Nuclear Physics (Lesson 2)

"Why nuclear physics is difficult"

$\boxed{\text{RECAP}}$

→ Scales are important to understand physics

→ Natural problems

Easy estimations of observables in terms of scales

[HYDROGEN ATOM]

$$a_B = \frac{1}{m_e \alpha} \simeq 5.4 \cdot 10^4 \text{ fm}$$

$$(0.54 \text{ Å})$$

"Bohr radius"

$$\left[-\frac{1}{2\mu} \vec{\nabla}^2 - \frac{\alpha}{r} \right] \psi(\vec{r}) = E_B \, \psi(\vec{r})$$

$$\left[-\vec{\nabla}^2 - \frac{2\mu\alpha}{r}\right]\psi(\vec{r}) = -\underbrace{\gamma_B^2}_{\text{wave number}}\,\psi(\vec{r})$$

"reduced potential"

$$E_B = -\frac{\gamma_B^2}{2\mu}$$

$$\frac{2\mu\alpha}{r} = \frac{2}{a_B r} \Rightarrow a_B = \frac{1}{\mu\alpha}, \quad \mu \simeq m_e$$

$$\underbrace{\left[-\vec{\nabla}^2 - \frac{2}{a_B r}\right]}_{\text{Input}} \psi(\vec{r}) = \underbrace{-\gamma_B^2 \psi(\vec{r})}_{\text{output}}$$ Solve it

Input $\longrightarrow$ output

(QM + Coulomb potential)

Input: $a_B \rightarrow [a_B] = L$

Output: $\gamma_B, \psi(\vec{r}) \rightarrow [\gamma_B] = [L]^{-1}$

$$\left[-\nabla^2 + U(r, \underbrace{a_1, \dots, a_n}) \right] \psi(\vec{r}) = -\gamma^2 \psi(\vec{r})$$

INPUT : $\boxed{a_1, a_2, \dots, a_n}$

OUTPUT : $\gamma, \psi(\vec{r})$

→ MULTISCALE PROBLEM

↓

STANDARD SITUATION

characteristic scales

HYDROGEN ATOM :

→ Expectation : $r_B \sim \frac{1}{a_B}$

$$r_B = \frac{c_B}{a_B}, \quad c_B \sim \mathcal{O}(1) \text{ "naturalness"}$$

→ Reality : (ground state, 1s)

$$\psi(\vec{r}) = \frac{1}{\sqrt{4\pi}} \frac{2}{a_B^{3/2}} e^{-r/a_B}$$

1) $\psi(\vec{r}) \underset{r \to \infty}{\longrightarrow} e^{-\gamma r}$

2) $\psi(\vec{r}) = \dfrac{1}{\sqrt{4\pi}} \dfrac{2}{a_B^{3/2}} e^{-r/a_B}$

$\longrightarrow \boxed{\gamma_B = \dfrac{1}{a_B}}$

$$E(1S) \simeq -\frac{1}{2m_e}\left(\frac{1}{a_B}\right)^2 \simeq -\frac{1}{2m_e}\gamma_B^2$$

1) Expectation: $\sqrt{\langle r^2 \rangle}$ (mean square radius)

$$[\sqrt{\langle r^2 \rangle}] = [L]$$

$$\sqrt{\langle r^2 \rangle} \sim a_B$$

$$\sqrt{\langle r^2 \rangle} = d_B a_B$$

$$d_B \sim O(1)$$

2) Reality:

$$\langle r^2 \rangle = \int d^3\vec{r}\, |\psi(\vec{r})|^2 r^2$$

$$(\langle \hat{O} \rangle = \langle \psi | \hat{O} | \psi \rangle)$$

$$\langle r^2 \rangle = \int d^3\vec{r}\, |\psi(\vec{r})|^2 r^2 = \int d^3\vec{r}\, \frac{1}{4\pi} \frac{4}{a_B^3} e^{-2r/a_B} r^2$$

$$= \left(\int d^3\vec{r} = 4\pi \int r^2 dr \right) = \int r^4 dr\, \frac{4}{a_B^3} \underbrace{e^{-2r/a_B}}_{e^{-x}}$$

$$= \left(x = \frac{2r}{a_B} \right) = \frac{a_B^2}{8} \int_0^\infty dx\, x^4 e^{-x} = \underline{\underline{3 a_B^2}}$$

$$\left(\int_0^\infty dx\, x^n e^{-x} = n! \right) ?$$

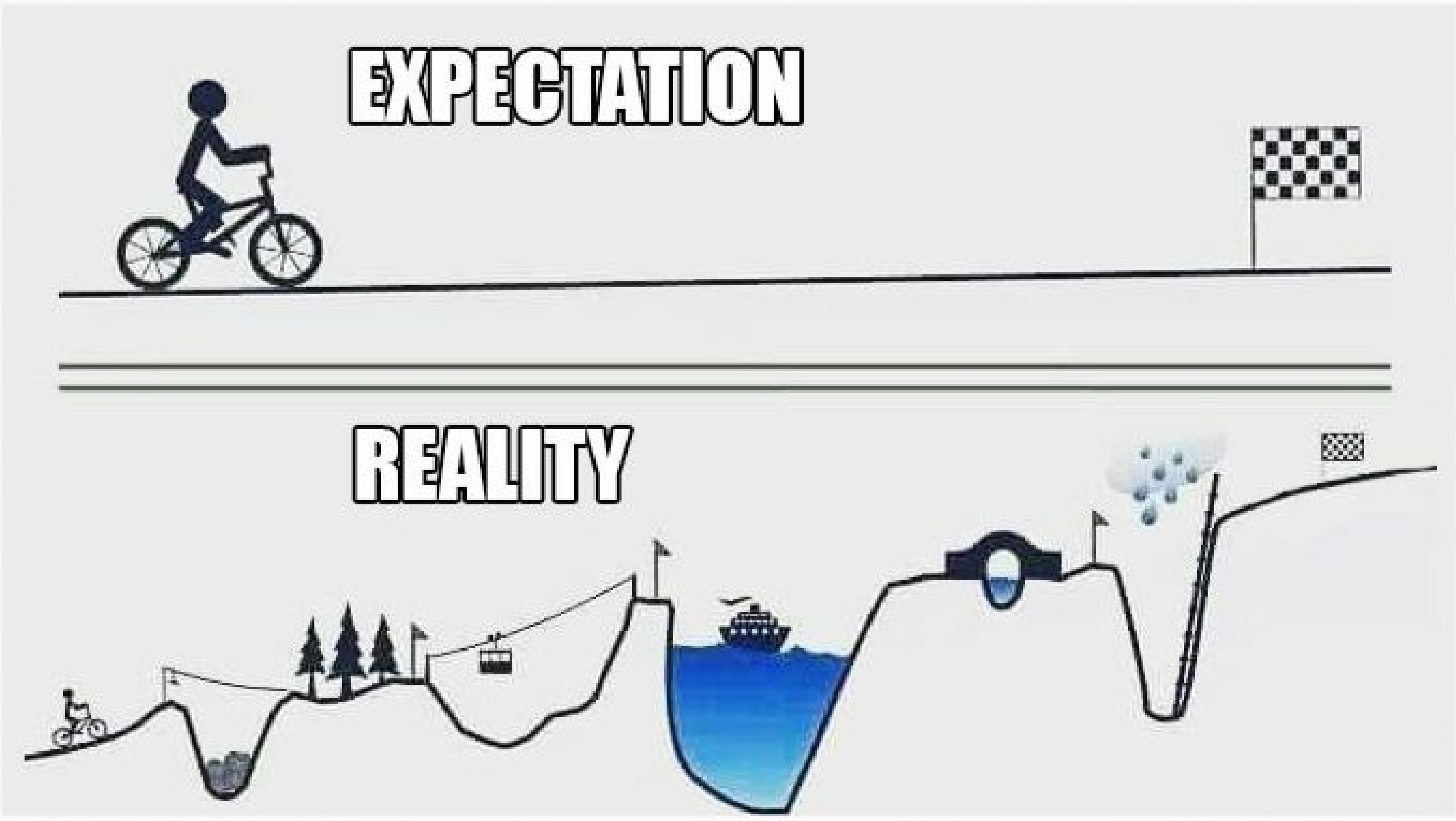
$$\langle r^2 \rangle = 3a_B^2 \Rightarrow \sqrt{\langle r^2 \rangle} = \sqrt{3}\, a_B$$

$$\sqrt{\langle r^2 \rangle} = d_B\, a_B \,,\; d_B \sim O(1)$$

$$\Rightarrow \boxed{d_B = \sqrt{3} \simeq 1.7\ldots} \checkmark$$

Expectation meets reality!

Life is not so easy



(in some cases it is easy; natural problems)

NATURAL PROBLEM

(i) Characteristic scale

Example: hydrogen atom (atomic physics)

$\propto$

(electron mass) $\times$ (strength of Coulomb)

$$Q_B = m_e \alpha = (0.511\ \text{MeV}) \times \frac{1}{137} \simeq 3.7\ \text{keV}$$

$$Q_B = 3{,}7\,\text{keV} \longrightarrow a_B = \frac{1}{Q_B} = \frac{\hbar c}{3{,}7\,\text{keV}} \simeq 0.54\,\text{Å}$$

natural problems have easy to identify scales

(ii) Everything else is $O(1)$ in terms of the characteristic scale

NATURAL PROBLEM:

(i) Characteristic scale (Q)

(ii) Everything else $O(1)$ in terms of Q

Example : hydrogen atom $\quad Q_B = \frac{1}{a_B}$

$r_B \sim Q_B \quad , \quad E_B \sim -\frac{1}{2m_e}(Q_B)^2 \quad , \quad \sqrt{\langle r^2 \rangle} \sim a_B$

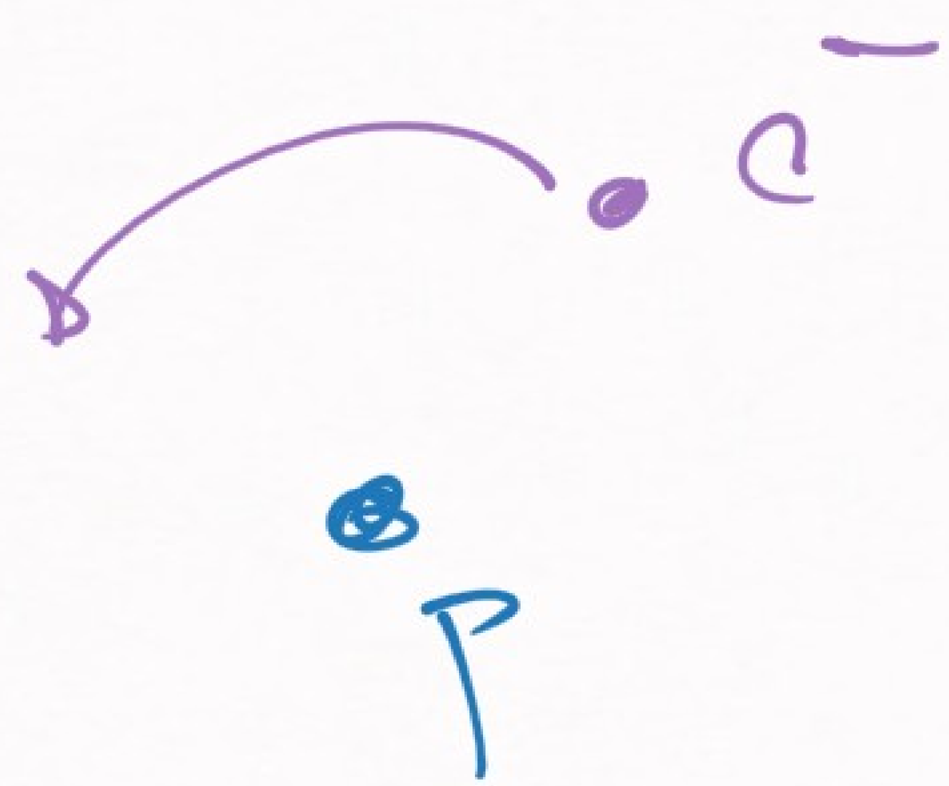
What is the speed of an electron inside the hydrogen atom?

$$\langle p \rangle \sim Q_B \quad \Rightarrow \langle v \rangle \sim \frac{Q_B}{m_e} \sim \frac{m_e \alpha}{m_e} \sim \alpha$$

$$p = m_e v \qquad \alpha \sim \frac{1}{137}$$

$$\langle v \rangle \sim \frac{1}{137} \quad \left(\left\langle \frac{v}{c} \right\rangle,\ c = 1\right)$$

$$\sim \frac{3 \cdot 10^5 \text{ km/s}}{137} \sim \underline{\underline{2200 \text{ km/s}}}$$



