NUCLEAR PHYSICS 16

Tensor forces

$\boxed{\text{RECAP}}$ $\longrightarrow$ Summary of T-matrix

1) T-matrix $\longrightarrow$ $T = V + V G_0 T$ (LSE)

(difficult in general)

2) T-matrix is easy to solve analytically for a few selected potentials

$\langle \vec{p}\,' | V | \vec{p} \rangle = \lambda\, g(p')\, g(p)$

(separable potentials)

2') $\langle \vec{p}' | T(E) | \vec{p} \rangle = \tau(E)\, g(p')\, g(p)$ ansatz for separable potentials

$$\tau(E) = \frac{1}{1/\lambda - I(E)} , \quad I(E) = \int \frac{d^3 \ell}{(2\pi)^3} \frac{g^2(\ell)}{E - \frac{\ell^2}{2\mu}}$$

Σ

$-\!\!\otimes\!\!-$

3) $\langle \vec{p}\,' | V | \vec{p} \rangle = \underbrace{C_0(\Lambda) f\left(\frac{\vec{p}\,'}{\Lambda}\right) f\left(\frac{\vec{p}}{\Lambda}\right)}$

contact range potential + separable regulator

$f(x) \xrightarrow[x \to 0]{} 1$ & $f(x) \xrightarrow[x \to \infty]{} 0$

3') $\Lambda \to$ auxiliary parameter

(a parameter of the theory, but not a physical parameter)

$$\boxed{\frac{d}{d\tau}\tau(t;\Lambda) \simeq 0} \longrightarrow \underline{C_0 = C_0(\Lambda)}$$

Bound states w/ T-matrix

5) $T(E)$ has poles at $E \to E_B$

$$T(E) \underset{E \to E_B}{\longrightarrow} \frac{\mathrm{Res}\, T(E)}{E - E_B} \qquad \mathrm{Res}\, T(E) = V|B\rangle\langle B|V$$

$$= G_0^{-1}(E_B)\,|B\rangle\langle B|\,G_0^{-1}(E_B)$$

6) $|B\rangle = G_0(E)\,V\,|B\rangle \to$ bound state equation (BSE)

7) $\langle \vec{p}' | V | \vec{p} \rangle = \lambda\, g(p)\, g(p')$ → separable V

$$\langle \vec{p} | B \rangle = N \frac{g(p)}{p^2 + \gamma^2}$$

$$E_B = -\frac{\gamma^2}{2\mu}$$

$$\int \frac{d^3\vec{p}}{(2\pi)^3} \left| \langle \vec{p} | B \rangle \right|^2 = 1$$ → obtain N

8) Pro tip: use the vertex functions
(instead of the wave function)

$$|B\rangle = C_0(E)\,|\phi_B\rangle$$

8.a) $\tau(E) \xrightarrow[E \to E_B]{} \dfrac{|\phi_B\rangle\langle\phi_B|}{E - E_B}$

8.b) $|\phi_B\rangle = V C_0\,|\phi_B\rangle \rightarrow$ easier

3') $|B\rangle = G_0 V |B\rangle$

$$\psi_B(\vec{q}) = \frac{1}{E_B - \frac{\vec{p}^2}{2\mu}} \int \frac{d^3\vec{e}}{(2\pi)^3} \langle \vec{p} | V | \vec{e} \rangle \psi_B(\vec{e})$$

real nuisance

$|\phi_B\rangle = V G_0 |B\rangle$

$$\phi_B(\vec{p}) = \int \frac{d^3\vec{e}}{(2\pi)^3} \frac{\langle \vec{p} | V | \vec{e} \rangle \phi_B(\vec{e})}{E_B - e^2/2\mu}$$

easier

9) More about the T-matrix

9.a) Partial wave expansion

9.b) Solving it by means of discretization methods

$\Rightarrow$ Advance topics (covered in lecture notes)

→ This is enough about the T-matrix at the moment

— ⊗ —

The tensor force → Difficult topic

WHAT WAS THE TENSOR FORCE? (previous lessons)

1) Yukawa's idea → exchange of a scalar meson

$$V_Y(\vec{r}) = -g^2 \frac{e^{-mr}}{4\pi r}$$

2) But the deuteron has a quadrupole moment; $Q_d \simeq 0.28 \text{ fm}^2 e$

3) Nuclear forces are not central at long distances

→ pseudoscalar particle (pion)

$g_A \simeq 1.26$

$f_\pi \simeq 92.4$ MeV

$m_\pi \simeq 138$ MeV

$$V_{OPE}(\vec{q}) = -\frac{g_A^2}{4 f_\pi^2} \vec{\tau}_1 \cdot \vec{\tau}_2 \frac{\vec{\sigma}_1 \cdot \vec{q} \; \vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^{\,2} + m_\pi^2}$$

$$\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q} \rightarrow \frac{1}{3}\vec{\sigma}_1\cdot\vec{\sigma}_2\,q^2 + \underbrace{\left(\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q} - \frac{1}{3}\vec{\sigma}_1\cdot\vec{\sigma}_2\,q^2\right)}_{\text{tensor force}}$$

tensor force

↳ mixes different partial waves

DEUTERON WILL HAVE A SMALL D-WAVE COMPONENT

→ We can Fourier-transform the previous potential & obtain:

$$V_{OPE}(\vec{r}) = \frac{g_A^2 m_\pi^3}{48\pi f_\pi^2} \vec{\tau}_1 \cdot \vec{\tau}_2 \times \left[\vec{\sigma}_1 \cdot \vec{\sigma}_2 \, W_C(m_\pi r) + S_{12}(\hat{r}) \, W_T(m_\pi r) \right]$$

$W_C(x), W_T(x) \rightarrow$ certain functions of $x = m_\pi r$

Bottom-line is: $V_{OPE}(\vec{r}) \propto \underbrace{\vec{\sigma}_1 \cdot \vec{\sigma}_2 W_C}_{\substack{\text{spin-spin} \\ \text{term} \\ (1)}} + \underbrace{S_{12}(\hat{r}) W_T}_{\substack{\text{tensor} \\ \text{term} \\ (2)}}$

(1) $\vec{\sigma}_1 \cdot \vec{\sigma}_2 = \begin{cases} 1 & S=1 \\ -3 & S=0 \end{cases}$

(S is the total spin)

(2) $S_{12}(\hat{r}) = 3\,\vec{\sigma}_1 \cdot \hat{r}\;\vec{\sigma}_2 \cdot \hat{r} - \vec{\sigma}_1 \cdot \vec{\sigma}_2$

→ tensor operator will generate some complication

$[S_{12}(\hat{r}), \vec{L}^2] \neq 0$ → it does not conserve angular momentum

Luckily we have the following:

$$[S_{12}(\hat{r}), \vec{S}^2] = 0$$

$$[S_{12}(\hat{r}), \vec{J}^2] = 0$$

} total spin & total angular momentum are conserved

$$\vec{J} = \vec{L} + \vec{S}$$

$\Rightarrow$

only true for $S < 2$

Simplification → the tensor force only acts on triplets ($S=1$)

1) Singlet wave ($S=0$)

$_S\langle 00 | S_{12}(\hat{r}) | 00 \rangle_S = 0$ ⟶ Why?

