NUCLEAR PHYSICS 12

GOLDSTONE THEOREM & CHIRAL SYMMETRY

ANNOUNCEMENT

→ We have the first excercise set out

→ Please look at it

→ Optional: you can present your solution for an exercise you like in the class (extra points)

$\boxed{\text{RECAP}}$ Linear sigma model (LσM)

Problem → why is the pion so light?

LσM proposed a Solution

1) Originally massless nucleon
(and massive boson fields)

2) Mexican hat potential

→ very important idea of the SM

3) For certain choices of the potential the minimum energy did not happen at $\phi_i = 0$ $(i = 0, 1, 2, 3)$

→ usually in QFT, one expands around $\phi_i = 0$ (peskin & schröder, Zee, etc.)

4) We end up w/massive nucleons ✓

w/ massless pions ✓

w/massive sigma ☹

→ development of the nonlinear sigma model (because for a long time people thought that the sigma didn't exist)

[GENERALIZE THE IDEA WITHIN THE LσM]

→ LσM contained this mexican hat potential

(one of the great ideas in QFT)

→ But this is just an especial case of

a more general idea → $\boxed{\text{GOLDSTONE THEOREM}}$

NAMBU-GOLDSTONE THEOREM

→ full generalization of the idea within the LSM (mexican hat potential)

[SIMPLE VERSION] (caveat: we will need a bit of group theory)

1) Hamiltonian H invariant under group G

$$[H, G] = 0$$

2) Hamiltonian has a "vacuum" state

vacuum → minimum energy state

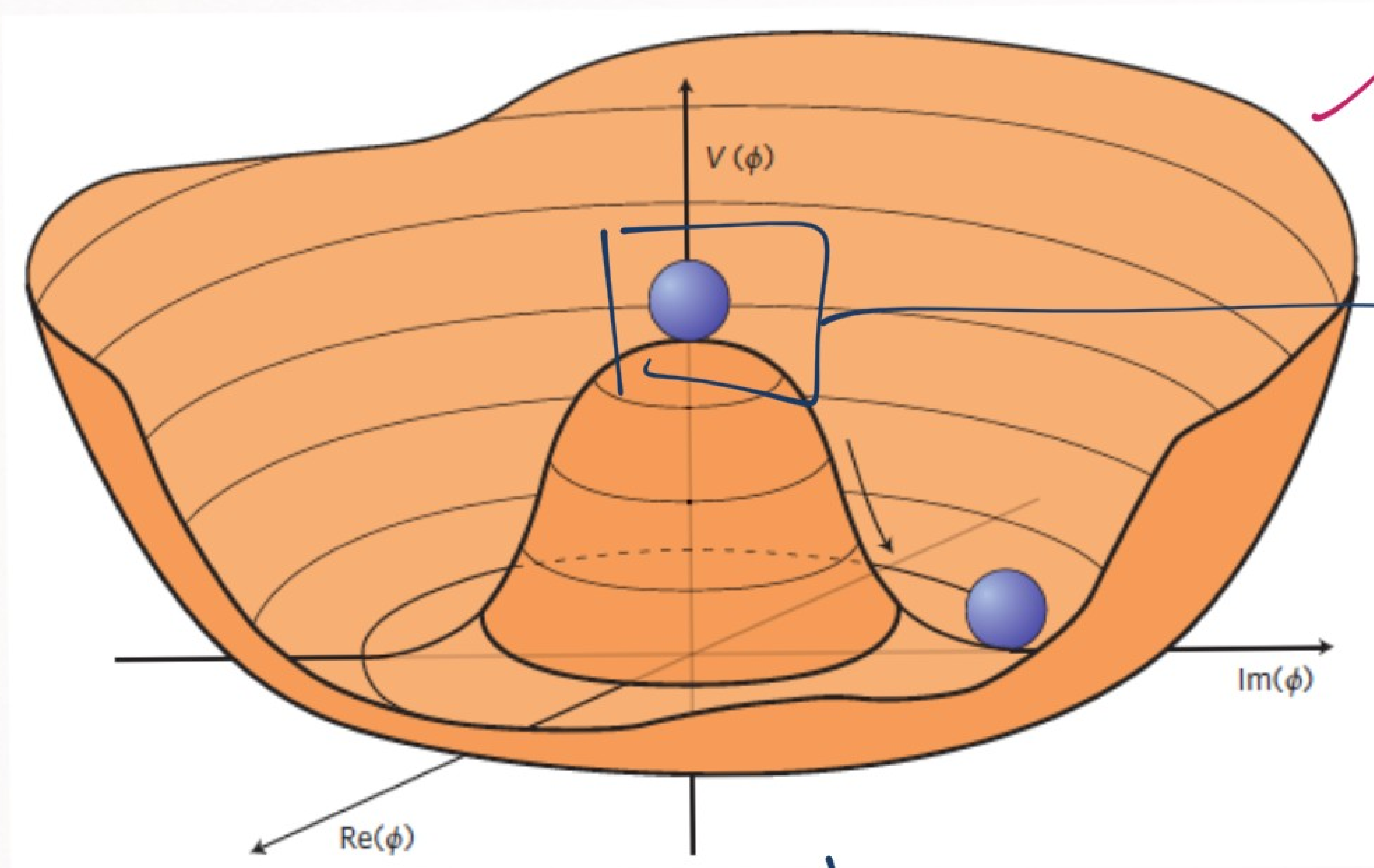
$H|0\rangle = E_0|0\rangle$ $|0\rangle$ → vacuum state