$\langle v \rangle \sim 2200 \text{ km/s}$

MESSAGE: $\boxed{\text{NATURAL PROBLEMS ARE EASY}}$

→ Really stupid (*) estimations of anything (and they will work)

(*) good sense of stupid

Example: Boiling point of Cesium
(铯)

~~FIVE MINUTE BREAK (YS)~~

OVER

CAVEAT → "characteristic scale" is ambiguous & requires physical intuition

good question (not trivial)

p e

$\underbrace{\text{size}}_{\text{scale}}$

^4He, $\boxed{\underbrace{\text{size} \simeq 1.7\ \text{fm}}_{\text{scale}}}$

Size of the electron → "point-like"

QFT → $\boxed{\dfrac{\hbar c}{m_e} \simeq 400\ \mathrm{fm}}$

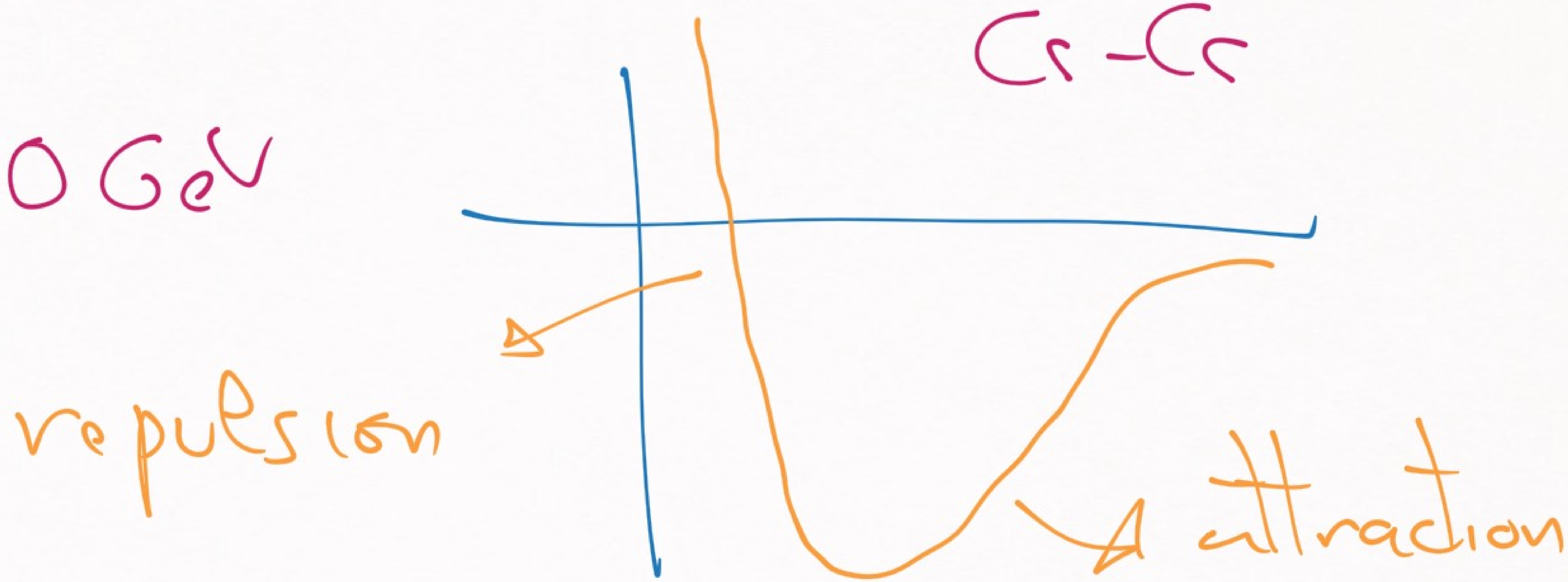
"characteristic scale"

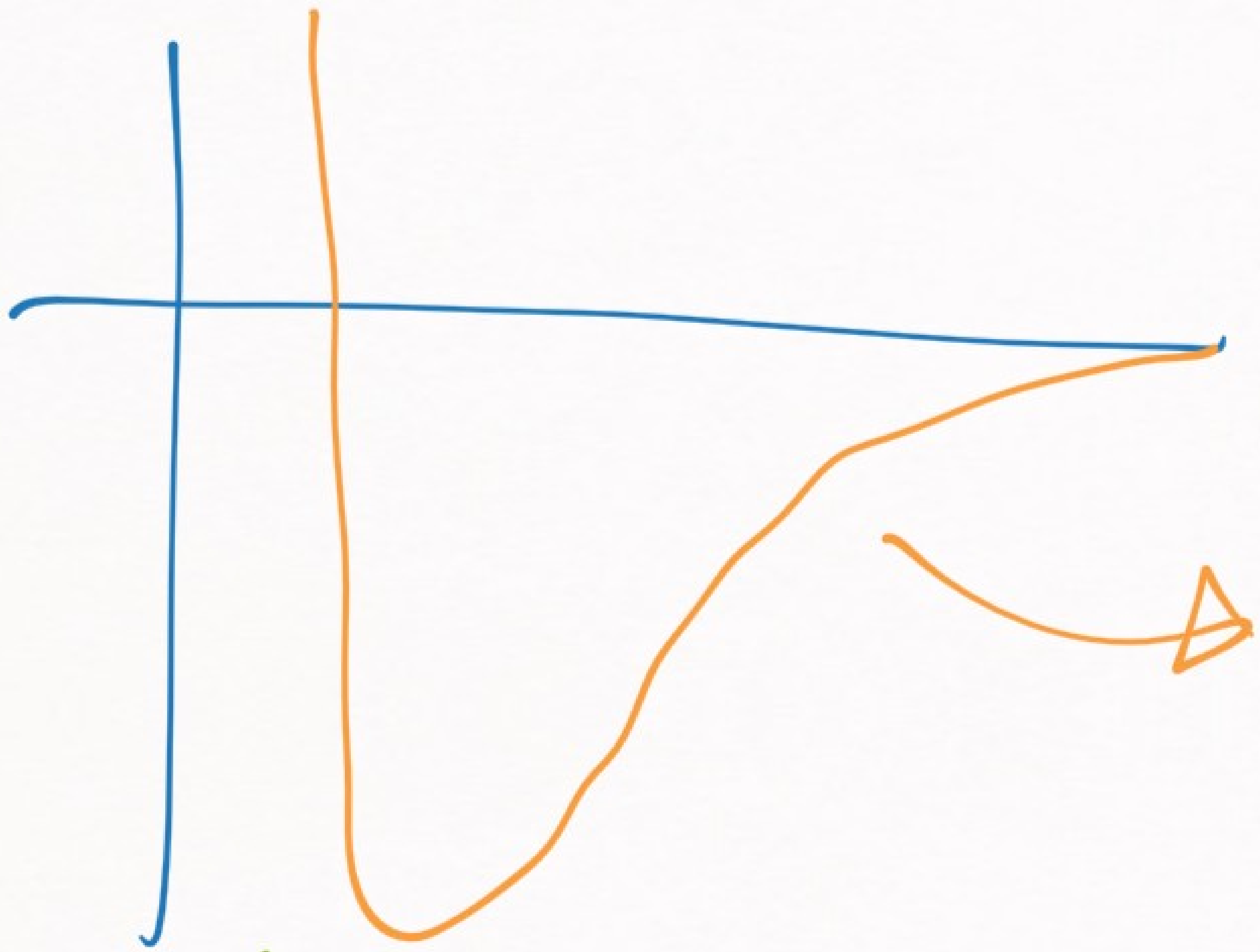
(less obvious example)

$\boxed{\text{BOILING POINT OF CS (毛色)}}$

Cs → 133 (atomic number)