$S_{12}(\hat{r})$ → symmetric w/ respect to changing particles 1 & 2

$$S_{12}(\hat{r}) = 3\,\bar{\sigma}_1\cdot\hat{r}\;\bar{\sigma}_2\cdot\hat{r} - \bar{\sigma}_1\cdot\bar{\sigma}_1 = S_{21}(\hat{r})$$

$$|00\rangle = \frac{1}{\sqrt{2}}\left[|\underset{1}{+}\underset{2}{-}\rangle - |-+\rangle\right]$$

$$S_{12}(\hat{r})|00\rangle_s = 0$$

$$1 \leftrightarrow 2 \qquad |00\rangle_s \to -|00\rangle_s$$

$$S_{12}(\hat{r})|00\rangle_s = -S_{12}(\hat{r})|00\rangle_s$$

1') For singlets ($S=0$) the tensor force vanishes

2) Triplet waves $\rightarrow$ no simplification possible

$[S_{12}(\vec{r}), \vec{J}^2] = 0$ $\rightarrow$ construct state w/ good $\vec{J}^2$

2a) $\underbrace{\mathcal{Y}_{\ell m_\ell}(\hat{r})\,|s m_s\rangle}_{\text{good } L \,\&\, S \text{ (but not good } J)}$ (w/o tensor force)

$$2.b)\ |j\,m\rangle = \sum_{\substack{s m_s \\ \ell m_\ell}} \mathcal{Y}_{\ell m_\ell}(\hat{r})\,|s m_s\rangle \times \underbrace{\langle \ell m_\ell\, s m_s | j m\rangle}_{\text{Clebsch-Gordan coefficient}}$$

3) $|jm\rangle = |(\underline{s\ell})jm\rangle$

$\rightarrow$ matrix element of $S_{12}(\hat{r})$ w/ these states

$$\langle(s'\ell')\underline{j'm'}|S_{12}(\hat{r})|(s\ell)\underline{jm}\rangle = S^{J}_{s'\ell's\ell}\,\delta_{jj'}\delta_{mm'}$$

$[S_{12}(\hat{r}), \vec{J}^2] = 0$

for NN $\rightarrow$ $S = S' = 1$ (we can ignore the s' part)

$$\langle (1 l') j' m' | S_{12}(\hat{r}) | (1 l) j m \rangle = S^{J}_{ll'} \delta_{jj'} \delta_{s's}$$

$l' = l, \; l \pm 2$ ($l' \neq l \pm 1$ breaks parity)

EXAMPLE : DEUTERON

1) No tensor force $\rightarrow$ $\psi_d(\vec{r}) = \frac{u(r)}{r} \underbrace{Y_{00}(\hat{r})}_{\text{angular momentum part}} \underbrace{|1 m_d\rangle}_{\text{spin part}}$

DECOUPLED

2) Tensor force :

S-wave component

$$\psi_d(\vec{r}) = \frac{u(r)}{r} \mathcal{Y}_{00}(\hat{r}) \, |1 \, m_d\rangle$$

$$+ \frac{w(r)}{r} \sum_{\substack{m_s \\ m_\ell}} \mathcal{Y}_{2 m_s}(\hat{r}) \, |1 \, m_s\rangle$$

$$\times \langle 2 m_\ell \, 1 m_s | 1 m_d \rangle$$

D-wave component

→ We can calculate the matrix elements of $S_{12}(\hat{r})$

$$|S(1 m_d)\rangle = Y_{00}(\hat{r})\,|1 m_d\rangle$$

$$|D(1 m_d)\rangle = \sum \mathcal{Y}_{2 m_s}(\hat{r})\,|1 m_s\rangle$$

$$\times \langle 2 m_s 1 m_s | 1 m_d \rangle$$

$$\mathcal{B} = \{ |S(1\,\mathrm{md})\rangle , |D(1\,\mathrm{md})\rangle \}$$

$$\downarrow$$

$$S_{12} = \begin{pmatrix} 0 & 2\sqrt{2} \\ 2\sqrt{2} & -2 \end{pmatrix} \longrightarrow S_{12}(\hat{r}) \text{ evaluated in the deuteron}$$

Schrödinger equation:

1) w/o $S_{12}(\hat{r})$ $\quad -u'' + 2\mu V u(r) = -\gamma^2 u(r)$

2) w/ $S_{12}(\hat{r})$

$$-u''(r) + 2\mu\left[V_C u(r) + 2\sqrt{2}\, V_T w(r)\right] = -\gamma^2 u(r)$$

$$-w''(r) + \frac{6}{r^2} w(r) + 2\mu\left[2\sqrt{2}\, V_T u(r) + (V_C - 2V_T) w(r)\right] = -\gamma^2 w(r)$$

Convention: $V = V_C \vec{\sigma}_1 \cdot \vec{\sigma}_2 + V_T S_{12}(\hat{r})$

$\boxed{\text{TENSOR FORCE \& PARITY}}$ $\longrightarrow L \longrightarrow P = (-1)^L$

$S_{12}(\hat{r}) = 3\, \vec{\sigma}_1 \cdot \hat{r}\, \vec{\sigma}_2 \cdot \hat{r} - \vec{\sigma}_1 \cdot \vec{\sigma}_2$

$r_i r_j \longrightarrow P = (+)$

$L = 0 \rightarrow L = 1$

$P = + \rightarrow P = -1$

$S_{12} \rightarrow$ two powers of r $(r_i r_j)$

$\langle S | \hat{O} | P \rangle \neq 0$

$\Downarrow$

$\hat{O} \rightarrow$ odd powers of r

$(r_i, r_i r_j r_k, \ldots)$

$$\langle S | S_{17}(\hat{r}) | P \rangle = 0$$

(one can check this explicitly)

$$\longrightarrow \int d^2\hat{r} \; \sum$$

→ We go back to the deuteron

$$-\begin{pmatrix} u \\ w \end{pmatrix}'' + 2\mu \begin{pmatrix} V_C & 2\sqrt{2}V_T \\ 2\sqrt{2}V_T & V_C - 2V_T \end{pmatrix} \begin{pmatrix} u \\ w \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & \frac{6}{r^2} \end{pmatrix} \begin{pmatrix} u \\ w \end{pmatrix} = -\gamma^2 \begin{pmatrix} u \\ w \end{pmatrix}$$

