Nuclear Physics 22

The Collective Model

Part 2: Vibrational modes

Curious observation!

—— 2^+ Excited state

—— 0^+ Ground state of even-even nuclei

$\rightarrow$ Almost universal pattern

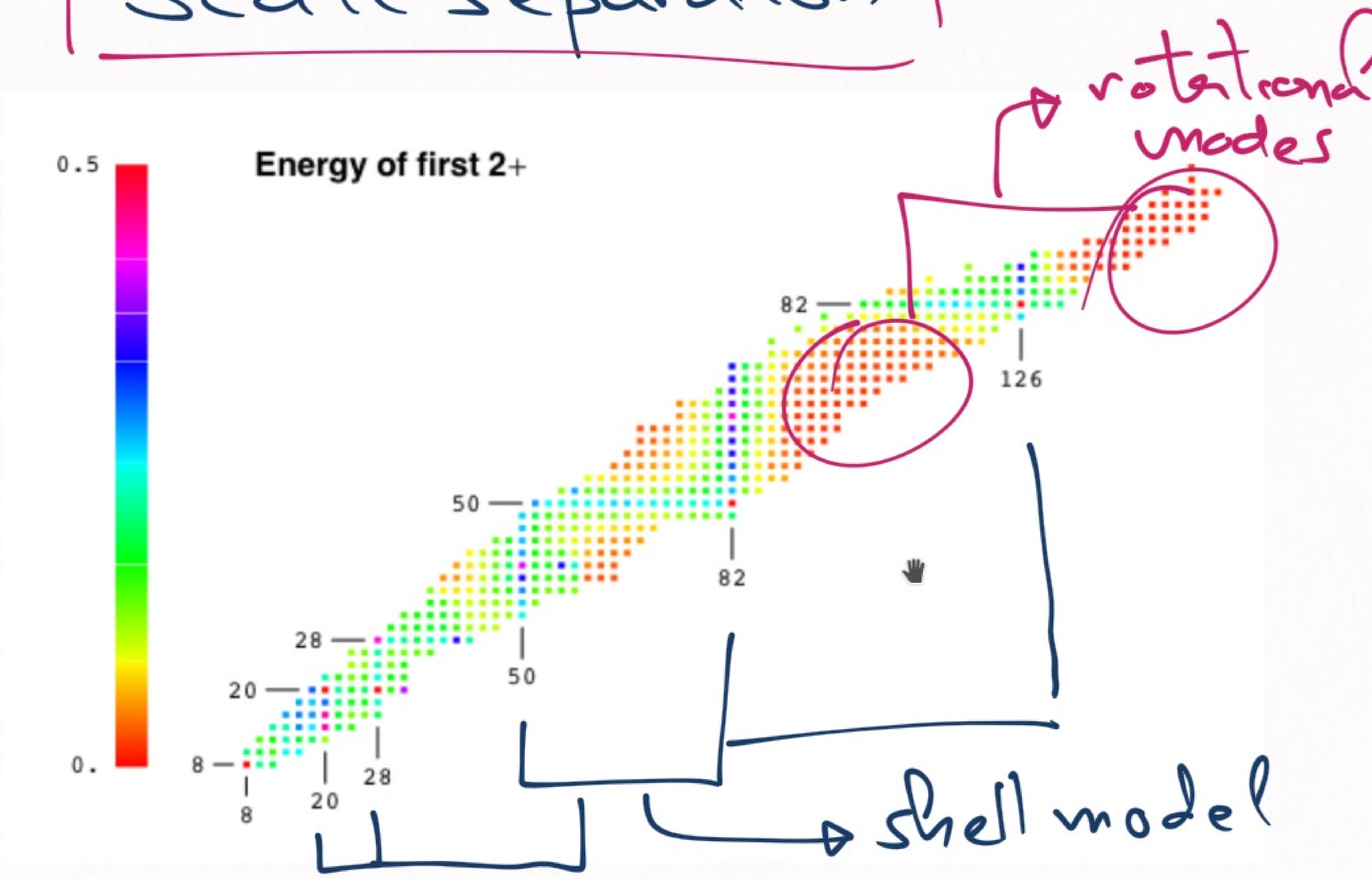
Explanations:

1) Shell model $\rightarrow$ 2^+ achievable
(but not a necessary consequence)

