NUCLEAR PHYSICS 8

EFFECTIVE FIELD THEORIES

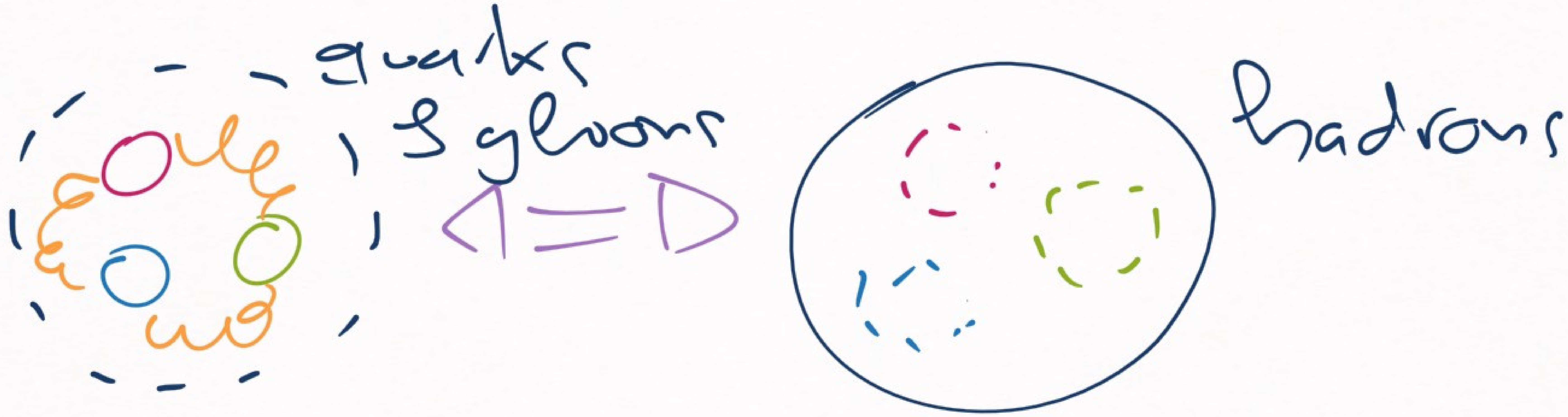
$\boxed{\text{RECAP}}$ Renormalization (basic idea)

"Physics at long-distances does not depend on short-distance details"

EXAMPLES $\longrightarrow$ 1) $\exists$ different scientific disciplines

Biology / Chemistry / Molecular physics / Atomic physics / Nuclear physics / ...

2)



EXAMPLE → [THEORY OF TEAPOTS & TEACUPS]

High energy / short distance theory : DIFFICULT

→ Fourier equation of heat conduction

→ Convection theory

→ $\boxed{\text{EFT APPROACH}}$

1) Some dynamics (Newton's law of cooling)

$$(T-T_{ext}) = (T_0-T_{ext})\, e^{-\lambda(t-t_0)}$$

2) Find the relevant degrees of freedom (d.o.f)

S, Σ → Exposed, ceramic surface

3) Find a small expansion parameter

$$x = \frac{1}{\ell}\frac{\Sigma}{S}$$ (ℓ a numerical factor) ($\ell \gg 1$)

4) Write down the theory (as an EXPANSION)

$$\lambda = S\,(c_0 + c_1 x + c_2 x^2 + \dots)$$

5) Choose some accuracy (if $x<1$, convergence)

$\mathcal{O}(x)$, $\mathcal{O}(x^2)$, $\mathcal{O}(x^3)$, ...

$$\lambda = c_0 S + \mathcal{O}(x)$$

6) Fit the LECs (low energy constants) to the exp. data

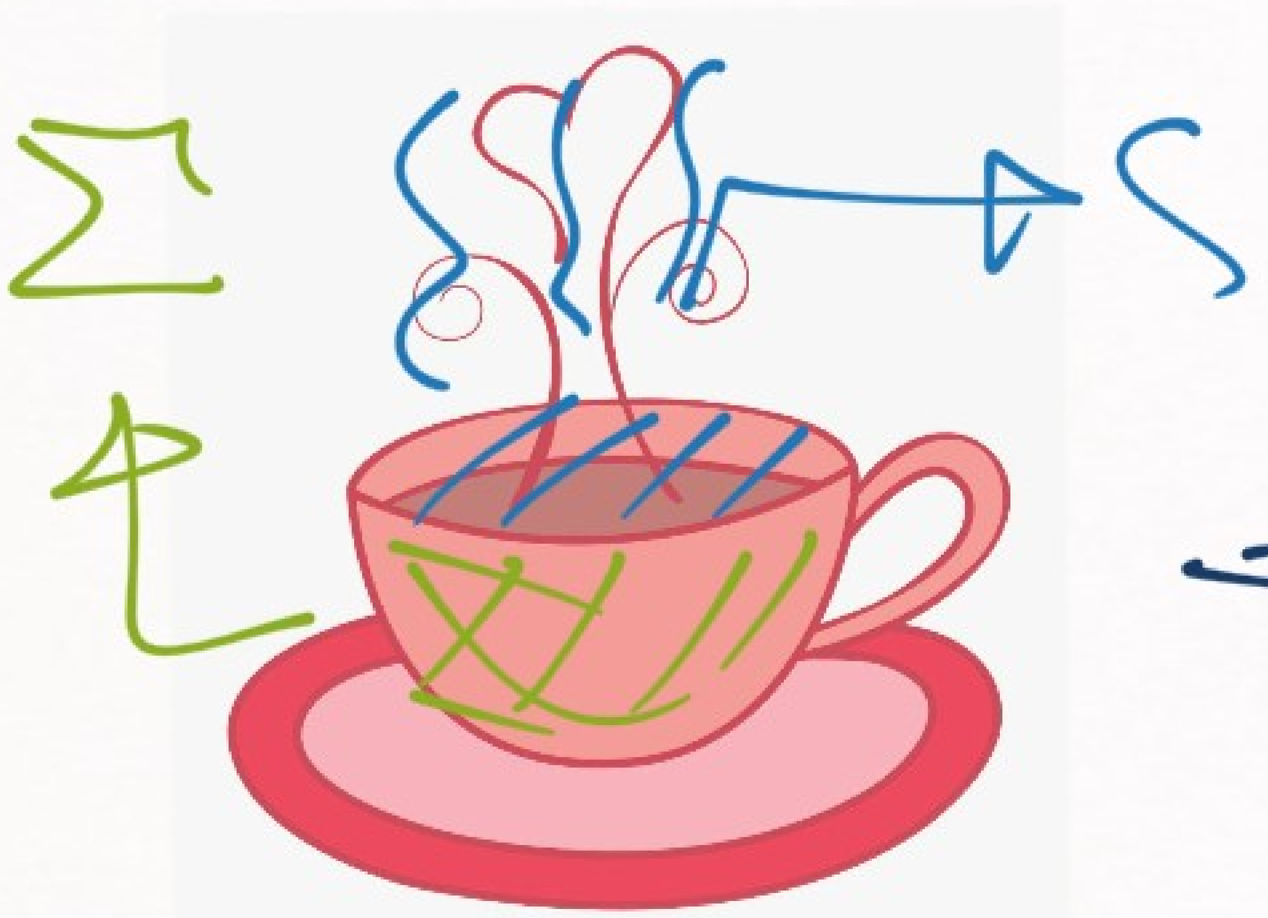
$[c_0, c_1, c_2, \dots]$

$\boxed{\text{EFT ALGORITHM}}$ → Simple

→ There is more than the previous

$\boxed{\begin{array}{l}\exists \text{ many possible expansions} \\ (\exists \text{ many power countings})\end{array}}$

TEAPOT & TEACUP THEORY → At least two power countings



→ $x = \frac{\ell}{\ell}\frac{\Sigma}{S}$, $\ell \gg 1$

↓

$\boxed{x < y}$? $\left[x,\ y = \frac{1}{x}\right]$

If $\ell \simeq 10$, $x < 1 \Rightarrow S < \frac{1}{10}\Sigma$

→ S very small, Σ very big

$\rightarrow \Sigma \gg \rho S$

(Σ wins $\rightarrow \Sigma$ becomes the way this will cool down)

$$x = \frac{\Sigma}{\rho S} > 1 \Rightarrow \boxed{\lambda = S \sum c_n x^n}$$ Divergent

$\rightarrow$ [This expansion/power counting fails]

$$\longrightarrow y = \frac{1}{x} = \frac{\rho S}{2}$$

$$\boxed{\lambda = \sum \left(d_0 + d_1 y + d_2 y^2 + \dots \right)}$$

New expansion in the new power counting

[POWER COUNTING IS NOT UNIQUE]

