NUCLEAR PHYSICS 5

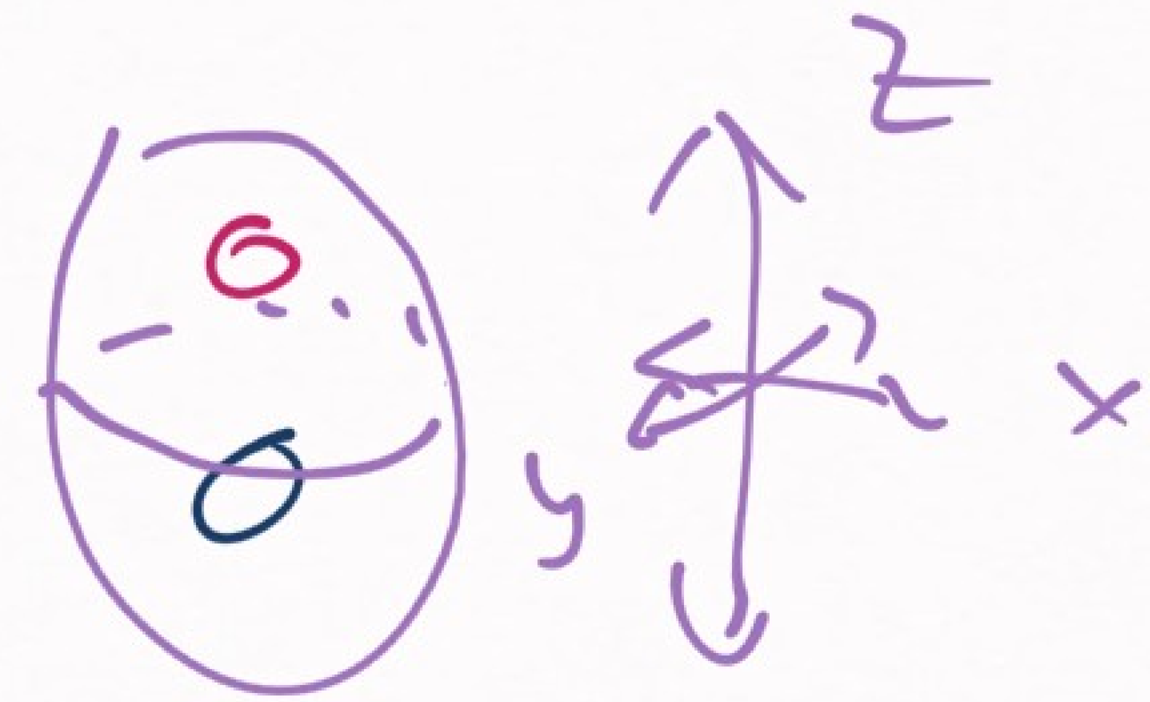
MORE ABOUT THE ORIGIN OF NUCLEAR FORCES

RECAP → Properties of the nuclear force

1) Short-ranged (finite-ranged)
2) Attractive at intermediate distances (1-2 fm)
3) Repulsive at short distances (0.5 fm)
4) Does not distinguish between proton & neutrons

5) Not central ← continue here

Not-central →



Deuteron has $Q_d > 0$

Quadrupolar moment:

$$Q_{ij} = \int d^3\vec{r}\, \underbrace{\rho(\vec{r})}_{\text{charge density}} (3 r_i r_j - \delta_{ij} r^2)$$

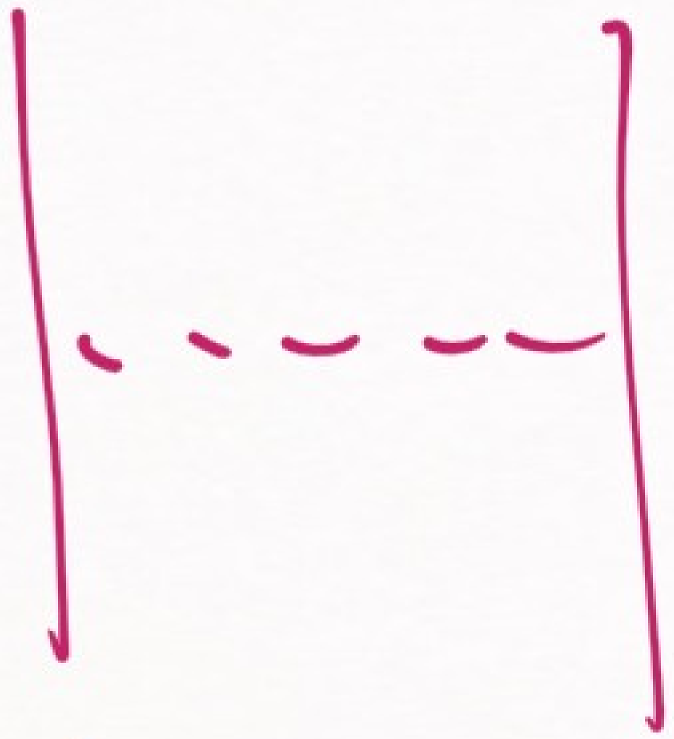
$$\boxed{Q = Q_{33}}$$

$Q_d = 0.286\ \mathrm{fm}^2\ (\times e)$

PROBLEM : $\boxed{\text{HOW TO EXPLAIN Q8 ?}}$

$\boxed{\text{Yukawa's idea}}$ → nuclear forces come from exchanging some meson (boson)

$\boxed{\text{QFT}}$ → only need to know a bit (any textbook will do)

 → $V(\vec{q})$ (non-relativistic limit)

↳ depends on $\underbrace{\text{type of exchanged boson}}_{J^P \text{ (spin \& parity)}}$

Yukawa's original proposal $\rightarrow J^P = 0^+$

(scalar)

Scalar: $\phi(x)$

$\mathcal{P}(\vec{x} \to -\vec{x})$: $\phi \longrightarrow \phi$

$\boxed{\text{QFT}} \longrightarrow \mathcal{L}_{int} = g \bar{\psi}_N \psi_N \phi$ (Peskin Schröder example)

(nucleon $\to$ Dirac fermion)

$$ig \left| \overset{\cdots\cdots}{\frac{1}{q^2+m^2}} \right| ig \longrightarrow V(\vec{q}) = -\frac{g^2}{q^2+m^2}$$

$$\Big\downarrow \text{FT}$$

$$\boxed{V(\vec{r}) = -\frac{g^2}{4\pi}\frac{e^{-mr}}{r}}$$

$\longrightarrow$ Spherically symmetric

$\downarrow$

PROBLEM

$\longleftarrow$

$$\boxed{Q_d = 0}$$

$\rightarrow \boxed{J^P = 0^+} \rightarrow$ no quadropolar moment for the deuteron

$\downarrow$

Try something else

Vector boson $\rightarrow \boxed{J^P = 1^-}$ spin-1 (3 components)

$A_\mu \rightarrow$ Lorentz index $\rightarrow \vec{A}$ $\quad \vec{A} \rightarrow -\vec{A}$

(under parity)

If pion is $J^P = 1^-$, the potential will be ...?

$$\mathcal{L}_{int} = g\,\bar{\psi}_N \gamma^\mu \psi_N V_\mu \longrightarrow \text{electric-type}$$

$$+ \frac{f}{2M}\bar{\psi}_N [\gamma^\mu, \gamma^\nu] \psi_N (\partial_\nu V_\mu - \partial_\mu V_\nu)$$

general structure

magnetic-type term

Where does this electric-/magnetic-thing comes from?

→ analogy w/ electromagnetism

photon → spin-1 vector boson

$\underline{\underline{e\vec{E}}} + \underline{\underline{\vec{\mu}\cdot\vec{B}}}$

$\vec{E} = -\partial_0 \vec{A} + \vec{\nabla} A_0$

$\vec{B} = (\vec{\nabla} \wedge \vec{A})$

$\rightarrow$ nucleons $1/2$

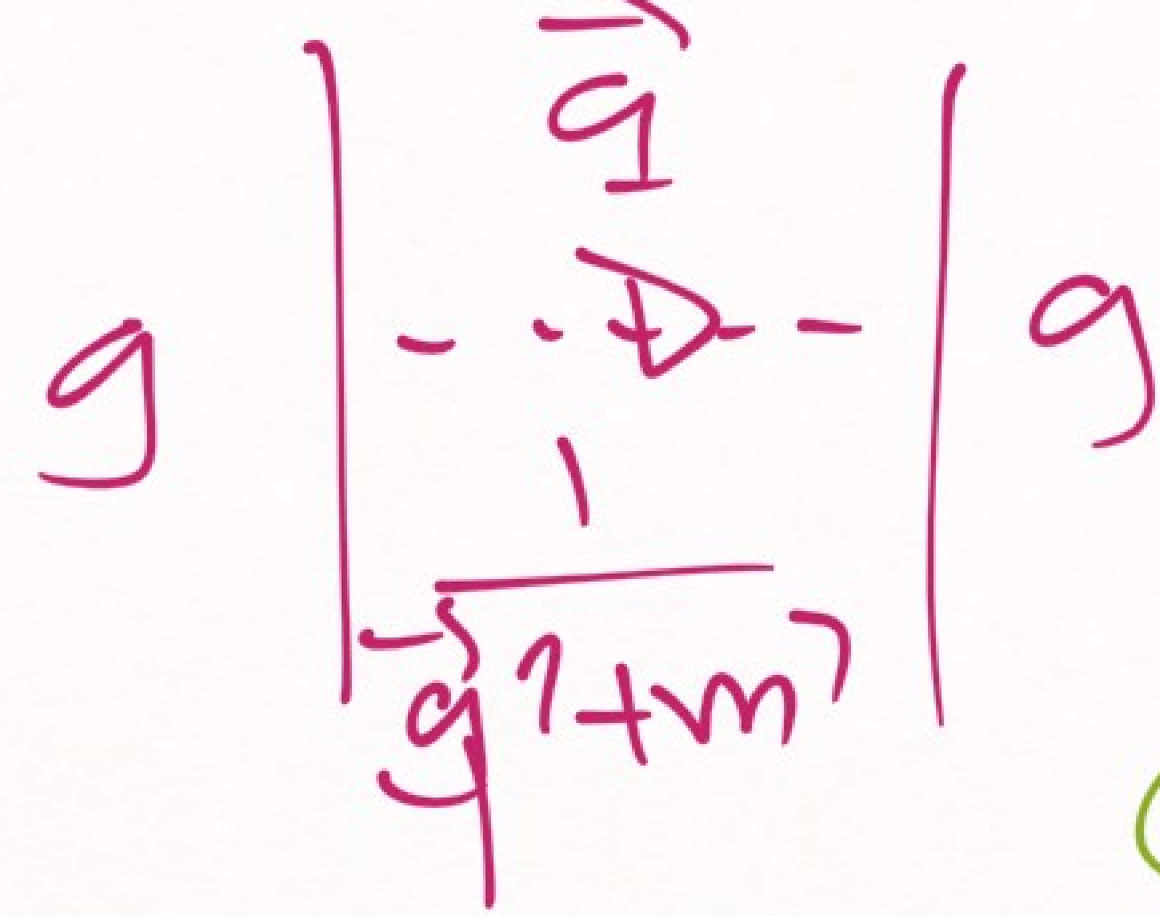
vector meson 1

$$\left.\begin{array}{l}\text{nucleons } 1/2 \\ \text{vector meson } 1\end{array}\right] \quad \frac{1}{2} \otimes 1 = \underbrace{\frac{1}{2} \oplus \frac{3}{2}}$$

two-terms

for the interaction

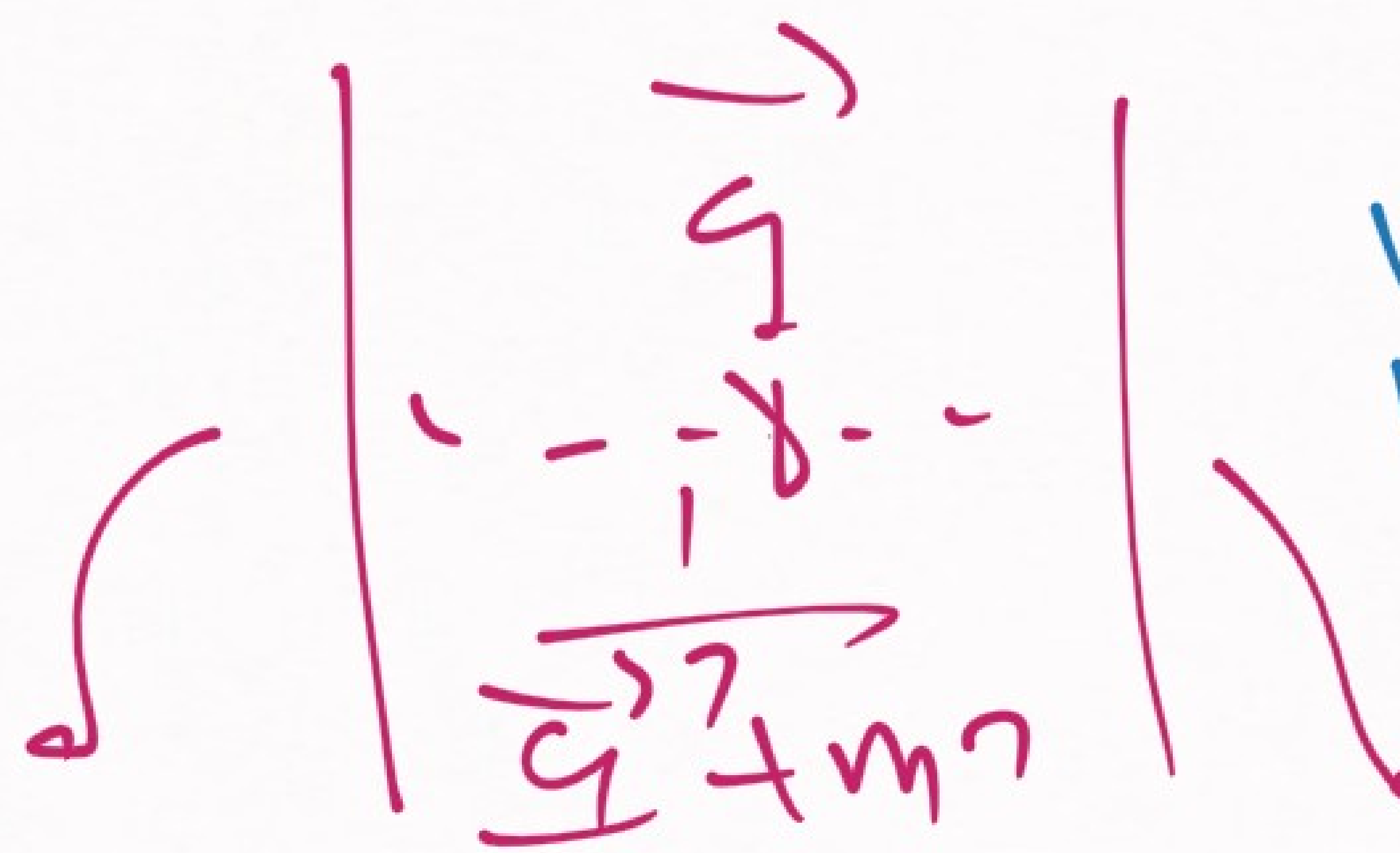
E0 piece
(electric)



$$V_{E_0}(\vec{q}) = \frac{g^2}{\vec{q}^2+m^2}$$

(repulsive Yukawa)