3) But $|0\rangle$ will not be invariant under G

$[H, G] = 0$

Expectation: $G|0\rangle = e^{i\alpha}|0\rangle$

Reality: $G|0\rangle \neq |0\rangle$



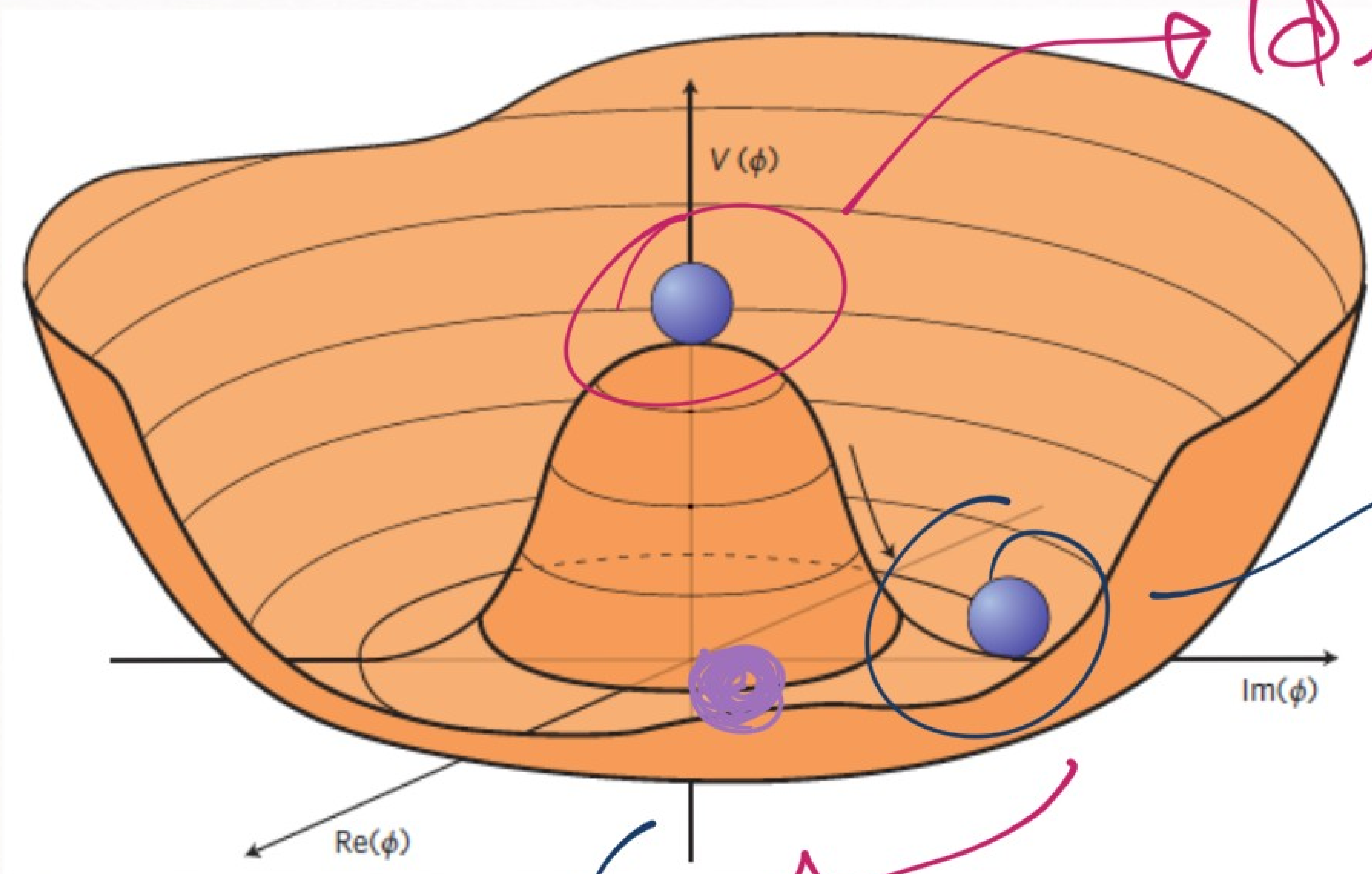
$[V, R] = 0$

$|\phi_1\rangle$

$R \in O(2)$

$R|\phi_1\rangle = e^{i\alpha}|\phi_1\rangle$ ✓

$O(2) \rightarrow$ Rotations + Reflections in $d=2$



$|\phi_1\rangle \,/\, R|\phi_1\rangle = e^{i\alpha}|\phi_1\rangle$ ✓

$|\phi_0\rangle$ (the vacuum)

$R|\phi_0\rangle = |\phi_0'\rangle$

R

$|\phi_0'\rangle$

VACUUM NOT INVARIANT
UNDER G ($[H,G]=0$)

3) But $|0\rangle$ not invariant under G

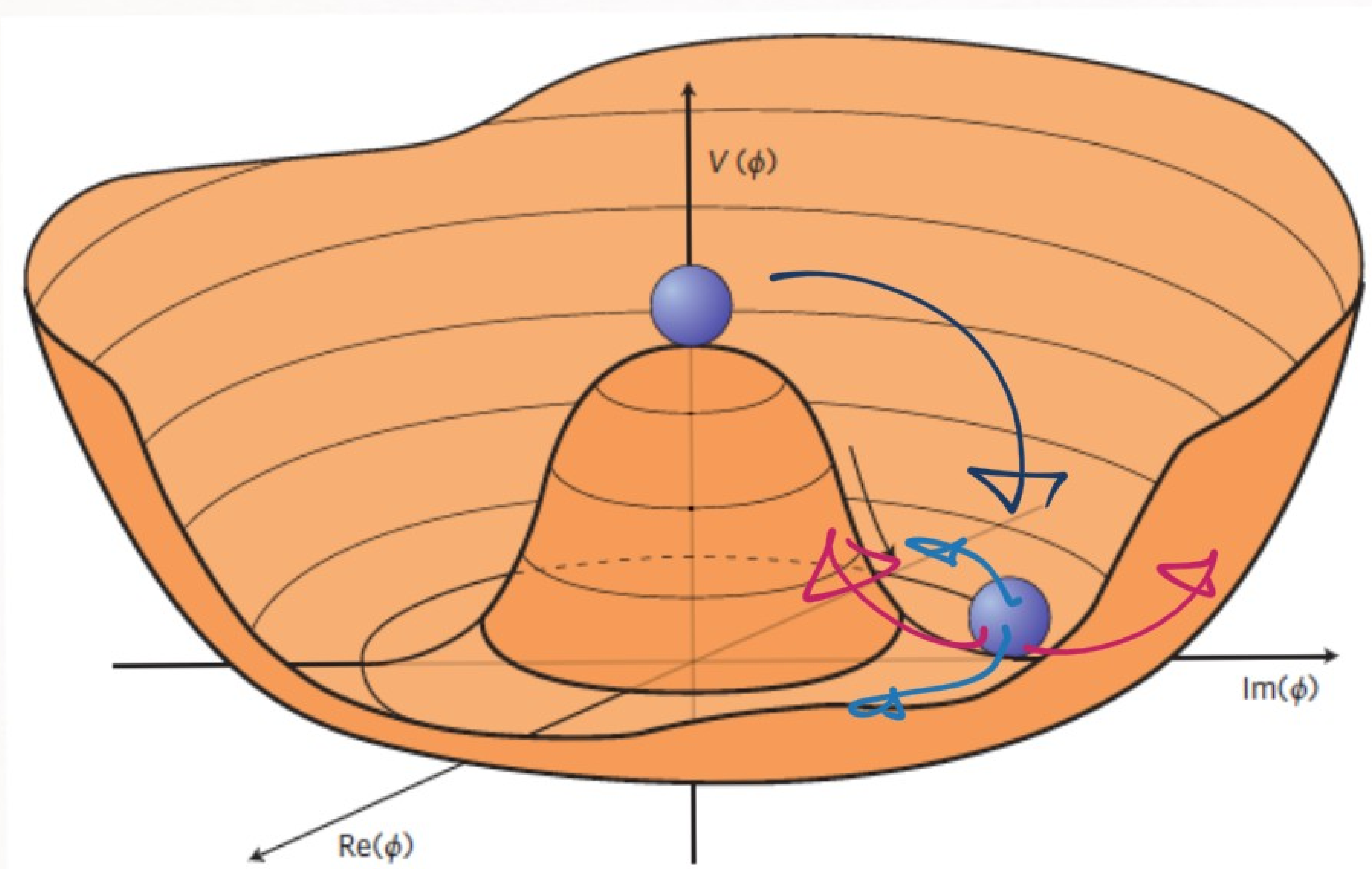
$\to$ Still invariant under $\underbrace{F \subset G}$

