NUCLEAR PHYSICS 4

→ MORE ABOUT NATURAL &
UNNATURAL SYSTEMS

→ GENERAL PROPERTIES OF
THE NUCLEAR FORCE

$\boxed{\text{RECAP}}$ → Natural & Unnatural systems

Natural:

1) Good scale separation
2) $\forall$ is $O(1)$ on Q (the characteristic momentum scale)

Unnatural:

1) Poor scale separation

or

2) not $\forall$ is $O(1)$ in Q (fine tuning)

[NEUTRON-PROTON SYSTEM]

→ fine-tuning   1/25 for the deuteron

1/1000 for the virtual state (singlet, S=1)

→ poor scale separation

$$R_\pi \sim \frac{1}{m_\pi} \sim \boxed{1.4\ \text{fm}} \quad \text{vs} \quad r_p \sim \boxed{(0.5-1.0)}\ \text{fm}$$

TWO TYPES OF FINE-TUNING:

1) FORTUITOUS (by chance)

→ up system → eclipses (relative size of moon & sun) ?

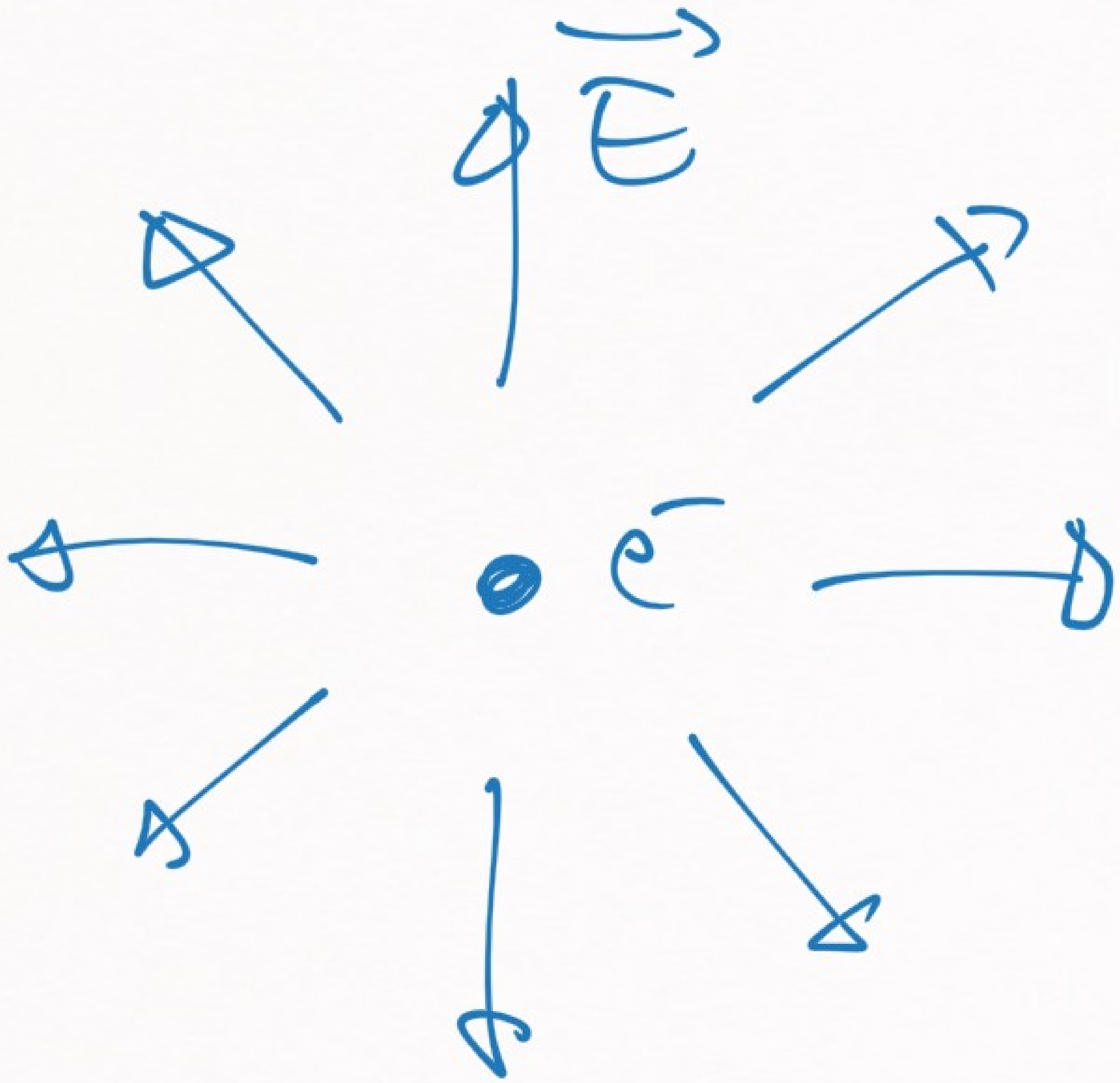
→ solar system

2) CONSPIRACY ($\exists$ a deeper reason)

→ classical electron radius ?

