

→ Don't be afraid to interrupt

→ Signal any problem

→ General introduction

Two-nucleon system

Three-nucleon system

Nuclear models

Easy

Indy

(difficult)

→ Geared towards theoreticians

Experimentalist → concentrate  
on the conceptual level

(ideas, etc)

techniques,  
formalism

→ Evaluation → Exercises (presentation)

Exercise ~~sheet~~ sets → choose (extra parts)

15 points

→ Permit correction

↳ no copying

↳ You can ask

2-3 minutes break now

(first, some questions)

→ begin the 1<sup>st</sup> lesson

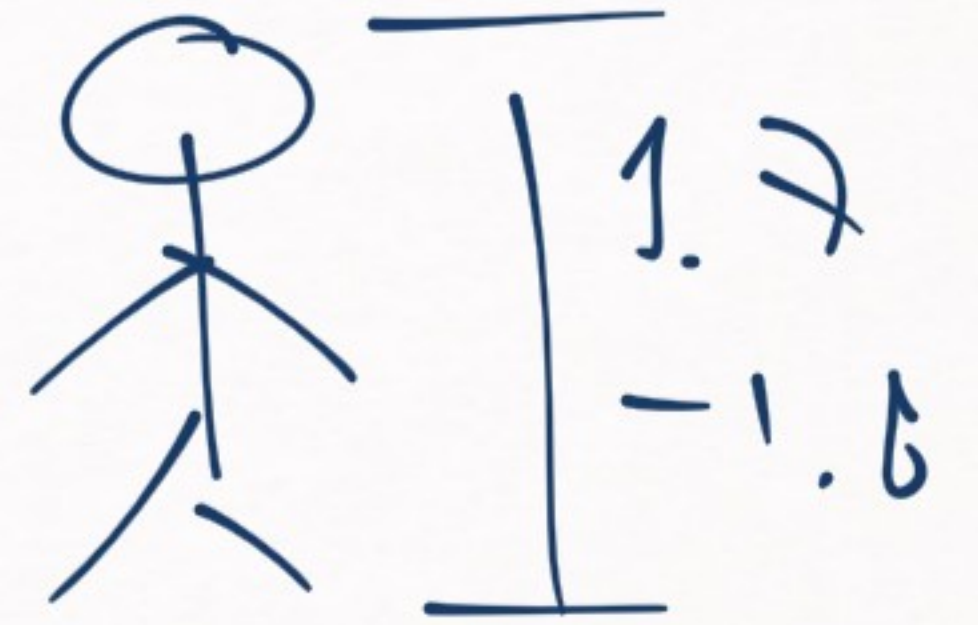
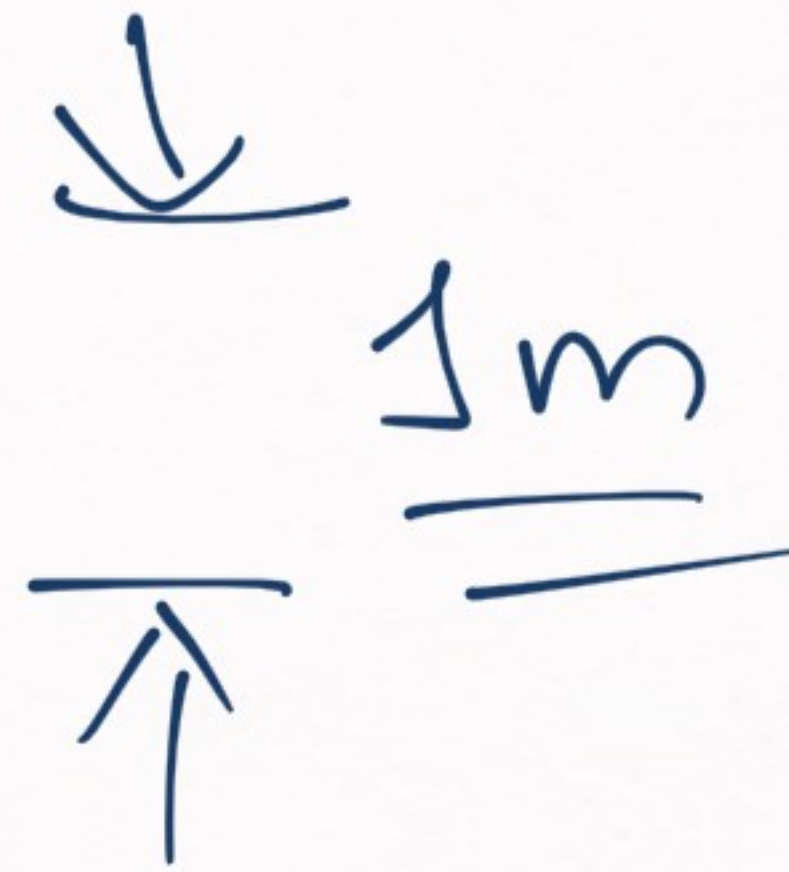
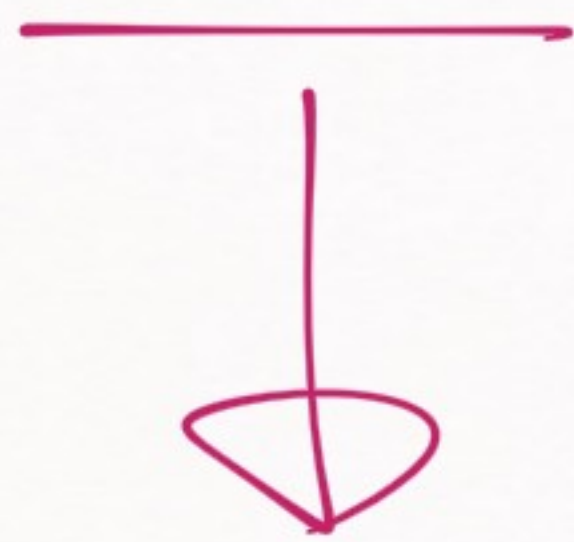
# NUCLEAR PHYSICS | ①

"a matter of scales"



