NUCLEAR PHYSICS 10

MORE ABOUT RENORMALIZATION

RECAP → Renormalization & EFT

EFT with NDA: the algorithm

1. Identify the relevant degrees of freedom
2. Identify high- and low-energy scales→expansion parameters x
3. Identify symmetries of low-energy theory
4. Choose the accuracy required. This, together with the size of x, tells you the order, n, to which you must calculate.
5. Write down all possible local operators, that have naive dimensions up that order, and are consistent with symmetries

   "NDA"

6. Derive the behaviour of loops, and calculate them.

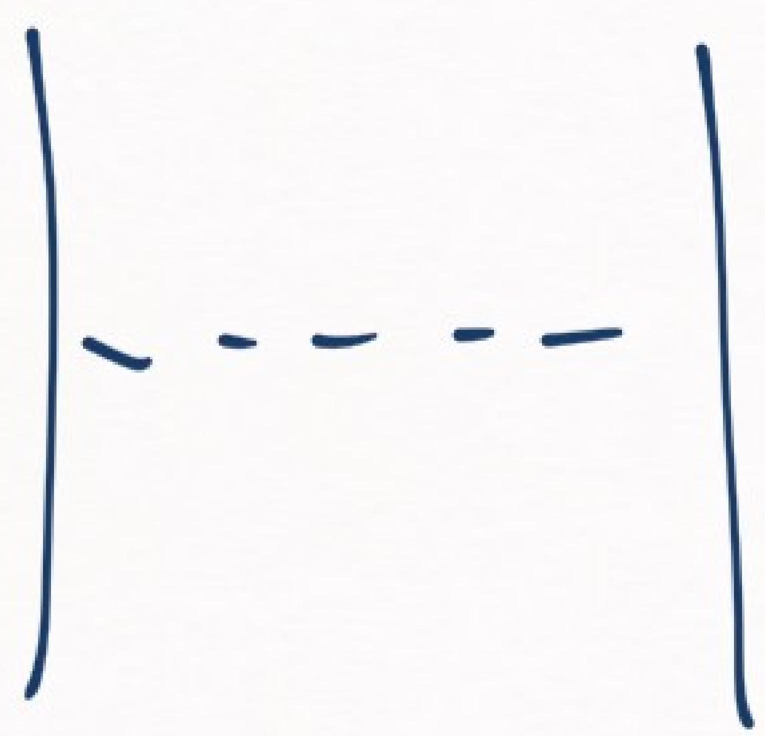
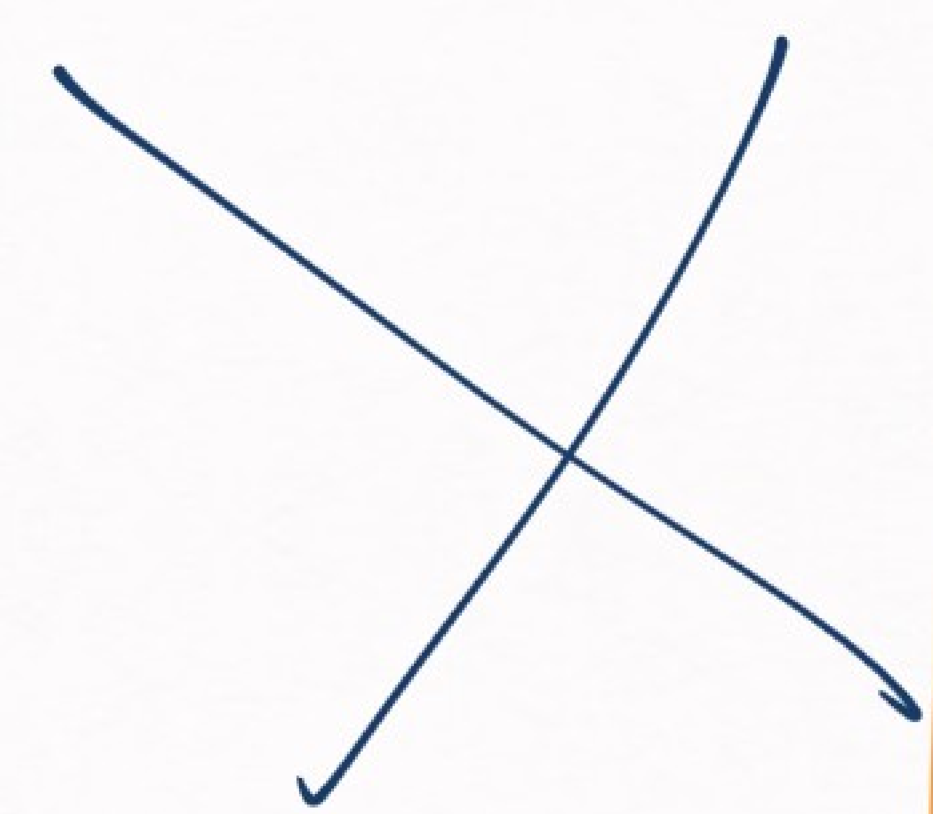
All operators needed for renormalization at this order should be present →Model independence

→ A talk by D. R. Phillips

(Other formulations in our previous lectures)

[Basic idea]

View at long-distances

 $\Rightarrow$ 

→ no details seen

$$V(\vec{q}) = c_0 + c_2 \vec{q}^{\,2} + \dots$$

View we have at short-distance → see detail

$$V(\vec{q}) = - \frac{g^2}{|\vec{q}|^2 + m^2}$$

Basic idea is simple

"PHYSICS AT LONG-DISTANCES DOES NOT DEPEND ON SHORT-RANGE DETAILS"

But implementation is difficult

→ Renormalization, EFTs

→ We tried different implementations

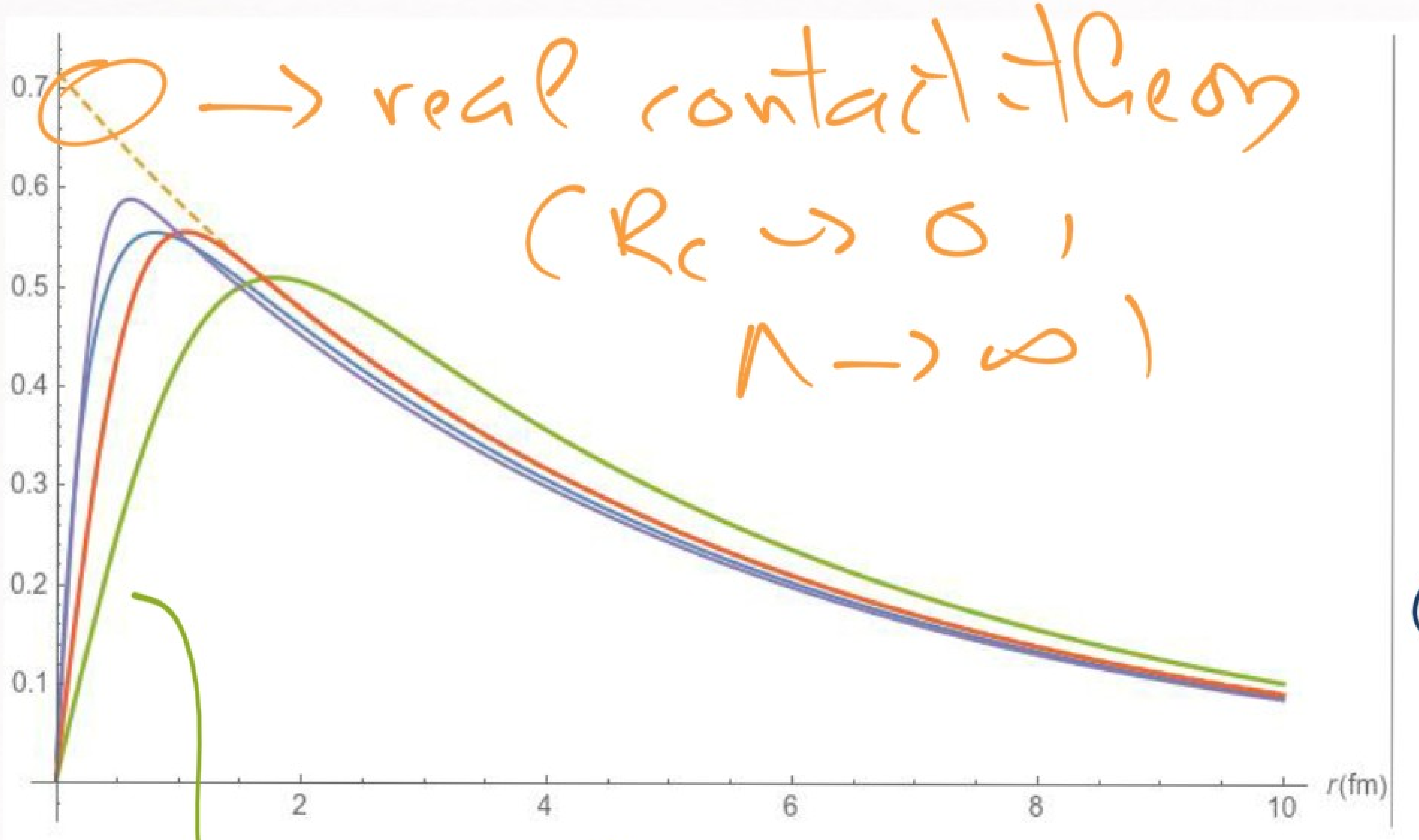
(1) Fundamental view of a problem

→ two-body system + Yukawa potential

Effective view of a problem

→ two-body system + contact interaction

Summary of results of view (1) (check previous lessons)



