NUCLEAR PHYSICS 6

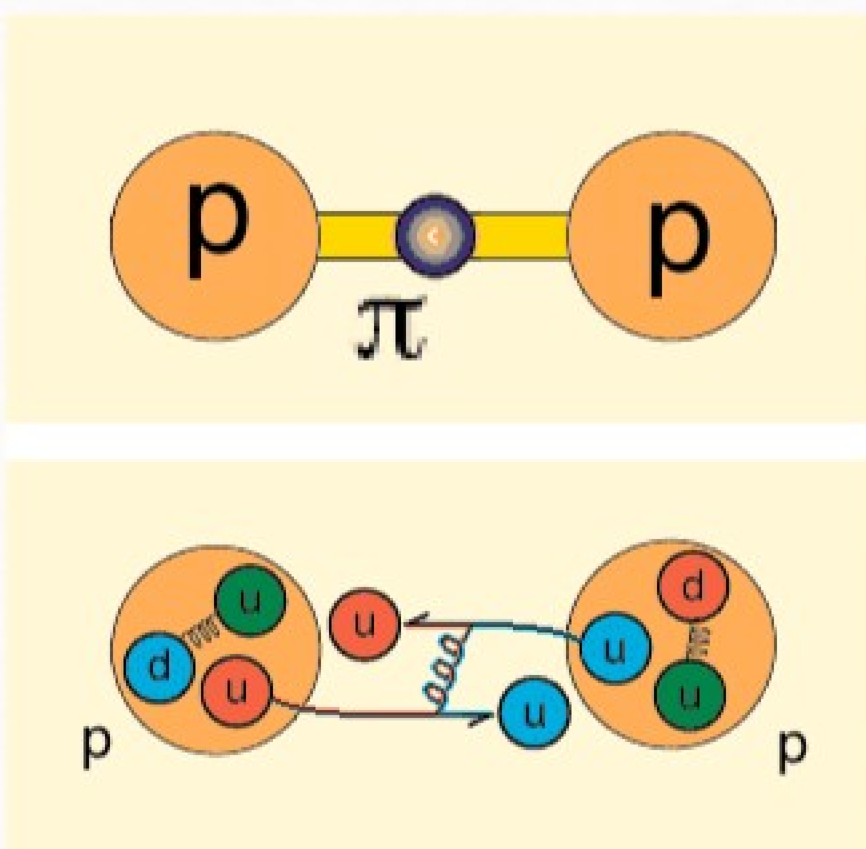
QUANTUM CHROMODYNAMICS

RECAP

nucleons are composite

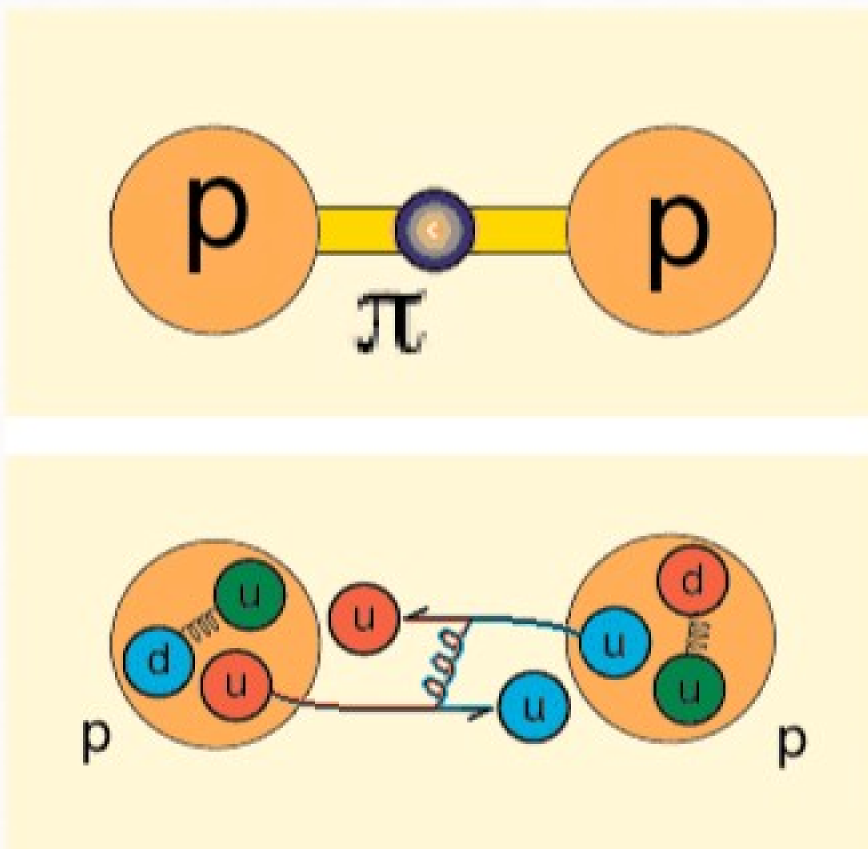
$\Downarrow$

nuclear forces are residual forces



$\rightarrow$ nucleons & pions

Ideally, we want to explain nuclear forces like this



Explain nuclear forces in terms of QCD

Quantum Chromodynamics

→ Fundamental theory of strong interactions

Quarks & Gluons

Quantum Chromodynamics (QCD)