M1 piece
(magnetic)



$$V_{M_1}(\vec{q}) = \frac{f^2}{4M^2}\,\frac{(\vec{\sigma}_1\wedge\vec{q})(\vec{\sigma}_2\wedge\vec{q})}{\vec{q}^2+m^2}$$

$$i\frac{f}{2M}(\vec{\sigma}_1\wedge\vec{q}) \qquad i\frac{f}{2M}(\vec{\sigma}_2\wedge(-\vec{q}))$$

$$V_{E1}(\vec{r}) = \frac{g^2}{4\pi}\frac{e^{-mr}}{r}$$

$$V_{M1}(\vec{r}) = \int \frac{d^3\vec{q}}{(2\pi)^3}\, e^{-i\vec{q}\cdot\vec{r}}\, V_{M1}(\vec{q})$$

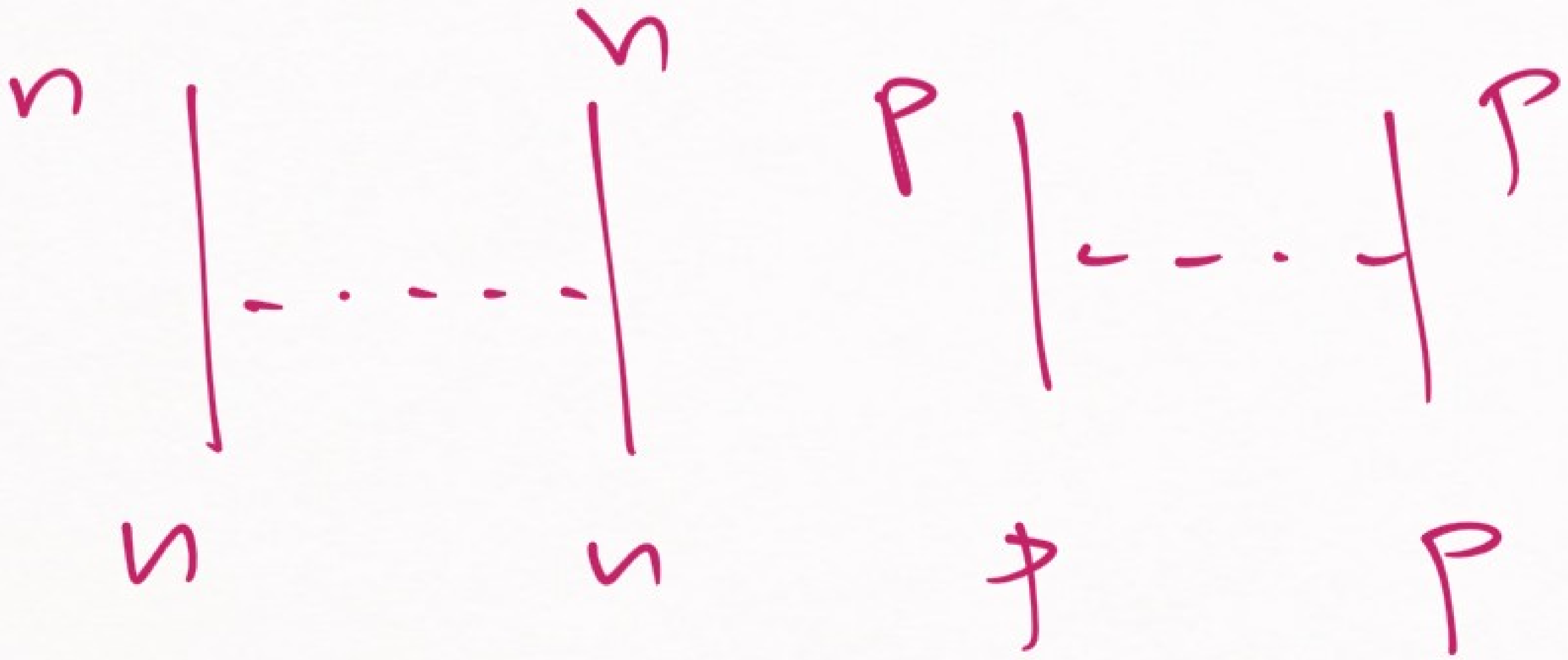
$$= \left(\frac{f}{2M}\right)^2 \Big[\frac{2}{3}\,\vec{\sigma}_1\cdot\vec{\sigma}_2\,\frac{e^{-mr}}{4\pi r} - \frac{1}{3}\underbrace{\left(3\,\vec{\sigma}_1\cdot\hat{r}\;\vec{\sigma}_2\cdot\hat{r} - \vec{\sigma}_1\cdot\vec{\sigma}_2\right)}_{\text{new : tensor force}}\frac{e^{-mr}}{4\pi r}(\ldots)\Big]$$

$\rightarrow \left[S_{12}(\hat{r}) = 3\,\vec{\sigma}_1 \cdot \hat{r}\,\vec{\sigma}_2 \cdot \hat{r} - \vec{\sigma}_1 \cdot \vec{\sigma}_2 \right]$

$\hookrightarrow$ generates a quadrupolar moment

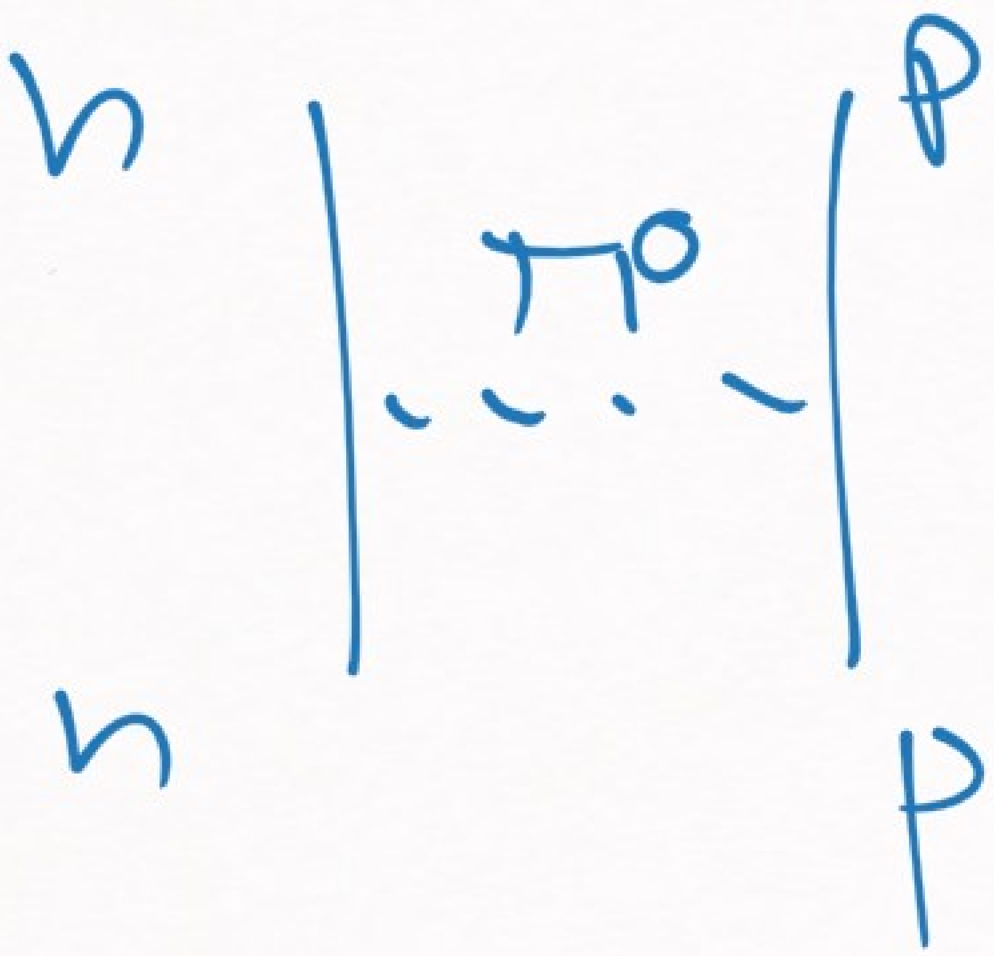
DOES A VECTOR MESON WORKS?

$\rightarrow \boxed{Q_d \neq 0} \rightarrow \boxed{Q_d < 0}$



$\rightarrow$ WHAT WE HAVE CALCULATED BEFORE

COMPLICATION: π^0, π^+, π^- (decleran: up)



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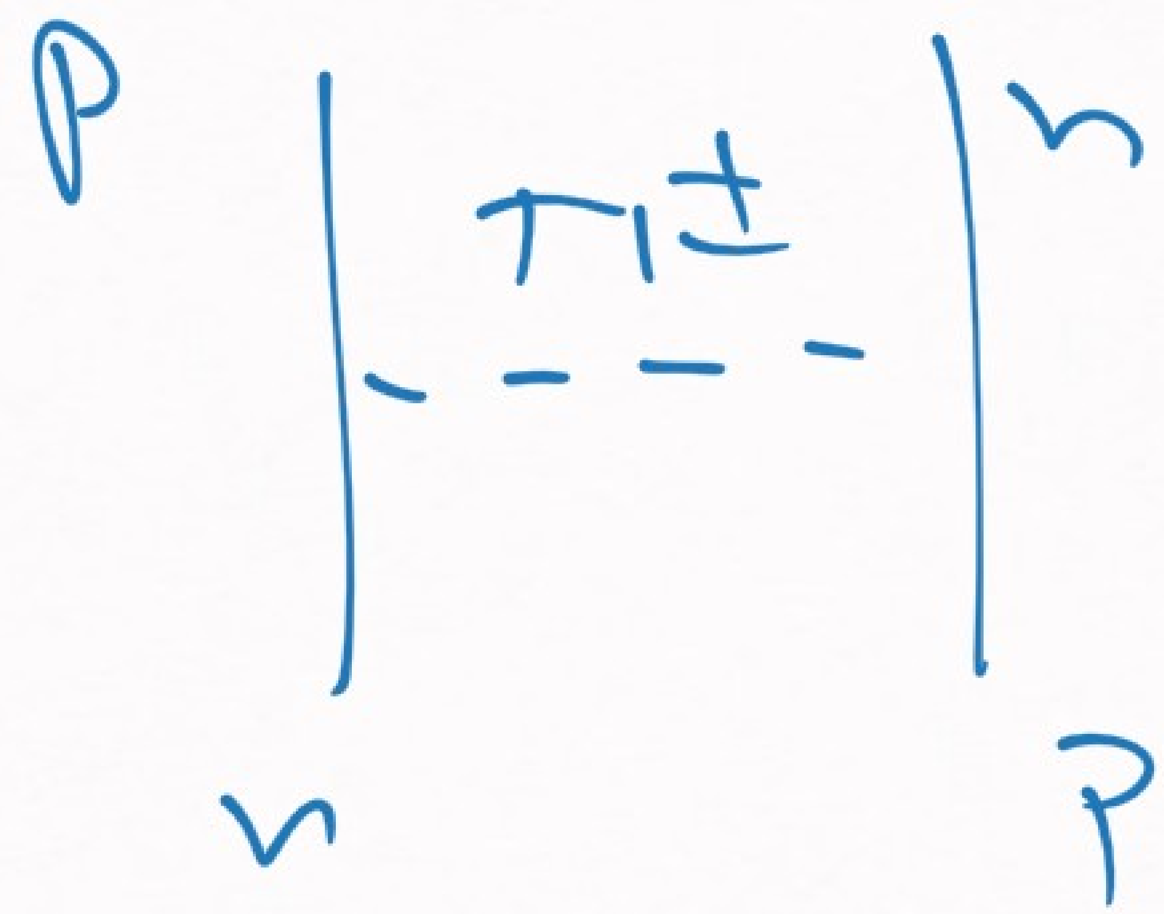
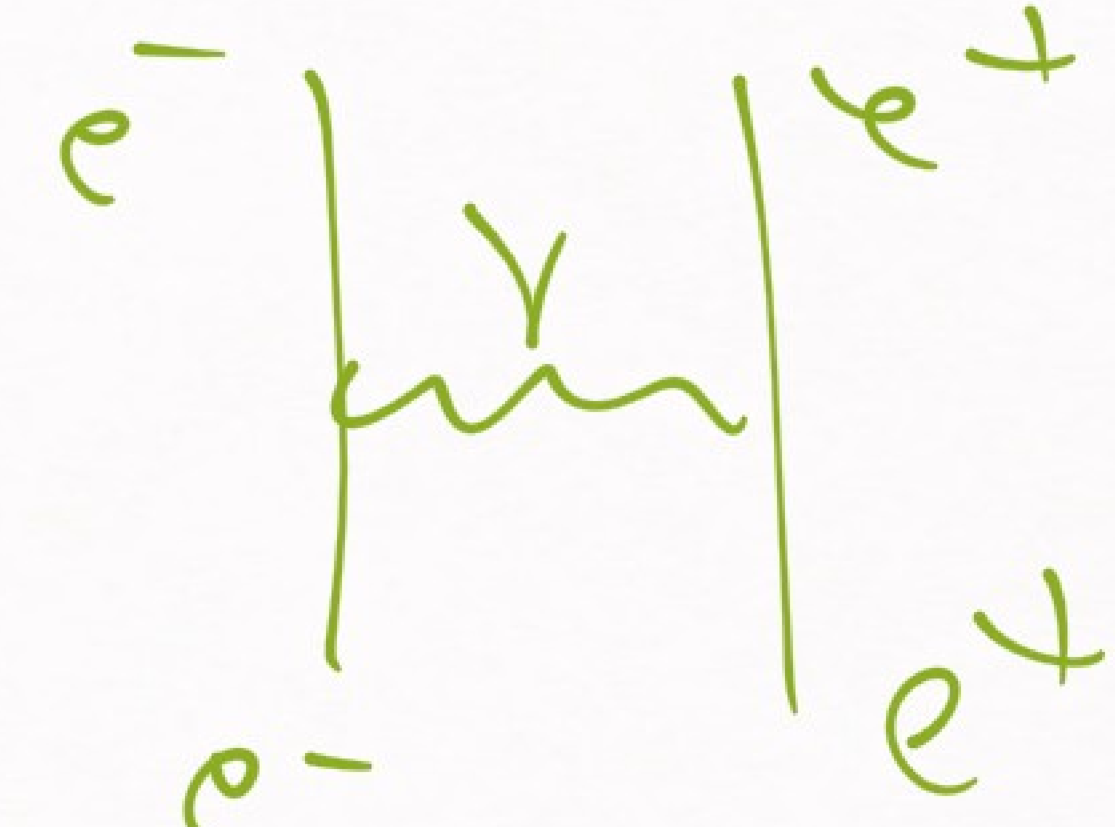