→ real contact theory ($R_c \to 0$, $\Lambda \to \infty$)

exception: too soft cutoff

Blue line → wave function of some Yukawa potential

Other lines → contact theories w/ different cutoffs hard/soft cutoffs

→ hard cutoff $R_c \to 0, \Lambda_c \to \infty$

→ soft cutoff $R_c \to \infty, \Lambda_c \to 0$

(ultraviolet / infrared limits)

(2) $\frac{d}{d\Lambda}\langle\psi|\hat{O}|\psi\rangle = 0$ → RG invariance

→ Equivalent to (1)

→ More abstract (also allows us to understand more things)

Ingredients for 2

2.a) Regulator (because R_c dependence)

2.b) Wave function $|\Psi\rangle$ (choice)

(comply w/ expectations for a system)

2.c) Observable $\hat{O}$

2. a) Regulators

$$\left.\begin{aligned} V_{contact}(\vec{r}) &= C_0\, \delta^{(3)}(\vec{r}) \\ V_{contact}(\vec{q}) &= C_0 \end{aligned}\right\}\ \text{singular}$$

$$\left.\begin{aligned} &\text{Delta-shell:} \quad C_0(R_c)\,\frac{\delta(r-R_c)}{4\pi R_c^2} \\ &\text{Gaussian:} \quad C_0(R_c)\,\frac{e^{-(r/R_c)^2}}{\pi^{3/2} R_c^3} \end{aligned}\right]\ \forall \text{ kind of possibilities}$$

2.5) Wave functions

(i) One option was $\psi(\vec{r}) = e^{i\vec{k}\cdot\vec{r}}$

(good for scattering by a weak potential)

(ii) Another option was $\psi(\vec{r}) = \dfrac{A_r}{\sqrt{4\pi}} \dfrac{e^{-\gamma r}}{r}$

(good for describing a bound state)

2.c) Observable → whatever we are interested in

$V(r)$ is a possibility

Caveat → $V(r)$ is not a QFT-observable

(only a QM observable)

$V(r) = C_0 \times$ regulator] → for testing purposes

(i) $\psi(\vec{r}) \to e^{i\vec{k}\cdot\vec{r}}$ (plane wave)

$$\langle\psi|V_c|\psi\rangle = \int d^3\vec{r}\, V_c(\vec{r})\underbrace{|\psi(\vec{r})|^2}_{=1} = V_c(|\vec{q}|=0)$$

If we use $V_c(\vec{q}) = C_0 \longrightarrow \langle\psi|V_c|\psi\rangle = C_0$

$$\frac{d}{dR_c}\langle\psi|V_c|\psi\rangle = 0 \longrightarrow \boxed{\frac{d}{dR_c}C_0(R_c) \simeq 0}$$

We actually checked this was correct:

$$\psi(\vec{r}) \to e^{i\vec{k}\cdot\vec{r}} + f(\hat{r})\frac{e^{ikr}}{r} \quad \text{(Scattering problem)}$$

$$f(\hat{r}) \simeq -\frac{\mu}{2\pi}\int d^3\vec{r}\, V(\vec{r})\, e^{i(\vec{k}-\vec{k}')\vec{r}} = -\frac{\mu}{2\pi} V(\vec{q})$$

Born approximation

Yukawa: $\rho_Y(\vec{r}) \simeq -\frac{M}{2\pi} V_Y(\vec{q}) = -\frac{M}{2\pi} \frac{(-g^2)}{|\vec{q}|^2 + m^2}$

$$\xrightarrow[|\vec{q}| \ll m]{} +\frac{M}{2\pi}\frac{g^2}{m^2}$$

We match

Contact (EFT): $\rho_c(\vec{r}) \simeq -\frac{M}{2\pi} C_0(R_c)\left(\frac{\sin(qR_c)}{qR_c}\right)$

$$\xrightarrow[|\vec{q}| \ll 1/R_c]{} -\frac{M}{2\pi} C_0(R_c)$$

$$C_0(R_c) \simeq -\frac{g^2}{m^2}$$

$\longrightarrow$ Explicit solution of EFT

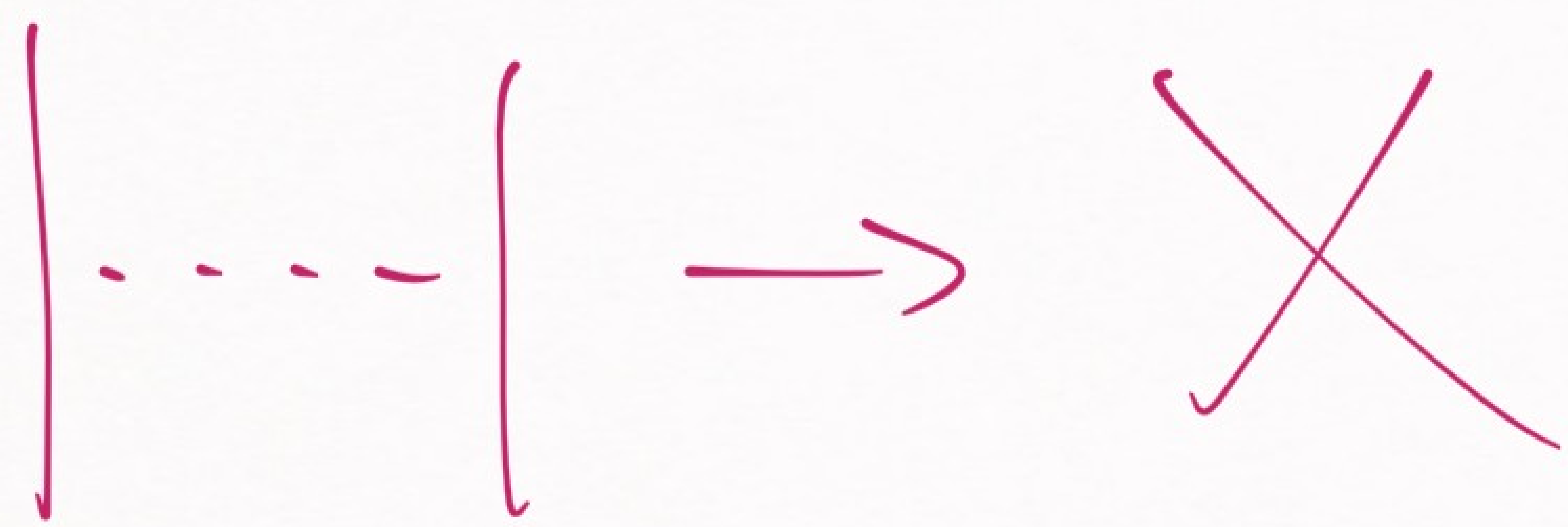
$\longrightarrow$ Confirms our calculation based on $\frac{d}{dR_c}\langle\psi|\hat{O}|\psi\rangle = 0$

$$\underbrace{\qquad\qquad\qquad}$$

no R_c-dependence

$\downarrow$

$$\frac{d}{dR_c}\left[C_0(R_c)\right] \simeq 0$$



$$V(\vec{q}) = -\frac{g^2}{|\vec{q}|^2+m^2} \longrightarrow V(\vec{q}) = -\frac{g^2}{m^2} + \frac{g^2}{m^2}\left(\frac{|\vec{q}|^2}{m^2}\right) + \cdots$$