what are "scales" in physics  
how to use them to understand  
physics

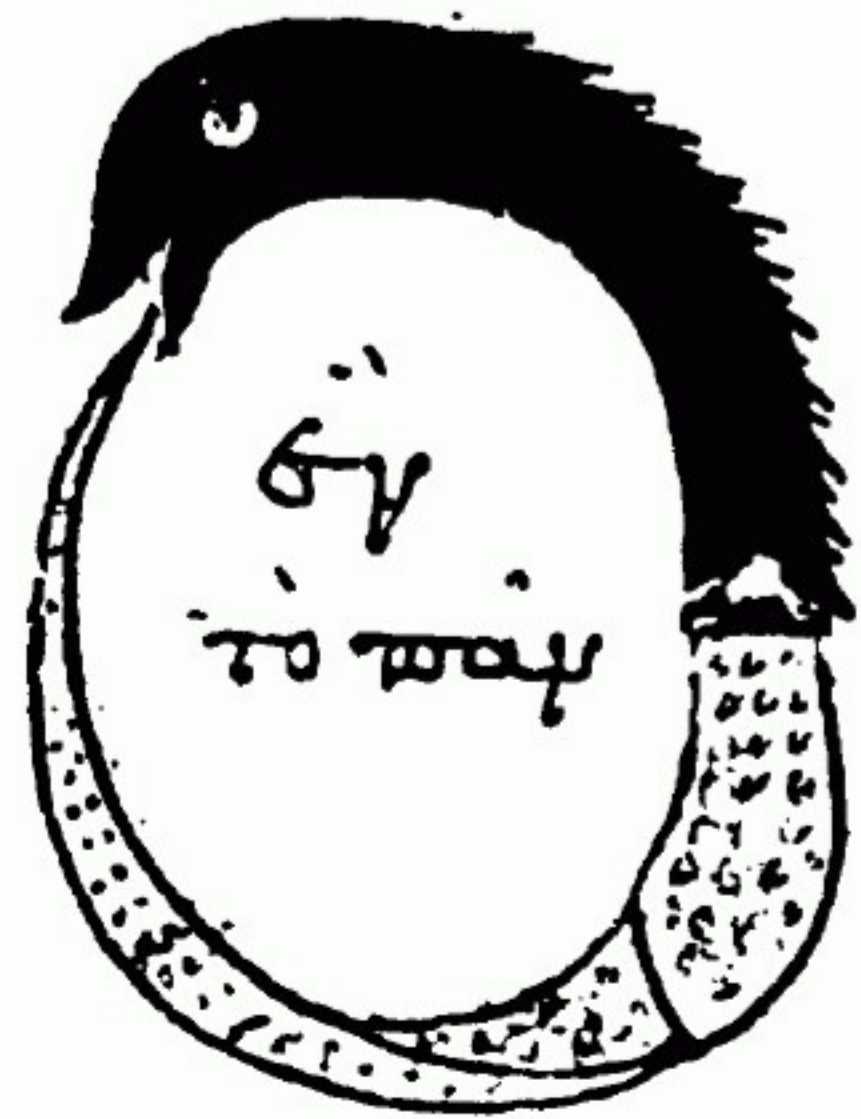
Scale (尺度)  $\rightarrow$



How we view physics depends on  
the scale



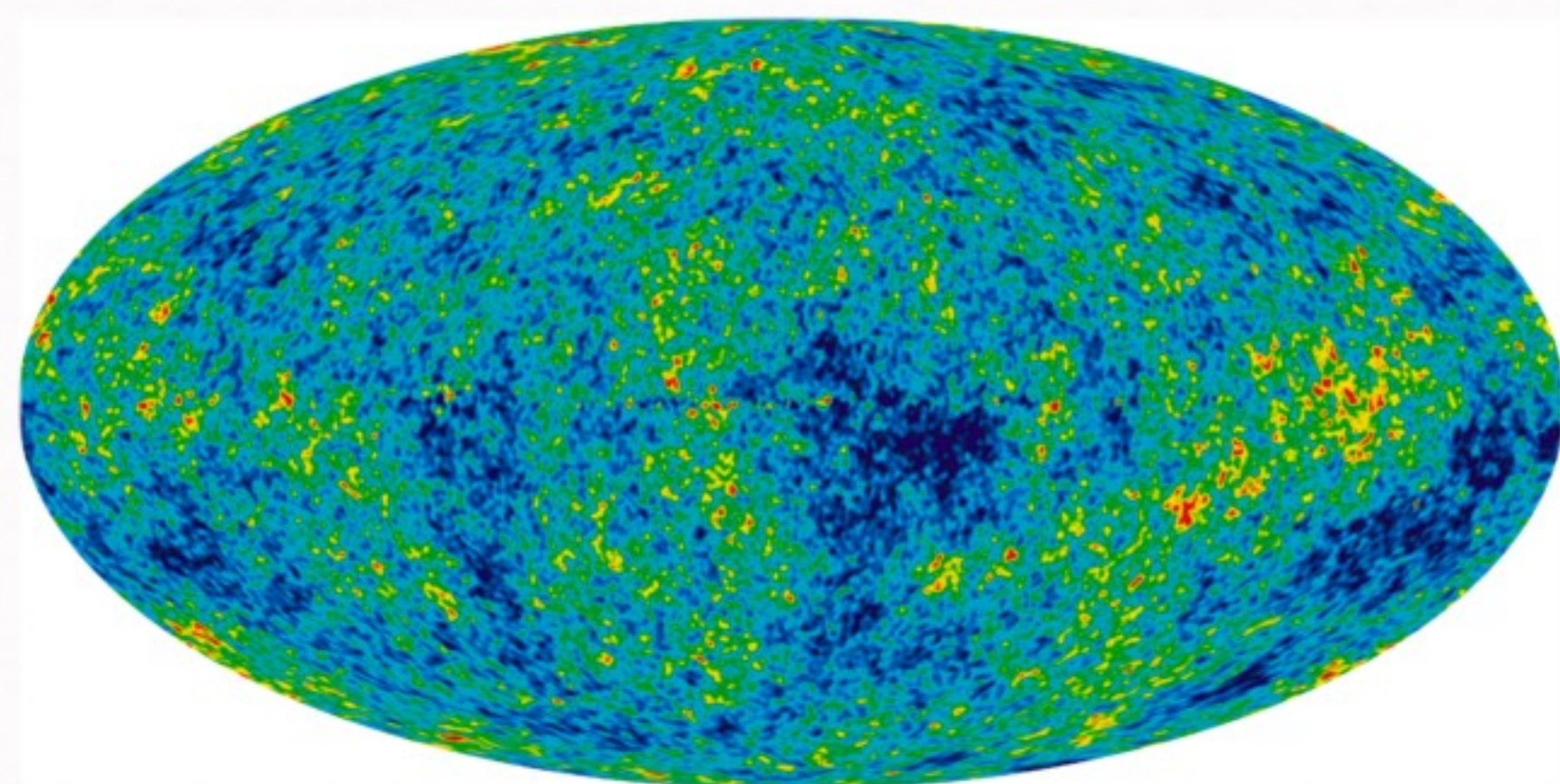
Scales  $\rightarrow$  from very big to very small  
Each scale  $\rightarrow$  different understanding  
of physics



metaphysics  
view to  $\pi\alpha\upsilon\upsilon$   
(call it one)

$10^9$  eight years

→ cosmology





$10^3 - 10^5$  light years

→ astrophysics



100 a.u. (astronomical units)