Quantum electrodynamis

(QED)

1) Easier

2) QCD as an extension of QED

more complicated version

$\boxed{\text{QED}}$ → theory of electrons & photons

↳ QFT → we begin w/ a Lagrangian

$$\mathcal{L}_{QED} = \bar{\psi}\,(i\not{D} - m)\,\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

↳ photon field

Dirac fields (electrons)

covariant derivative

Details not important, except for one:

$\rightarrow$ QED as a $\boxed{\text{gauge theory}}$

QCD

1) Dirac field (e^-) $\rightarrow$ $\psi(x)$ (4-components)

$$\mathcal{L} = \bar{\psi}(x)(i\not{\partial} - m)\psi(x) \quad , \quad \not{\partial} = \gamma^\mu \partial_\mu$$

$$\mathcal{L}_{Dirac} = \bar{\psi}(i\not{\partial} - m)\psi \rightarrow \text{[symmetries]}$$

GLOBAL U(1) SYMMETRY :

$$\psi(x) \rightarrow e^{ie\alpha}\psi(x)$$

$$\bar{\psi}(x) \rightarrow e^{-ie\alpha}\bar{\psi}(x)$$

$$\left.\begin{aligned} \bar{\psi}\psi &\rightarrow \bar{\psi}\psi \\ \bar{\psi}\not{\partial}\psi &\rightarrow \bar{\psi}\not{\partial}\psi \end{aligned}\right] \Rightarrow \boxed{\mathcal{L}_{Dirac} \rightarrow \mathcal{L}_{Dirac}}$$

REMINDER OF GROUP THEORY:

$$U(N) = \{ N \times N \text{ matrices} \,/\, U^{\dagger} U = 1 \}$$

$$SU(N) = \{ N \times N \text{ matrices} \,/\, U^{\dagger} U = 1,\ \det U = 1 \}$$

$$U(1) \longrightarrow z^{*} z = 1 \longrightarrow |z|^{2} = 1$$

$$\longrightarrow \boxed{z = e^{i\beta}}$$

GLOBAL U(1): $\mathcal{L}_{Dirac} \to \mathcal{L}_{Dirac}$ ✓

$\downarrow$

LOCAL U(1) $\to$ "local" means that it can be different at each point in space-time

$\psi = \psi(x)$ $\quad \psi(x) \to e^{ie\underbrace{\alpha(x)}_{\text{Local}}} \psi(x)$

$$\left.\begin{aligned}\psi(x) &\to e^{ie\alpha(x)}\psi(x)\\ \bar\psi(x) &\to e^{-ie\alpha(x)}\bar\psi(x)\end{aligned}\right]\ \text{basic rules}$$

$$\bar\psi(x)\psi(x) \longrightarrow \bar\psi(x)\psi(x) \quad \checkmark \quad \text{Local } U(1)$$

$$\left.\begin{aligned}\bar\psi(x)\partial_\mu\psi(x) \longrightarrow\ & \bar\psi(x)\partial_\mu\psi(x)\\ & + ie\,\partial_\mu\alpha\,\bar\psi(x)\psi(x)\end{aligned}\right]\ \times$$

LOCAL U(1) : $\mathcal{L}_{Dirac} \rightarrow \mathcal{L}_{Dirac} - e\,\partial_\mu \alpha\, \bar{\psi}(x)\psi(x)$

(violates local U(1) symmetry)

$\rightarrow$ [BUT WE CAN FIX THIS PROBLEM]

We can modify $\mathcal{L}_{Dirac}$ to be local U(1) symmetric $\rightarrow$ $\boxed{\text{interesting}}$

$\mathcal{L}_{Dirac}$ not local U(1) symmetric

↳ can be modified to be U(1) symmetric

↳ electromagnetism appears as a consequence of this modification

↳

SOLUTION

1) Introduce a new field : $\boxed{A_\mu \to A_\mu + \partial_\mu \alpha}$

2) Define a new type of derivative

$$\cancel{\partial_\mu} \longrightarrow \boxed{D_\mu \equiv \partial_\mu - ieA_\mu}$$

$\boxed{2)} \longrightarrow \underbrace{\bar{\psi} D_\mu \psi}_{=\ \bar{\psi}(\partial_\mu - ieA_\mu)\psi} \longrightarrow \bar{\psi}(\partial_\mu - ieA_\mu - ie\cancel{\partial_\mu \alpha})\psi$

$+ ie\cancel{\partial_\mu \alpha}\,\bar{\psi}\psi$

$=$

$\bar{\psi}(\partial_\mu - ieA_\mu)\psi$

$=$

$\bar{\psi} D_\mu \psi$

$$\boxed{\bar{\psi} D_\mu \psi \to \bar{\psi} D_\mu \psi}$$

$\hookrightarrow$ $\boxed{\text{Local } U(1)}$

$$\mathcal{L}_{QED} = \bar{\psi}(i\not{D} - m)\psi + \underbrace{(\ldots)}_{\text{contain } A_\mu}$$

$$A_\mu \to A_\mu + \partial_\mu \alpha \quad \times$$

$\Downarrow$

build it carefully

(respect local $U(1)$)

