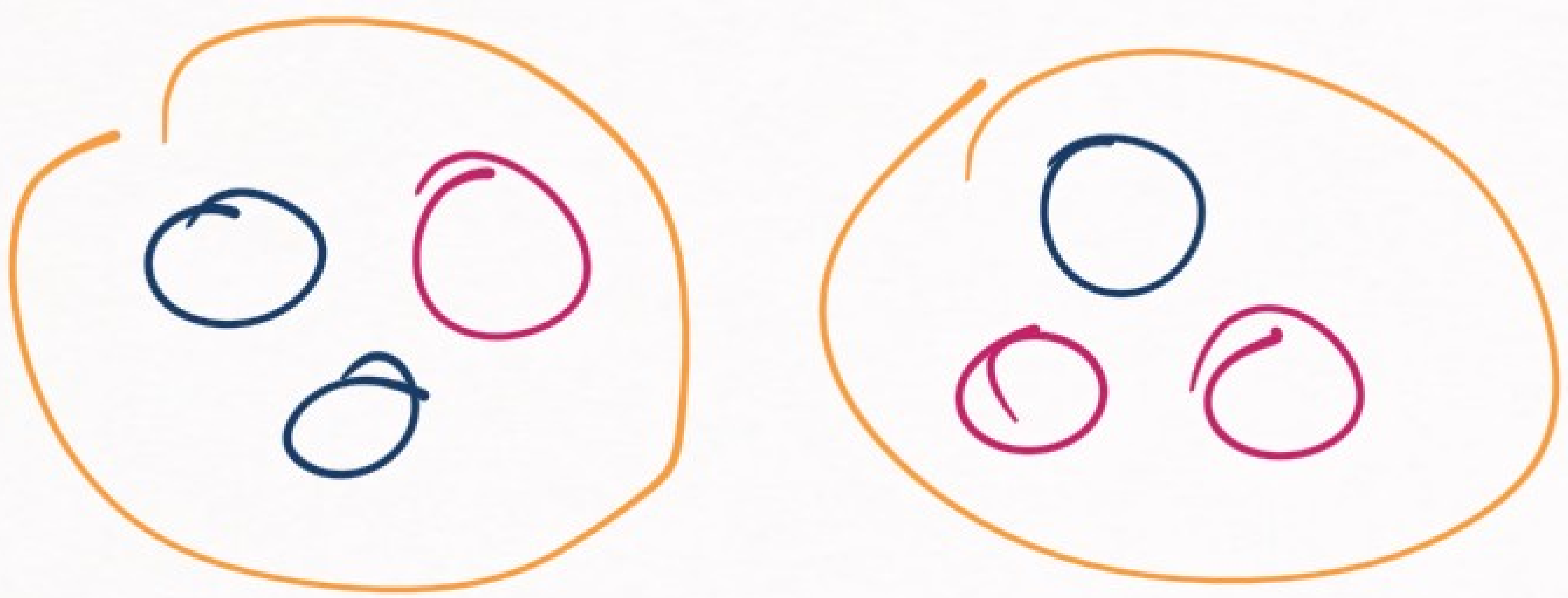
→ a bit advance for this course

→ NUCLEAR STRUCTURE

Why nuclear structure?