2) Rotational modes
$\rightarrow$ 2^+ necessary

3) Vibrational modes
$\rightarrow$ 2^+ necessary

Scale separation



1) Shell model $E(2^+) \sim (1-2)\,\mathrm{MeV}$

2) Rotational modes

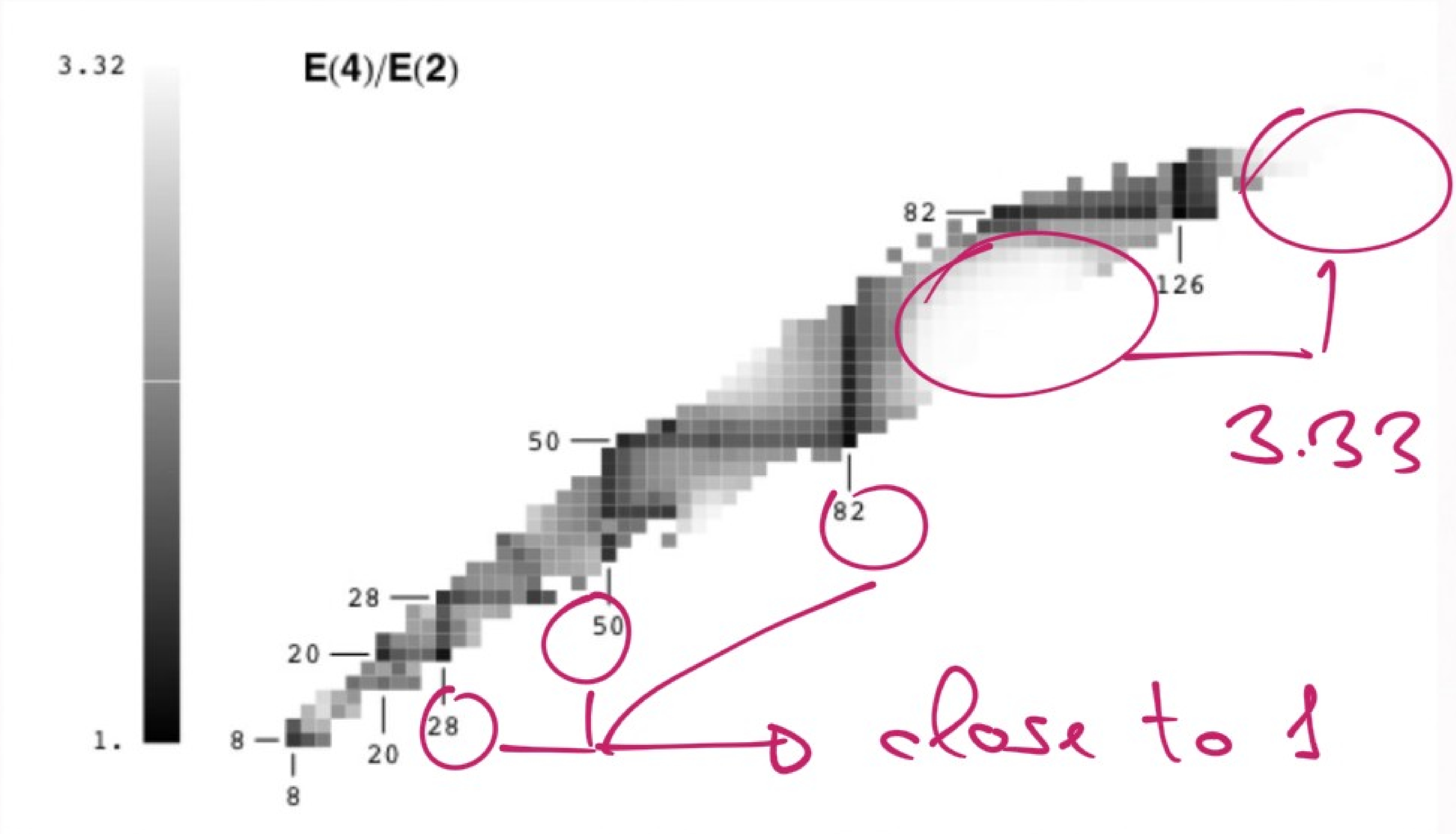
$E(2^+) \sim 0.1\ \mathrm{MeV}$

$\Rightarrow$ There is a word in between

3) Vibrational modes ?

$\rightarrow$ might this be the answer?

We also have $E(4^+)/E(2^+)$



1) Shell model : if $J_{shell} > 2$

$\Rightarrow$ $2^+, 4^+$ could have similar energies (except for residual ΔV)

2) Rotational modes:

$E(4^+)/E(2^+) \sim 3.33$

3) Vibrational modes:

$E(4^+)/E(2^+) \sim 2$

A few pieces of the nuclear structure puzzle:

1) $\frac{\overline{}\ 2^+}{\overline{}\ 0^+}$ } powerful pattern

2) $E(2^+)$ $\longrightarrow$ scales matter

3) $\dfrac{E(4^+)}{E(2^+)}$ $\longrightarrow$ requires explanation

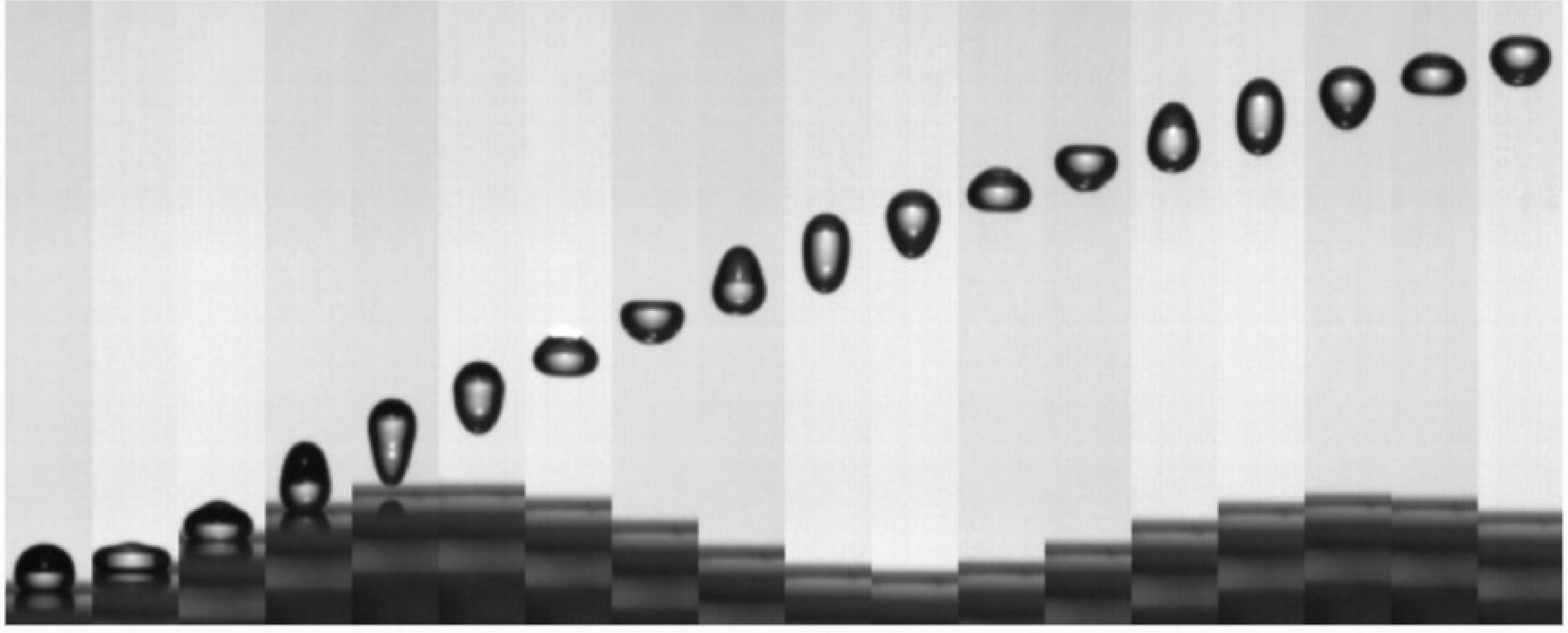
— ⊗ —

LET'S GO BACK TO THE COLLECTIVE MODEL

COLLECTIVE MODEL

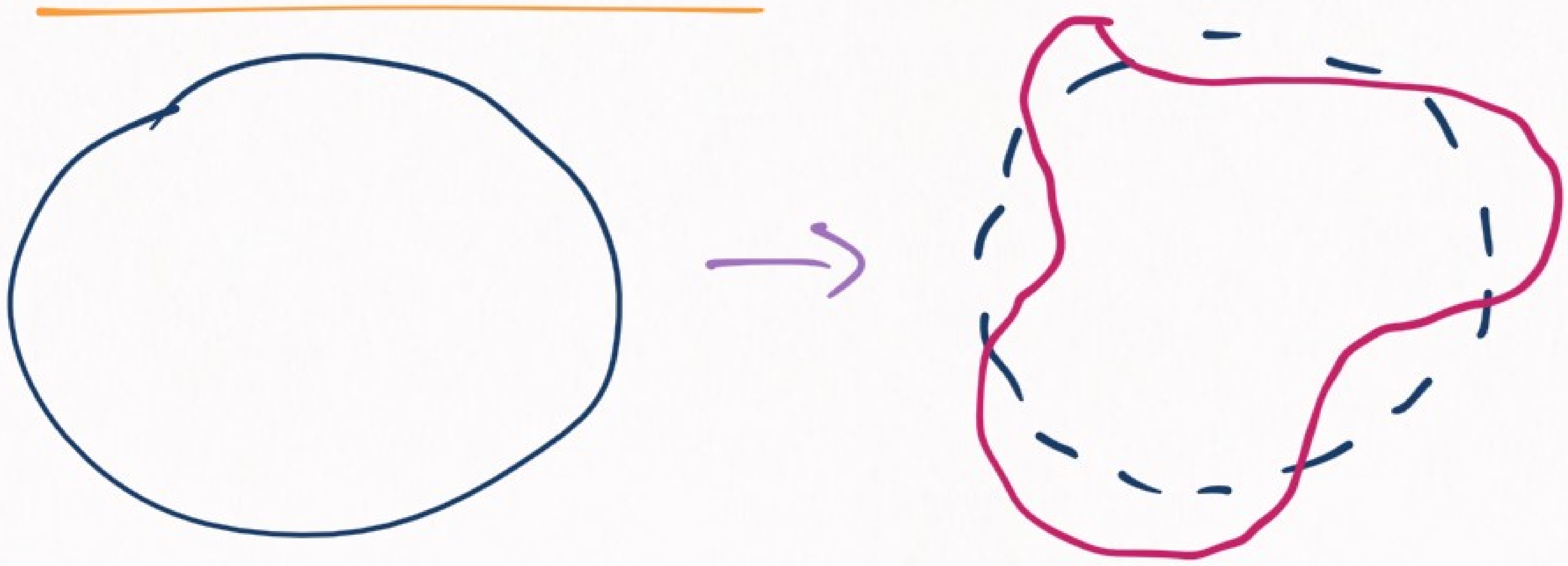
1) Rotational modes:
   - → non-spherical nucleus
   - → rigid body