[F is a subgroup of G]

4) Then we will have $(n_G - n_F)$ massless fields in the theory

$n_G \to$ # of generators of group G

$n_F \to$ # of generators of subgroup F



$G = O(2)$ (rotations)

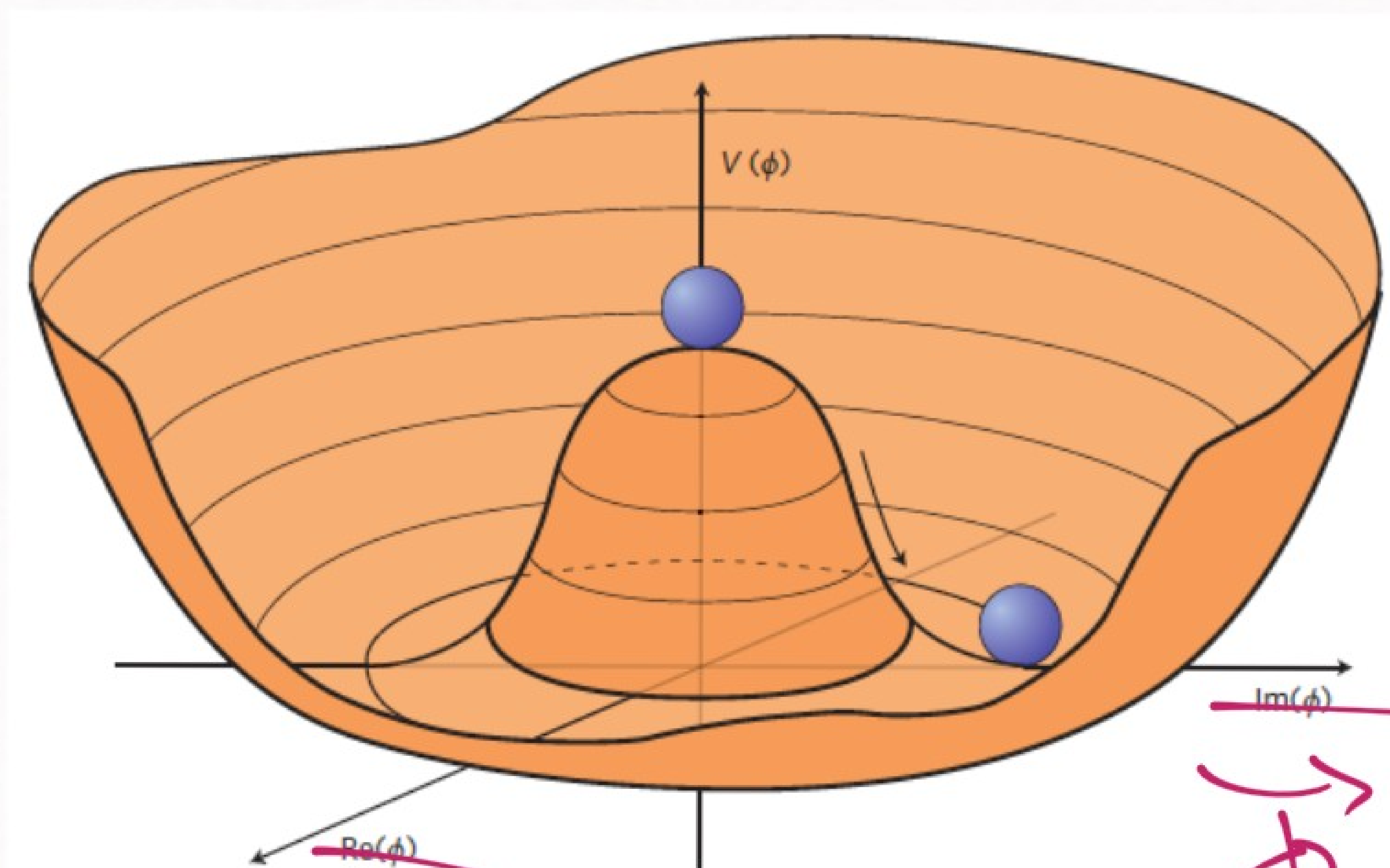
$O(n) \to \frac{n(n-1)}{2}$ generators

$F = O(1)$ (trivial group)

$n_G - n_F = 1 - 0 = 1$ massless field

$\frac{G}{F}$ acting on the vacuum just gives you another different type of vacuum

$\boxed{\text{LSM}}$



ϕ_0

$G = O(n)$

$F = O(n-1)$

$|\vec{0}\rangle \xrightarrow{F} |\vec{0}'\rangle$

$\vec{\phi} = \{\phi_1, \phi_2, \phi_3, \dots, \phi_{n-1}\}$

$$G = O(n) \qquad n_G = \frac{n(n-1)}{2}$$

$$F = O(n-1) \qquad n_F = \frac{(n-1)(n-2)}{2}$$

$n_G - n_F = n-1$ massless bosons

$\boxed{\text{L}\sigma\text{M}} \rightarrow n=4 \rightarrow \boxed{\text{3 massless bosons}}$

Remember that in LSM one can also add a litter perturbation to $V(\phi)$ in order to obtain a small mass for the pions

$$\delta V(\phi) = -\epsilon v^3 \phi_0 \longrightarrow \text{mass for pions}$$

$$\langle \phi_0 \rangle = v \quad (\text{minimum of } V(\phi))$$

NOW WE KNOW THE GOLDSTONE THEOREM

→ WE CAN EXTEND ITS APPLICATION

RECAP

1) LoM is only a especial case of a more general result $\boxed{\text{THE GOLDSTONE THEOREM}}$

2) If a hamiltonian is invariant under G (a group), but vacuum invariant under F (subgroup of G) $\Rightarrow$ $n_G - n_F$ massless boson

Quarks (u, d, s) → CHIRAL SYMMETRY

SPONTANEOUSLY BROKEN

WE WILL BE ABLE TO APPLY THE GOLDSTONE THEOREM

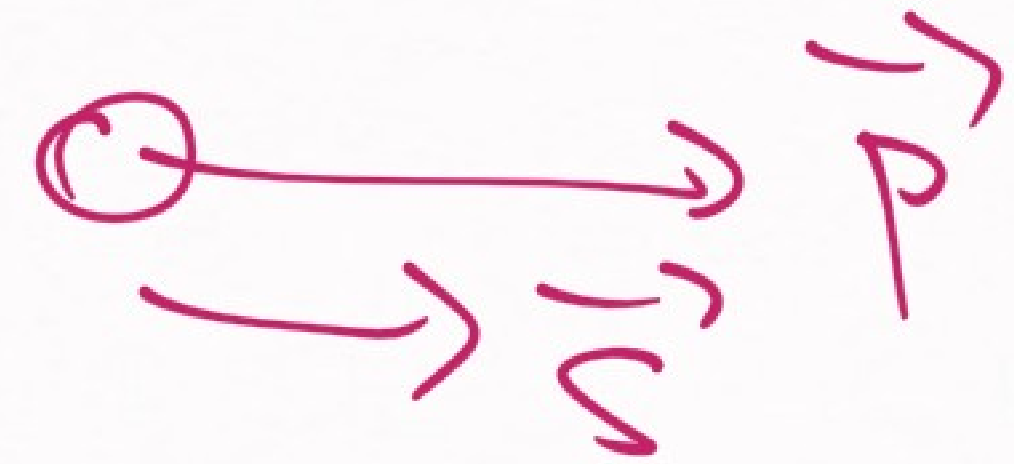
CHIRAL SYMMETRY → what is it?