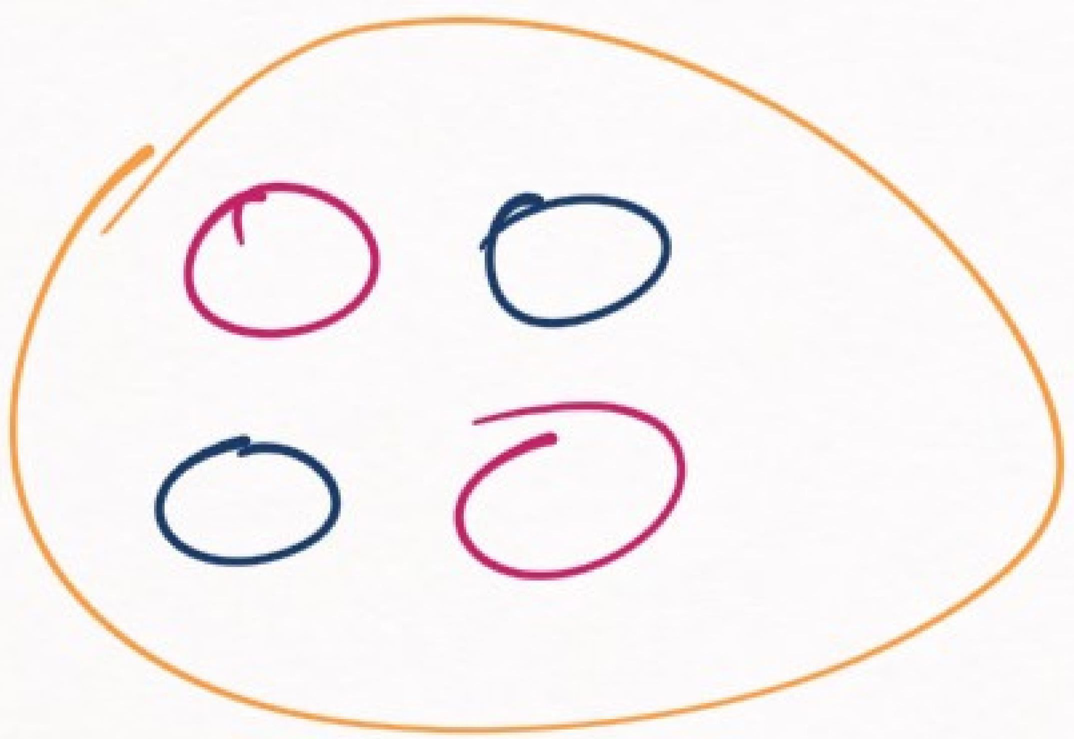


A = 2 Nucleons (Deuteron)

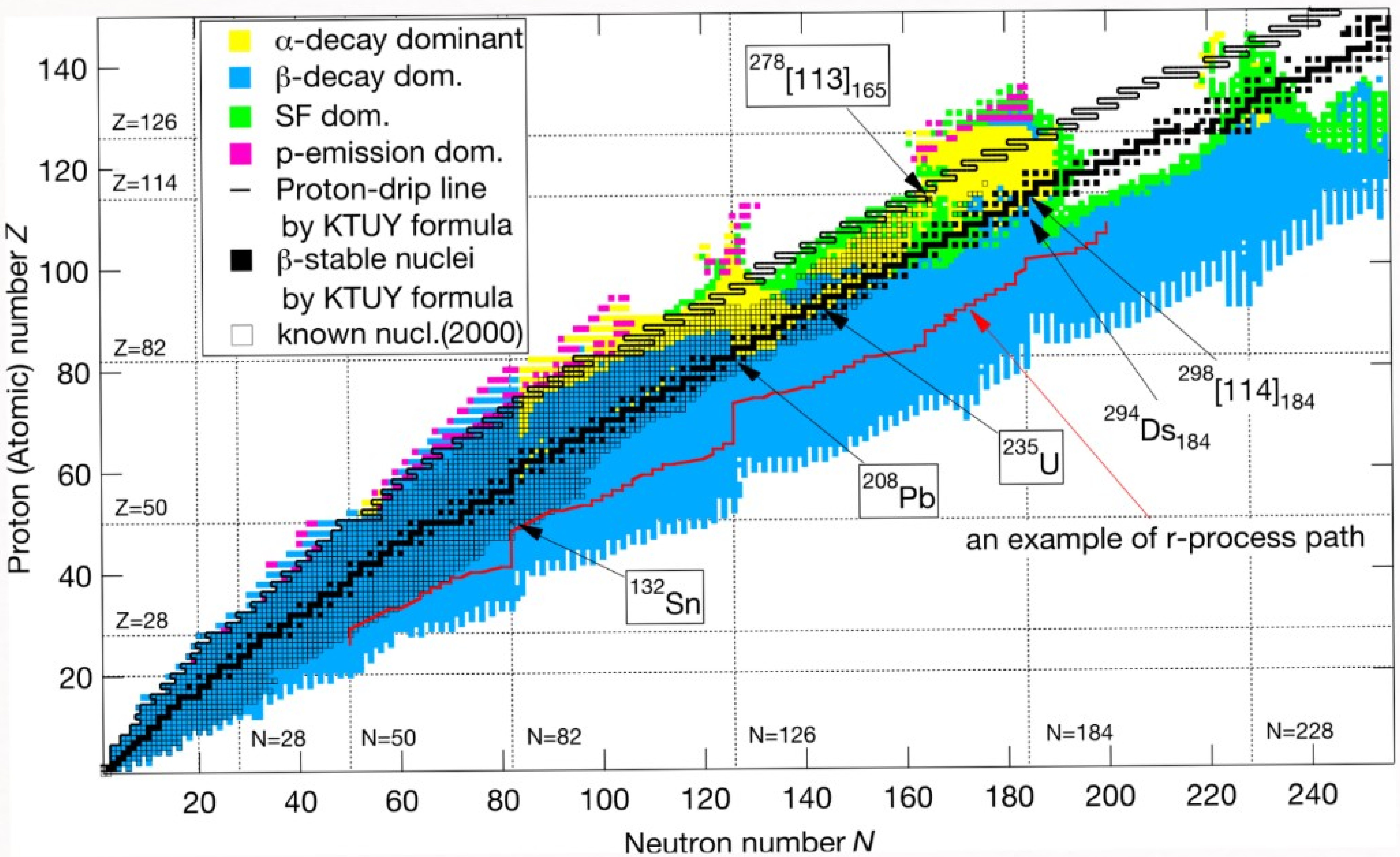


A = 3 Nucleons

(Triton & Helium-3)



→ A=4 (Helium-4)



→ How to describe all these nuclei?