→ higgs mass ⟷ cosmological constant

FINE TUNING 1 → "CONSPIRACY" → NEW PHYSICS



$\vec{E}$ → energy → mass

$$m_{phys}(e^-) = m(\vec{E}) + m_{bare}(e^-)$$

If the electron is point-like

$\Rightarrow m(\vec{E}) \to \infty \quad \Rightarrow m_{bare}(e^-) \to -\infty$

$$m(\vec{E}) + m_{bare}(e^-) = m_{phys}(e^-)$$

$$\infty \quad - \quad \infty \qquad = 0{,}5 \text{ MeV}$$

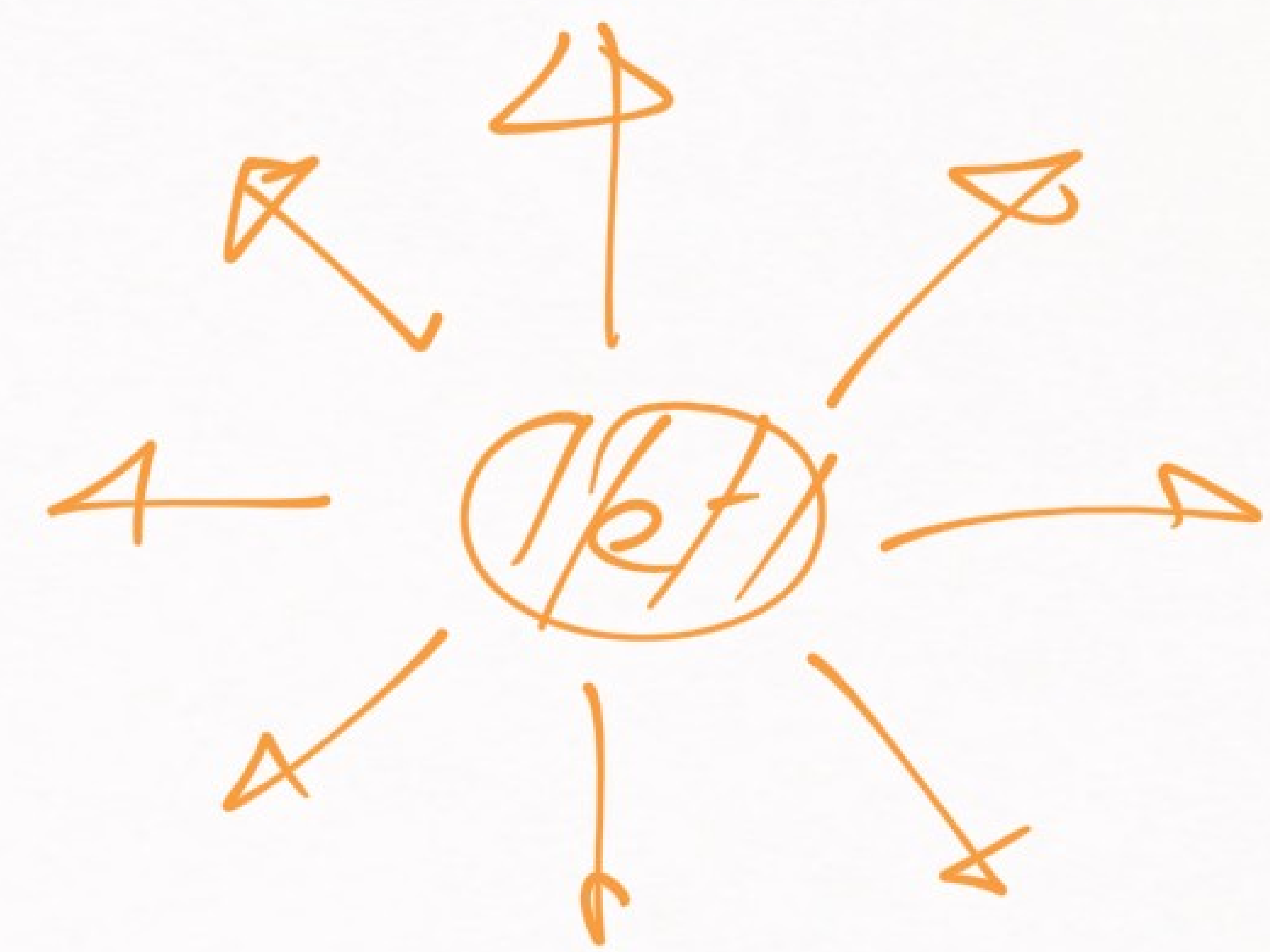
$\to$ infinite level of fine-tuning

infinite fine-tuning → ~~chance?~~ → new physics

→ Classical radius of the electron

r_e / $m(\vec{E}) = m_e$

$m_{bare}(e^-) = 0$



$$r_{classical}(e^-) = \frac{\alpha}{m_e} \simeq 2.8\,\text{fm}$$

~~chance~~ $\longrightarrow$ NEW PHYSICS HAPPENING BEFORE $r \simeq r_{cl}(e^-)$

1) Quantum mechanics $\quad r \sim a_B \sim 0.5\,\mathring{A}$

$(r \gg r_{cl}(e^-))$

2) Quantum field theory $\quad r \sim \dfrac{1}{m_e} \sim 400\,\mathrm{fm}$

$\hookrightarrow$ pair creation

QFT $\to$ $m_{phys}(\bar{e}) = m_{bare}(\bar{e}) +$ loop corrections

Renormalization

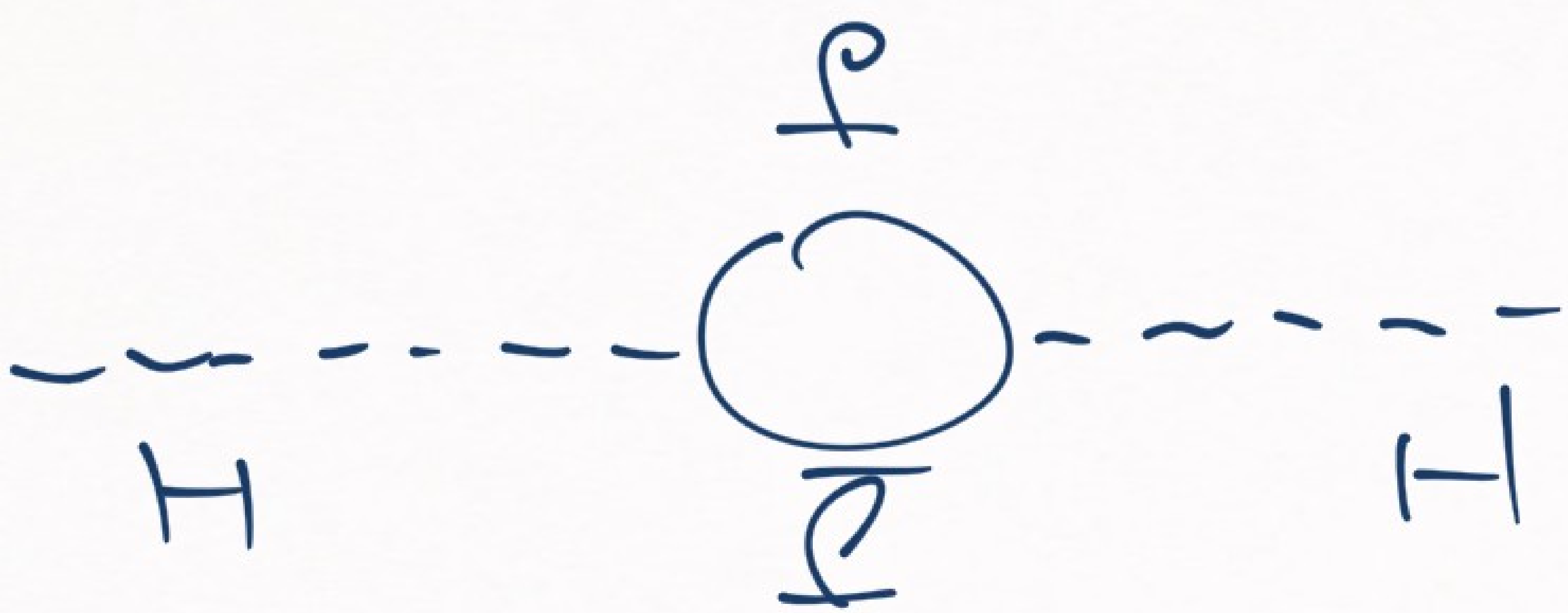
New scale for fine-tuning

(but happens at really short-distances)

$(\Lambda_{UV} > M_{pl})$

FINE-TUNING (2): HIGGS MASS

$$m_H^2 = m_{bare}^2 + \Delta m_H^2 \rightarrow \text{loops/reno.}$$



cutoff of the theory

$$\Delta m_H^2 \propto -\frac{|\lambda_f|^2}{8\pi^2}\left[\Lambda_{UV}^2 + \ldots\right]$$

$\Lambda_{UV} \rightarrow$ energy at which we expect the SM (standard model) to fail

Amount of fine-tuning $\rightarrow$ $\boxed{\dfrac{m_H}{\Lambda_{UV}}}$

$\Lambda_{UV} \rightarrow M_{Pl}$ ($\sim 10^{18}$ GeV)

$$\boxed{\frac{m_H}{\Lambda_{UV}} \sim 10^{-16}}$$

This is why people believe in the existence of new physics before M_{Pl}

$\longrightarrow$ SUPERSYMMETRY (if M_{susy} not too large)

Fine-tuning: $\boxed{\dfrac{m_H}{M_{susy}}}$ (instead of $\dfrac{m_H}{M_{Pl}}$)

$\rightarrow$ less fine tuning

FINE-TUNING 3 → COSMOLOGICAL CONSTANT

(most extreme example)

Λ_c

1) Naturalness :

appears in Einstein equations for gravity

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda_c g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$[\Lambda_c] = [M]^4$

Naturalness:

$$\Lambda_c \sim [M]^4 \sim M_{Pl}^4 \sim (10^{27}\, eV)^4$$

$$\sim 10^{108}\, eV^4$$

Planck mass → natural scale for quantum gravity

$$10^{18}\, MeV \simeq 10^{27}\, eV \qquad \boxed{\Lambda_c \sim 10^{108}\, eV^4}$$

"Experimentally"

$$\Lambda_c \sim (10^{-3}\,eV)^4$$
$$\sim 10^{-12}\,eV$$

Fine-tuning $\rightarrow$ $\boxed{\dfrac{\Lambda_c}{M_{Pl}^4} \sim \dfrac{1}{10^{120}}}$

dimensional expectation is "a bit" off

COSMOLOGICAL CONSTANT PROBLEM:

$$\boxed{\frac{\Lambda_c}{M_{pl}^4} \sim \frac{1}{10^{120}}} \longrightarrow \text{Solutions?}$$

the landscape

→ Anthropic principle

→ Multiverse → "fortuitous"

→ New physics (eternal inflation, string theory vacua)

+

$$\frac{\Lambda_c}{M_{Pl}} \sim \frac{1}{10^{120}} \longrightarrow \text{we don't know why}$$

MESSAGE TO TAKE

FINE-TUNING

↓

SOMETHING ELSE IS GOING ON

PART 2 → PROPERTIES OF THE NUCLEAR FORE

→ properties of this potential

1) Short-ranged
2) Attractive at "intermediate distances"
3) Repulsive at short-distances
4) Does not distinguish protons & neutrons

nucleons

5) Not central

1) SHORT-RANGED ($\neq$ SHORT RANGE)

Two types of potentials

1.a) Long-ranged $\rightarrow$ effects of $V(\vec{r})$ extend to infinity

$$V(r) \sim \frac{1}{r^n}$$ (power-law) $\rightarrow$ Coulomb, gravity, van der Waals

1.b) Short-ranged $\rightarrow$ effects decay exponentially

$$V(r) \sim \frac{e^{-\lambda r}}{r^n}$$ (e.g. Yukawa)

Short-ranged potentials are easier

Good for theoreticians

Short-ranged vs Short-range

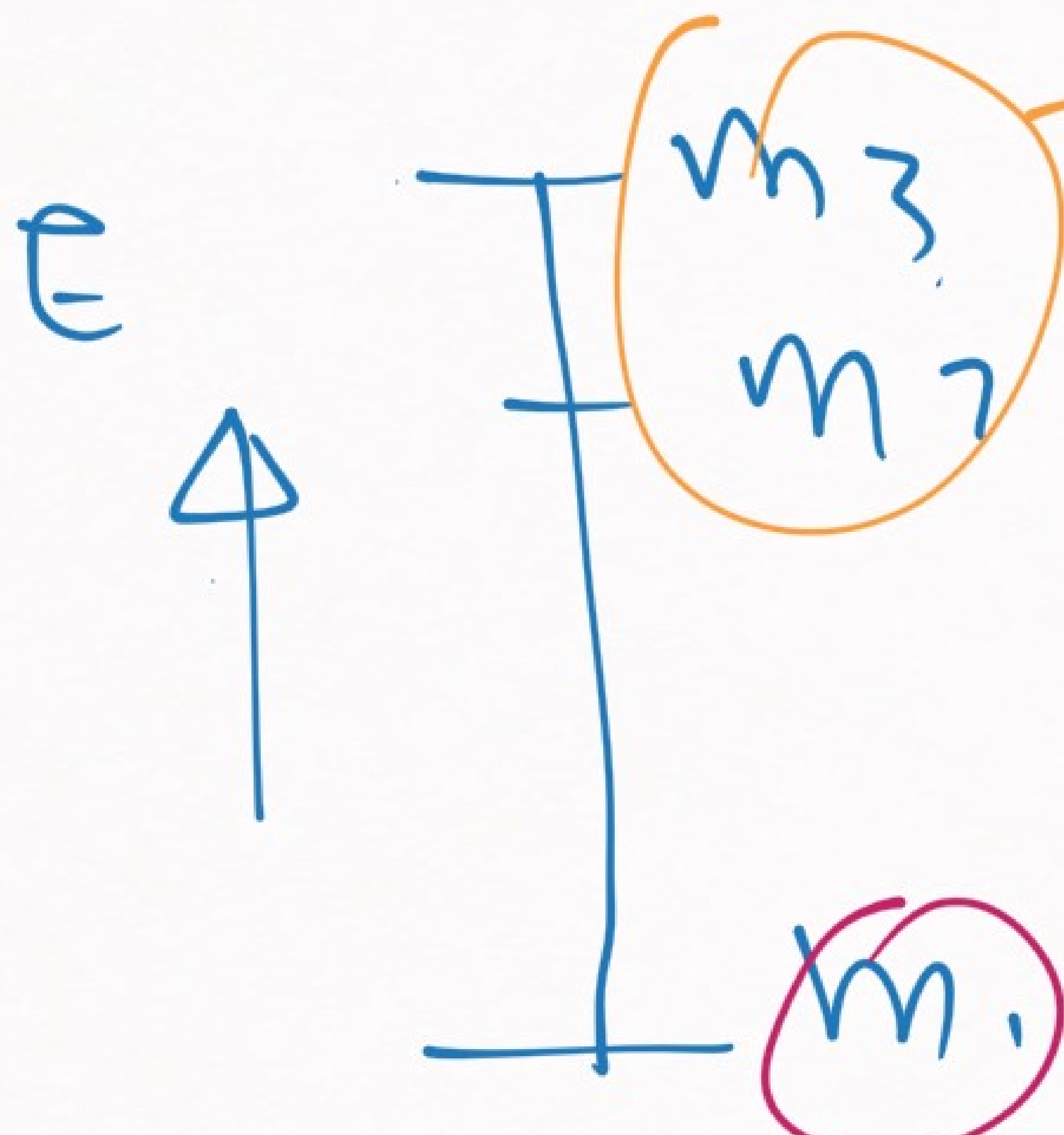
exponential damping

separation of scale

EXAMPLE : YUKAWA POTENTIAL

$$V(r) \sim \frac{e^{-mr}}{4\pi r}$$

$\longrightarrow$ finite-ranged (short-ranged)