→ solar system

classical  
mechanics



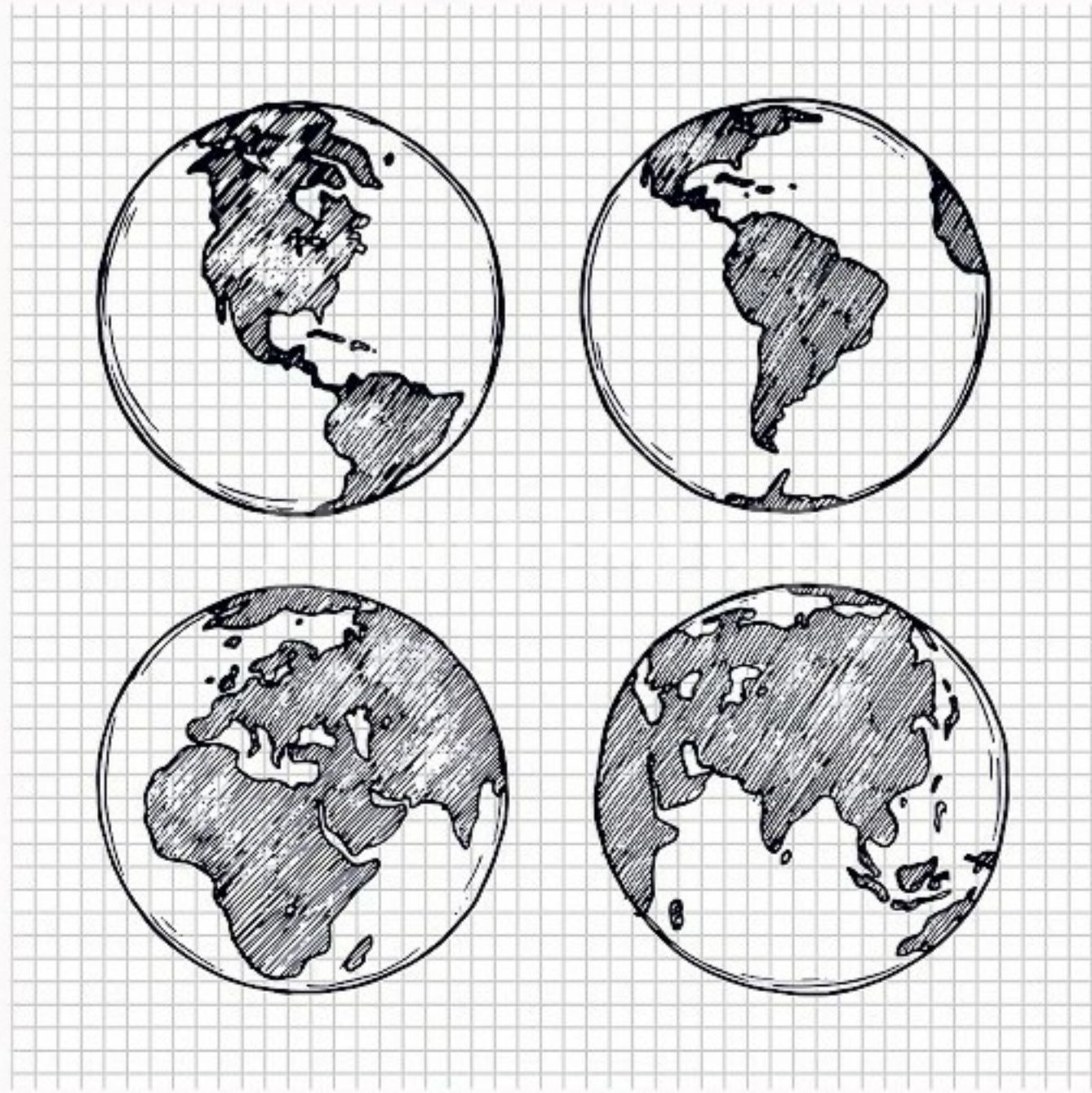
10<sup>4</sup> Km



geography,

ecology,

climate



$10^3$  km



countries  
economy  
sociology

concepts change  
w/ the scale

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skm



social life

