NUCLEAR PHYSICS 7

QCD can't be solved analytically ;

what are the alternatives ?

$\boxed{\text{RECAP}}$ → Last lesson about $\boxed{\text{QCD}}$

QCD → a bit like QED

but w/ different gauge group

QED → U(1) local symmetry

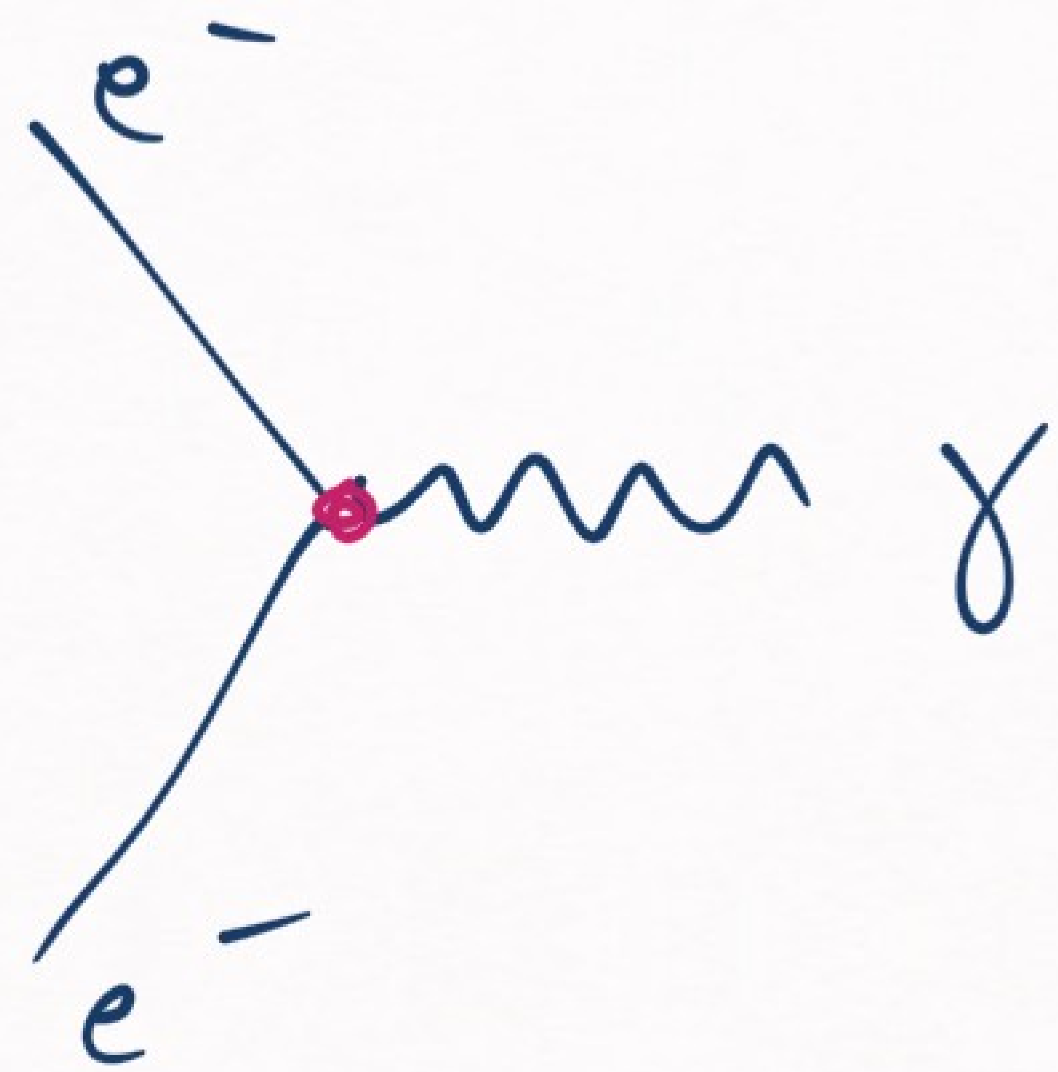
QCD → SU(3) local symmetry

QED & QCD behaviors are very different

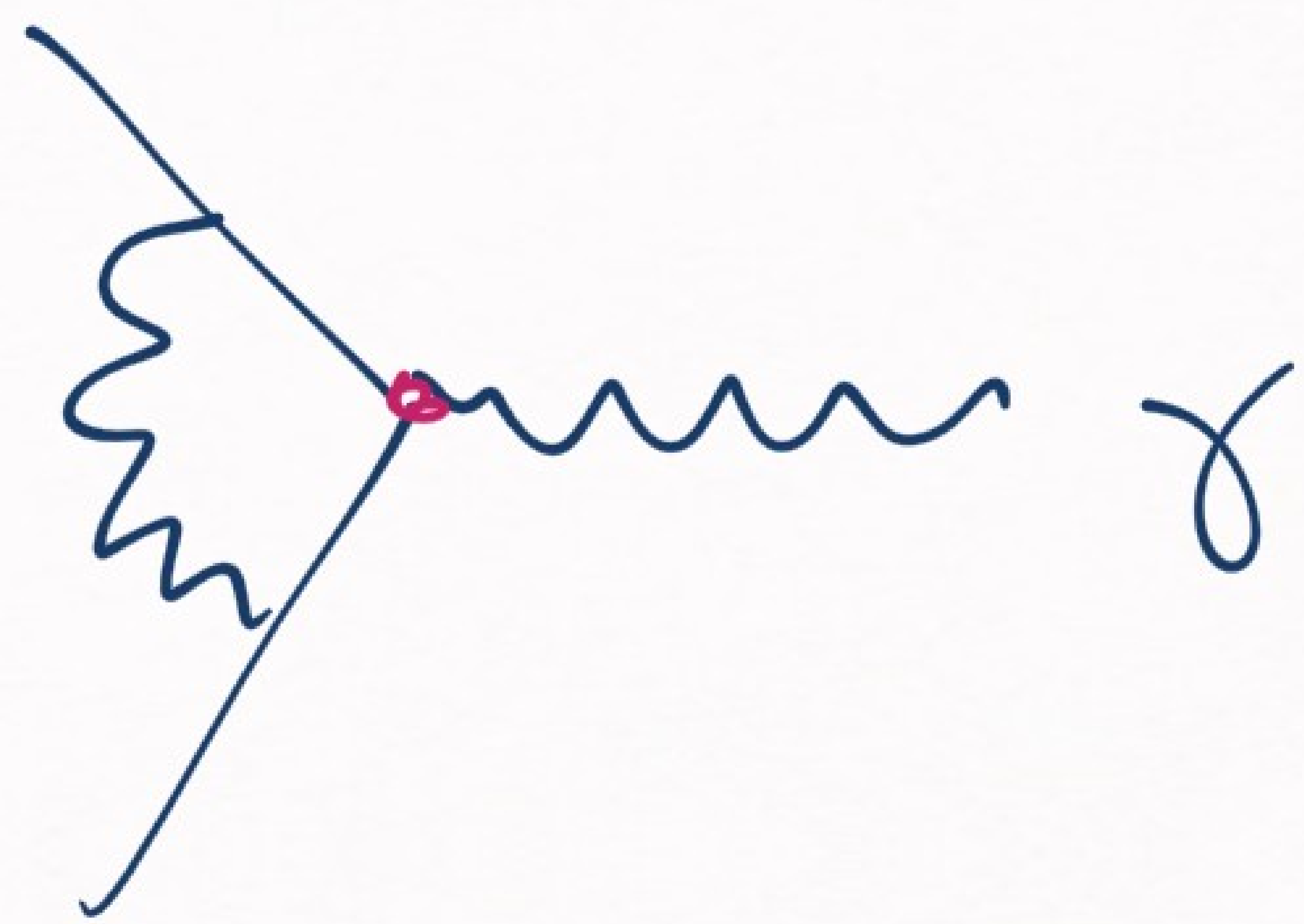
QED → photon (gauge boson of QED)
doesn't carry charge

QCD → gluons (gauge bosons of QCD)
carry strong charge
(color)