m(Cs) ≃ 130 GeV

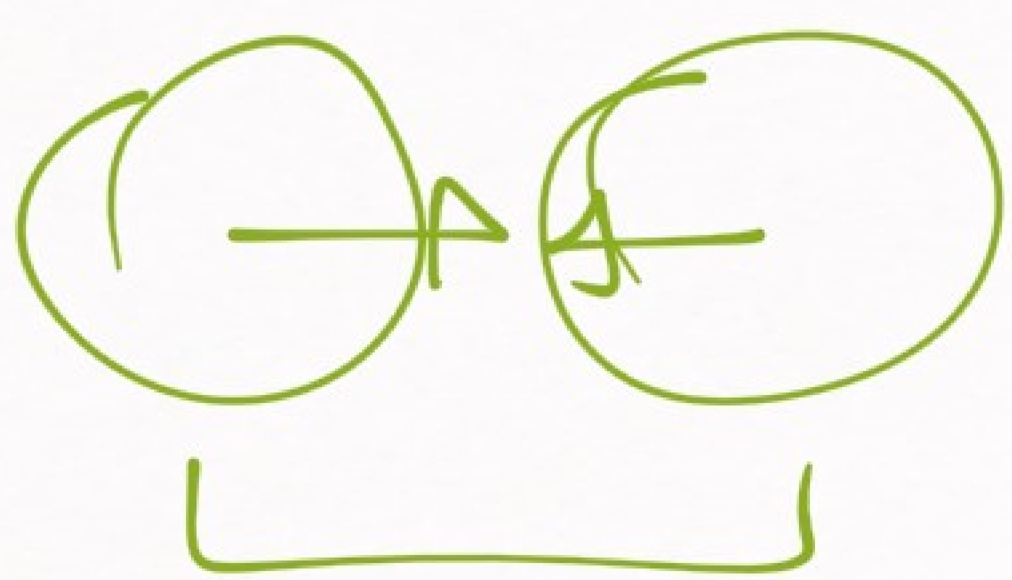




"van der Waals" potential

$$V_{vdW} = -\frac{C_6}{r^6} = -\frac{1}{2\mu}\frac{R_6^4}{r^6}$$

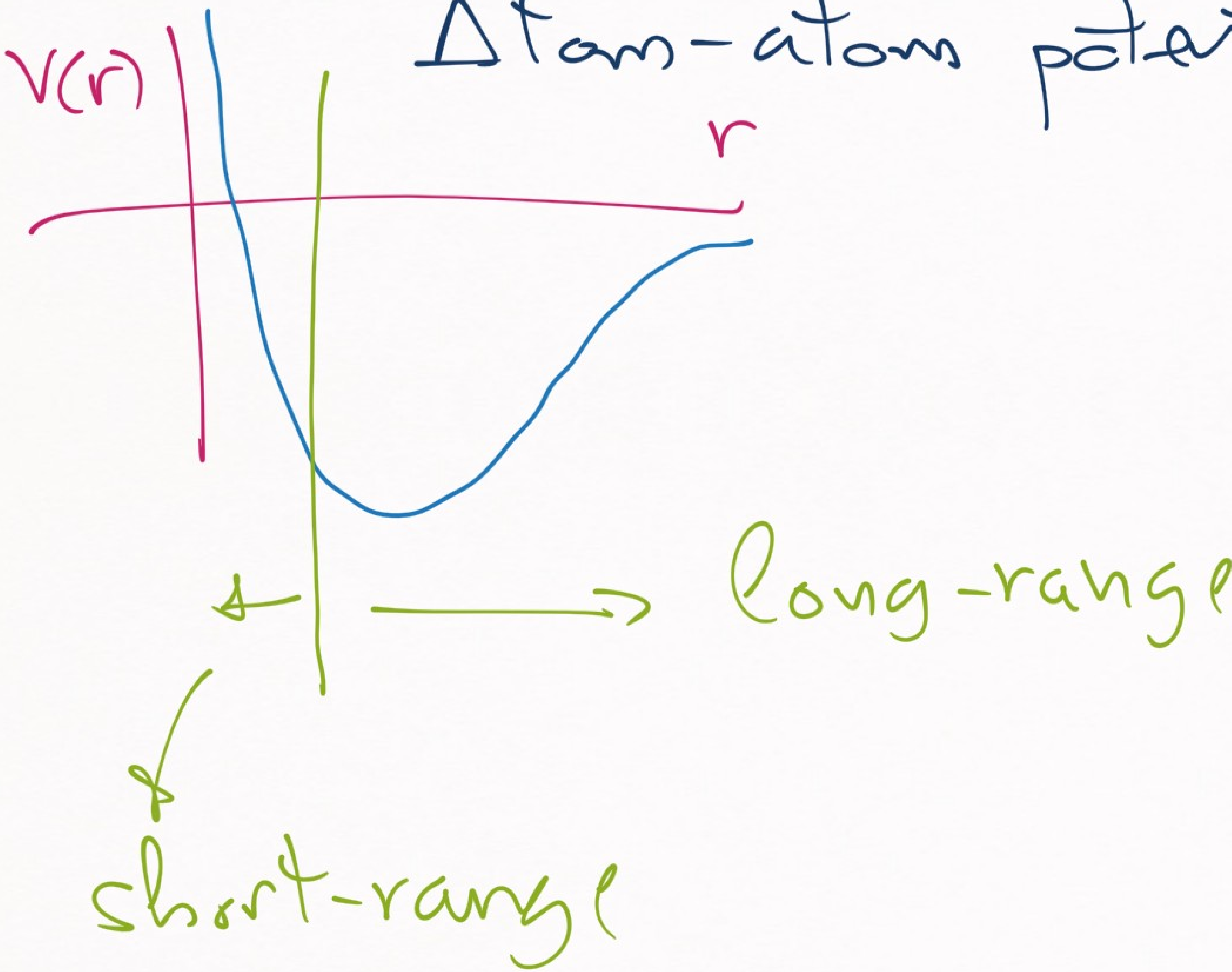
short-range



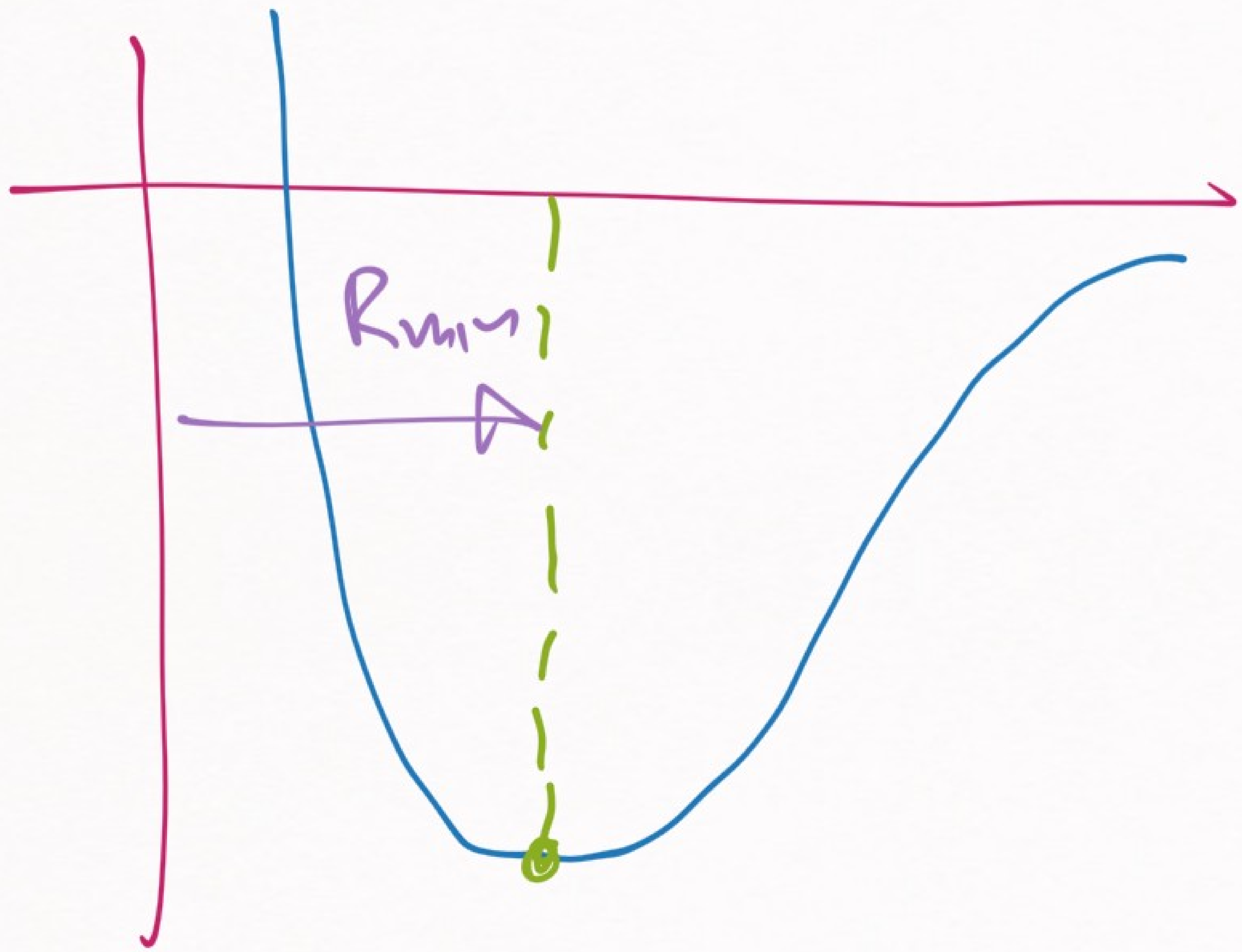
repel

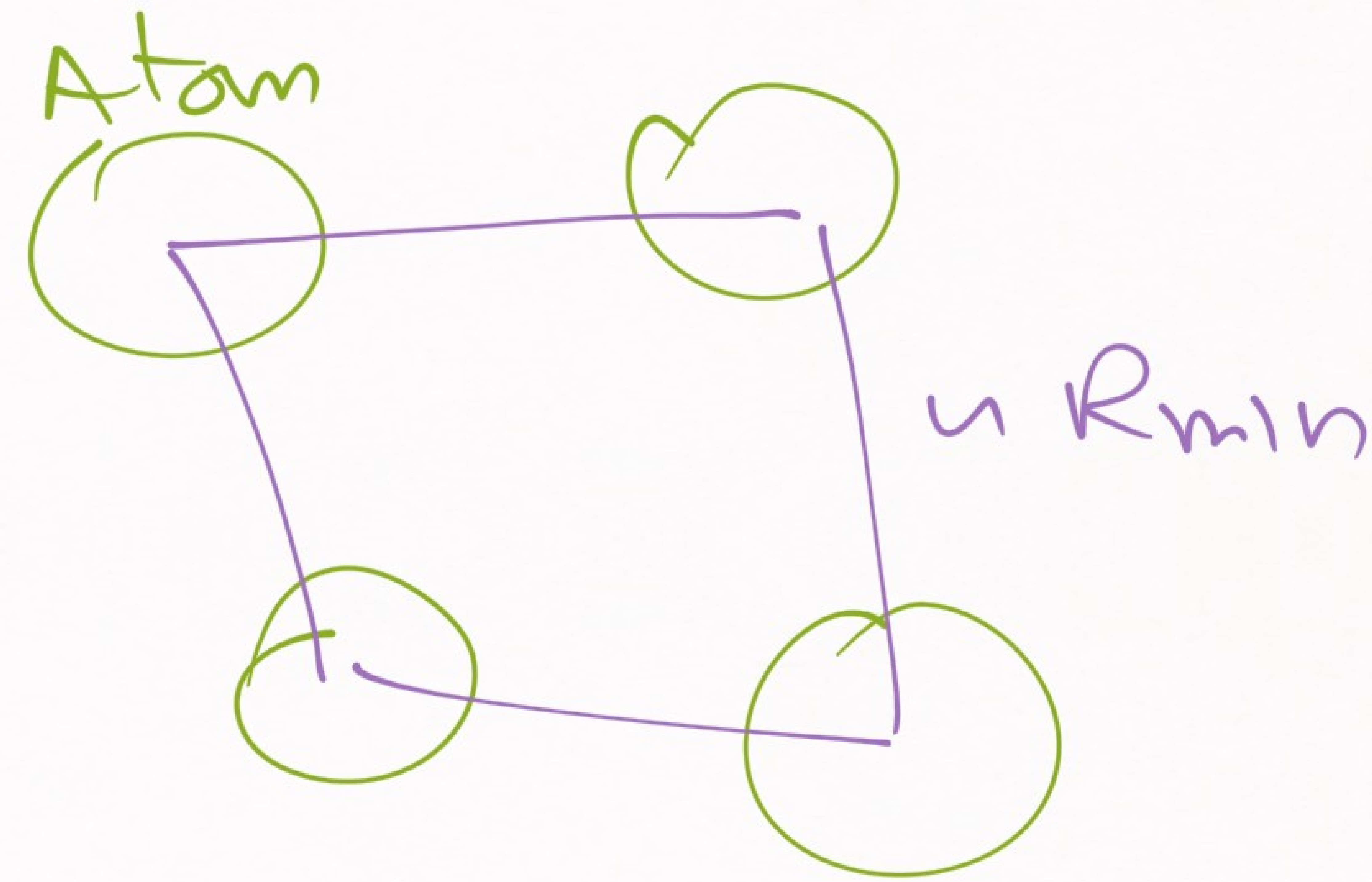
$$(2\mu V = U \to [L]^{-2}$$

Atom-atom potential



Nature likes minimum energy configurations


Rmin


Atom
~ Rmin

R_{min}

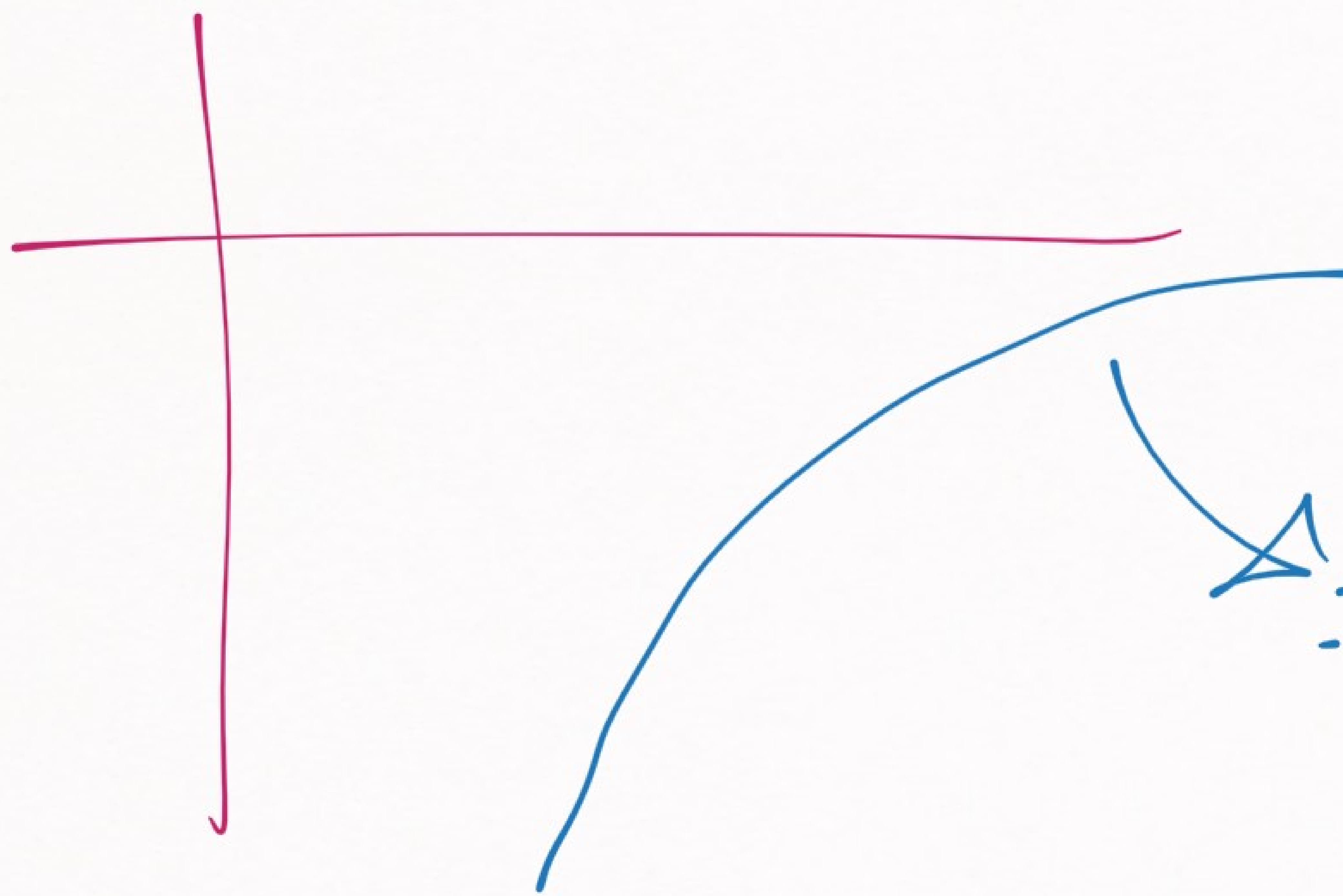
$\Rightarrow$ What is their speed at the minimum energy configuration?