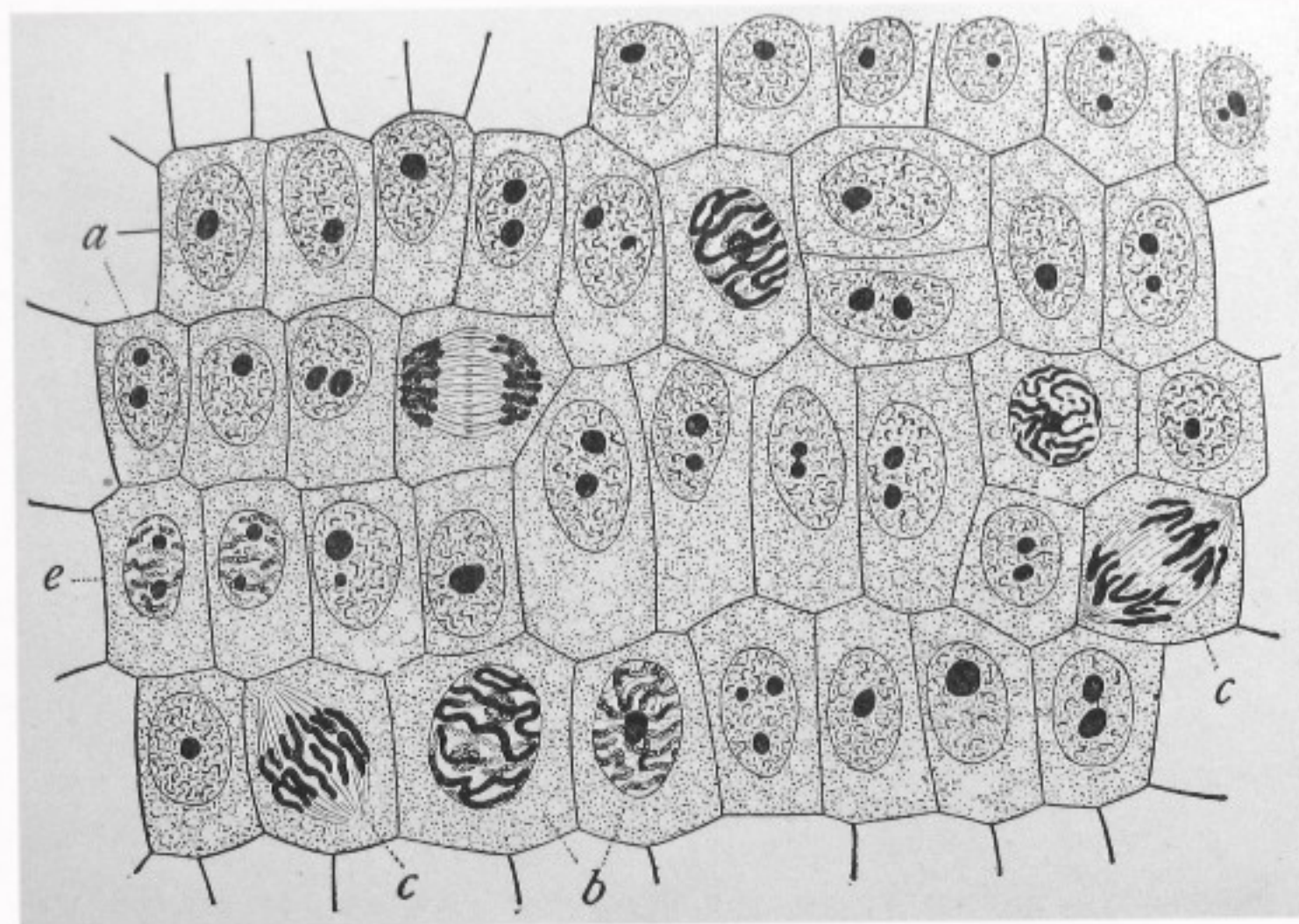
△ Beihang campus size

1m  $\rightarrow$  classical mechanics  
people



$10^{-5}$  m

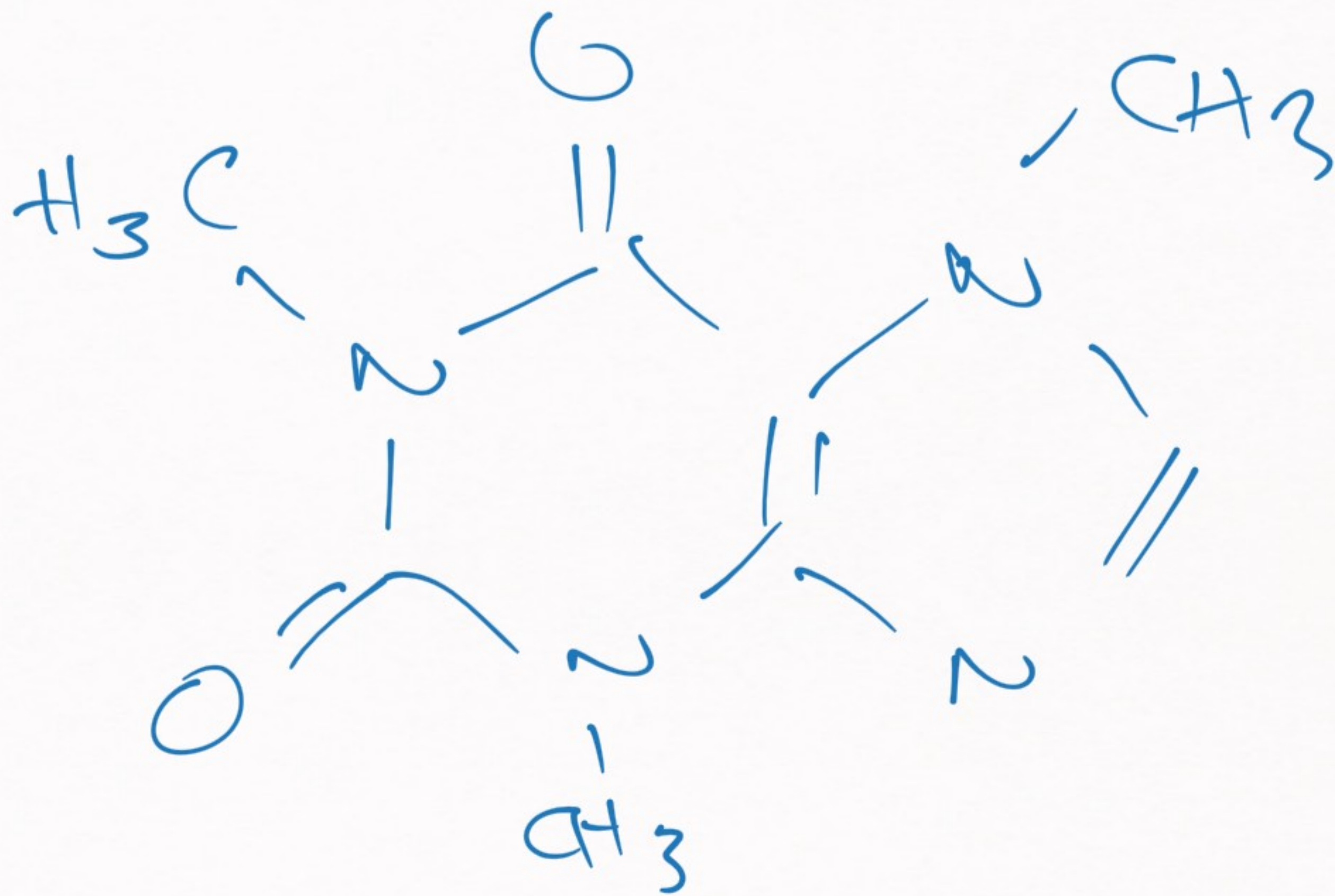
$\rightarrow$  biology



$10^{-9} \text{ m}$

$\approx 10 \text{ \AA}$

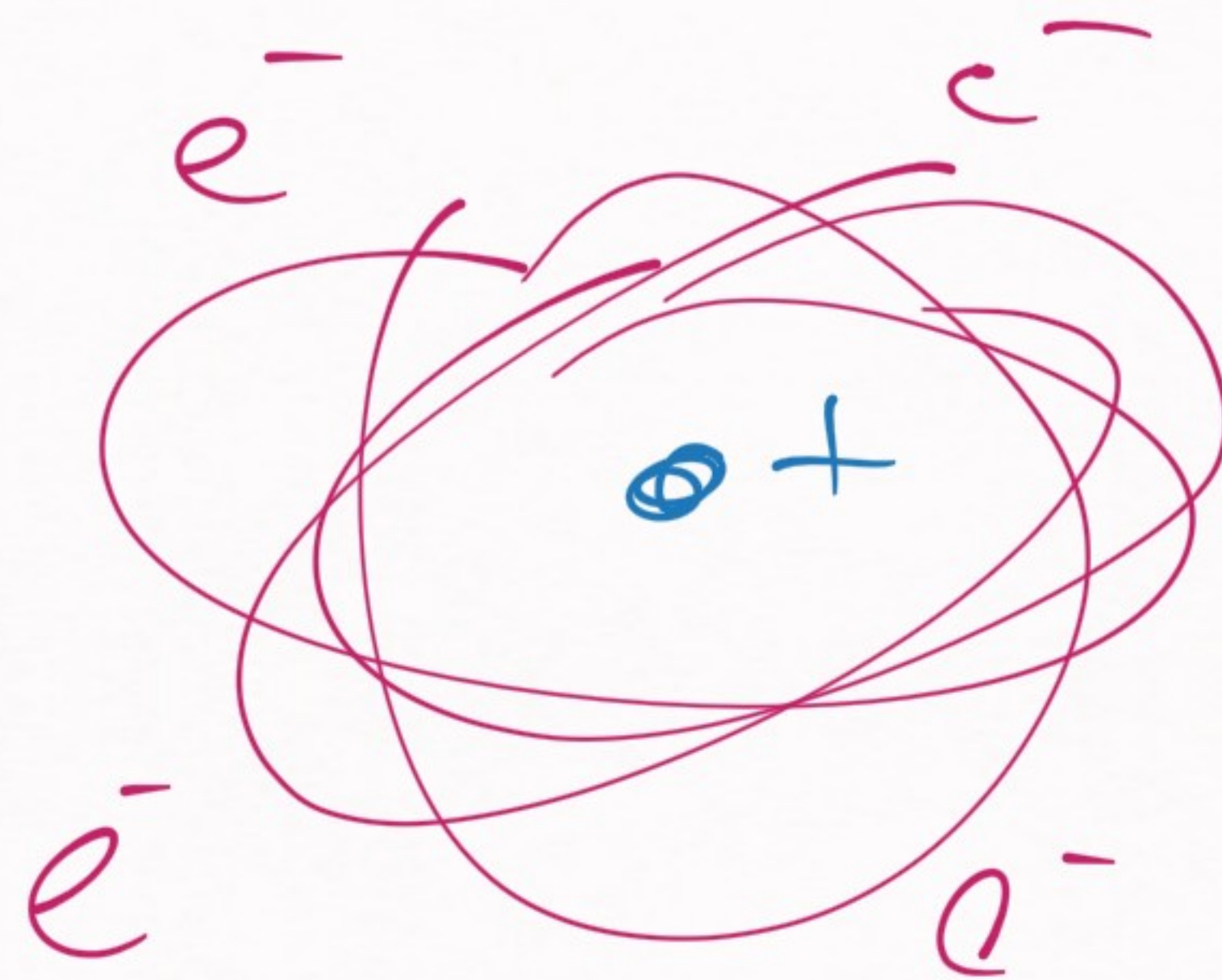
Chemistry



(caffeine)