$$\boxed{\begin{aligned} &\partial_\nu A_\mu - \partial_\mu A_\nu \\ &\to \partial_\nu A_\mu - \partial_\mu A_\nu \end{aligned}} \quad \checkmark$$

$$+ \partial_\nu \partial_\mu \alpha - \partial_\mu \partial_\nu \alpha$$

$$0$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \longrightarrow \text{bc/ local } U(1)$$

$$\mathcal{L}_{QED} = \bar{\psi}(i\not{D} - m)\psi + \underbrace{(\ldots\ldots)}_{\propto \, \underbrace{F_{\mu\nu}F^{\mu\nu}}_{\text{bc/index contraction}}}$$

$$\mathcal{L}_{QED} = \bar{\psi}(i\not{D} - m)\psi + \lambda\, \underbrace{F_{\mu\nu}F^{\mu\nu}}_{\circledast}$$

How to determine λ ?

$\circledast$ $\rightarrow$ contain a kinetic-term

$$\mathcal{L}_{KG} = \underbrace{\frac{1}{2}\partial_\mu \phi\, \partial^\mu \phi}_{\text{kinetic-term}} - \frac{1}{2} m^2 \phi^2$$

$$\frac{1}{2}\partial_\mu\phi\,\partial_\mu\phi \rightarrow -\frac{1}{2}\phi\,\Box^2\,\phi$$

(integration by part / $\int d^4x\,\mathcal{L}$ does not change)

$$+\frac{1}{2}A_\mu\,\Box^2 A^\mu$$

$$g_{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

Condition $\rightarrow$ $\mathcal{L}_{kin}(A_\mu) = +\frac{1}{2} A_\mu \Box^2 A^\mu$

$\Downarrow$

$$\boxed{\lambda = -1/4}$$

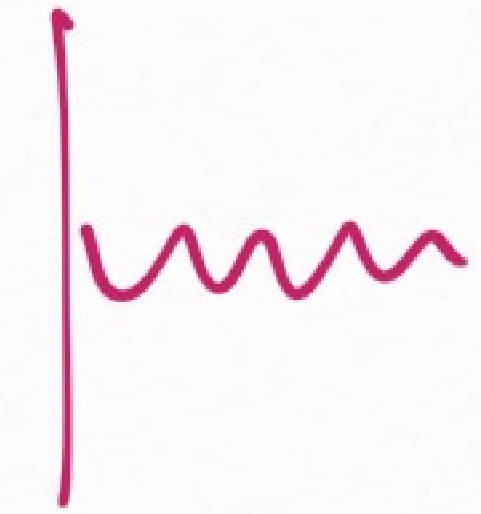
$$\boxed{\mathcal{L}_{QED} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}}$$

IMPOSING LOCAL $U(1)$ ON $\mathcal{L}_{Dirac}$

$\Downarrow$

APPEARANCE OF ELECTROMAGNETISM

(RYDER'S BOOK)

LOCAL U(1) → $\mathcal{L}_{QED}$ →  → $\frac{\alpha}{r}$

(Coulomb potential)

QED → electrons & photons

1 type of charge

QCD → quarks & gluons

3 types of charge

□ → spin-$\frac{1}{2}$ fermions

○ → spin-1 bosons

3 types of charge $\rightarrow$ $\boxed{\text{colors}}$

$\underbrace{\text{Red Green Blue}}_{\text{non-trivial consequences}}$

Quarks $\rightarrow$ 6 types (flavors)

$\boxed{u, d, s, c, b, t}$

$$\mathcal{L}_{quarks} = \sum_{i=1}^{n_f} \overline{\psi}_i (i\not{\partial} - m_i)\psi_i \rightarrow \text{add local symmetry}$$

1) Begin w/ global symmetry

3 charges $\rightarrow$ Lie Group w/ 3 dimensions

2) Go local

$\hookrightarrow \underbrace{SU(3)}_{\text{8 generators}}, \underbrace{U(3)}_{\text{9 generators}}, \ldots$

Between $SU(3)$ & $U(3)$ $\longrightarrow$ $SU(3)$

$\cancel{\det U} \neq 1 \Rightarrow \exists$ of some type of strong charge for hadrons

Hadrons are colorless

$\det U = 1 \Rightarrow \boxed{SU(3)}$

QCD → 3 charges, SU(3), 6 flavors (i.e. types of fermions)

$$\mathcal{L}_{QCD} = \sum_{i=1}^{n_F} \bar{q}_i \left(i\not{D} - m_i \right) q_i - \frac{1}{4} \underbrace{G^a_{\mu\nu} G^{a\mu\nu}}_{\text{gluon fields}}$$

quark fields

gluon fields

$$D_\mu = \partial_\mu - ig \sum_a \frac{\lambda^a}{2} A_\mu^a \quad , \; a = 1, \ldots, 8$$

(generators of $SU(3)$)

$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f_{abc} A_\mu^b A_\nu^c$$

$$\underbrace{\left[\frac{\lambda_b}{2}, \frac{\lambda_c}{2} \right] = i f^{abc} \frac{\lambda^a}{2}}_{\text{Lie Algebra of the group}}$$