2) Vibrational modes:
   - → spherical nucleus
   - → liquid



↳ Like this, but quantum-mechanical

BASIC IDEA



GROUND STATE

VIBRATION ($l=3$)

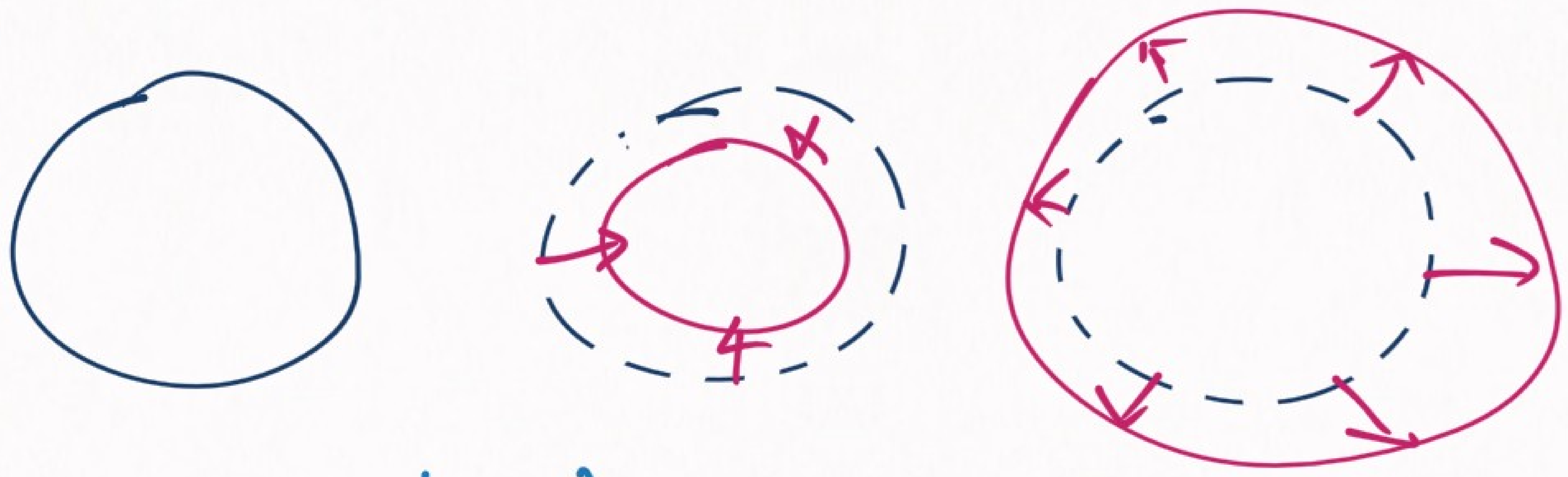
→ $\boxed{\text{QUANTIZED VIBRATIONS}}$

↳ Phonons (a type of boson)

(Maybe you have already meet them in Statistical Mechanics)

Types of vibrations/phonons:

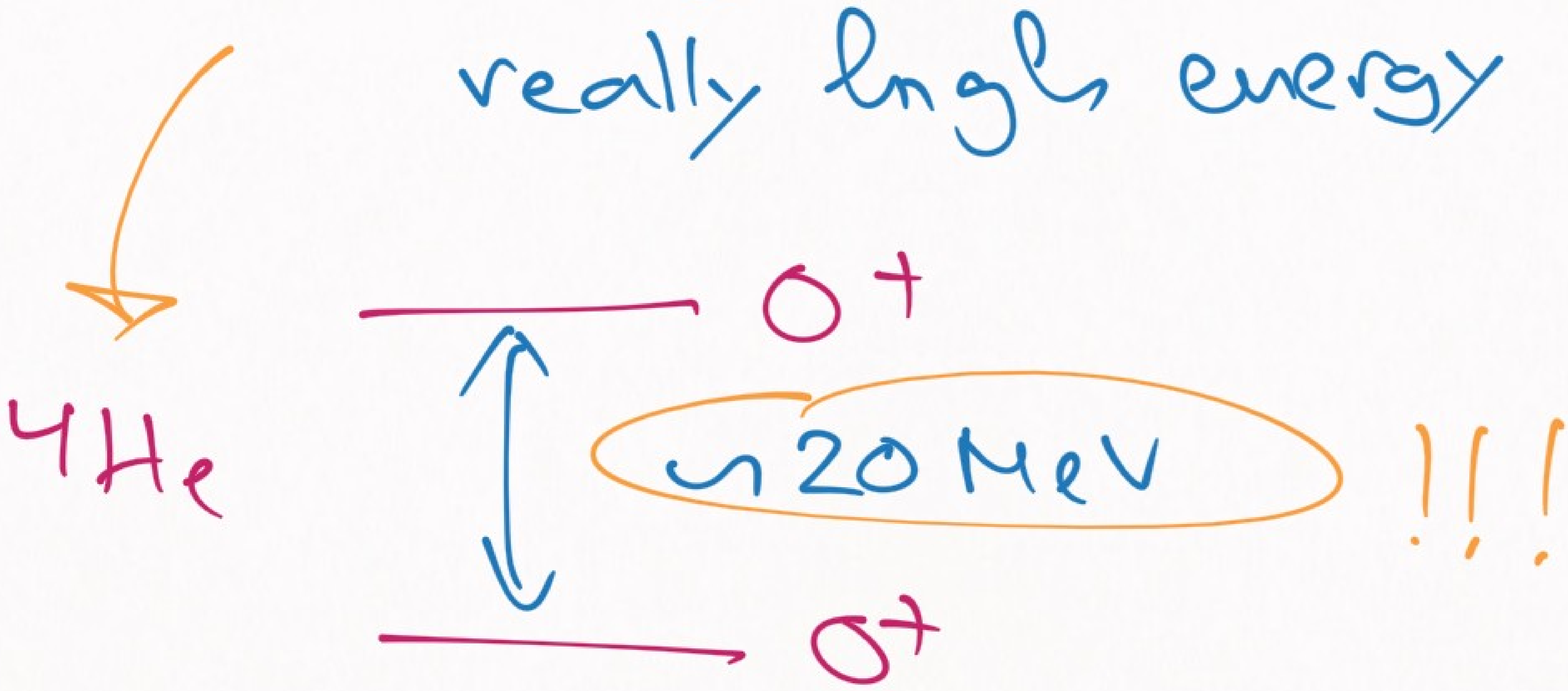
$\boxed{1}$



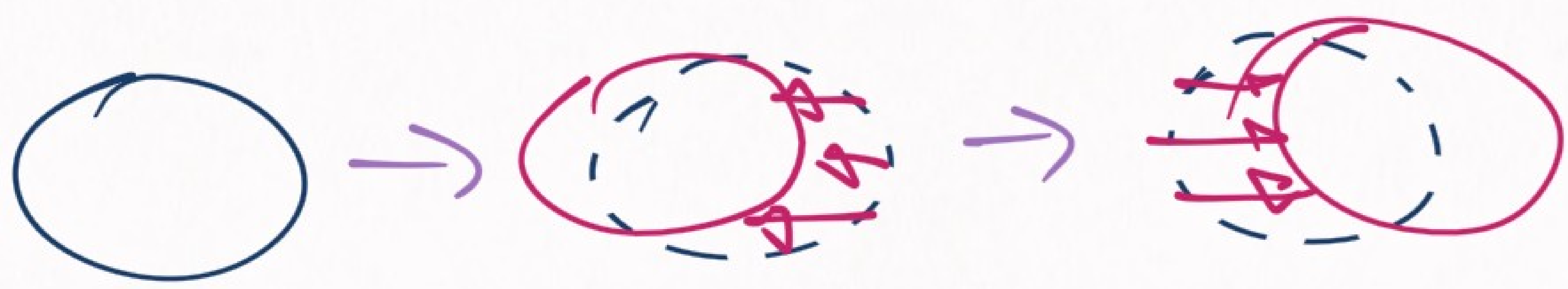
L=0 vibration
(breathing mode)

→ Related to compressibility
(but liquid are not particularly compressible)

→ Breathing mode is really high energy



12 Dipolar ($L = 1$)



$L = 1$ vibration

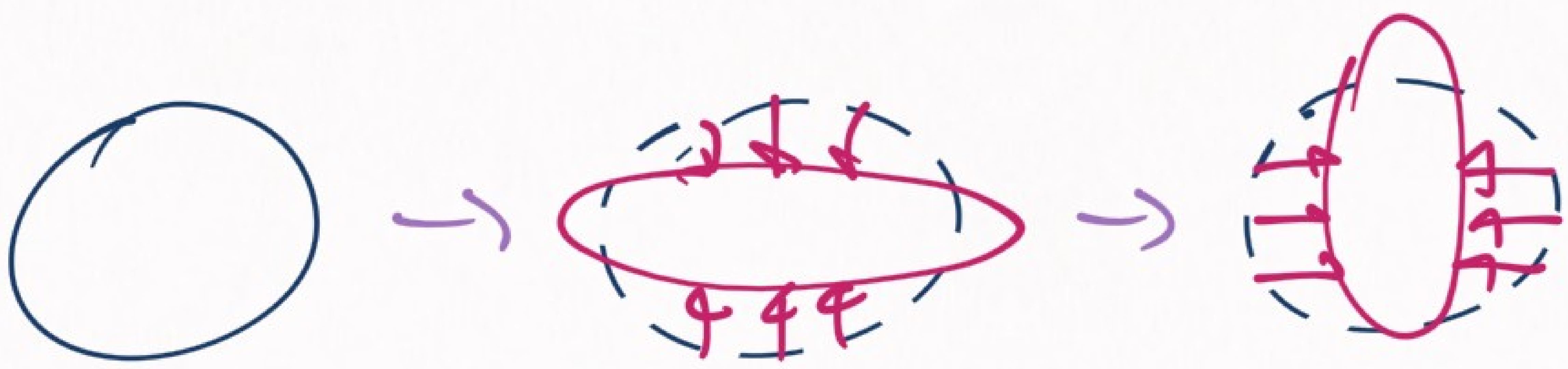
→ Does not count!