QED

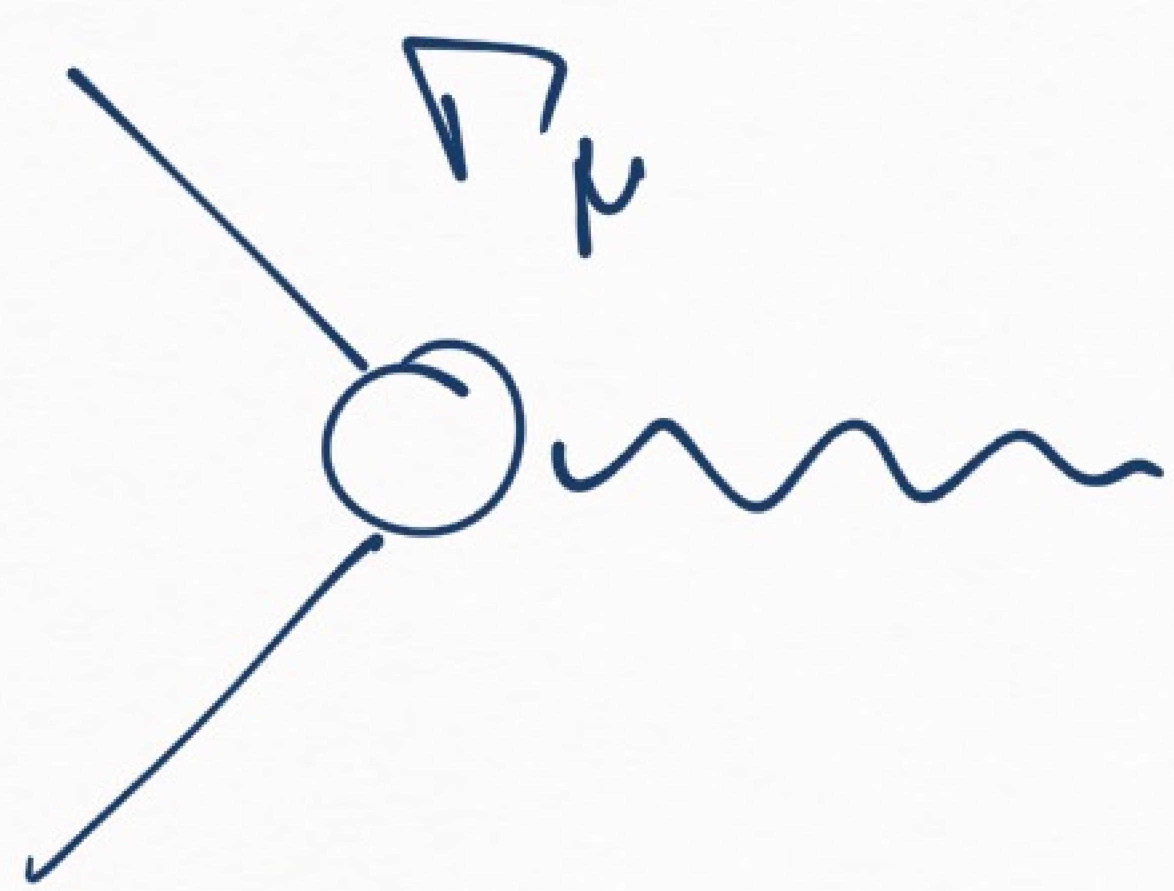


$+$



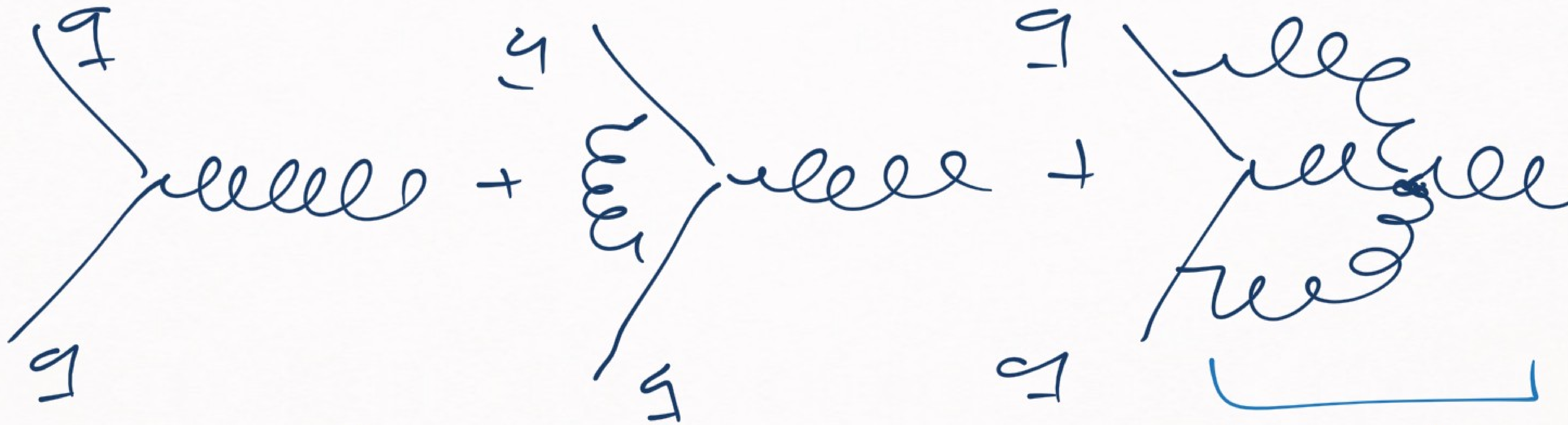
$+ \ldots$

$=$



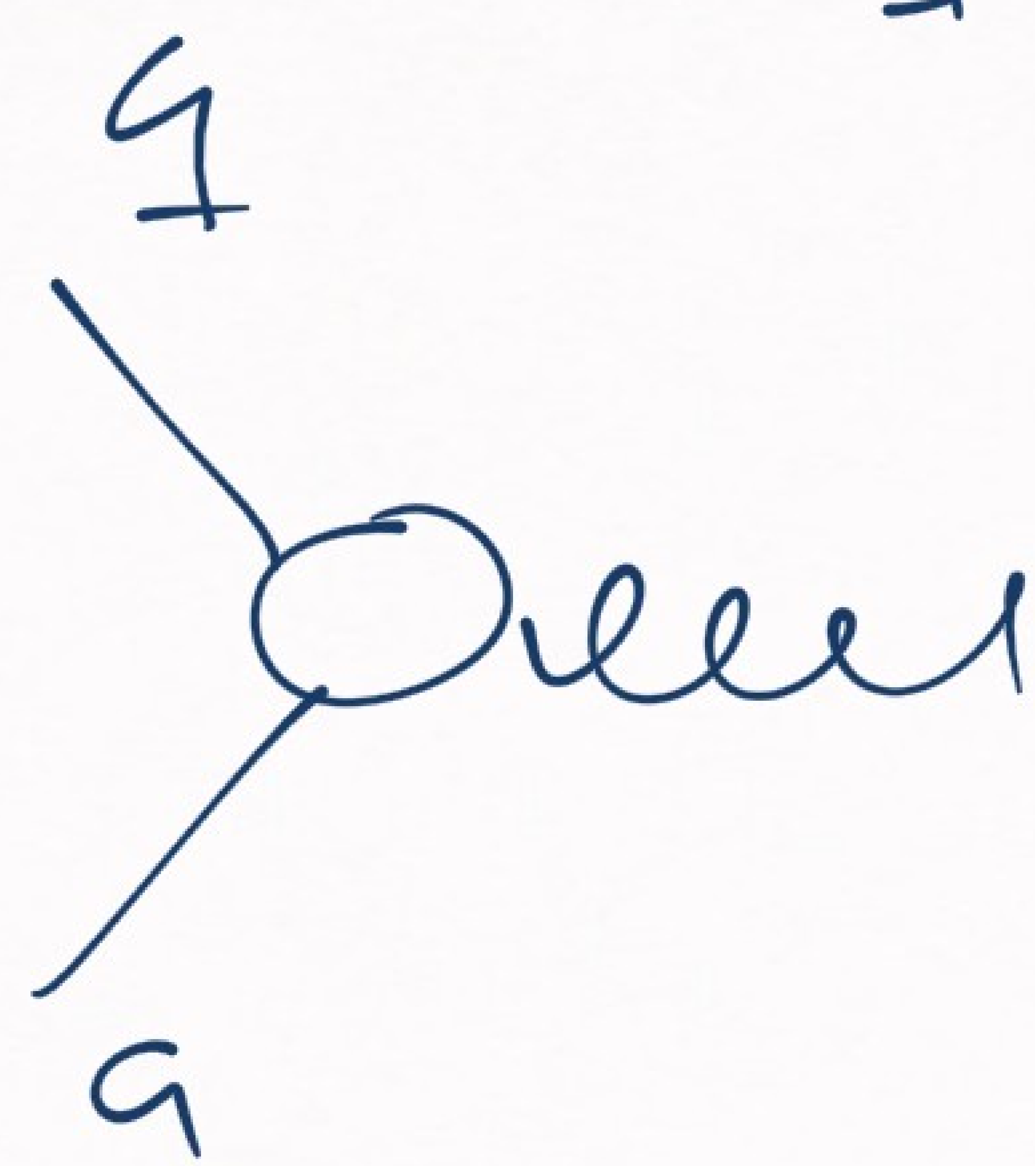
$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi}\log\left(\frac{q^2}{\mu^2}\right)}$$

QCD

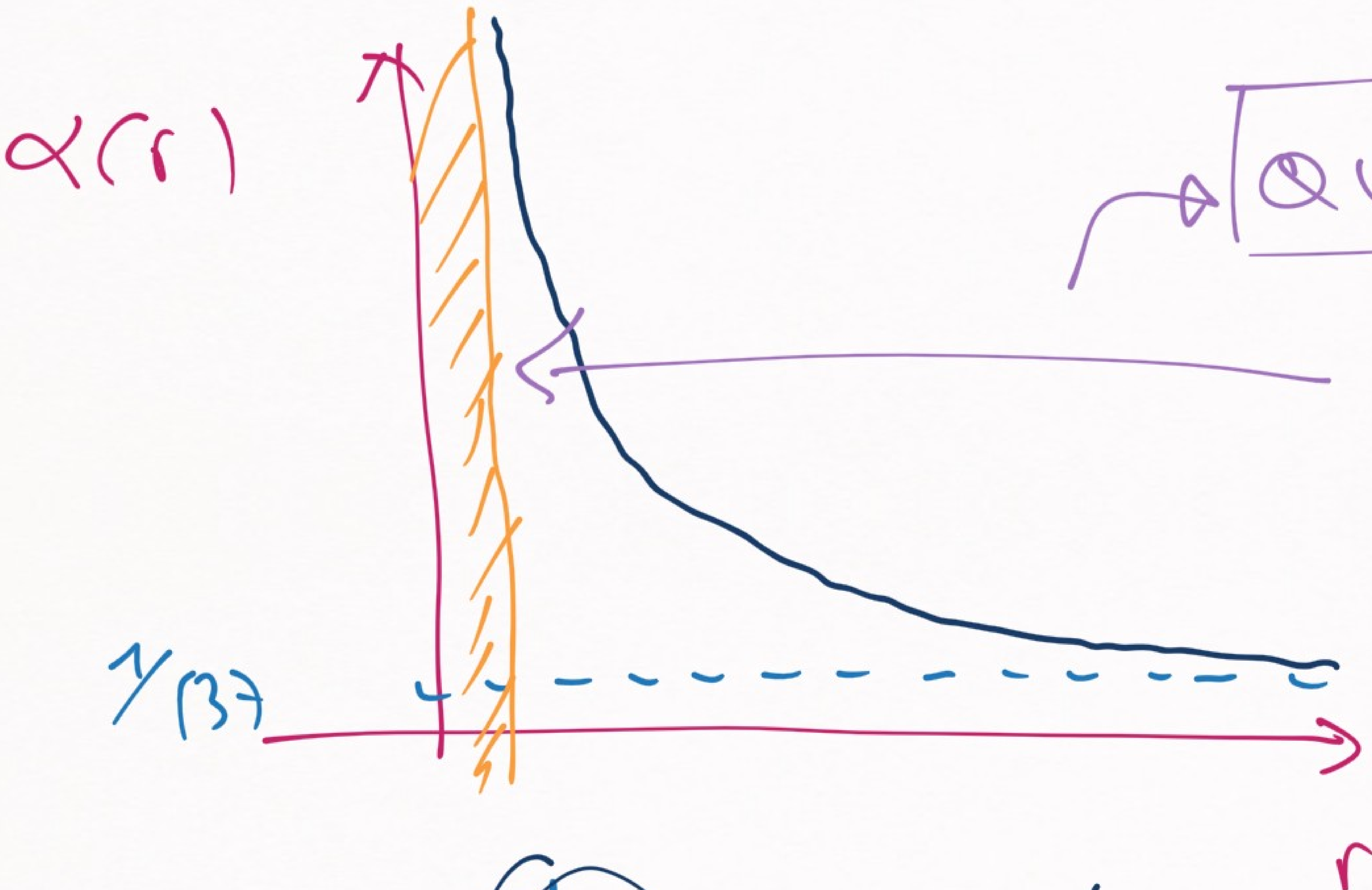


new diagrams

$+ \dots =$



$$\alpha_s(Q^2) = \frac{12\pi}{(33-2n_f)\log\frac{Q^2}{\Lambda^2_{QCD}}}$$



QED perturbative

$r \to \infty$

not a practical issue

$r_0 \sim \frac{1}{\Lambda_0}$ → location of Landau pole ($\Lambda_0 \sim 10^{280}$ MeV)

