Nuclear Physics (27)

The Collective Model

Part 1: Rotational modes

Curious observation

(from previous lesson)

—— 2^+ First excitation

—— 0^+ Ground state

(even-even nuclei)

Concrete example → $^{38}_{18}Ar$

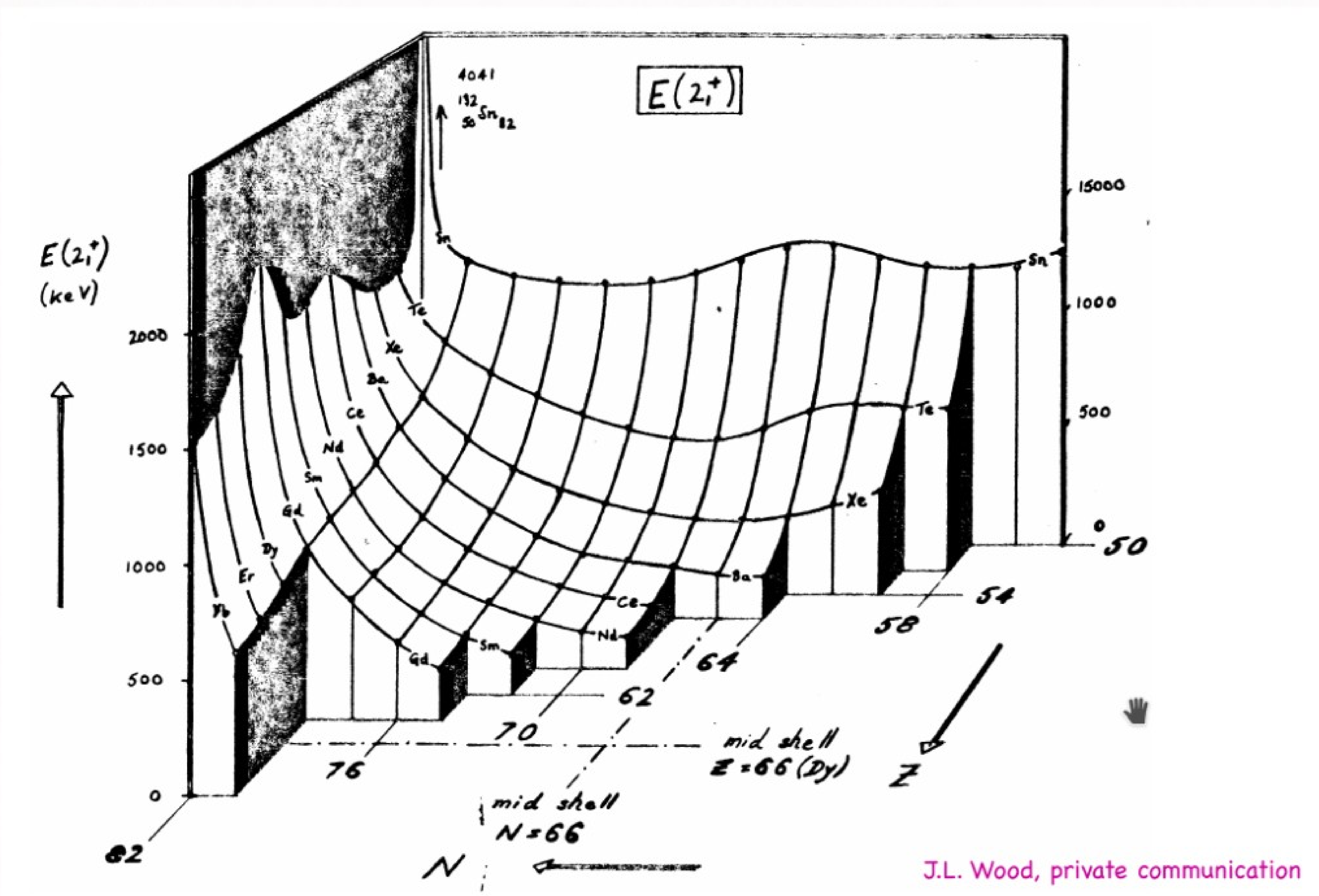
↙

[But happens in almost all 0^+ nuclei]

→ **EXPLANATION WANTED**

—— 2+ First
—— 0+ Grond

} shell model can explain this, but...