1) CUP - COUNTING $\quad x = \frac{\ell}{\rho}\frac{\Sigma}{S}$

$$\lambda = S\left(c_0 + c_1 x + c_2 x^2 + \ldots\right)$$

2) POT-COUNTING $\quad y = \rho\frac{S}{\Sigma}$

$$\lambda = \Sigma\left(d_0 + d_1 y + d_2 y^2 + \ldots\right)$$

$\boxed{\text{EFTs}}$ → power counting not unique

technical names

→ Universality classes

→ $\boxed{\text{Infrared}}$ fixed points

Infrared $\rightarrow$ $\boxed{\Lambda \rightarrow 0}$

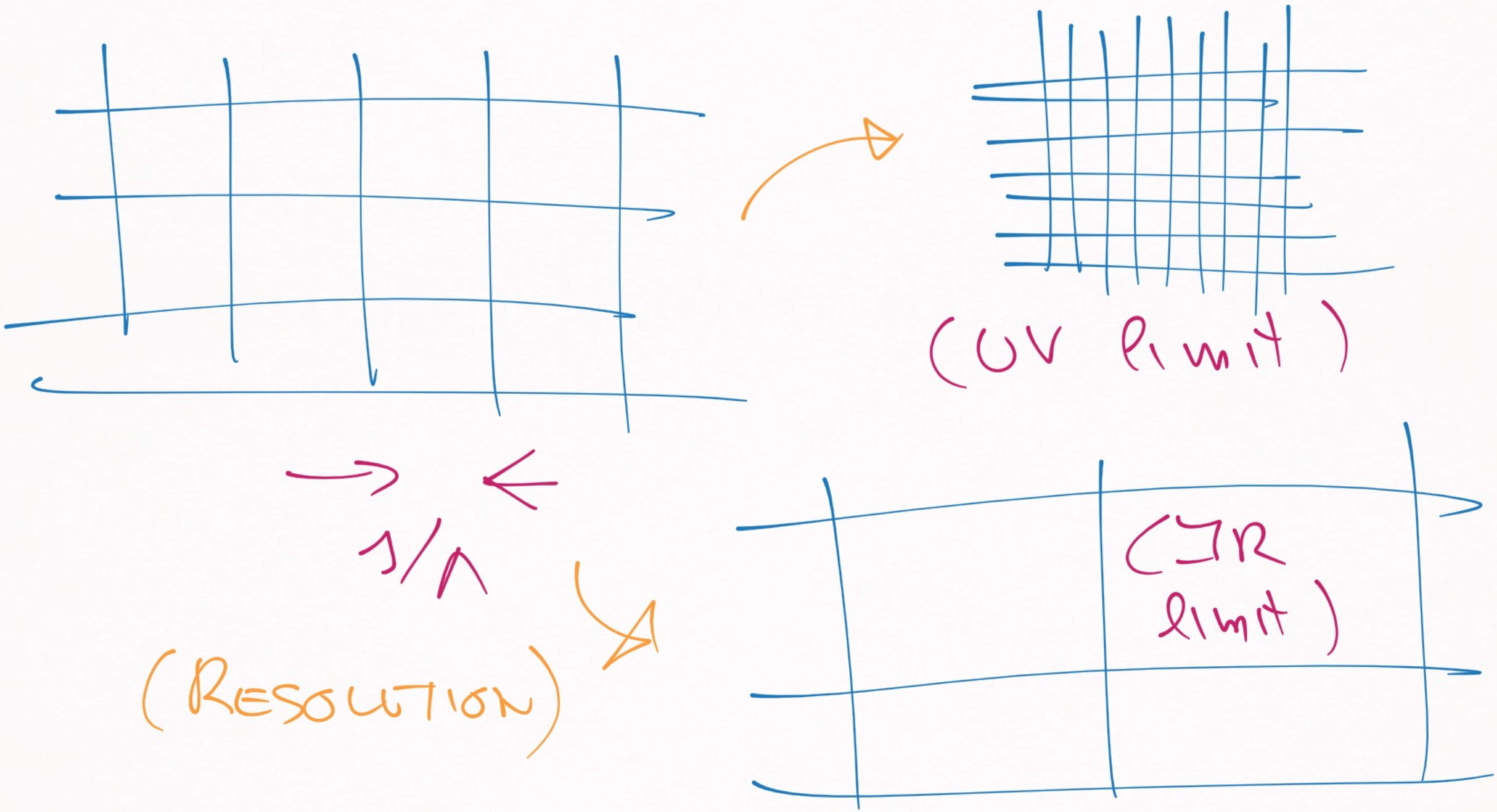
Ultraviolet $\rightarrow$ $\boxed{\Lambda \rightarrow \infty}$

[$\Lambda \rightarrow$ resolution of our theory]

[CONGRATULATIONS!] → Renormalize your tea set!

RENORMALIZED ✓

→ Renormalize some QM problem


(UV limit)
1/Λ
(RESOLUTION)
(IR limit)

→ JUST LANGUAGE

ULTRAVIOLET ⟷ HIGH RESOLUTION

INFRARED ⟷ LOW RESOLUTION

λ

LITERATURE ABOUT RENORMALIZATION

2. **The Renormalization group and the epsilon expansion**
(2700) K.G. Wilson (Princeton, Inst. Advanced Study & Cornell U., LNS), John B. Kogut (Princeton, Inst. Advanced Study). Jul 1973. 126 pp.
Published in **Phys.Rept. 12 (1974) 75-199**
DOI: 10.1016/0370-1573(74)90023-4
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record - Cited by 2700 records 1000+

→ classic (background on solid state physics)

4. **Renormalization and Effective Lagrangians**
Joseph Polchinski (Harvard U.). Apr 1983. 27 pp.
Published in **Nucl.Phys. B231 (1984) 269-295**
HUTP-83-A018
DOI: 10.1016/0550-3213(84)90287-6
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
KEK scanned document
Detailed record - Cited by 1120 records 1000+

→ VERY MUCH RECOMMENDED (ϕ^4-theory in QFT)

1. **Building light nuclei from neutrons, protons, and pions**
(45) Daniel R. Phillips (Ohio U.). Mar 2002. 54 pp.
Published in **Czech.J.Phys. 52 (2002) B49**
DOI: 10.1007/s10582-002-0079-z
To appear in the proceedings of Conference: C01-07-09.13 Proceedings
e-Print: **nucl-th/0203040** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
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→ GOOD & EASY
(QM & nuclear physics)

1. **Effective field theory for few nucleon systems**
(618) Paulo F. Bedaque (LBL, Berkeley), Ubirajara van Kolck (Arizona U. & RIKEN BNL). Mar 2002. 55 pp.
Published in **Ann.Rev.Nucl.Part.Sci. 52 (2002) 339-396**
DOI: 10.1146/annurev.nucl.52.050102.090637
e-Print: **nucl-th/0203055** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record - Cited by 618 records 500+

→ GOOD & STILL EASY

3. **A Renormalization group treatment of two-body scattering**
(196) Michael C. Birse, Judith A. McGovern, Keith G. Richardson (Manchester U.). Jul 1998. 4 pp.
Published in **Phys.Lett. B464 (1999) 169-176**
MC-TH-98-11
DOI: 10.1016/S0370-2693(99)00991-0
e-Print: **hep-ph/9807302** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record - Cited by 196 records 100+

← MORE ABSTRACT
(you can try, not easy, but doable for you)

[RENORMALIZATION IN QM & NUCLEAR PHYSICS]

→ Renormalization:

"Physics at long-distances does not depend on short-distance details"

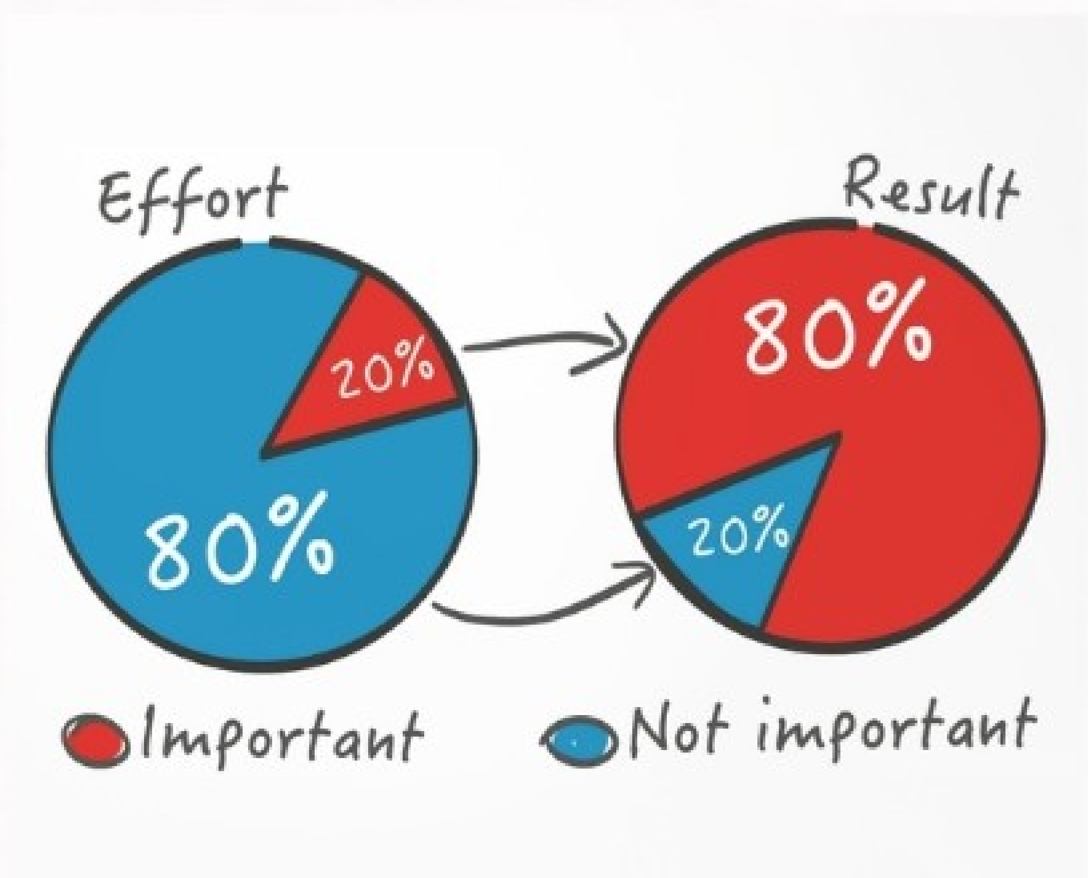
→ RGA (Renormalization Group Analysis)

$$\boxed{\frac{d}{d\Lambda}\langle \Psi | \hat{G} | \Psi \rangle = 0}$$

→ EXAMPLE OF "EFT THINKING"