$\boxed{\text{PyQ}}$

$$2\mu V_{vdW} = \frac{-R^4_{vdW}}{r^6}$$

You could try to guess $Q \sim \dfrac{1}{R_{vdW}}$

(wrong)



R_{vdW} is the scale of the tail of the potential

RELATED TO THE "CHARACTERISTIC SCALE"

IDENTIFICATION PROBLEM

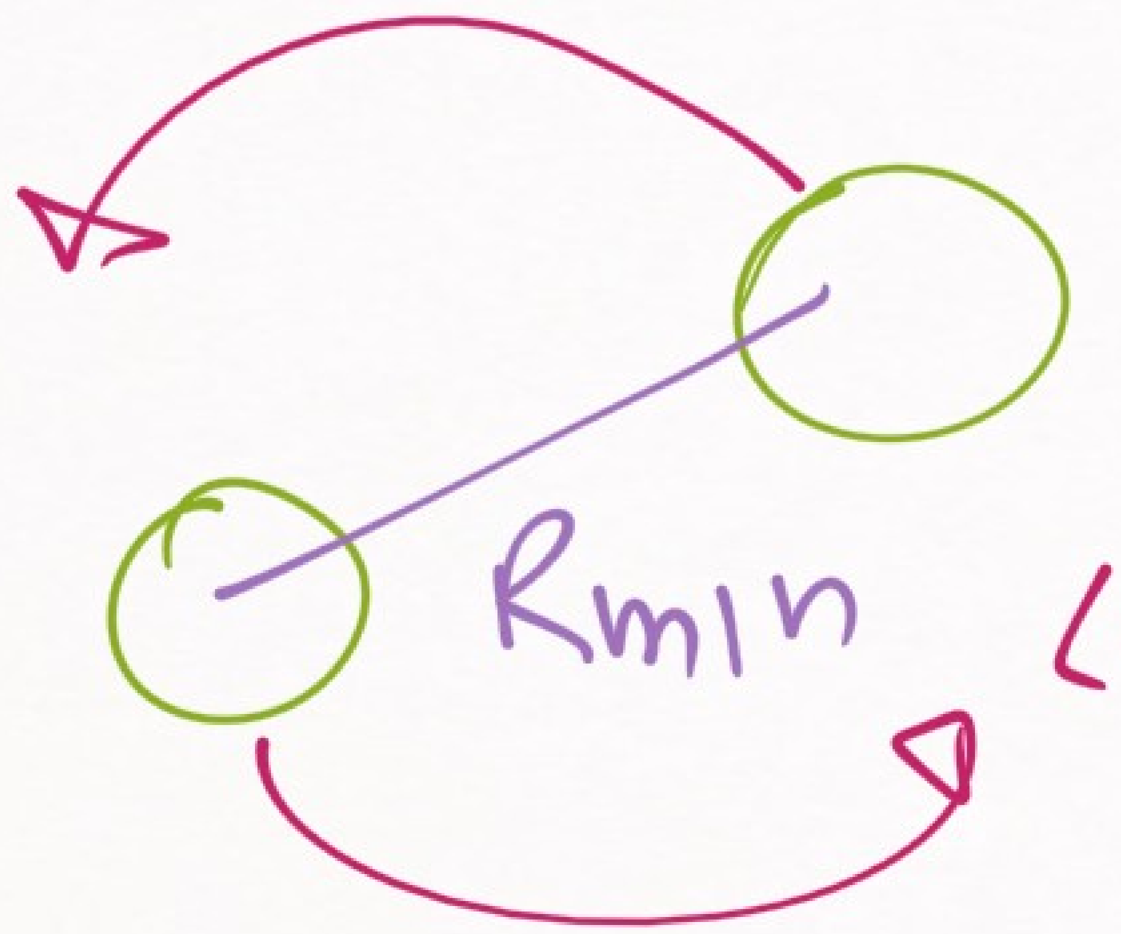
→ it's not trivial

$1\ a.u = 1\ a_B$

(Bohr radius)

$R_{vdW}(Cs-Cs) \sim 203\ a_B$

203 a.u. (atomic units)



$\langle v \rangle$?

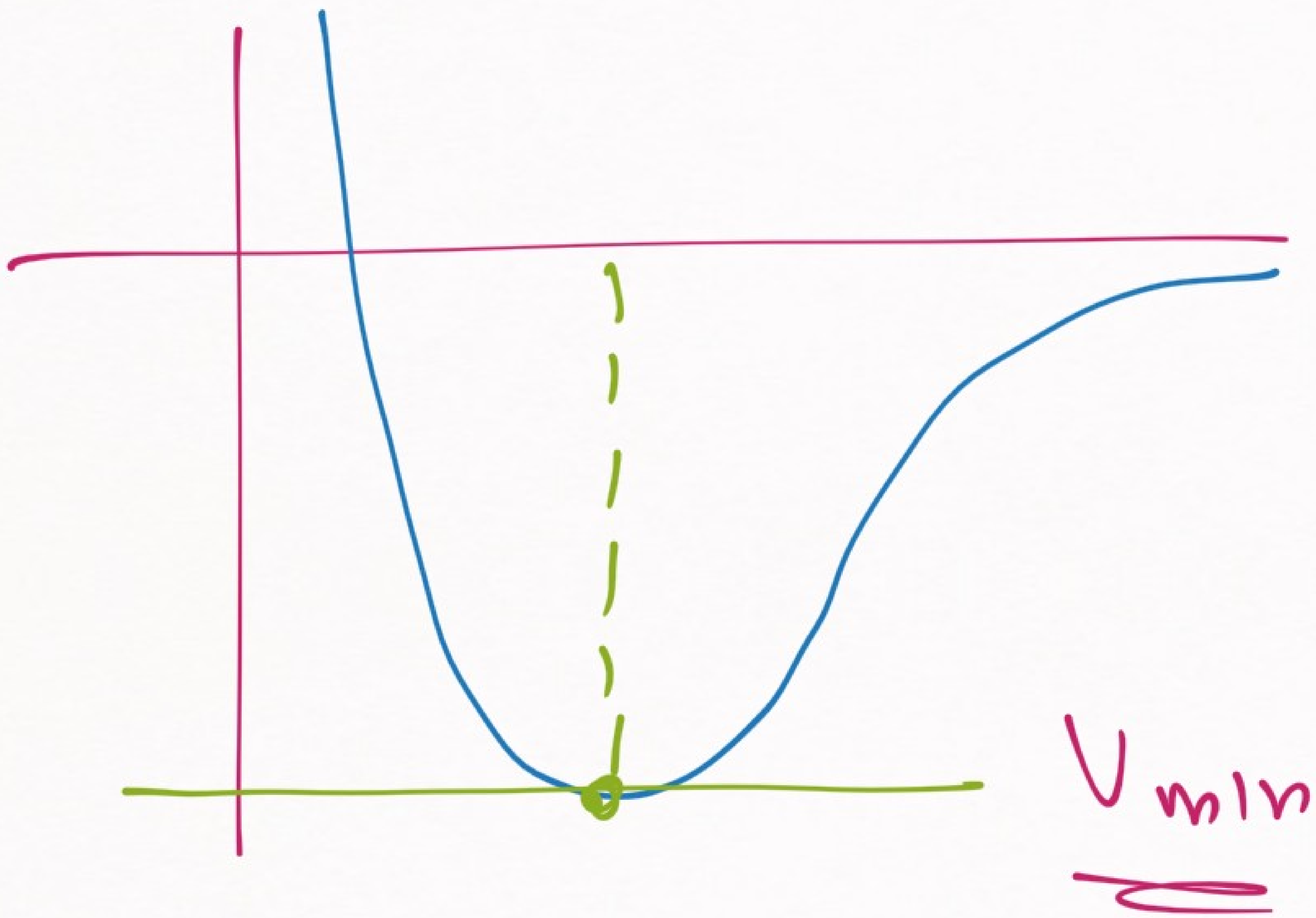
→ Identify the correct scale

$E_T = k_B T$

→ Boltzmann constant

$E_T \simeq \frac{1}{2} m \langle v \rangle^2$

(boiling point)



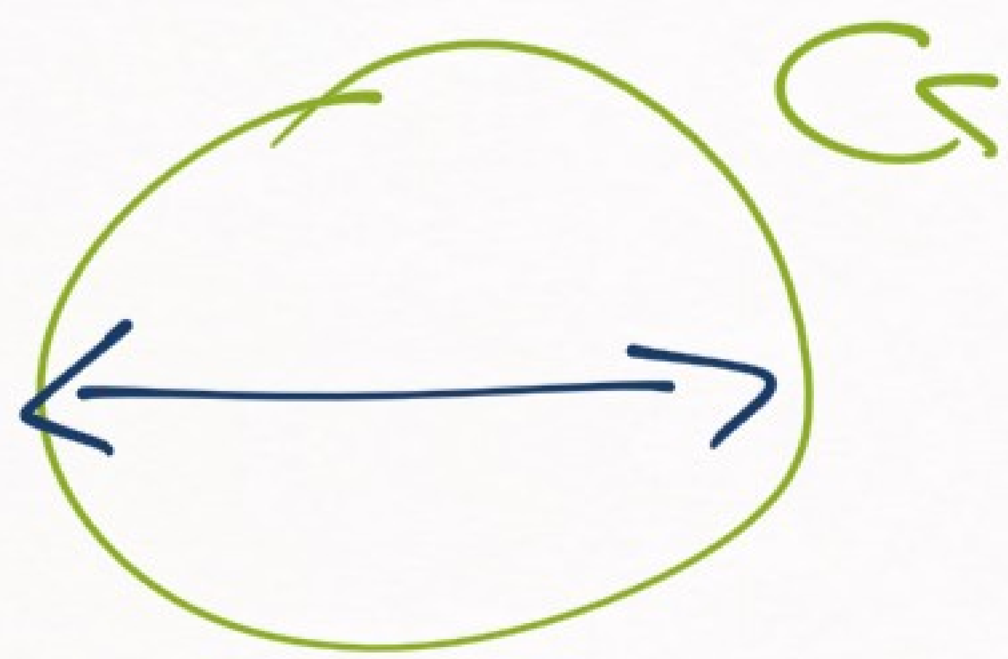
$$V_{min} \simeq -\frac{1}{2\mu}\left(\frac{1}{R_{cs}}\right)^2$$