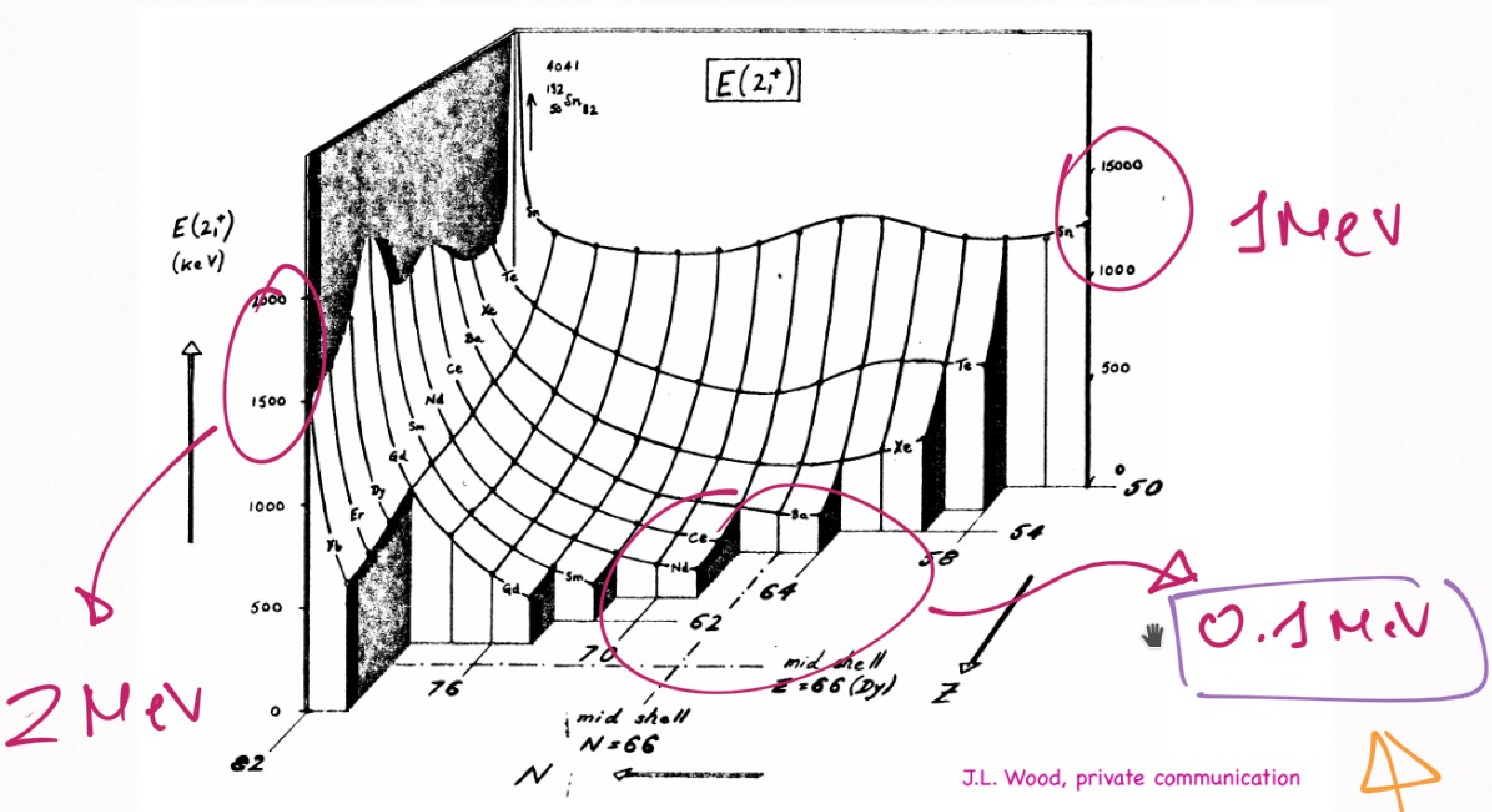
J.L. Wood, private communication

... far away from shell closures

$E(2_1^+)$ is sort of small

[SCALE ARGUMENT]

Let's have a closer look:



SHELL MODEL → $E(2_1^+) \sim (1-2)$ MeV

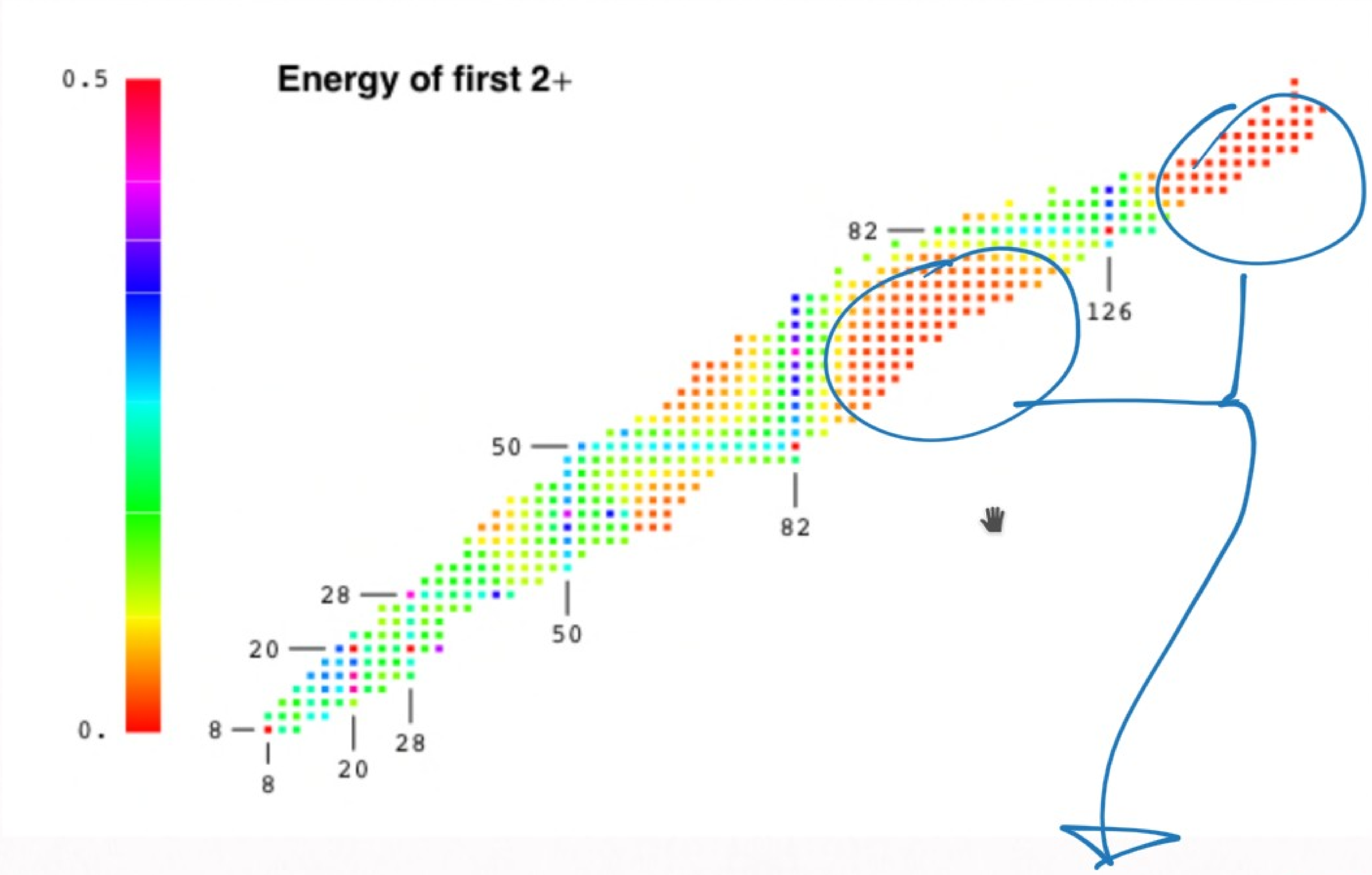
(N, Z ~ magic)

natural scale ~ few MeV

$$\hbar\omega \sim \frac{40}{A^{1/3}} \text{ MeV} \sim 8 \text{ MeV}$$

$$\frac{\hbar^2}{\mathcal{J}} \sim \frac{20}{A^{2/3}} \text{ MeV} \sim 1 \text{ MeV}$$

But far away from N, Z magic, $E(2_1^+)$ much smaller



[Particularly here]

Energy scale too small for being natural in the shell model

Thus, we need a different explanation

↓

COLLECTIVE MODEL

↓

nucleus as a "whole"