→ useful for physics

$10^{-10}$  m

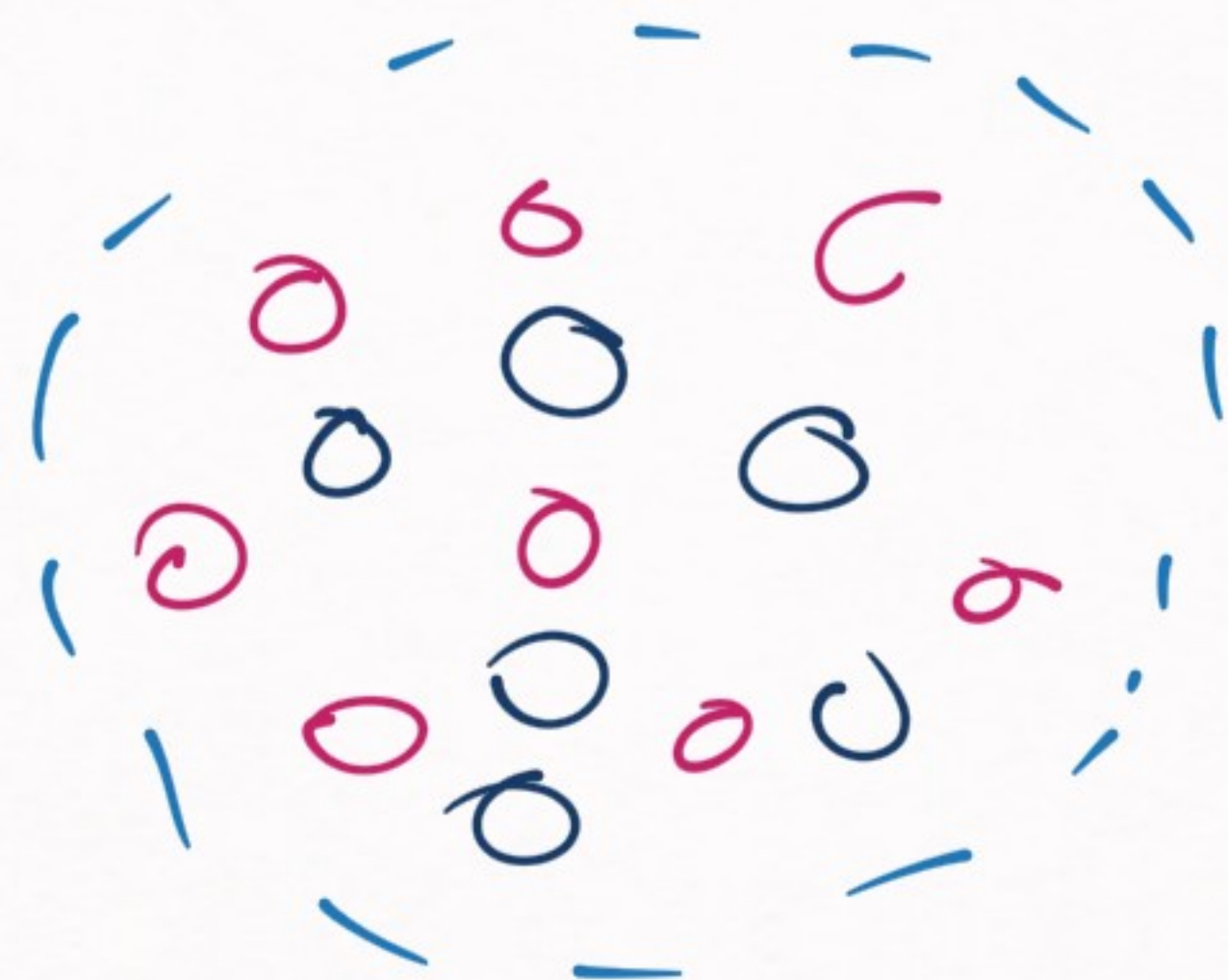


atomic  
physics

different scales  $\rightarrow$  different world  
- news



$10^{-15}$  m



nucleus

nuclear  
physics

units of fm

our course

$10^{-16}$  m

proton

Quantum  
Chromodynamics  
(QCD)



quarks  
&  
gluons

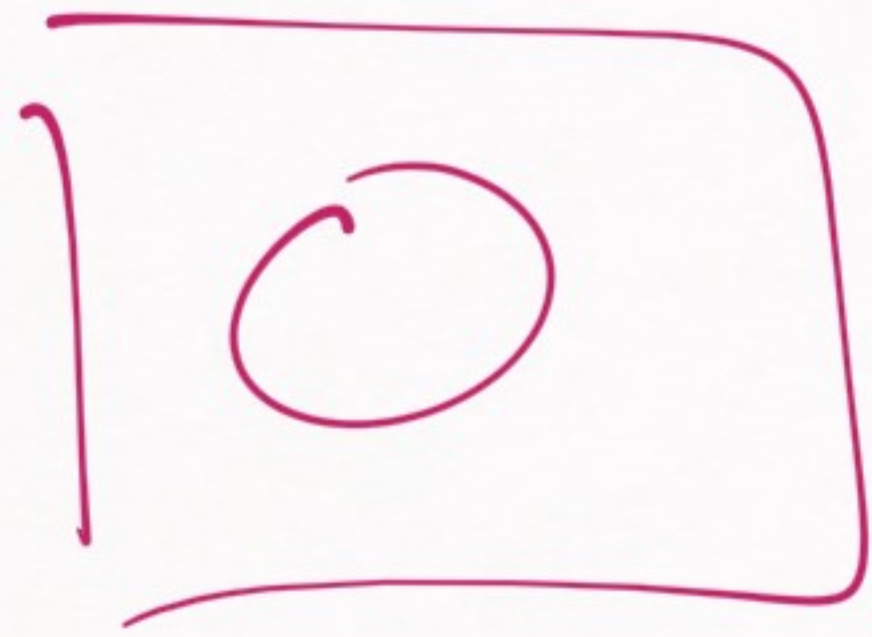
(is very important)