we can't use perturbation theory

$$\boxed{r \sim 0.5\,\text{fm}}$$

QCD unsolvable w/ standard methods

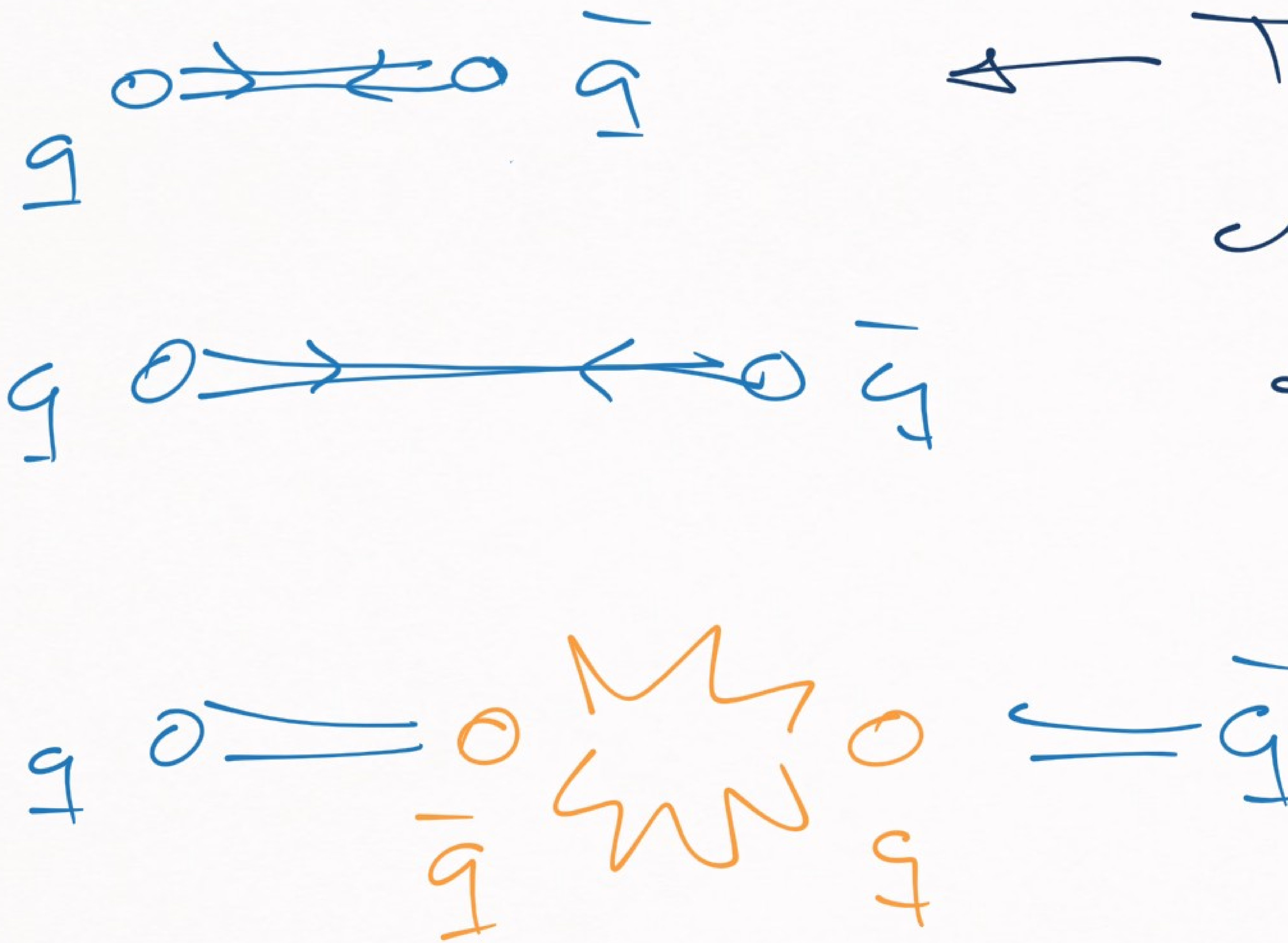
$\boxed{\text{PROBLEM:}}$

nuclear physics happens at $r > 0.5\,fm$

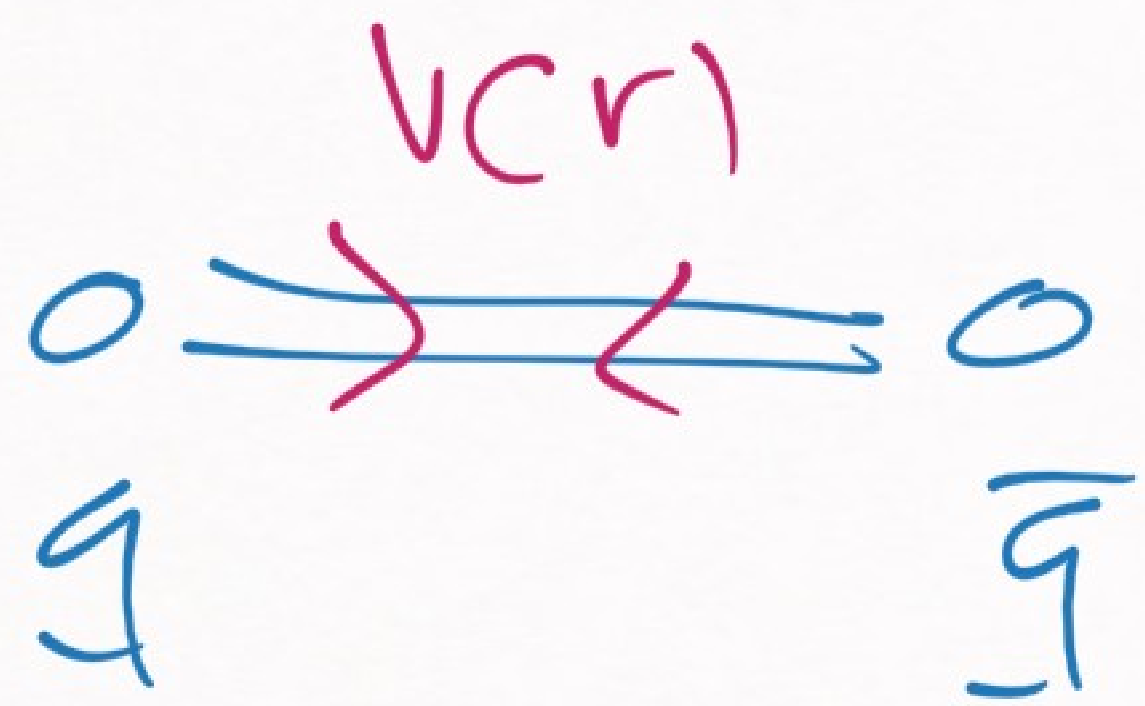
$$\boxed{\text{FUNDAMENTAL PROBLEM OF NUCLEAR PHYSICS}}$$

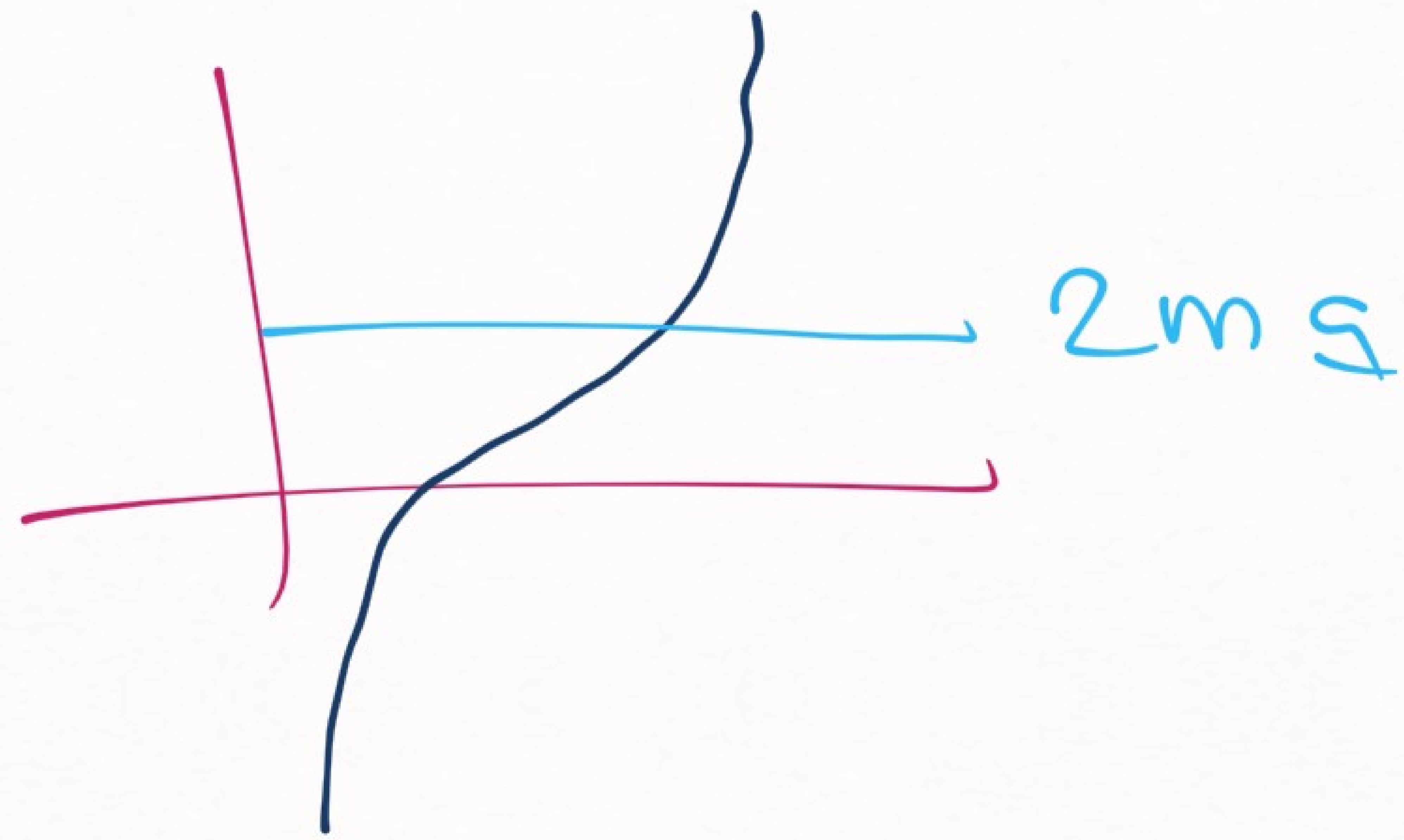
How to derive nuclear physics from QCD ?

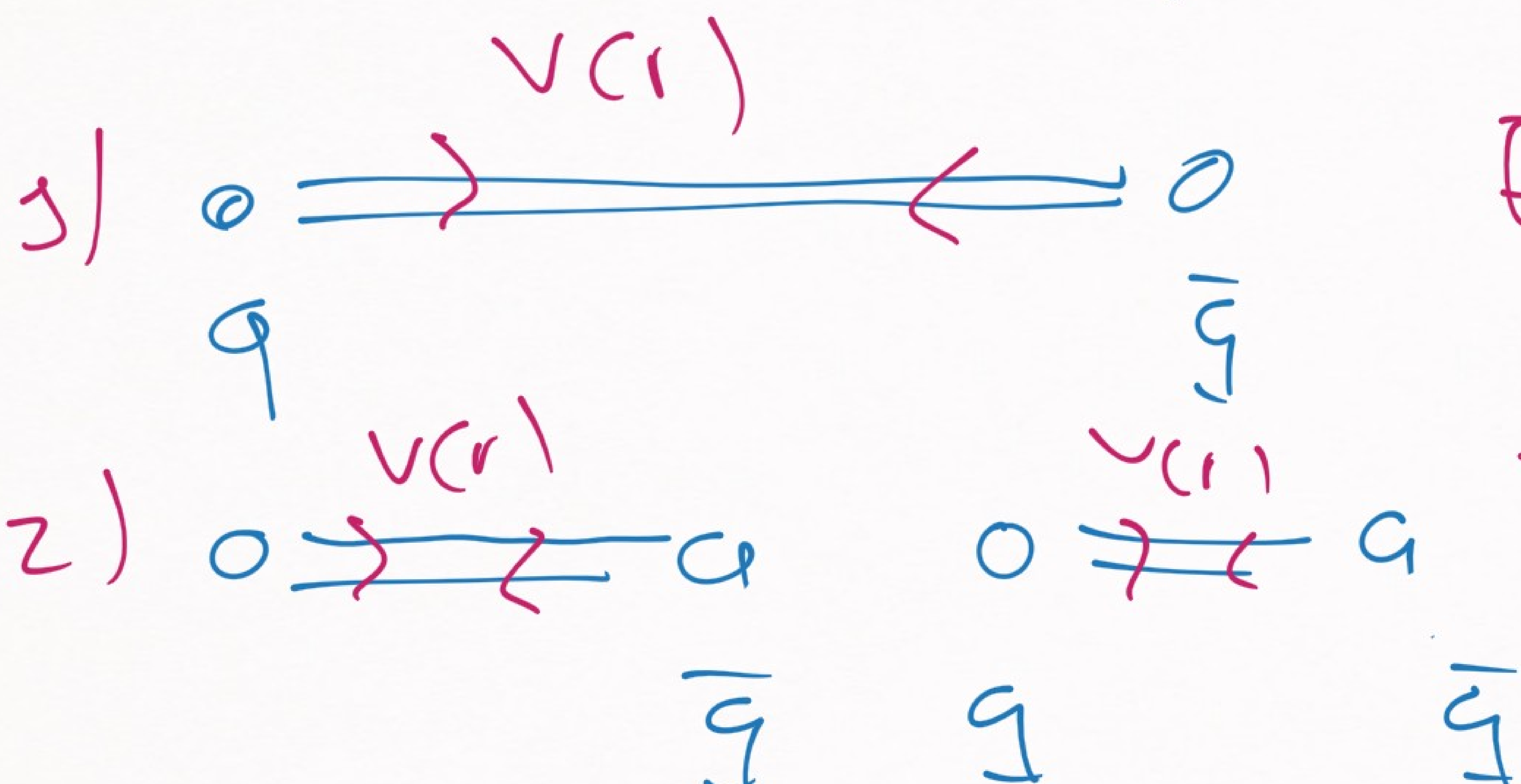
QCD → confinement (no free quarks)