$$V_d(\vec{r}) = -3\,[V_{EO}(\vec{r}) + V_M(\vec{r})]$$

isospin factor

generalizing spin

$$|p\rangle = |\tfrac{1}{2}\ \tfrac{1}{2}\rangle_I$$

$$|n\rangle = |\tfrac{1}{2}\ {-\tfrac{1}{2}}\rangle_I$$

$$|N\rangle = |\tfrac{1}{2}\ m_I\rangle_I$$

$$\Rightarrow \boxed{Q_d < 0} \longrightarrow \text{exercise}$$

$$V_d(\vec{r}) = -3\Big[\underbrace{\frac{g^2}{4\pi}\frac{e^{-mr}}{r} + \frac{2}{3}\vec{\sigma}_1\cdot\vec{\sigma}_2\frac{e^{-mr}}{4\pi r}}_{\text{same sign for deuteron}}$$

$$-\frac{1}{3}S_{12}(\hat{r})\frac{e^{-mr}}{4\pi r}(\cdot - \cdot)\Big]$$

RECAP

$0^+ \longrightarrow Q_d = 0$

$1^- \longrightarrow Q_d < 0$

$\longrightarrow$ discarded (don't work)

$\longrightarrow$ different type of particle

pseudovector $\nearrow$ vector

Classical physics $\longrightarrow$ $\vec{r}, \vec{p} \longrightarrow -\vec{r}, -\vec{p}$

pseudovector $\longleftarrow$ $(\vec{r} \wedge \vec{p}) \longrightarrow (\vec{r} \wedge \vec{p})$

Pseudovector → $J^P = 1^+$ (axial vector)

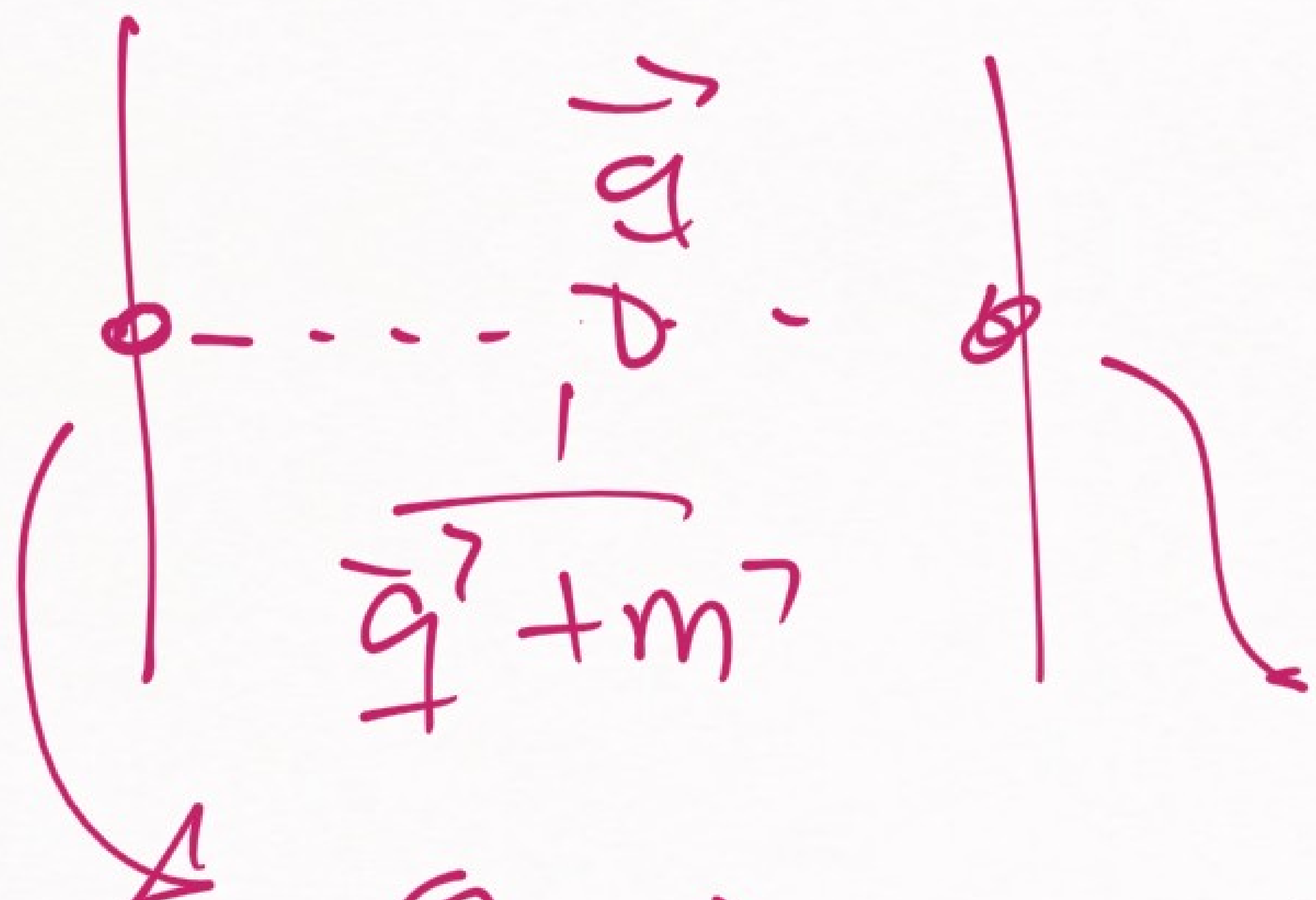
More simple → Pseudoscalar

$$\boxed{J^P = 0^-}$$

ϕ_P (no index), P: $\vec{r} \longrightarrow -\vec{r}$

$\phi_P \rightarrow -\phi_P$

$$\mathcal{L}_{int} = g \bar{\psi}_N \gamma^5 \psi_N \phi_\pi$$



$$V_\pi(\vec{q}) = -\left(\frac{g}{2M_N}\right)^2 \frac{\vec{\sigma}_1 \cdot \vec{q}\; \vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^2 + m^2}$$

$$\frac{g}{2M_N} \vec{\sigma}_1 \cdot \vec{q} \qquad \frac{g}{2M} \vec{\sigma}_2 \cdot (-\vec{q})$$

$$V_P(\vec{r}) = \left(\frac{g}{2M}\right)^2 \left[\vec{\sigma}_1 \cdot \vec{\sigma}_2 \frac{e^{-mr}}{4\pi r} + S_{12}(\hat{r}) \frac{e^{-mr}}{4\pi r} (\dots) \right]$$

$$V_d(\vec{r}) = \underbrace{-3}_{\text{isospin factor}} V_P(\vec{r})$$

compare it w/ vector boson

$$\underset{\text{" + "}}{(\dots)} \frac{e^{-mr}}{4\pi r} - \frac{1}{3} S_{12}(\hat{r}) \frac{e^{-mr}}{4\pi r} (\dots)$$

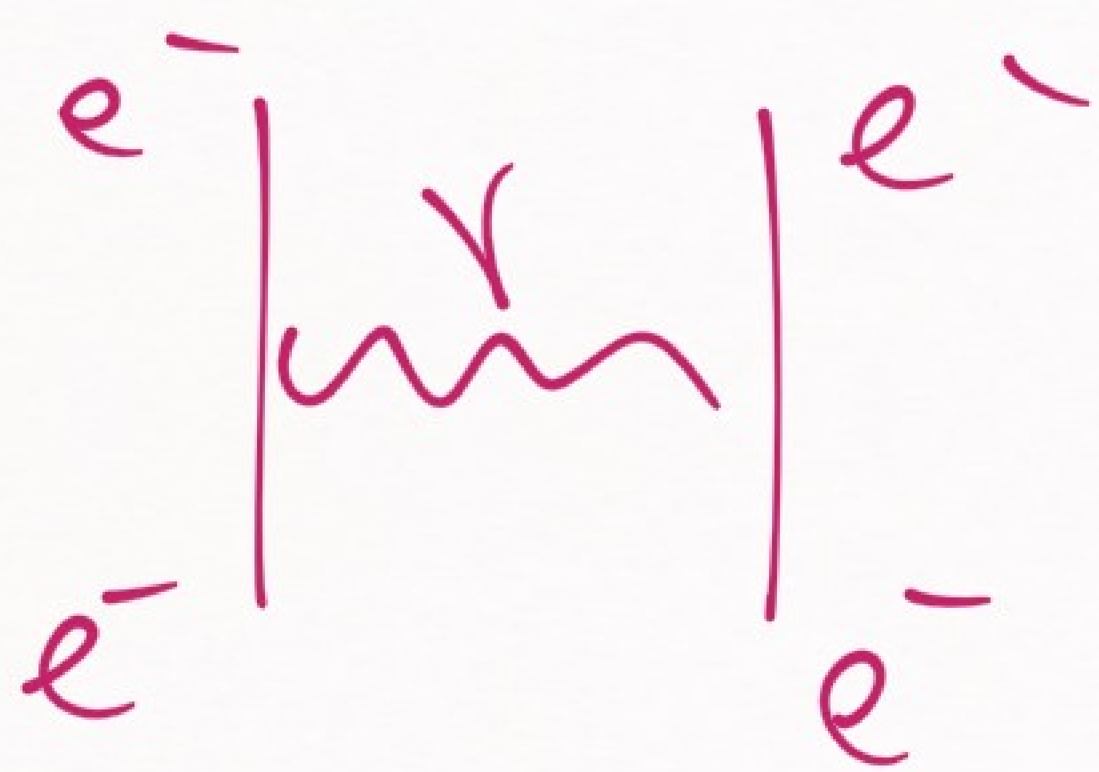
$$\boxed{J^P = 0^-} \longrightarrow \boxed{Q_d > 0} \quad \text{(excersise)}$$

$$\downarrow$$

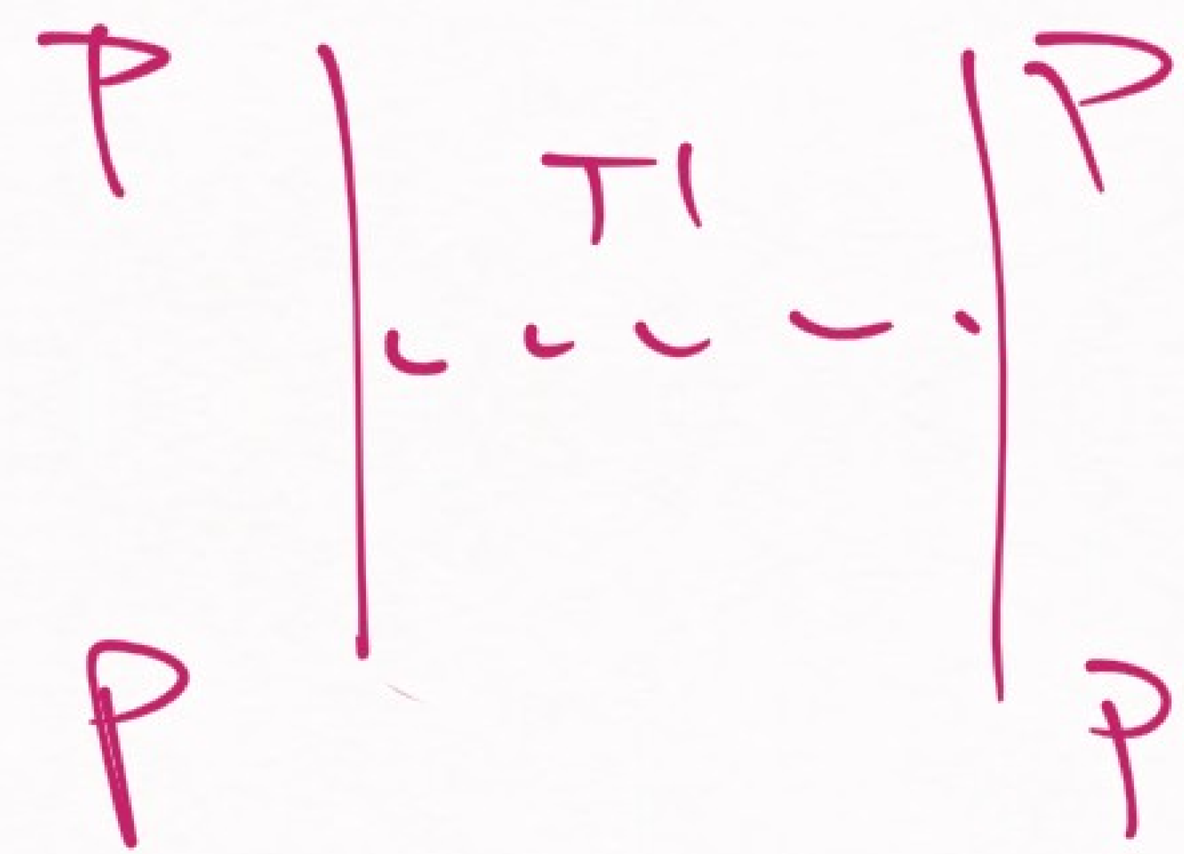
$$\boxed{\text{CORRECT}}$$

$$\text{DEUTERON } Q_d \Rightarrow \boxed{\text{PION IS } J^P = 0^-}$$

A FEW RESULTS FROM ISOSPIN:

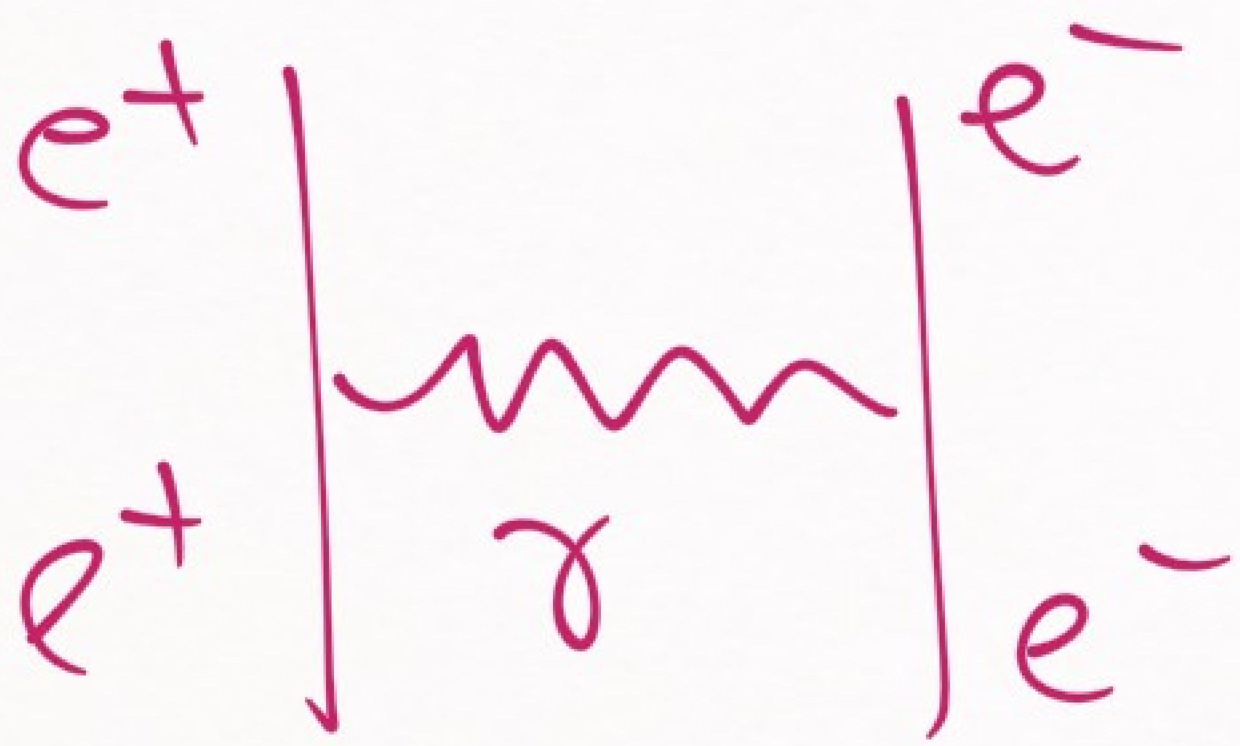


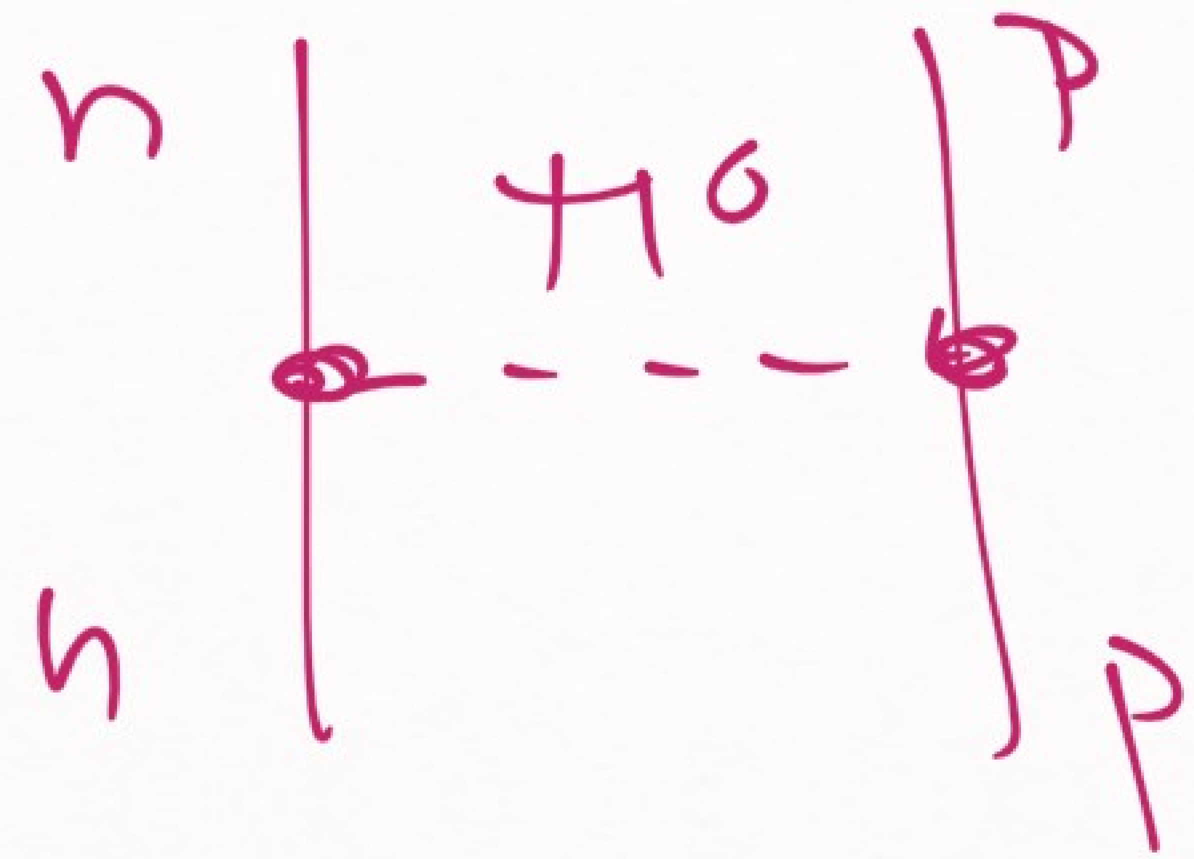
$\simeq$

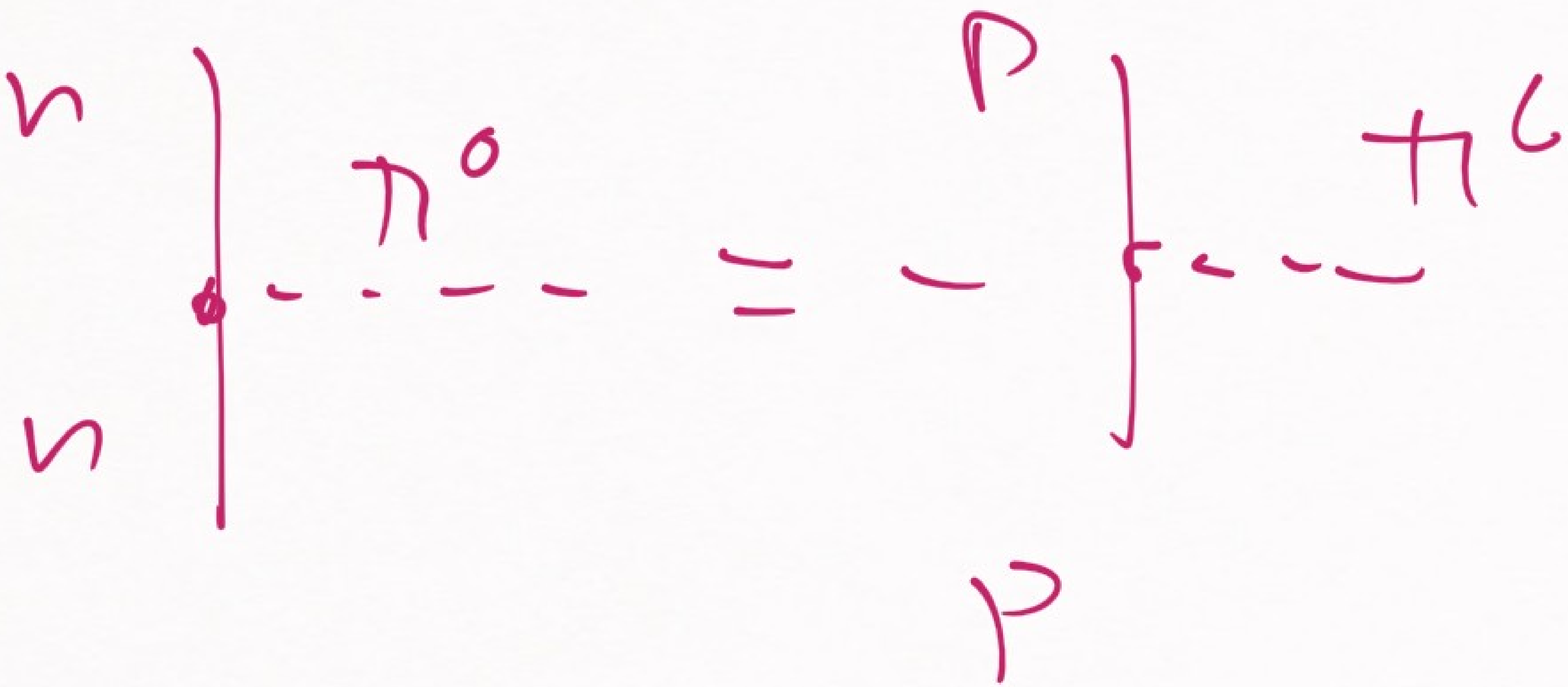


$\longrightarrow$ same rules

nuclear force $\longrightarrow$ (n, p) $\longrightarrow$ $|N\rangle = |\tfrac{1}{2}\, \underline{\underline{m_I}}\rangle_I$







on isospin space

$\pi^0 \sim \boxed{\tau^3}$

$\tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

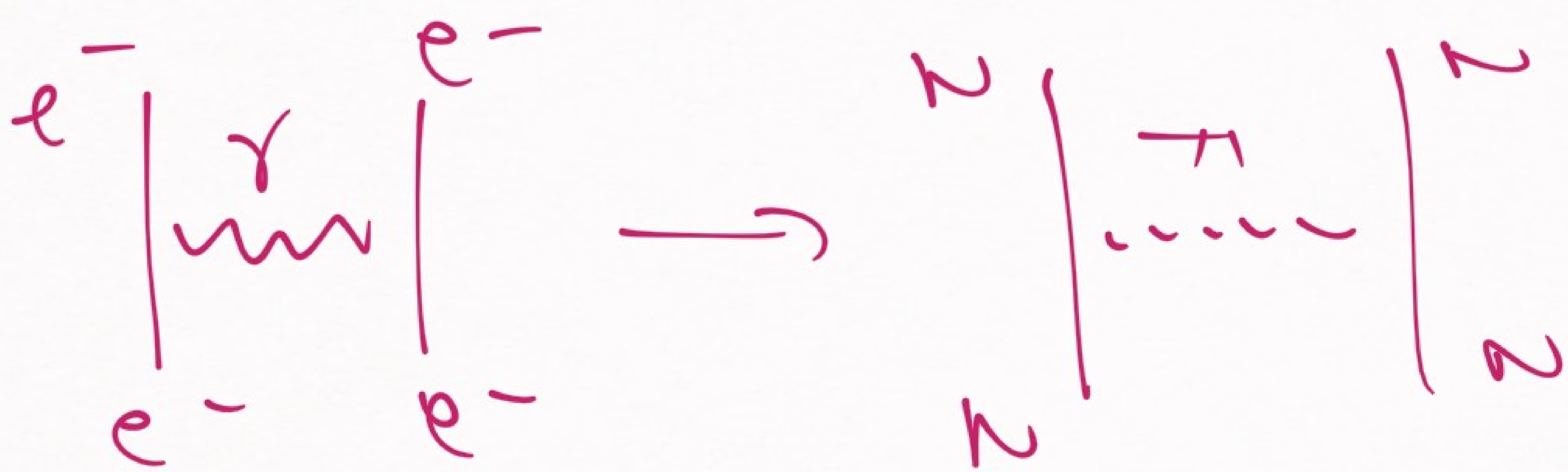
$$\begin{array}{c} p \\ \\ n \end{array}\left|\; \pi^{\pm} \;\right| \begin{array}{c} n \\ \\ p \end{array} \qquad \begin{array}{c} p \\ \\ n \end{array}\left|\; \pi^{-} \;=\; \pm\sqrt{2} \;\right| \begin{array}{c} p \\ \\ p \end{array} \pi^{0}$$

$$\begin{array}{c} n \\ \\ n \end{array}\left|\; \cdots \;\right| \begin{array}{c} p \\ \\ p \end{array} , \quad \begin{array}{c} p \\ \\ n \end{array}\left|\; \cdots \;\right| \begin{array}{c} n \\ \\ p \end{array} \sim \begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix}$$

$$-1 \quad +1, \quad -3$$

→ $\boxed{\text{MORE ABOUT THE ORIGIN OF NUCLEAR FORCES}}$

RECAP: YUKAWA'S IDEA → $\boxed{\text{PION HAS } J^P = 0^-}$



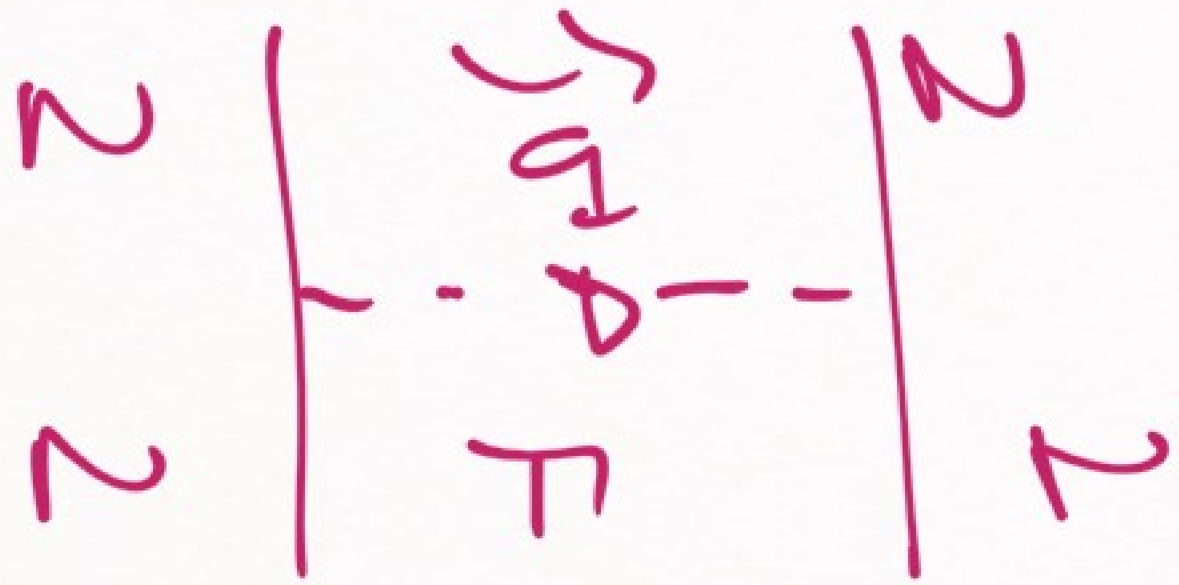
PHYSICS → $\boxed{\text{TRYING TO IMPROVE OUR EXPLANATIONS}}$