1) TOY THEORY OF CUPS & POTS

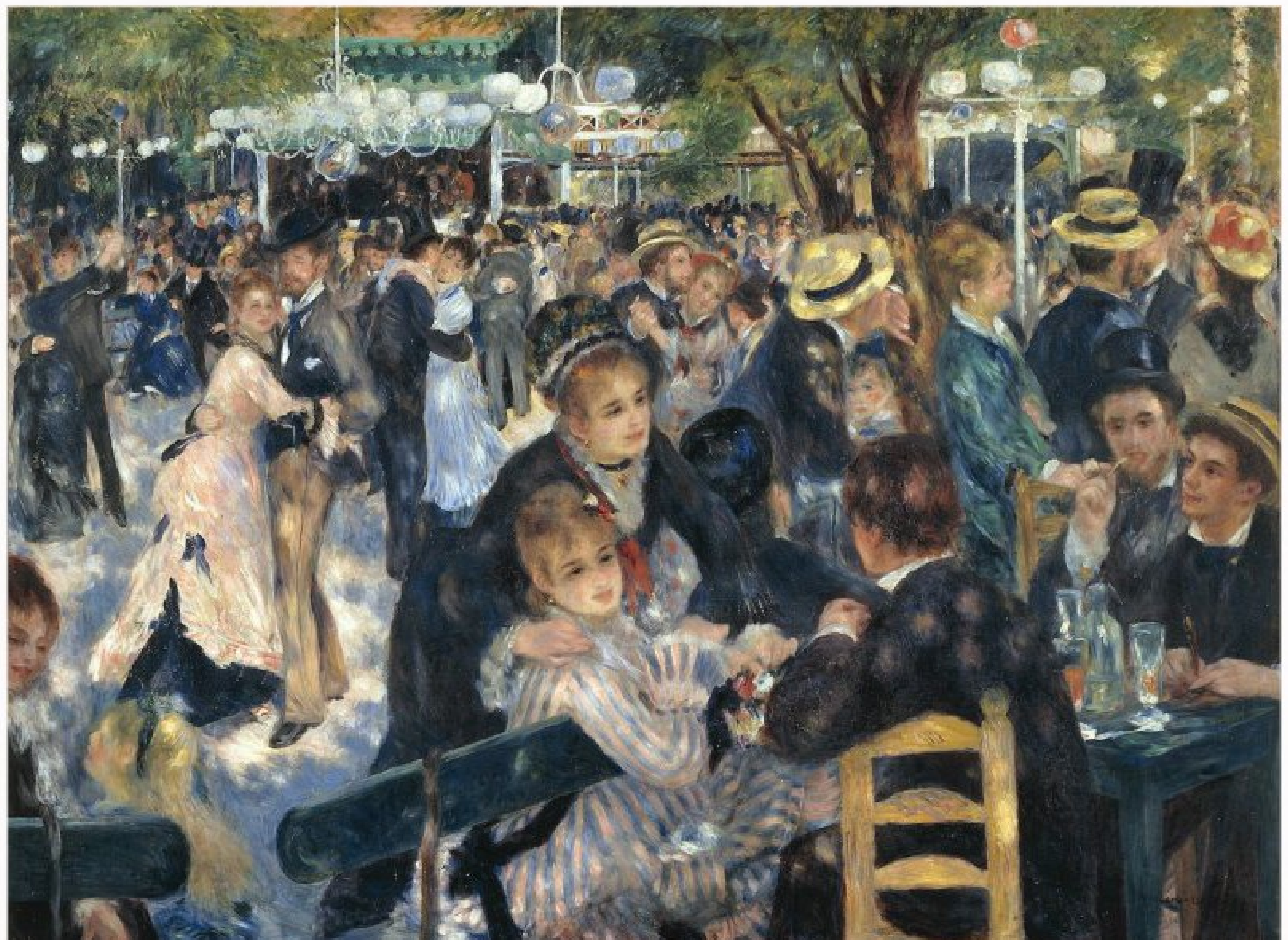
2) PARETO PRINCIPLE



"20% of the effort gives 80% of the results"

(power counting)

3) IMPRESIONISM (ART) 印象派

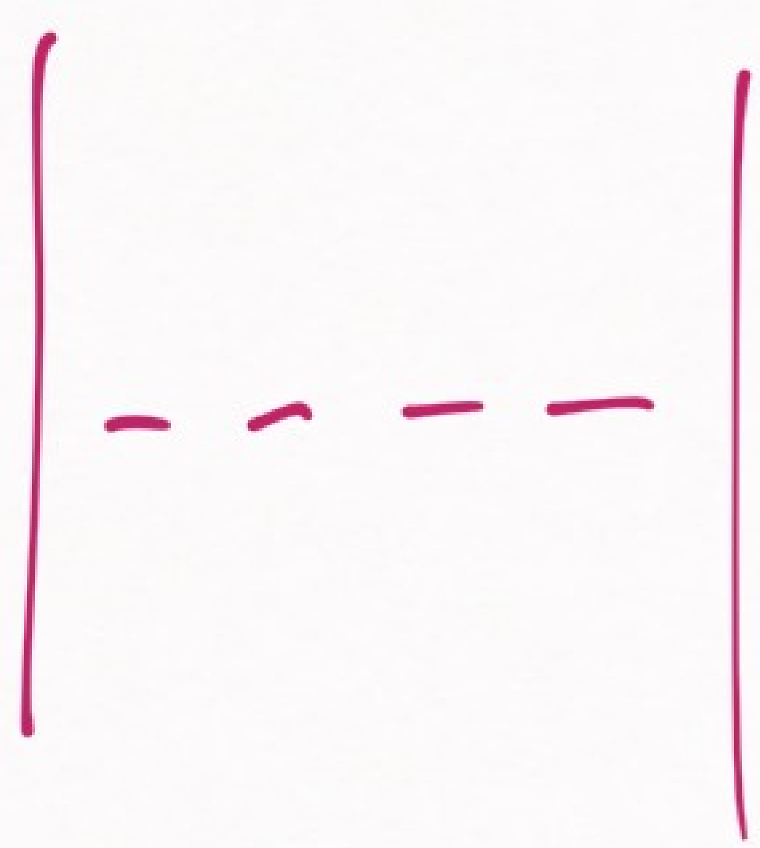


1) Big dots
(A → Q) ("akin of A → 6")
2) No short-distance detail
3) Great painting
(Effective painting)

→ [This is the way we think in EFT]

[GOING BACK TO PHYSICS] → How do I apply these ideas?

Yukawa



$$V(r) = -\frac{g^2}{4\pi}\frac{e^{-mr}}{r}$$

$$V(\vec{q}) = -\frac{g^2}{q^2+m^2}$$

Two-body problem w/ Yukawa potential:

1) two-particles of mass M
2) they exchange a scalar boson of mass m
3) the potential is a Yukawa
4) $\left[-\frac{\nabla^2}{M} + V(r)\right]\psi(\vec{r}) = E\,\psi(\vec{r})$

Interesting facts:

→ binding depends on $\lambda = \frac{M}{m}\frac{g^2}{4\pi} \geq 1.68$

$(\lambda_c = 1.68)$

1) $\lambda < \lambda_c \rightarrow$ no binding

2) $\lambda = \lambda_c \rightarrow$ zero-energy bound state
(unitary limit)

3) $\lambda > \lambda_c \rightarrow$ binding

[1,2,3 relate to EFT]