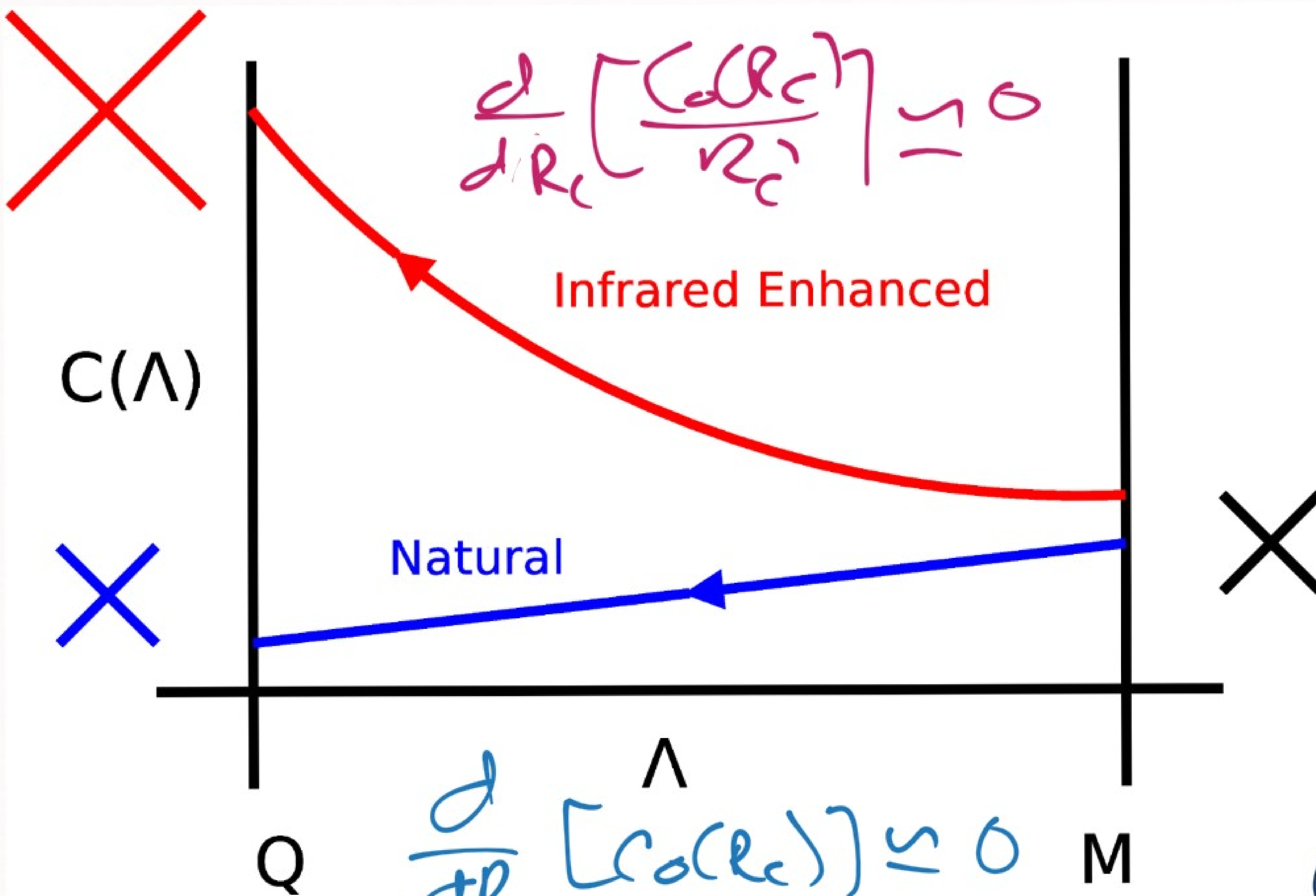
[The previous EFT reproduces this term]

(ii) $\psi(\vec{r}) = \frac{A_s}{\sqrt{4\pi}} \frac{e^{-\gamma r}}{r}$ (bound state)

$$\langle \psi | V_c | \psi \rangle \propto \frac{C_0(R_c)}{R_c^2}$$

$$\frac{d}{dR_c}\langle \psi | V_c | \psi \rangle \propto \boxed{\frac{d}{dR}\left[\frac{C_0(R_c)}{R_c^2}\right] \simeq 0}$$

NON-UNIQUENESS OF EFTs / POWER COUNTING



$$\frac{d}{dR_c}\left[\frac{C_0(R_c)}{R_c}\right] \simeq 0$$

$$\frac{d}{dR_c}\left[C_0(R_c)\right] \simeq 0$$

(Go back to lessons 1 & 2)

$\downarrow$

Depends on long-distance physics

(NATURAL / NON-NATURAL)

Helps to guess the size of $C_0(R_c)$:

→ We begin at $\Lambda \sim M$ $(MR_c \sim 1)$

$$C_0(\Lambda \sim M) \sim \frac{1}{M^2}$$

$$\underset{\text{hard}}{\underline{M}} \gg \underset{\text{soft}}{\underline{Q}}$$

→ We evolve towards $\Lambda \sim Q$

$$C_0(\Lambda \sim M) \sim \frac{1}{M^2}$$

$C_0(\Lambda \sim Q) \sim \frac{1}{M^2}$ (natural)

$C_0(\Lambda \sim Q) \sim \frac{1}{Q^2}$ (unnatural)

RECAP

1) [The $\frac{d}{dR_c} C(R_c) = 0$ RGE describes a "NATURAL SYSTEM" (a system for which the Born approximation works well]

2) [The $\frac{d}{dR_c}\left[\frac{C(R_c)}{R_c^2}\right] = 0$ RGE describes a "FINE-TUNED SYSTEM" (a system for which we have a shallow-bound state for instance)

IF YOU ARE ADVENTUROUS, YOU CAN TRY THIS PAPER:

3. **A Renormalization group treatment of two-body scattering**
(196) Michael C. Birse, Judith A. McGovern, Keith G. Richardson (Manchester U.). Jul 1998. 4 pp.
Published in **Phys.Lett. B464 (1999) 169-176**
MC-TH-98-11
DOI: 10.1016/S0370-2693(99)00991-0
e-Print: **hep-ph/9807302** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record - Cited by 196 records 100+

→ more abstract [p-space
RGE

→ will help you understand RG

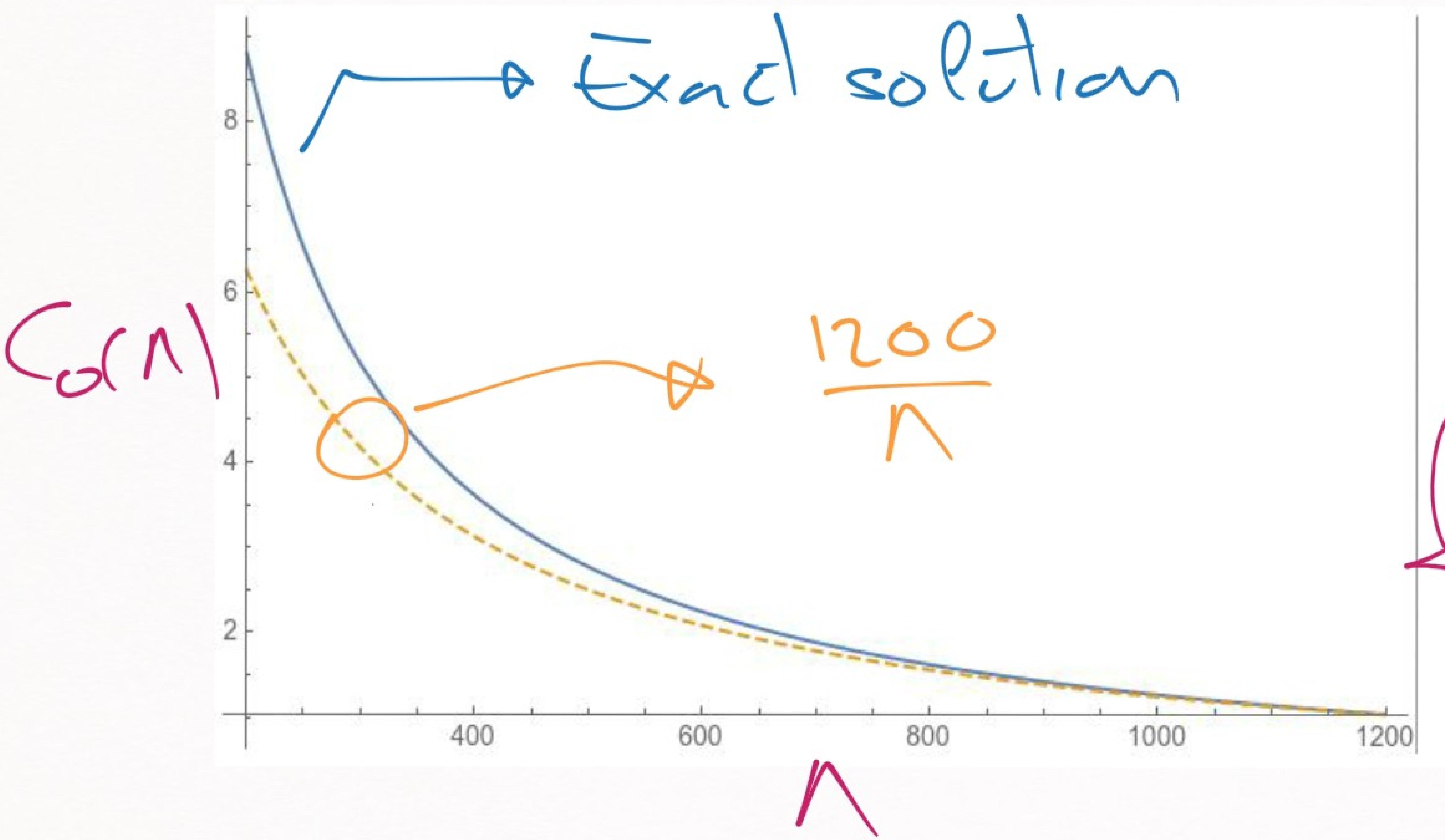
VIEWS 1 & 2 → [inconsistency]

page 5

page 9

Regards the counting for unnatural systems

VIEW 1 $\longrightarrow$ RGE for C_0 is



$$\frac{d}{d\Lambda}\left[\Lambda C_0(\Lambda)\right] \simeq 0$$

VIEW 2

?

$$\frac{d}{d\Lambda}\left[\Lambda^2 C_0(\Lambda)\right] \simeq 0$$

$\boxed{\text{MISMATCH}}$

VIEW 1 $\rightarrow$ $\frac{d}{d\Lambda}\left[\Lambda C_{\gamma}(\Lambda)\right] \succeq 0$

VIEW 2 $\rightarrow$ $\frac{d}{d\Lambda}\left[\Lambda^{2} C_{G}(\Lambda)\right] \preceq 0$

CLUE: CONCEPTUAL PROBLEM

CHALLENGE $\rightarrow$ Why is that?