Trying to separate a quark & an anti quark



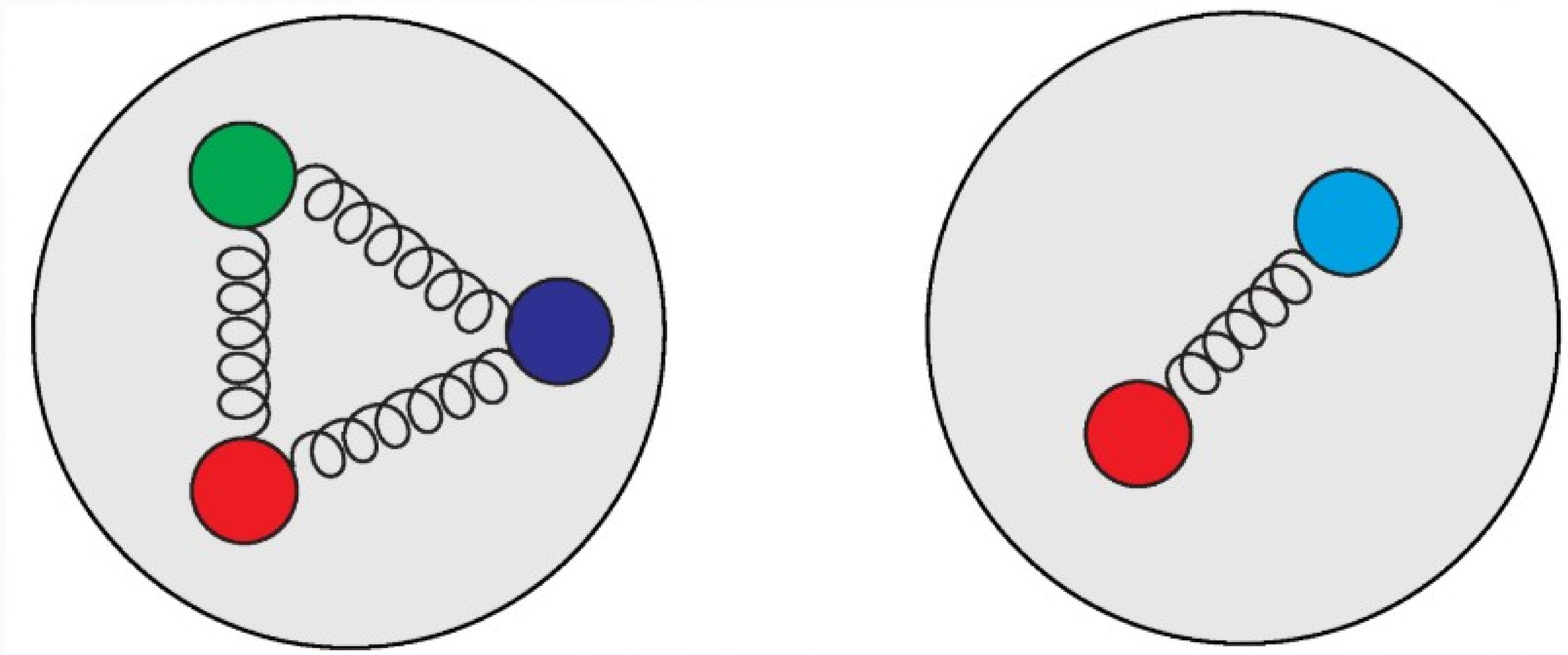




E_1

E_1

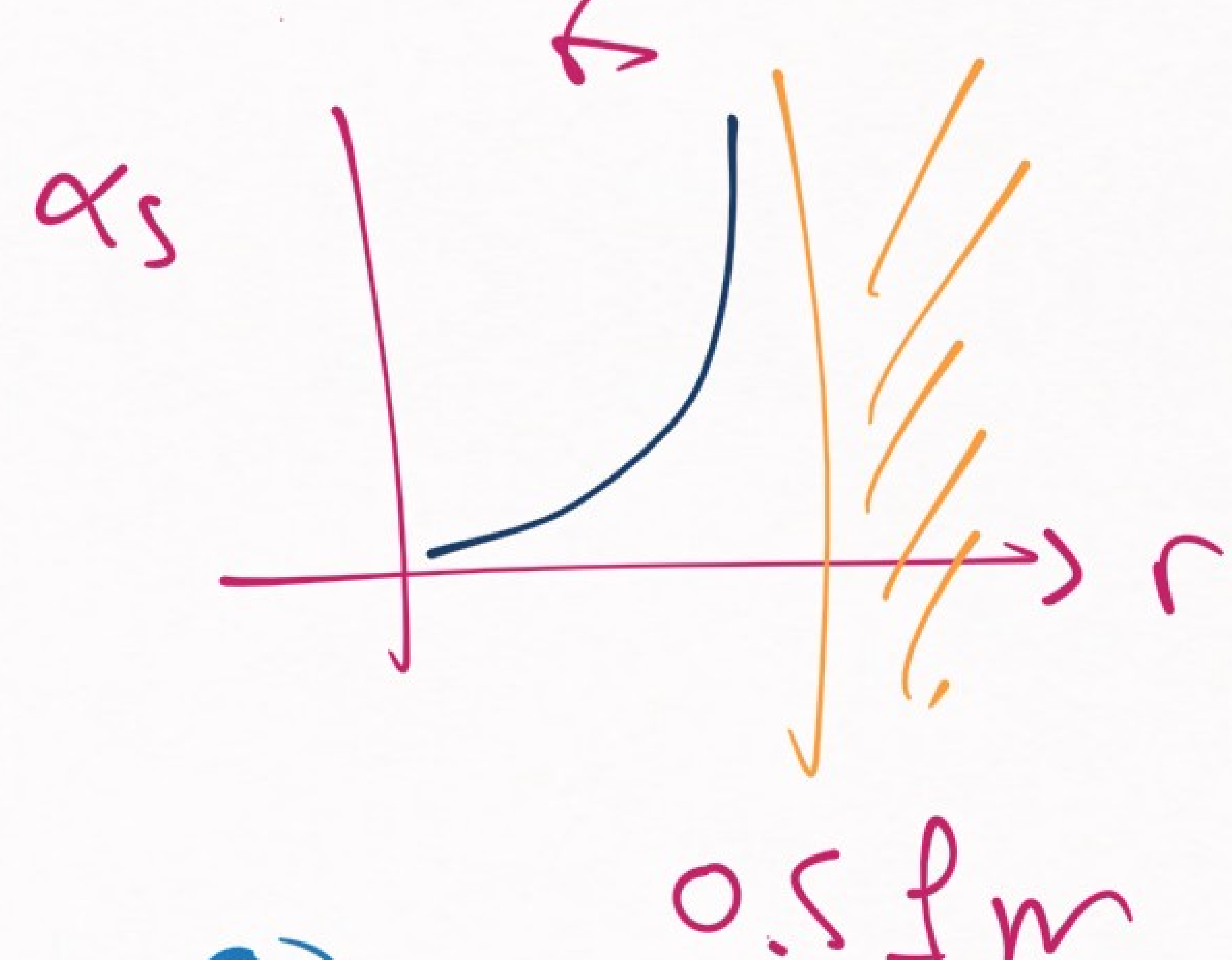
$E_2 < E_1$



BARYON
(qqq)

MESON
$(q\bar{q})$

1 & 2 → $\boxed{\text{PROBLEM}}$



proton

1

$r_p \sim 0.85$ fm

pion

2

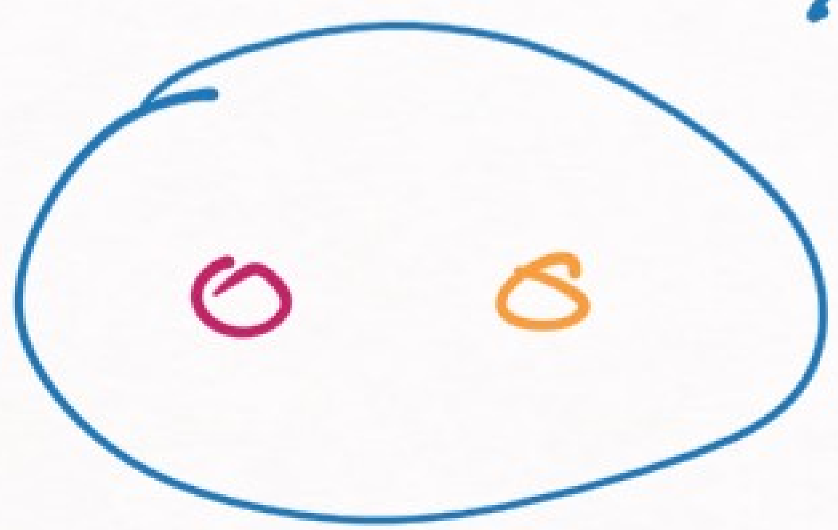
$r_\pi \sim 0.65$ fm

EXCEPTION:

QUARKONIUM

(CHARMONIUM → $c\bar{c}$

BOTTOMONIUM → $b\bar{b}$)



$c \quad \bar{c}$

$b \quad \bar{b}$

$\boxed{0.2-0.4 \text{ fm}}$

↓

PERTURBATIVE QCD

↓

POTENTIAL

$\boxed{\text{QUARKONIUM}} \rightarrow V_S(r) = -\frac{a}{r} + br$

confinement

Coulomb-like

It's smaller than the size at which QCD becomes unsolvable

$m_c \sim 1,2$ GeV

$m_b \sim 4$ GeV

w/ the exception of charmonium/bottomonium

→ $\boxed{\text{QCD is not solvable analytically}}$

↳ What are the solutions?