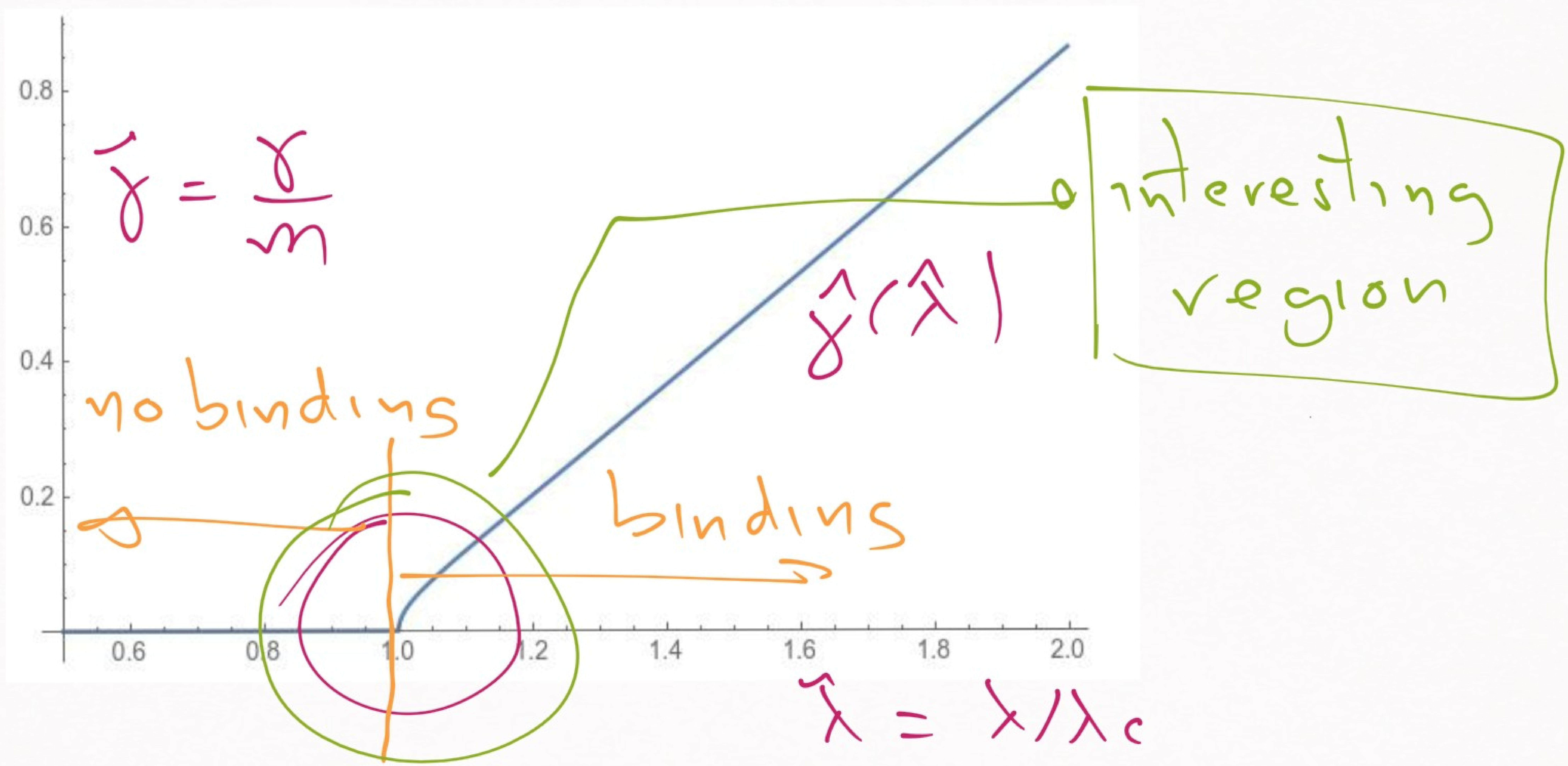
Some calculations $\rightarrow$ $M = 1\ \text{GeV}$, $m = 0.5\ \text{GeV}$

$E_B = -\gamma^2/M \rightarrow \gamma = \gamma(\lambda)$ $\quad \lambda = \frac{M}{m}\frac{g^2}{4\pi}$

Better yet $\rightarrow$ remove all dimensionful quantities

$$\gamma = \gamma(\lambda) \longrightarrow \hat{\gamma} = \frac{\gamma}{m}, \quad \hat{\lambda} = \frac{\lambda}{\lambda_c}, \quad \boxed{\hat{\gamma} = \hat{\gamma}(\hat{\lambda})}$$

Computing $\hat{\gamma} = \hat{\gamma}(\hat{\lambda})$ is actually easy:



[WHY $\exists$ AN INTERESTING REGIME ?]

(why $\hat{\lambda} \sim 1$ is interesting ?)

If $\hat{\lambda} \simeq 1 + \Delta$ ($\Delta \ll 1$, $\Delta > 0$)

$\Rightarrow \boxed{\hat{\gamma} \ll 1} \longrightarrow \psi_B(\vec{r}) = \frac{1}{\sqrt{4\pi}} \frac{u(r)}{r}$

$u(r) \longrightarrow A_s e^{-\gamma r}$

$r \gg 1/m$

$\boxed{\gamma \ll m}$

$$\psi_B(\vec{r}) \longrightarrow \frac{1}{\sqrt{4\pi}} A_s \frac{e^{-\gamma r}}{r}$$

$\Rightarrow$ for $\gamma \ll m$

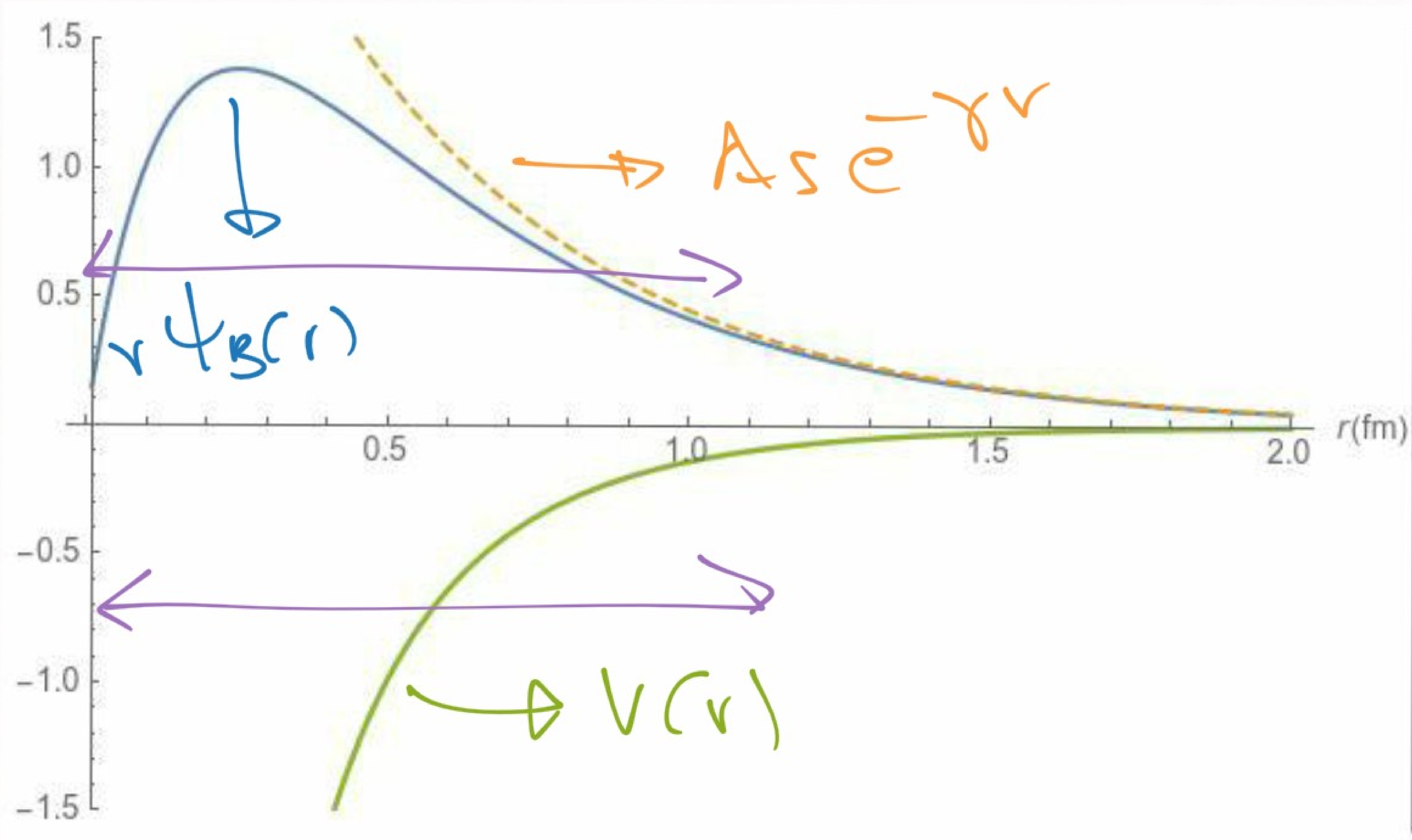
$\psi_B(\vec{r})$ is going to be much bigger than the potential

$$\boxed{\begin{aligned} \psi_B(r) &\sim e^{-\gamma r} \\ V(r) &\sim e^{-mr} \end{aligned}}$$