→ Simplification

A small $\Rightarrow$ exact methods to solve these nuclei

($A=2 \rightarrow$ Schrödinger,

$A=3 \rightarrow$ Faddeev, ...)

A big $\Rightarrow$ we need to think about the general properties/structure of these nuclei

(simplifications that make sense)

SHOPPING LIST → WHAT FEATURES OF NUCLEI ARE WORTH DESCRIBING?

1) Binding energy

2) Size

3) Angular momentum & parity

4) Magnetic dipole moments

5) Stability / Decay

[NUCLEAR PROPERTIES]

1) BINDING ENERGY

Deuteron → $B_d \simeq (m_p + m_n) - m_d \simeq 2.2\ MeV$

Triton → $B_t \simeq (m_p + 2m_n) - m_t \simeq 8.5\ MeV$

$$B(Z,N) = (Z m_p + N m_n) - M(Z,N)$$

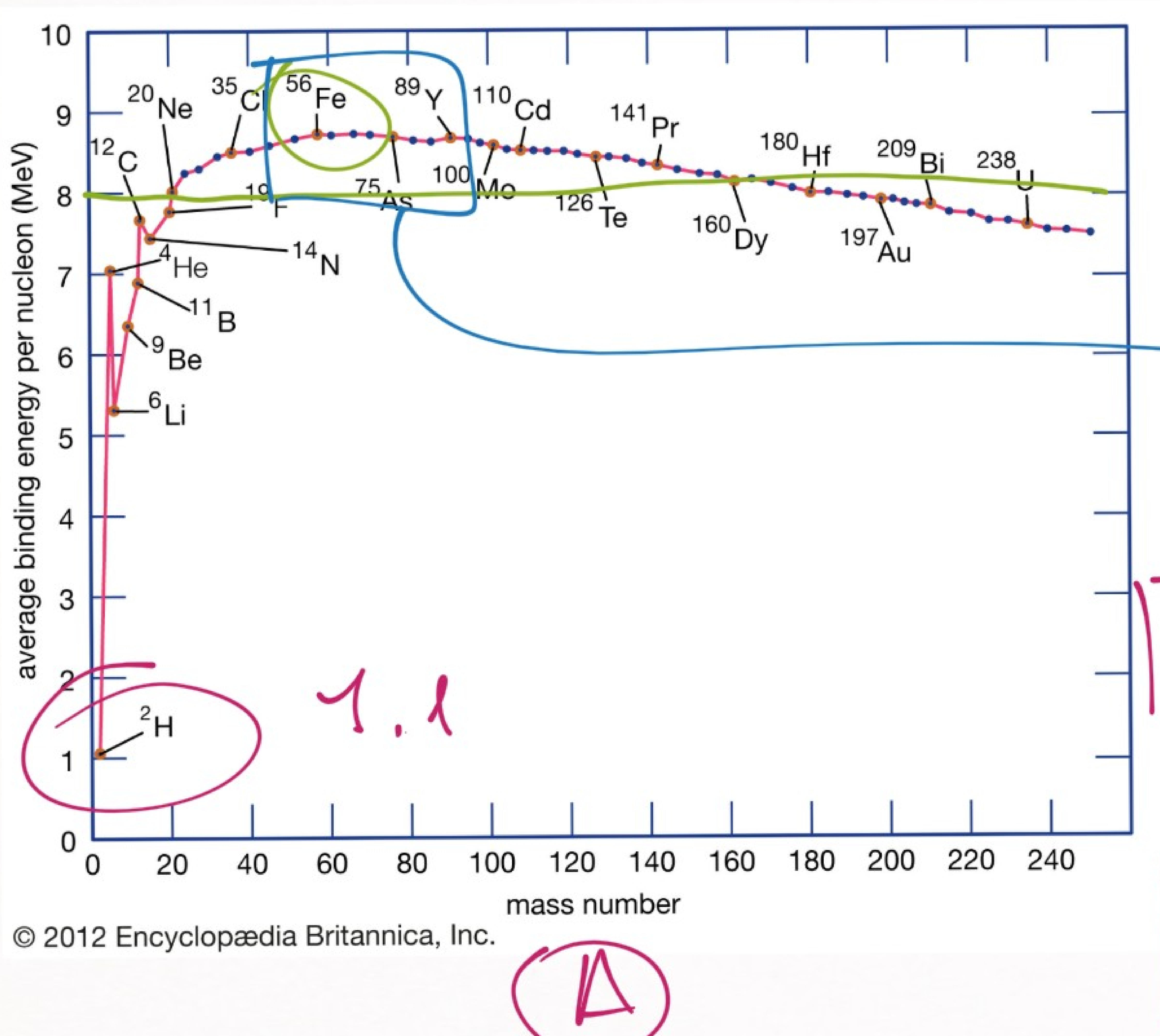
$$B(Z,N) = (Z m_p + N m_n) - M(Z,N)$$

$Z \rightarrow$ # of protons
$N \rightarrow$ # of neutrons

$\Big\}$ $\underline{\underline{A = Z + N}}$

Interesting finding $\rightarrow$ [SATURATION] $\left[\frac{B}{A} \sim 8 \text{ MeV/nucleon} \right]$