$\lambda^a \rightarrow$ Gell-Mann matrices

Basically $\boxed{\text{QCD}}$ is a complicated version of $\boxed{\text{QED}}$

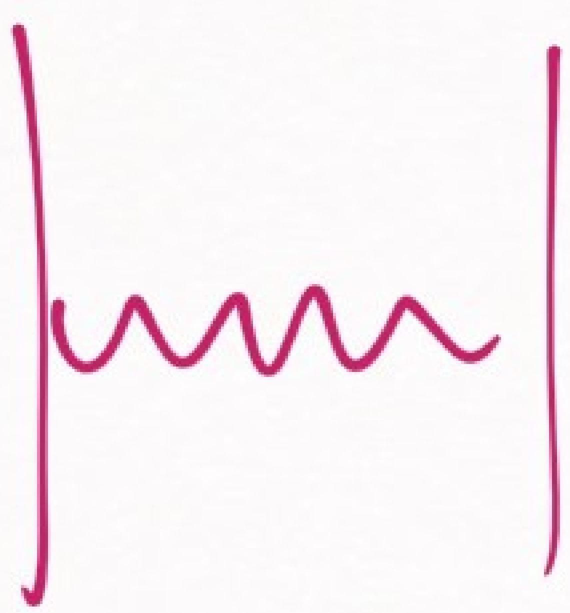
Important difference $\rightarrow$ N_c & N_F

N_c $\rightarrow$ colors (or charges)

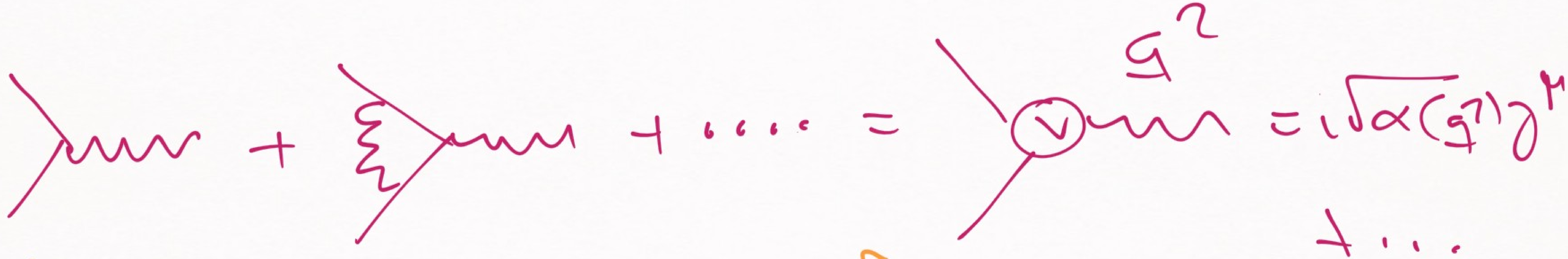
N_F $\rightarrow$ flavors

But ∃ a surprise → $\boxed{\text{asymptotic freedom!}}$

→ FIRST, GO BACK TO QED

 $\rightarrow 4\pi \frac{\alpha}{|q|^2}$ $\Rightarrow$  $= \underbrace{i\sqrt{\alpha}}\,\gamma^\mu$

$\boxed{\text{QFT}} \rightarrow \exists$ quantum corrections



$\forall$ the "stories" contributing to $\rightarrow$ effective strength coupling electrons & photons

$$\left.\begin{array}{l} 4\pi \dfrac{\alpha}{|q|^2} \\[2ex] \alpha \simeq \dfrac{1}{137} \end{array}\right] \xrightarrow[\text{corrections}]{\text{quantum}} 4\pi \frac{\alpha(q^2)}{|q|^2} + \ldots$$

$\alpha = \alpha(q^2) \longrightarrow$ coupling depends on energy

$$\alpha(q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi} \log\left(\frac{q^2}{\mu^2}\right)}$$

RUNNING OF $\alpha(q^2)$

RUNNING $\longrightarrow$ relates α for $\underbrace{q^2 \;\&\; \mu^2}$

two different energy scales

if $q^2 > \mu^2 \Rightarrow \alpha(q^2) > \alpha(\mu^2)$

$\hookrightarrow$ STRENGTH OF E.M. INTERACTIONS GROW W/ ENERGY

LANDAU POLE

$$\alpha(q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi}\log\left(\frac{q^2}{\mu^2}\right)}$$

$\to 0$

$$\alpha(m_e^2) \simeq 1/137$$

$$\Downarrow$$

$$\boxed{\alpha(\Lambda_0^2) \to \infty}$$

LANDAU POLE $\longrightarrow$ $\boxed{\begin{aligned}\Lambda_0 &= m_e \exp\left(\frac{3\pi}{2\alpha(m_e^2)}\right) \\ &\simeq 10^{280}\ \text{MeV}\end{aligned}}$

$\boxed{\Lambda_0 \gg M_{Planck}}$

mostly a theoretical problem

energy at which EM breaks down (i.e. QED stops making sense)

HISTORICAL INTERMEZZO :