1) Helicity

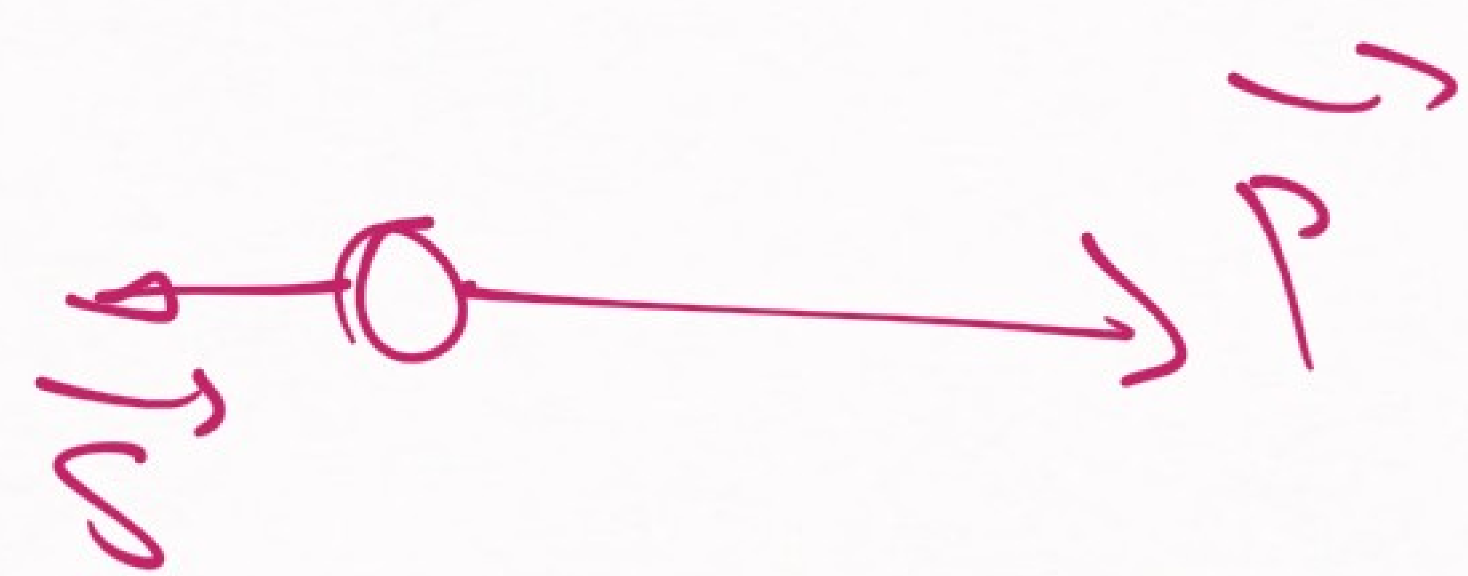
(for spin-½ fermions)

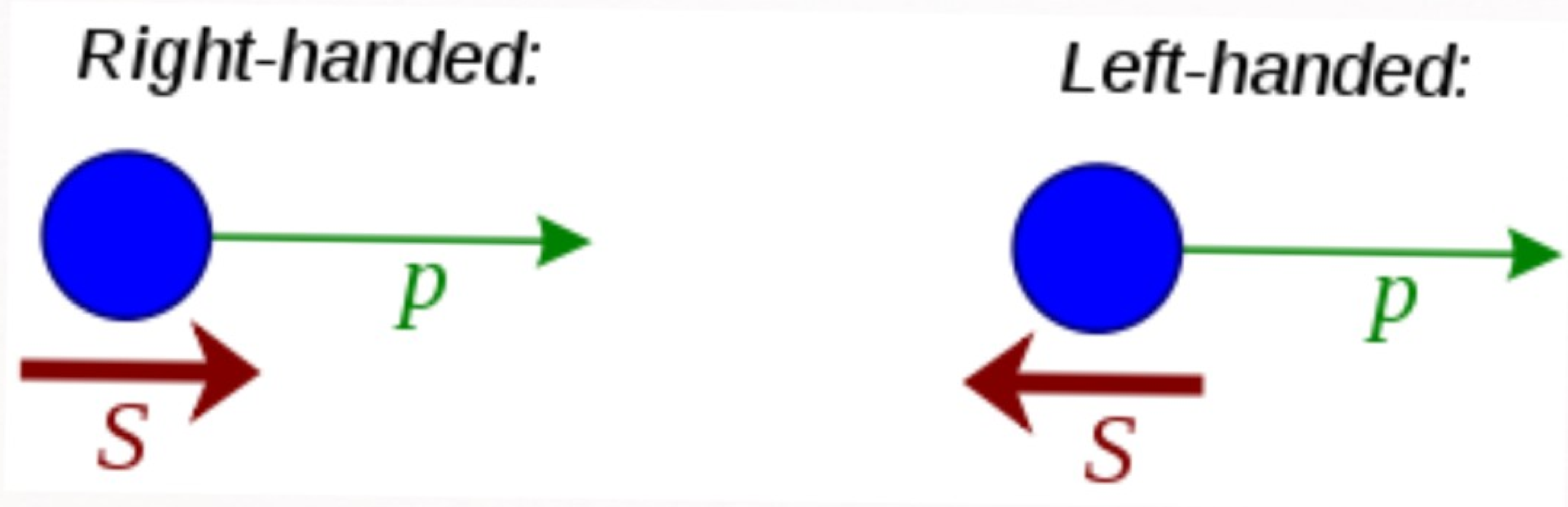
$$\lambda = \frac{\vec{\sigma}\cdot\vec{p}}{|\vec{p}|} \rightarrow \boxed{\lambda = \pm 1}$$