$(A \gtrsim 20\text{-}30)$



→ Observation 1

- 56 Fe has the largest B/A ratio (approx)

A < 56 → fusion is possible

A > 56 → fission is possible

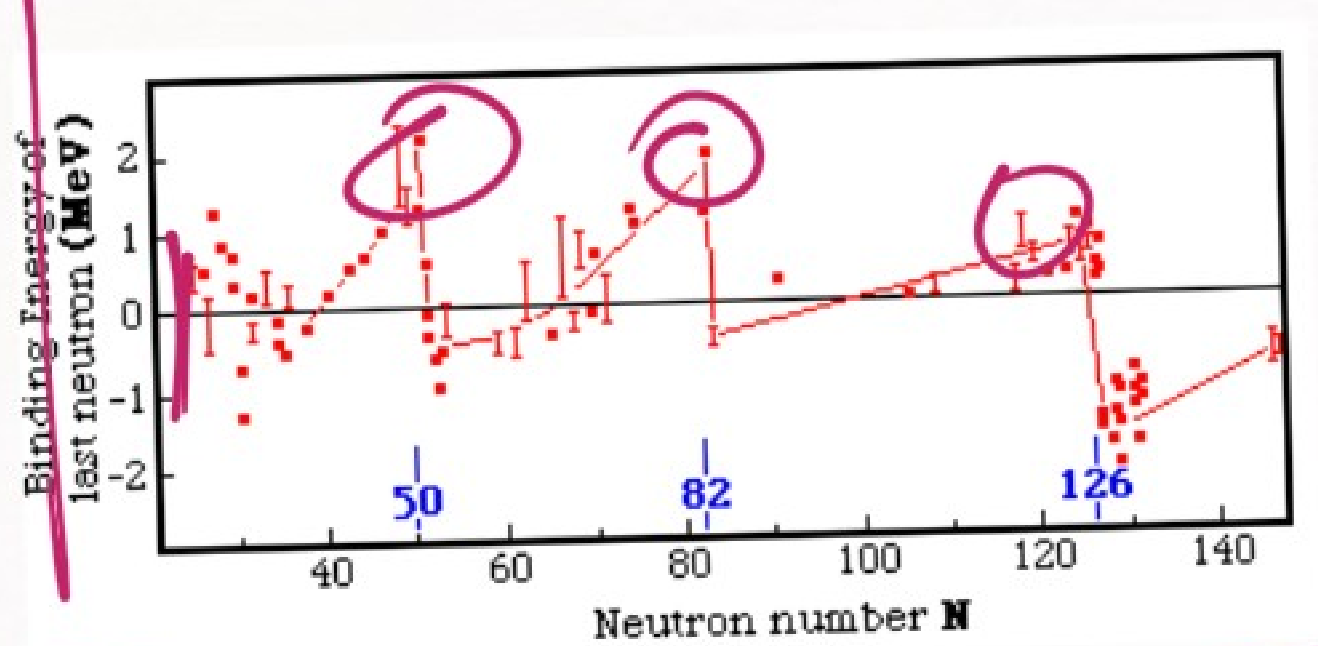
Separation energy → energy required to knock off a nucleon

↓

$$S_p(Z,N) = B(Z,N) - B(Z-1,N)$$

$$S_n(Z,N) = B(Z,N) - B(Z,N-1)$$

→ $\exists$ a pattern



$S_n(Z, N)$ → Special values of N for which S_n is very large

—— ——

→ The same will be true for the proton separation energy
Z

(MAGIC NUMBERS) → $N = 2, 8, 20, 28, 50, 82, 126, \ldots$

↙

Will give us clues about nuclear structure

(Shell model)