1st person solving it $\rightarrow$ 8 pts 2nd $\rightarrow$ 5 pts

AMASS MORE VIEWS ON RG

1) Fitting $V_c = C_0(\Lambda) e^{-(\vec{q}^{\,2}/\Lambda^2)}$ to bound state /data

2) Solving

$\frac{d}{d\Lambda}\langle\psi|V_c|\psi\rangle \simeq 0$ w/ certain assumptions

But $\exists$ more ways $\rightarrow$ review quickly

3) Directly counting the powers of a given scale (counting powers of Q & M)

→ classical or original understanding of power counting

3. **Effective field theory of short range forces**
U. van Kolck (Caltech, Kellogg Lab & Washington U., Seattle). Aug 1998. 38 pp.
Published in **Nucl.Phys. A645 (1999) 273-302**
KRL-MAP-230, NT-UW-98-01
DOI: 10.1016/S0375-9474(98)00612-5
e-Print: **nucl-th/9808007** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record - Cited by 331 records 250+

→ GOOD EXAMPLE OF THIS TYPE OF THINKING

Short-version:

$V_c = C_0 \longrightarrow$ Scattering amplitude / cross section

$$\frac{d\sigma}{d\Omega} = \left| \frac{\mu C_0}{2\pi} \right|^2 \longrightarrow \boxed{C_0 \sim \frac{2\pi}{\mu} a_0}$$

$= a_0^2$ (scattering length)

$\boxed{C_0 \sim \frac{2\pi}{\mu} a_0}$ $\longrightarrow$ natural system

$a_0 \sim 1/\mu \longrightarrow \boxed{C_0 \sim \frac{2\pi}{M^2}}$

$(\mu \sim M)$

$\downarrow$

unnatural system, $a_0 \sim 1/a$

$\longrightarrow \boxed{C_0 \sim \frac{2\pi}{M a}} \longrightarrow$ Infrared enhancement

4) Removing infinities

→ usual one in QFT

→ useful for IFTs containing a long-range potential

→ only Co and the like
(this will look a bit forced)

Example for NN w/ pion exchanges

7. **Perturbative renormalizability of chiral two pion exchange in nucleon-nucleon scattering**
(90) M.Pavon Valderrama (Julich, Forschungszentrum & JCHP, Julich). Dec 2009. 4 pp.
Published in **Phys.Rev. C83 (2011) 024003**
FZJ-IKP-TH-2009-37
DOI: 10.1103/PhysRevC.83.024003
e-Print: **arXiv:0912.0699 [nucl-th] | PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service; Link to Fulltext
Detailed record - Cited by 90 records 50+

(not contact-theory)

5) LOOKING AT RESIDUAL CUTOFF DEPENDENCE

Example: $f_c(\hat{r}) \simeq -\frac{M}{2\pi} G(R_c)\left[\frac{\sin(qR_c)}{qR_c}\right]^2$

$\frac{d}{dR_c}[f_c(\hat{r})] \neq 0$ $(\simeq 0)$ → add new terms to make all the cutoff dependence disappears

$$\rho_c(r) = -\frac{\mu}{2\pi} C_0(R_c)\left[1 - \frac{1}{3}(qR_c)^2 + \dots\right]$$

$$+ \left(-\frac{\mu}{2\pi}\right) C_2(R_c)\left[\# (qR_c)^2 + \dots\right]$$

You reduce when to include new terms

Good example of using "residual cutoff dependence" arguments

1. **Renormalizing Chiral Nuclear Forces: Triplet Channels**
(68) Bingwei Long (Jefferson Lab), C.J. Yang (Arizona U. & Ohio U., Inst. Nucl. Part. Phys.). Nov 2011. 20 pp.
Published in **Phys.Rev. C85 (2012) 034002**
JLAB-THY-11-1464, INT-PUB-11-038
DOI: 10.1103/PhysRevC.85.034002
e-Print: **arXiv:1111.3993 [nucl-th] | PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service; OSTI.gov Server; JLab Document Server
Detailed record - Cited by 68 records 50+

→ Also advance $\rightsquigarrow$ pion exchanges and the like

6) Consider the matrix elements of some short-range potential w/ effective wave function $\longrightarrow$ similar to 2) & 3)

$$\langle \psi_L | V_S | \psi_L \rangle \longrightarrow \begin{cases} 1)\ \psi_L \to \text{long-range wf} \\ 2)\ V_S \to \text{"true" short range potential} \end{cases}$$

→ play w/ $|\psi_c\rangle$ & V_s and deduce things

$V_s(r) = M f(Mr)$ → why?

short-range scale

(i) V_s has units of energy

$[V_s] = [\text{energy}]$ → M

(ii) $f(\underline{\underline{M}}r)$ dimensionless function

↳ short-range scale

$$|\psi_L\rangle \longrightarrow \psi_L(r) \sim (Qr)^n$$

a general ansatz (minimal assumption)

Example: Yukawa + shallow bound state

$$V_Y(r) = -\frac{g^2}{4\pi}\frac{e^{-Mr}}{r} = -\frac{g^2}{4\pi} M \frac{e^{-Mr}}{Mr} = M f_Y(x)$$

$$x = Mr$$

$M = m$ (mass of exchanged boson)

$$V_Y(r) = M f_Y(x) \quad , \quad f_Y(x) = -\frac{g^2}{4\pi} \frac{e^{-x}}{x}$$

for $\psi_L \rightarrow$

$$\psi(r) = \frac{\Delta_c}{\sqrt{4\pi}} \frac{e^{-\gamma r}}{r} \quad (\gamma \sim \alpha)$$

$$\propto \left(\frac{e^{-\alpha r}}{\alpha r} \right) \sim (\alpha r)^{-1}$$

EVALUATE MATRIX ELEMENT & COUNT POWERS OF Q & M :

$$\langle \psi_i | V_S | \psi_i \rangle \simeq \left(\frac{Q}{M}\right)^{2n} I_S(2n)$$

$$I_S(k) = \int_0^\infty dx\, \rho(x)\, x^k$$ → dimensionless

(we can ignore it)

$$\langle \psi_L | V_S | \psi_L \rangle \sim \left(\frac{\alpha}{M}\right)^{2n} \times$$ (factors to correct the dimensionality)

$$\psi_L(r) \sim \frac{e^{-\gamma r}}{r} \propto (\alpha r)^{-1} \rightarrow n = -1$$

$$\langle \psi_L | V_S | \psi_L \rangle \sim \left(\frac{M}{\alpha}\right)^{2} \sim \frac{1}{\alpha^2}$$

→ WE RECOVER VIEW 2

$$\psi_\ell(r) \sim e^{iKr} \sim (Qr)^\ell$$

$$\frac{\langle \psi_\ell | V_S | \psi_\ell \rangle}{\langle \psi_\ell | \psi_\ell \rangle} \sim \left(\frac{Q}{M}\right)^{0} \times \frac{1}{M^2}$$

→ Also coincides w/ view 2

→ Seen six possible ways to understand the RGE of $C_0(R_0)/C_0(\Lambda)$

- Therefore psychologically we must keep all the theories in our heads, and every theoretical physicist who is any good knows six or seven different theoretical representations for exactly the same physics.
    - chapter 7, “Seeking New Laws,” p. 168

→ THIS WILL BE USEFUL FOR THOSE OF YOU DOING EFT

NUCLEAR PHYSICS 10 1/2

[ISOSPIN & SU(3)-FLAVOR SYMMETRIES]

[ISOSPIN SYMMETRY]

p, n $\rightarrow$ $M_p = 938.272$ MeV $\boxed{M_p \simeq M_n}$

$M_n = 939.563$ MeV

(similar to this $\rightarrow$ $\pi^\pm, \pi^0$ / energies of a few nuclei)

$\rightarrow$ We can treat p, n as two states of the same particle (N: nucleon)

The NUCLEON

$N = \begin{pmatrix} p \\ n \end{pmatrix}_I$ → [Isospin is actually formulated in analogy to spin]

⇒ spin → $|S M_S\rangle$

Isospin → $|I M_I\rangle$ $\begin{cases} |\tfrac{1}{2}\ {+\tfrac{1}{2}}\rangle = |p\rangle \\ |\tfrac{1}{2}\ {-\tfrac{1}{2}}\rangle = |n\rangle \end{cases}$

Isospin algebra $\to$ spin algebra

Example $\to$ two-nucleons

$$|NN\rangle_I \longrightarrow \frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$$

coupling of two isospin-1/2 states

$|NN\rangle_I \,,\ \frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$ (isospin)