$\lambda = +1$



$\lambda = -1$



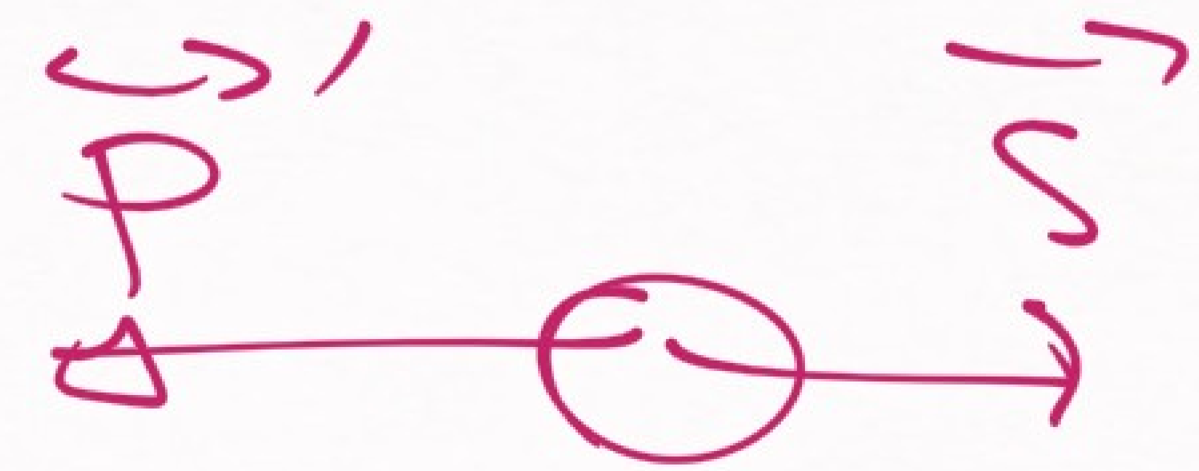


$\lambda = +1$ $\qquad$ $\lambda = -1$

A detail about helicity $\rightarrow$ massive particles

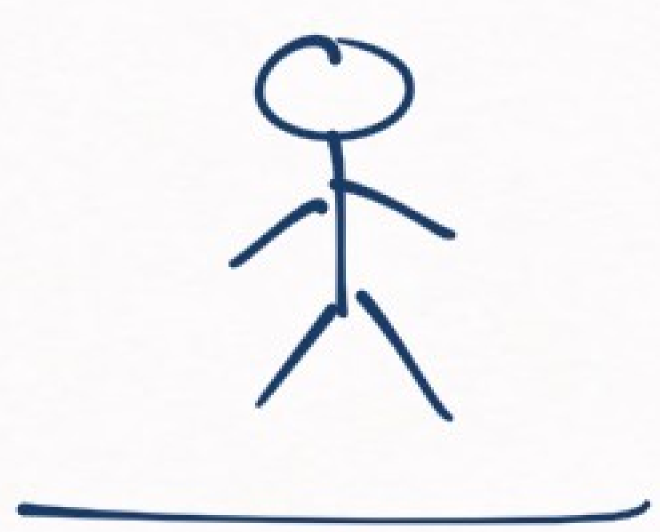
1) for a $m \neq 0$ particle $\rightarrow$ speed is relative