Another type of separation energy

$$S_\alpha(N,Z) = B(Z,N) - B(Z-2,N-2) - B(2,2)$$

→ energy require to extract an α-particle (^4He nucleus) from a nucleus

$A > 150 \Rightarrow \boxed{S_\alpha < 0}$

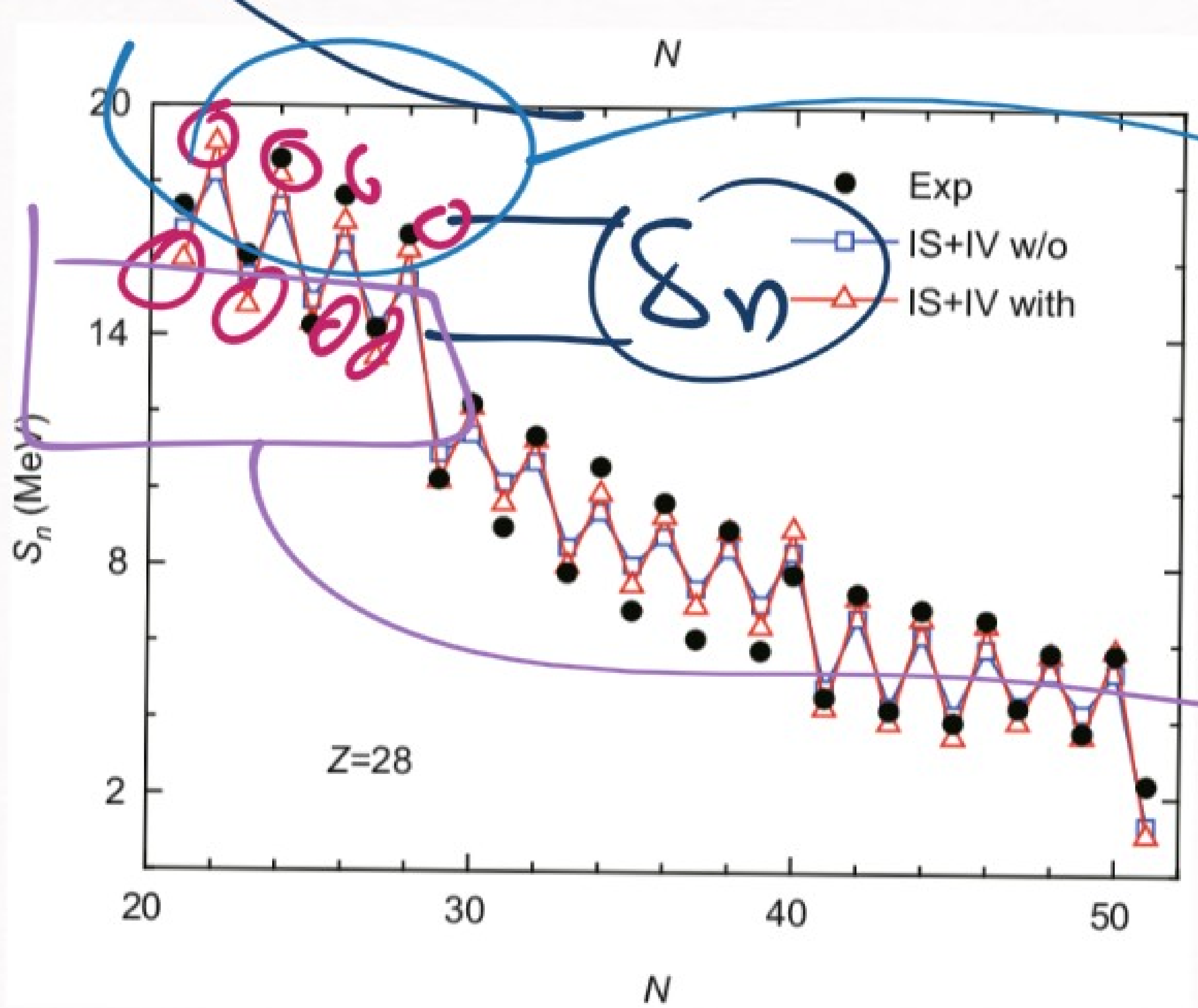
↙