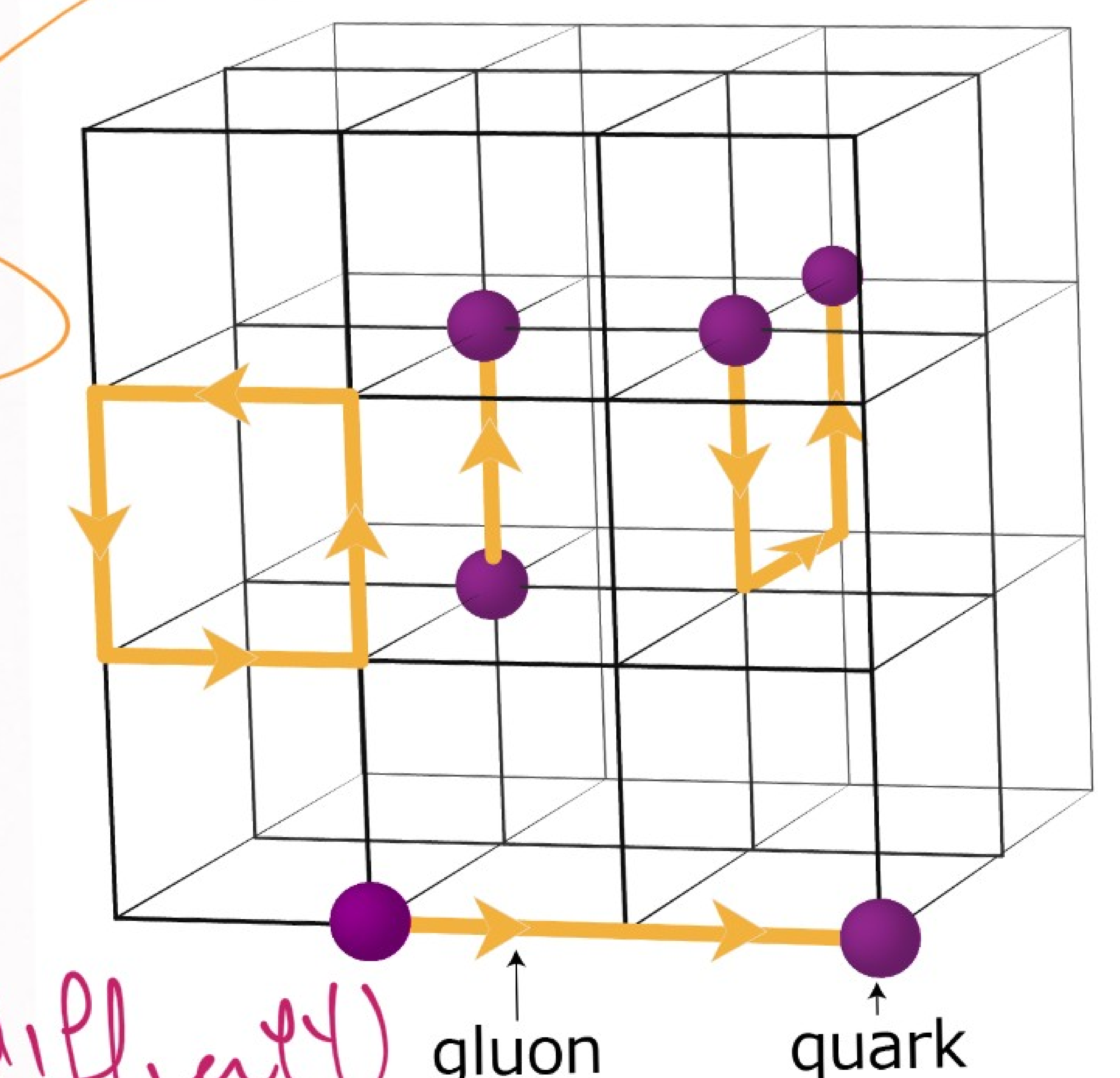
TWO POSSIBILITIES

I don't have one

1) Lattice QCD

Use a supercomputer to solve QCD numerically

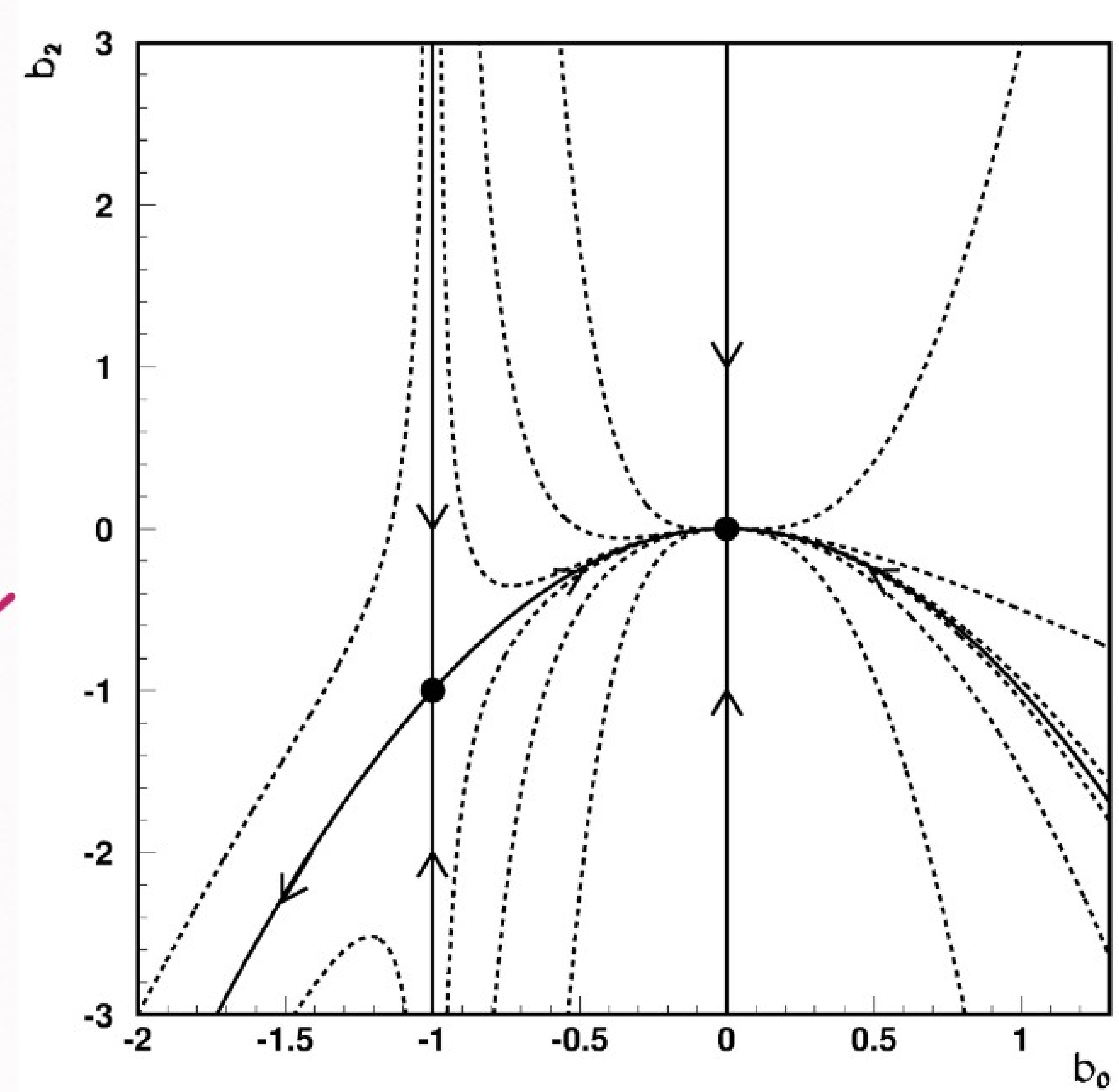
$(m_g \to 0 \Rightarrow$ super difficulty$)$



2) Effective field theory (EFT)

Use renormalization group analysis (RGA) to solve QCD indirectly

↳ this is what I will do here



EFFECTIVE FIELD THEORIES (EFT)

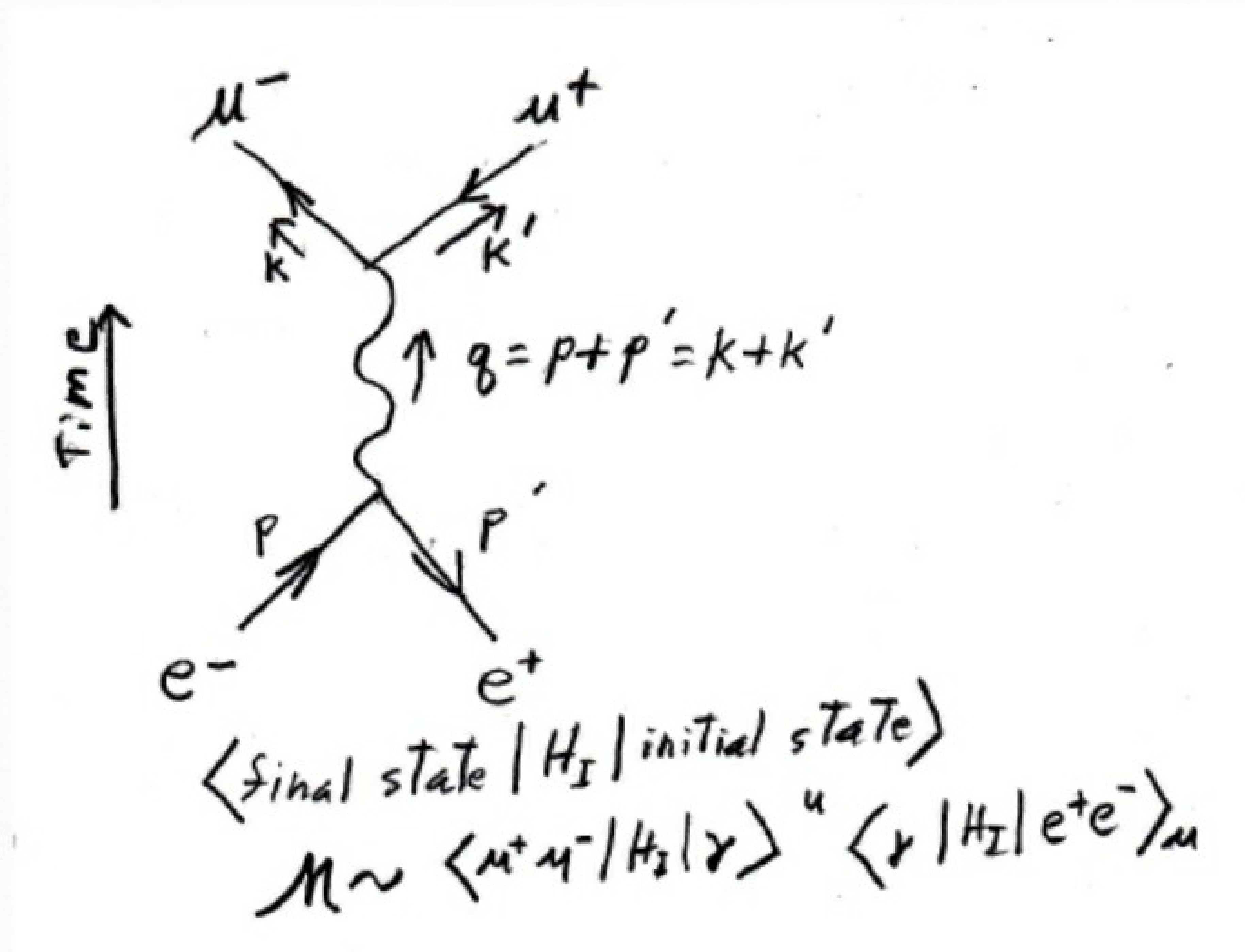
→ What are EFTs?

→ What is RENORMALIZATION?

1) Old ideas about renormalization

1948 → conference in POCONO

FEYNMAN & SCHWINGER



$\rightarrow$ $\boxed{\text{QED}}$

$\downarrow$

infinities

present some really strange methods to solve these infinities

RENORMALIZATION (OLD SCHOOL SENSE)

→ a set of arcane rules to remove these infinites → at that time they were not really understood

Why arcane? E.g. harmonic series

$$H(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n}$$

$\hookrightarrow \lim_{n \to \infty} H(n) \to \infty \quad (O(\log n))$

Renormalized sum: $\boxed{\lim_{n \to \infty} H(n) = -\frac{1}{12}}$

(e.g. calculation of Casimir effect)

How this is explained (old school):

$$H(n) = \sum_{k=1}^{n} \frac{1}{k}$$

Riemann ζ-function: $\zeta(s) = \sum_{k=1}^{\infty} \frac{1}{k^s}$

$$\zeta(1) = \lim_{n \to \infty} H(n) \qquad (\mathrm{Re}(s) > 1)$$

$$\zeta(1) = -\frac{1}{12} \quad \Rightarrow \quad \lim_{n \to \infty} H(n) = -\frac{1}{12}$$

→ this way of understanding Reno is somewhat weird

我们为求出n和j所玩的壳层游戏，在专业上叫做“重正化”（renormalization）。但是，不管这个词听来多聪明，我却说这个过程是蠢笨的！求助于这类戏法妨碍了我们去证明量子电动力学在数学上的自洽性（self-consistent）。令人不解的是，尽管人们用了各种办法，这个理论至今仍未被证实是自洽的；我猜想，重正化在数学上是不合法的。我们还没有一种好的数学方法描述量子电动力学，这是肯定的——像这样描述n、j同m、e之间关系的语言不是好的数学。[23]

QED: the strange theory of light & matter

（QED：光和物质的奇妙理论）

- So it appears that the only things that depend on the small distances between coupling points are the values for n and j-theoretical numbers that are not directly obseroable any- way; everything else, which can be observed, seems not to be affected. The shell game that we play to find n and j is technically called "renormalization." But no matter how clever the word, it is what I would call a dippy process! Having to resort to such hocus-pocus has prevented us from proving that the theory of quantum electrodynamics is mathematically self-consistent. It's surprising that the theory still hasn't been proved self-consistent one way or the other by now; I suspect that renormalization is not mathematically legitimate. What is certain is that we do not have a good mathematical way to describe the theory of quantum electrodynamics: such a bunch of words to describe the connection between n and j and m and e is not good mathematics.
    - Richard Feynman, *QED: The Strange Theory of Light and Matter* (1985), Chap. 4. Loose Ends