$$10^{-35} \text{ m}$$

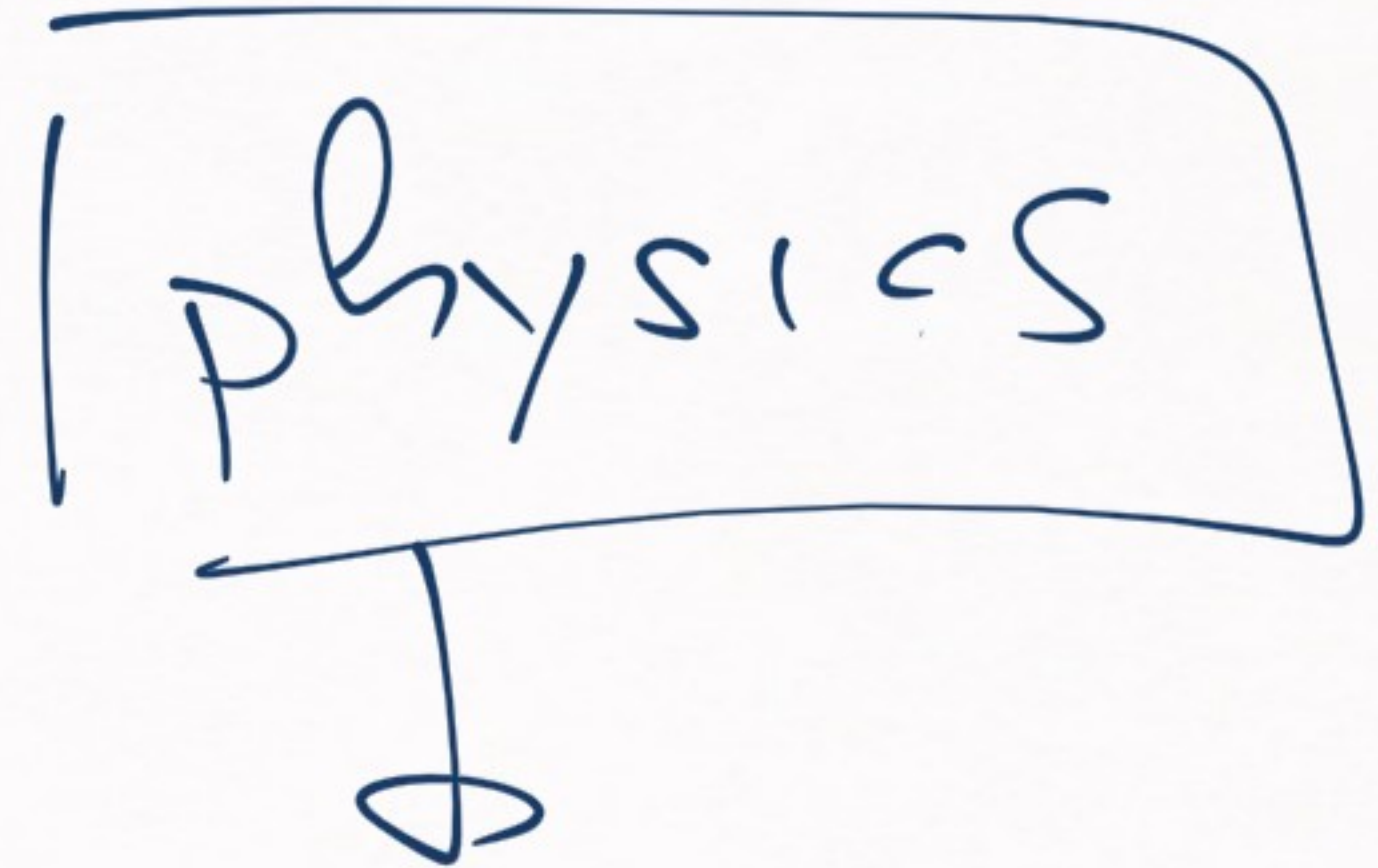
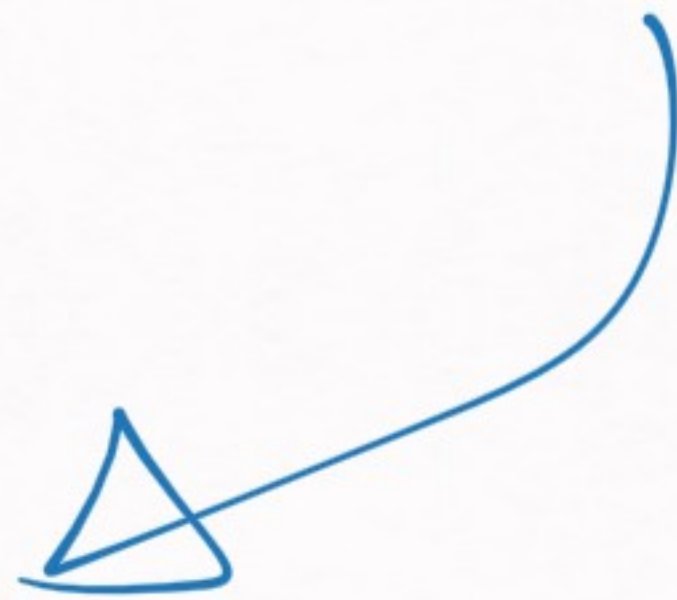
→ Planck scale

(quantum gravity)

(nobody knows how it works,  
yet)



metaphysics



[only  $0/\infty$  do not  
have onts]

→ Here we are interested in  
 $10^{-(15-16)}$  m scales

(nuclear physics & QCD)

→ Back to atomic physics  
(to understand scale)

→ Question (you guys are shy,  
but don't be afraid)

→ Rest a pair of minutes  
before going for atomic  
physics

# ATOMIC PHYSICS

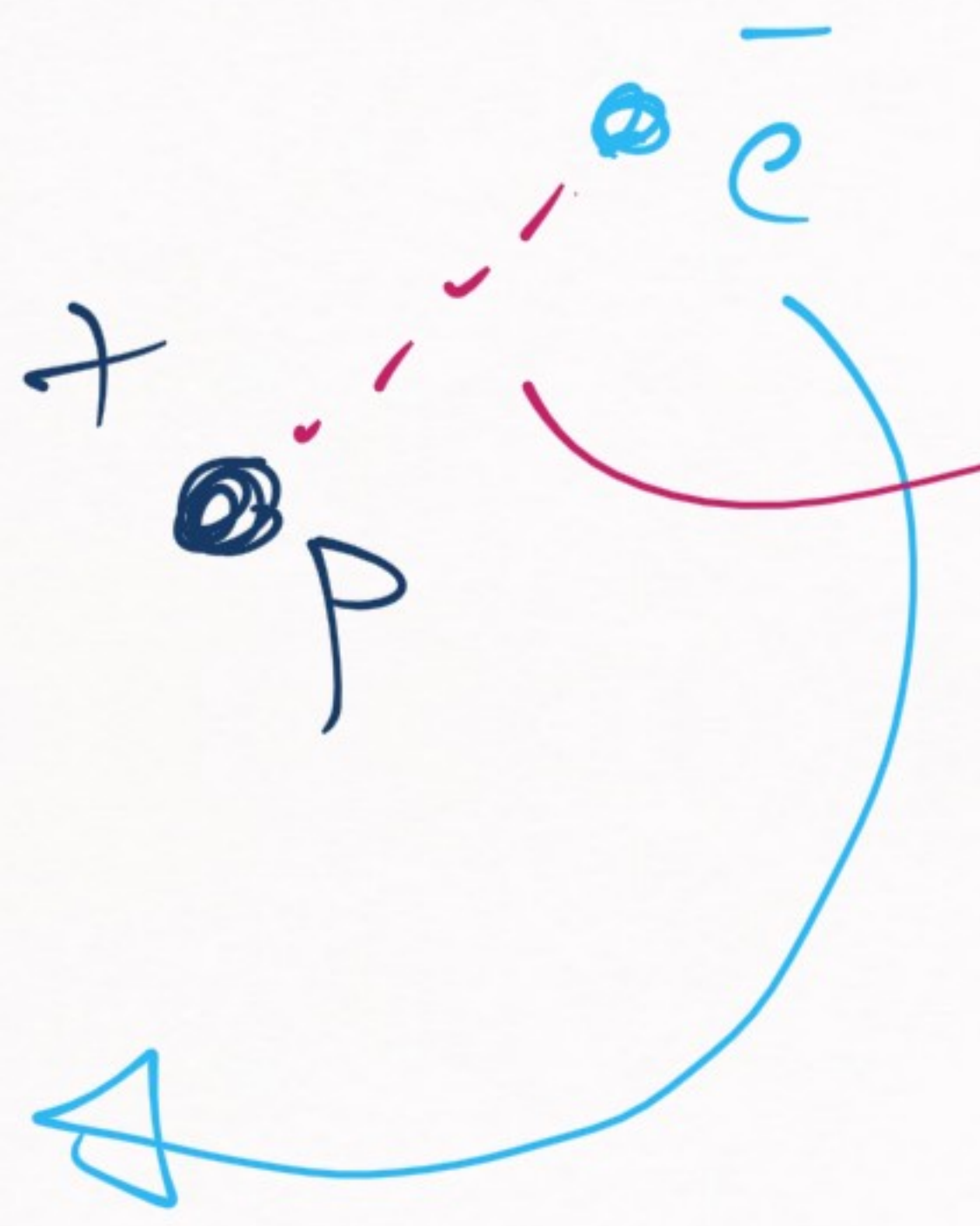
→ THE HYDROGEN ATOM

(FROM THE SCALE VIEWPOINT)