α-particles can be spontaneously emitted from most heavy nuclei

Another type of separation energy

$$\delta_n = S_n(Z,N) - S_n(Z,N-1)$$

→ difference between N even/odd



Even N

Odd N

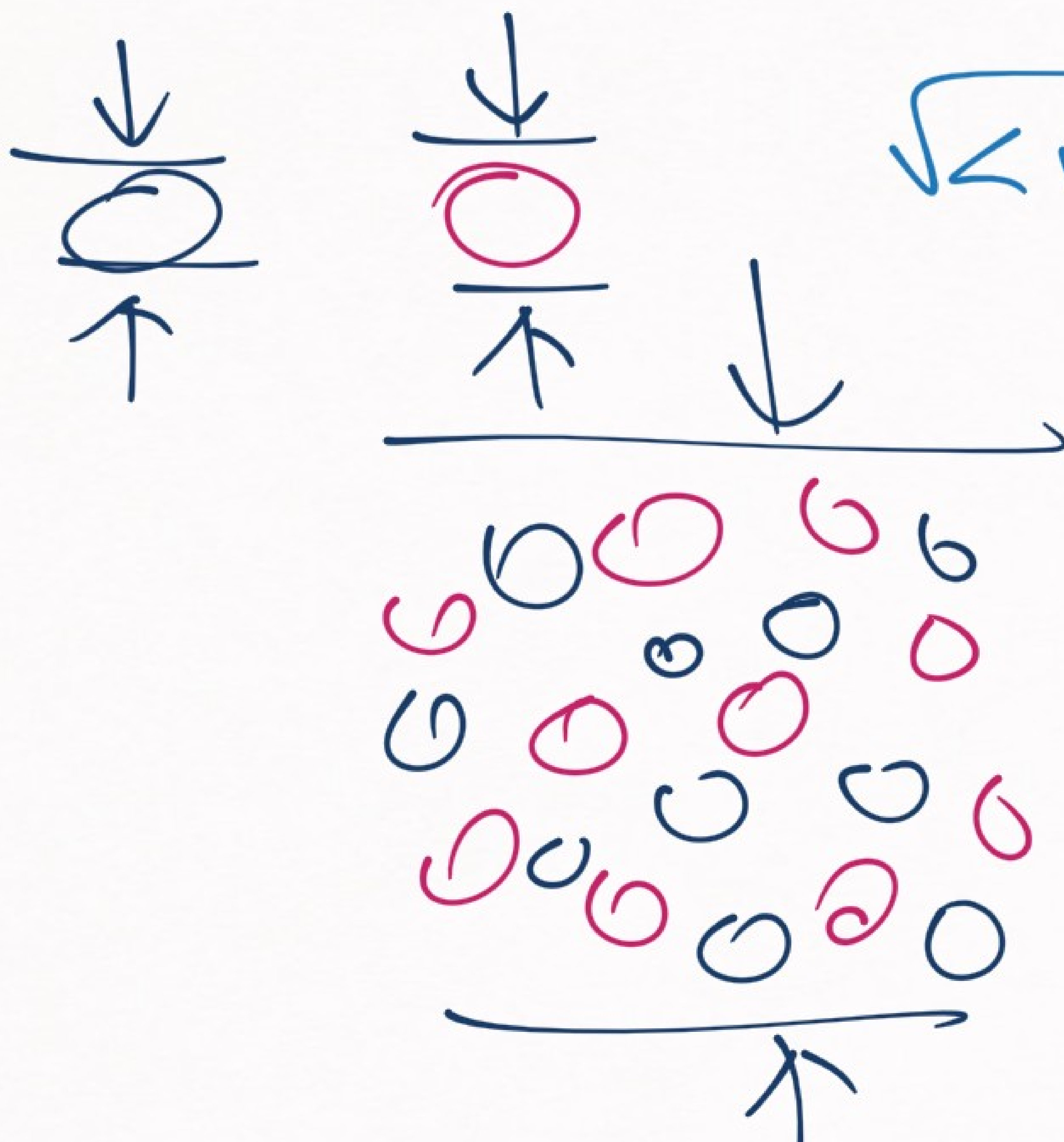
CONFIGS W/ EVEN N, Z

→ more stable (more bound)