ORIGINALLY THEY WERE NOT SURE
ABOUT THE MEANING OF RENO

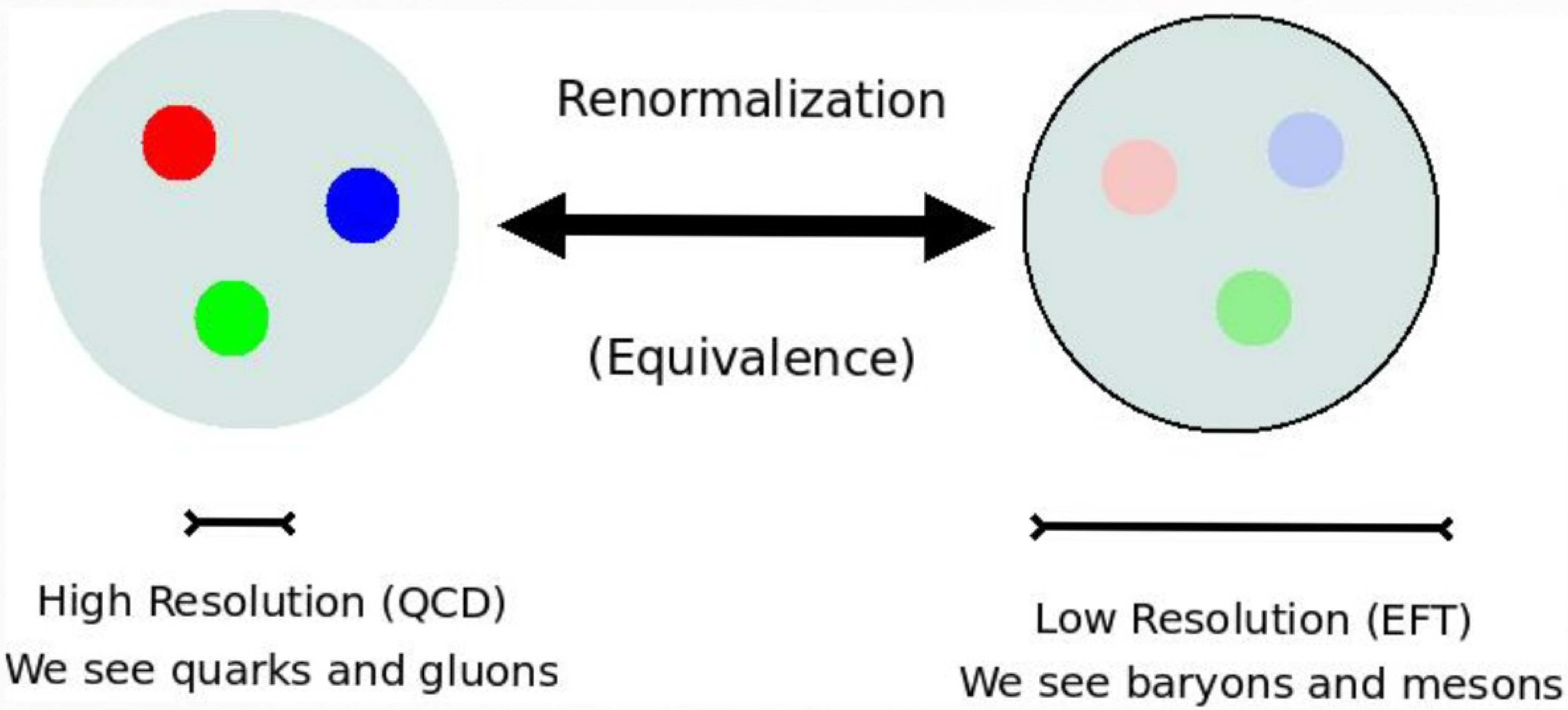
70 YEARS HAVE PASSED SINCE POCONO

$\rightarrow$ much better understandings

$-\otimes-$ $\rightarrow$ Scale separation

BASIC IDEA $\rightarrow$ $\boxed{\text{Physics at long-distances does not depend on short-distance details}}$

$\boxed{\text{MODERN RENO}}$



HADRONS

→ two possible pictures

① short-distances (quarks & gluons)

② long-distances (hadrons)

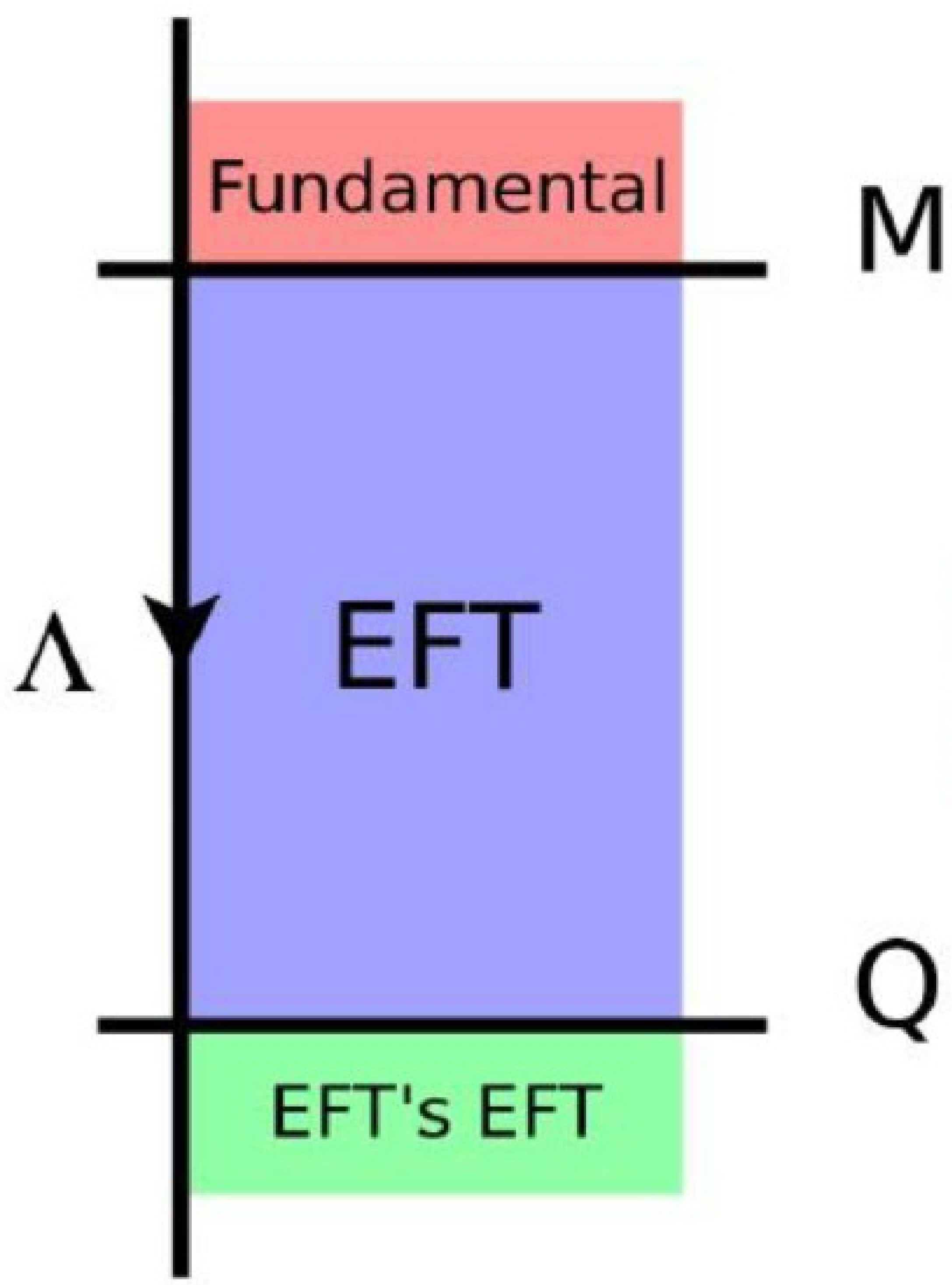
① & ② EQUIVALENT

RENORMALIZATION

↳ the mathematical way to connect these two views/pictures/explanations in a rigorous way

QUARK & GLUONS $\xleftrightarrow{\text{RENO}}$ HADRONS

EFTs



Physics is unique, but choice of theory depends on resolution Λ:

- $\Lambda \geq M$: Fundamental
- $M \geq \Lambda \geq Q$: EFT

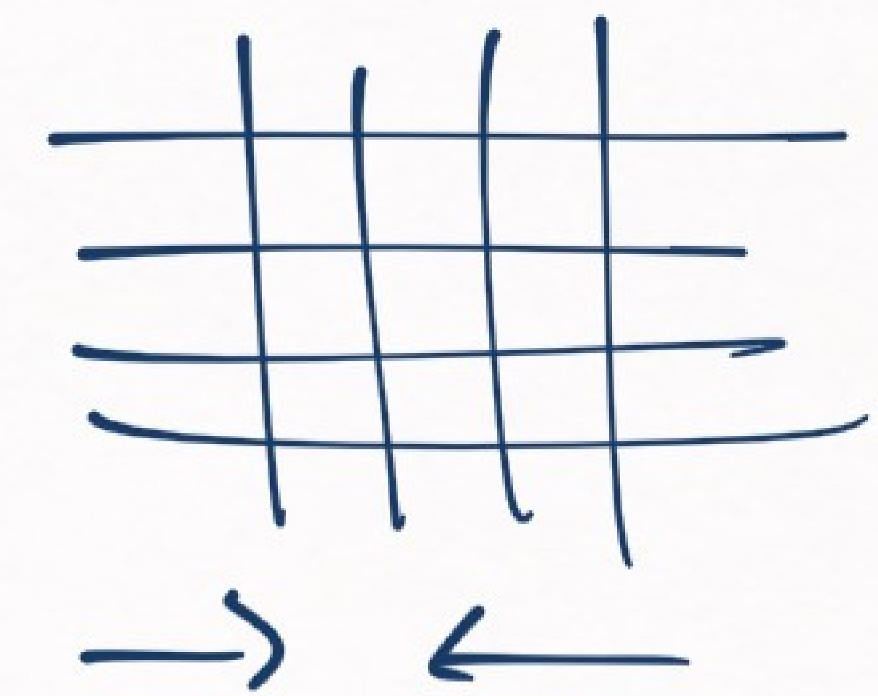
For equivalent descriptions:

$$\boxed{\frac{d}{d\Lambda}\langle\Psi|\mathcal{O}|\Psi\rangle = 0}$$

Renormalization group invariance

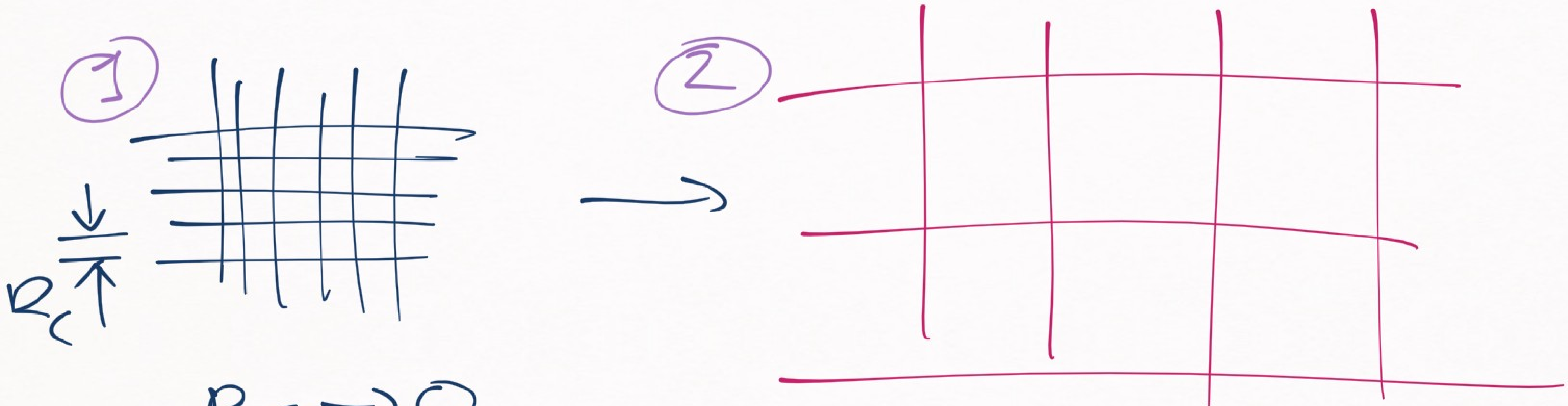
(1) $\Lambda \to \infty$

($R_c \to \infty$)



$1/\Lambda$ or R_c

"Fundamental theory"



$R_c \to 0$

(Idealization)

$R_c \to \infty$

(Idealization)

1 → Fundamental

2 → Effective

$\boxed{\text{FOR QCD}}$

① FUNDAMENTAL → $M \sim 1\,\text{GeV}$ $\left(\frac{1}{M} \sim 0.2\,\text{fm}\right)$

→ QCD is the theory we will use

② EFFECTIVE → $Q \sim m_\pi \sim 0.14\,\text{GeV}$

$\left(1/m_\pi \sim 1.4\,\text{fm}\right)$

→ intuitive understanding of nuclei

Λ → my choice (scale at which I choose to see nature)