LANDAU REALLY IMPORTANT IN USSR

AFTER DISCOVERY OF LANDAU POLE

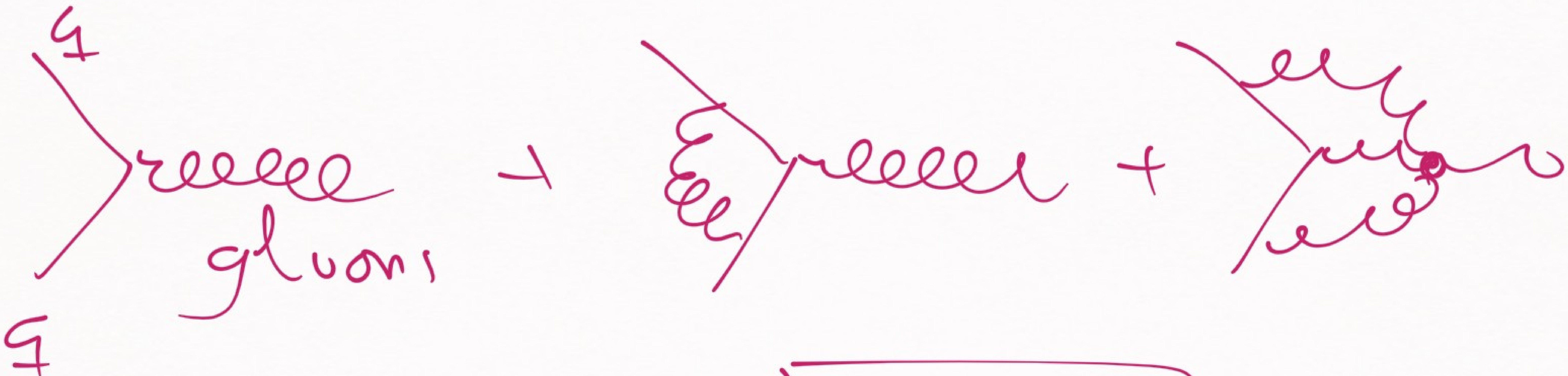
A LOT OF RUSSIAN PHYSICISTS STOPPED STUDYING QFT BECAUSE THEY THOUGHT IT WAS WRONG

THEY FELL BEHIND

LESSON → NOT TAKE COMPLETELY SERIOUSLY SOME INCONSISTENCY IN ANY THEORY

(BUT YOU SHOULD THINK OF THESE INCONSISTENCIES)

[WHAT IS THE EQUIVALENT OF Λ_0 IN QCD?]



$+ \ldots \rightarrow \boxed{\text{RUNNING}}$

RUNNING OF $\alpha_S(q^2)$ IS DIFFERENT THAN
(IN QCD) IN QED

$\boxed{\text{Why?}}$ $\longrightarrow$ gluons carry color
(photons are electrically neutral)

$\rightarrow$ this gluon is $G\bar{B}$

gluons carry color(s) $\rightarrow$ gluons can interact w/ each other

$\boxed{\text{RUNNING}}$

$$\alpha_s(q^2) = \frac{\alpha_s(\mu^2)}{1 + \frac{\alpha_s(\mu^2)}{12\pi}(33 - 2n_f)\log\frac{q^2}{\mu^2}}$$