NUCLEI W/ ODD N, Z ARE MORE RARE

NUCLEAR PROPERTY #12

2) NUCLEAR SIZE



$\sqrt{\langle r^2 \rangle} \sim 0.5 - 1.0$ fm

$R^3 \sim A\, r_0^3$ — avg separation of nuclei

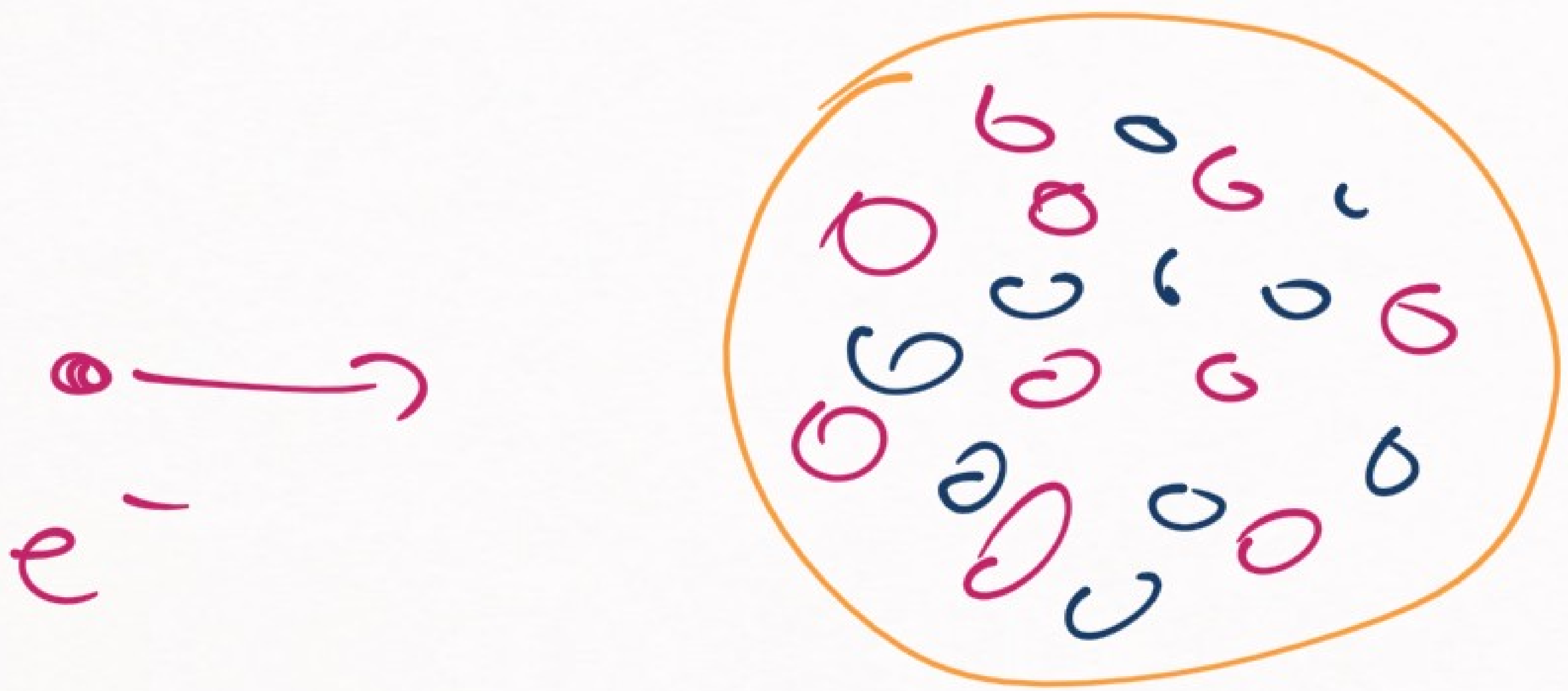
w/ $r_0 \sim 1.2 - 1.3$ fm

$R \sim A^{1/3} r_0$ → find size of most nuclei

→ How do we know this?

Hofstadter experiment

↳ Form factors of nuclei



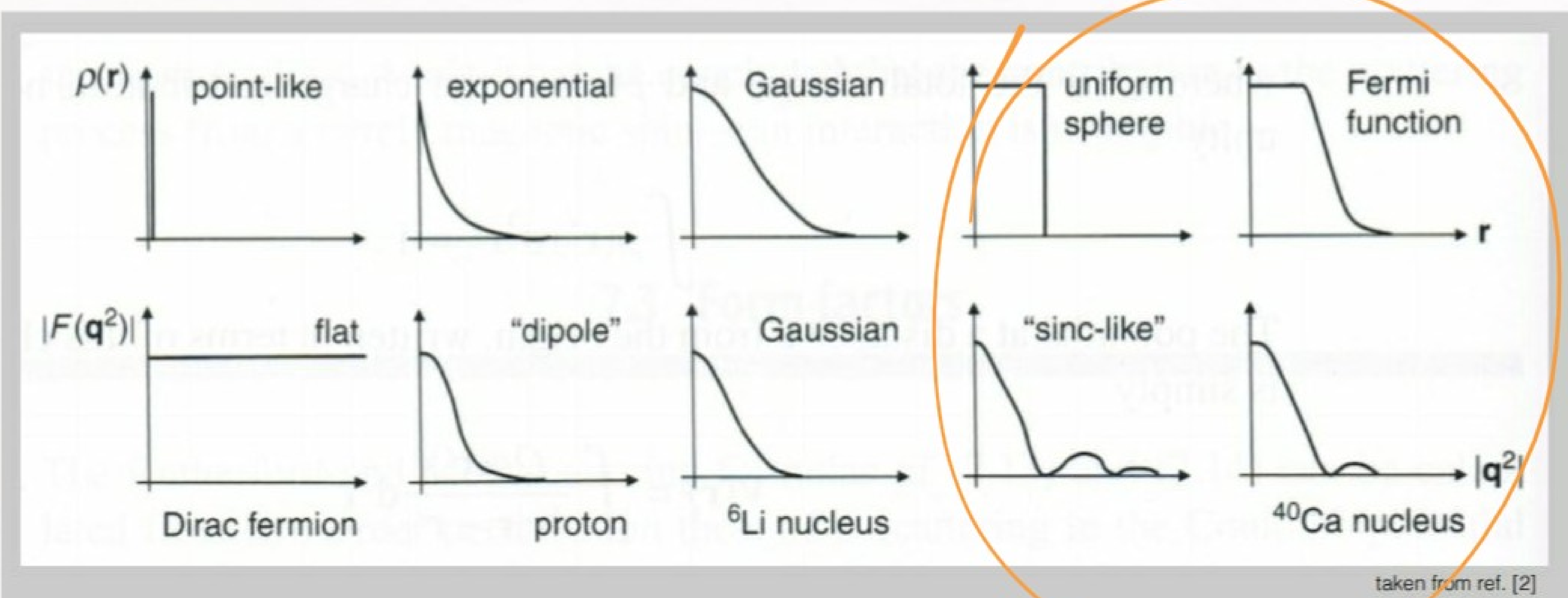
$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{point-like}} |F(\vec{q}^{\,2})|^2$$