$(|\vec{p}| < E)$



1 $\lambda = +1$

2 $\lambda = -1$



1

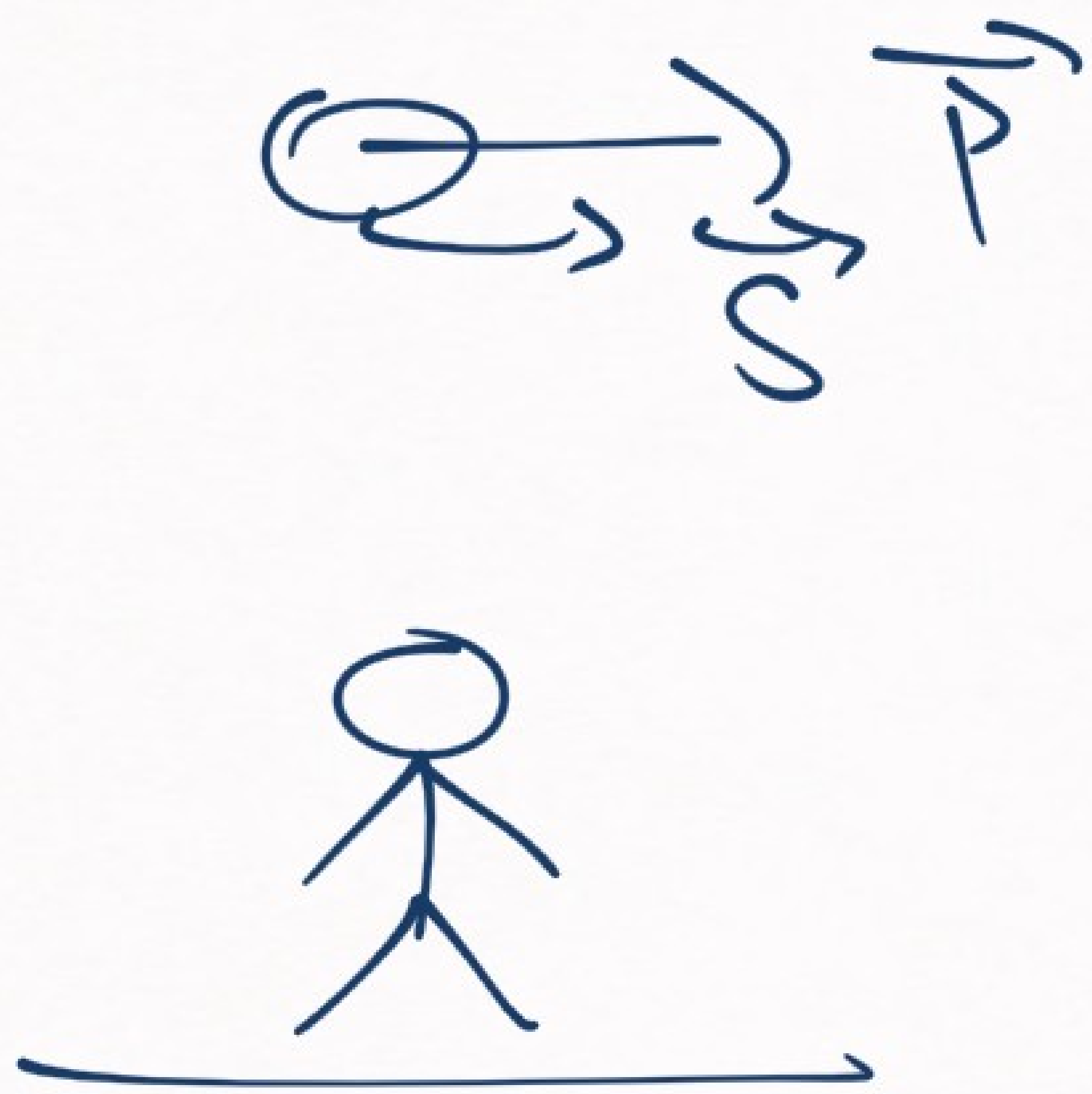
$\Rightarrow$

2

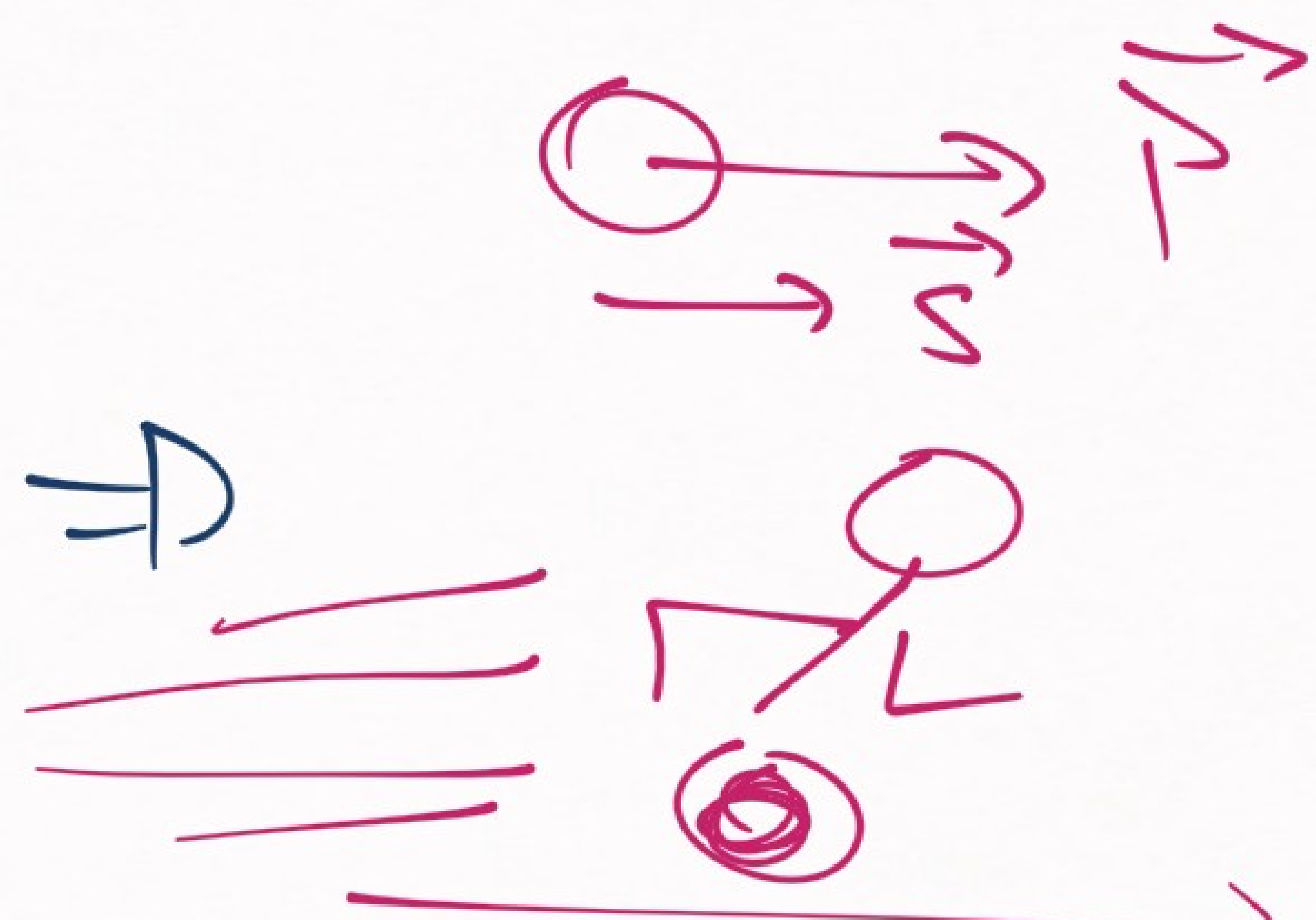
If I run fast enough → helicity changes

2) massless ($m=0$) particles

→ travel at $v=1$ (light-speed)



1



2

1 $\lambda = +1$

2 $\lambda = +1$

for massless particles

$\longrightarrow$ $\boxed{\text{helicity is conserved}}$

[The QFT version of helicity :]

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - \underbrace{m}_{\text{mass}})\underbrace{\psi}_{\text{field}} \rightarrow \boxed{\text{Dirac field}}$$

$$\downarrow$$

[$\exists$ global symmetry]

Global symmetry : U(1) symmetry

$\psi(x) \to e^{i\alpha}\psi(x)$ $\quad$ $\left(\psi^\dagger(x) \to e^{-i\alpha}\psi^\dagger(x)\right)$

U(1) transformation

$\Rightarrow \boxed{\mathcal{L} \to \mathcal{L}}$

$\bar{\psi}\psi = \psi^\dagger\gamma^0\psi$

$\psi \rightarrow$ has four components (two for particle states, two for antiparticles)

We can defined right and left-handed fields:

$$\psi_R = \underline{\underline{P_R}} \psi = \frac{1}{2}(\underline{\underline{1+\gamma_5}})\psi \quad \text{(right-handed)}$$

$$\psi_L = \underline{\underline{P_L}} \psi = \frac{1}{2}(\underline{\underline{1-\gamma_5}})\psi \quad \text{(left-handed)}$$

$$P_L^\dagger = P_L \qquad P_L^2 = P_L \qquad P_L P_R = 0$$

$$P_R^\dagger = P_R \qquad P_R^2 = P_R \qquad (P_L + P_R = 1)$$

Sounds familiar?

$\boxed{\text{Projectors}}$

$$\boxed{\psi = \psi_L + \psi_R} \quad (P_L + P_R = 1)$$

$$\psi_L = P_L \psi$$

$$\psi_R = P_R \psi$$

Dirac field can be decompose into a R- and L-subfields

It's possible that there are new symmetries related to this L+R decomposition

$$\left.\begin{aligned} \psi_L &\to e^{i\alpha_L}\psi_L \\ \psi_R &\to e^{i\alpha_R}\psi_R \end{aligned}\right\} \text{ analogous to } \psi \to e^{i\alpha}\psi$$

$$\underline{\underline{U(1)_L,\ U(1)_R}} \qquad\qquad \underbrace{U(1)}_{U(1)_{L+R}}$$

$$\psi = \psi_L + \psi_R$$

We can explicitly check $U(1)_R, U(1)_L$:

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - m)\psi = \underbrace{\bar{\psi} i \not{\partial} \psi}_{1} - \underbrace{m\bar{\psi}\psi}_{2}$$

1 $\bar{\psi}\not{\partial}\psi = \bar{\psi}_L \not{\partial} \psi_L + \bar{\psi}_R \not{\partial} \psi_R$ (check)

$$\boxed{\bar{\psi}\not{\partial}\psi \to \bar{\psi}\not{\partial}\psi} \text{ under } U(1)_L, U(1)_R$$