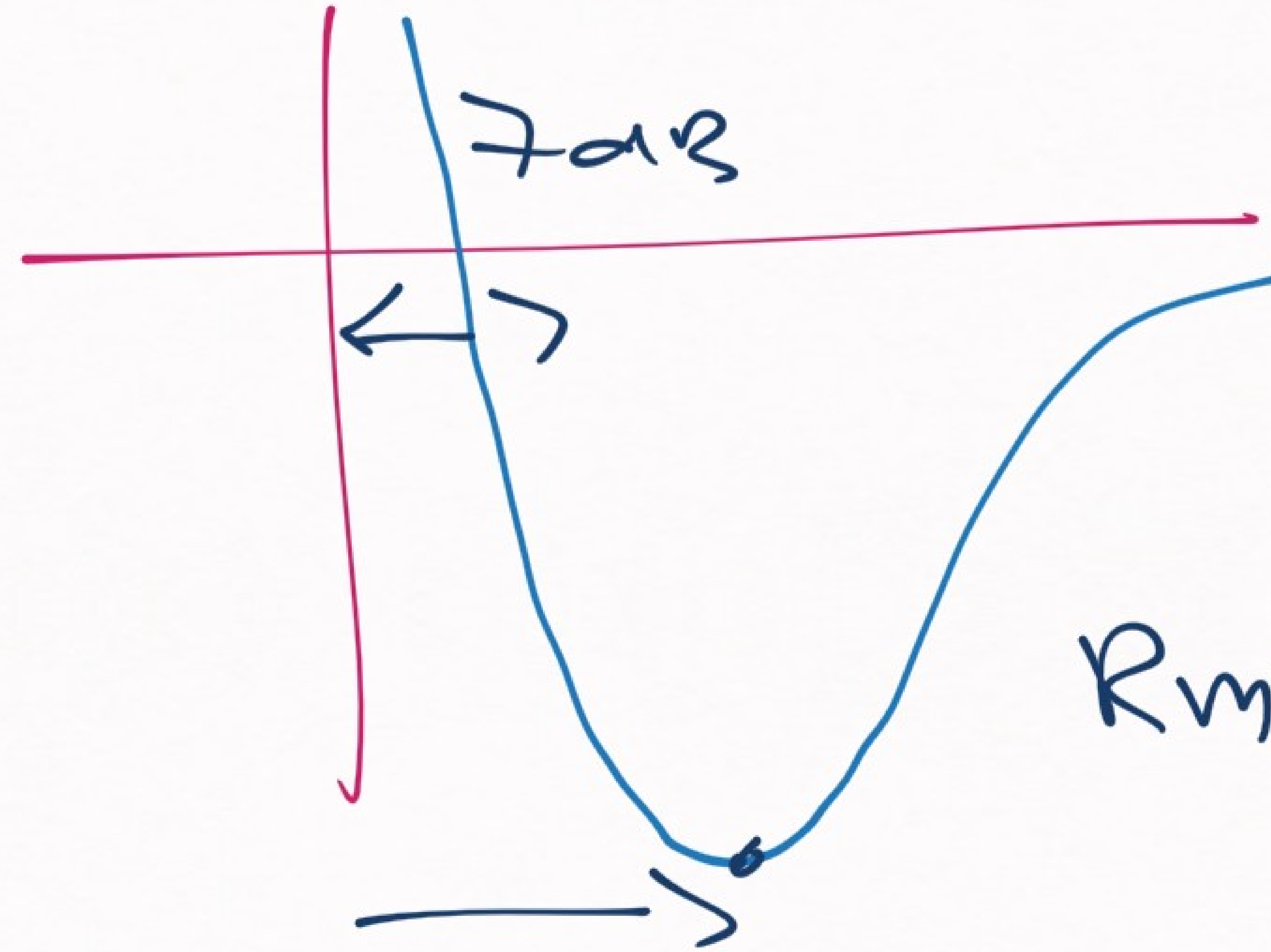
why?

$$2\mu V = [L]^{-2}$$

$$|E_{kin}| \simeq |V_{min}|$$

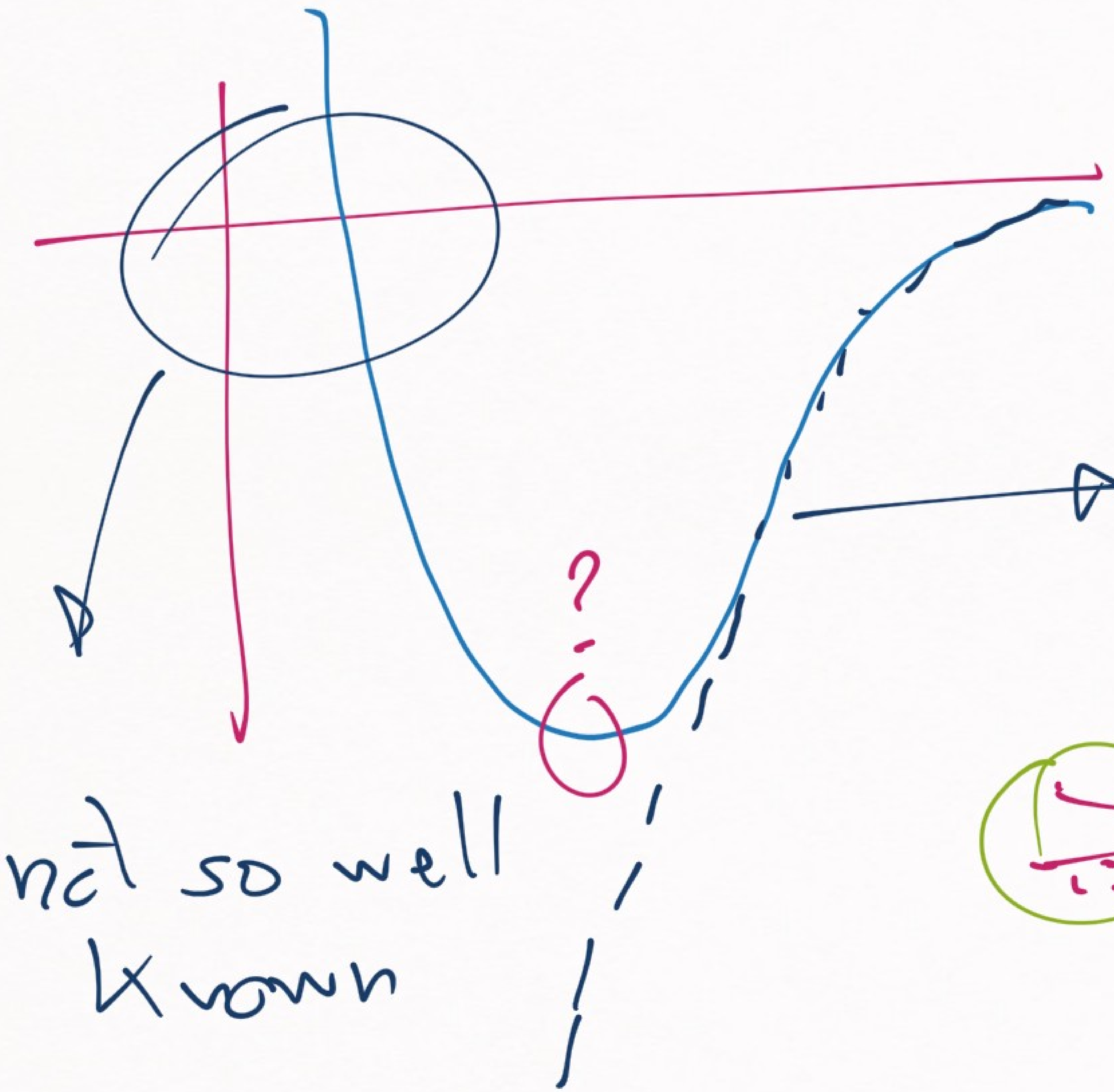
$\boxed{\text{GUESS } V_{min}}$





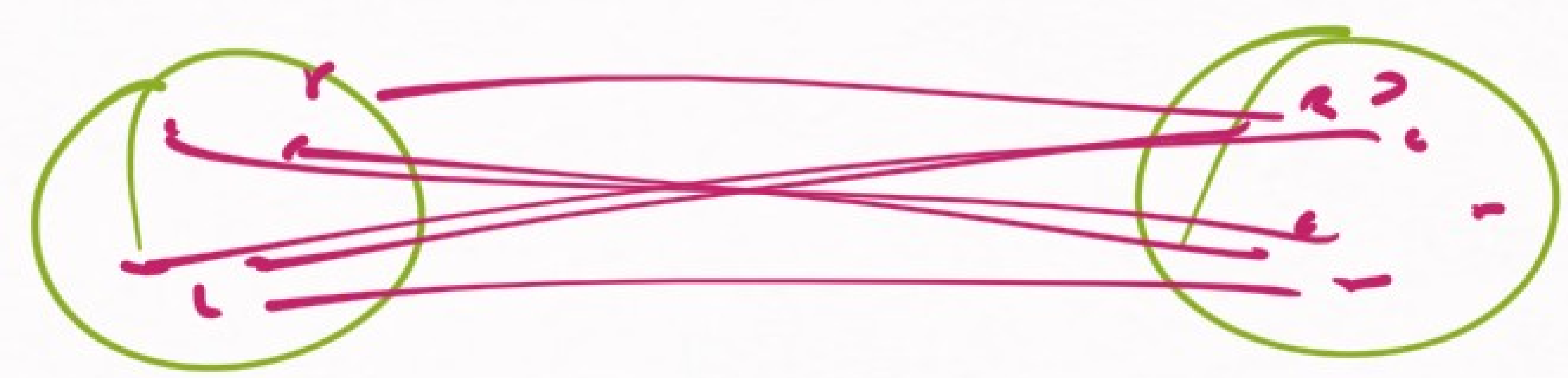
1) Guess R_{min}

2) Guessing V at R_{min}



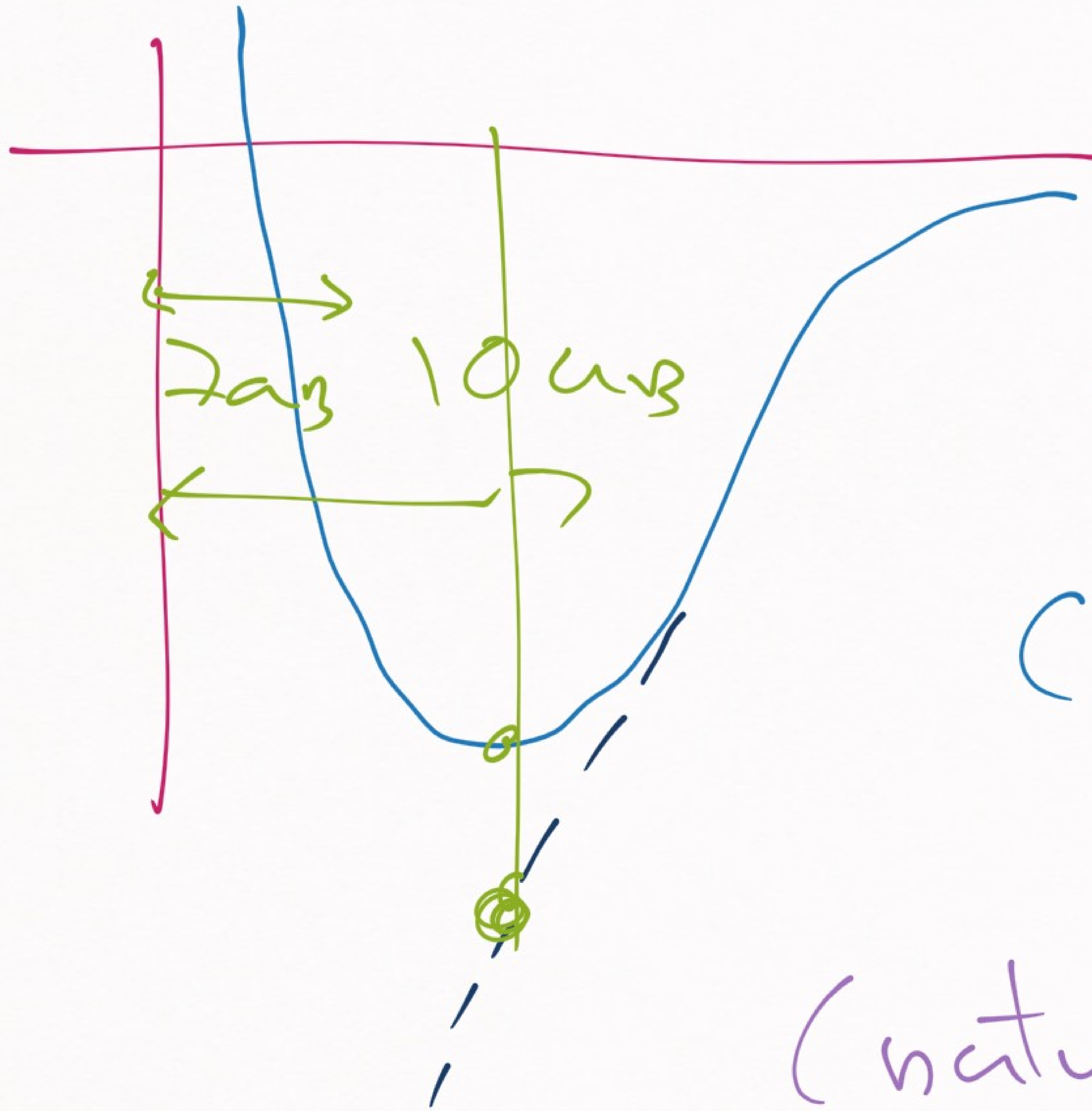
not so well known

VdW well known (because it is an average)



→ compute it

→ experiment



Approximation:

$V_{min} \simeq V_{vdW}(R = 10 a_B)$

(Simple & Crude, maybe not good)

(natural problem)

$\boxed{\text{RECAP (Cs)}}$

1) Cs - Cs → minimum energy config (V_{min})

2) $|E_{kin}| \simeq |V_{min}|$

3) $V_{min} \simeq V_{vdW}(1.5 \times \text{size}(Cs))$

4) V_{min} → scale → $\langle v \rangle$ (now)