→ not part of physics (part of theory)

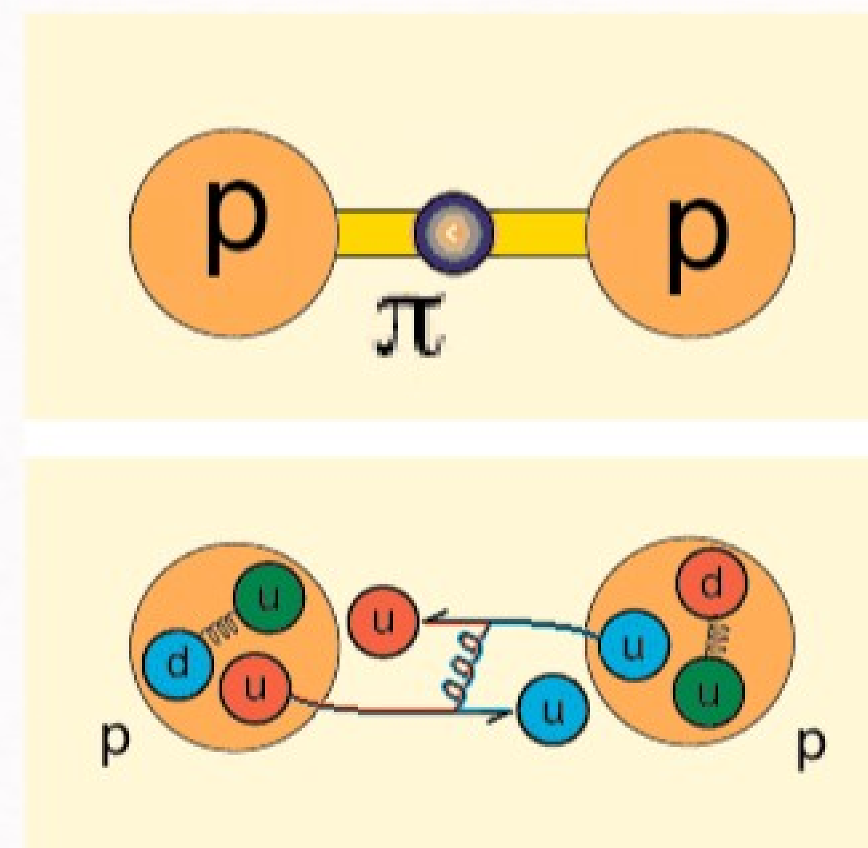
1) $\Lambda > M$ (large resolution) → quarks & gluons

2) $\Lambda < M$ (poor resolution) → baryons & mesons

3) $\Lambda \sim M$ → both descriptions are possible

3) → [How I will try to build an EFT]

3) $\boxed{\Lambda \sim M}$



BOTH ARE USEFUL

EFT $\Lambda \to Q \Rightarrow$ my theory will still be equivalent to QCD if and only if observable are independent of Λ

$$\frac{d}{d\Lambda}\langle \Psi | \hat{O} | \Psi \rangle = 0$$

from this we can build an EFT

Operator for an observable

wave function

cutoff (it is a parameter of our theories, not a parameter from nature)

$\frac{d}{d\alpha}\langle \mathcal{O}^{1} \rangle = 0 \rightarrow$ PROBLEM : $\boxed{\text{PREDICTIVE POWER}}$

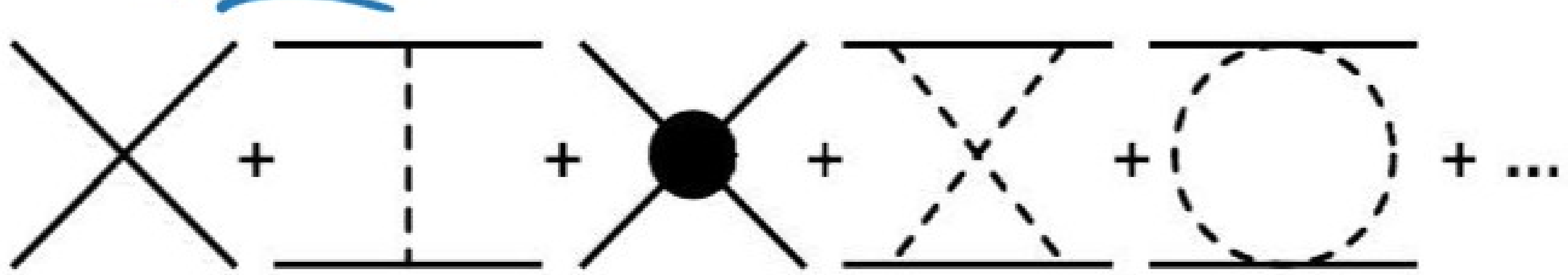
Begin at $\Lambda = M$, two equivalent descriptions

$$\boxed{\underbrace{\text{quarks \& gluons}}_{\text{high energy}}} \iff \underbrace{\text{hadrons } (N, \Delta, \pi)}_{\text{low energy}}$$

The hadron description equivalent if and only if

(1) Include low energy symmetries (particularly **chiral symmetry**)

(2) Consider infinite set of Feynman diagrams consistent with (1)



Problem: **infinite diagrams imply no predictive power**

1) Fundamental description (QCD)

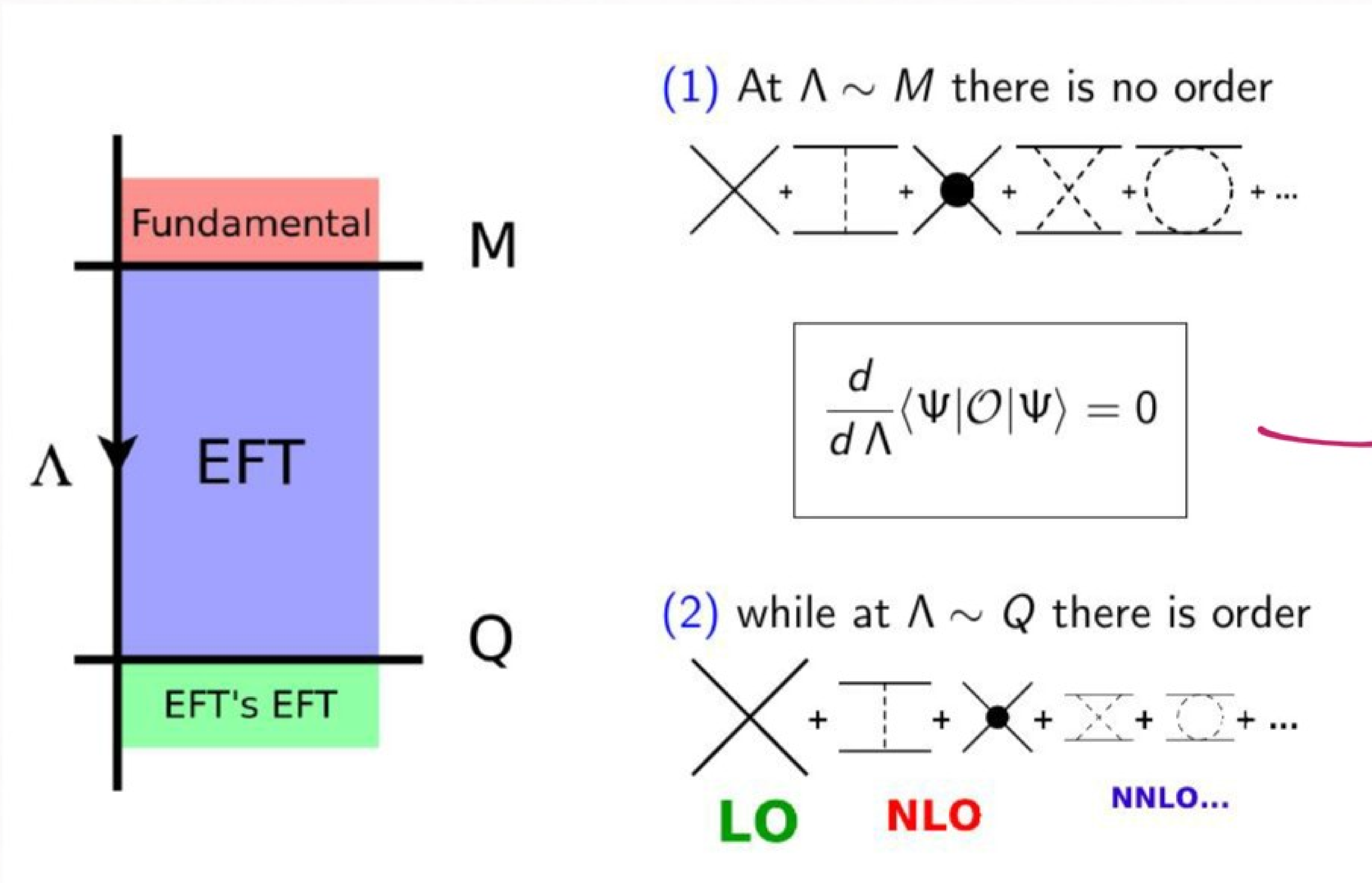
2) EFT description

1) $\hat{=}$ 2)

infinite diagrams

$\rightarrow$ infinite parameters

PREDICTIVE POWER (PROBLEM) → POWER COUNTING (SOLUTION)



→ Infinite diagrams

RG Equation

Some diagrams are bigger than others at low energy

$$V(r) = \underbrace{\sum_{n=n_0}^{\infty} \frac{c_n}{r^n}}_{\text{very simple example of power counting}}$$

$r \to \infty$ $\Rightarrow$ as n grows, each term will be smaller

$$V_c(r) = \pm \frac{\alpha}{r} + (\text{corrections})$$

EXAMPLE: $\sim \dfrac{e^{-2m_e r}}{r^{5/2}}$

check

vacuum polarization

$\boxed{\text{RECAP}}$

1) RENORMALIZATION → long-distance explanations/theories independent of short-distance theories

2) EFT → a way to implement 1)

2.a) RGE $\frac{d}{d\Lambda}\langle\psi|\hat{O}|\psi\rangle = 0$

2.b) POWER COUNTING → $\exists$ order of contributions to physics

→ really abstract (EFT's are based on conceptual ideas of how to build physical theories)

$$\boxed{\text{THEORY OF TEACUPS \& TEAPOTS}}$$

[THEORY OF TEAPOTS & TEACUPS]

1 2

QUESTION

1 → TEAPOT (壶)

2 → TEACUP (杯)

WHICH ONE COOLS FASTER?

CONGRATULATIONS, IT'S 2

(YOU HAVE ALREADY FORMULATED A POWER COUNTING)

→ WHY IS THIS SO?