⇒

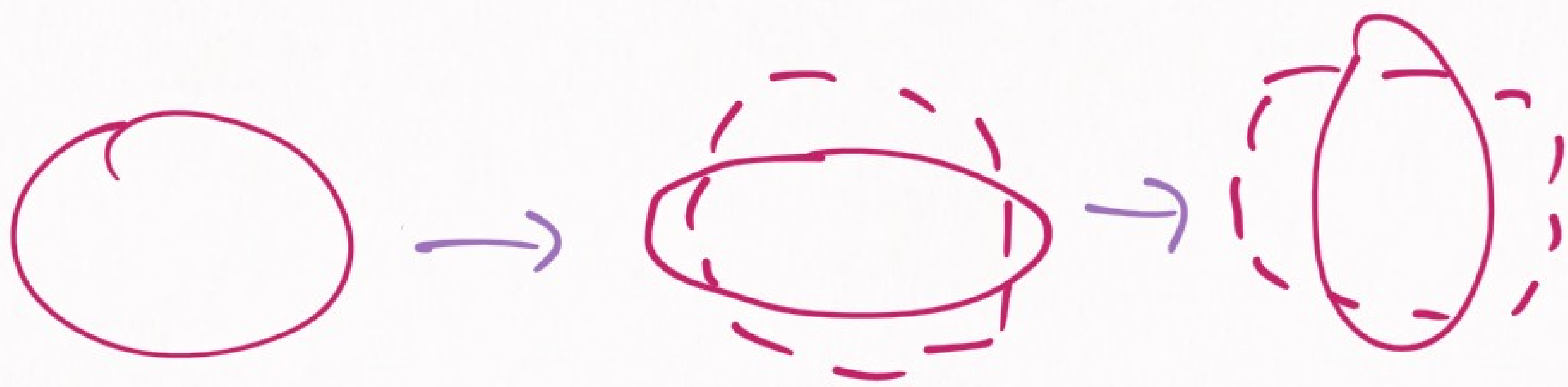
1) Liquid drop model
   → nucleus as a liquid (collective)

2) Shell model
   → individual nucleons in a mean field

3) Collective model
   → we go back to the liquid analogy

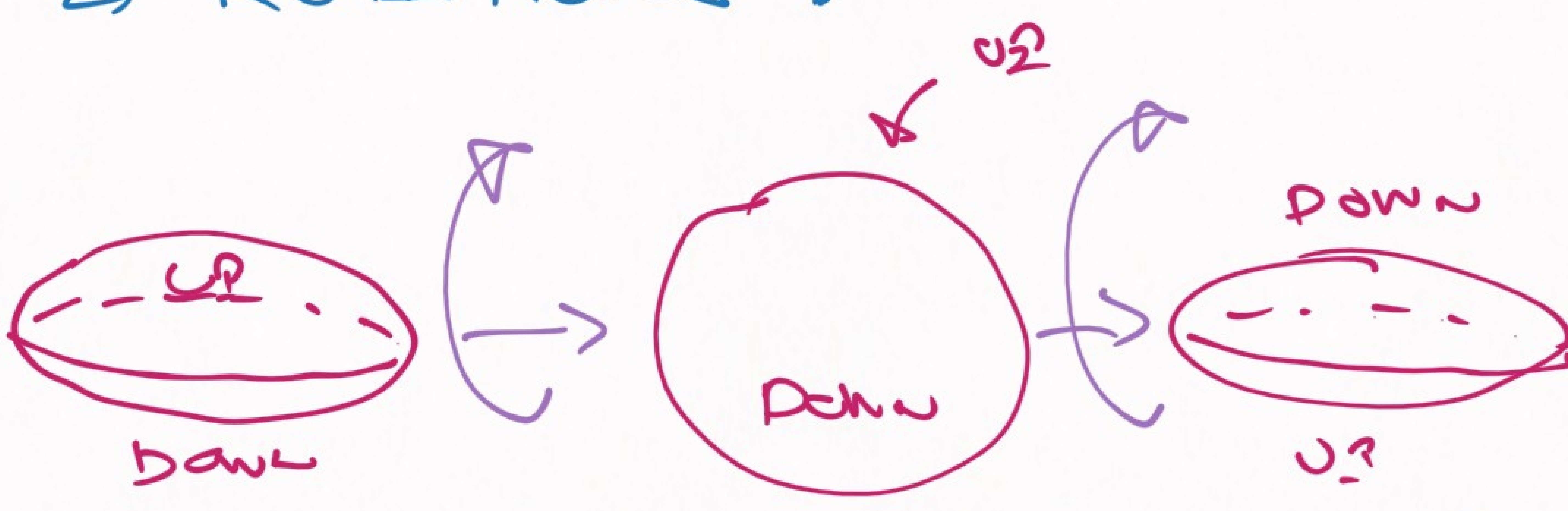
COLLECTIVE MODEL:

1) VIBRATIONS:



a liquid or elastic nucleus vibrating around its basic (spherical) form

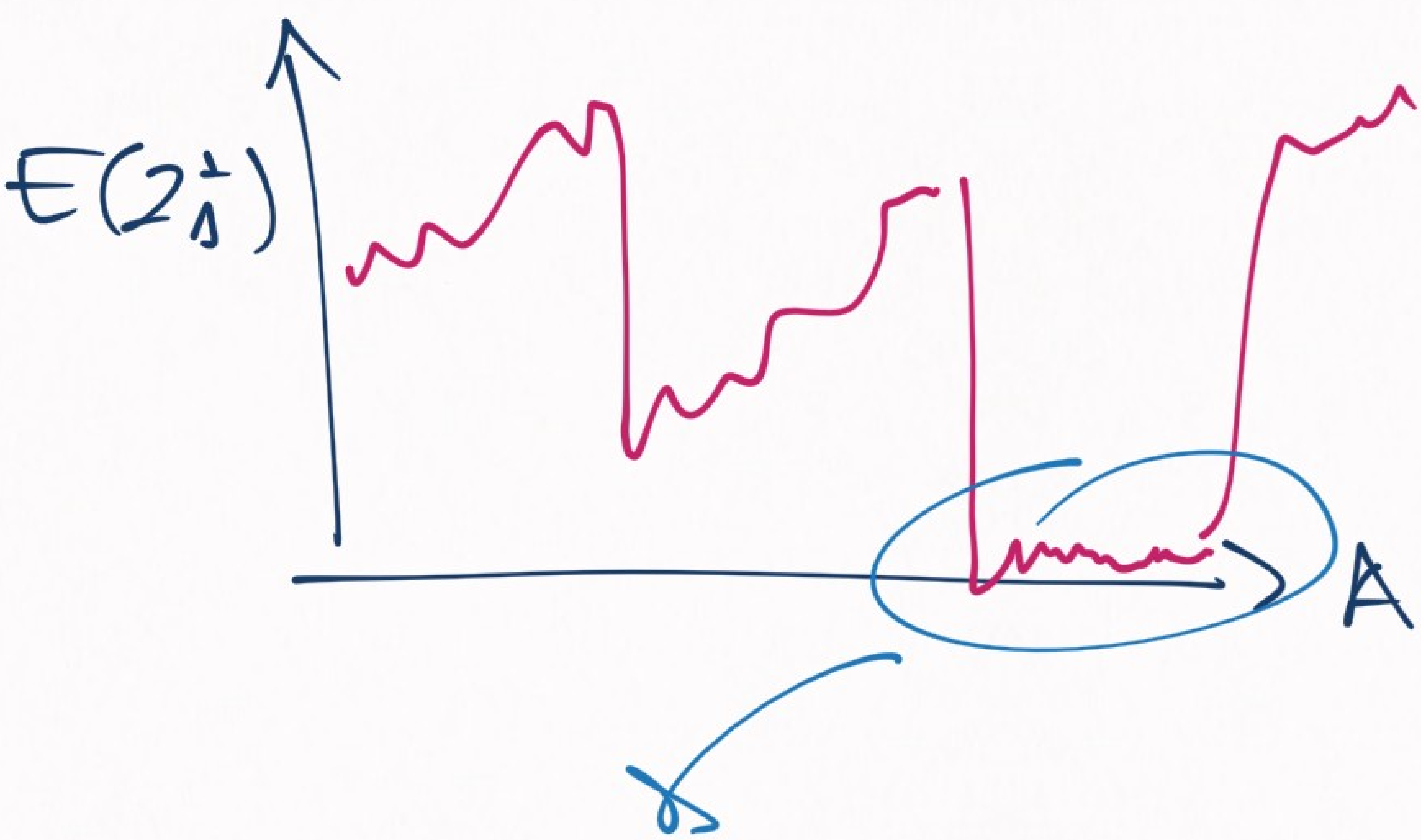
2) ROTATIONS:



a rigid nucleus (non-sperical) rotates around some axis

Today → **ROTATIONS**

(not the standard order, but they are easier)



$A \sim 150\text{-}190$ $E(2^+_1)$ really small (about ~ 100 keV)

→ [Very unusual feature]

$A \sim 150-190$ → very deformed nuclei (large quadrupolar moments)

1) Vibrations →  (spherical) nope

2) Rotations →  ✓ yes (non-spherial)

Maybe they can be explained as rotations

→ **QUANTUM ROTATOR**

QUANTUM ROTATOR

Remember the solid rigid & moments of intertia? (high school physics)

$$H = \frac{1}{2} I \vec{\omega}^2$$

$\vec{\omega}$ → rotation frequency

I → moment of inertia

⇒ let's quantize it!

1) $\vec{L} = I \vec{\omega}$ (angular momentum)

2) $$H = \frac{1}{2I} \vec{L}^2$$

→ We know how to quantize this!

[Quantum rotator :]

$$H = \frac{1}{2I}\vec{L}^2 \quad w/\ \vec{L}^2 = L(L+1)$$

$$L = 0, 1, 2, 3, \ldots$$

→ Really simple !!

But there are complications...
(as usual)

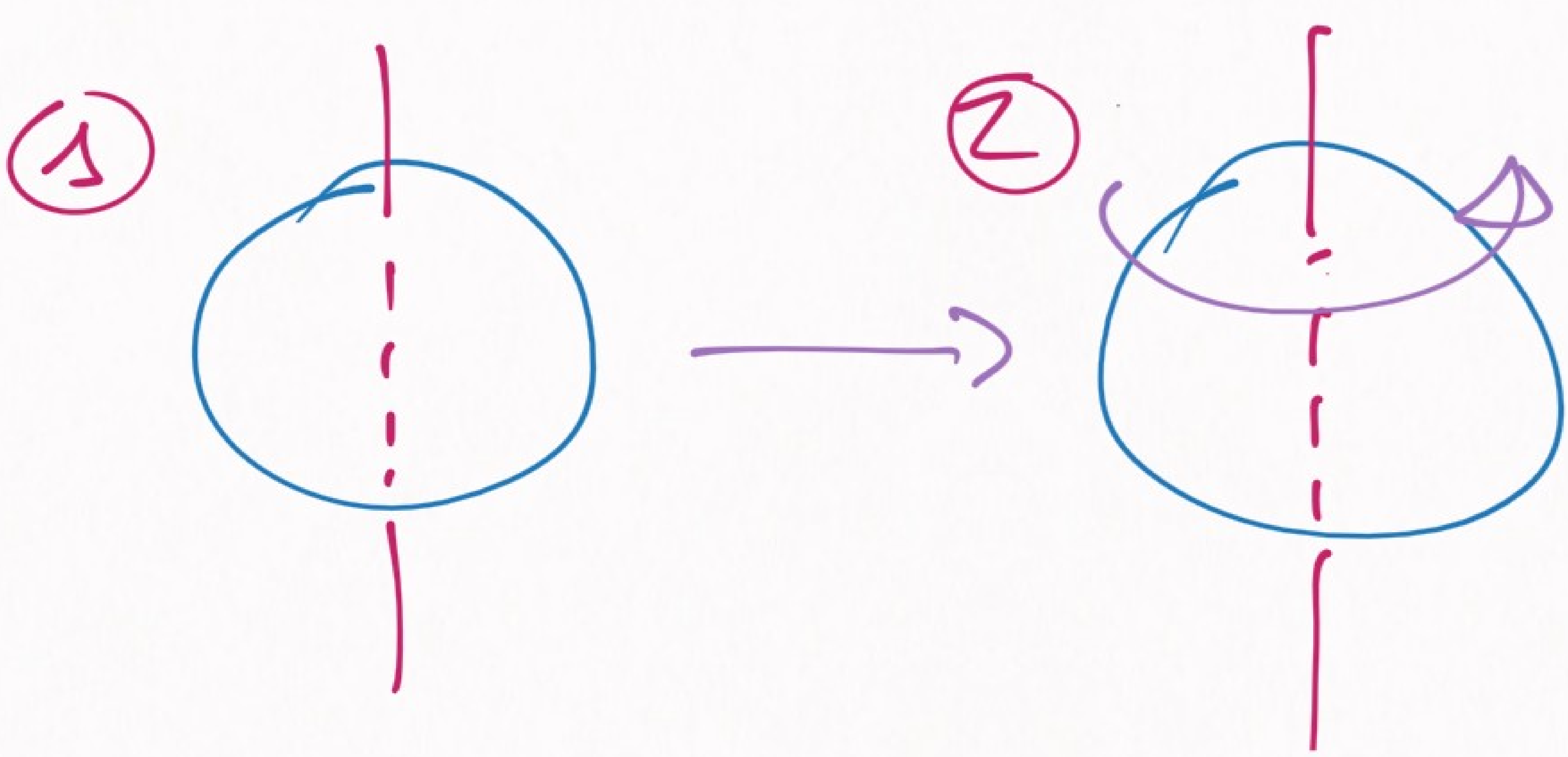
COMPLICATION (1)

↳ The rotation must do something to the system

Must do something?

What do you mean?