Form-factor

internal structure of a nucleus

$$F(\vec{q}^2) = \int d^3\vec{r}\, |\psi(\vec{r})|^2\, e^{-i\vec{q}\cdot\vec{r}}$$

$\rho(\vec{r}) \rightarrow$ charge distribution



$\rightarrow \rho$

$\rightarrow F(\vec{q}^2)$

4 point-like

→ THE END ←

(for today)

Next pages → EFT readings →

Approaches using NDA (not renormalizable):

#1
Accurate charge dependent nucleon nucleon potential at fourth order of chiral perturbation theory
D.R. Entem (Idaho U. and Salamanca U.), R. Machleidt (Idaho U.) (Apr 8, 2003)
Published in: *Phys.Rev.C* 68 (2003) 041001 • e-Print: nucl-th/0304018 [nucl-th]
pdf DOI cite 1,194 citations

→ the classic potential

#2
The Two-nucleon system at next-to-next-to-next-to-leading order
E. Epelbaum (Jefferson Lab), W. Glockle (Ruhr U., Bochum), Ulf-G. Meissner (Bonn U., HISKP and Julich, Forschungszentrum) (May 19, 2004)
Published in: *Nucl.Phys.A* 747 (2005) 362-424 • e-Print: nucl-th/0405048 [nucl-th]
pdf links DOI cite 596 citations

→ the other classic potential

#2
Local chiral effective field theory interactions and quantum Monte Carlo applications
A. Gezerlis (Guelph U.), I. Tews (Darmstadt, Tech. Hochsch. and Darmstadt, EMMI), E. Epelbaum (Ruhr U., Bochum), M. Freunek (Ruhr U., Bochum), S. Gandolfi (Los Alamos) et al. (Jun 2, 2014)
Published in: *Phys.Rev.C* 90 (2014) 5, 054323 • e-Print: 1406.0454 [nucl-th]
pdf DOI cite 139 citations

→ the easy-to-program potential

Precision nucleon-nucleon potential at fifth order in the chiral expansion #8

E. Epelbaum (Ruhr U., Bochum), H. Krebs (Ruhr U., Bochum), U.G. Meißner (Bonn U., HISKP and IAS, Julich and JCHP, Julich and Julich, Forschungszentrum) (Dec 15, 2014)

Published in: *Phys.Rev.Lett.* 115 (2015) 12, 122301 • e-Print: 1412.4623 [nucl-th]

pdf DOI cite 244 citations

→ one of the latest implementations

→ [The problem w/ Weinberg counting] ←

Renormalization of one-pion exchange and power counting

A. Nogga (Julich, Forschungszentrum), R.G.E. Timmermans (Groningen, KVI), U. van Kolck (Arizona U.)

Jun 2, 2005

19 pages

Published in: *Phys.Rev.C* 72 (2005) 054006

e-Print: nucl-th/0506005 [nucl-th]

DOI: 10.1103/PhysRevC.72.054006

Report number: FZJ-IKP-TH-2005-19

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pdf cite

→ the big problem

Nucleon - nucleon scattering from effective field theory #5

David B. Kaplan (Washington U., Seattle), Martin J. Savage (Carnegie Mellon U.), Mark B. Wise (Caltech) (May 3, 1996)

Published in: *Nucl.Phys.B* 478 (1996) 629-659 • e-Print: nucl-th/9605002 [nucl-th]

pdf DOI cite 335 citations

→ But ∃ a previous problem discovered long ago

→ Renormalizable implementations ←

Perturbative renormalizability of chiral two pion exchange in nucleon-nucleon scattering

M.Pavon Valderrama (Julich, Forschungszentrum and JCHP, Julich) (Dec 3, 2009)

Published in: *Phys.Rev.C* 83 (2011) 024003 • e-Print: 0912.0699 [nucl-th]

pdf DOI cite 91 cita

→ r-space (by me)

Short-range nuclear forces in singlet channels

#6

Bingwei Long (Jefferson Lab), C.J. Yang (Jefferson Lab) (Feb 20, 2012)

Published in: *Phys.Rev.C* 86 (2012) 024001 • e-Print: 1202.4053 [nucl-th]

pdf links DOI cite 54 citations

Renormalizing Chiral Nuclear Forces: Triplet Channels

#7

Bingwei Long (Jefferson Lab), C.J. Yang (Arizona U. and Ohio U., Inst. Nucl. Part. Phys.) (Nov 17, 2011)

Published in: *Phys.Rev.C* 85 (2012) 034002 • e-Print: 1111.3993 [nucl-th]

pdf links DOI cite 68 citations

Renormalizing chiral nuclear forces: a case study of 3P0

#8

Bingwei Long (Jefferson Lab), C.J. Yang (Arizona U. and Ohio U., Inst. Nucl. Part. Phys.) (Aug 4, 2011)

Published in: *Phys.Rev.C* 84 (2011) 057001 • e-Print: 1108.0985 [nucl-th]

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Feedback

→ p-space implementation (by Bingwei & Jerry)