IMPROVEMENTS OVER YUKAWA :

1) more pions
2) more mesons

→ consequences

1) MORE PIONS

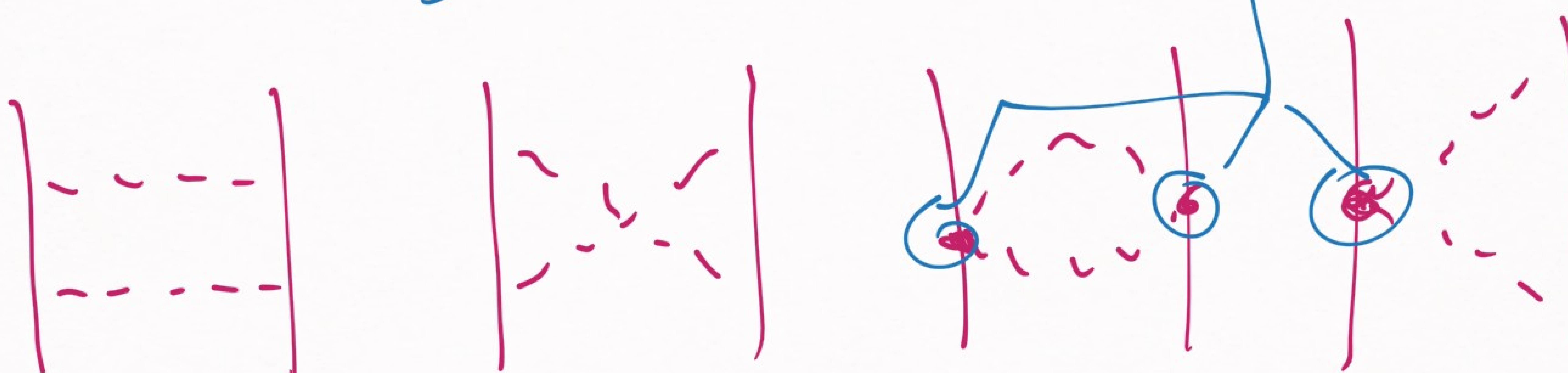


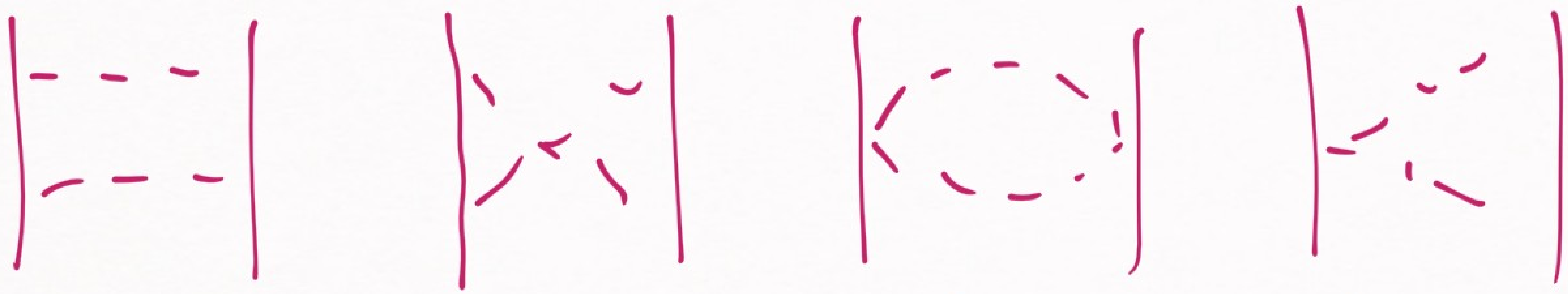
→ one pion exchange (OPE)

QFT/QM → loops

Two-pion exchange (TPE)

→ WT term





box crossed box football triangle

TRE has a long story → began in the 50's

1950's → two-pion theories (failure)

↓

problems everywhere

↳ full of infinities

⇓

Why? → 1) Wrong pion dynamics

2) Renormalization

（重正化）

[WRONG PION DYNAMICS]

"Inverse scattering problem"

↳ [Different potentials leading to the same observable consequences]

PION DYNAMICS

no derivative (no ∂_μ)

$$\mathcal{L}_{int} = g\,\bar{\psi}_N \gamma^5 \vec{\tau}\cdot\vec{\pi}\,\psi_N$$

$\tau_1 = \sigma_1$

$\tau_2 = \sigma_2$

$\tau_3 = \sigma_3$

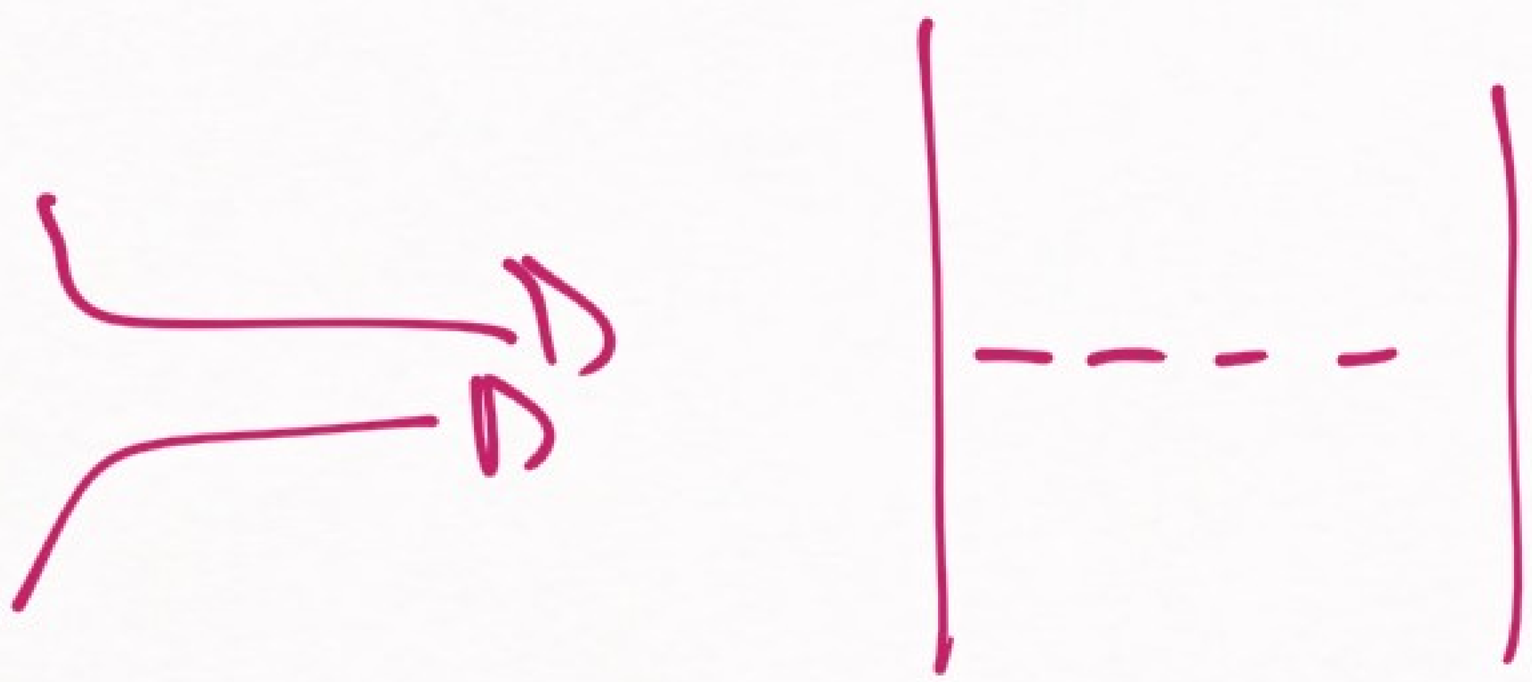
→ Simplest pseudoscalar theory we can write (not the only one)

$$\mathcal{L}'_{int} = \frac{g}{2f_\pi}\,\bar{\psi}_N \gamma^5 \gamma^\mu \vec{\tau}\cdot\partial_\mu\vec{\pi}\,\psi_N$$

derivative interaction

$\mathcal{L}$

$\mathcal{L}'$



$\longrightarrow$ same potential

Different example: $V(\vec{q}) = -\dfrac{g^2}{\vec{q}^{\,2}+m^2}$

$$V'(\vec{q}) = +g^2 \frac{|\vec{q}|^2}{\vec{q}^{\,2}+m^2}$$

$$V(\vec{r}) = -\frac{g^2}{4\pi}\frac{e^{-mr}}{r}$$

$(\vec{r} \neq 0)$

1950 $\rightarrow$ They use $\mathcal{L}_{int}$ (non-derivative pion interaction)

 correct

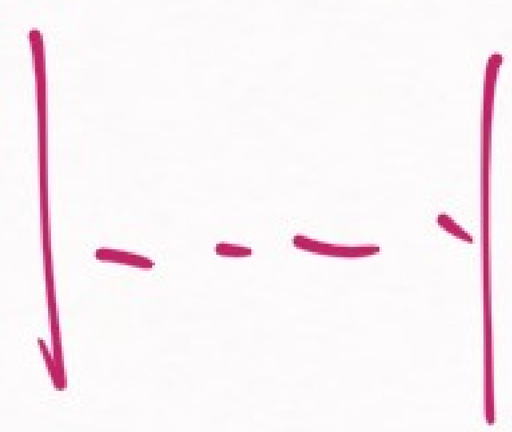
 incorrect

Later $\rightarrow$ $\boxed{\text{CHIRAL SYMMETRY}}$

$\Downarrow$

Pion interactions have to be derivative

(∂_μ's)

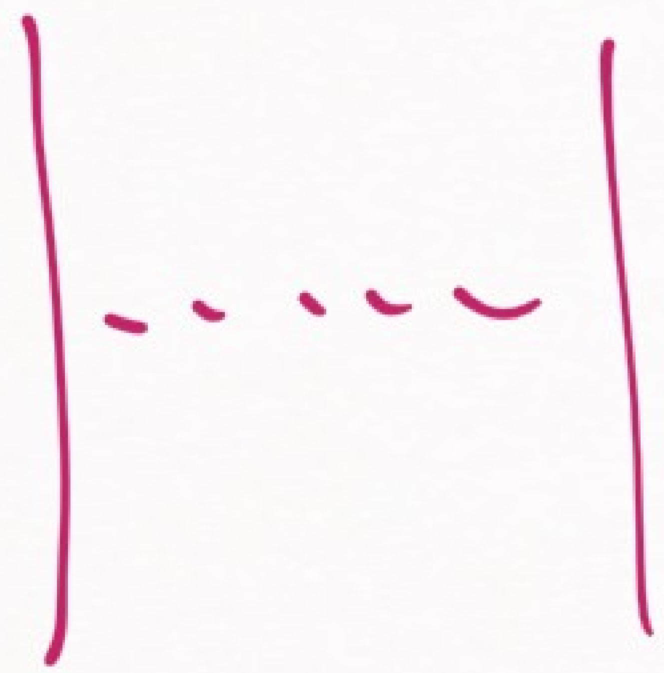
 $\rightarrow$ no difference

 $\rightarrow$ big difference

2) MORE MESONS $\rightarrow$ One boson exchange (OBE) model

1950 $\rightarrow$ multi-pions don't work $\rightarrow$ let's try something <u>different</u>

Besides the pion, $\exists$ other mesons



$\pi : J^P = 0^-$, $m_\pi = 140\,\text{MeV}$

$\rho, \omega : J^P = 1^-$, $m_\rho \simeq 770\,\text{MeV}$

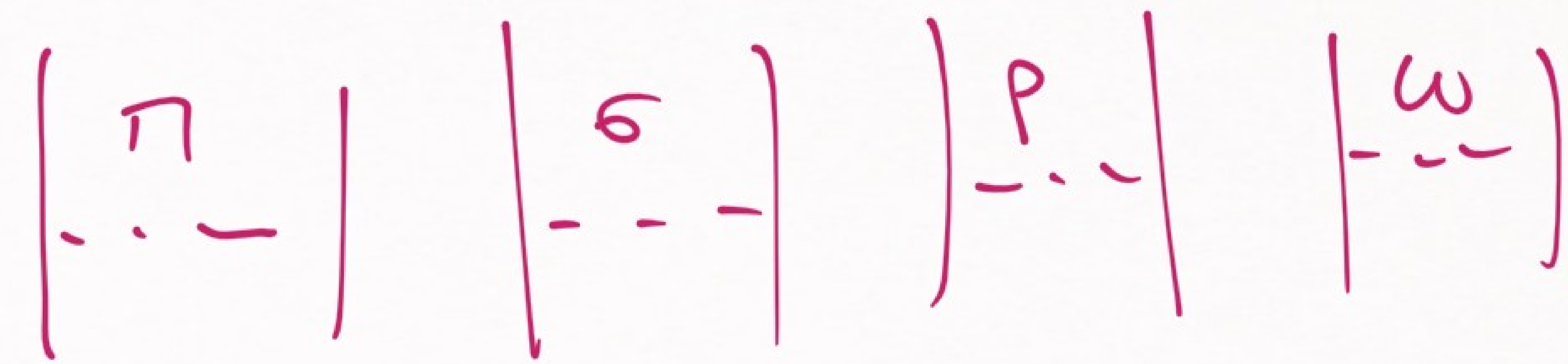
$m_\omega \simeq 780\,\text{MeV}$

$\sigma : J^P = 0^+$, $m_\sigma \simeq 550\,\text{MeV}$

$\boxed{\text{OBE}}$ $\longrightarrow$ $V_{NN}(\vec{r}) = V_\pi + V_\sigma + V_\rho + V_\omega$