→ Try to rotate this :



①, ② → [NO DIFFERENCE]

Spherical nucleus

$\cong$ Rotated spherical nucleus

$\Rightarrow$ [No difference in their <u>internal</u> energy levels]

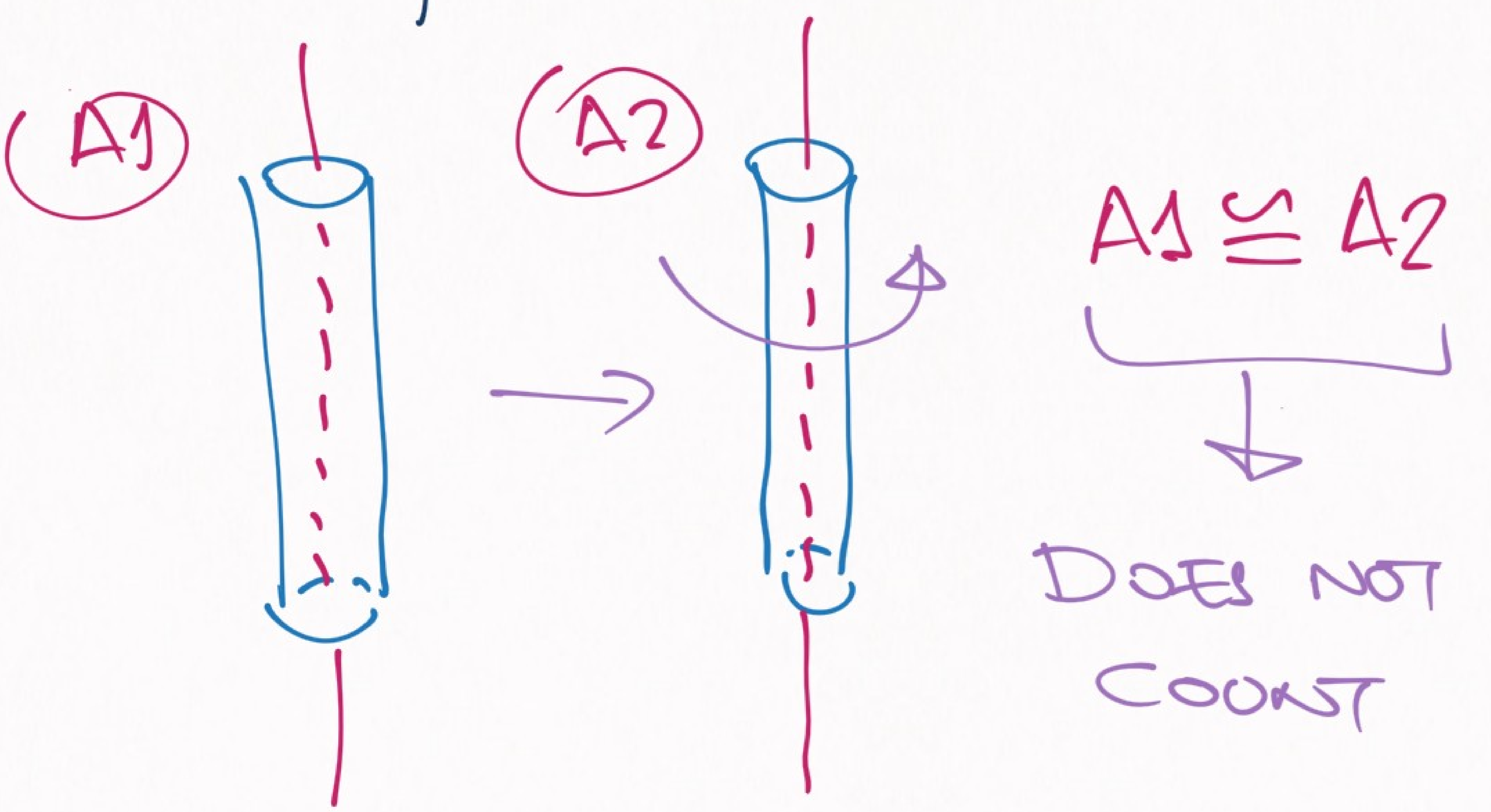
$\Rightarrow$ NO ROTATIONAL LEVELS

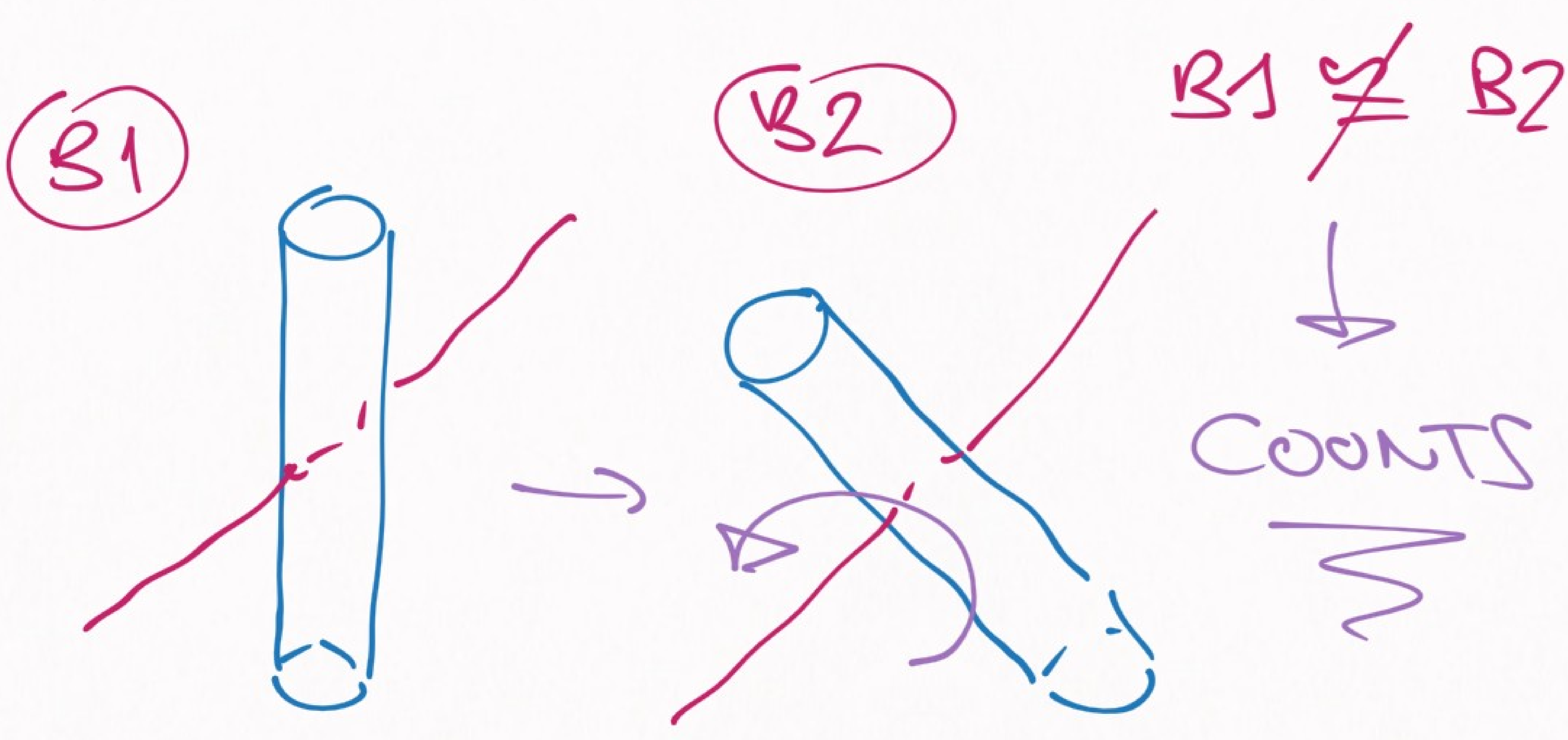
— o —

WHAT ABOUT NON-SPHERICAL SHAPES ?