Deuteron → [S & D wave components]

Asymptotic behaviour :

$$u(r) \longrightarrow A_S e^{-\gamma r}$$

$$w(r) \longrightarrow \underline{A_D} e^{-\gamma r} \left(1 + \frac{3}{\gamma r} + \frac{3}{(\gamma r)^2} \right)$$

$$r \to \infty$$

Observables/quantities that depend on D-waves:

1) Normalization

$$\int_0^\infty dr\, u^2(r) = 1 \longrightarrow \int_0^\infty dr \left(u^2(r) + w^2(r) \right) = 1$$

2) The mean square radius & the quadrupole moment

$$r_m^2 = \frac{\langle r^2 \rangle}{4} = \frac{1}{4}\int_0^\infty r^2 (u^2 + w^2)\, dr$$

(matter radius)

$$Q_d = \frac{1}{20}\int_0^\infty r^2 w (2\sqrt{2}\, u - w)\, dr$$

← (quadrupole moment)

$\boxed{20}$

3) D-wave probability (probability of the deuteron to be in D-wave)

$$P_D = \int_0^\infty dr\, w^2(r) \simeq 0.03 - 0.05$$

(3-5 % probability)

→ not an observable (actually)

Experimental values:

$r_m = 1.9754(9)\ \text{fm}$

$Q_d = 0.2859(3)\ \text{fm}^2$

→ fitting a deuteron in an e.m. field

$P_D \sim (3-5)\%$ → not really observable

$\eta = \dfrac{A_D}{A_S} = 0.0256(4)$ → from scattering at low energies

TENSOR FORCE → CHAGES THE SCATTERING AMPLITUDES

We have to take spin & angular mometum into account

[SCATTERING WAVE FUNCTION]

1) NO SPIN $\rightarrow$ $\psi(\vec{r}) \underset{r\to\infty}{\longrightarrow} e^{i\vec{k}\cdot\vec{r}} + f(\Omega)\frac{e^{ikr}}{r}$

$$\frac{d\sigma}{d\Omega} = |f(\Omega)|^2$$

2) SPIN (WO/TENSOR FORCES)

$$\psi(\vec{r}) \underset{r\to\infty}{\longrightarrow} e^{i\vec{k}\cdot\vec{r}}\,|s\,m_s\rangle + \sum_{m_s'} f_{m_s' m_s}\frac{e^{ikr}}{r}\,|s\,m_s'\rangle$$

same s

$$\frac{d\sigma}{d\Omega} = |f_{m_s' m_s}(\Omega)|^2$$

3) SPIN (W/TENSOR FORCE IN NN)

$$\psi(\vec{r}) \to e^{ikr}|s\,m_s\rangle + \sum_{m_s'} f_{m_s' m_s}(\Omega)\frac{e^{ikr}}{r}|s\,m_s'\rangle$$

$s = 0, 1$ (no change in $\psi(\vec{r})$ asymptotic form)

2) & 3) different once we consider the partial wave expansion

$\boxed{\text{NO SPIN}}$

$$f(\Omega) = \sum_{\ell} (2\ell+1) f_\ell P_\ell(\cos\theta)$$

$$f_\ell = \frac{e^{2i\delta_\ell} - 1}{2ik}$$

$\to$ S-matrix (in partial waves)

SPIN (w/o TENSOR FORCE)

$$f^J_{\ell s}(k) = \frac{e^{2i\delta^J_{\ell s}(k)} - 1}{2ik}$$

more coefficient here

diagonal in ℓ

SPIN w/ TENSOR FORCE

$$f_\ell(k) = \frac{e^{2i\delta_\ell} - 1}{2ik} \longrightarrow f^{SJ}_{\ell\ell'} = \frac{1}{2ik}\left(S^{SJ}_{\ell\ell'}(k) - \delta_{\ell\ell'}\right)$$

The S-matrix is now a matrix in angular momentum space

→ We are not going to explain the details

(JUST THE OUTCOME)

$$f^{(s)}_{\ell\ell'}(k) = \frac{S^{(s)}_{\ell\ell'}(k) - \delta_{\ell\ell'}}{2ik}$$

$$S S^{+} = 1$$

is an extension of $e^{2i\delta_\ell}$

For the NN case:

$$\boxed{\begin{array}{c} S = 1, \; J = L \pm 1 \\ L = J \pm 1 \end{array}} \longrightarrow \text{mixing of partial waves}$$

(Single channel $S(k) = e^{2i\delta_l(k)}$)

$$S^{J}(k) = \begin{pmatrix} \cos\epsilon_J & -\sin\epsilon_J \\ \sin\epsilon_J & \cos\epsilon_J \end{pmatrix} \begin{pmatrix} e^{2i\delta_1} & 0 \\ 0 & e^{2i\delta_2} \end{pmatrix} \begin{pmatrix} \cos\epsilon_J & \sin\epsilon_J \\ -\sin\epsilon_J & \cos\epsilon_J \end{pmatrix}$$

Uncoupled $\longrightarrow$ Coupled channel case

$$\underbrace{\begin{pmatrix} e^{2i\delta_1} & & & \\ & e^{2i\delta_2} & & \\ & & \ddots & \\ & & & e^{2i\delta_n} \end{pmatrix}}_{n \text{ phase shifts}} \longrightarrow R \begin{pmatrix} e^{2i\delta_1} & & & \\ & e^{2i\delta_1} & & \\ & & \ddots & \\ & & & e^{2i\delta_n} \end{pmatrix} R^{-1}$$

rotation matrix

$\frac{n(n-1)}{2}$ angles

$n + \frac{n(n-1)}{2}$ phases

NN scattering $\begin{cases} S=0 \text{ singlets} \rightarrow \text{normal scattering theory} \\ S=1 \text{ triplet} \end{cases}$

a different definition of phase shifts (that includes a series of mixing angles)

→ It's complicated (you only need to understand the general idea)

→ Even more complicated because

∃ DIFFERENT DEFINITIONS OF THE COUPLED-CHANNEL PHASE SHIFTS

→ It's just how the field developed

Two phase shift definitions:

1) Eigen phase shifts (Blatt-Biedenharn phase shift)

This idea of a rotation

$$S = \begin{pmatrix} \cos\epsilon_J & -\sin\epsilon_J \\ \sin\epsilon_J & \cos\epsilon_J \end{pmatrix} \begin{pmatrix} e^{2i\delta_{J-1}} & 0 \\ 0 & e^{2i\delta_{J+1}} \end{pmatrix} \begin{pmatrix} \cos\epsilon_J & \sin\epsilon_J \\ -\sin\epsilon_J & \cos\epsilon_J \end{pmatrix}$$