$$\alpha(q^2) = \frac{\alpha(q^2)}{1 - (\ldots)\log\frac{q^2}{\mu^2}}$$

$\longrightarrow$ sign difference

If $q^2 > \mu^2 \Rightarrow \alpha_S(q^2) < \alpha_S(\mu^2)$

$(\alpha(q^2) > \alpha(\mu^2))$

$\Rightarrow$ Strong force becomes weaker at higher energies

$$\Lambda_{QCD} = \mu \exp\left[-\frac{12\pi}{(33-2n_f)\alpha_s(\mu^2)}\right]$$

$$\alpha_s(q^2) = \frac{\alpha_s(\mu^2)}{1 + (\ldots)\log\frac{q^2}{\mu^2}}$$

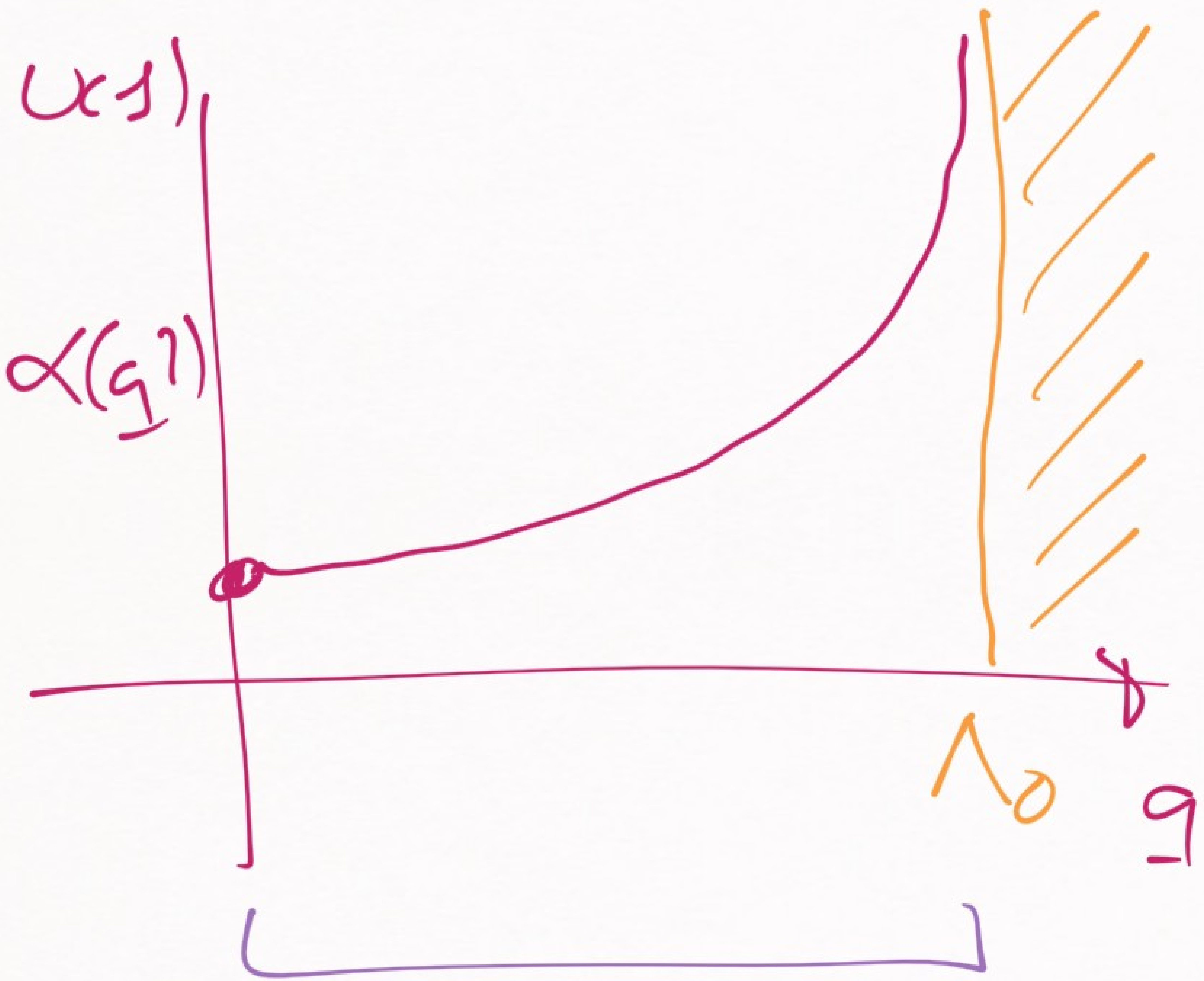
$$\boxed{\alpha_s(\Lambda_{QCD}^2) \to \infty}$$

$$\alpha(\Lambda_0^2) \to \infty$$

$$\Rightarrow \quad \alpha_s(q^2) = \frac{12\pi}{(33 - 2n_f)\log\frac{q^2}{\Lambda_{QCD}^2}}$$

$\Lambda_{QCD} \sim (200 - 300)\ \text{MeV}$

only one parameter



QED is solvable

QCD is solvable

QED $\rightarrow$ $\alpha(Q^2) \rightarrow \infty$ for $Q = \Lambda_0$

QCD $\rightarrow$ $\alpha_s(Q^2) \rightarrow \infty$ for $Q = \Lambda_{QCD}$

QED $\rightarrow$ $\alpha(Q^2)$ grows w/ Q^2

QCD $\rightarrow$ $\alpha_s(Q^2)$ decreases w/ Q^2

$\boxed{\Lambda_{QCD}}$ → Why it is important?

→ only dynamical parameter of $\underline{\underline{QCD}}$

→ QCD also has other six parameters

(the quark masses)

$$\alpha_s(Q^2) = \frac{12\pi}{(33-2n_f)\log\frac{Q^2}{\Lambda_{QCD}}}$$

[Λ_{QCD} is the natural scale of QCD]

$$\underbrace{m_u, m_d, m_s}_{\text{light quarks}} \ll \Lambda_{QCD} \ll \underbrace{m_c, m_b, m_t}_{\text{heavy quarks}}$$

SCALE SEPARATION

In practice we can make these approximations:

1) $\frac{m_u}{\Lambda_{QCD}}, \frac{m_d}{\Lambda_{QCD}}, \frac{m_s}{\Lambda_{QCD}} \to 0$ (CHIRAL SYMMETRY)

2) $\frac{\Lambda_{QCD}}{m_c}, \frac{\Lambda_{QCD}}{m_b}, \frac{\Lambda_{QCD}}{m_t} \to 0$ (HEAVY QUARK SYMMETRY)

CAVEAT: t quark is super short live $\to$ we just ignore it

Λ_{QCD} is the natural scale : $\boxed{\Lambda_{QCD} \gg m_u, m_d, m_s}$

1) ρ-meson mass $(u\bar{d})$ $\boxed{m_\rho \simeq 0.77 \text{ GeV}}$

$m_\rho \neq m_u + m_d \rightarrow \boxed{m_\rho \simeq 2\Lambda_{QCD} \simeq 0.6 \text{ GeV}}$

2) proton mass (uud) $\boxed{m_p = 0.940 \text{ GeV}}$

$m_p \simeq 3\Lambda_{QCD} \simeq \boxed{0.9 \text{ GeV}}$

3) D-meson ($c\bar{u}$) $m_c \simeq 1.2$ GeV

$$m(D) \simeq m_c + 2\Lambda_{QCD} \simeq 1.8 \text{ GeV}$$

(1.87 GeV)

4) η, J/ψ ($c\bar{c}$ states)

$$m(J/\psi) \simeq 2m_c + 2\Lambda_{QCD} \simeq 2.4 + 0.6 \text{ GeV}$$

$$\simeq 3.0 \text{ GeV}$$

[Λ_{QCD} is really the natural scale of QCD]

↳ what happens w/ the pion?