$\boxed{I = 0}$ (Isoscalar channel)

$$|00\rangle_I = \frac{1}{\sqrt{2}}\left[|+-\rangle_I - |-+\rangle_I\right]$$

$$= \frac{1}{\sqrt{2}}\left[|pn\rangle - |np\rangle\right]$$

$\boxed{\text{the deuteron}}$

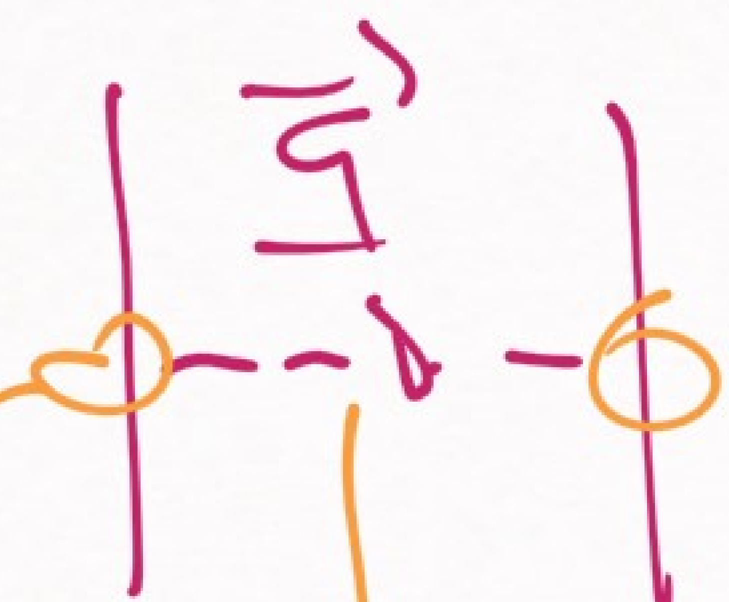
$\boxed{I = 1}$ (Isovector)

$$\underbrace{\begin{aligned} |1\,1\rangle_I &= |pp\rangle \\ |1\,0\rangle_I &= \frac{1}{\sqrt{2}}\left[\,|pn\rangle + |np\rangle\,\right] \\ |1\,-1\rangle_I &= |nn\rangle \end{aligned}}$$

A the singlet virtual state

$\boxed{\text{ISOSPIN}}$ → OPE (one pion exchange) potential

1) BEFORE ISOSPIN (check previous lectures)



$$\frac{g_{\pi NN}}{2M}\vec{\sigma}_2 \cdot (-\vec{q})$$

$$= -\left(\frac{g_{\pi NN}}{2M}\right)^2 \frac{\vec{\sigma}_1\cdot\vec{q}\;\vec{\sigma}_2\cdot\vec{q}}{q^2+m_\pi^2}$$

$$\frac{1}{\vec{q}^2+m^2}$$

$$\frac{g_{\pi NN}}{2M}\vec{\sigma}_1\cdot\vec{q}$$

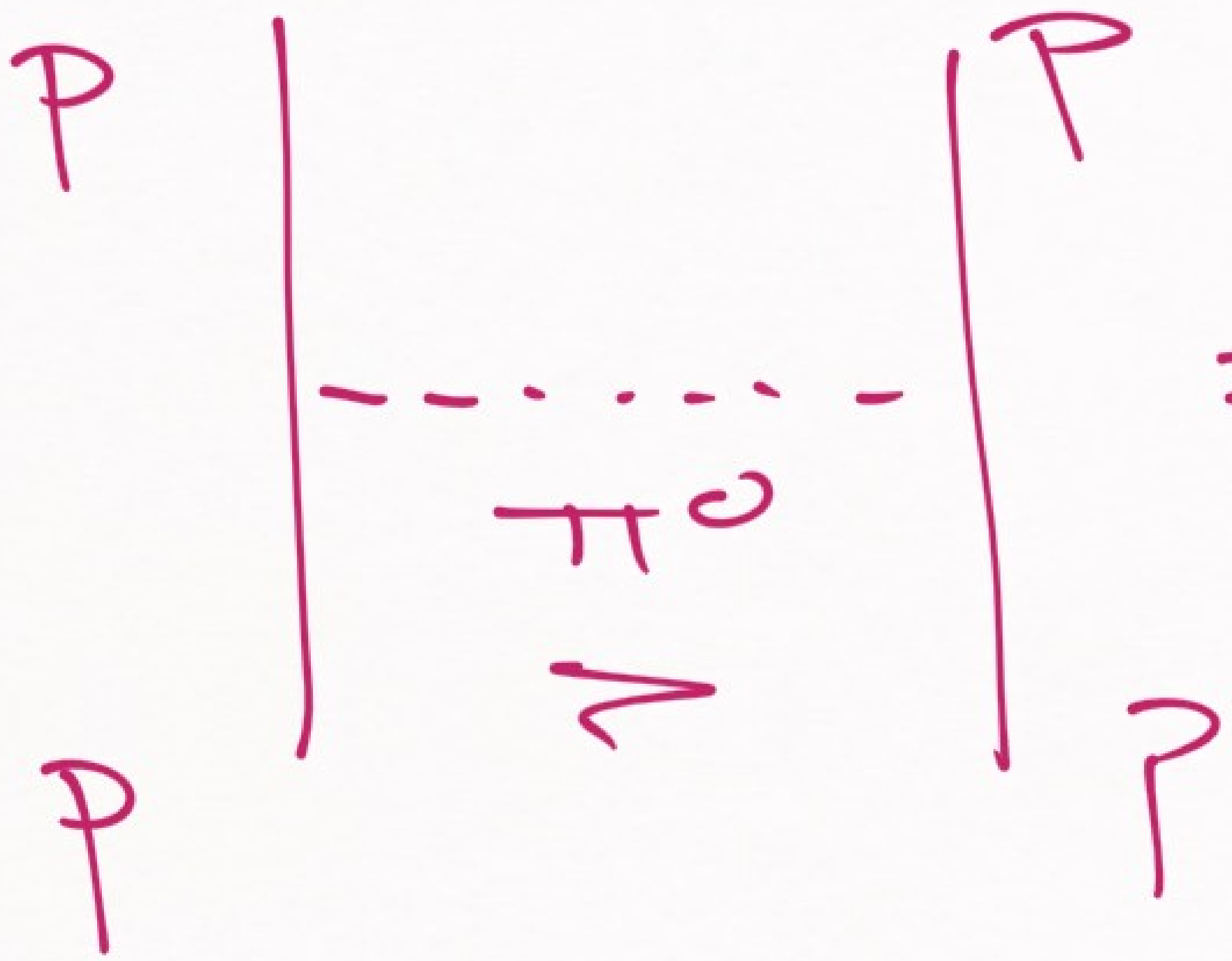
Then we said → for getting the deuteron right,

we had to multiply by (-3)

(not explained then)