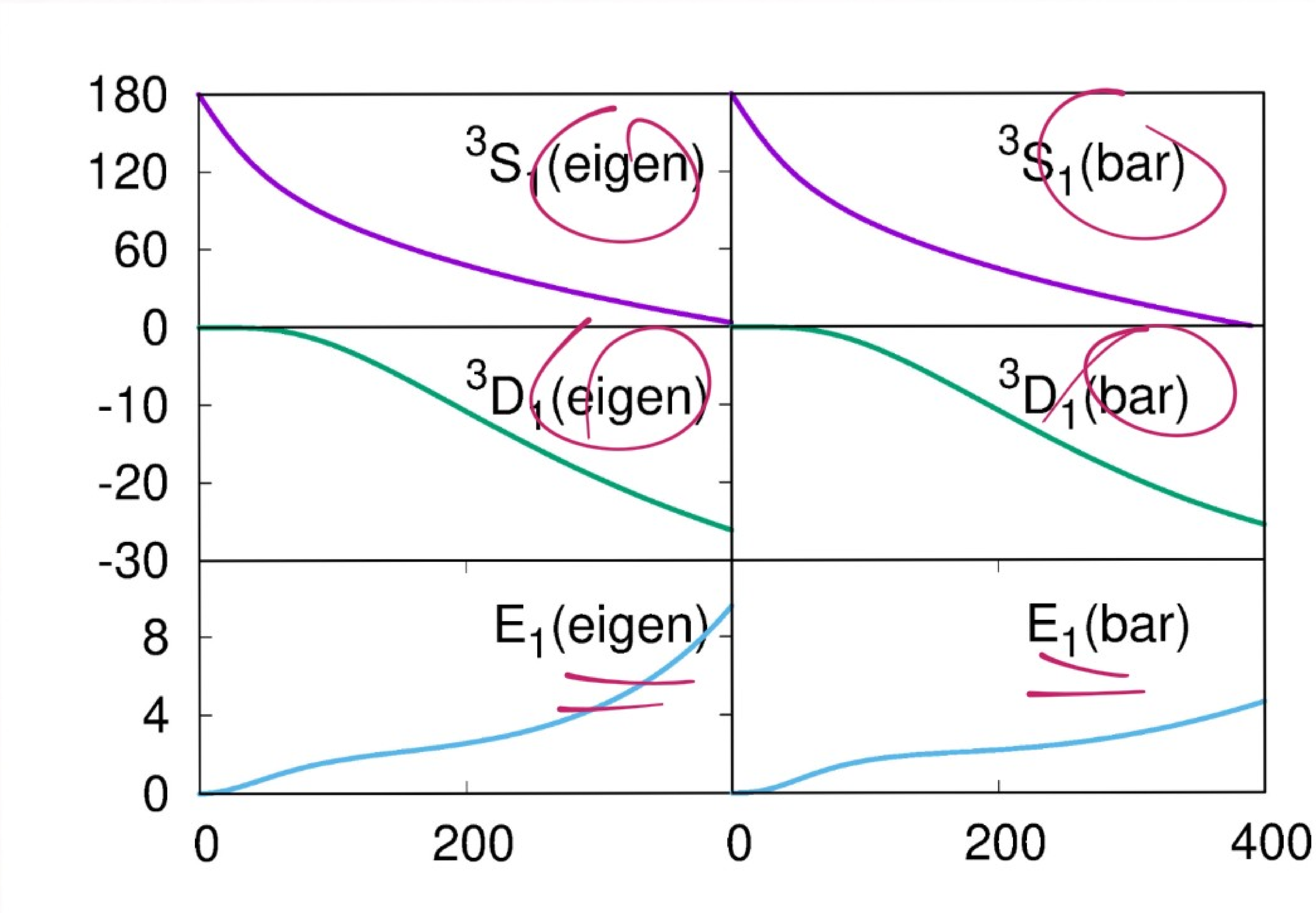
2) Nuclear-bar phase shifts (Stapp-Ypsilantis-Metropolis phase shifts)

$$S = \begin{pmatrix} \cos(2\bar{\epsilon}_J)\, e^{2i\bar{\delta}_{1J}} & i\sin(2\bar{\epsilon}_J)\, e^{i(\bar{\delta}_{1J}+\bar{\delta}_{2J})} \\ i\sin(2\bar{\epsilon}_J)\, e^{i(\bar{\delta}_{1J}+\bar{\delta}_{2J})} & \cos(2\bar{\epsilon}_J)\, e^{2i\bar{\delta}_{2J}} \end{pmatrix}$$

→ two most commonly use parametrizations

→ don't worry if you don't understand this (this requires a heavy investment of time doing all the coupled-channel scattering math) → lectures notes

EXAMPLE: TRIPLET J=1 S & D-WAVE PHASES



(Nymegen II potential)

small

mixing angles (ϵ_J, $\bar{\epsilon}_J$)

SUMMARY

→ TENSOR FORCE IMPORTANT PART OF NUCLEAR FORCES

→ IT CREATES SOME TECHNICAL COMPLICATIONS (IN CASE YOU DO NN, YOU WILL HAVE TO MASTER THESE COMPLICATIONS)

[ONE BOSON EXCHANGE (OBE) MODEL]

→ this is easy stuff

OBE MODEL → first qualitatively successful description of the nuclear force

↓ problem addressed

WHAT IS THE ORIGIN OF NUCLEAR FORCES?

1) Before QCD :

1.a) Pion theories (50's) → failed

1.b) OBE model → succeeded

2) Post QCD :

2.a) Quark-model related

2.b) Effective field theory methods

(ongoing)

?

Why did people propose the OBE model?