→ It depends ...

Example: a (quantum) rod





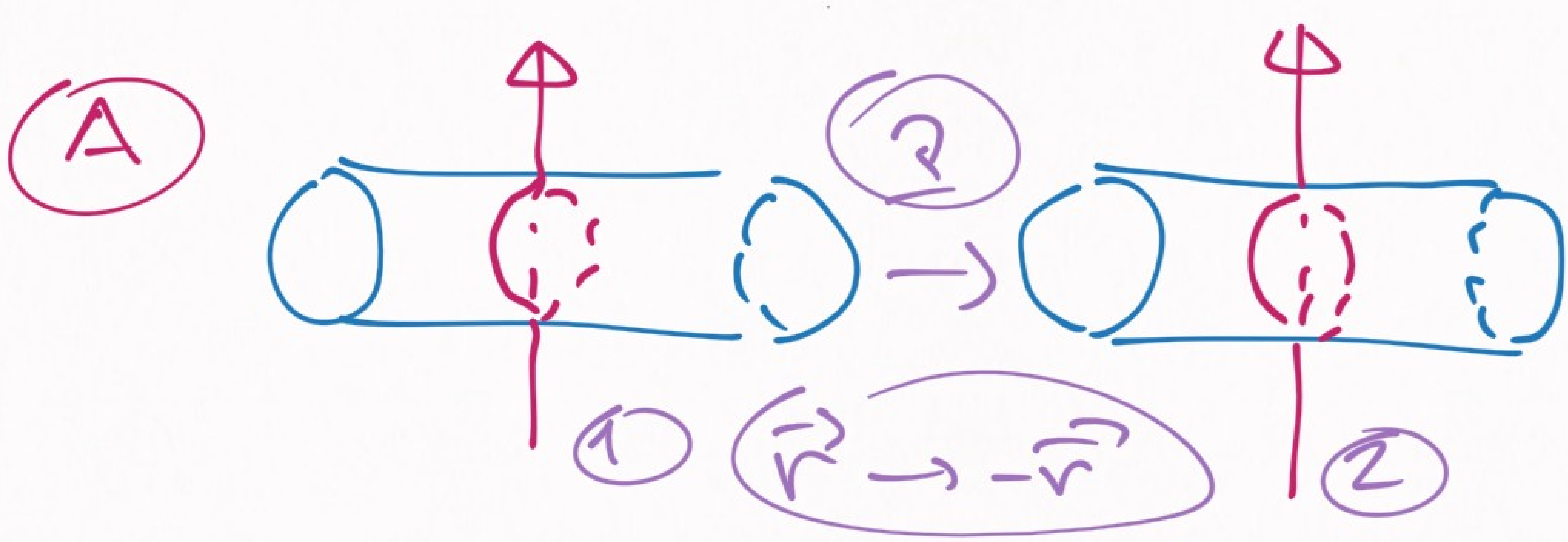
→ Depends on axis of rotation

CONJUGATION (2)

→ the rotation must have even L (L = 0, 2, 4, ...)

↓

WHY? Parity problems



$P|1\rangle = |2\rangle$ → identical configurations

↓

Does not count as a different state w/ different energy

Notice $\vec{r} \to -\vec{r}$ is identical to a $\theta = \pi$ rotation w/

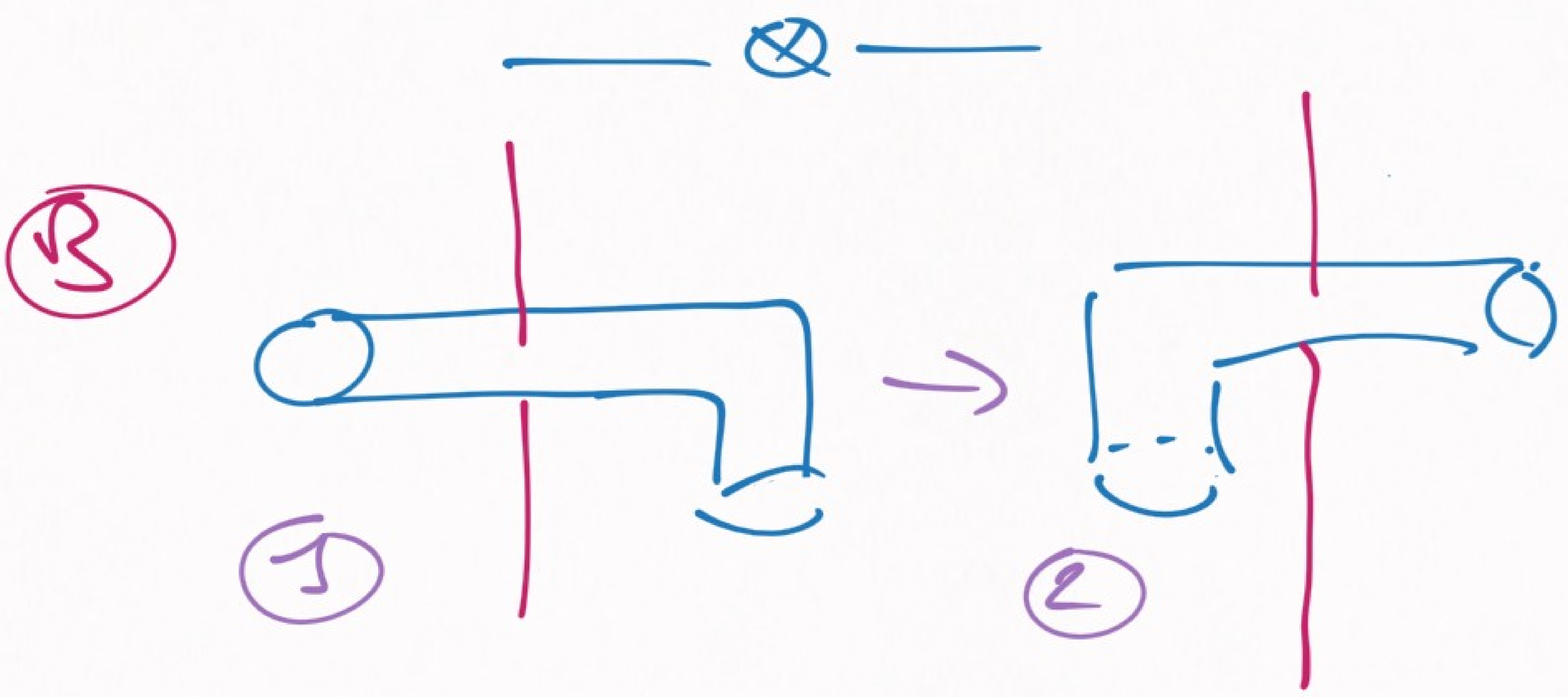
$L = 1, 3, 5, \ldots$

$R(\theta) = e^{iL\theta} \to R(\pi) = (-1)^L$

$\Rightarrow L = 1, 3, 5, \ldots$ works as a parity transformation for $\vec{r}$ ($L=1$ object) and the solid rigid