$+ \ldots$

$\boxed{\text{Really easy idea}}$



[MOTIVATION FOR OBE]

1950 → multi-pion theories failing

Why? → TPE two types:

1) $\boxed{\text{correlated TPE}}$ → important

2) uncorrelated TPE → 

→ not important

hypothesis:

OBE → $\boxed{\text{PHENOMENOLOGICAL MODEL}}$

?

it works, but we don't worry about its theoretical basis

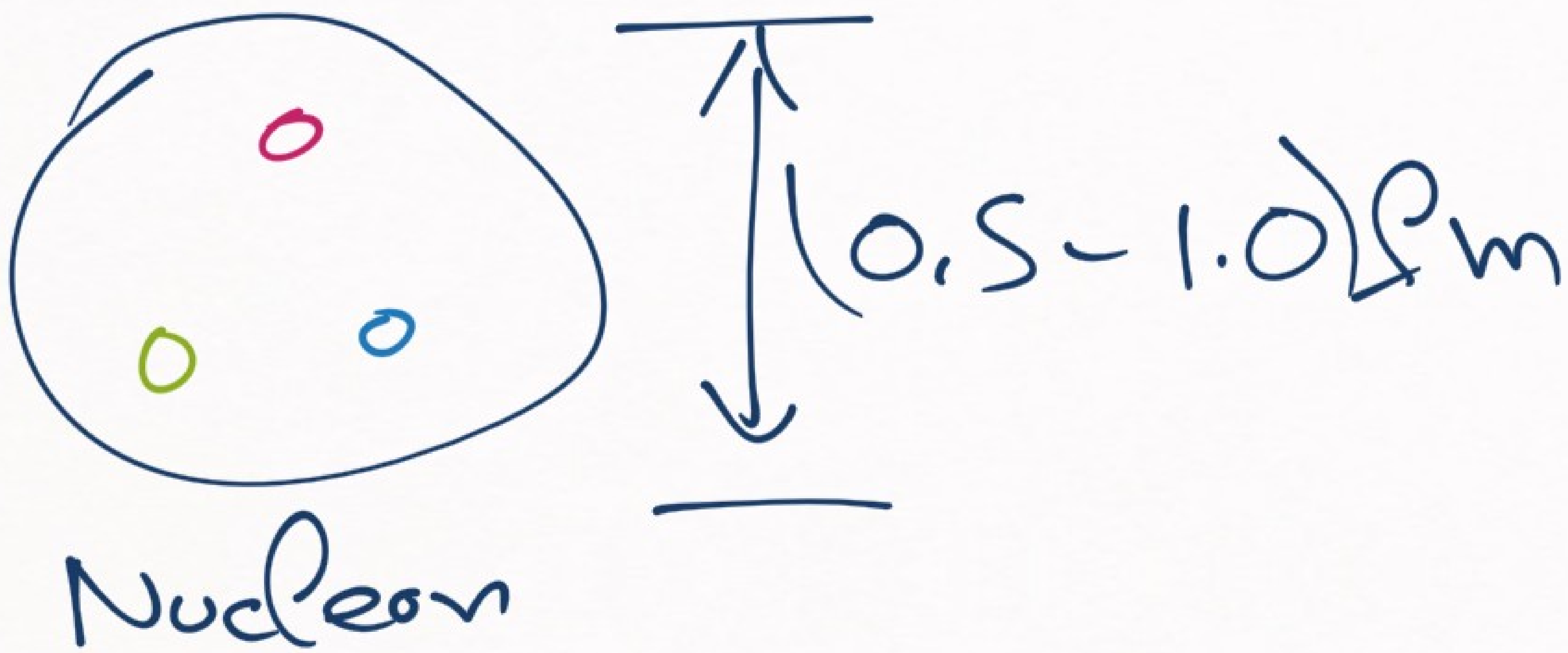
→ $\boxed{\text{CONVENIENT}}$

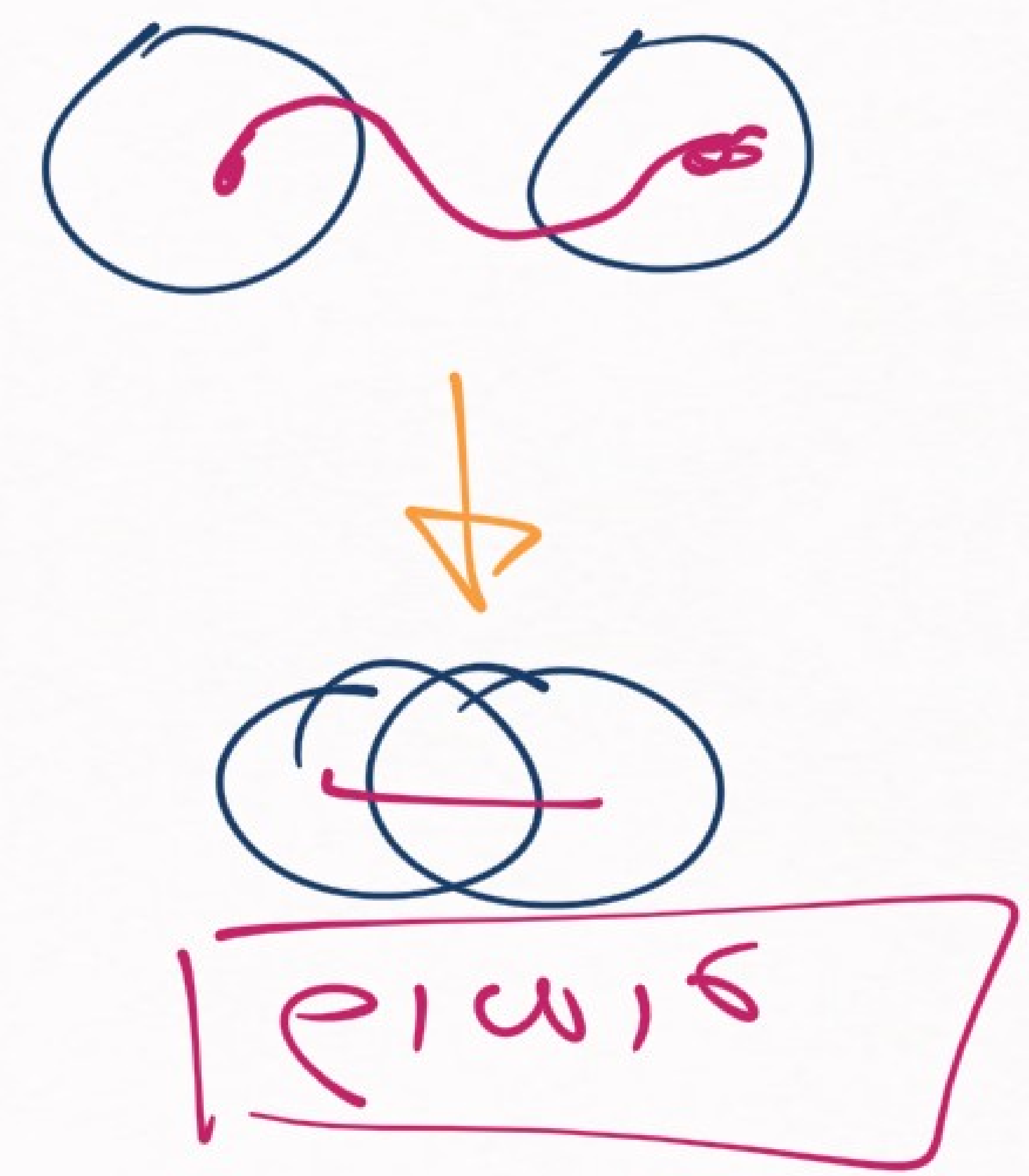
OBE $\longrightarrow$ theoretical problems

1) range of $\sigma/\rho/\omega$ exchanges

$$\sim \frac{e^{-m_B r}}{r} \Rightarrow \frac{1}{m_B} \text{ range}$$

$\longrightarrow$ heavy $\longrightarrow$ $\frac{1}{m_B} \sim 0.2\text{-}0.3\,\mathrm{fm}$





$$\frac{1}{m_B} \ll \sqrt{\langle r^2 \rangle}_{nucleon}$$

PARTIAL SOLUTIONS $\longrightarrow$ range is not $\frac{1}{m_B}$

but something like $\frac{\pi}{m_B}$

2) Singular potentials

$$\left| \overset{\pi/\rho/\omega}{\text{- - - - - -}} \right| \sim S_{12}(\hat{r})\,\frac{e^{-mr}}{r}\left(1+\frac{3}{mr}+\underbrace{\frac{3}{(mr)^2}}\right)$$

$$\lim_{r\to 0} r^2 V(r) \neq 0$$

super strong at short distance

VARIATIONAL PRINCIPLE (FOR BOUND STATES IN QM)

$$\underline{\underline{E_B}} \leq \langle \underbrace{\psi_{test}} | H | \psi_{test} \rangle$$

test wave function

$$-\frac{C_3}{r^3} \longrightarrow \langle \psi_{test} | H | \psi_{test} \rangle \to -\infty$$

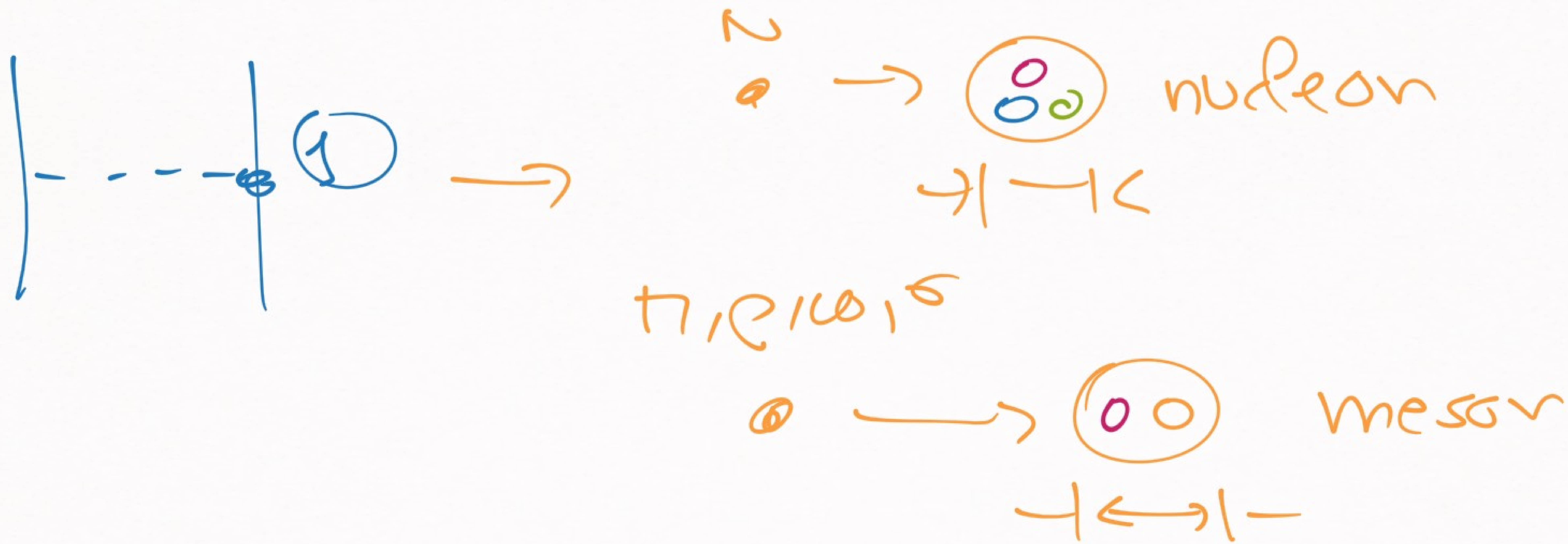
Attractive singular potential

↳ Does not have a ground state

→ Deuteron should collapse

$E_B \neq -2.2\,\text{MeV} \rightarrow \boxed{E_B \rightarrow -\infty}$

2) Singular interactions → form factors



finite size → potential not singular
at short distance!