(2) $\bar{\psi} m \psi = \bar{\psi}_L m \psi_R + \bar{\psi}_R m \psi_L$ (check)

$$\psi m \psi \to e^{i(\alpha_R - \alpha_L)} \bar{\psi}_L m \psi_R$$

$$+ e^{-i(\alpha_R - \alpha_L)} \bar{\psi}_R m \psi_L$$

$\bar{\psi} m \psi \not\to \bar{\psi} m \psi$ under $U(1)_R, U(1)_L$

$\bar{\Psi} m \Psi \to \bar{\Psi} m \Psi$ only for $\alpha_L = \alpha_R$

$(\cong U(1)_{L+R})$

a) If $m \neq 0 \Rightarrow U(1)_{L+R}$ symmetry

b) If $m = 0 \Rightarrow U(1)_L, U(1)_R$ symmetries $\to$ $\boxed{\text{CHIRALLY SYMMETRIC}}$

How does this apply to $\boxed{\text{QCD}}$

$$\mathcal{L}_{QCD} = \underbrace{-\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}}_{\text{gloons}} + \bar{q} i \not{D} q \underbrace{- \bar{q} M q}_{\text{quark masses}}$$

$$\underline{q} = \begin{pmatrix} u \\ d \\ s \\ c \\ b \\ t \end{pmatrix}, \quad \mathcal{M} = \begin{pmatrix} m_u & & & & & \\ & m_d & & & 0 & \\ & & m_s & & & \\ & & & m_c & & \\ & 0 & & & m_b & \\ & & & & & m_t \end{pmatrix}$$

Question: can we ignore the masses?

When is $m_{\underline{q}} \simeq 0$?

QCD $\rightarrow$ natural scale $\rightarrow$ $\boxed{\Lambda_{QCD} \simeq 200\text{-}300 \text{ MeV}}$

1) $\dfrac{m_q}{\Lambda_{QCD}} \ll 1 \longrightarrow m_q \simeq 0$ approximation is expected to be valid

2) $\dfrac{m_q}{\Lambda_{QCD}} \gtrsim 1 \rightarrow m_q \neq 0$

Let's see the quark masses:

$m_u \sim 2\,MeV$ ✓ CHECK!

$m_d \sim 5\,MeV$ ✓ CHECK!

$m_s \sim 95\,MeV$ ∼ MAYBE

LIGHT QUARKS Σ

$m_c \sim 1.3\,GeV$ ✗ NO!

$m_b \sim 4.2\,GeV$

HEAVY QUARKS Σ

LIGHT QUARKS $\rightarrow$ $m_q \simeq 0$ is a good approximation (probably)

two possible approximations:

a) u, d quarks light $(n=2)$

b) u, d, s quarks light $(n=3)$

a) $m_q \simeq 0 \rightarrow$ CHIRAL TRANSFORMATIONS

$$\left.\begin{array}{l} \psi_L \rightarrow e^{i\alpha_L} \psi_L \\ \psi_R \rightarrow e^{i\alpha_R} \psi_R \end{array}\right\} \text{ 1 field}$$

$$\psi_{L,R} \rightarrow \begin{pmatrix} u \\ d \end{pmatrix}_{L,R} \qquad \begin{pmatrix} u \\ d \end{pmatrix}_{L/R} \rightarrow \underbrace{\begin{pmatrix} 2\times 2 \end{pmatrix}}_{\text{equivalent of } e^{i\alpha_{L/R}}} \begin{pmatrix} u \\ d \end{pmatrix}_{L,R}$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_{L/R} \to V_{L/R} \begin{pmatrix} u \\ d \end{pmatrix}_{L/R}$$

$U(n) = \{ n \times n \text{ matrices} \; / \; U U^{+} = 1 \}$

$V_{L/R}$ is a 2×2 matrix $\in U(2)_{L/R}$

$$g_L \to V_L \, g_L \qquad g_L \in U(2)_L$$

$$g_R \to V_R \, g_R \qquad g_R \in U(2)_R$$

b) $\begin{pmatrix} u \\ d \\ s \end{pmatrix} \to V_L, V_R \in U_R(3), U_L(3)$

---

n light quarks $\to$ $\boxed{\begin{array}{l} q_L \to V_L q_L \\ q_R \to V_R q_R \\ V_L \in U_L(n) \\ V_R \in U_R(n) \end{array}}$ We should have this symmetry

For n light-quark $\Rightarrow$ $\mathcal{L}_{QCD}$ invariant under

$$\boxed{G = U_L(n) \otimes U_R(n)}$$

$\exists$ complication

$\exists$ anomaly called chiral anomaly

ANOMALY → CLASSICAL SYMMETRY BROKEN BY THE QUANTIZATION PROCESS

EXAMPLE IN QM → $\boxed{V(r) = -\frac{g}{r^2} \text{ in } D=3}$

QFT w/ chiral symmetry ($n \geq 2$)

have the chiral anomaly

$G = U_L(n) \otimes U_R(n) \longrightarrow \boxed{G = SU_L(n) \otimes SU_R(n) \otimes U(1)_{L+R}}$

$L+R \to$ vector

$L-R \to$ axial

SYMMETRY WE EXPECT FOR HADRONS CONTAINING LIGHT QUARK IS

$$\boxed{G = SU(2)_L \otimes SU(2)_R \otimes U(1)_{L+R}}$$

Prediction → $\boxed{m(H, J^+) = m(H, J^-)}$

Reality → this does not happen

EXAMPLES

1) LoM → $\pi(0^-)$, $\sigma(0^+)$ $m_\pi \neq m_\sigma$

2) $N(940)$, $N^*(1535)$ $m_N \neq m_{N^*}$
$J = 1/2^+$ $J = 1/2^-$

3) $\rho(770)$, $a_1(1260)$ $m_\rho \neq m_{a_1}$
$J = 1^-$ $J = 1^+$
(careful w/ this example)

What we have found here :

$$\boxed{\begin{array}{c}\text{FUNDAMENTAL QCD STATE} \\ \text{NOT CHIRALLY SYMMETRIC}\end{array}}$$

→ Chiral symmetry is broken

→ opportunity to use GOLDSTONE THEOREM

RECAP

GOLDSTONE THEOREM

$$H \,/\, [H, G] = 0$$

$$|0\rangle \,/\, G|0\rangle \neq |0\rangle$$

but still $F|0\rangle = |0\rangle$ ($F \subset G$)

QCD LAGRANGIAN

$$\mathcal{L} \,/\, [\mathcal{L}, G] = 0$$

$$G = SU_L(n) \otimes SU_R(n) \otimes U(1)_{L+R}$$

$$|0\rangle \,/\, G|0\rangle \neq |0\rangle$$

(masses $\neq$)

QCD LAGRANGIAN
& QCD SPECTRUM
(GROUND STATE)

$\rightarrow$ DIFFERENT SYMMETRIES

$\boxed{\text{WE CAN USE THE GOLDSTONE THEOREM}}$

1) $\mathcal{L}_{QCD} \rightarrow G = SU(n)_L \otimes SU(n)_R \otimes U(1)_{L+R}$