5) π-mass $(u\bar{d}) \rightarrow m(\pi) \simeq 2\Lambda_{QCD} \simeq 0.6$ GeV

Reality $\rightarrow m(\pi) \simeq 0{,}14$ GeV

$$\boxed{\frac{m_\pi}{2\Lambda_{QCD}} \simeq \frac{1}{6} - \frac{1}{5}} \rightarrow \exists \text{ FINE TUNING}$$

π mass $\to$ fine-tuning $\to$ $\begin{cases} \text{coincidence} & \times \\ \text{conspiracy} & \checkmark \end{cases}$

$$\boxed{\text{CHIRAL SYMMETRY}}$$

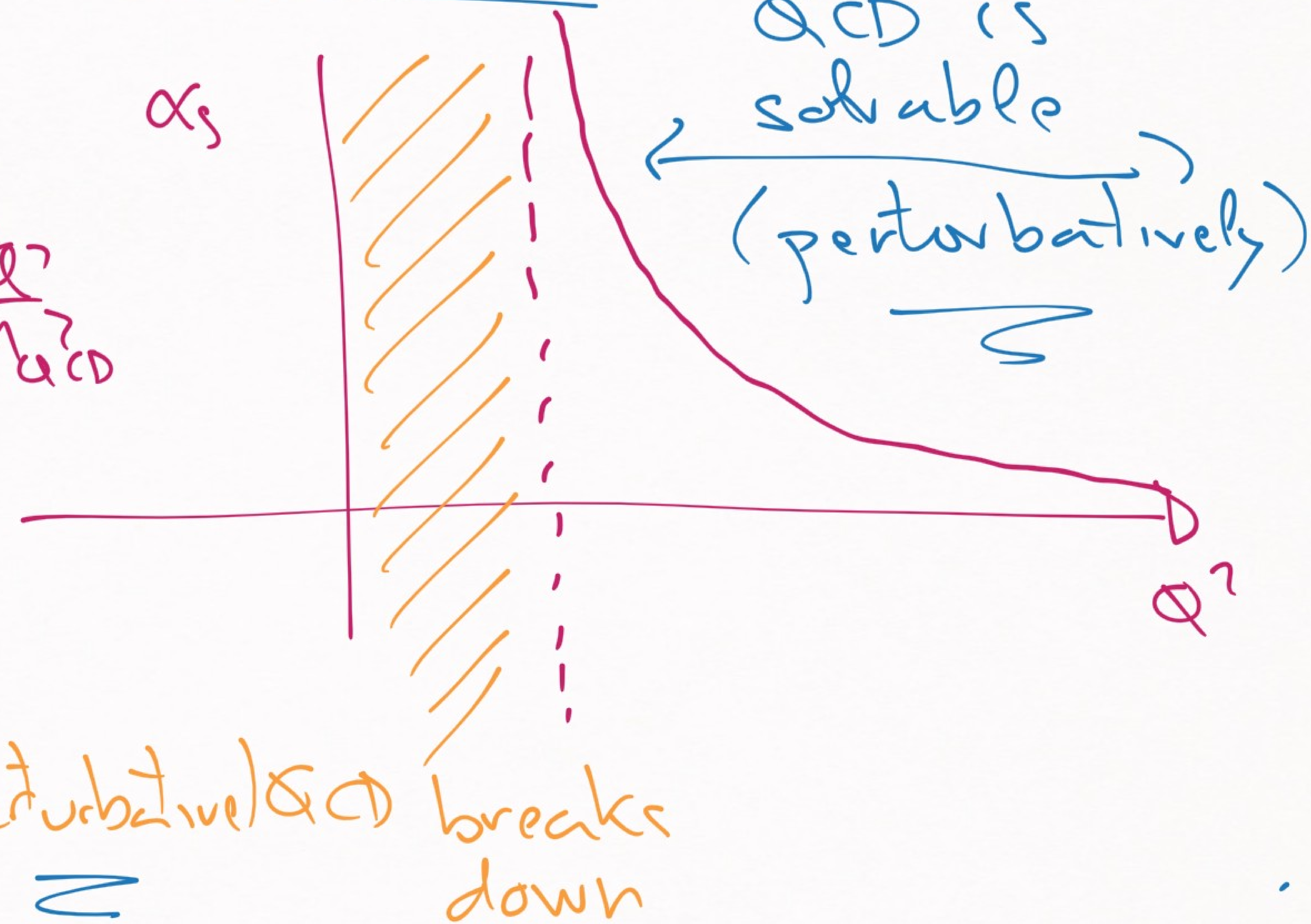
CONSEQUENCE OF

$$\left[\begin{array}{l} \dfrac{m_u}{\Lambda_{QCD}} \to 0 \\ \dfrac{m_d}{\Lambda_{QCD}} \to 0 \end{array}\right]$$

$$\frac{m_s}{\Lambda_{QCD}} \to 0 \quad \left(\text{in this case, } \frac{m_s}{\Lambda_{QCD}} \sim \frac{1}{3} - \frac{1}{2}\right)$$