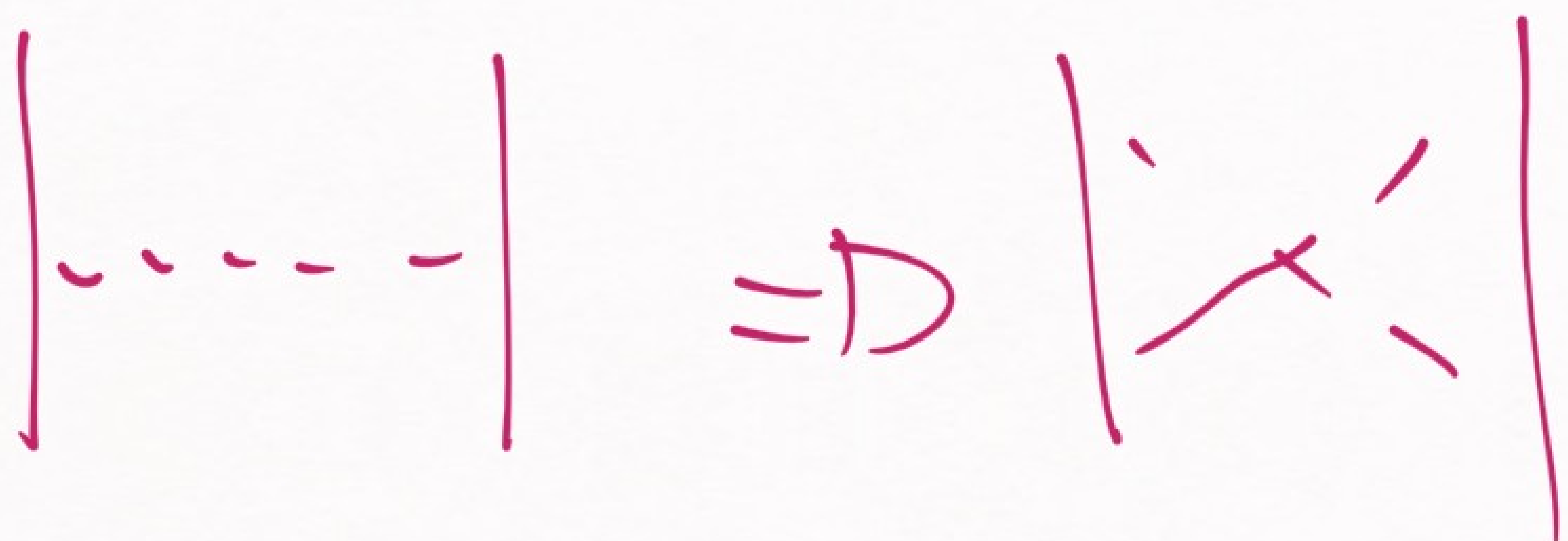
1) Yukawa's idea → one pion exchange (OPE)

→ first step in describing NN

2) How to extend OPE?

↳ this will lead to problem

2.a) Multi-pion theories



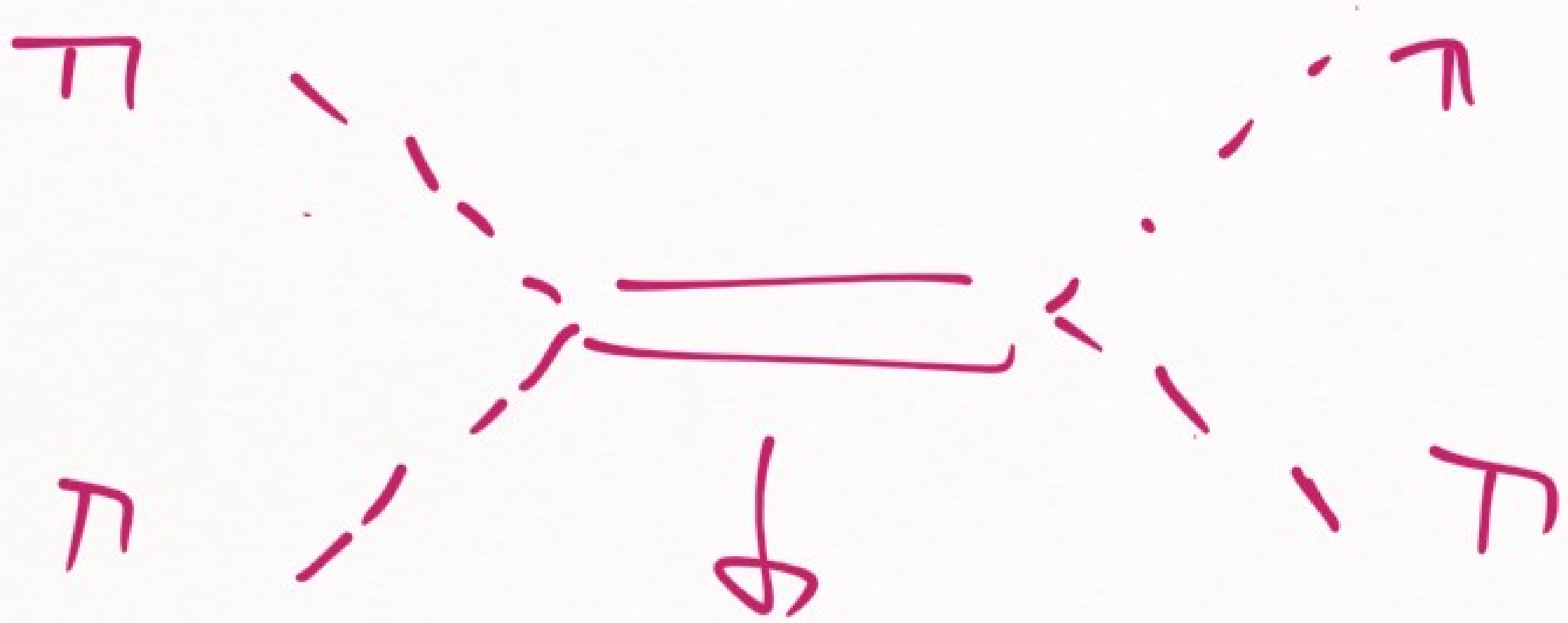
more pions!

~~problem solved!~~

In the 50's we didn't understand pion dynamics

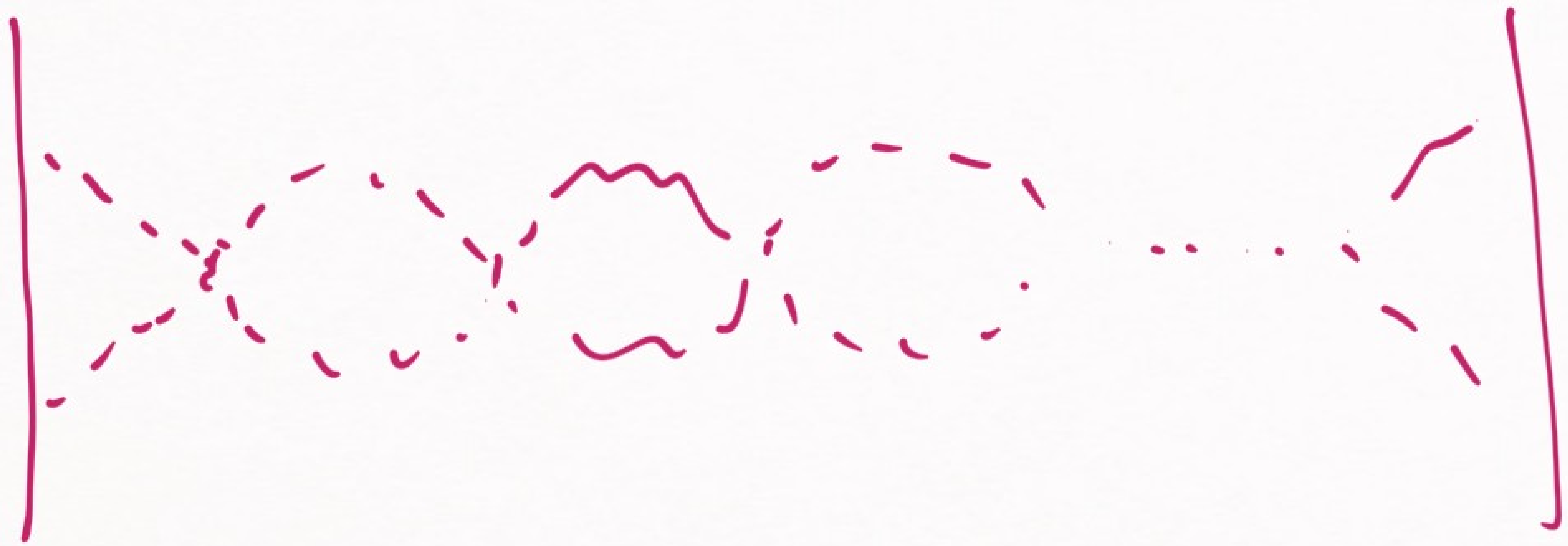
(no chiral symmetry, no renormalization)