Why? → Because it is actually a translation

(not an internal vibration)

→ $\boxed{\text{WE IGNORE IT}}$

3) Quadrupolar ($l=2$)

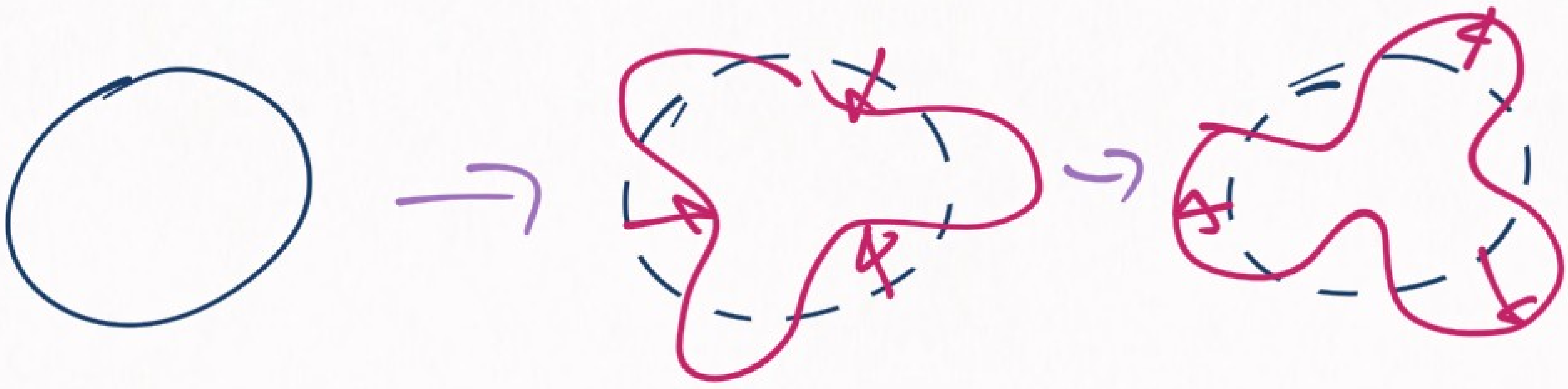


→ $l=2$ vibration

→ lower energy (a fraction of a MeV)

→ most important type of vibration

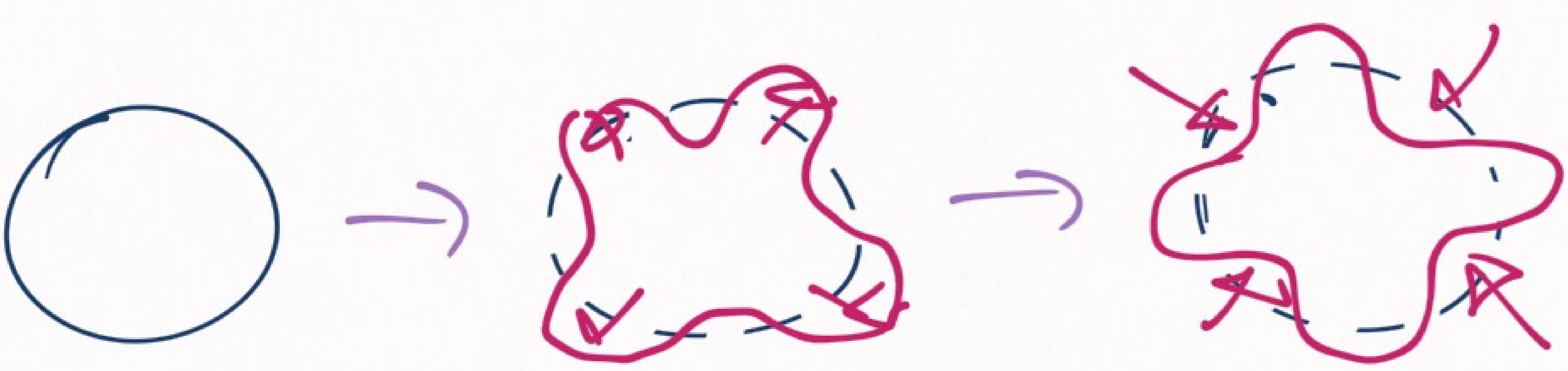
14 Octupolar ($L=3$)



→ $L=3$ vibration

→ $\omega_3 \simeq (2\text{-}3)\,\omega_2$
(higher energy)

15 Hexadecapolar ($L=4$)



→ $L=4$ vibration

→ even higher energy

RECAP

$$R(\theta,\varphi) = R_0\left[1 + \sum_{\lambda\mu} \alpha^*_{\lambda\mu} Y_{\lambda\mu}(\theta,\varphi)\right]$$

$\lambda = 0 \rightarrow$ compression (high energy $\rightarrow$ ^4He)

$\lambda = 1 \rightarrow$ translation (not a deformation)