3) σ meson was not found in experiments

$\pi, \rho, \omega \rightarrow$ were known/observed in experment

$\boxed{\sigma \rightarrow ?}$ $\rightarrow$ 2000's σ is a resonce in $\pi\pi$ scatterings

$\downarrow$

how we know it exists

RECAP

old pion theories
(diverge)

$\downarrow$

OBE model
(range, form factors, σ)

$\downarrow$

$\rightarrow$ modern pion theories
(chiral symmetry, renormalization)

PATTERN → 1) we find a good idea

2) good idea fails

3) we reformulate the idea / find a new idea

4) repeat the process

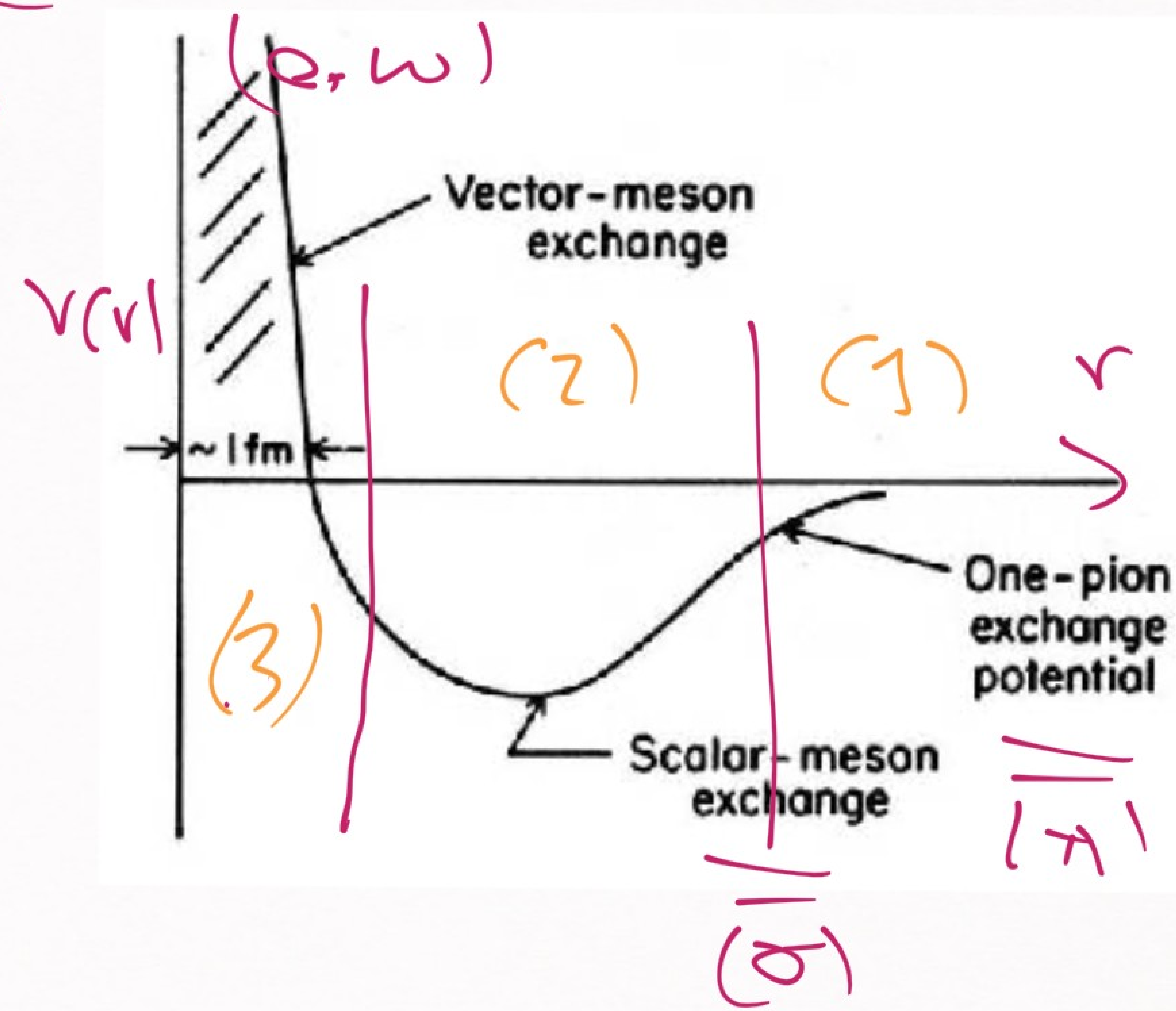
(it is difficult)

USEFUL IDEA → TAKETANI, NAKAMURA, SASAKI (TNS)

SCALE SEPARATION

↓

(3 REGIONS)



TNS CLASSIFICATION/ SEPARATION :

(1) CLASSICAL ZONE ($r \geq 2$ fm)

→ OPE most important factor

(2) DYNAMICAL ZONE ($2\,\text{fm} \geq r \geq 1$ fm)

→ TPE, σ most important factors

(3) $\boxed{\text{PHENOMENOLOGICAL}}$ ZONE ($r \leq 1$ fm)

→ multipions, ρ, ω, weird stuff

$\boxed{\text{TNS idea}}$ → interesting

why?, very modern

(scale separation)

long / medium / short distances

↓ well known

↓ some information

↓ poorly known

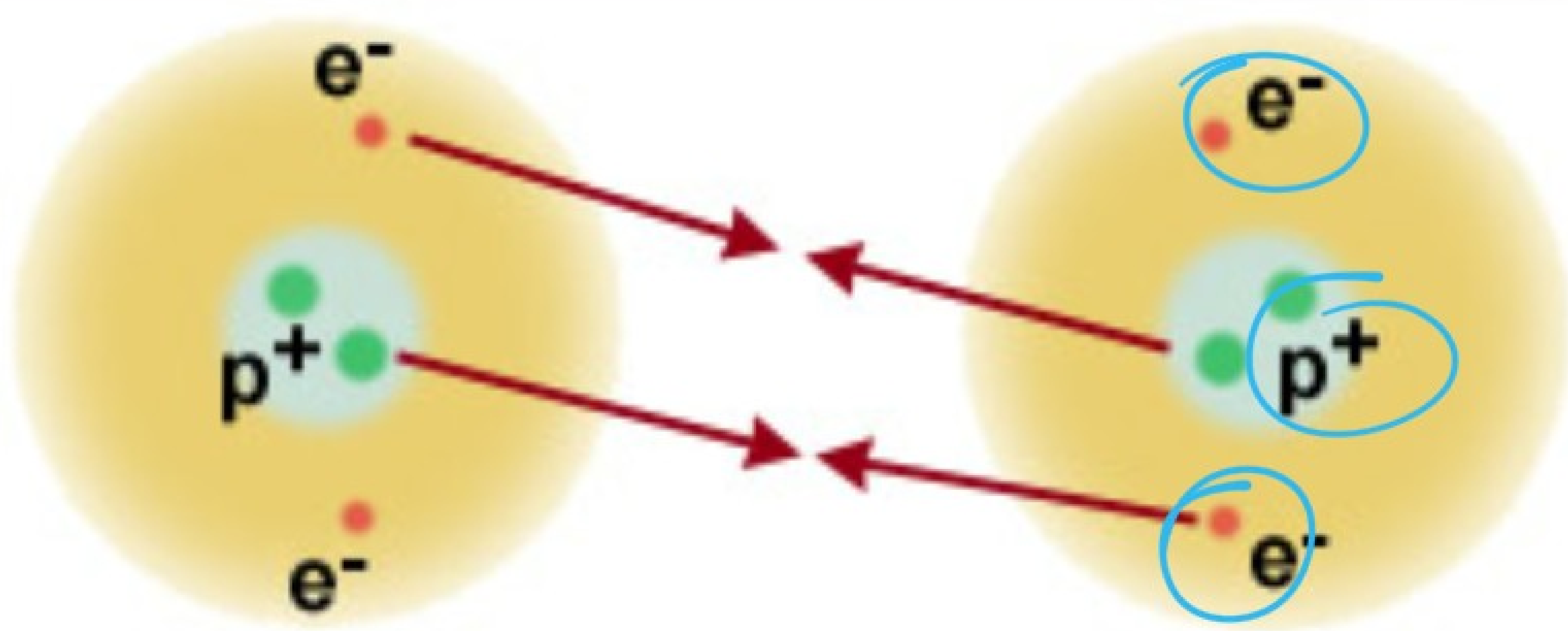
TNS → distinguish between
knowns & unknowns

Another idea: $\boxed{\text{NUCLEAR FORCES ARE RESIDUAL FORCES}}$

What is a $\boxed{\text{RESIDUAL FORCE}}$

atoms → electrically neutral ($V_C = 0$)

→ but still ∃ potential (V_{vdW})



Residual E-M force in action: the atoms are electrically neutral, but the electrons in one are attracted to the protons in another, and vice versa!

→ atoms are neutral

→ composed of non-neutral parts

complicated calculation

(Born-Oppenheimer approximation)

$V(\vec{r}) \neq 0$ → van der Waals potential

RESIDUAL FORCES

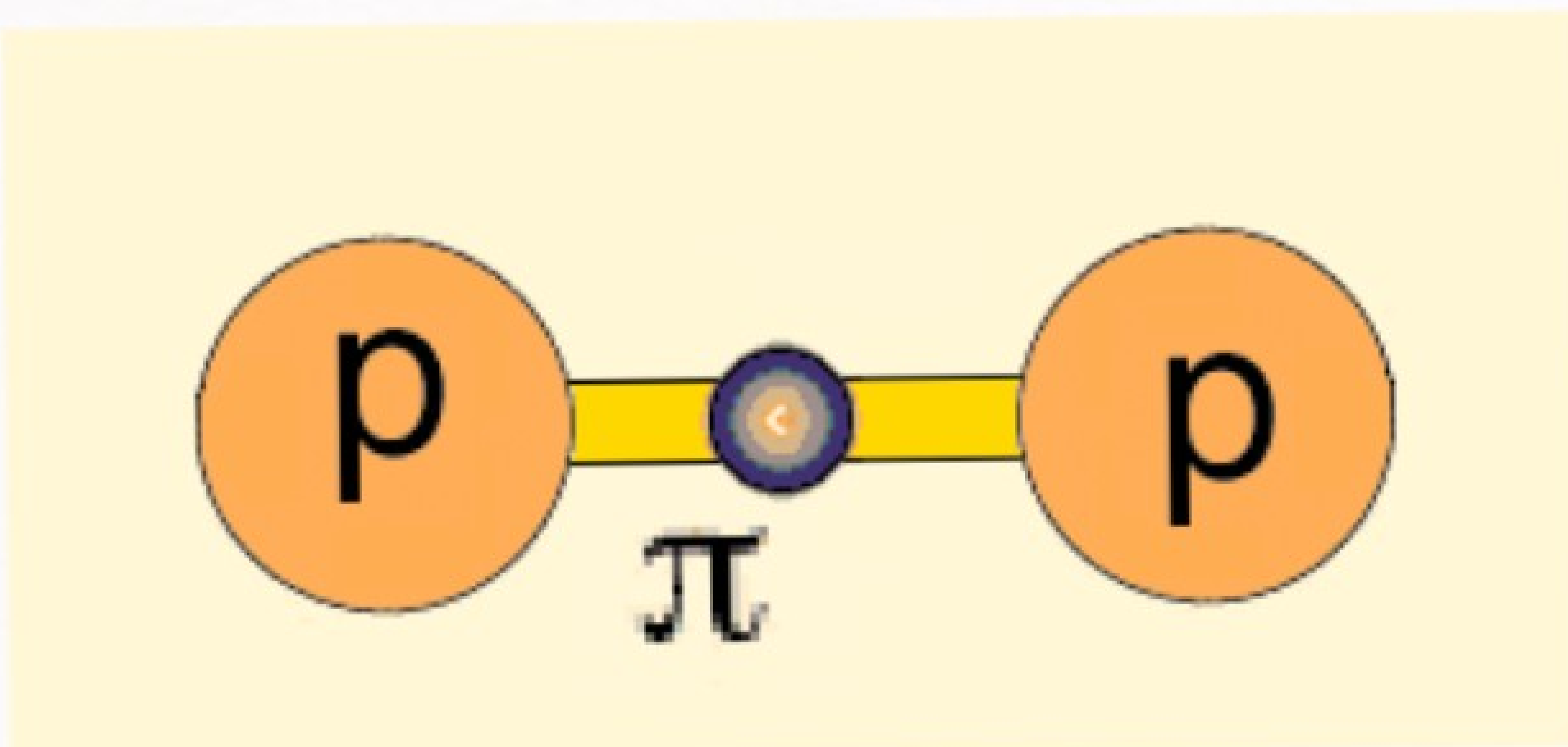
Example → atom-atom potential

$$\underbrace{V(r) \simeq -\frac{C_6}{r^6} - \frac{C_8}{r^8} - \frac{C_{10}}{r^{10}}}_{\text{Residual}} - \ldots\ldots$$

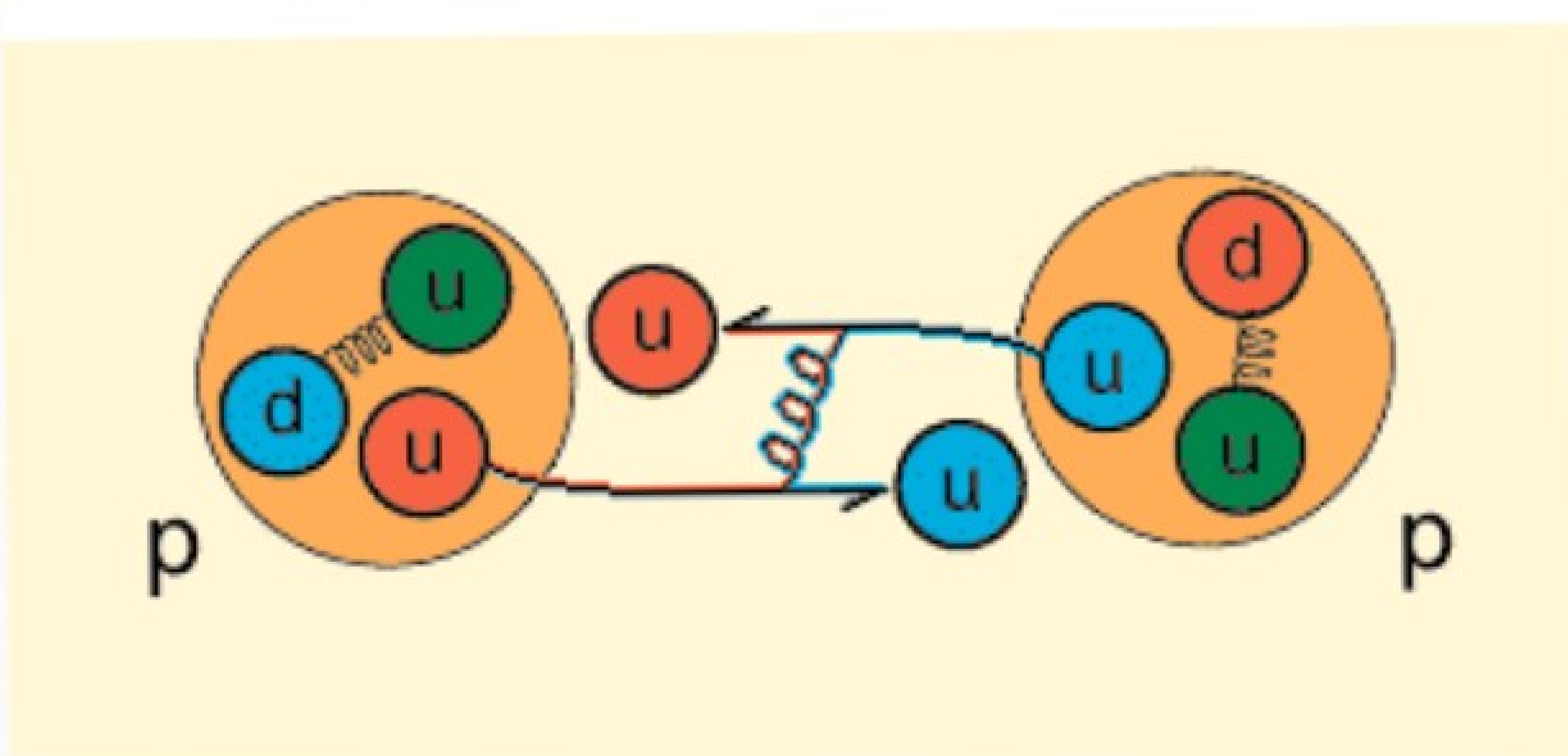
Residual ← van der Waals

in contrast to Coulomb → Fondamental

Nuclear forces:



→ Yokawa's idea

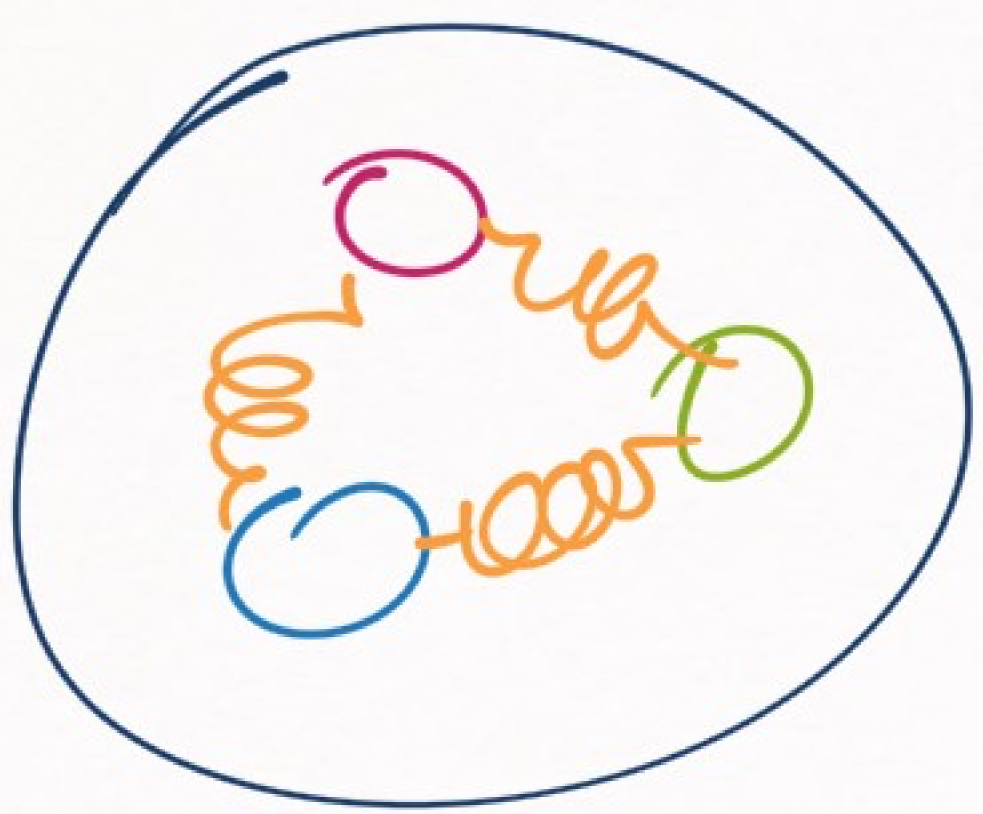


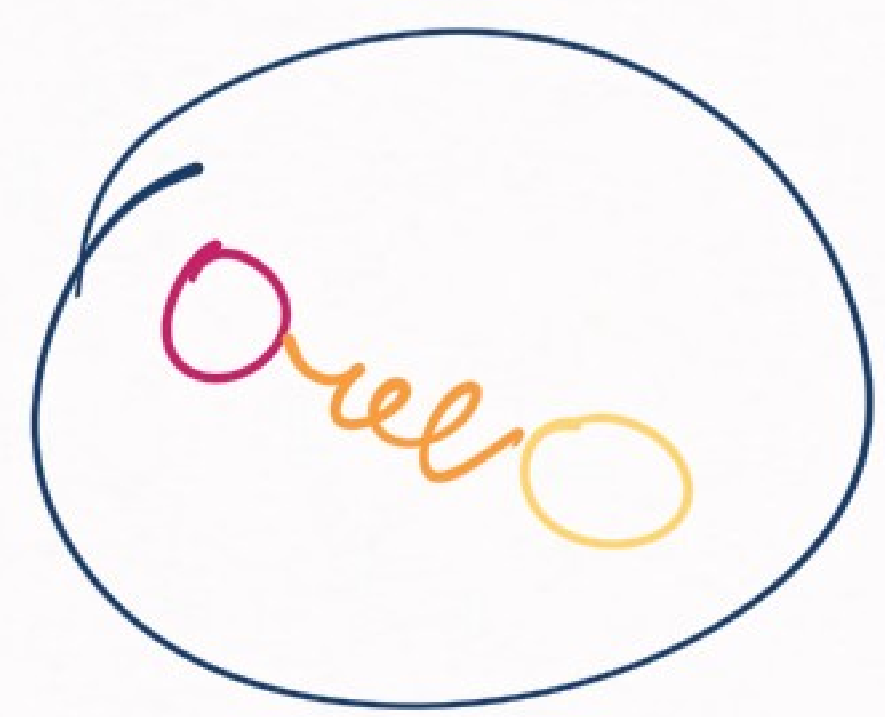
→ What is "really" happening

proton is not fundamental

(not point-like)

pion is not fundamental





IDEALLY WE SHOULD BE ABLE TO EXPLAIN V_{NN} IN TERMS OF QUARKS & GLUONS

PROBLEM $\rightarrow$ nucleons, pions, σ's, ρ's, ω's

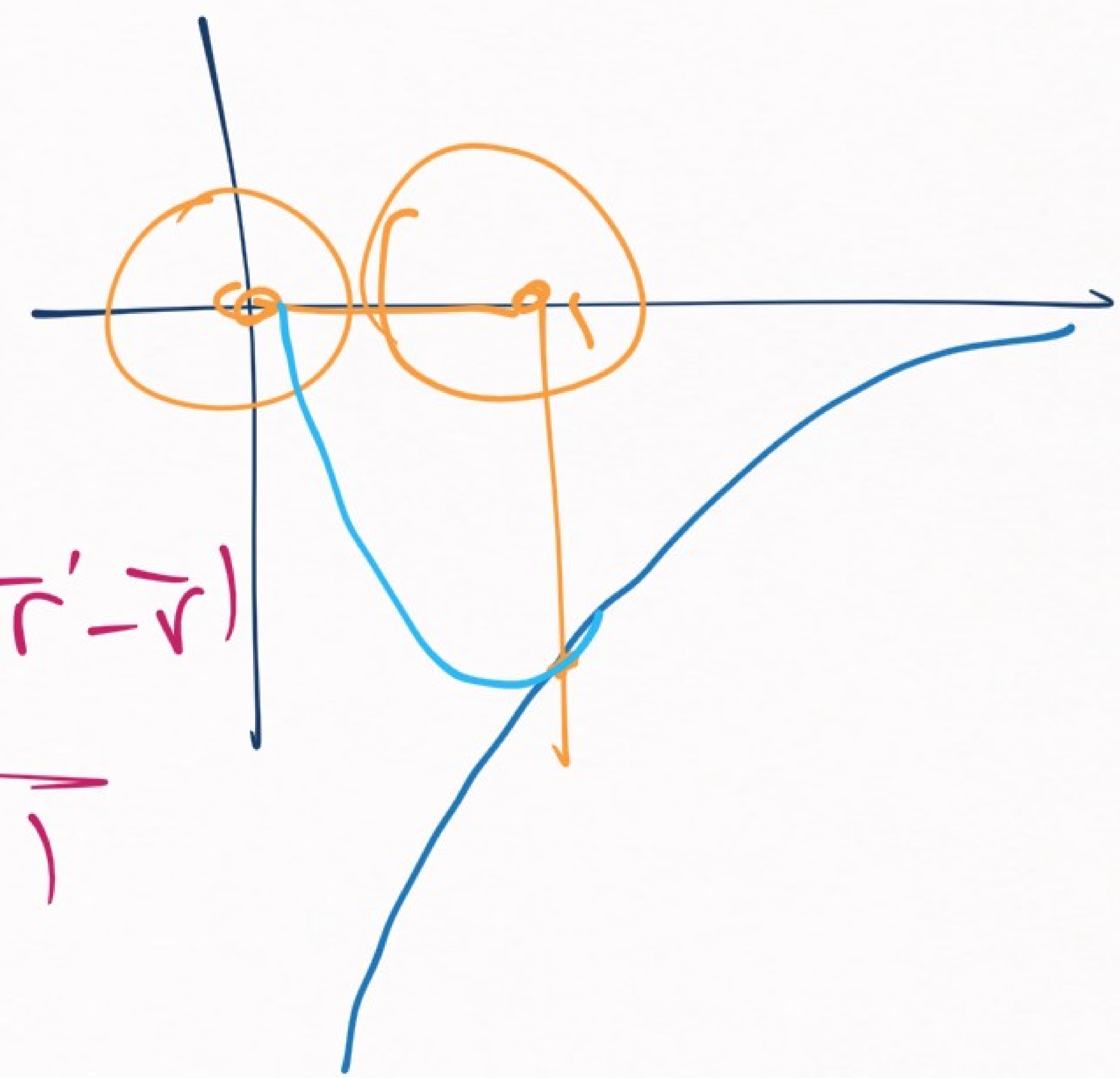
quarks & gluons

residual effect

NEXT LECTURE

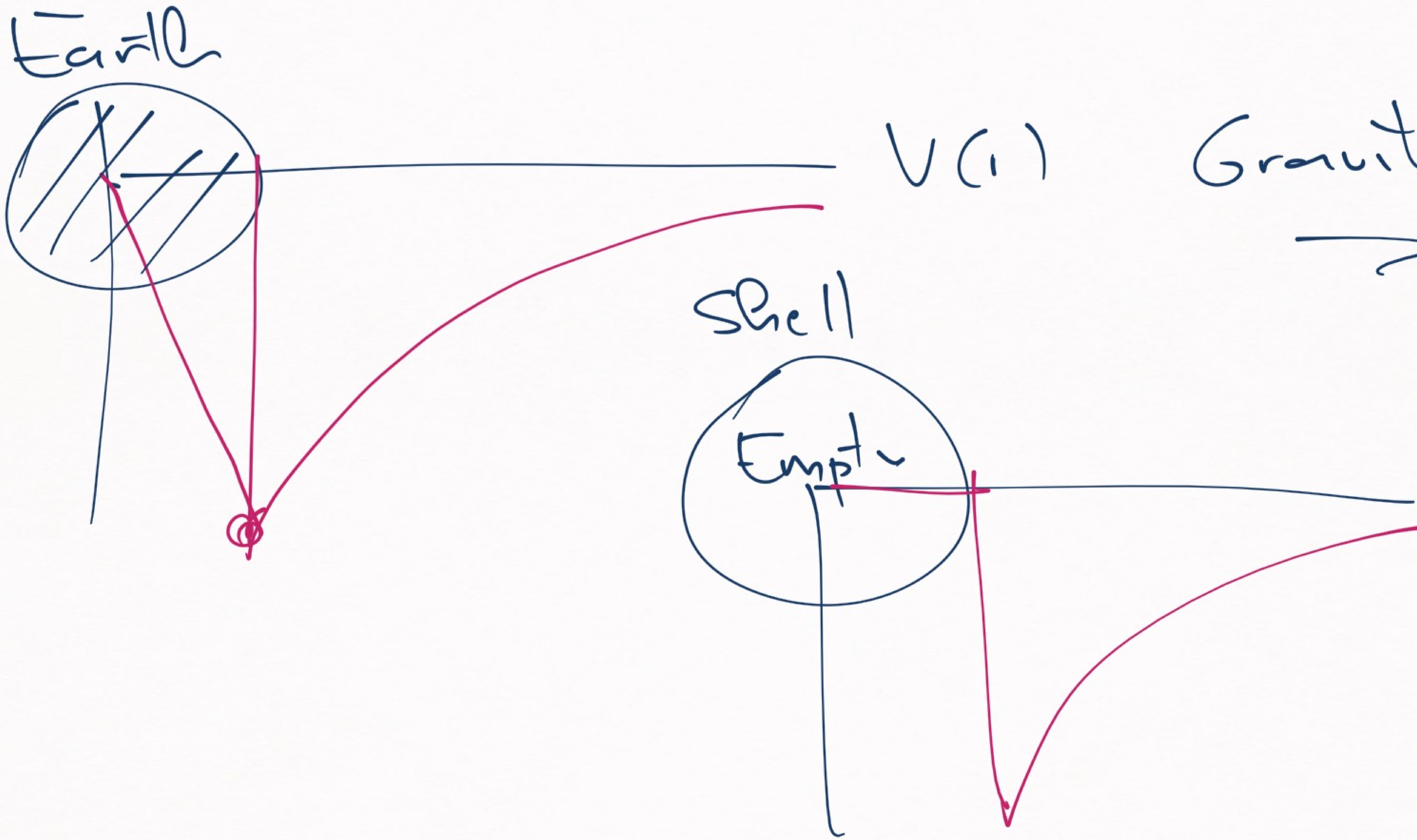
SINGULAR INTERACTION:

$$V_3(\vec{r}) = -\frac{C_3}{r^3}$$



$$\frac{1}{r} \to \int d^3\vec{r}\,' \frac{\rho(\vec{r}\,')\rho(\vec{r}\,'-\vec{r})}{|\vec{r}\,'-\vec{r}|}$$

Gravitation →



$\frac{1}{r^n}$, $n \geq 2$ $\longrightarrow$ SINGULAR