$\Rightarrow$ this will have consequences

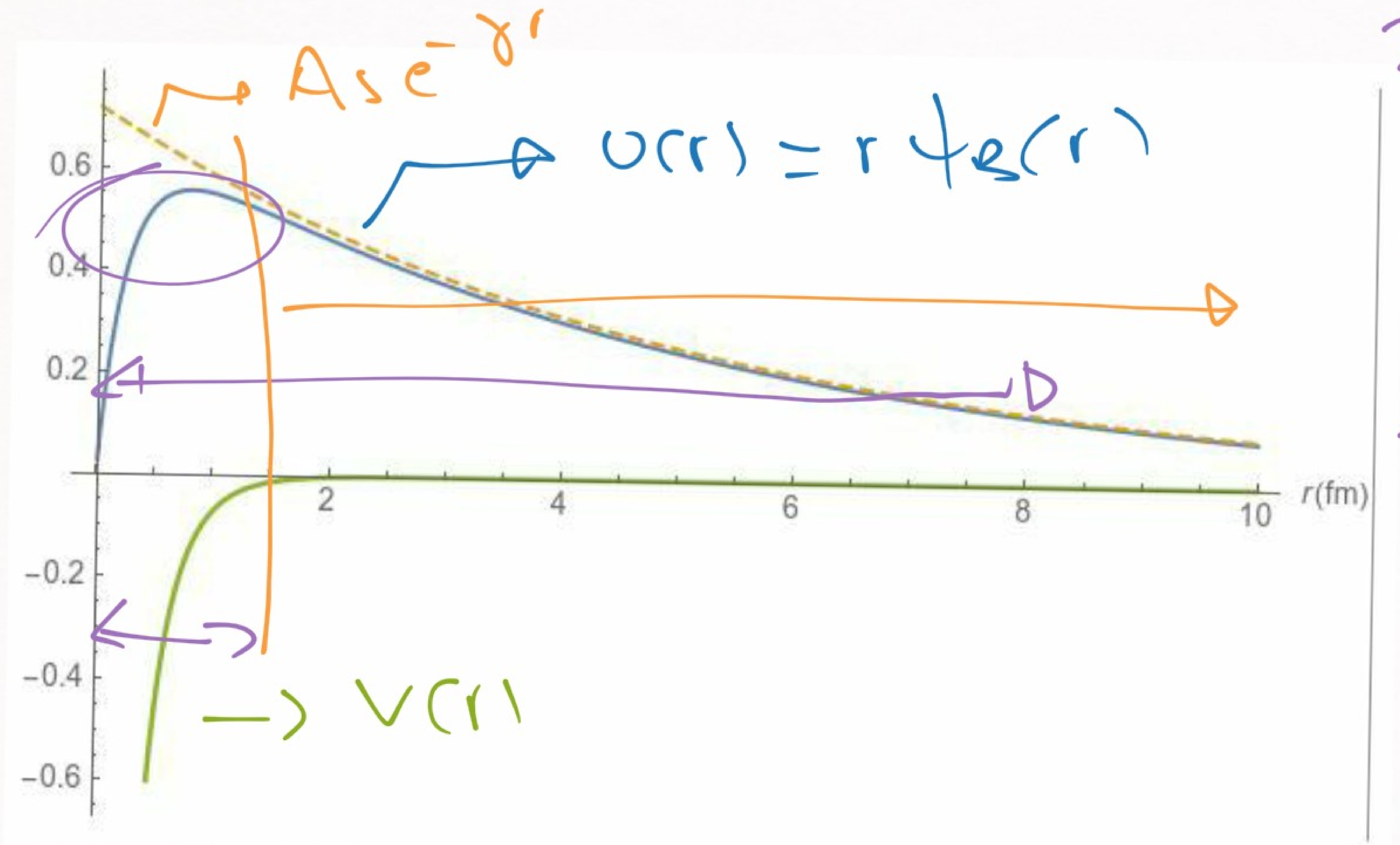
($\Rightarrow$ Simplifications)

1) $\lambda = 2\lambda_c$, $\gamma = 0.36\, m$ $(\gamma \sim \underline{\underline{m}})$ (NATURAL THEORY)



$\boxed{\text{Size of } \psi_B \text{ similar to Size of } V(r)}$

2) $\lambda = 1.05\,\lambda_c$, $\gamma = 0.095\,m$



2.a) Size of $\psi_B(r)$ is much larger than size of $V(r)$

2.b) For $mr \gg 1$, $\psi_B(r)$ looks like

$$\psi_B(r) \sim \frac{A_s}{\sqrt{4\pi}} \frac{e^{-\gamma r}}{r}$$

What do we learn from this example?

1) Natural case

1.a) Wave function/potential same size

1.b) Wave function depends on form of the potential

2) Unnatural case 2.b → RENORMALIZATION

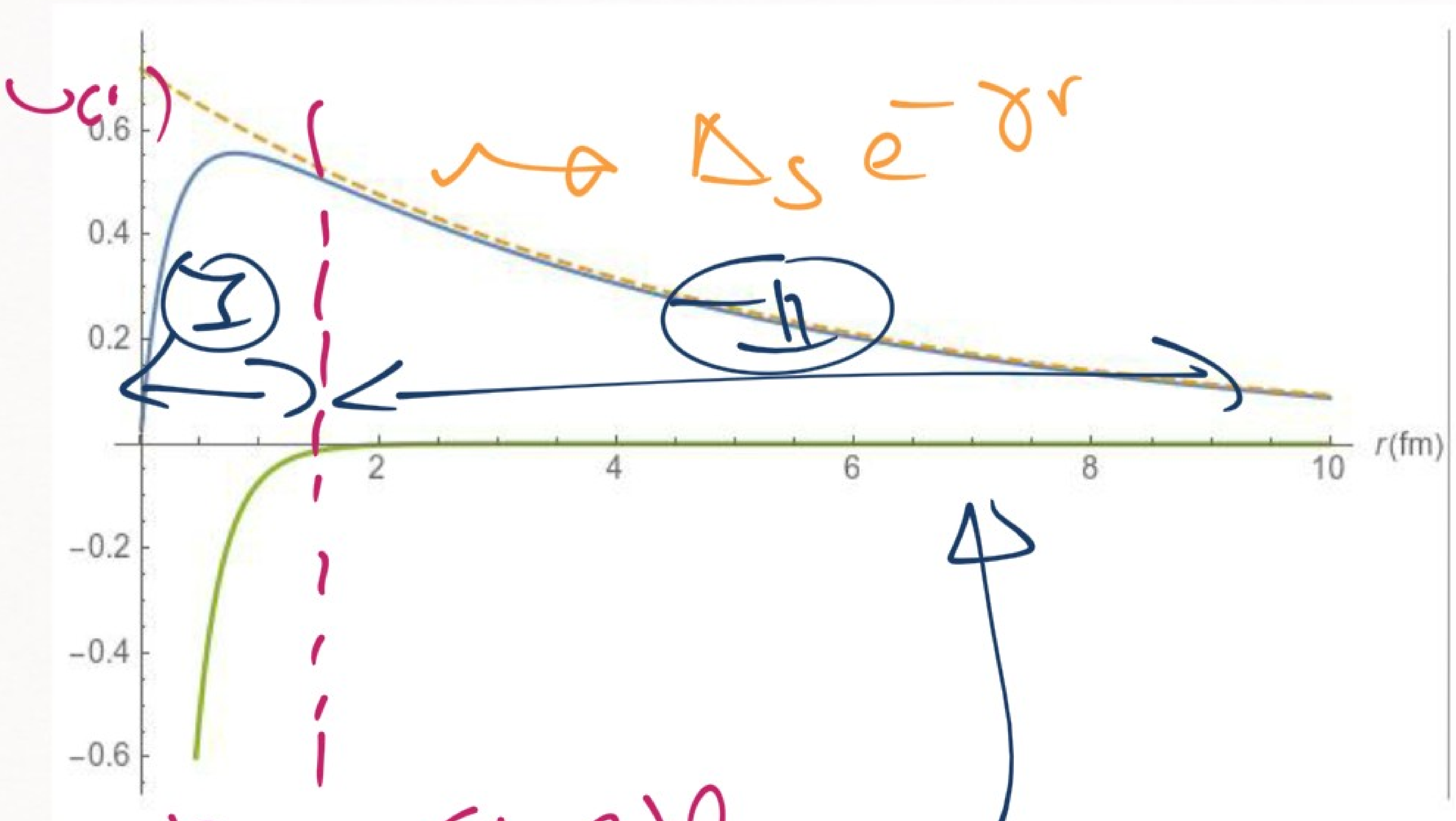
2.a) Wave function much larger than potential

2.b) At long distances, wave function independent of $V(r)$

FOR 2) WE CAN BUILD
AN EFFECTIVE DESCRIPTION

$$\boxed{\frac{\lambda}{\lambda_c} \sim 1}$$

HOW TO DO IT?