(Anch $\rightarrow$ chiral anomaly)

2) Hadrons $\rightarrow$ $m(\rho) < m(a_1)$

$J = 1^-$ $\qquad$ $J = 1^+$

ground state $\leftarrow$ vector $\qquad$ axial vector

"L+R" $\qquad$ "L−R"

$F = SU(n)_{L+R} \otimes U(1)_{L+R}$

GOLDSTONE THEOREM W/

$$G = SU(n)_L \otimes SU(R)_R \otimes U(1)_{L+R}$$

$$F = SU(n)_{L+R} \otimes U(1)_{L+R}$$

REMINDER $\rightarrow$ $SU(n)$ has n^2-1 generators

$$\left.\begin{aligned} n_G &= 2(n^2-1)+1 = 2n^2-1 \\ n_F &= n^2 \end{aligned}\right] \Rightarrow$$

$\Rightarrow$ We should have $n_G - n_F$ massless bosons in the theory

$$\boxed{n_G - n_F = n^2 - 1}$$

Two versions of chiral symmetry $\begin{cases} n=2 \\ n=3 \end{cases}$

(depends on how we treat the strange quark)

a) $n=2 \to$ 3 massless bosons

b) $n=3 \to$ 8 massless bosons

a) $\to$ pions ($m_\pi \simeq 140$ MeV)

$m_\pi \ll m_\sigma$ ($m_\sigma \simeq 400-550$ MeV)

b) pions, kaons, eta (8 in total)

$m_K \simeq 495 \text{ MeV}$ } $m_K \ll m_{K^0}$ (σ^+ version)

$m_\eta \simeq 550 \text{ MeV}$ } $m_\eta \ll m_{a_0}$ (σ^+ version)

Hadronic physics → scale separation is bad, things are approximate, etc

Which are the quantum numbers of these "massless" bosons?

$\underline{\underline{10)}}$ $\longrightarrow$ $\frac{G}{F} \sim (L-R) \longrightarrow \bar{q}\, i\gamma_5\, q$

$\sim$ QCD analogy of pions

$\longrightarrow$ γ_5 indicates non-natural parity $\to$ $\boxed{0^-}$

WE CAN APPLY GOLDSTONE THEOREM TO QCD

$\Rightarrow$ MASSLESS PIONS

$\Downarrow$

$\bar{q}Mq$ as perturbation $\Rightarrow$ will give the pion a small mass

What are the consequences of this for nuclear physics?

$\boxed{\text{pion interactions will be derivative}}$

this means that the interaction lagrangian contains ∂_μ factor

→ pion interactions become weaker at low energies

important for two-pion exchange diagrams in the nucleon-nucleon system

1) Before chiral symmetry:

$$\mathcal{L}_{\pi NN} = g_{\pi NN}\, \bar{\psi}_N\, i\gamma_5\, \vec{\tau}\cdot\vec{\pi}\, \psi_N$$ (not derivative)

2) After chiral symmetry:

$$\mathcal{L}_{\pi NN} = \frac{g_A}{2f_\pi}\, \bar{\psi}_N\, \gamma^5 \gamma_\mu\, \vec{\tau}\cdot\underbrace{\partial_\mu \vec{\pi}}_{\text{derivative interaction}}\, \psi_N$$

Comparison of 1 & 2:

$$\boxed{\frac{g_A}{f_\pi} = \frac{g_{\pi NN}}{M_N}}$$ → Golgberger-Treimann relation

appears in $n \to p e \bar{\nu}_e$

NOTE → this is for $f_\pi = 93$ MeV

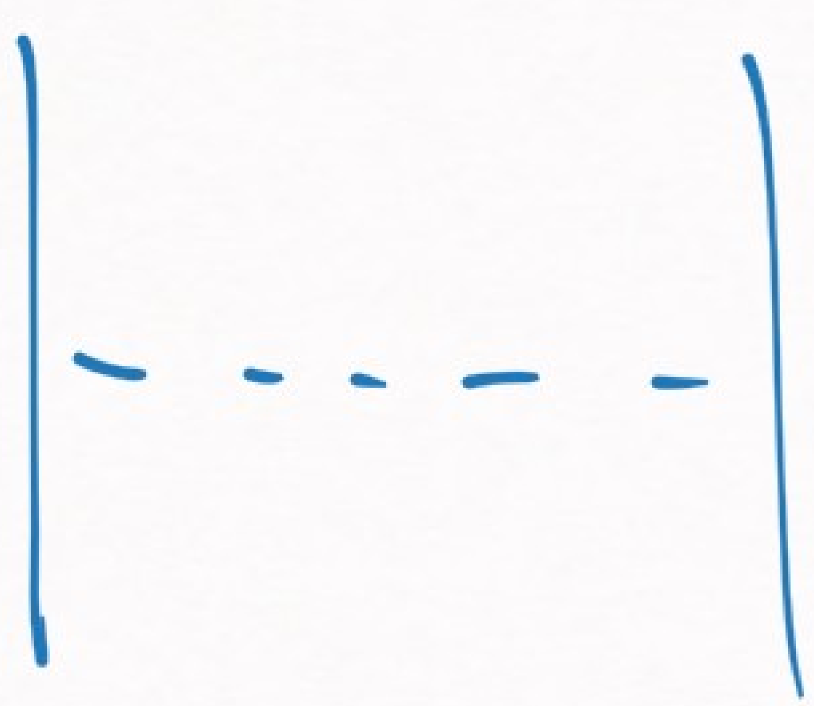
($\exists$ different normalizations for f_π)

(generates a lot of confusion)

Prediction of chiral symmetry

$\rightarrow$ Derivative interactions

1) OPE (one pion exchange)

 $\rightarrow$ this diagram is identical for derivative / non derivative pions

2) TPE (two pion exchange)

→ big difference between derivative & non-derivative pions

planar box crossed box triangle football

People abandoned TDE in the late 50's because they did not get results that made sense

REASON: THEY DID NOT KNOW ABOUT CHIRAL SYMMETRY

(THEIR PION DYNAMICS WAR WRONG)

1) Easy intro to chiral symmetry

(INSPIRE-HEP)

4. **Chiral effective Lagrangians**
H. Leutwyler (Bern U.). Jul 1991. 55 pp.
Published in **Lect.Notes Phys. 396 (1991) 1-37**
BUTP-91-26
DOI: 10.1007/3-540-54978-1_8
Conference: C91-06-02, p.97-138, Conference: C91-02-27
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KEK scanned document
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2) Classical review

1. (1099) **Chiral dynamics in nucleons and nuclei**
V. Bernard (Strasbourg, CRN), Norbert Kaiser (Munich, Tech. U.), Ulf-G. Meissner (Bonn U.).
Jan 1995. 153 pp.
Published in **Int.J.Mod.Phys. E4 (1995) 193-346**
CRN-95-3, TK-95-1
DOI: 10.1142/S0218301395000092
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