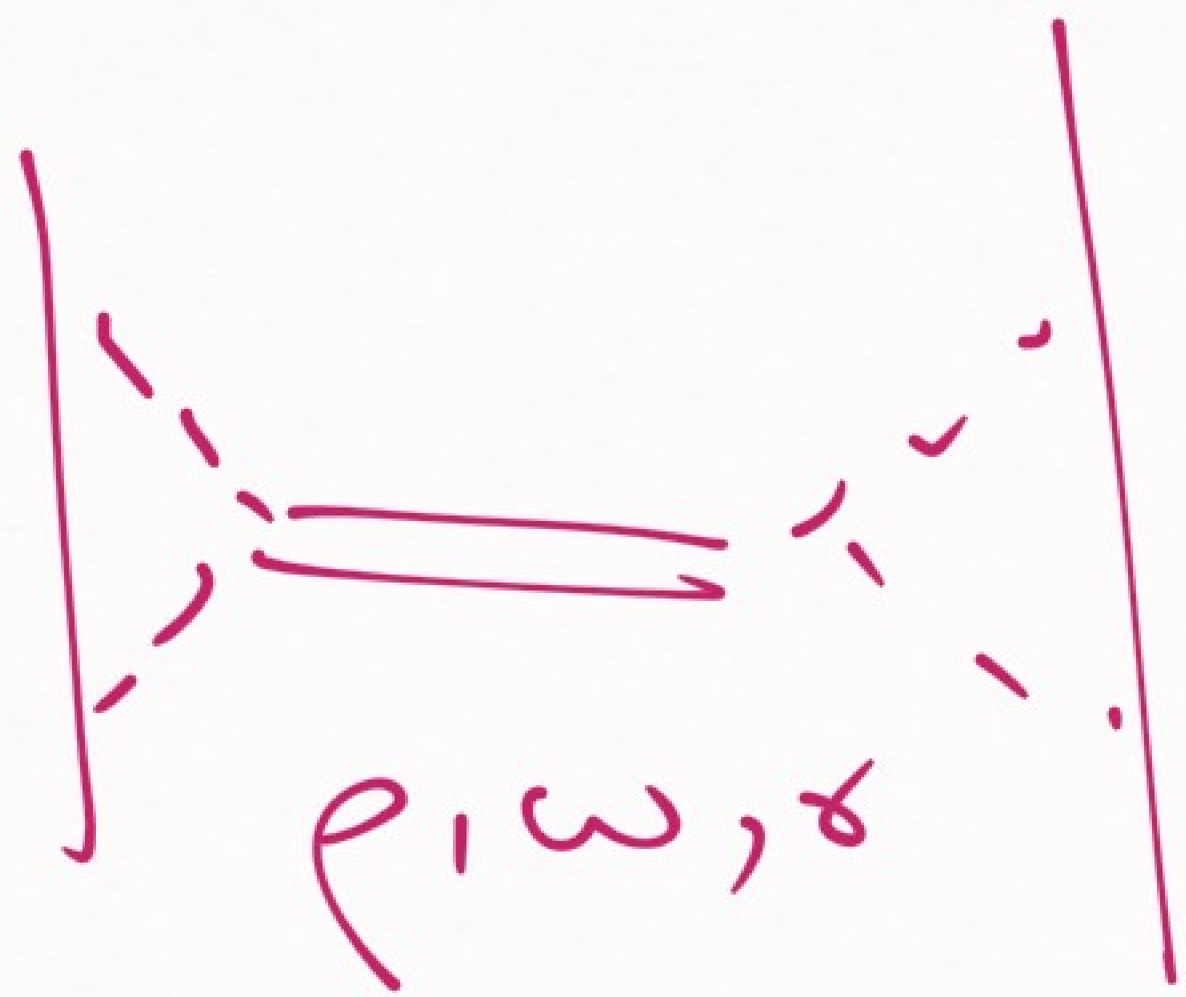
2.b) $\pi\pi$ scattering $\rightarrow$ resonances



resonance
(σ, ρ, ω)