$R_c \sim (1-2)\,\text{fm}$

[Most of u(r) lives in II]

CLOSER LOOK :

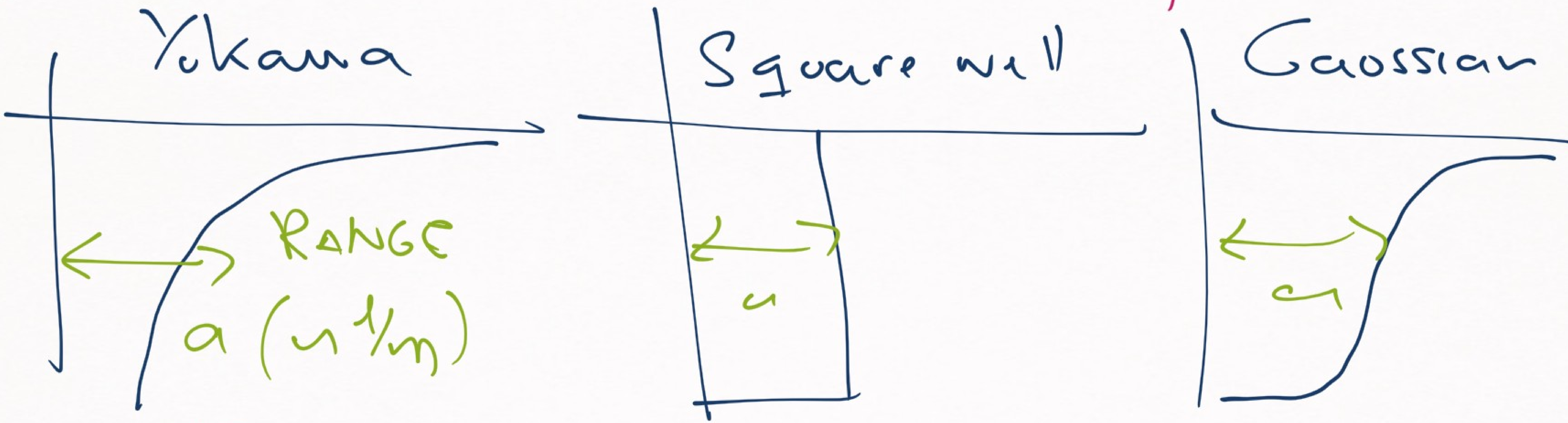
If $r > R_c$

$\Rightarrow \boxed{u(r) \simeq A_s e^{-\gamma r}}$

$u(r)$ & $A_s e^{-\gamma r}$ basically coincide at long distances

[We can ignore region I]

[Whatever happens in REGION I not very important]



$\Rightarrow$ [ψ given $u(r) \simeq A_s e^{-\gamma r}$ for $r \gg a$]

(γ for $\frac{1}{\gamma} \gg a$)

If $\frac{1}{\gamma} \gg a \Rightarrow$ [THE POTENTIAL DOES NOT MATTER]

Most potentials will generate almost exactly the same wave function

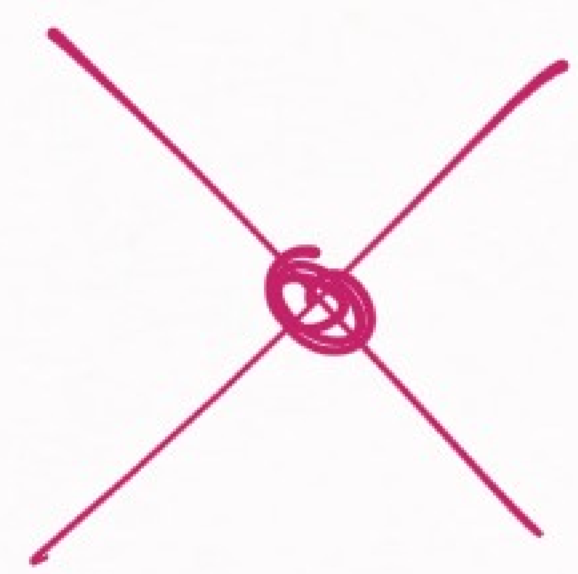
["Physics at long-distances does not depend on short-distance details"]

If $\frac{1}{\gamma} \gg a \; (\gamma \ll m) \Rightarrow$ [WE DON'T NEED TO KNOW THE TRUE/EXACT POTENTIAL]

(SHALLOW BOUND STATE)

$\rightarrow$ Substitute the true potential by something simpler

Yukawa

 $\Rightarrow$  ? (contact interaction)

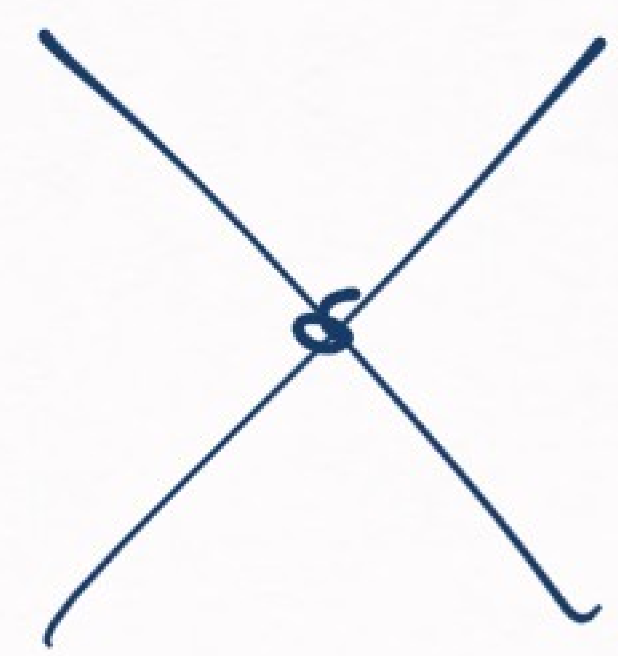
$\boxed{\text{YUKAWA}}$ $V(\vec{q}) = -\dfrac{g^2}{|\vec{q}|^2 + m^2} \xrightarrow[|\vec{q}|^2 \ll m^2]{} -\dfrac{g^2}{m^2}\left[1 - \dfrac{q^2}{m^2} + \dfrac{q^4}{m^4} - \dfrac{q^6}{m^6} + \dots\right]$

$$V(\vec{q}) \xrightarrow[|\vec{q}|^2 \ll m^2]{} -\frac{g^2}{m^2}\left(1 + \mathcal{O}\left(\frac{|\vec{q}|^2}{m^2}\right)\right)$$

$$V(\vec{q}) \xrightarrow[|\vec{q}|^2 \ll m^2]{} C_0 \quad \text{(just a number)}$$

At long distances / low energies, we don't know if two particles exchange a boson

$\Rightarrow$



(short-distance description) (long-distance description)

$$V(\vec{q}) = -\frac{g^2}{m^2 + |\vec{q}|^2} \longrightarrow \left[V(\vec{q}) = c_0 + c_2 \vec{q}^{\,2} + \ldots \right]$$

(Short distance)

$\downarrow$

(Details of $V(\vec{q})$ are important)

(Long distance)

$\downarrow$

(Details of $V(\vec{q})$ not important)

$\hookrightarrow$ [We can use a generic potential]

$$V(\bar{q}) = -\frac{g^2}{q^2+m^2} \quad \text{or anything else}$$

$$|\bar{q}^2| \ll m^2 \Rightarrow V(\bar{q}) = c_0 + c_2 q^2 + c_4 q^4 + \dots$$

(Taylor expansion)

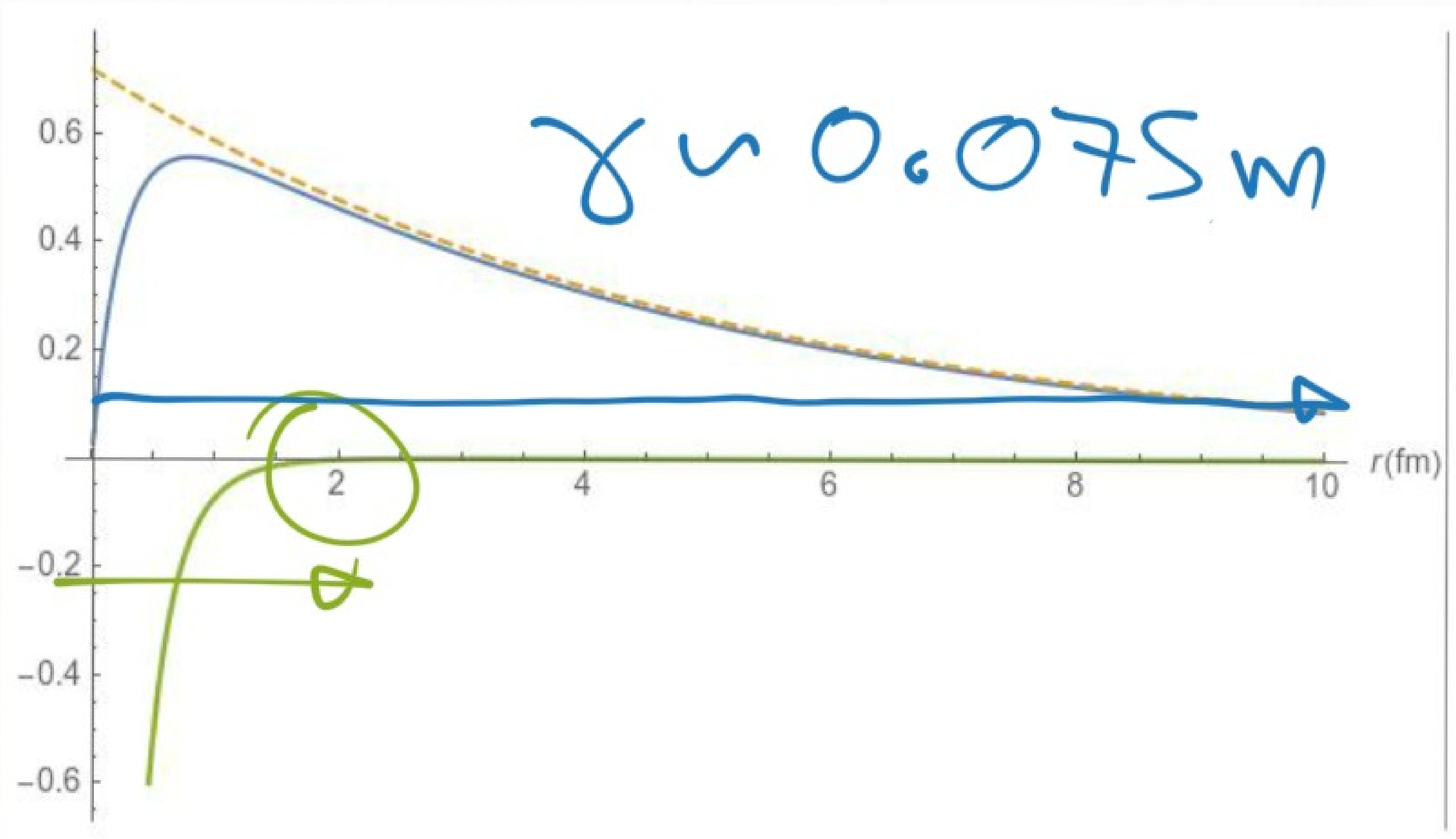
$\boxed{\text{BREAK}} \longrightarrow \boxed{17:41}$