$m_1 < m_2 < m_3$

$$V(r) = -\sum_i g_i^2 \frac{e^{-m_i r}}{4\pi r}$$

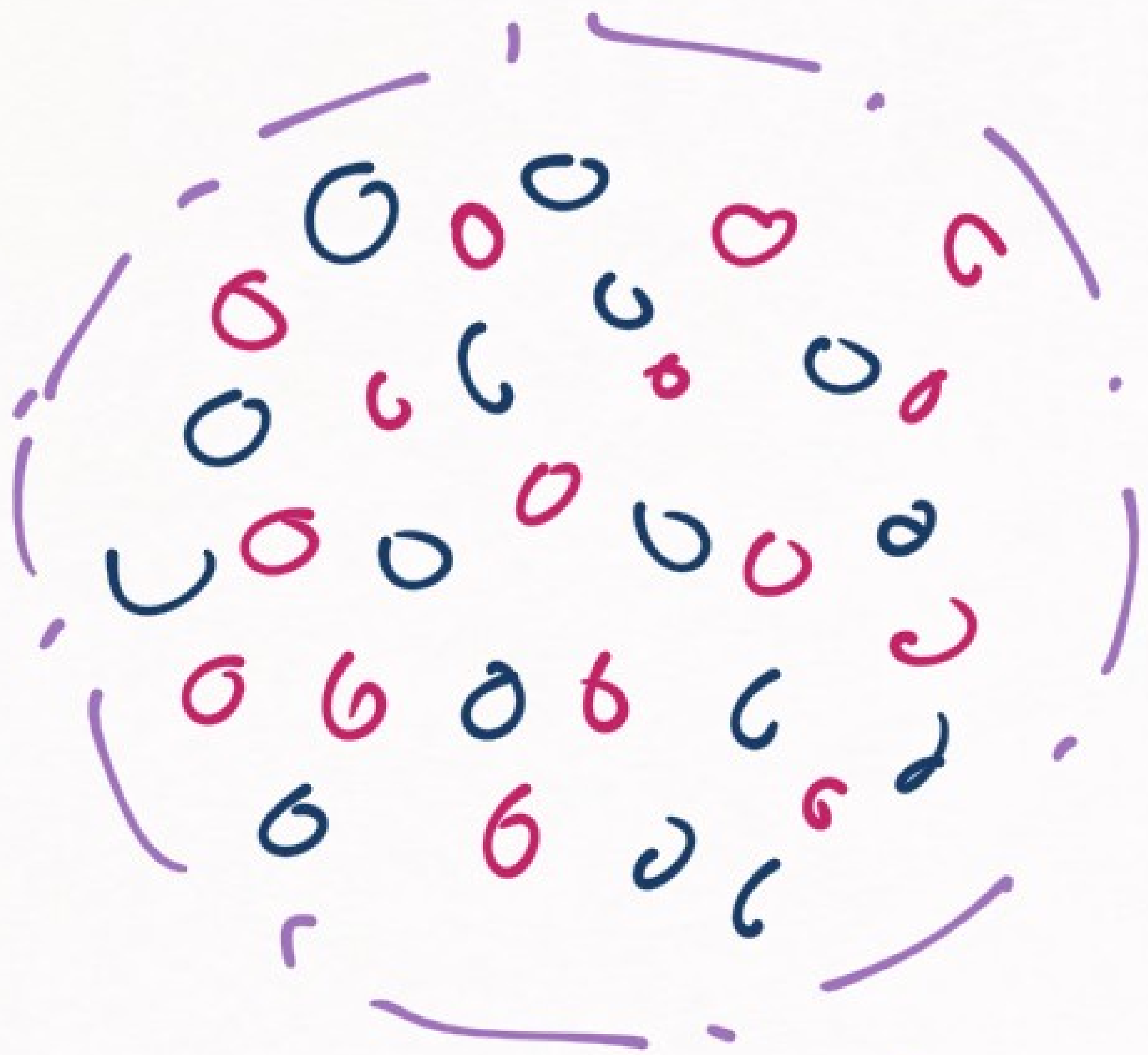
$m_2, m_3 \longrightarrow$ short-range part of $V(r)$

$m_1 \longrightarrow$ long-range part of $V(r)$

WHY IS THE NUCLEAR FORCE SHORT-RANGED ?

$$\boxed{\frac{B}{A} \sim 8\,\text{MeV}}$$

$\boxed{\text{SATURATION}}$



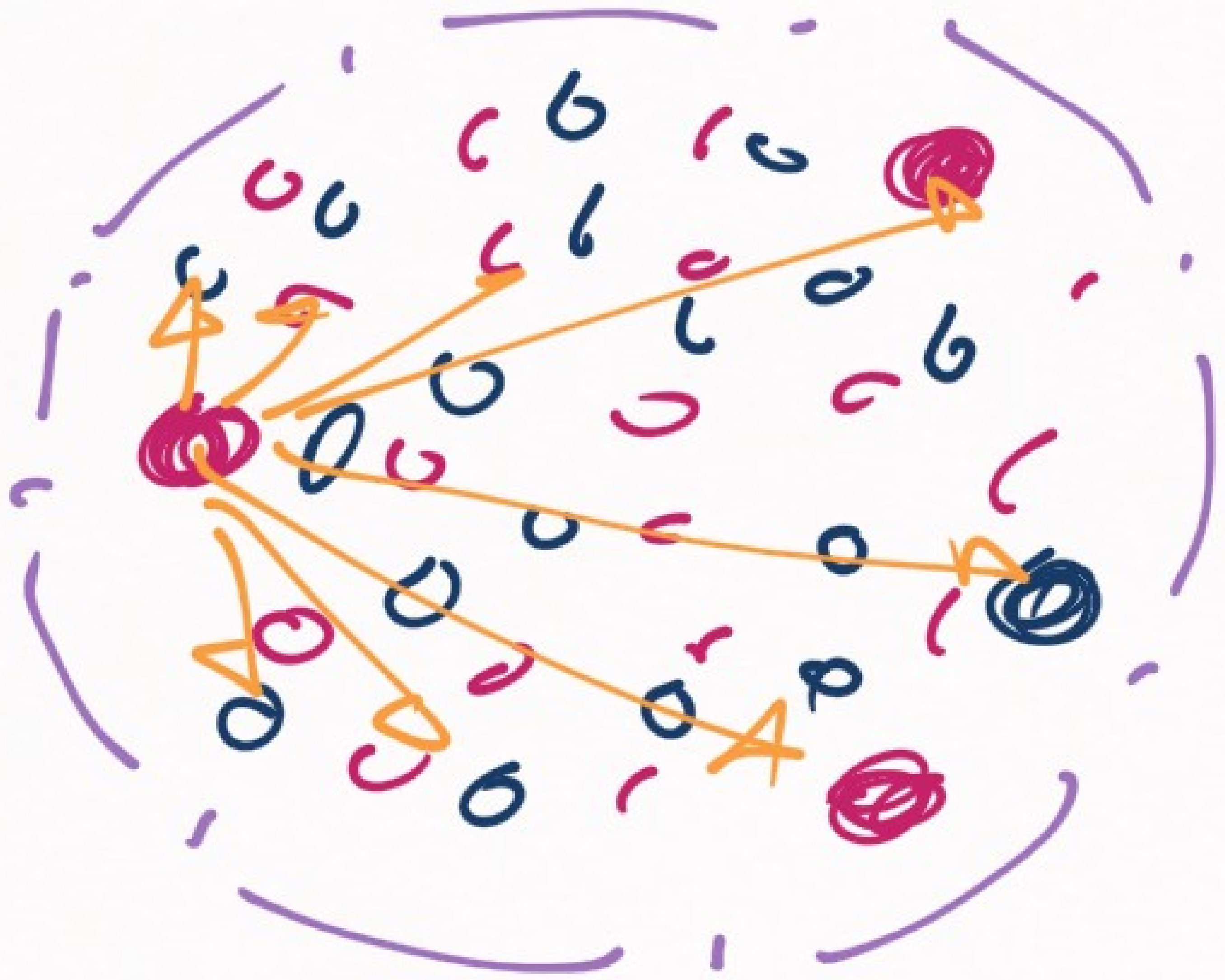
NUCLEUS

$B \rightarrow$ binding energy

$A \rightarrow$ # of nucleons

(p or n = N)

Long-ranged force $\rightarrow$ $\boxed{\frac{B}{A^2} \sim \text{constant}}$

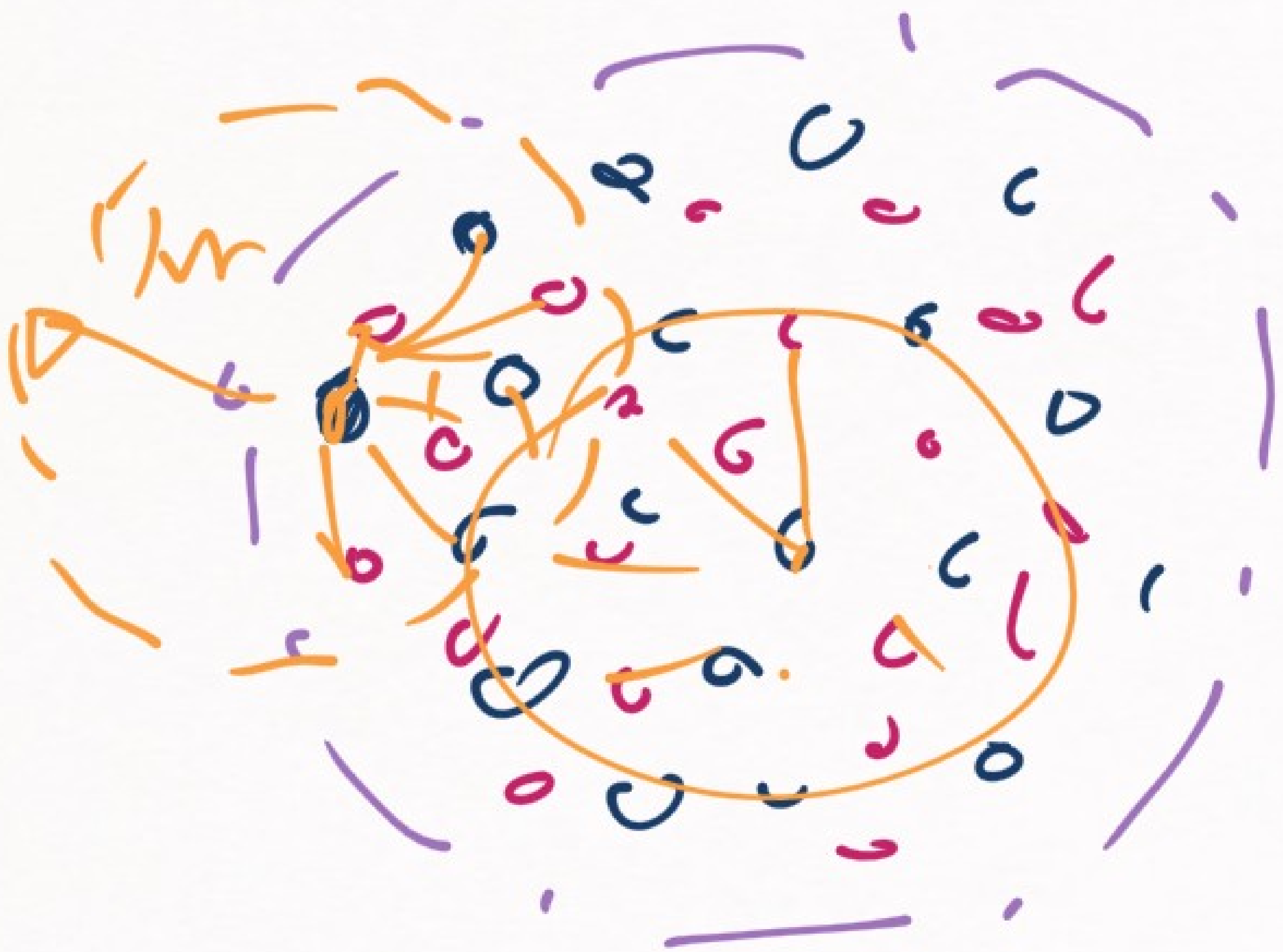


$\rightarrow$ each nucleon interacts w/ $A-1$ nucleons

$$B \propto \frac{1}{2} A(A-1) \langle V \rangle \propto A^2$$

Gravity $\rightarrow$ $U \sim -G \frac{M^2}{R}$ $\left(\underline{A^2}\right)$

Short-ranged force $\rightarrow$ $\boxed{\frac{B}{A} \sim \text{constant}}$



$\rightarrow$ potential range $\sim \frac{1}{m}$

each nucleon will only interact w/ neighbors (a fixed number)

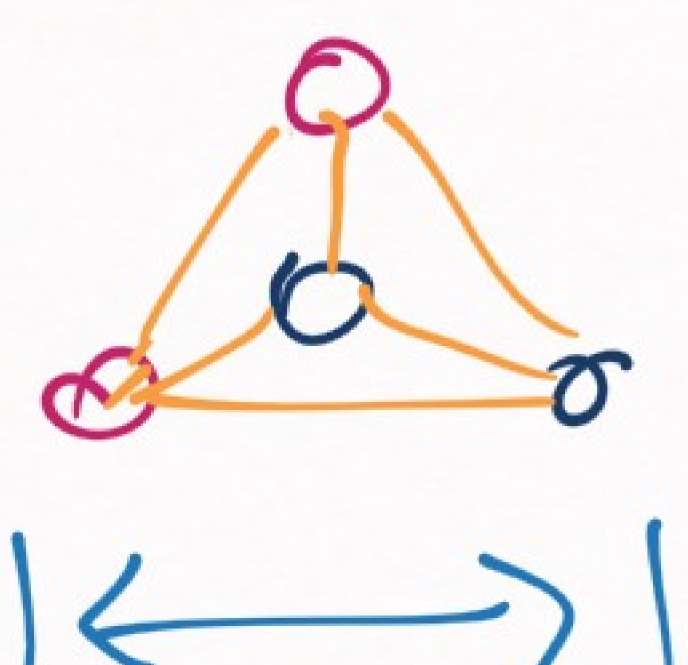
$$B \sim A \times N_0 \times \langle \bar{V} \rangle \sim A$$

$\boxed{\frac{B}{A} \sim 8\,\text{MeV}}$ $\Rightarrow$ [Nuclear force has a finite range]

What is the finite range?