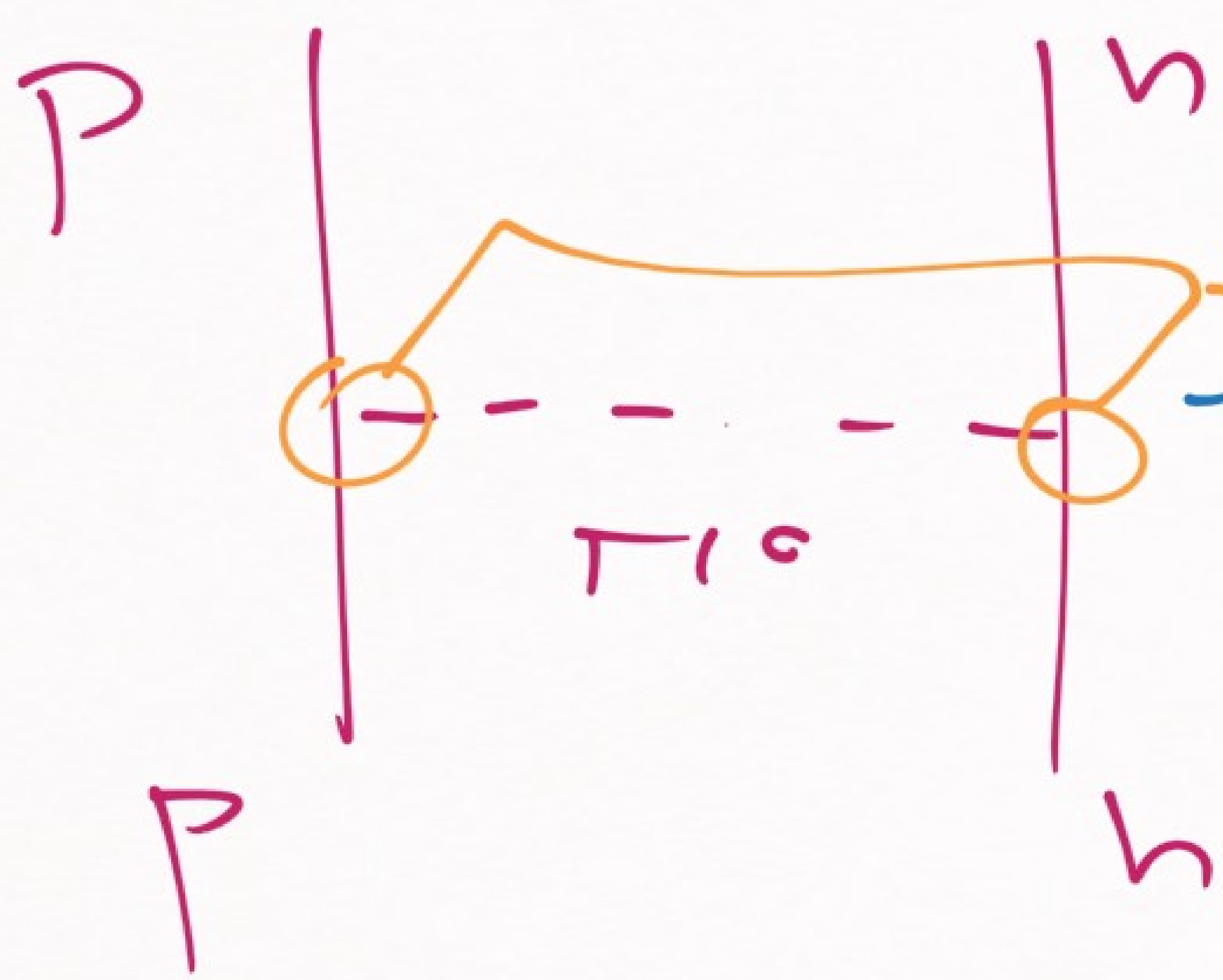
↳

[NOW WE CAN EXPLAIN THIS]



$$= -\left(\frac{g_{\pi NN}}{2M}\right)^2 \frac{\vec{\sigma}_1\cdot\vec{q}\;\vec{\sigma}_2\cdot\vec{q}}{q^2+m^2} = V_0(\vec{q})$$

How to do this?



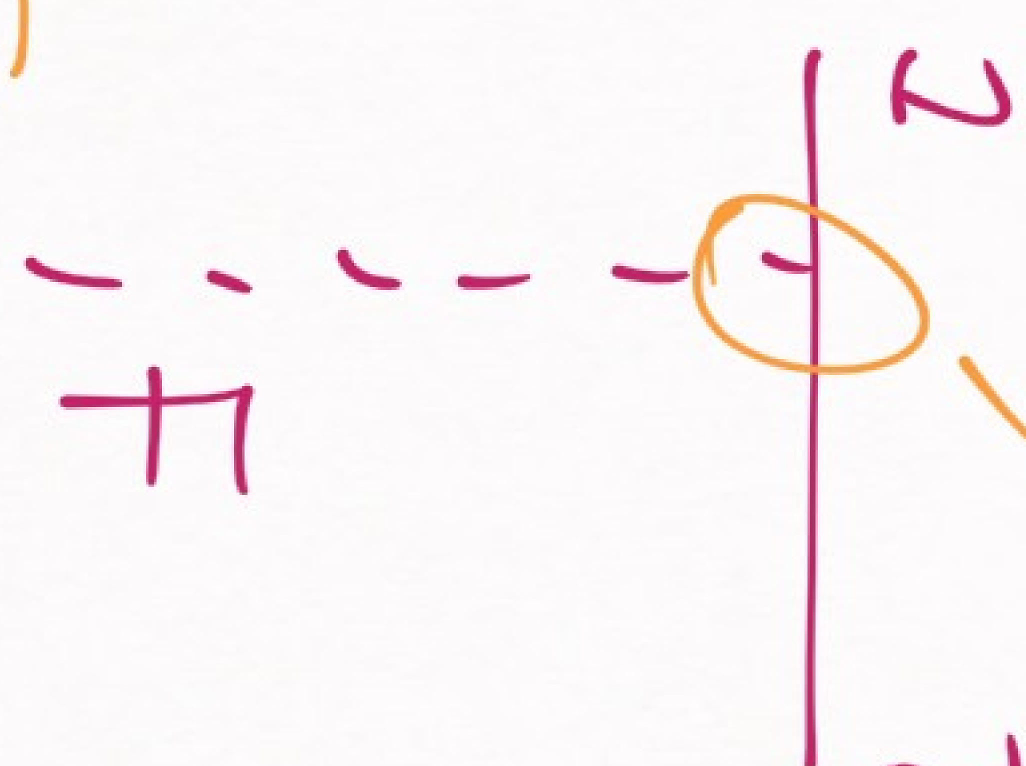
Some isospin dependence

[NOT DIFFICULT TO GUESS]

$$\left.\begin{array}{l}\pi^0 \\ \pi^\pm\end{array}\right\} \rightarrow \text{isospin vector } (\pi^a,\ a=1,2,3)$$

cartesian basis

(spherical basis)



→ final state (2 states)

2X2 matrix

→ initial state (2 state)

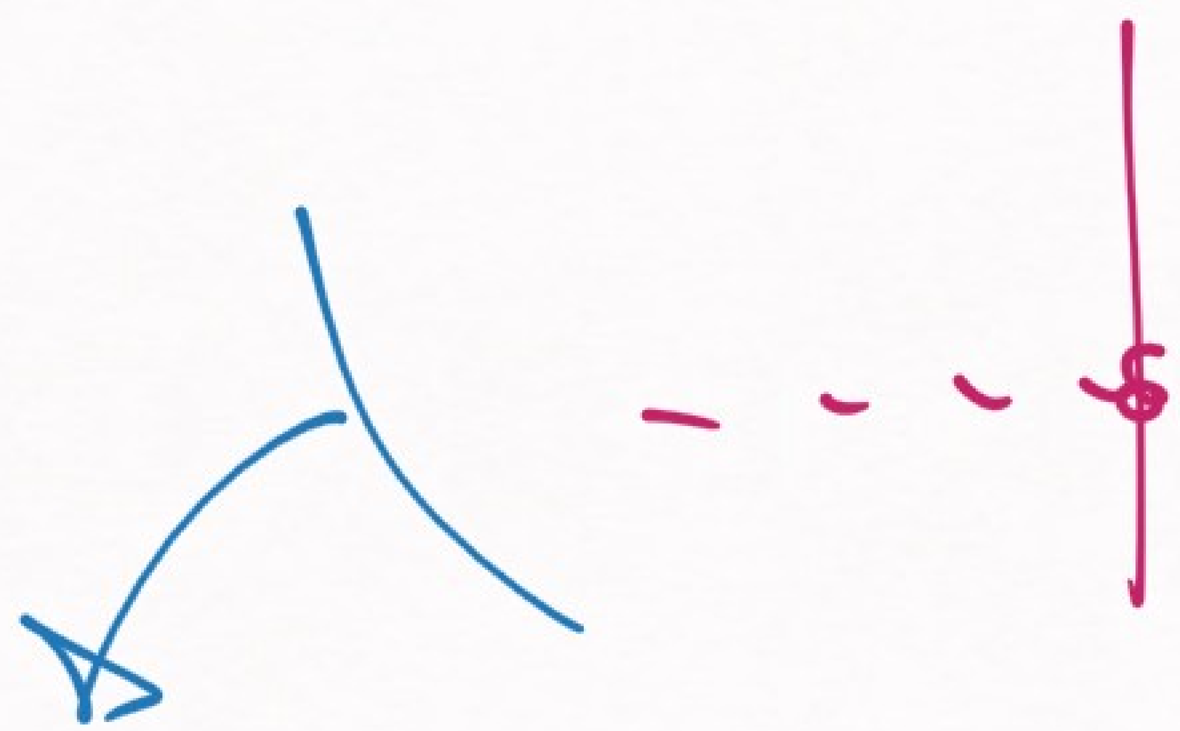
[2x2 matrices, 3 of them] → familiar

→ same as with spin (Pauli matrices)