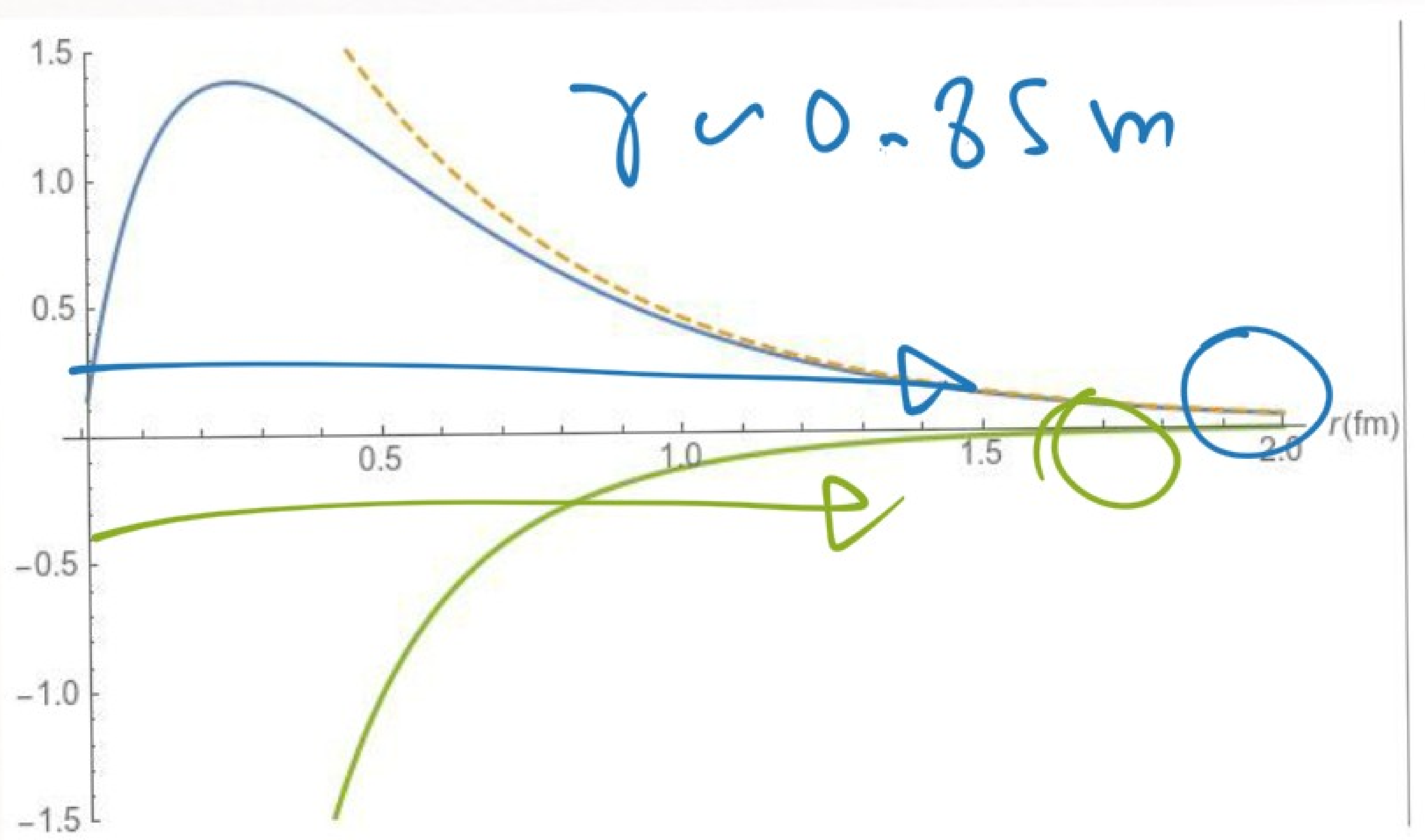
QUESTIONS

?

$$\psi_B(r) = \frac{1}{\sqrt{4\pi}} \frac{u(r)}{r}$$

$\longrightarrow$ ψ_B or u (doesn't matter)





→ COMPARE THE SIZES OF Σ AND V

$\boxed{\text{EFT}} \longrightarrow$ [TWO-BODY SYSTEM IN QM]

If wave function bigger than potential

$\Rightarrow$ exact form of potential not important

$$V(\vec{q}) = -\frac{g^2}{\vec{q}^2 + m^2} \longrightarrow \boxed{V(\vec{q}) = C_0 + C_2 \vec{q}^2 + C_4 \vec{q}^4 + \dots}$$

GENERIC POTENTIAL

If the bound state is big, then we can use the potential:

$$\boxed{V(\xi) = c_0 + c_2\, \xi^2 + \cdots}$$

Taylor series

Teacup & teapot theory

power counting

$$V(\bar{g}) = c_0 + c_2 \bar{g}^2 + c_4 \bar{g}^4 + \dots$$

$$= \hat{c}_0 + \hat{c}_2 \left(\frac{g^2}{m^2}\right) + \hat{c}_4 \left(\frac{g^2}{m^4}\right) + \dots$$

[Expansion parameter]

$$\lambda = S\left[c_0 + c_1 x + c_2 x^2 + \dots\right]$$

TRY THIS GENERIC POTENTIAL
AT LOWEST ORDER (LEADING ORDER)

$$\boxed{V_c^{LO}(\vec{q}) = C_0}$$

$\rightarrow$ C_0 can be determined from experiment (from γ / E_B)

$\boxed{\exists \text{ PROBLEM}}$ $\longrightarrow$ Singular potential

$V_c^{LO}(\vec{q}) = C_0 \rightarrow \left[V_c^{LO}(\vec{r}) = \int \frac{d^3\vec{q}}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} V_c^{LO}(\vec{q}) \right]$

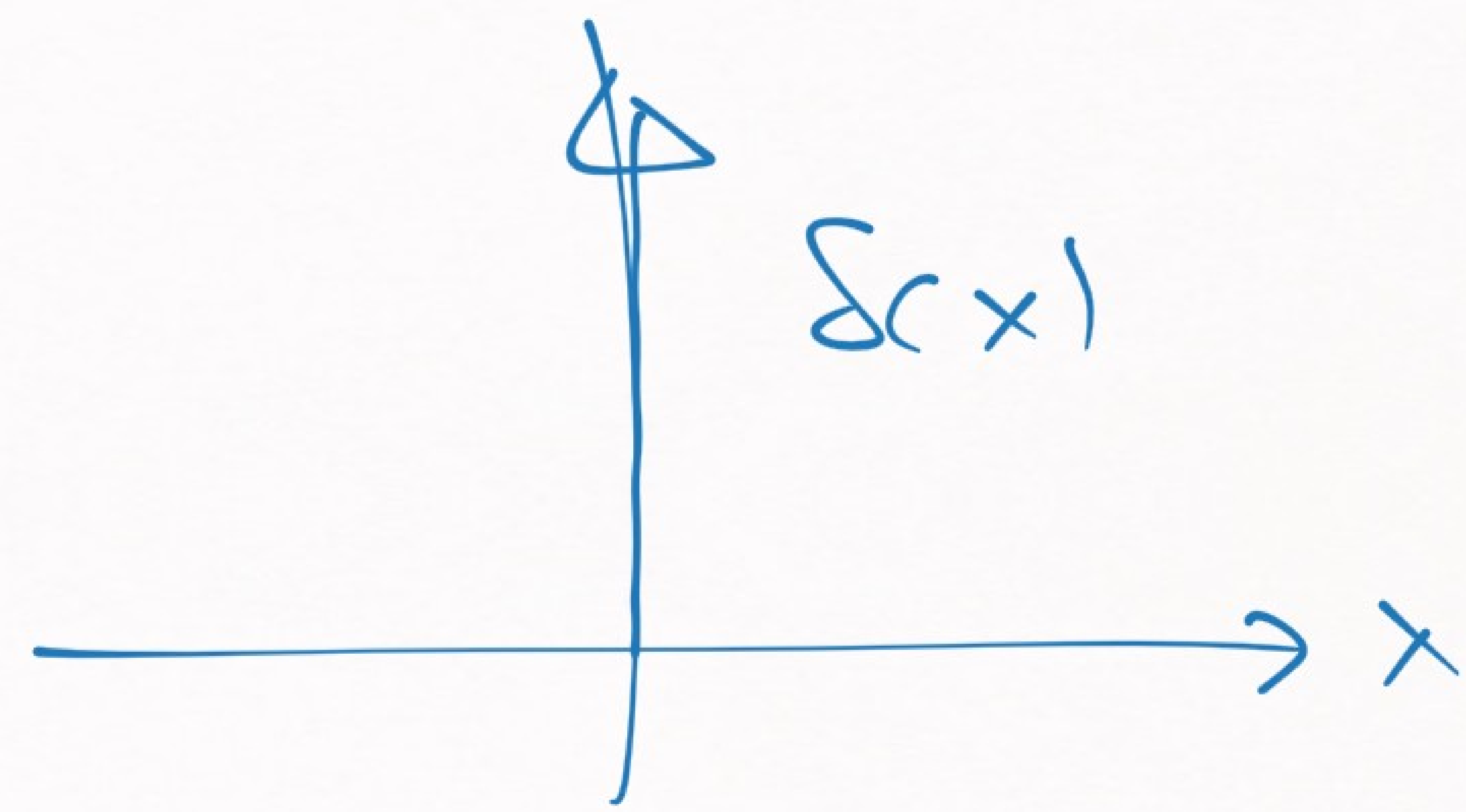
$V_c^{LO}(\vec{r}) = C_0 \underbrace{\delta^{(3)}(\vec{r})}_{\text{Dirac-delta}}$