4)

Wigner

 $\rightarrow$ ^4He $\hookrightarrow$ its size $\hookrightarrow$ $\boxed{1{,}7\ \text{fm}}$

range

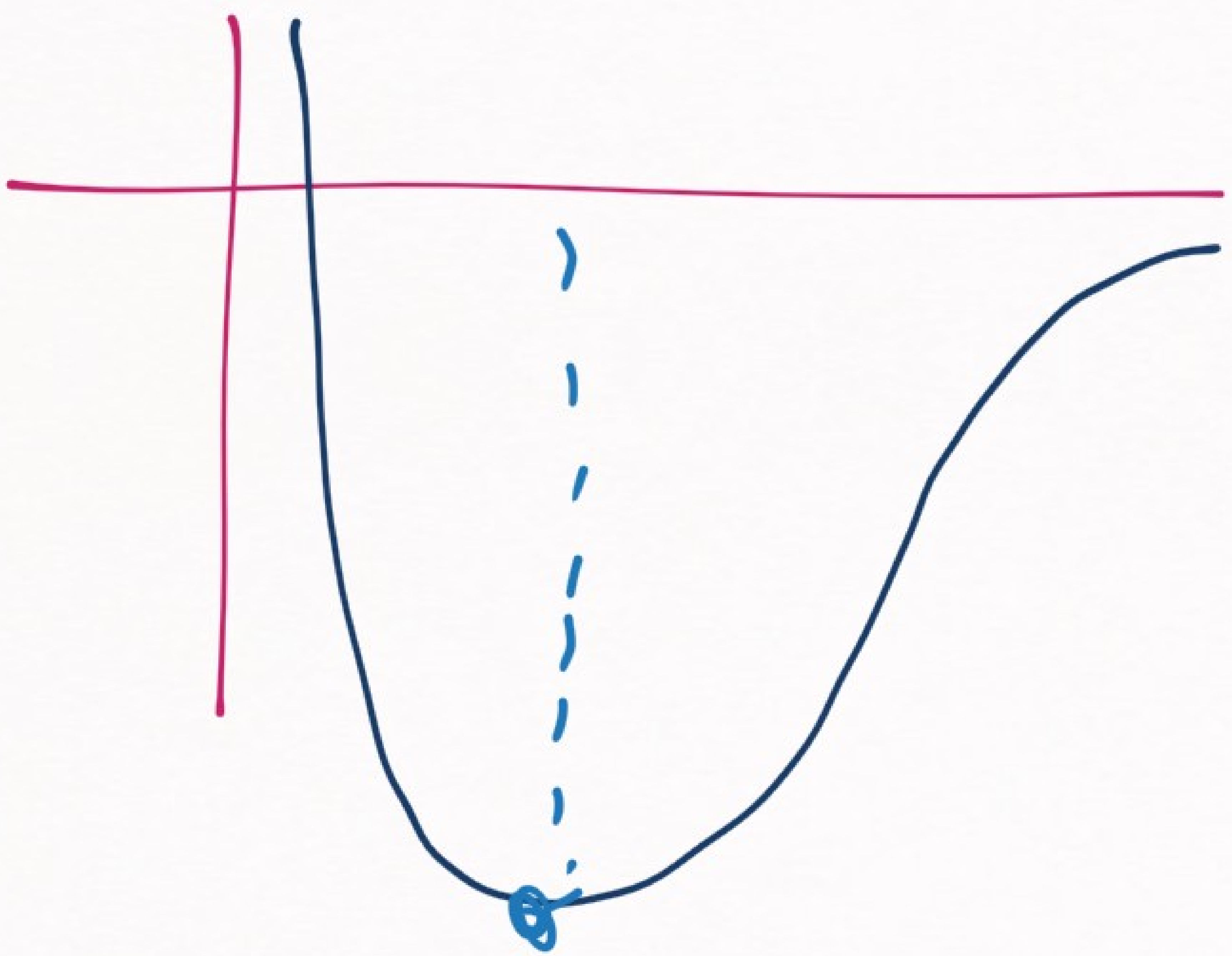
1,7 fm

2) Pion $\rightarrow$ $V(r) \sim \frac{e^{-m_\pi r}}{r}$ $\longrightarrow$ $\frac{1}{m_\pi}$

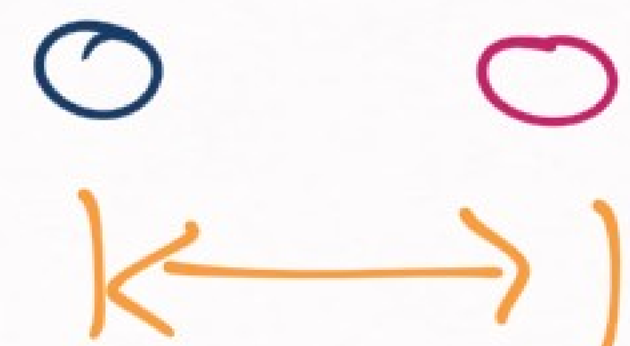
$m_\pi = 140\ \text{MeV}$, $\frac{\hbar c}{m_\pi} \sim 1.4\ \text{fm}$

2) ATTRACTIVE AT INTERMEDIATE DISTANCES

POTENTIAL → MINIMUM



$\boxed{R_{min} \sim 1{,}8\,\text{fm}}$



R_{min}

$R > R_{min}$

$R < R_{min}$

1) $\boxed{\frac{B}{A} \sim 8\,\text{MeV}}$ $\longrightarrow$ 1) attraction at medium distances

2) repulsion at short distance

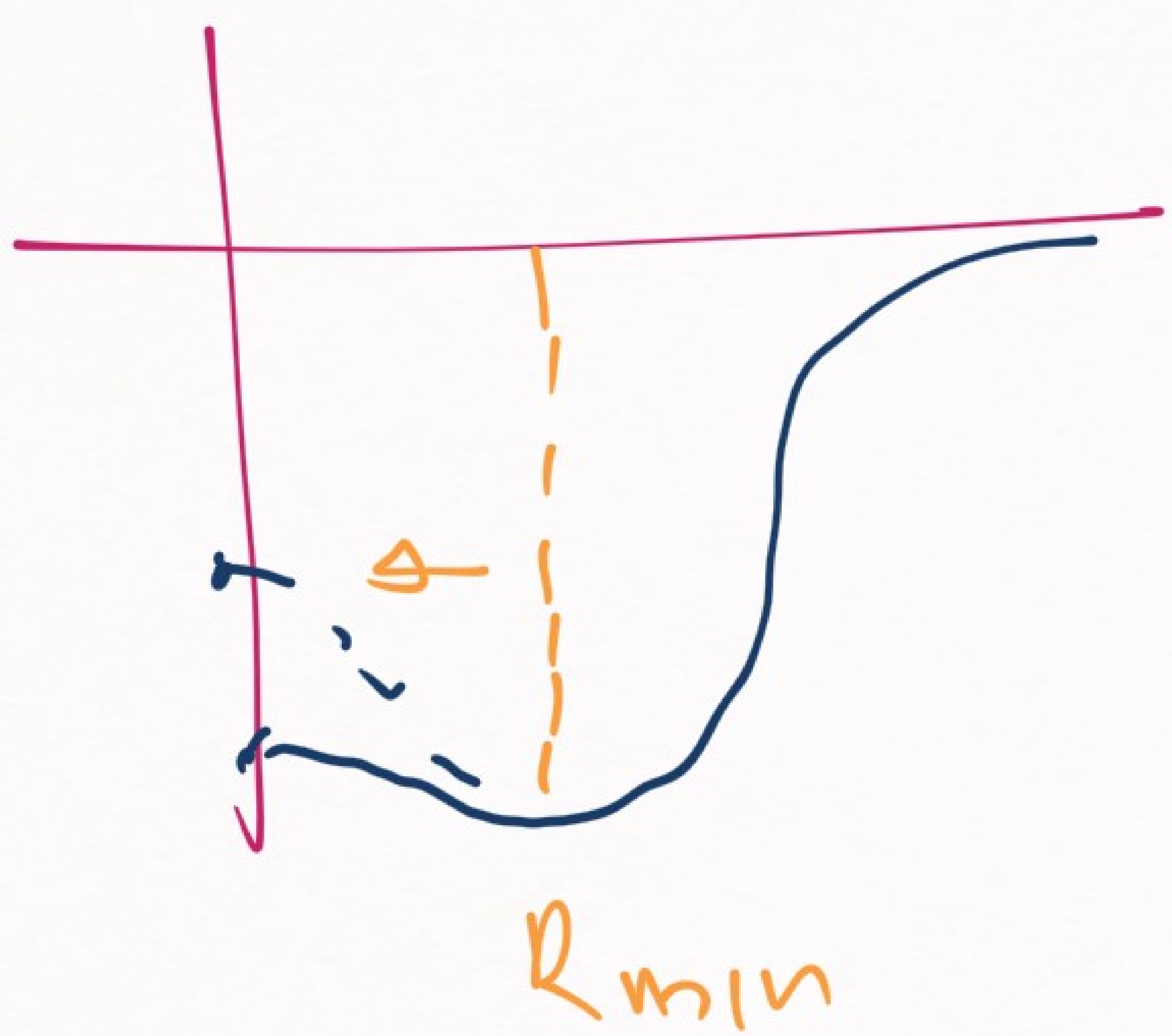
2) Density of heavy nuclei

~ 0.17 nucleons$/\text{fm}^3$

$\sim \left(\frac{1}{R_{min}}\right)^3$ nucleons $\Rightarrow$ $\boxed{R_{min} \sim 1.8\ \text{fm}}$