5) $K_B T \simeq \frac{1}{2} m \langle v \rangle^2$ (now)

$R_{min} \sim 10\, a_B$

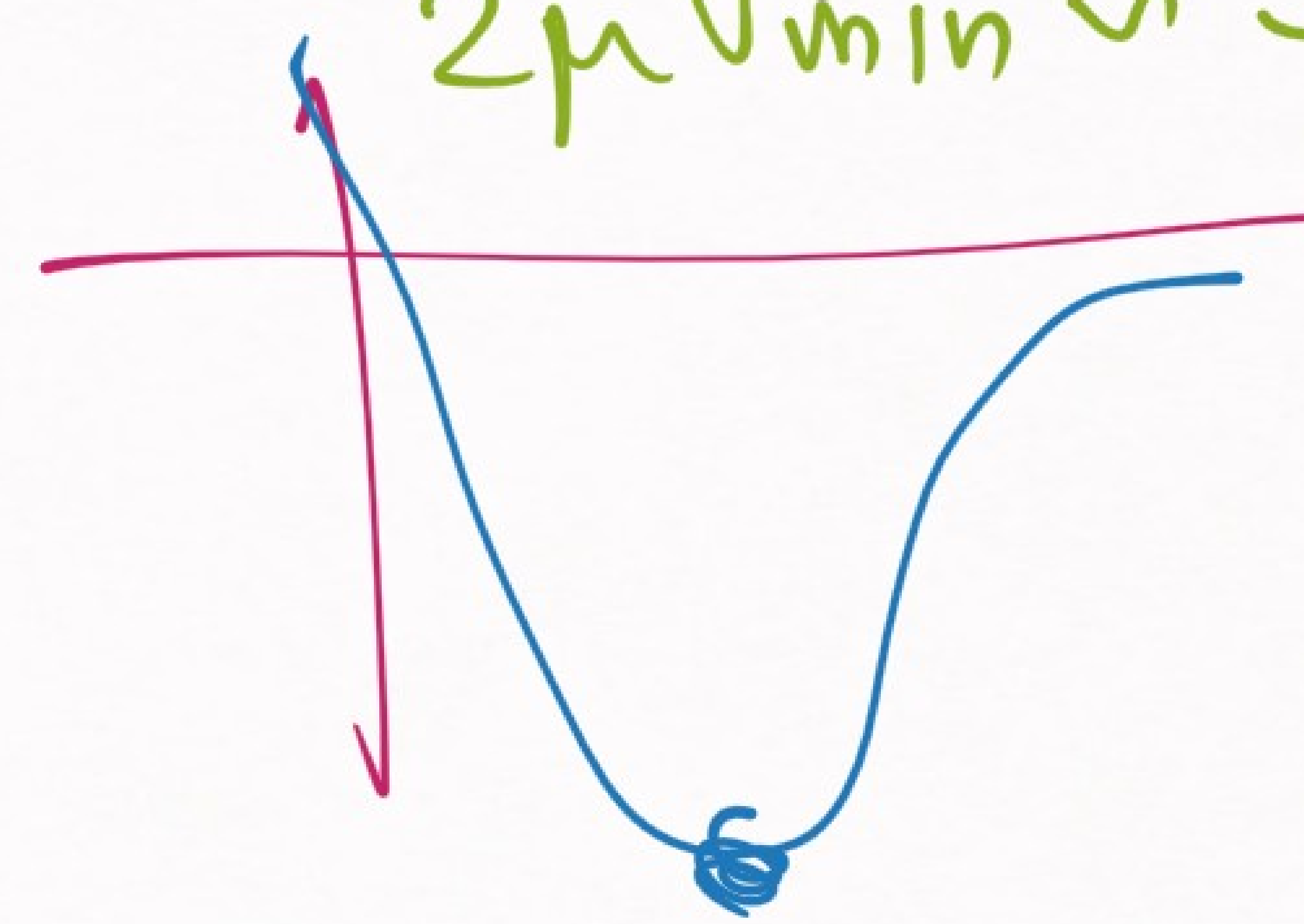
$R_{vdW} \sim 203\, a_B$

$$V_{vdW} = -\frac{1}{2\mu}\left(\frac{R_{vdW}^4}{r^6}\right) \to r = R_{min}$$

$$V_{min} = -\frac{\ell}{2\mu}\left(\frac{1}{\underline{\underline{R_{cs}^2}}}\right) \longrightarrow \langle p \rangle \sim \frac{1}{R_{cs}} \longrightarrow \langle v \rangle$$

$$2\mu V_{min} \sim -\frac{1}{R_{cs}^2}$$

$$R_{cs} \sim R_s \left(\frac{R_s}{R_{vdW}}\right)^2$$

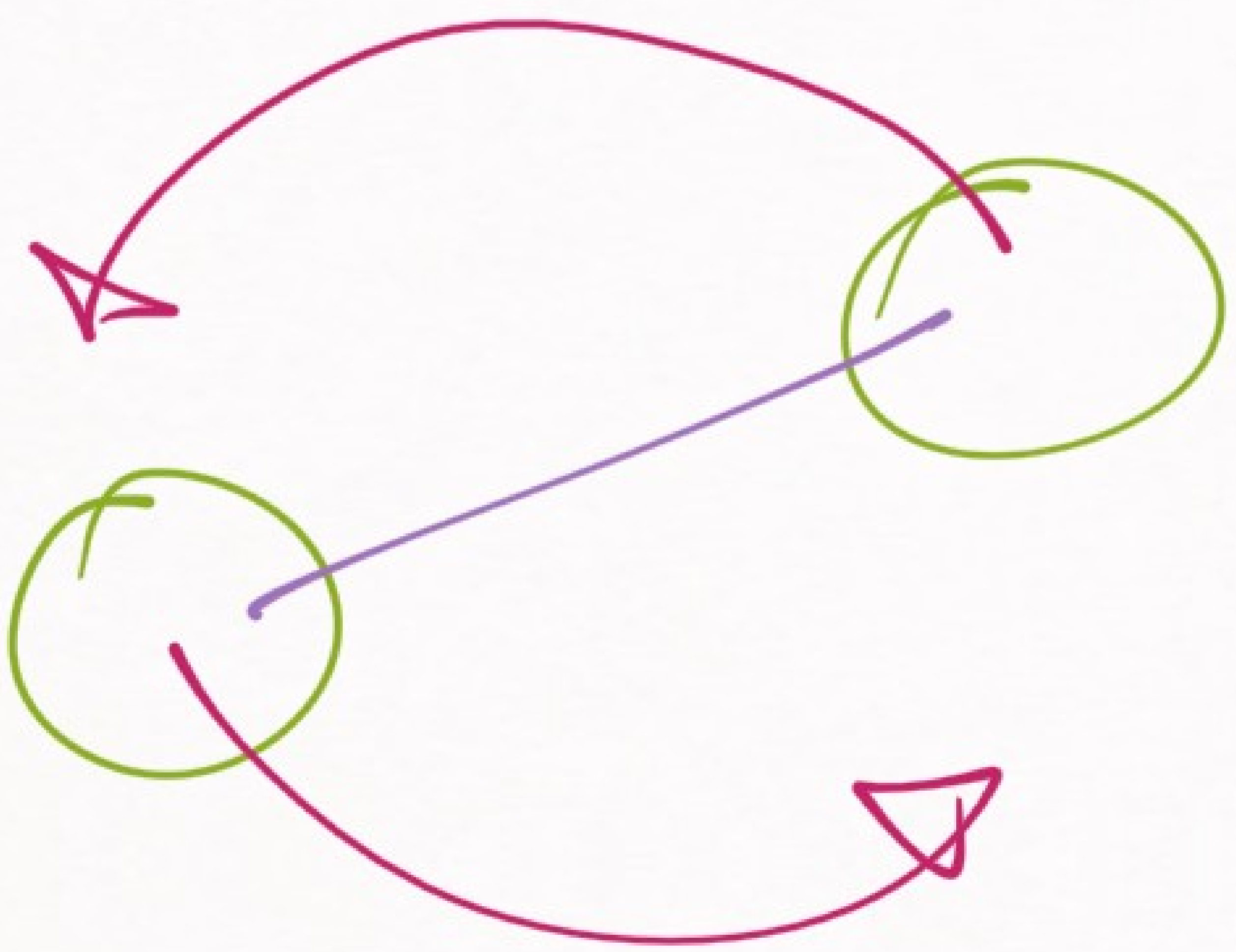


$$\Rightarrow \quad R_{cs} \sim R_s \left(\frac{R_s}{R_{vdW}}\right)^2$$

$$\Rightarrow \quad Q \sim \frac{1}{R_{cs}} \qquad \Rightarrow \quad \langle v \rangle \sim \frac{Q}{m} \sim 1{,}16 \cdot 10^{-6}$$

$$\left\langle \frac{v}{c} \right\rangle$$

$$\boxed{\langle v \rangle \sim 350 \, m/s}$$



$\langle v \rangle \sim 350\ m/s$

$E_{thermal} = k_B T$

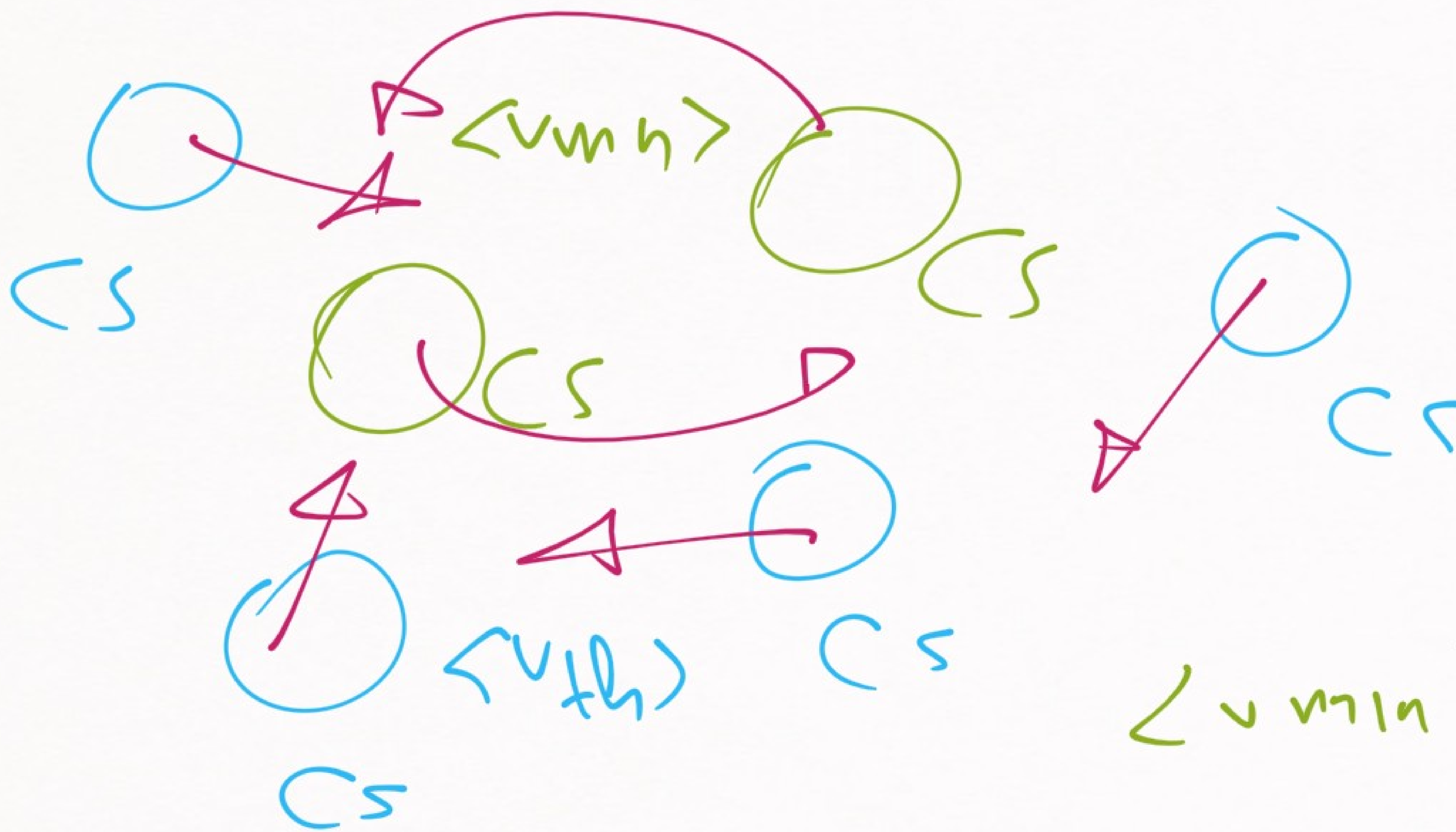
$k_B \simeq \dfrac{1\ eV}{11000\ K^{0}}$

temperature

Room temperature $\rightarrow$ $\boxed{50\ meV}$

$$\frac{1}{2} m \langle v \rangle^2 = k_B T \Rightarrow v = \sqrt{\frac{2 k_B T}{m}}$$

$$\boxed{\begin{array}{l} T \simeq 1000\ K^{\circ} \\ v_{th} \simeq 350\ m/s \end{array}}$$



$$\langle v_{min} \rangle \simeq \langle v_{th} \rangle$$

$\boxed{\text{RECAP}}$ 1) Long calculation (many steps)