$\delta^{(3)}(\vec{r}) = 0 \text{ for } |\vec{r}| > 0$

$\int d^3\vec{r}\, \delta^{(3)}(\vec{r}) = 1$

1-dimensional Dirac delta:

$$\delta(x) = \begin{cases} 0 & \text{for } x \neq 0 \\ \underline{\underline{\infty}} & \text{for } x = 0 \end{cases}$$



$\rightarrow$ $\boxed{\text{Singular}}$ $\rightarrow$ [As a potential, a δ cannot be solved]

$$\left.\begin{array}{l} V_c(\vec{r}) = C_0\, \delta^{(3)}(\vec{r}) \\ C_0 < 0 \end{array}\right] \rightarrow E_B \le \langle \psi_{trial} | H | \psi_{trial} \rangle$$

$$E_B \le -\infty$$

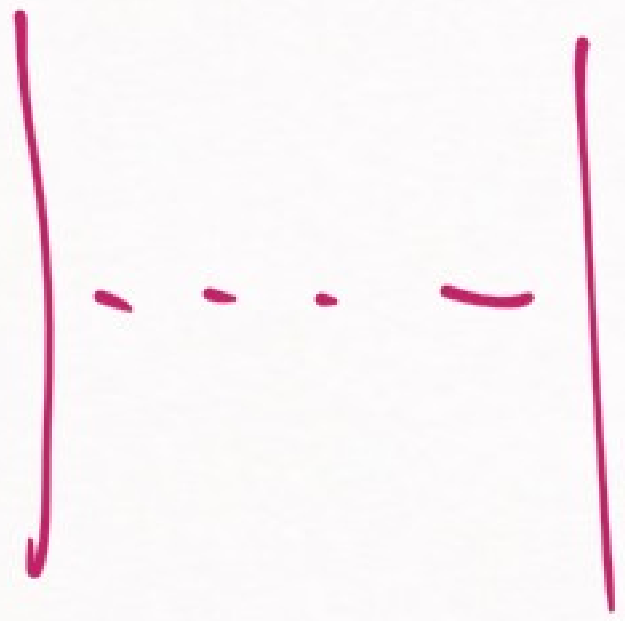
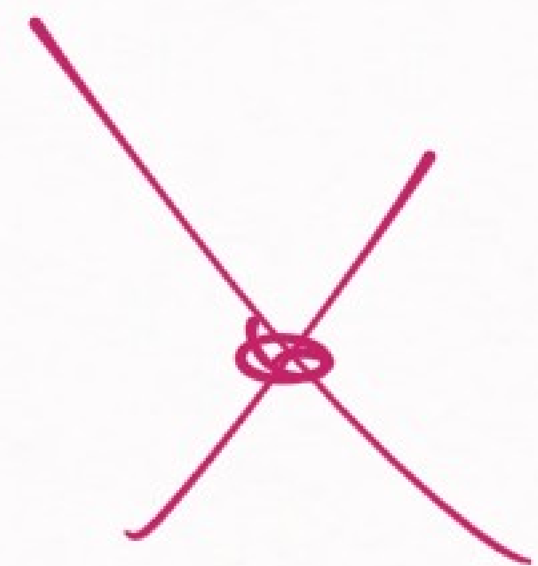
$$\boxed{E_B \rightarrow -\infty}$$

We have to do something about this problematic potential

New concept $\longrightarrow$ $\boxed{\text{REGULARIZATION}}$

$\rightarrow$ We have to regularize $\delta^{(3)}(\vec{r})$

"make something regular (finite)"

 $\Rightarrow$  $\Rightarrow$ infinite! $\Rightarrow$ Regularize

(Original potential) (Taylor expansion)

Λ enters here

$V_c(\vec{q}) = C_0 \Rightarrow$ (REGULARIZE) $\Rightarrow V_c(\vec{q};\Lambda)$

$= C_0 \underbrace{f\left(\frac{|\vec{q}|}{\Lambda}\right)}_{b}$

EXAMPLE $\rightarrow$ GAUSSIAN CUTOFF

$V_c(\vec{q};\Lambda) = C_0\, e^{-(q/\Lambda)^2}$

$f(x) \rightarrow 1 \quad x \rightarrow 0$

$f(x) \rightarrow 0 \quad x \rightarrow \infty$

1) $\Lambda \neq \infty \Rightarrow V_c$ is regular

2) $\lim_{\Lambda \to \infty} V_c^{LO} = C_0$

$$V_c^{LO}(\vec{q}) = C_0 e^{-(\vec{q}/\Lambda)^2}$$

$\boxed{\text{REGULARIZATION}}$

(Fourier transform)

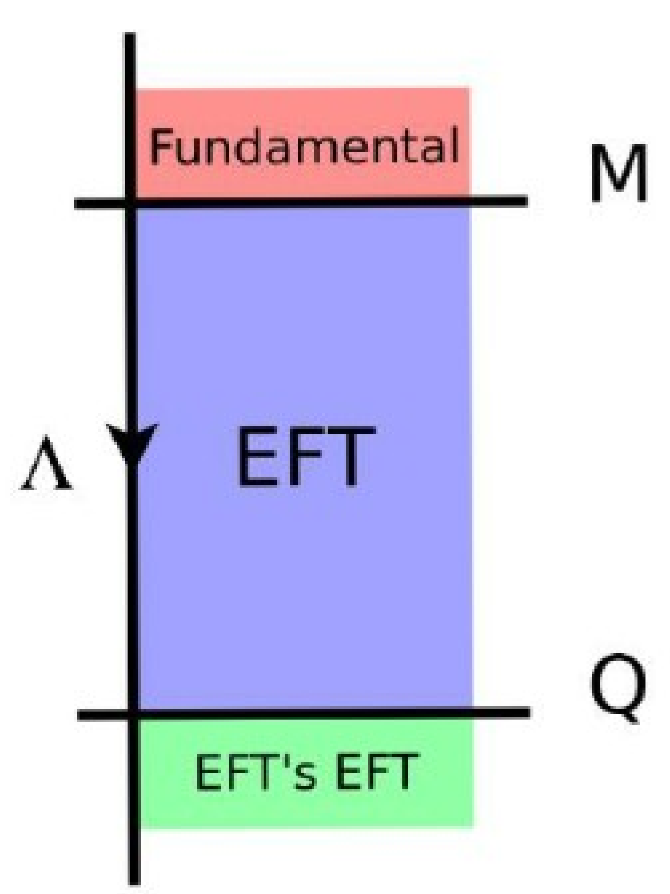
$$V_c^{LO}(\vec{r}) = C_0 \frac{\Lambda^3}{8\pi^{3/2}} e^{-\frac{1}{4}\Lambda^2 r^2}$$

If $\Lambda \gg \gamma$, the bound state will not notice the difference between a true Dirac-delta and a regularized one

REGULARIZATION:

→ we included Λ (a cutoff) to make calculations finite

But what we want is ... $\boxed{\text{RENORMALIZATION}}$



Physics is unique, but choice of theory depends on resolution Λ:

- $\Lambda \geq M$: Fundamental
- $M \geq \Lambda \geq Q$: EFT

For equivalent descriptions:

$$\boxed{\frac{d}{d\Lambda}\langle\Psi|\mathcal{O}|\Psi\rangle = 0}$$

Renormalization group invariance

→ $\forall$ to be cutoff independent

$$\to \boxed{\frac{d}{d\Lambda}(\dots) \simeq 0}$$

→ [RENORMALIZATION IS MORE RESTRICTIVE THAN REGULARIZATION]

AIM → TO RENORMALIZE

TO REGULARIZE → AN INTERMEDIATE STEP

NEXT LESSON $\rightarrow$ $\boxed{\text{RENORMALIZATION}}$