3) REPULSIVE AT SHORT-DISTANCES

3.a) no repulsion at short-distance



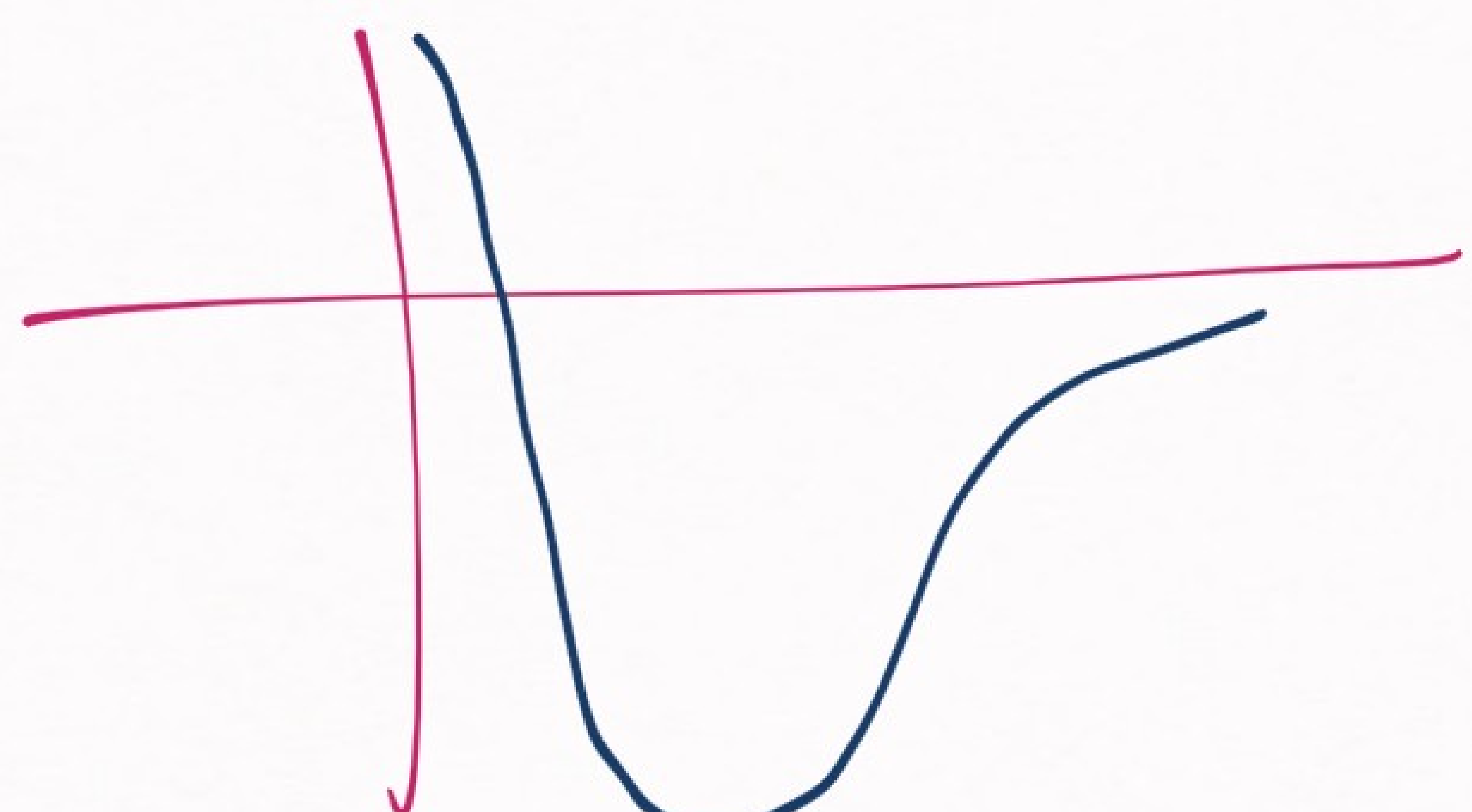
→ increase the nucleon density

↓

increase the binding energy per nucleon

$\frac{B}{A} \sim 8\,\text{MeV}$ → constant

→ good reason why we can't pack the nucleons closer together

→ 

3.b) $^1S_0$ phase shifts

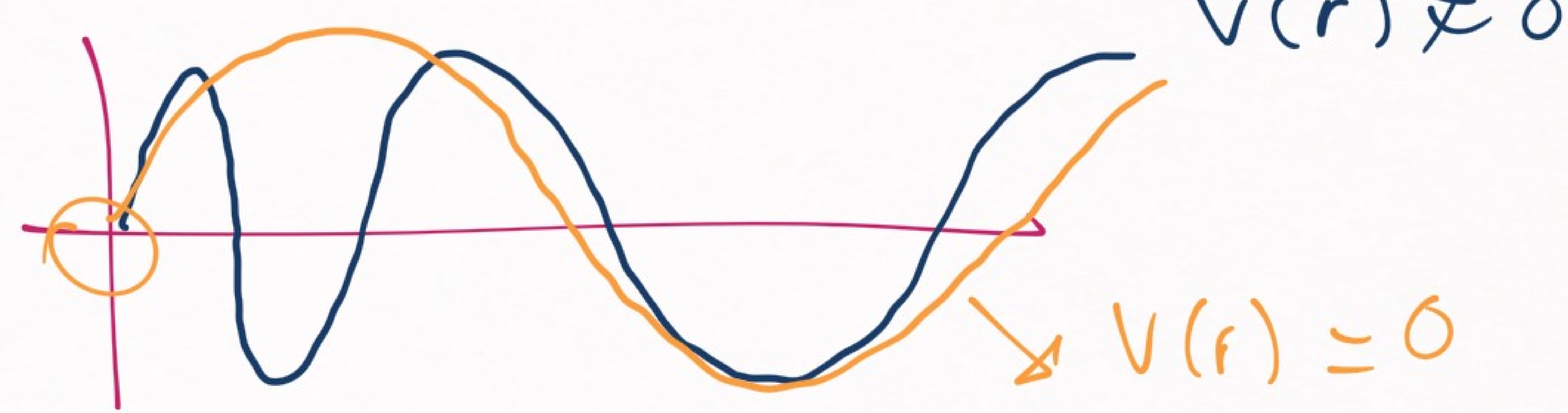
?

$k \to$ momentum

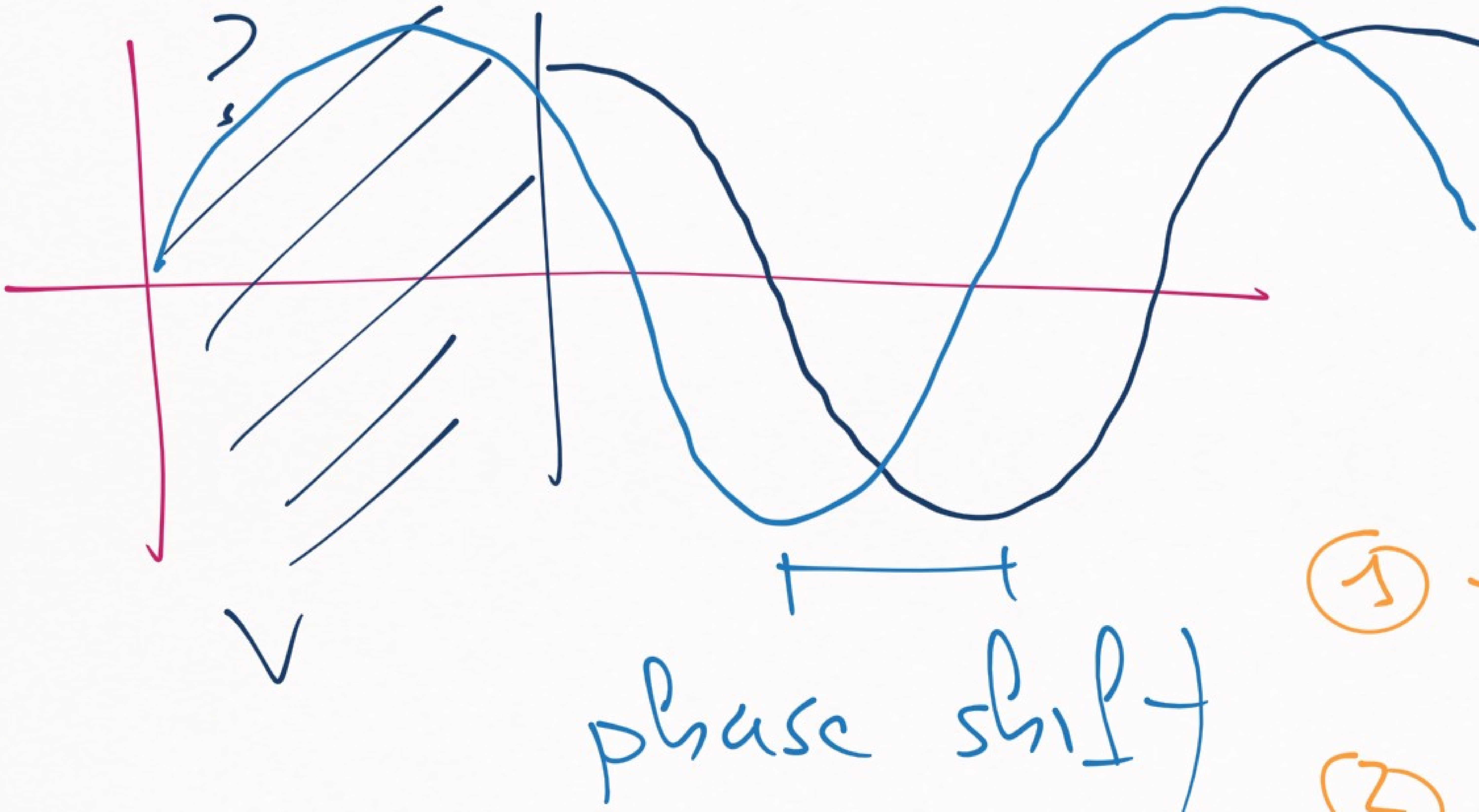
$$\psi(\vec{r}) = \frac{u(r)}{r} \longrightarrow V(r) = 0 \longrightarrow u(r) = \sin kr$$

& $u(0) = 0$

$V(r) \neq 0$



$V(r) \to 0$ for $mr \gg 1$ (short-ranged potential)



$V(r) \neq 0 \rightarrow$ 1

$V(r) = 0 \rightarrow$ 2

1 $\rightarrow \sin(kr + \underline{\underline{\delta}})$

2 $\rightarrow \sin kr$

$V(r) < 0 \rightarrow \delta > 0$

$V(r) > 0 \rightarrow \delta < 0$

(w/ caveats)