Why scales are important





Coulomb force

$$V(r) = -\frac{\alpha}{r}$$

radius

$$\alpha \approx \frac{1}{137}$$

(fine structure constant)



Use the Schrödinger equation:

$$\left[ -\frac{\hbar^2}{2\mu} \nabla^2 + V(\vec{r}) \right] \psi(\vec{r}) = (E_B + \epsilon) \psi(\vec{r})$$

Annotations:  
-  $-\frac{\hbar^2}{2\mu} \nabla^2$  is labeled as **reduced mass**.  
-  $V(\vec{r})$  is labeled as **Coulomb potential**.  
-  $\psi(\vec{r})$  is labeled as **wave function**.  
-  $E_B + \epsilon$  is labeled as **binding energy**.

reduced mass  $\Rightarrow \frac{1}{\mu} = \frac{1}{m_e} + \frac{1}{m_p}$   
electron & proton mass

$$E_B = - \frac{\gamma_B^2}{2\mu} \longrightarrow \text{wave number}$$

$\underbrace{\hspace{10em}}_{\text{reduced mass}}$

Rewrite the equation:

$$\underbrace{\left[ -\nabla^2 - \frac{2\mu a}{r} \right]}_{\text{INPUT}} \psi(\vec{r}) = \underbrace{-\gamma_B^2}_{\text{OUTPUT (prediction)}} \psi(\vec{r})$$

$$\left[ -\nabla^2 - \frac{Z\mu\alpha}{r} \right] \psi(\vec{r}) = -\epsilon_B^2 \psi(\vec{r})$$

$$\mu\alpha = \frac{1}{a_B} \rightarrow a_B \text{ : Bohr radius (length scale)}$$

$$\alpha = \frac{1}{137} \text{ (pure number)}$$

$$\mu \text{ (mass)} \rightarrow [\epsilon] = [U]$$

$$\Delta x \Delta p \sim \hbar/2 \quad \Delta E \Delta t \sim \hbar/2$$

$[E]^{-1} = [L]$   $\rightarrow$  conversion factor

$$\hbar c = 197.3 \text{ MeV} \cdot \text{fm}$$

Example:  $m_{\pi} \approx 140 \text{ MeV} \rightarrow \frac{e^{-mr}}{r}$

$$\frac{1}{m_{\pi}} \approx \frac{\hbar c}{m_{\pi}} \approx \frac{200 \text{ MeV} \cdot \text{fm}}{140 \text{ MeV}} \approx 1.4 \text{ fm}$$

WE USE IT ALL THE TIME

$$M_{\text{scale}} \rightarrow \alpha_{\text{scale}} = \frac{1}{M_{\text{scale}}} \left( \frac{\hbar c}{M_{\text{scale}}} \right)$$

Planck energy  $\rightarrow$  Planck length

$$1.2 \cdot 10^{19} \text{ GeV}$$

$$1.2 \cdot 10^{22} \text{ MeV}$$

$$10^{-35} \text{ m}$$

$$\left[ -\nabla^2 - \frac{Z}{a_B r} \right] \psi(\vec{r}) = -\gamma_B^2 \psi(\vec{r})$$

$a_B$ : Bohr radius  $\rightarrow a_B = \frac{1}{\mu \alpha} \approx \frac{137}{0.5} \text{ MeV}^{-1}$

$\frac{1}{\mu} = \frac{1}{m_e} + \frac{1}{m_p} \approx \frac{1}{m_e} \approx 270 \text{ MeV}^{-1} \times (\hbar c)$   
 $\approx 5.4 \cdot 10^4$

$m_e \ll m_p$  ( $0.5 \text{ GeV} \ll 940 \text{ MeV}$ )

$\left[ 0.54 \overset{6}{\text{Å}} \right] \xrightarrow{\text{fm}}$

$$\left[ -D^2 - \frac{2}{a_B r} \right] \psi(r) = -\gamma_B^2 \psi(r)$$



INPUT



OUTPUT:  $\gamma_B$

$$([E] - [U])$$

SCALES:



$$a_B$$



$$\gamma_B \propto \frac{1}{a_B}$$

ONLY POSSIBILITY

SCALES → hydrogen atom only has

one scale

$n=1$

→ every property of the hydrogen atom will only depend on  $n$



$$\gamma_B = \frac{C}{a_B}$$

$$\left( -\nabla^2 - \frac{Z}{a_B r} \right) \psi = -\gamma_B^2 \psi$$

$$E_B = -\frac{1}{2\mu} \left( \frac{C}{a_B} \right)^2$$

no  $\mu$  (explicitly)  
inert scale

$$\sqrt{\langle r^2 \rangle} = \sqrt{\langle \psi | r^2 | \psi \rangle} = d a_B$$



NATURAL PROBLEM

→ easy problems

SYSTEM → A SCALE ( $a_s$ )