$\rightarrow$ inspired to consider this idea in NN

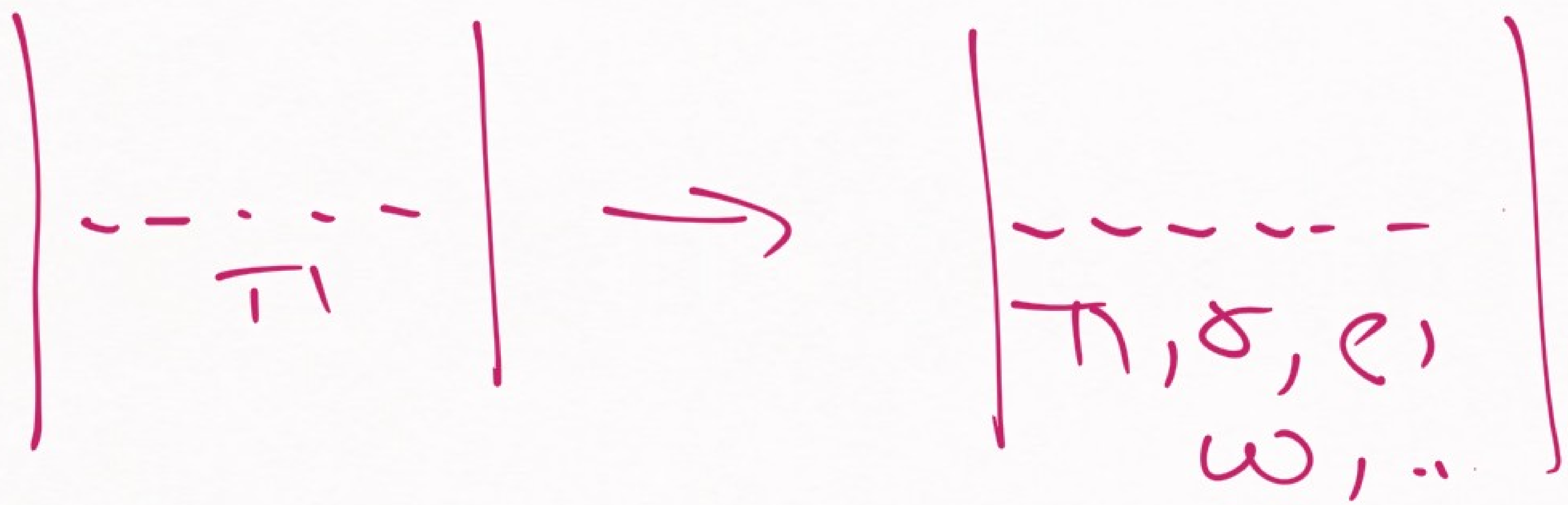


$\Rightarrow$ 

$\boxed{\text{IDEA}}$

→ substite multi-pion exchanges by the exchange of resonance appearing in $\pi\pi$ scattering

3) THE FINAL RESULT IS VERY INTUITIVE



Yukawa's idea (π)

[Natural extension of Yukawa's idea]

What are the important bosons in OBE?

1) The pion (π): $J^P = 0^-$, $I = 1$, $m_\pi = 140\,\text{MeV}$

(explains the deuteron's quadrupole moment)

2) The sigma (σ): $J^P = 0^+$, $I = 0$, $m_\sigma \sim 500\,\text{MeV}$

(strong mid-range attraction)

3) The rho (ρ) : $J^P = 1^-$, $I = 1$, $m_\rho \simeq 770$ MeV

(counters the excessive strength of the pion's tensor force at short distances)

4) the omega (ω) : $J^P = 1^-$, $I = 0$, $m_\omega \simeq 780$ MeV

(provides short-range repulsion)

The OBE potential:

$$\underbrace{V_{OBE}}_{} = V_\pi + V_\rho + V_\omega + V_\sigma \underbrace{+ \dots}_{}$$

Really simple description of the nuclear force

optionally we can add more bosons

$V_\pi(\vec{q}) \to$ you already know

$$V_\sigma(\vec{q}) = -\frac{g_\sigma^2}{q^2 + m_\sigma^2}$$

$$V_\omega(\vec{q}) = \frac{g_\omega^2}{q^2 + m_\omega^2} - \frac{(f_\omega + g_\omega)^2}{4M_N^2} \frac{(\vec{\sigma}_1 \times \vec{q}) \cdot (\vec{\sigma}_2 \times \vec{q})}{q^2 + m_\omega^2}$$

$$V_\rho(\vec{q}) = \vec{\tau}_1 \cdot \vec{\tau}_2 \left[g_\omega \to g_\rho \quad m_\omega \to m_\rho \quad f_\omega \to f_\rho \right]$$

(same potential form)

Traditional simplifications:

$$\left[\begin{array}{l} f_\varepsilon \gg g_\varepsilon \Rightarrow g_\varepsilon \simeq 0 \\ f_\omega \ll g_\omega \Rightarrow f_\omega \simeq 0 \end{array}\right]$$

sometimes people use these

$\boxed{\text{OBE Model}}$ $\longrightarrow$ exhibits singular potentials

$(V_\pi, V_\rho, V_\omega \sim 1/r^3)$

Renormalizations was not yet properly understood

$\boxed{\text{Form factors}}$

$\boxed{\text{FORM FACTORS}}$ → take into account the finite size of hadrons

Σ

$V_\pi, V_\rho, V_\omega \xrightarrow[r \to 0]{} \frac{1}{r^3}$] → only true if hadrons are point-like

$\xrightarrow[r \to 0]{}$ finite] → finite-sized hadrons

→ NUCLEAR PHYSICISTS DECIDED TO INCLUDE FORM FACTORS TO REGULARIZE V_{OBE}

$$V_M(\vec{q}) \longrightarrow V_M(\vec{q})\, F_M^2(\vec{q})$$

$$M = \pi, \sigma, \rho, \omega, \ldots$$

$$F_M(\vec{q}\,;\Lambda) = \left(\frac{\Lambda^2 - m^2}{\Lambda^2 + \vec{q}^{\,2}}\right)^{a}$$

MULTIPOLAR FF (POPULAR CHOICE)

→ [RELATIVISTIC EFFECTS]