$^1S_0$ phase shift $\rightarrow$ two-nucleons

L, S, J

$\rightarrow \boxed{^{2S+1}L_J}$

"partial wave"

two-nucleon

s-wave

$L = 0$

$S = 0, 1$

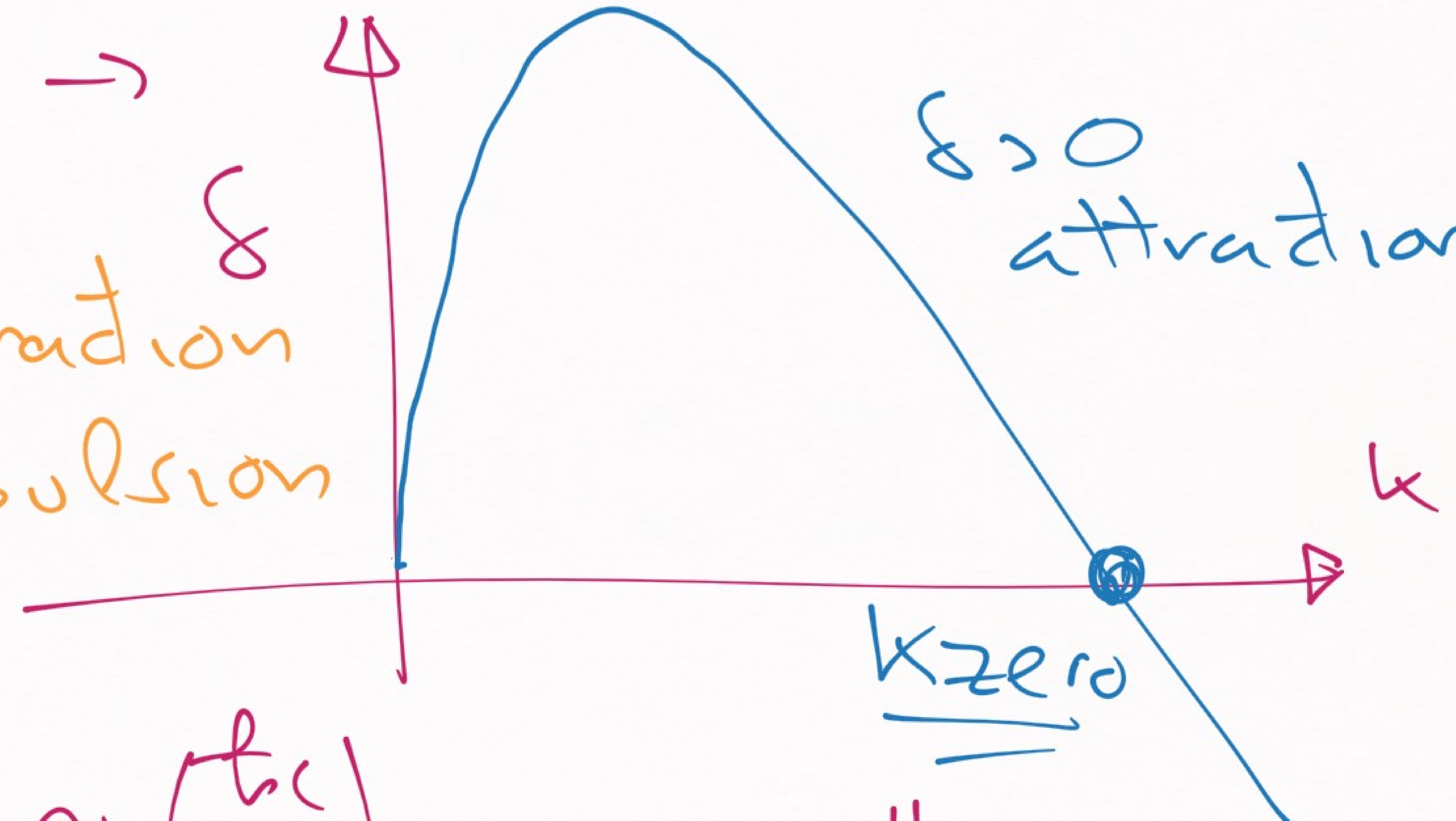
$J = S$

$S = 0 \rightarrow {}^1S_0 \rightarrow$ "singlet"

$S = 1 \rightarrow {}^3S_1 \rightarrow$ "triplet"

(deuteron)

$^1S_0$ phase shift $\rightarrow$



$k < k_{zero} \rightarrow$ attraction

$k > k_{zero} \rightarrow$ repulsion

$$K_{zero} \sim \frac{1}{R_{zero}} \sim \left(\frac{\hbar c}{360}\right)$$

$\sim (0.6 - 0.7)$ fm

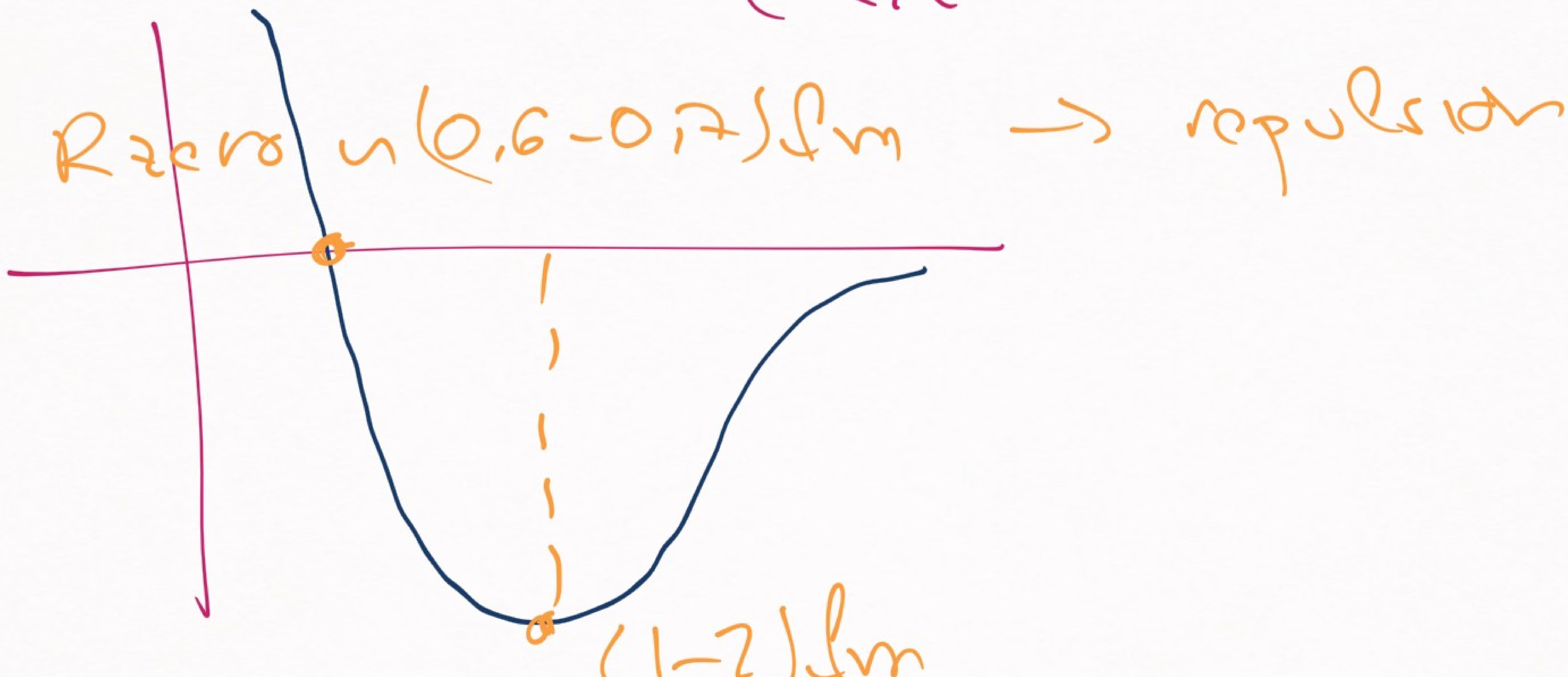
$k_{zero} = 300\,\mathrm{MeV}$

$\delta < 0$

repulsion

$\Rightarrow \boxed{r < R_{zero}} \rightarrow$ repulsion

($R_{zero} \sim 0.6-0.7$ fm)



$$pp \rightarrow \tfrac{1}{2} \otimes \tfrac{1}{2} = 0 \oplus 1$$

singlet triplet

fermions $\rightarrow$ antisymmetric $\rightarrow (-1)^{L+S+1} = (-1)$

$$L = 0 \Rightarrow \boxed{S = 0} \rightarrow \boxed{{}^1S_0}$$

singlet

$$\left.\begin{array}{l} nn \\ pp \end{array}\right\} L=0 \rightarrow \text{only singlet}$$ (because fermions)

$np \; \} \; L=0 \rightarrow S=0,1 \rightarrow$ singlet / triplet

$$\left[-\frac{1}{2\mu}\vec{\nabla}^2 + V(\vec{r})\right]\psi(\vec{r}) = E\psi(\vec{r})$$

$$\downarrow$$

$$-\nabla^2\psi(\vec{r}) = \underbrace{2\mu E}_{k^2}\psi(\vec{r}) \Rightarrow \psi(\vec{r}) = e^{i\vec{k}\cdot\vec{r}}$$

(plane wave)

$$\psi(\vec{r}) = \sum_{\ell m} Y_{\ell m}(\hat{r})\,\frac{u_\ell(r)}{r}$$

(partial wave decomposition)

$$\rightarrow \quad -u_\ell'' + \frac{\ell(\ell+1)}{r^2} u_\ell(r) = k^2 u_\ell(r)$$

$$\ell = 0 \rightarrow -u'' = k^2 u \longrightarrow$$

$$u(r) = A\sin(kr) + B\cos(kr)$$

$$= A\sin(kr + \delta)$$

no potential $\rightarrow$ $u(0) = 0$

$$\downarrow$$

$$\boxed{u(r) = A\sin(kr)}$$

4) THE NUCLEAR FORCE DOES NOT DISTINGUISH NEUTRONS & PROTONS

Why do we know this?

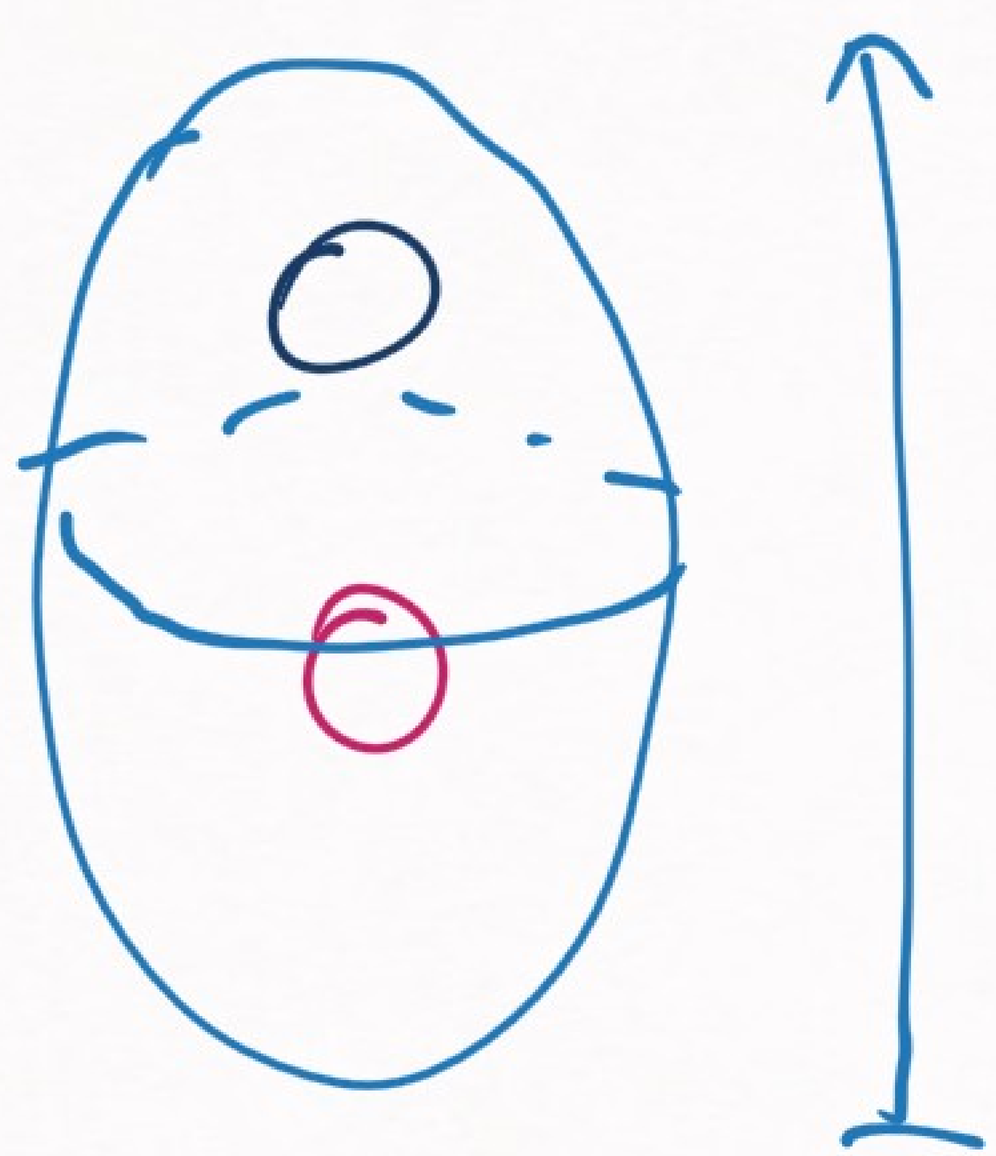
^3He $\quad$ ^3H (triton)