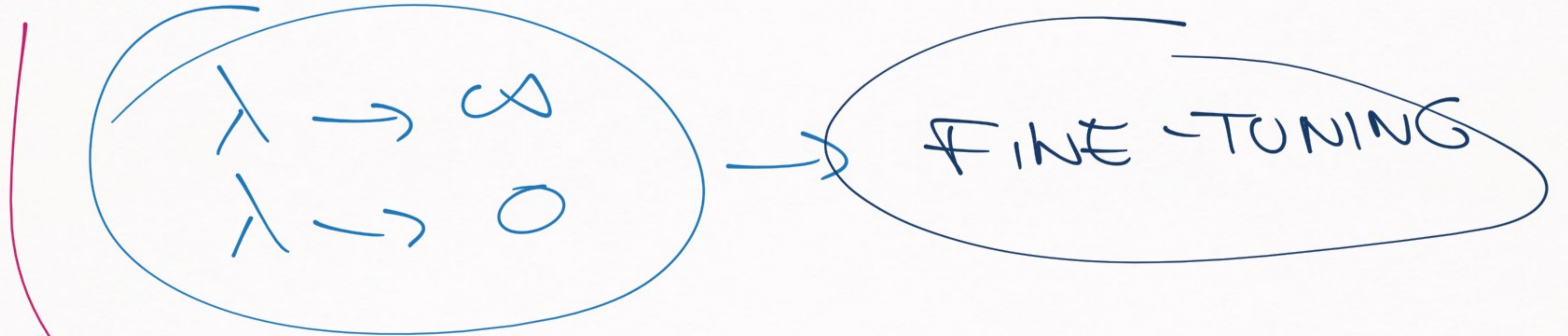
Observable  $\hat{\Theta}$

$$[\hat{\Theta}] = [L]^d$$

$$\langle \hat{\Theta} \rangle = \lambda a_s^d$$

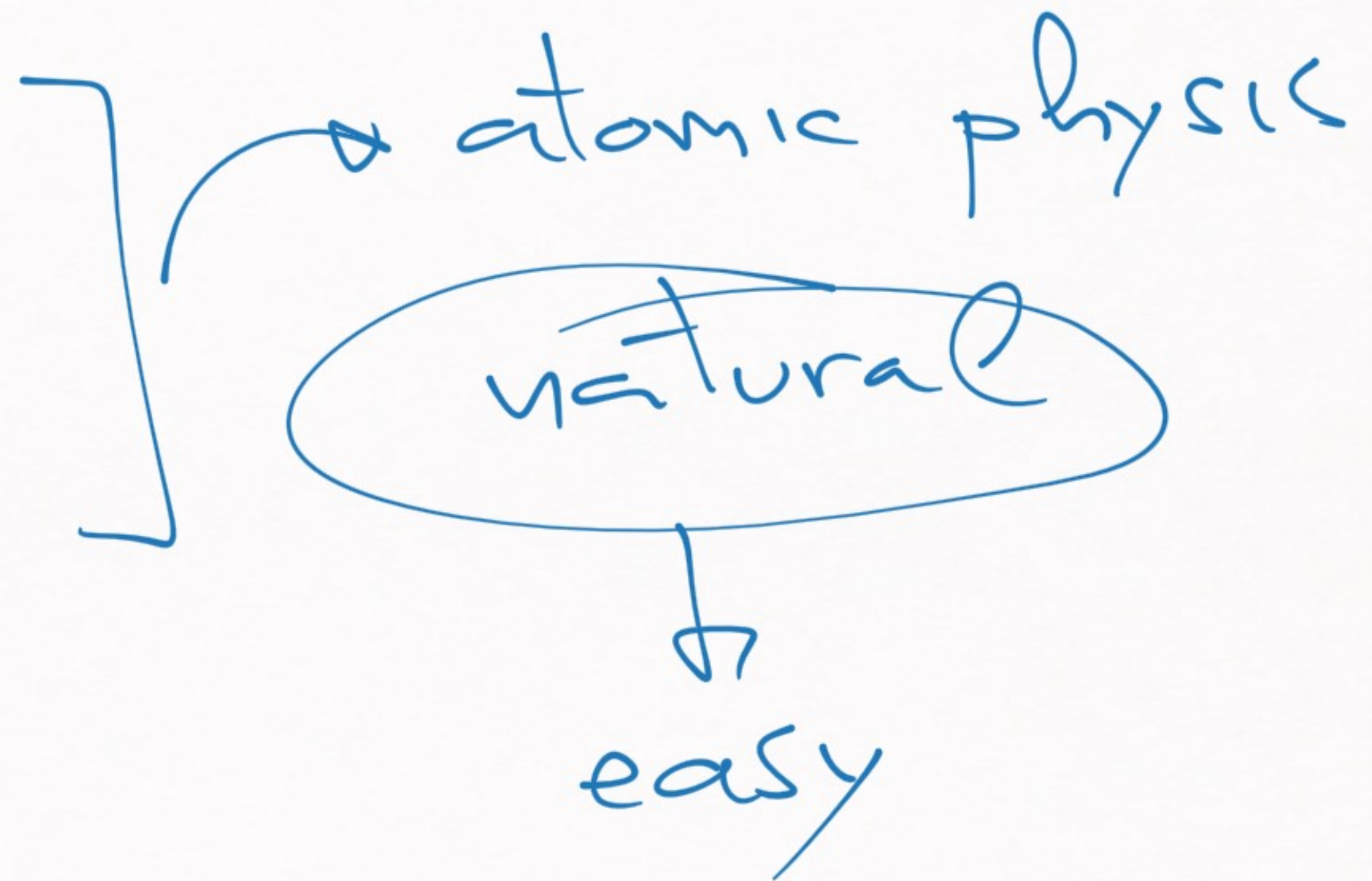
$$\Rightarrow \lambda \sim \mathcal{O}(1)$$

# UNNATURAL PROBLEMS



hard problems

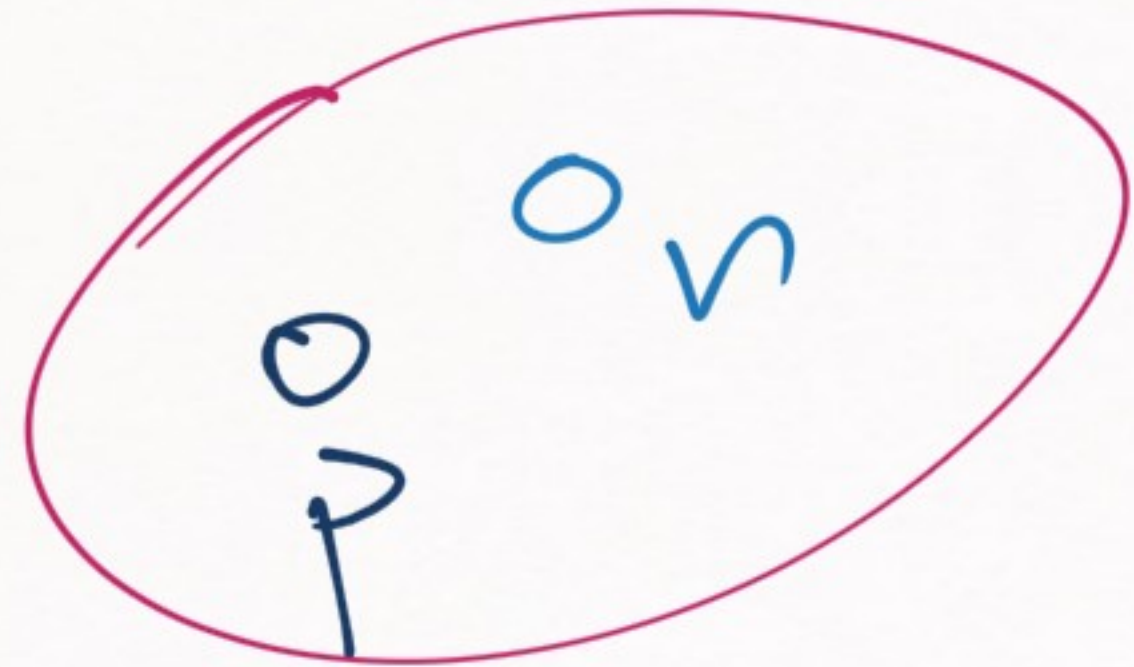
$$\gamma_B = \frac{1}{a_B}$$
$$\sqrt{\langle r^2 \rangle} = \sqrt{3} a_B$$



undergraduates  $\rightarrow$  atomic properties w/  
a precision of  $\frac{1}{10^4} - \frac{1}{10^6}$

# NUCLEAR PHYSICS

$$\frac{1}{m\pi} \approx 1.4 \text{ fm}$$



$35_1$

$$\gamma \approx 0.23 \text{ fm}^{-1}$$

$$\frac{1}{\rho} \approx 5.4 \text{ fm} \quad \left( \frac{3}{m\pi} \right)$$

(explained better next lesson)

$\begin{matrix} 0 \\ p \end{matrix} \rightarrow \begin{matrix} 0 \\ s \end{matrix} \quad (s=0) \rightarrow \text{singlet state}$

virtual state:  $\gamma \approx 0.04$  ↗ fine-tuning

$$\frac{1}{\gamma} \approx 8 \text{ MeV}$$

$\ll \underline{140 \text{ MeV}}$

virtual state (np) → reason why  
thermal neutrons  
are bad  
for your health





EXTRA:  $\left[ \nabla^2 - \frac{2}{ar} \right] \psi(r) = - \gamma_B^2 \psi(r)$

RESULT

$\gamma_B = \gamma_B(\dots)$

$\nabla^2$   
 $\frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} + \dots$

part of the equation

$$\left( -\frac{d^2}{dr^2} - \frac{2}{ar} \right) u(r) = -\gamma_B^2 u(r)$$

(S-wave)

$$\frac{u(r)}{r} = \psi(r)$$

$$\gamma_B^2 = \gamma_B^2(\underline{a_B})$$

$$\gamma_B = \gamma_B(\underline{a_B})$$

$$\psi^2(x)$$

$$\psi(x)$$

$$\psi(x, y)$$

$$\psi(x)$$

$$\gamma_B^2 = \frac{c^2}{a_B^2}$$