$\vec{\sigma} = \{\sigma_1, \sigma_2, \sigma_3\}$

$\vec{\tau} = \{\tau_1, \tau_2, \tau_3\}$

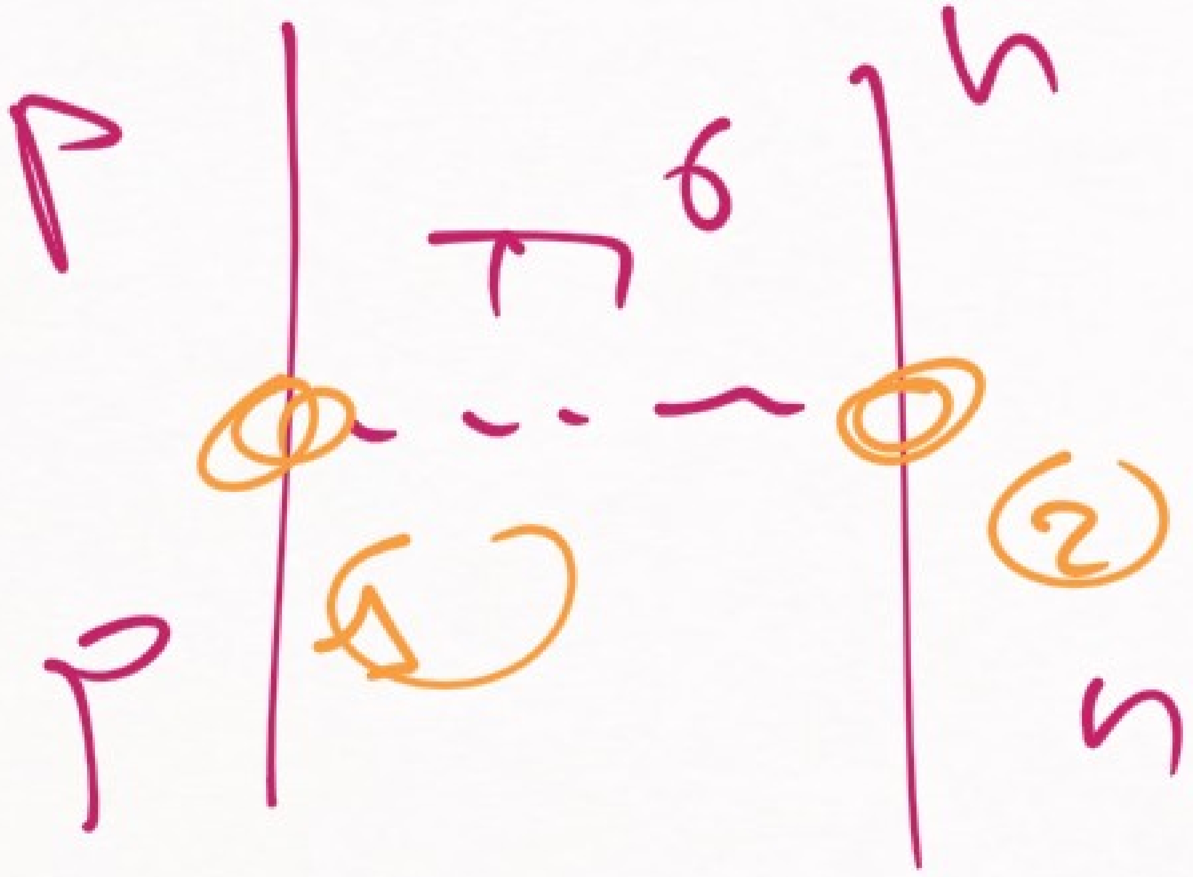
Mathematically: $\pi^a \longrightarrow \tau^a$



Pauli matrix

$\vec{\tau}_1 \cdot \vec{\tau}_2 \, V_0(\vec{q}^2)$

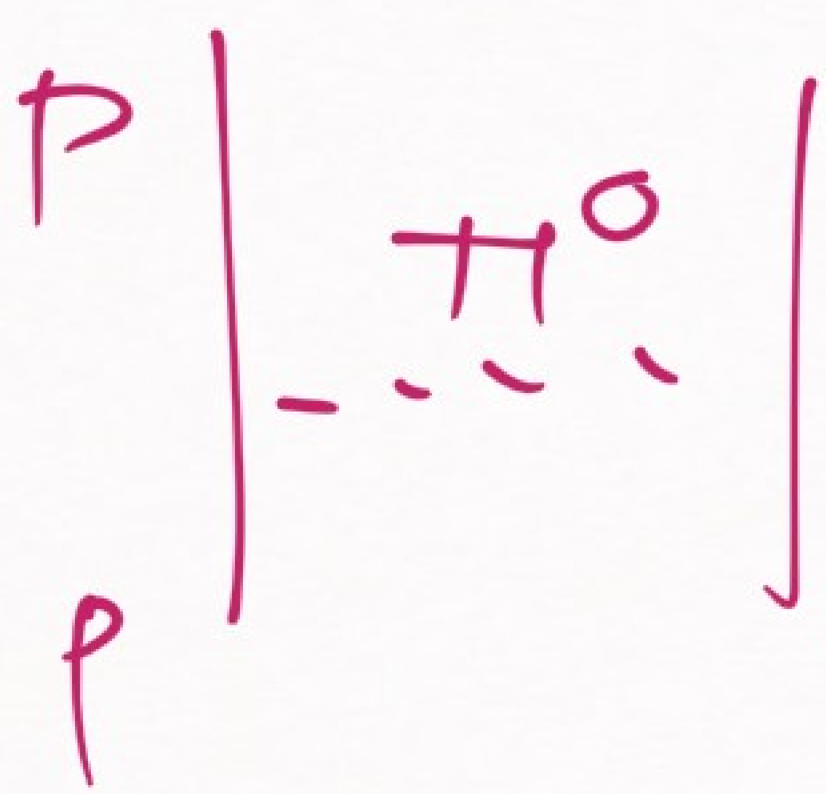
$\tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$



$$① \longrightarrow \langle p|\tau_3|p\rangle = 1$$

$$|p\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

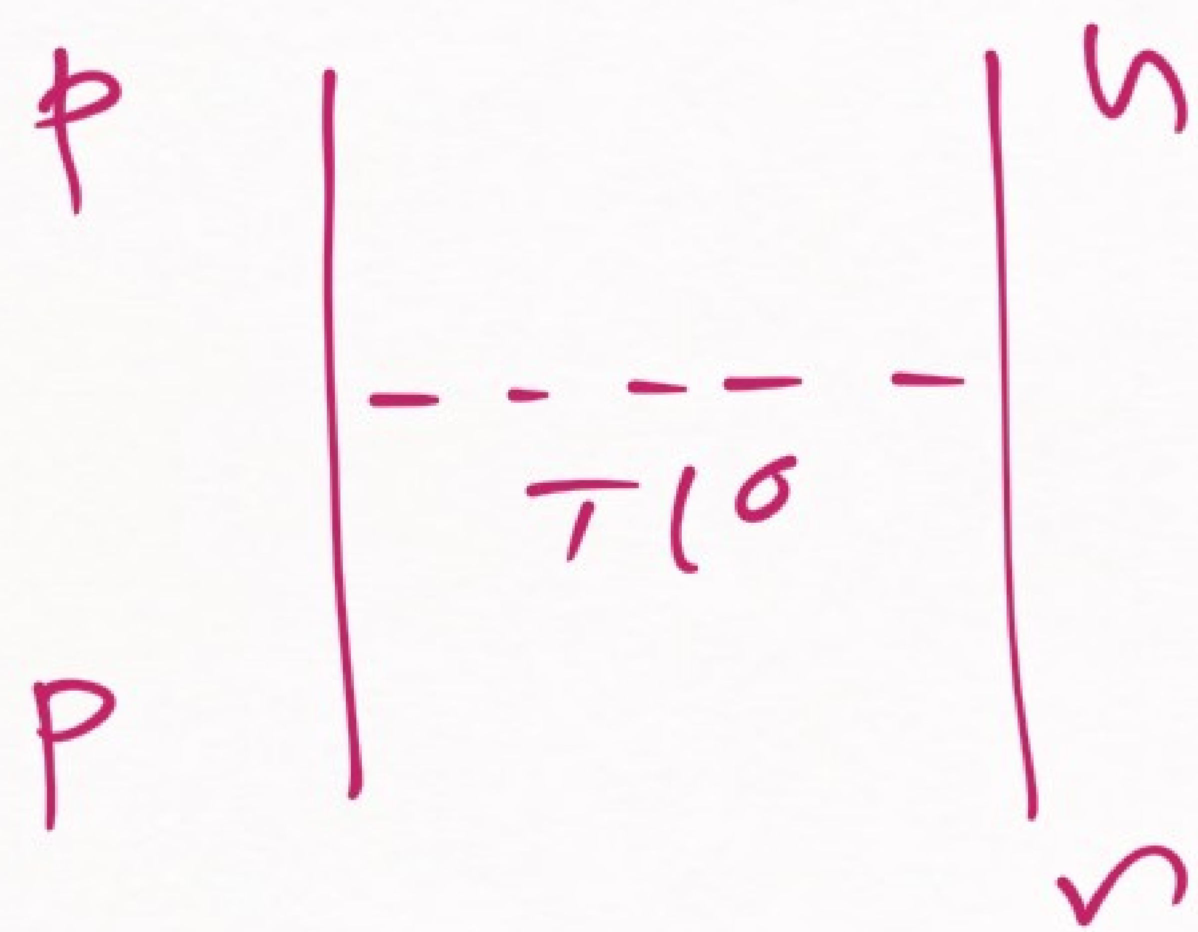
$$② \longrightarrow \langle n|\tau_3|n\rangle = -1$$