$$\underbrace{B(^3He)}_{7.72\,MeV} \sim \underbrace{B(^3H)}_{8.48\,MeV}$$

Coulomb repulsion in ^3He

5) NUCLEAR FORCE NOT CENTRAL

→ central force $V(\vec{r}) = V(|\vec{r}|)$



$V \neq V(|\vec{r}|)$

→ origin of nuclear forces → later

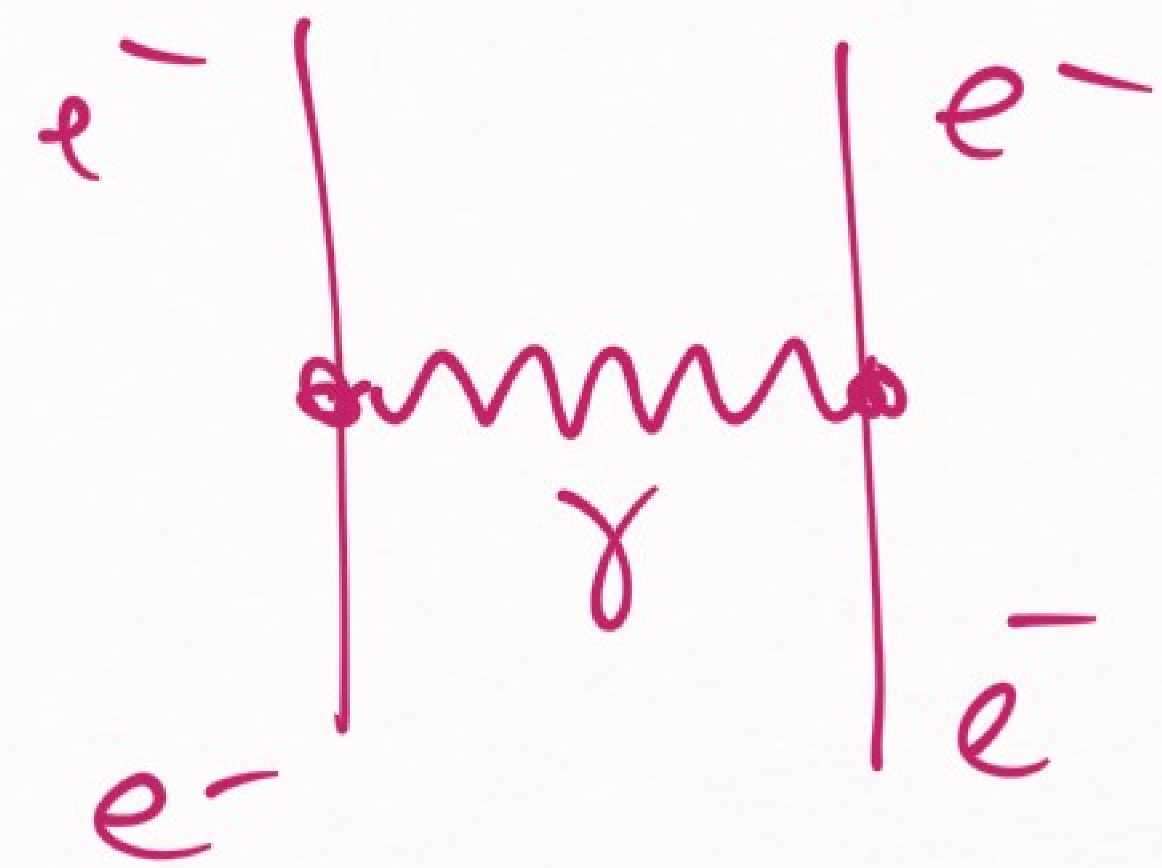
→ $\boxed{\text{ORIGIN OF NUCLEAR FORCES}}$

→ Quantum field theory

$\boxed{V(\vec{r})}$ → "just there" in classical physics or quantum mechanics

QFT → Coulomb

time ↑

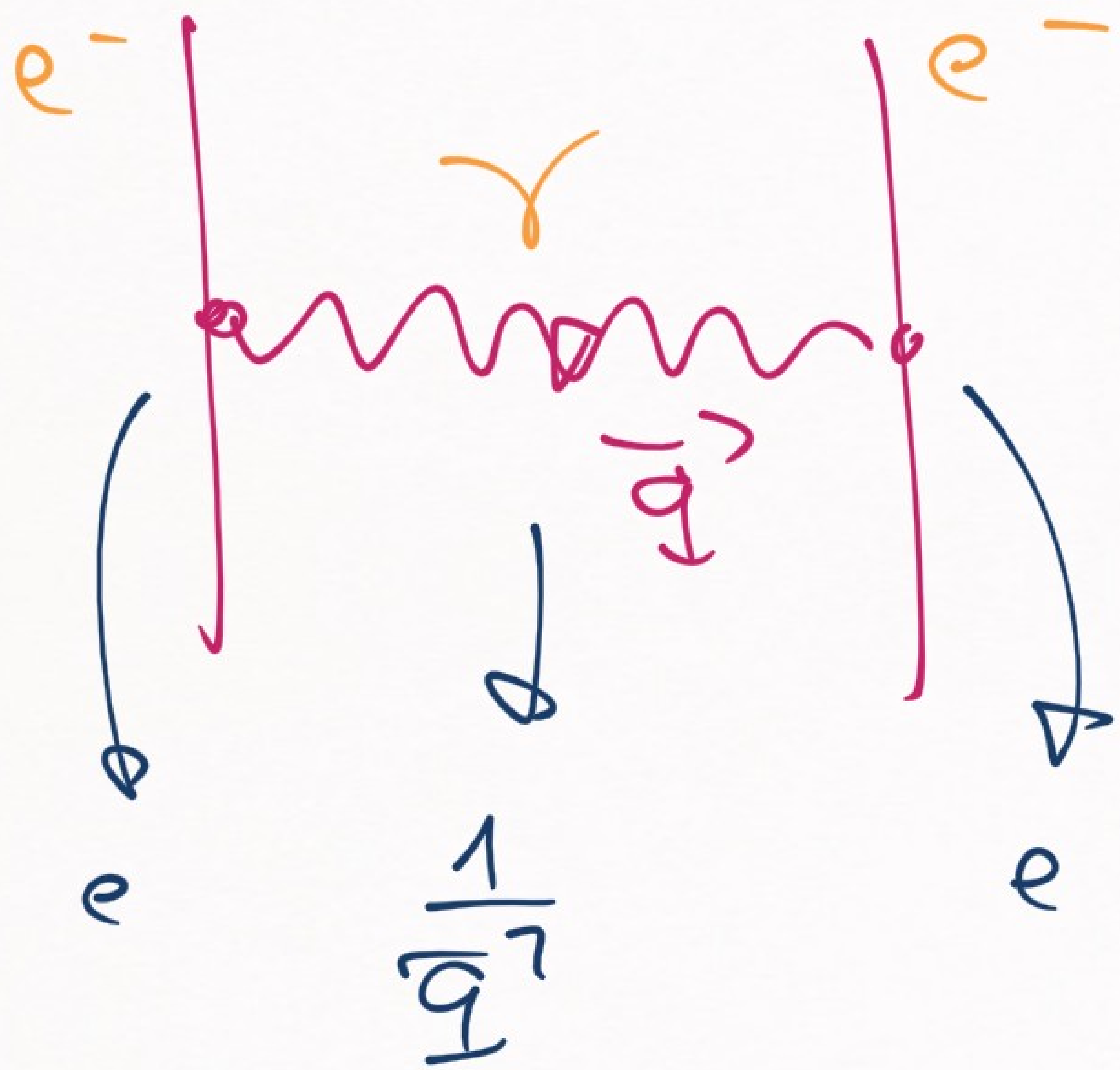


FEYNMAN DIAGRAM

"exchange of a photon"

Feynman rules

(Sect. 4.7 of Peskin & Schröder)



Non-relativistic limit

$$V(\vec{q}) = \frac{e^2}{|\vec{q}|^2}$$

$\leftrightarrow$ Fourier transform this

$$V(\vec{r}) = \int \frac{d^3\vec{q}}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}}\, V(\vec{q}) = \frac{e^2}{4\pi r}$$

$$\frac{e^2}{4\pi} = \alpha \simeq \frac{1}{137} \Rightarrow \boxed{V(r) = \frac{\alpha}{r}}$$

FOR NUCLEONS $\rightarrow$ SHORT-RANGED FORCE

YUKAWA $\rightarrow$ [EXCHANGE OF A MASSIVE BOSON]

$\boxed{\text{YUKAWA}}$ $\rightarrow$ $J^P = 0^+$ (SCALAR BOSON) $\rightarrow$ "pion"

angular momentum

parity

Use Feynman rules $\longrightarrow$

$$\underset{N}{ig} \quad \overset{\pi}{\frac{1}{m^2+q^2}} \quad \underset{N}{ig} = \frac{(ig)^2}{q^2+m^2}$$

$\boxed{\text{YUKAWA}}$ $\rightarrow$ $J^P = 0^+$ "pion" gives me:

$$V(\vec{q}) = -\frac{g^2}{\vec{q}^2 + m^2} \rightarrow V(\vec{r}) = -\frac{g^2}{4\pi}\frac{e^{-mr}}{r}$$

finite-range

1935: $m \sim (100-200)$ MeV

[YUKAWA + $J^P = 0^+$] $\longrightarrow$ $V(\vec{r}) = -\frac{g^2}{4\pi}\frac{e^{-mr}}{r}$

A set of the properties of the nuclear force (not all of them)

$\longrightarrow$ gives you a central force

($\Rightarrow$ deuteron?)