(easy steps)

2) $v_{th} = \sqrt{\frac{2k_B T}{m}}$ vs $V_{min} \simeq 350\,m/s$

$\boxed{\text{REALLY GOOD APPROX.}}$

$T_{Boiling} \simeq \underline{\underline{1000\ K^\circ}}$

Reality: $T_{Boiling}(Cs) \simeq \underline{\underline{944\ K^\circ}}$

LESSON → Natural problems are really easy

This is why atomic physics is so easy

Next part: Why nuclear physics
is difficult

(scales)

$\boxed{\text{~~5 minute break~~}}$ (35)

over

NATURAL PROBLEMS → EASY
(usually only one relevant scale)

→ most problems in physics involve
several scales

→ corrections

Hydrogen atom:

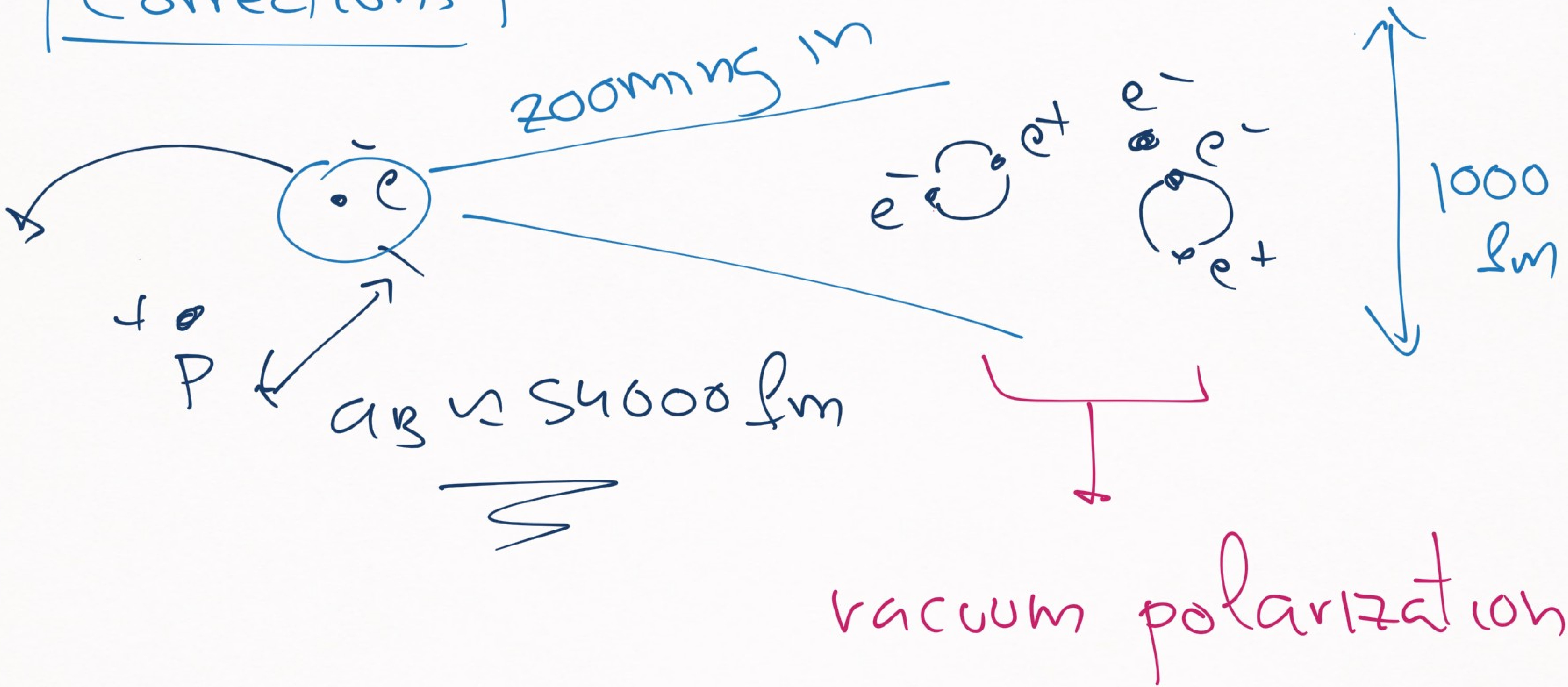
1) most important scale $a_B = \frac{1}{m_e \alpha}$

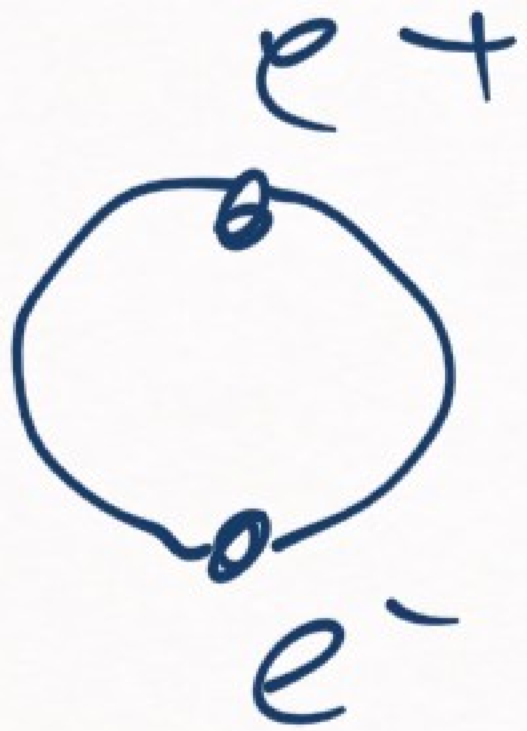
$(Q_B = m_e \alpha)$

2) other scales:

$r_e, r_p \longrightarrow$ electron & proton size

$\boxed{\text{Corrections}}$





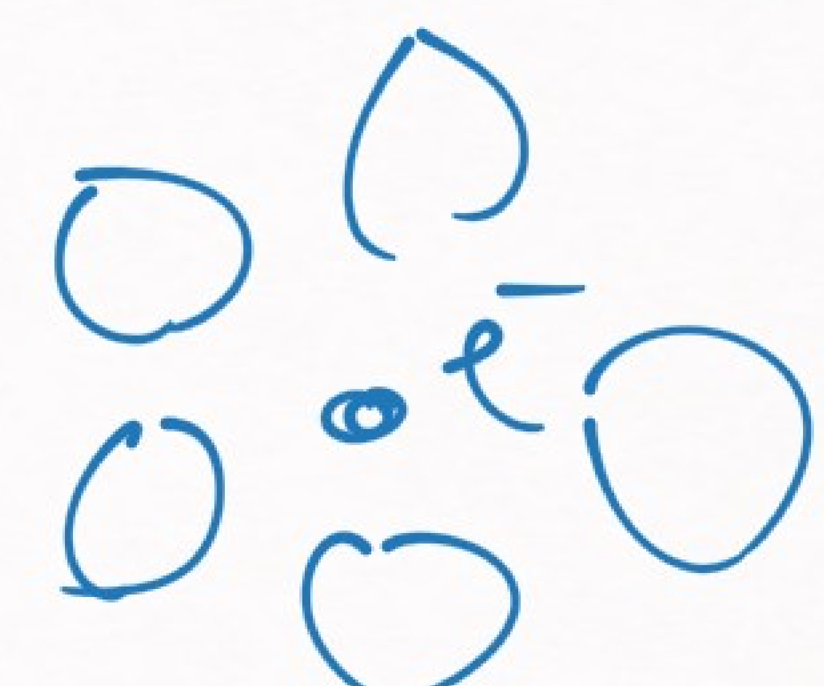
$$\Delta E \Delta t \simeq \hbar/2 \qquad (\Delta x \Delta p)$$

$$\rightarrow \frac{1}{m_e} \simeq \frac{\hbar c}{0.511\,\mathrm{MeV}} \simeq 400\,\mathrm{fm}$$

$$\hbar c = 197.3\,\mathrm{MeV\,fm}$$

54000 fm $\longrightarrow$ 400 fm

e^-

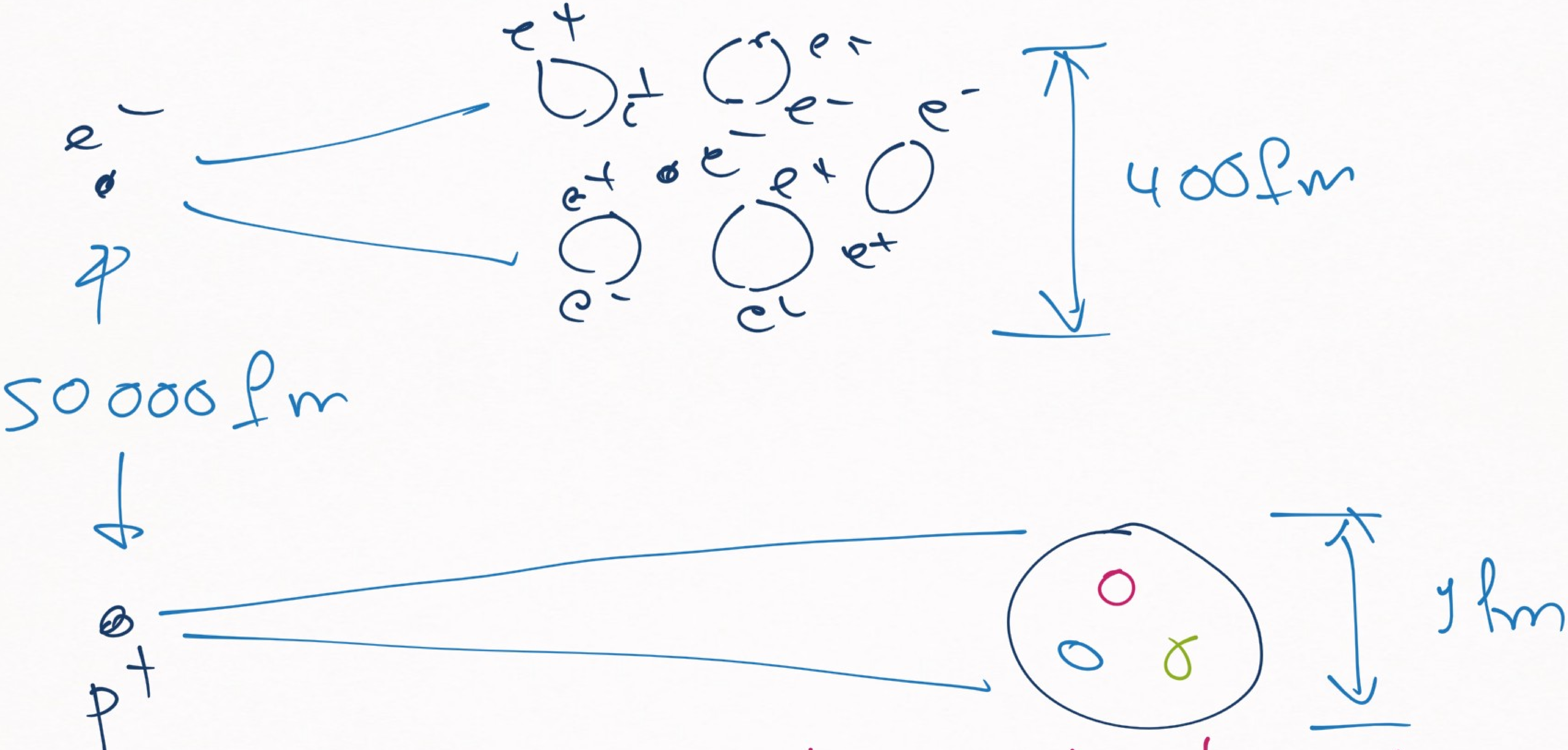


The way you see the world depends on
the distance scale

$\Rightarrow$

Long distance $\rightarrow$ e^- has no structure
(point)