1 TEAPOT :

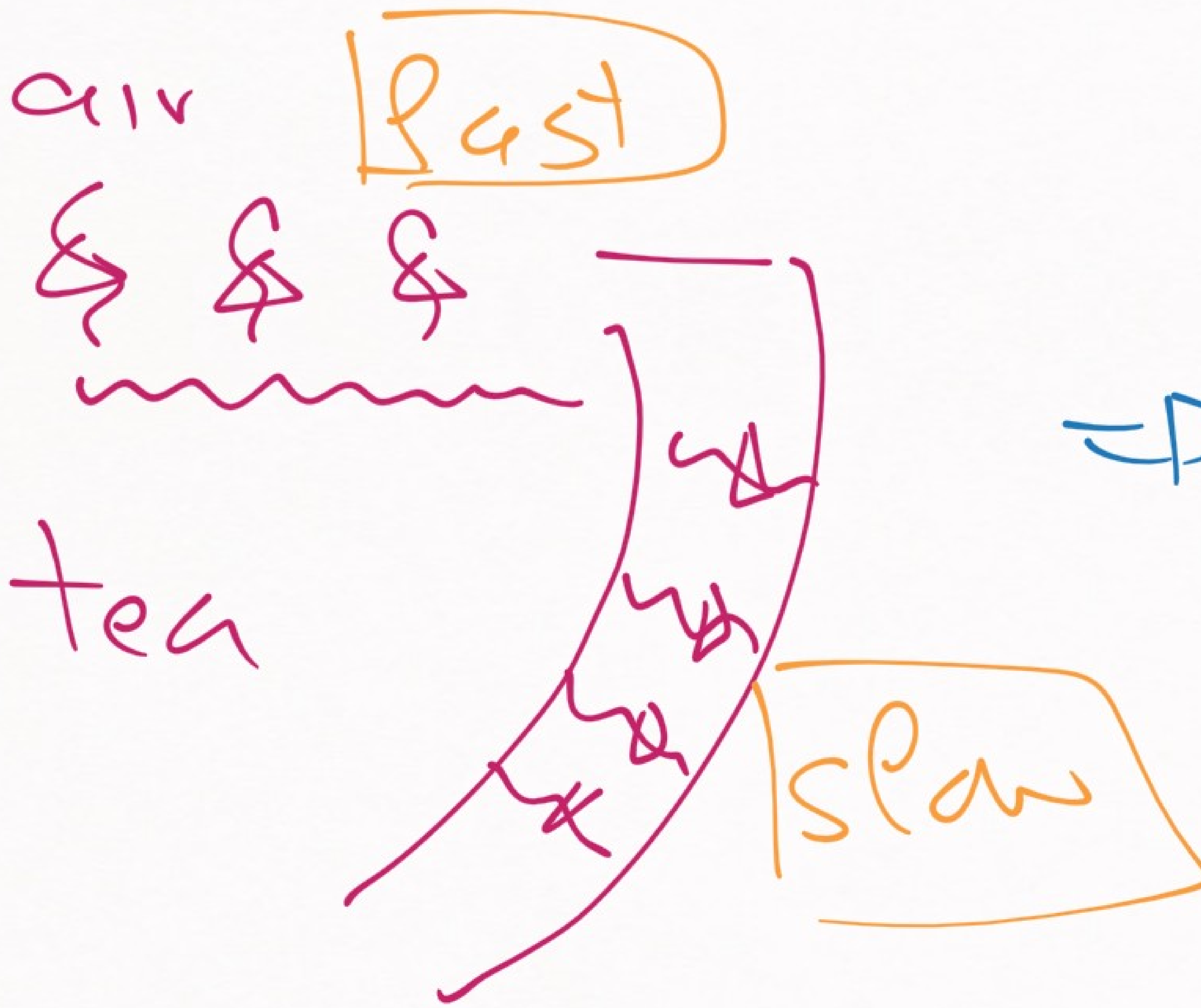
a) ceramic surface large

b) exposed (to the air) surface is small

2 TEACUP → OPPOSITE AS 1

HEAT TRANSFER

→ intuitively, convection is faster than conduction



⇒ $\boxed{\text{CUP COOLS FASTER THAN THE POT}}$

WE KNOW THIS WITHOUT SOLVING ANY EQUATION $\rightarrow$ HEAT TRANSFER EQUATIONS

$\rightarrow$ Fondamental theory ($\Lambda \sim M$)

(analogous of QCD)

EFT ARGUMENT

FUNDAMENTAL THEORY ($\Lambda \leftrightarrow M$) :

1) Fourier's law of heat conduction

$$\boxed{\underline{q} = -k \nabla T} \longrightarrow \text{ceramic wall}$$

2) Convection-diffusion equation

$$\boxed{\frac{\partial c}{\partial t} = \nabla \cdot (D \nabla c) - \nabla \cdot (vc) + R}$$

3) Use some computer to solve these equations & determine which one (pot/cup) cools faster

→ STRAIGHTFORWARD

(Bc I'm a lazy guy)

EFT $\rightarrow$ lazy way to solve this problem

1) Some dynamics (but keep them to the minimum)

Newton's law of cooling

$$\boxed{T = T_0 e^{-\lambda(t - t_0)}}$$

$\lambda \rightarrow$ coefficient

$T, T_0 \rightarrow$ final & initial temperature

$t, t_0 \rightarrow$ final & initial time

2) Find relevant degrees of freedom

Surface exposed to the air (S)

Ceramic surface (Σ)

3) Propose a "power counting"

(try to determine which are the most important factors)

$\boxed{\text{CUP}} \longrightarrow \boxed{S}$ is more important than $\boxed{\Sigma}$

Taylor-expand in powers of $x = \left(\dfrac{S}{f\Sigma} \right)$ $(f \gg 1)$

$(x \ll 1)$

$x \ll 1 \Rightarrow x^2 \ll x \Rightarrow$ Taylor expansion

4) Write down the theory:

$$T = T_0 e^{-\lambda(t-t_0)}$$

we want to describe this

$$\lambda = S\underbrace{(c_0 + c_1 x + c_2 x^2 + \dots)}$$

Low energy constants (LECs)

Power series (power counting)

5) Choose the accuracy we want for our calculation

$$\lambda = S\sum_{n=0}^{\infty} c_n x^n$$ → infinite parameters

→ no predictive powers

↳ solution → choose the accuracy we want

$O(x^0)$: $\boxed{\lambda = c_0 S}$ → leading order approximation

$\mathcal{O}(x^2)$; $\boxed{\lambda \simeq c_0 S + c_1 S x}$

→ next-to-leading order (NLO) approximation

$\mathcal{O}(x^2)$: $\boxed{\lambda = c_0 S + c_1 S x + c_2 S x^2}$

→ next-to-next-to-leading order (N^2LO) approximation

More terms → more accuracy

(but more parameters)

Accuracy is set by $\mathcal{O}(x^{n+1})$

Example: $\boxed{x \simeq 0.1}$ → LO → 10%

NLO → 1%

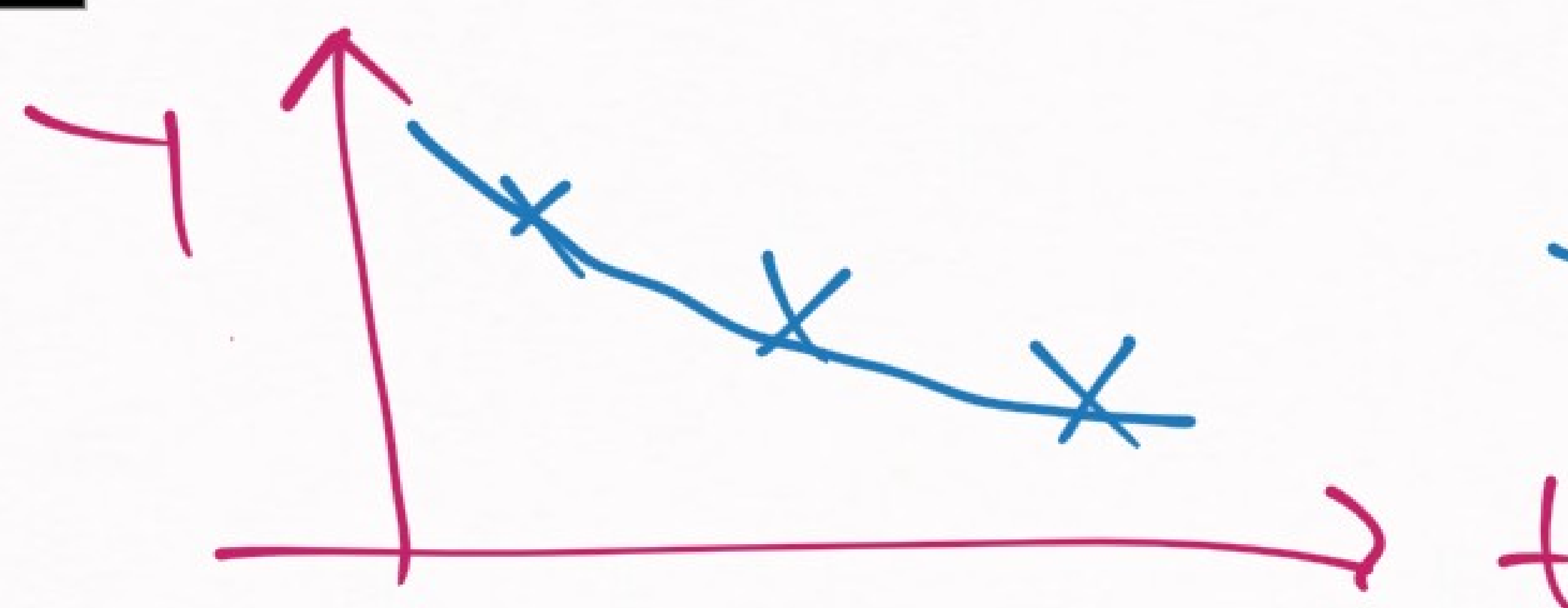
N^2LO → 0.1%

Error

6) Fit the LECs $(c_0, c_1, c_2, \ldots)$ to experimental data

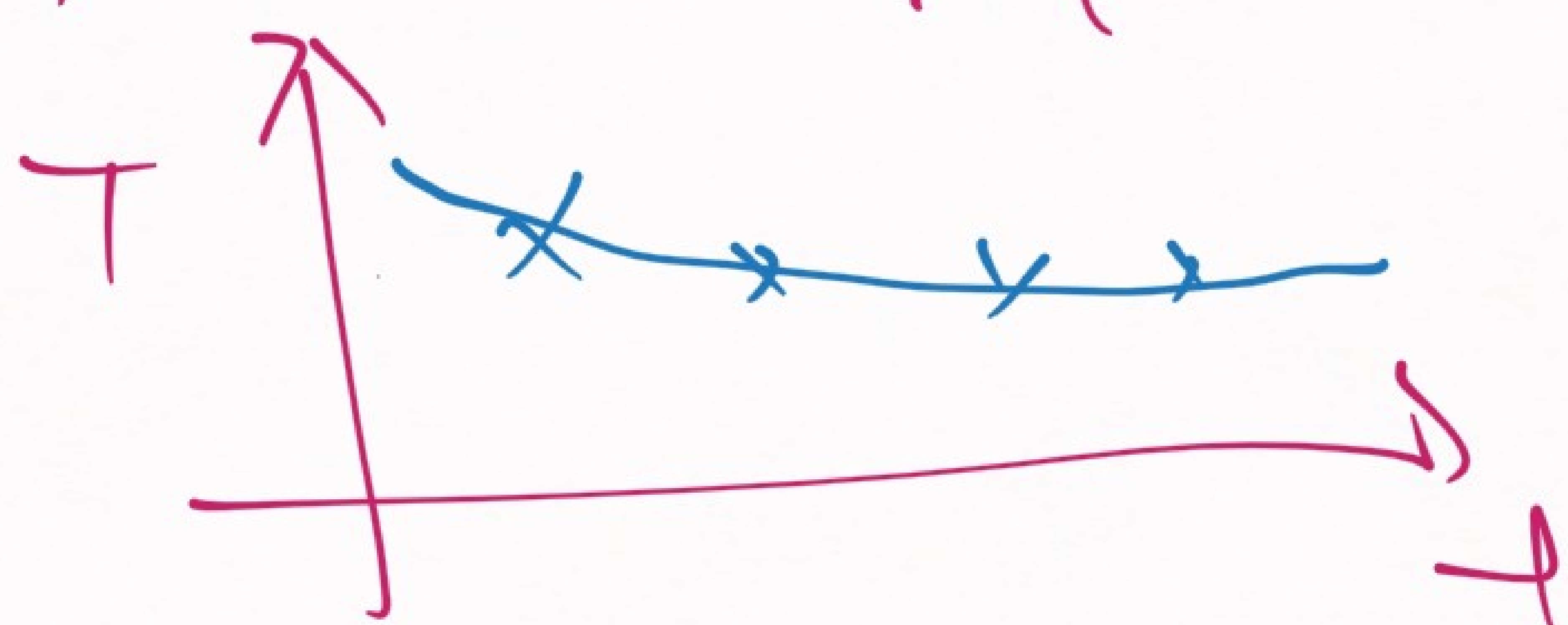
$\rightarrow$ Instruments

(1)



$\rightarrow S_1, \Sigma_1$

(2)



$\rightarrow S_2, \Sigma_2$

$\boxed{\text{RECAP}}$ EFT ALGORITHM

1) Some dynamics (cooling's law / QM / QFT)
2) Some degrees of freedom (surfaces's type / types of particles / ...)
3) Power counting ($x \ll 1$, $\sum c_n x^n$)
4) Write down the theory ($\rightarrow$ LECs)
5) Choose the accuracy (LO, NLO, N^2LO, ...)
6) Fit the LECs to experiment

SEE YOU NEXT MONDAY