Some partial waves require a strong spin-orbit potential

$$\boxed{\underbrace{V_{SO}\,\vec{l}\cdot\vec{s}}}$$

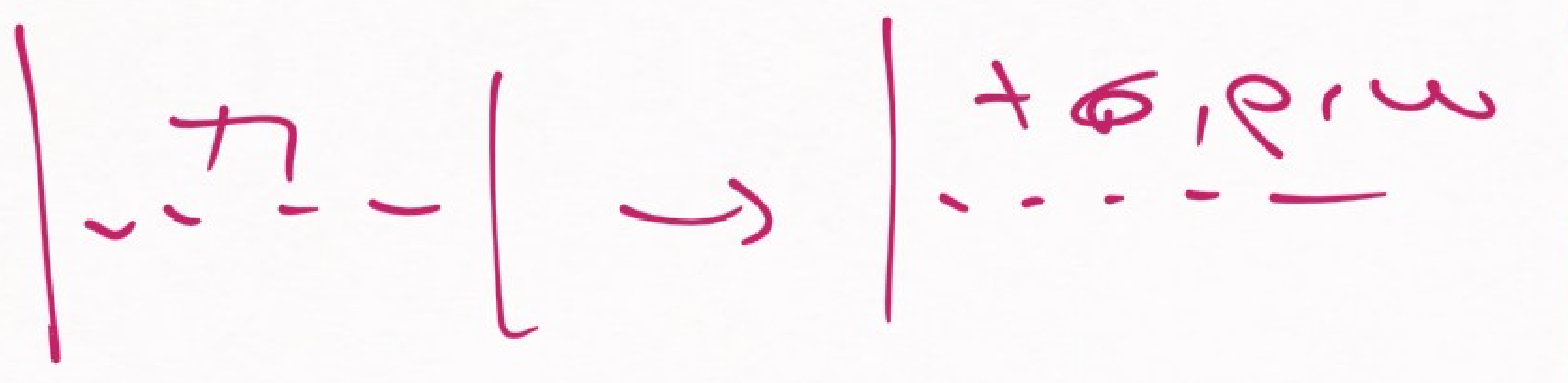
NON-RELATIVISTIC POTENTIALS DO NOT HAVE THIS

RELATIVISTIC CORRECTIONS

$\rightarrow \sigma, \omega \rightarrow$ good $\vec{L}\cdot\vec{S}$ force

$\boxed{\text{RECAP}}$ OBE MODEL

1) NATURAL EXTENSION OF YUKAWA'S IDEA



2) FORM FACTORS

Otherwise we end up w/ $1/r^3$ potentials

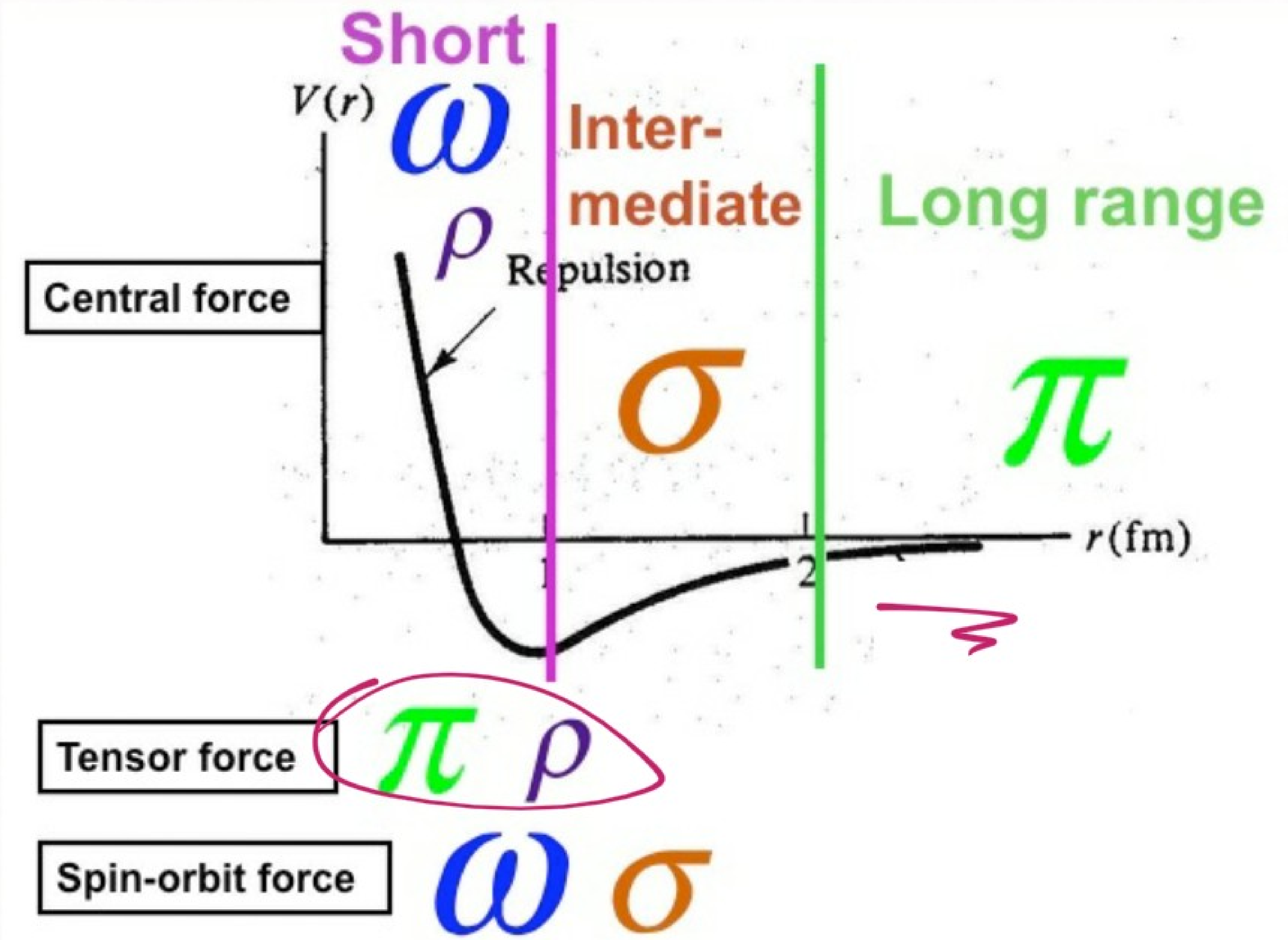
3) RELATIVISTIC CORRECTIONS

→ Spin-orbit force

4) EACH MESON HAS A JOB

↳ Let's check this

ROLE OF DIFFERENT MESONS:



→ Long/medium/short range

SCHOLARPEDIA ARTICLE ON NUCLEAR FORCES (BY MACHLEIDT)

Table 1: The four most important mesons and the main characteristics of their contributions to components of the nuclear force.

Meson	Central	Spin-Spin	Tensor	Spin-Orbit
$\pi(138)$	---	weak, long-ranged	**strong**, long-ranged	---
$\sigma(500)$	**strong, attractive**, intermediate-ranged	---	---	moderate, intermediate-ranged
$\omega(782)$	**strong, repulsive**, short-ranged	---	---	**strong**, short-ranged, coherent with σ
$\rho(770)$	---	weak, short-ranged, coherent with π	moderate, short-ranged, **opposite to** π	---

→ GOOD SUMMARY (64 & 65)

pseudoscalar octet

→ USUAL PARAMETERS IN OBE MODEL

Table 5

Meson parameters used in the relativistic (energy-independent) momentum space one-boson-exchange potential (OBEPQ)

	$g_\alpha^2/4\pi$; $[f_\alpha/g_\alpha]$	$g_\alpha^2/4\pi(k^2=0)$	m_α [MeV]	Λ_α [GeV]	n_α
π	14.6	14.27	138.03	1.3	1
ρ	0.81; [6.1]	0.43	769	2.0	2
η	5	3.75	548.8	1.5	1
ω	20; [0.0]	10.6	782.6	1.5	1
δ	1.1075	0.64	983	2.0	1
σ	8.2797 [a]	7.07	550 [a]	2.0	1

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Nuclear mass: $m = 938.926$ MeV. For notation and empirical values see table 4. Here, there are NN vertices only.

[a] The parameters for the σ-boson given in the table apply only to the $T=1$ NN potential. For $T=0$ we have: $m_\sigma = 720$ MeV, $g_\sigma^2/4\pi = 16.9822$ and $\Lambda_\sigma = 2$ GeV. The parameters for the other mesons in the table are the same for $T=0$ and $T=1$.

$a_0(980)$

$\boxed{\text{THE END}}$

(FOR TODAY)