Short distance $\rightarrow$ e^- has structure
(cloud of e^+e^- pairs)



(internal structure of proton)

HYDROGEN ATOM → MULTISCALE PROBLEM

$a_B \sim 54000$ fm (bigger one)

→ very small

$1/m_e \sim 400$ fm

$r_p \sim 1$ fm

→ generate physical effects

Bigger scale, smaller scales

$\rightarrow$ good separation of scale

1) $\frac{r_e}{a_B} \simeq \frac{m_e \alpha}{m_e} \simeq \alpha \simeq \frac{1}{137}$ (but smaller because of dynamics)

2) $\frac{r_p}{a_B} \simeq \frac{1}{56000} \simeq 2 \cdot 10^{-5}$ (it is actually smaller)

Extra scales $\rightarrow$ CORRECTIONS

1) Electron "size" $\longrightarrow$ $\dfrac{r_e}{a_B} \sim \dfrac{1}{137}$

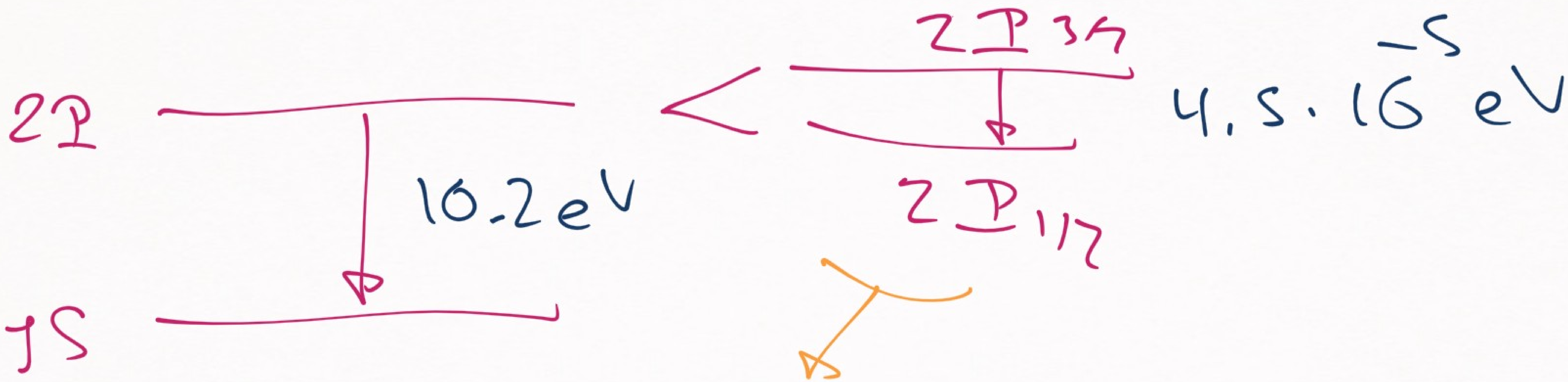
(relativistic effect)

$$\sqrt{p^2+m^2} = m + \frac{p^2}{2m} + O(p^4)$$

$$\left(\frac{r_e}{a_B}\right)^2 \sim \frac{1}{19000} \sim 5 \cdot 10^{-5}$$

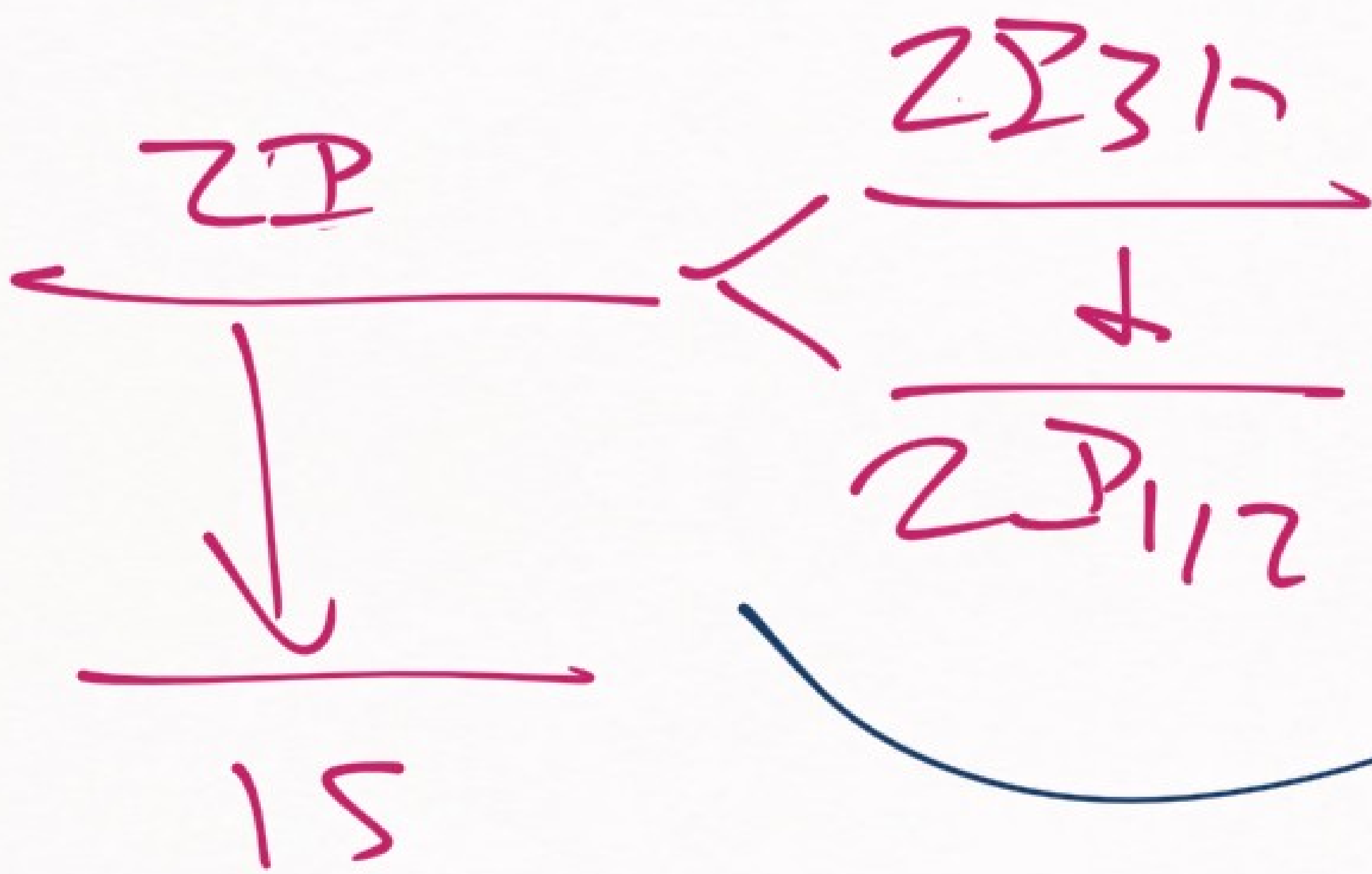
EXPECTATION → $\left(\frac{r_e}{a_B}\right)^2 \sim 5\cdot10^{-5}$

REALITY : fine structure corrections



$\vec{L}\cdot\vec{S}$ correction → $j = 1/2, 3/2$
(p-wave)

fine structure 2P :



$$\frac{4.5\cdot 10^{-5}\,eV}{10.2\,eV} \sim \boxed{5\cdot 10^{-6}}$$

REALITY

EXPECTATION $\boxed{5\cdot 10^{-5}}$

NOT BAD

2) Proton size correction

$$\left(\frac{r_p}{a_B}\right) \sim \boxed{2\cdot 10^{-6}}$$ (EXPECTATION)

NOT BAD

2) Hyperfine correction

1S

↑↑

(−13.6 eV) $<$

↑↓

$$-\frac{1}{2m_e}\left(\frac{1}{a_B}\right)^2$$

$$\frac{5.9\cdot 10^{-6}\,eV}{13.6\,eV} \simeq \boxed{5\cdot 10^{-7}}$$

(REALITY)

$\boxed{\text{LESSONS}}$ Systems that:

1) are natural ($\sim O(1)$)

2) have a good separation of scales

$$\boxed{\text{THEY ARE ALL VERY EASY}}$$

hydrogen atom → incredibly accurate predictions <u>easy</u>

$\boxed{\text{COROLARY}}$ Systems that

1) are not natural or
2) have a poor separation of scales

$\boxed{\text{THEY ARE A NIGHTMARE}}$

Good thing → wonderful research projects ↯

What about nuclear physics?

1) Deuteron size:

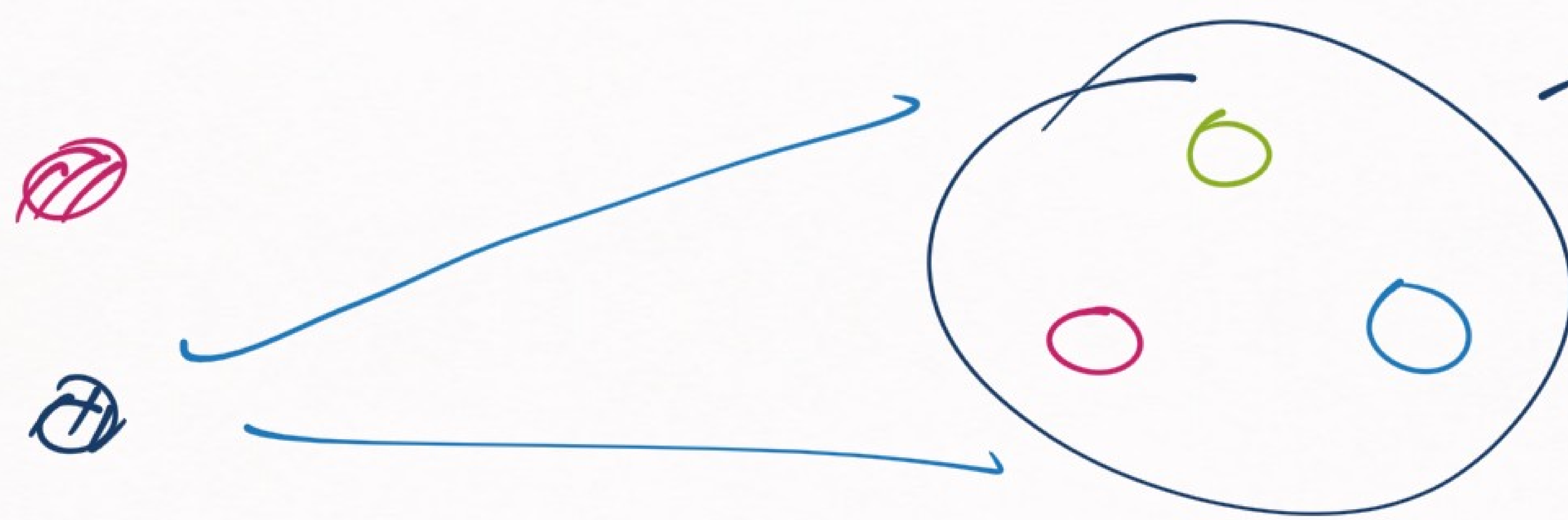
$n \circ \quad {}^{\circ p} \quad \updownarrow \quad r_d \sim 5\,\text{fm}$ $\quad 1/3$ (not that bad)

2) Range of nuclear forces:

$\underset{p}{\circ} \overset{}{- - - - -} \underset{n}{\circ}$ $\quad \frac{1}{m_\pi} \sim \frac{\hbar c}{140\,\text{MeV}} \rightarrow 1.4\,\text{fm}$

π (pion)

3) Size of the nucleons



→ quarks & gluons

$\rightarrow| \quad 1\,fm \quad |\leftarrow$

GOOD

$r_d \simeq 5\,fm$

OOPS!

$\frac{1}{m_\pi} \simeq 1{,}4\,fm$

$1/3$

3

Nuclear physics hard because:

1+2) (r_d vs $1/m_\pi$) deuteron kind of big

(A) not too natural

2+3) ($\frac{1}{m_\pi}$ vs r_N, w/ N = n, p)

(B) poor scale separation

$A + B \longrightarrow \boxed{\text{DIFFICULT PROBLEM}}$

This is why nuclear physics
is going to be difficult