WHEN IS QCD SOLVABLE ?

$$\alpha_s(Q^2) = \frac{12\pi}{(33-2n_f)\log\frac{Q^2}{\Lambda^2_{QCD}}}$$



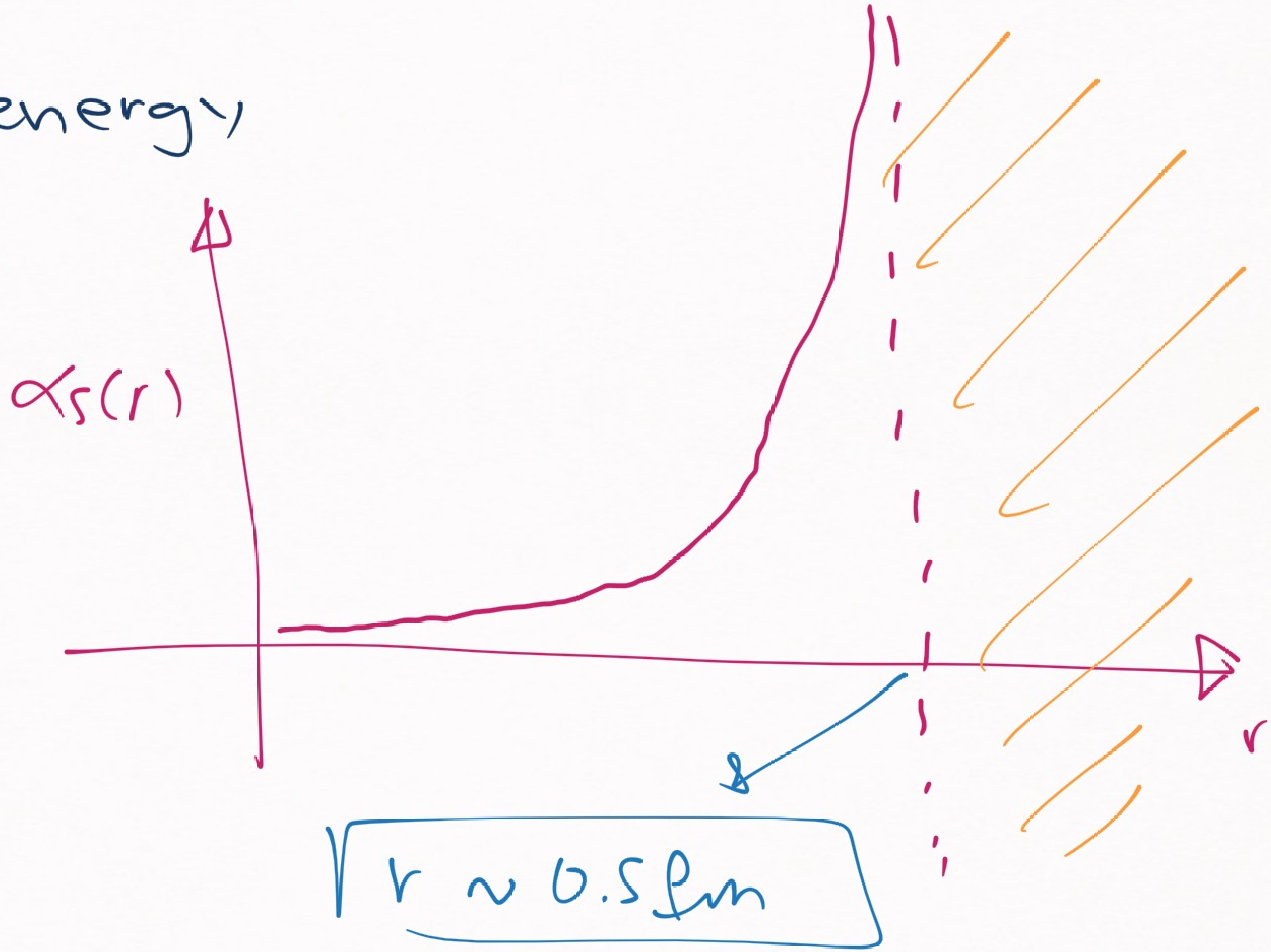
QFT book → Feynman diagrams to solve QFT

Perturbative technique

only valid for weak coupling

Before → energy

Distance →



If $\underbrace{r > 0.5\ \text{fm}}_{\text{approximate}}$ $\Rightarrow$ $\boxed{\text{WE CANNOT SOLVE QCD}}$

$\boxed{\text{NUCLEAR PHYSICS}}$ $\rightarrow$ most things happen at $r > 1\ \text{fm}$

FUNDAMENTAL PROBLEM OF NUCLEAR PHYSIC

$\boxed{\text{How do we derive nuclear physics from quantum chromodynamics?}}$

↳ BAD NEWS: WE CAN'T DO THIS DIRECTLY

We can't go and write Feynman diagrams and find what QCD predicts for the deuteron ...

→ INDIRECT METHODS

(NEXT LESSON)

Two main solutions:

1) Brute force methods → lattice QCD

2) Indirect method → effective field theory

$\boxed{}$