DEUTERON → [QUADRUPOLE MOMENT]

inside an electric field

↓

Electromagnetism 101

dipole moment

$$V = q\Phi(\vec r) + \vec d \cdot \vec\nabla \Phi(\vec r) + \frac{1}{6} Q_{ij} \partial_i \partial_j \Phi(\vec r) + \dots$$

charge

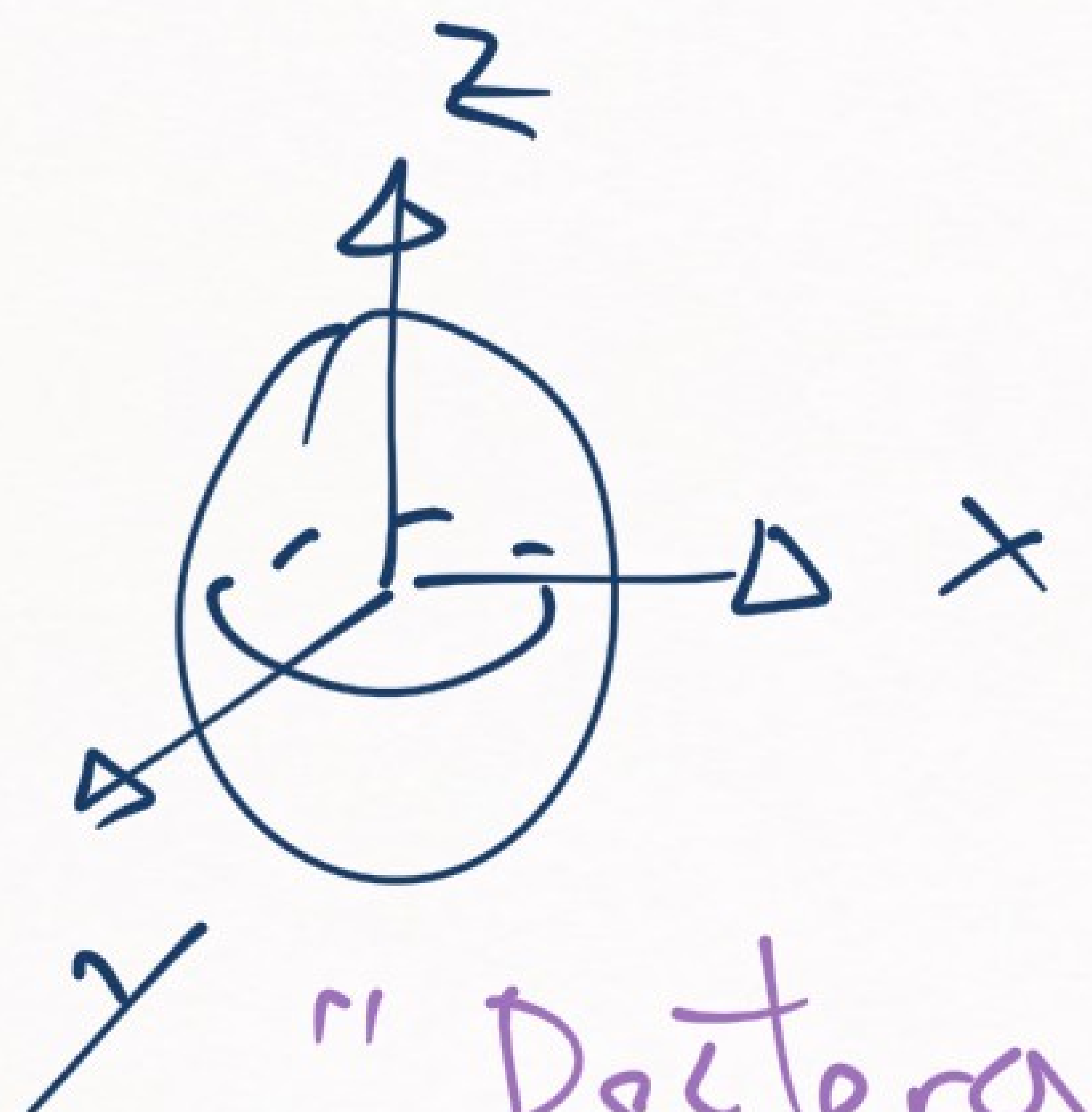
Charge → $q = \int d^3\vec{r}\, \boxed{\rho(\vec{r})}$ → charge distribution

Dipole moment → $\vec{d} = \int d^3\vec{r}\, \vec{r}\, \rho(\vec{r})$

Quadrupole moment → $Q_{ij} = \int d^3\vec{r}\, \rho(\vec{r}) \left(3 r_i r_j - r^2 \delta_{ij}\right)$

Go back to EM textbook and check

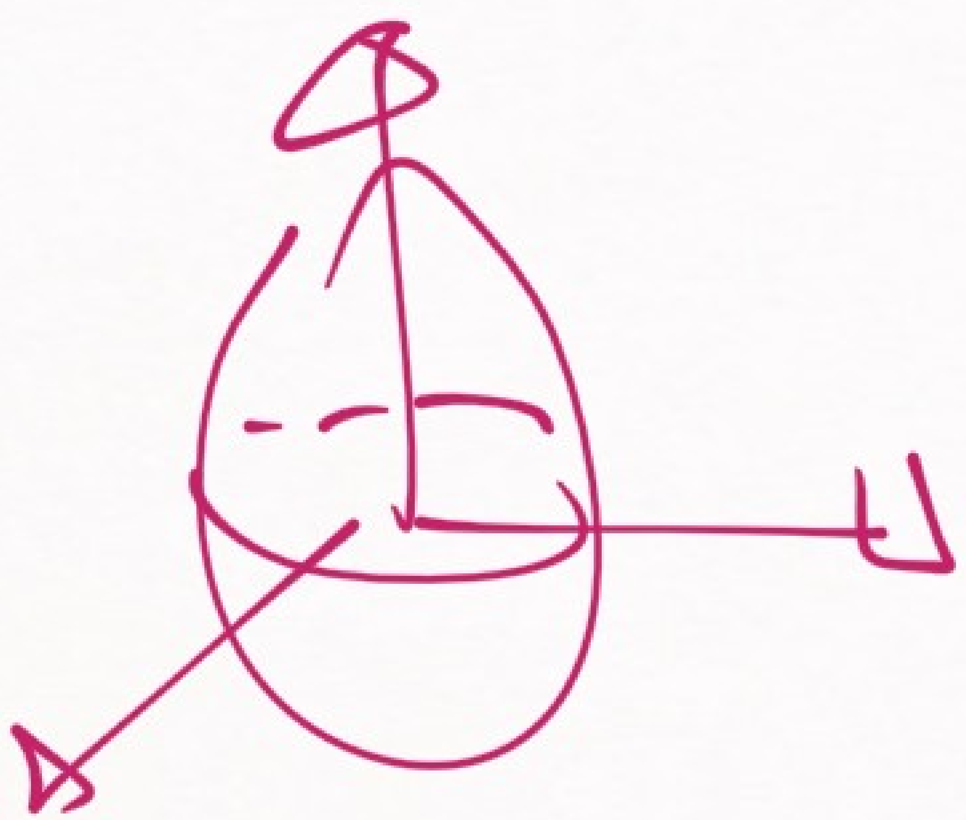
Quadrupolar moment $\rightarrow$ $\underbrace{Q \doteq Q_{33}}_{\text{definition}}$



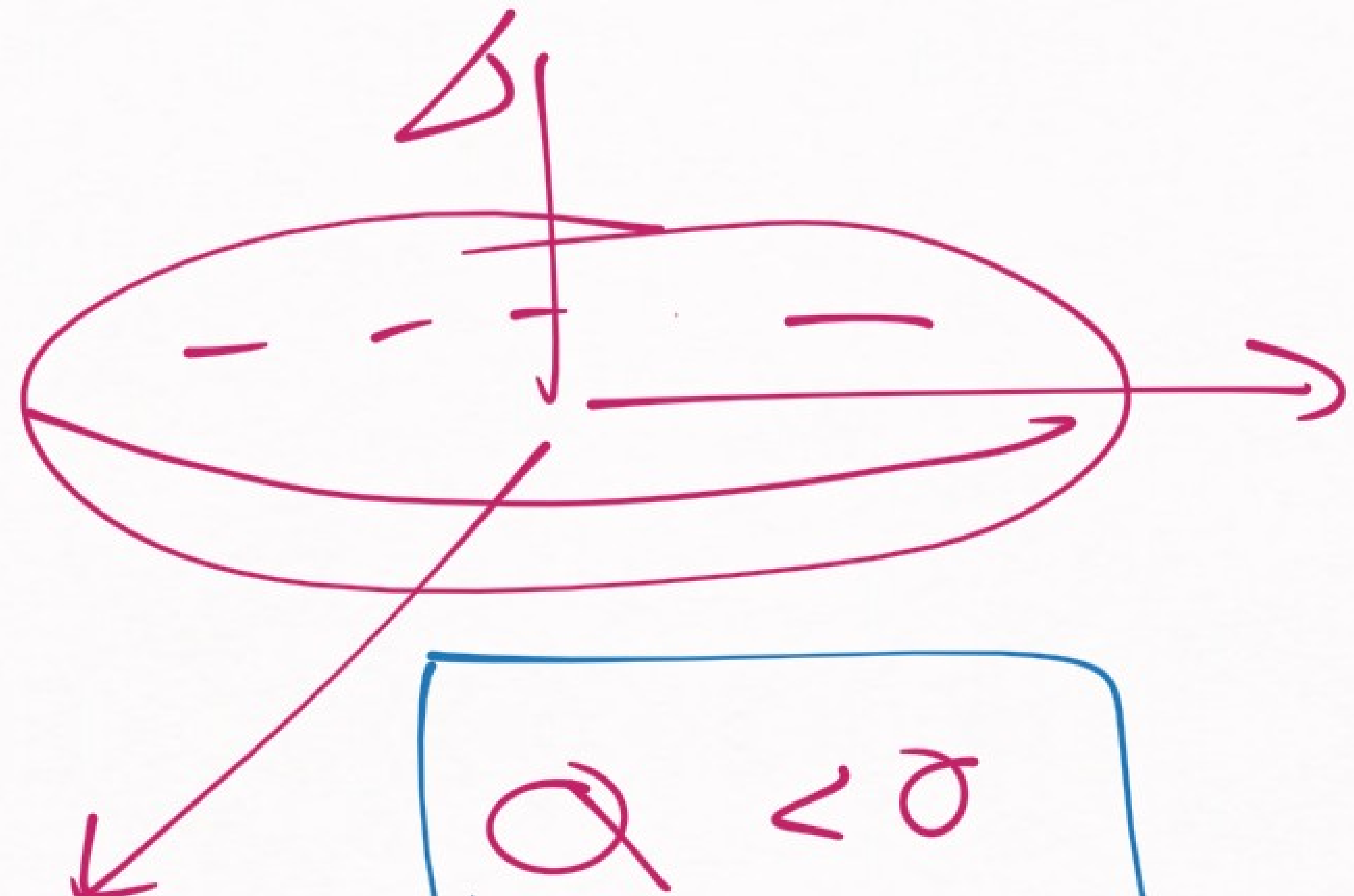
"Deuteron"

$$\boxed{Q > 0}$$

$$Q_d = 0.286\ fm^2$$



$\boxed{Q > 0}$



$\boxed{Q < 0}$

→ Why the deuteron has $Q > 0$?

NEXT LESSON