$\lambda = 2 \rightarrow$ quadrupole deformation

$\lambda = 3 \rightarrow$ octupole deformation

$\lambda = 4 \rightarrow$ hexadecapole

$\lambda = 5 \rightarrow$ triacontadipole (32-pole 也可以)

The Quantization Process

→ Second quantization

(just like number operators in harmonic oscillator)

$$H|n_0, n_2, n_3, \ldots\rangle =$$

$$\hbar(n_0\omega_0 + n_2\omega_2 + n_3\omega_3 + \ldots)$$

$$\times |\underbrace{n_0, n_2, n_3}, \ldots\rangle$$

# of phonons w/ $L = 0, 2, 3, \ldots$

(they are bosons)

→ $\boxed{\text{Easy-peasy}}$

Only one complication :

J^P → monopole : 0^+

quadrupole : 2^+

octupole : 3^-

⇒ If more than one phonon,

we couple angular momenta

n-monopoles : $0^+ \otimes 0^+ \otimes 0^+ \dots$

$= 0^+$

2-quadrupoles :

$$2^+ \otimes 2^+ = 0^+ \oplus 2^+ \oplus 4^-$$

Exercise : why not $1^+, 3^+$?

(1 point)

What type of spectrum should we obtain?

3^- $\qquad \omega_3 \sim (2-3)\,\omega_2$

$0^+, 2^+, 4^+ \qquad 2\omega_2$

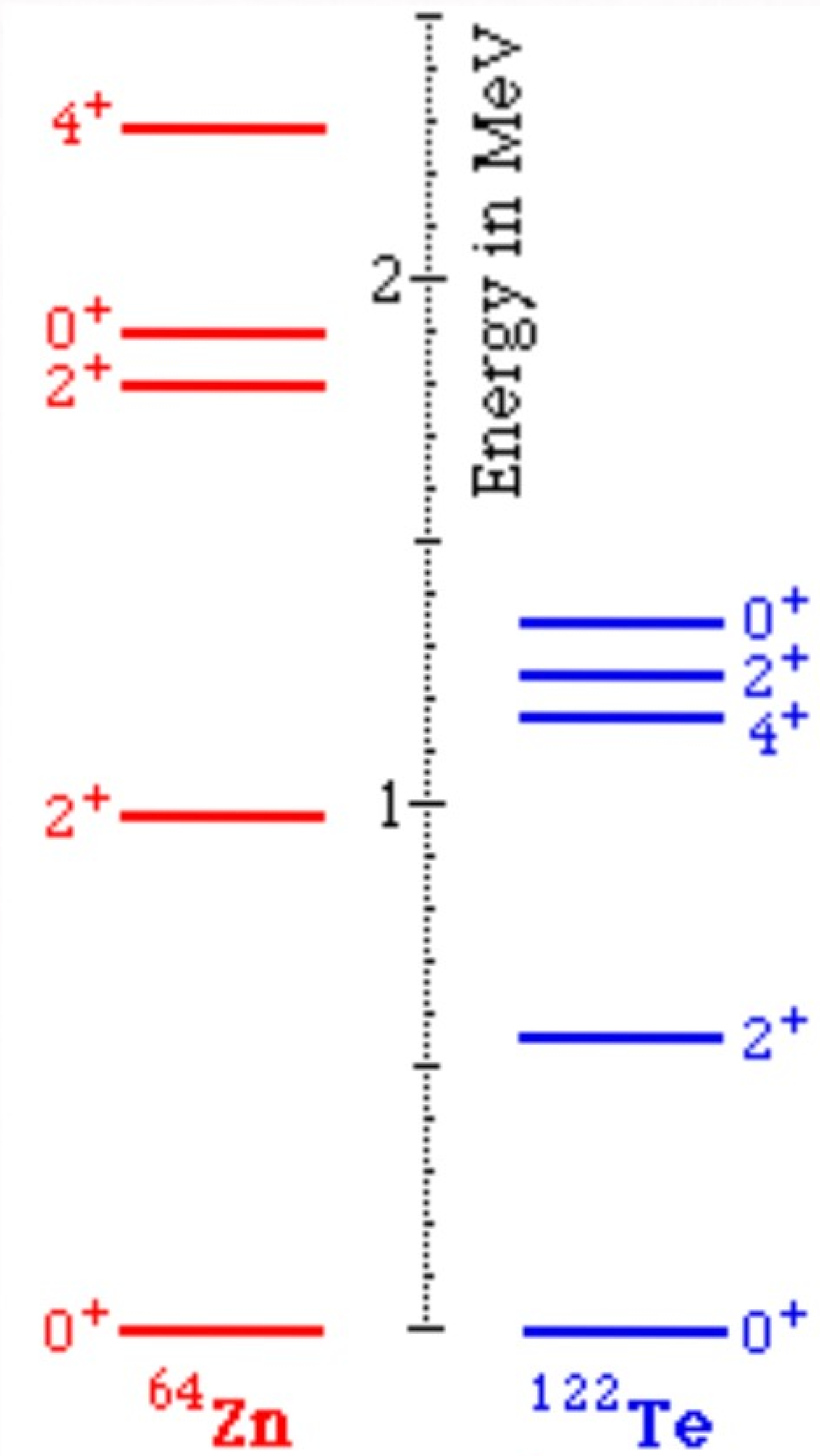
$2^+ \qquad \omega_2$

$0^+ \qquad 0$

$$\Rightarrow \boxed{\frac{E(4^+)}{E(2^+)} \simeq 2}$$

Our theoretical prediction

EXAMPLES



Typical vibrational spectrum

$\omega_2 \sim 0.5\,\text{MeV}$

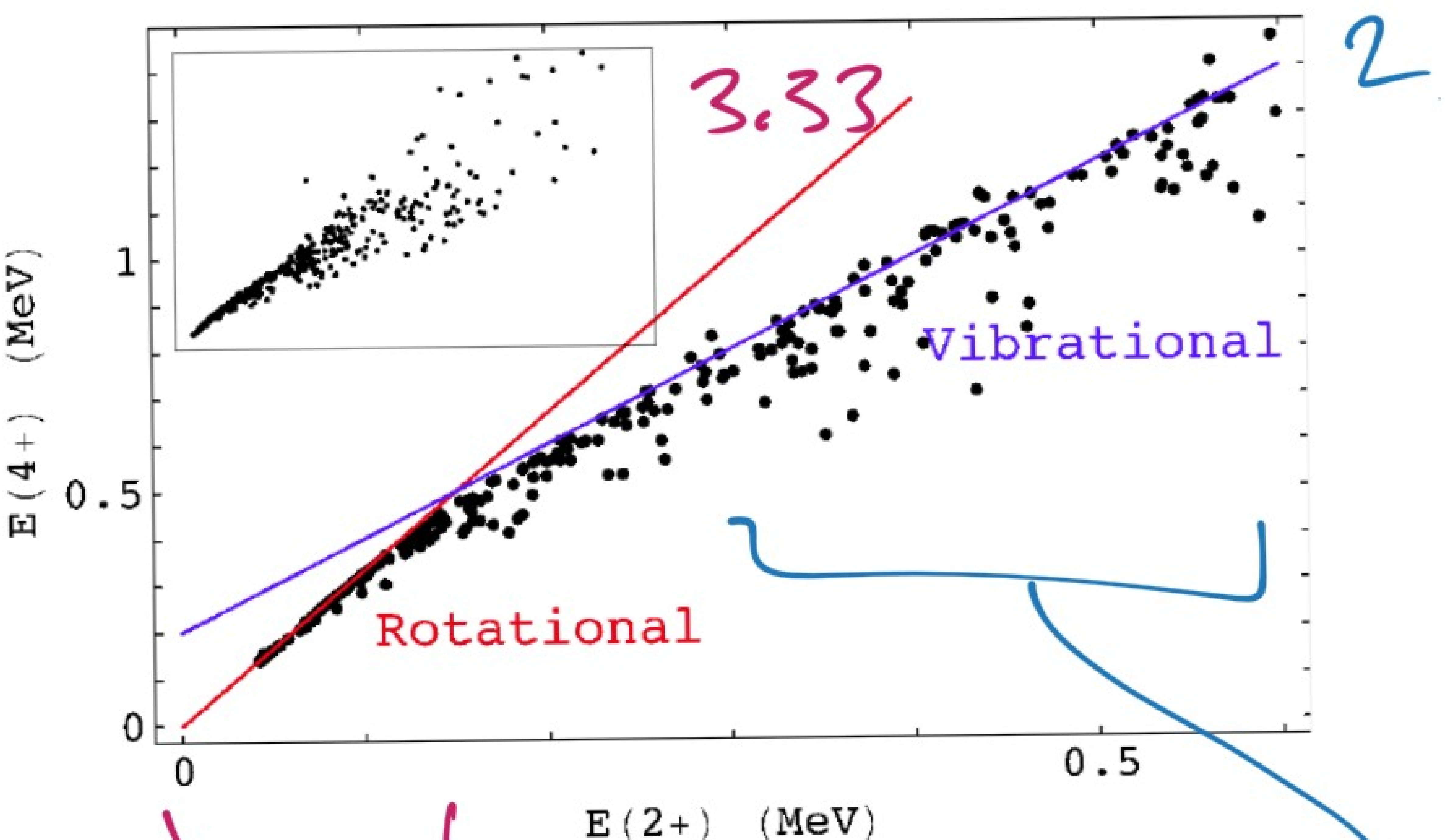
In between shell-model and vibrational

$E(2^+) \sim 1\,\text{MeV}$

$E(4^+) > 2E(2^+)$

Still, looks very vibrational

TRANSITION FROM ROTATIONAL TO VIBRATIONAL:



N.V. Zamfir *et al.*, Phys. Rev. Lett. 72 (1994) 3480

$$E(2^+) \sim 0.1 \text{ MeV}$$

$$\frac{E(4^+)}{E(2^+)} \sim 3.33$$

$$E(2^+) \sim (0.3-0.6) \text{ MeV}$$

$$\frac{E(4^+)}{E(2^+)} \sim 2$$

$\boxed{\text{NEXT LESSON :}}$

Including interactions in these models