$= V_0 \longrightarrow$

$= (-) V_0$

$np \rightarrow$ two types of diagrams



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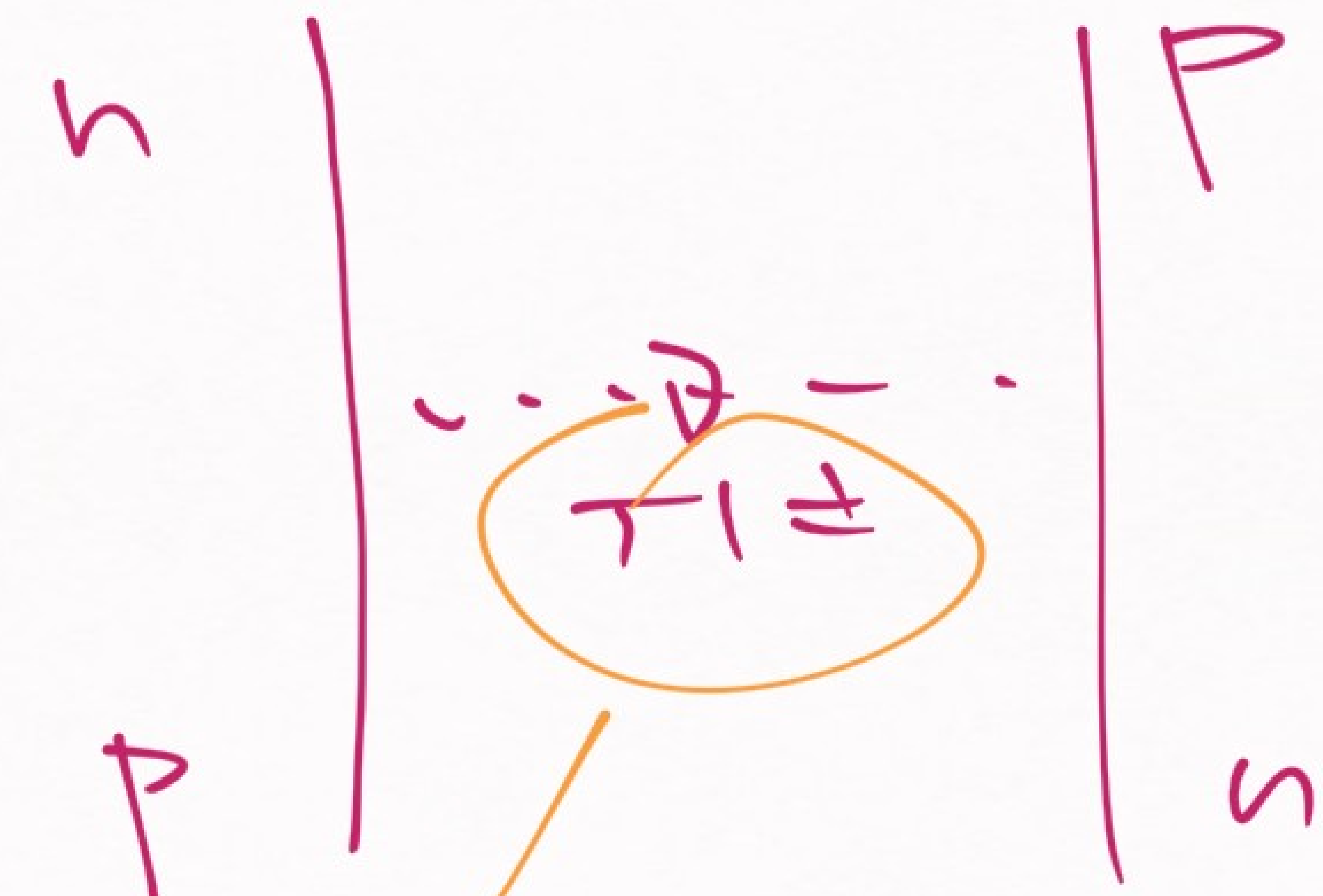


$$\tau^+ = \begin{pmatrix} 0 & \sqrt{2} \\ 0 & 0 \end{pmatrix}$$

$$\tau^- = \begin{pmatrix} 0 & 0 \\ \sqrt{2} & 0 \end{pmatrix}$$

$= -V_0$

$$\tau^{\pm} = \left(\frac{\tau_1 \pm i\tau_2}{\sqrt{2}}\right)$$

Sign?

Write the potential as a matrix:

$$\begin{array}{c} p \\ \\ p \end{array}\Big|\overset{\cdots\cdots}{+1\mathrm{c}}\Big|\begin{array}{c} n \\ \\ n \end{array} \quad , \quad \begin{array}{c} n \\ \\ p \end{array}\Big|\overset{\cdots\cdots}{\pi^\pm}\Big|\begin{array}{c} p \\ \\ n \end{array} \rightarrow \boxed{\begin{pmatrix} -1 & +2 \\ +2 & -1 \end{pmatrix} V_0}$$

$$\beta = \{|np\rangle, |pn\rangle\}$$

Eigenvectors

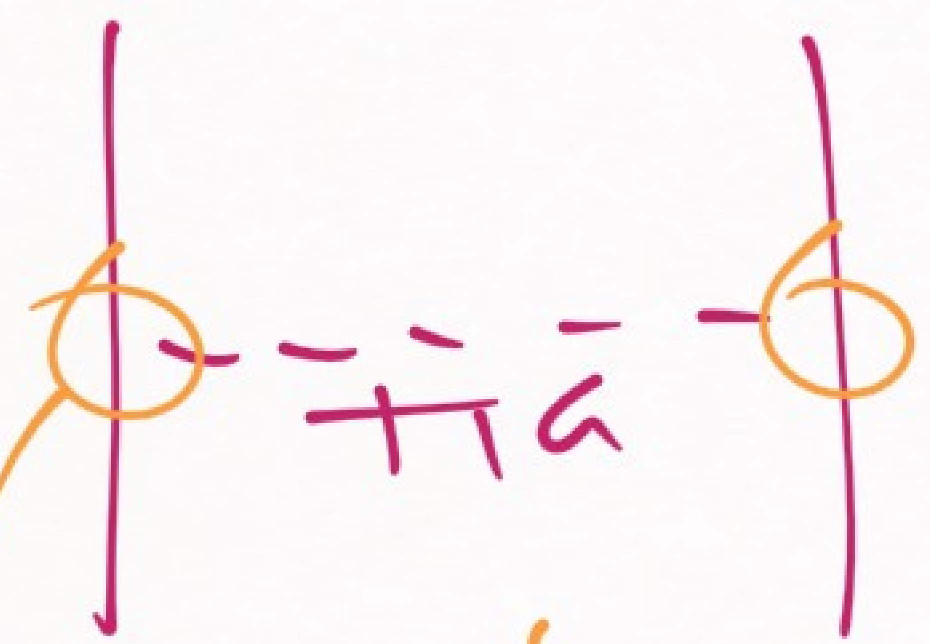
$$\frac{1}{\sqrt{2}}\{|np\rangle \pm |pn\rangle\}$$

Eigenvalues

$$\boxed{1, -3}$$

$+ \quad -$

→ [WRITE THIS DOWN IN ISOSPIN FORMALISM]



$$\longrightarrow \frac{g_{\pi NN}}{2M}\,\vec{\sigma}_2\cdot(-\vec{q})\,\tau_2^b$$

$$\frac{g_{\pi NN}}{2M}\,\vec{\sigma}_1\cdot\vec{q}\,\tau_1^a \qquad \frac{\delta_{ab}}{q^2+m^2}$$

$$= -\left(\frac{g_{\pi NN}}{2M}\right)^2 \vec{\tau}_2\cdot\vec{\tau}_1\,\frac{\vec{\sigma}_1\cdot\vec{q}\;\vec{\sigma}_2\cdot\vec{q}}{q^2+m^2}$$

NOTICE THE ANALOGY W/ SPIN

$$\vec{\tau}_1 \cdot \vec{\tau}_2 \rightarrow \vec{\sigma}_1 \cdot \vec{\sigma}_2 \leftarrow -3, +1$$

$S=0 \quad S=1$

$$\parallel$$

$-3, +1$

$I=0 \quad I=1$

CROSS-CHECKED W/
PREVIOUS SLIDES

$$\pi^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\pi^+ = \begin{pmatrix} 0 & \sqrt{2} \\ 0 & 0 \end{pmatrix}$$

$$\pi^- = \begin{pmatrix} 0 & 0 \\ \sqrt{2} & 0 \end{pmatrix}$$

Repeat the previous calculation

$$|\cdots| = V_0 \text{ (no isospin)} \longrightarrow |\cdots| = \vec{\tau}_1 \cdot \vec{\tau}_1 \, V_0 \text{ (isospin)}$$

Extended Dirac-Fermi statistics :

→ Isospin formalism : isospin just another quantum number

→ $\boxed{\begin{array}{l} p \to \text{Fermions} \\ n \to \text{Fermions} \end{array}} \longleftrightarrow \boxed{\text{NUCLEONS ?}}$

↳ Fermions

Nucleons as Fermions

→ Full wave functions
has to be antisymmetric
(with respect to all quantum
numbers)

NN $\rightarrow$ wave function

$= (\text{isospin wf}) \times (\text{spin wf}) \times (\text{spatial wf})$

$\rightarrow$ Symmetry : $(-1)^{I+1} \times (-1)^{S+1} \times (-1)^{l}$

$I = 1 \rightarrow$ symmetric

$I = 0 \rightarrow$ antisymmetr.

Symmetry $= (-1)^{L+S+I}$

Fermions $\rightarrow$ $\boxed{L+S+I \text{ is odd}}$

$(1, 3, 5, \ldots)$

Deuteron: $L=0$, $S=1$ $\rightarrow$ $I=0$

$\rightarrow$ $\emptyset$

$$\underbrace{\frac{1}{\sqrt{2}}\left[|pn\rangle - |np\rangle\right]}$$

this is why, this piece is antisymmetric

SEE YOU ON THURSDAY