$\Rightarrow L = 0, 2, 4, \ldots$ we have $R\left((2n+1)\frac{\pi}{L}\right) = -1$ for the solid rigid, but this type of rotation does not do $\vec{r} \to -\vec{r}$ but $\vec{r} \to e^{i(2n+1)\frac{\pi}{L}} \vec{r}$

Subtle point → $L = 1, 3, 5, \ldots$ rotations contain an implicit parity transformation



$P|1\rangle = |2\rangle \neq |1\rangle$ which is not allowed in quantum mechanics, because

$P^2 = 1 \Rightarrow P|1\rangle = \pm|1\rangle$

$\Rightarrow \boxed{L \neq 1, 3, 5, \ldots}$ same conclusion

RECAP

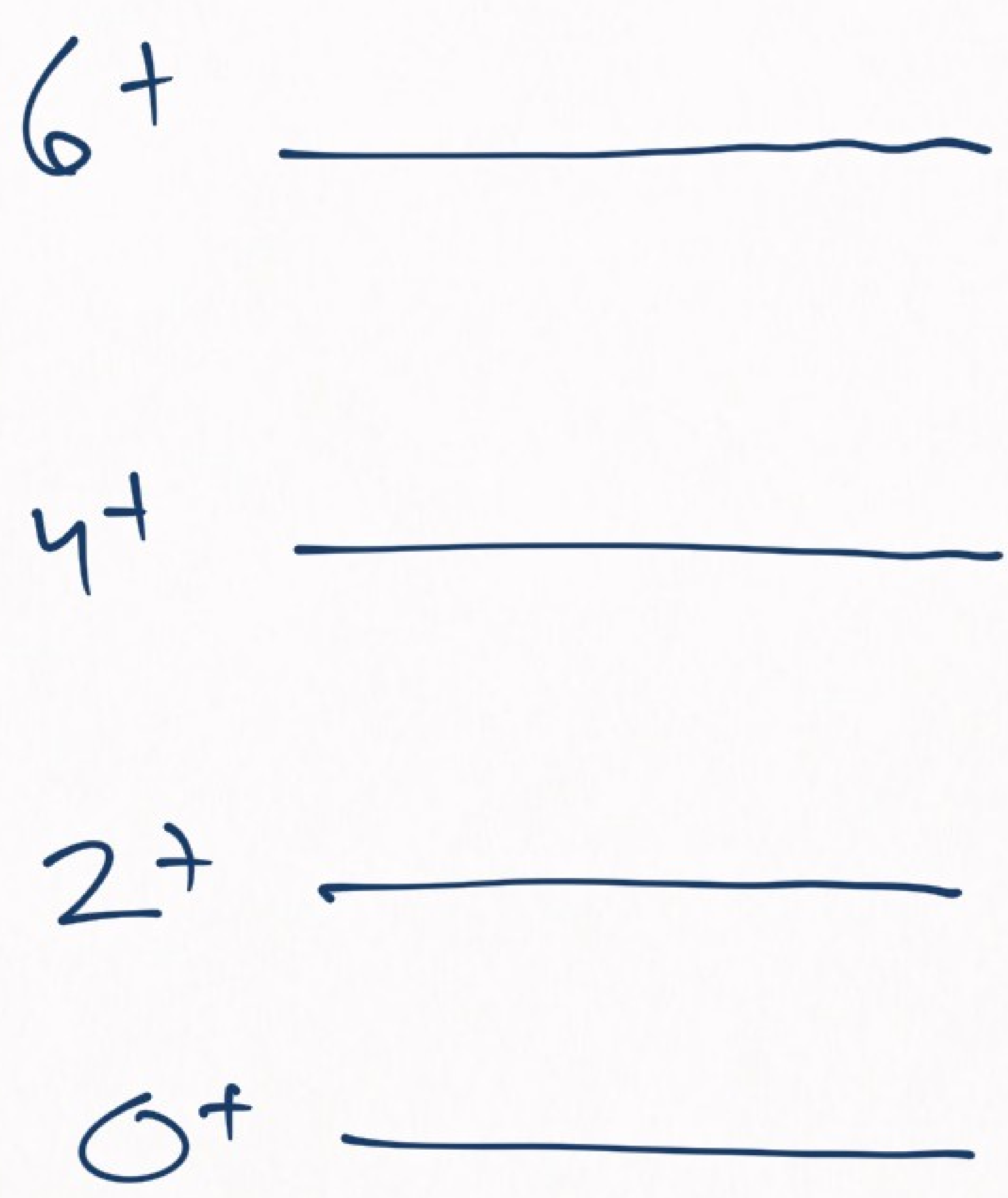
Rotational levels require:

1) Non-spherical nucleus

2) $L = 0, 2, 4, \ldots$

$\Rightarrow$ Energy levels given by:

$$E(L) = \frac{1}{2I} L(L+1)$$

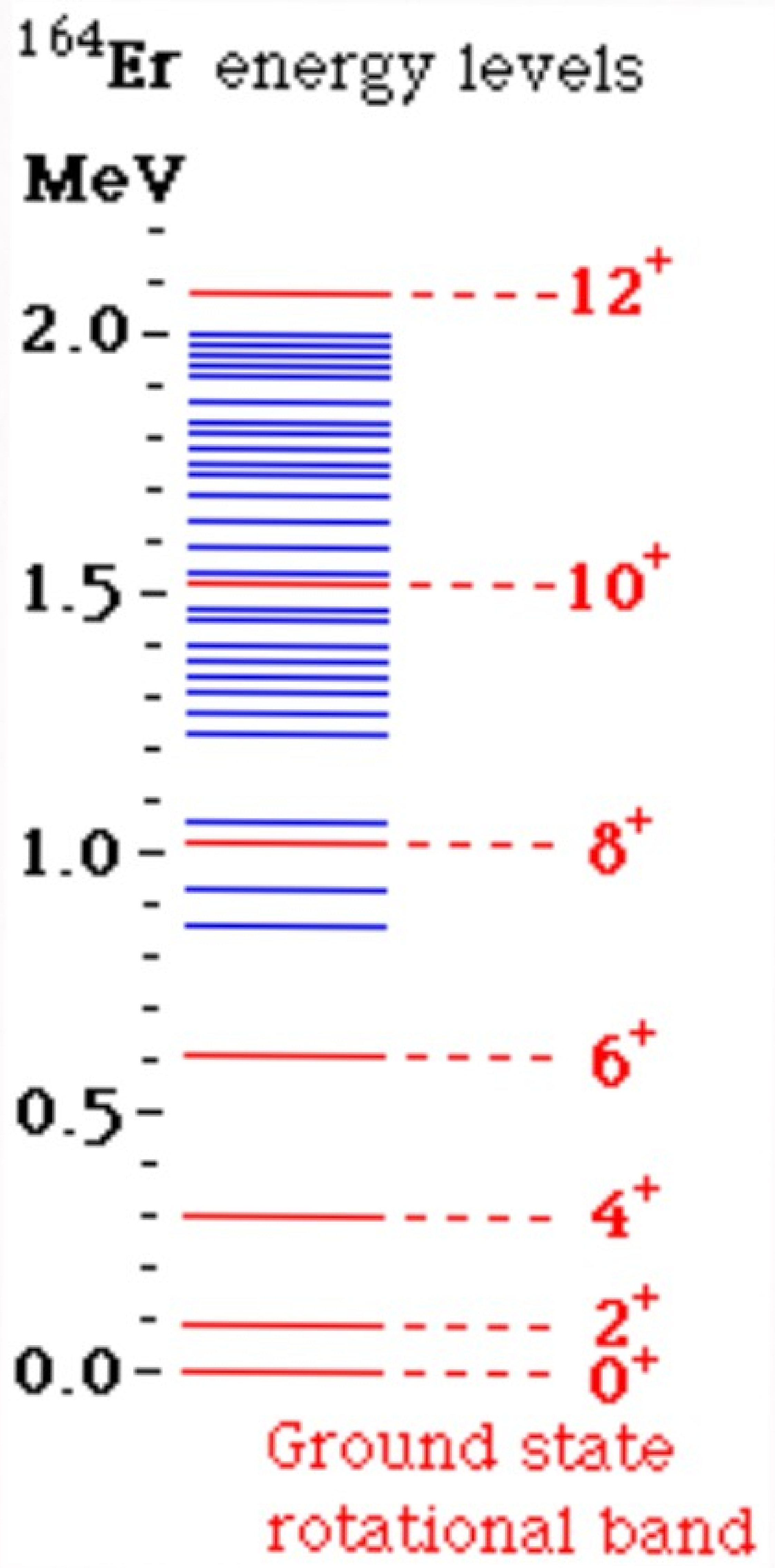


w/ Ratios:

$$\frac{E(6^+)}{E(2^+)} = 7$$

$$\frac{E(4^+)}{E(2^+)} = \frac{20}{6} \simeq 3.33$$

A possible example is ^{164}Er:

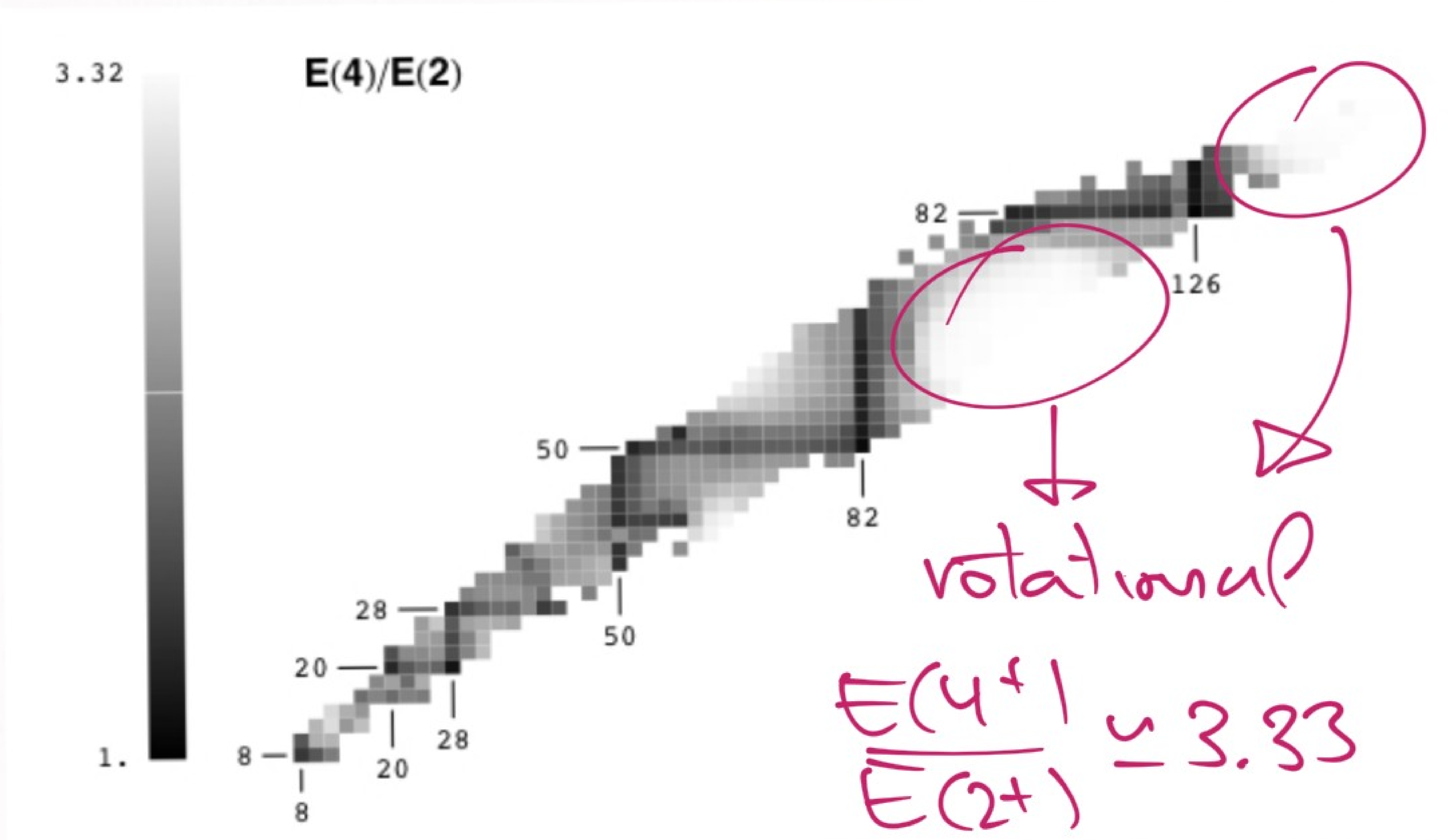


Energies proportional to $L(L+1)$

^{164}Er → large A ✓

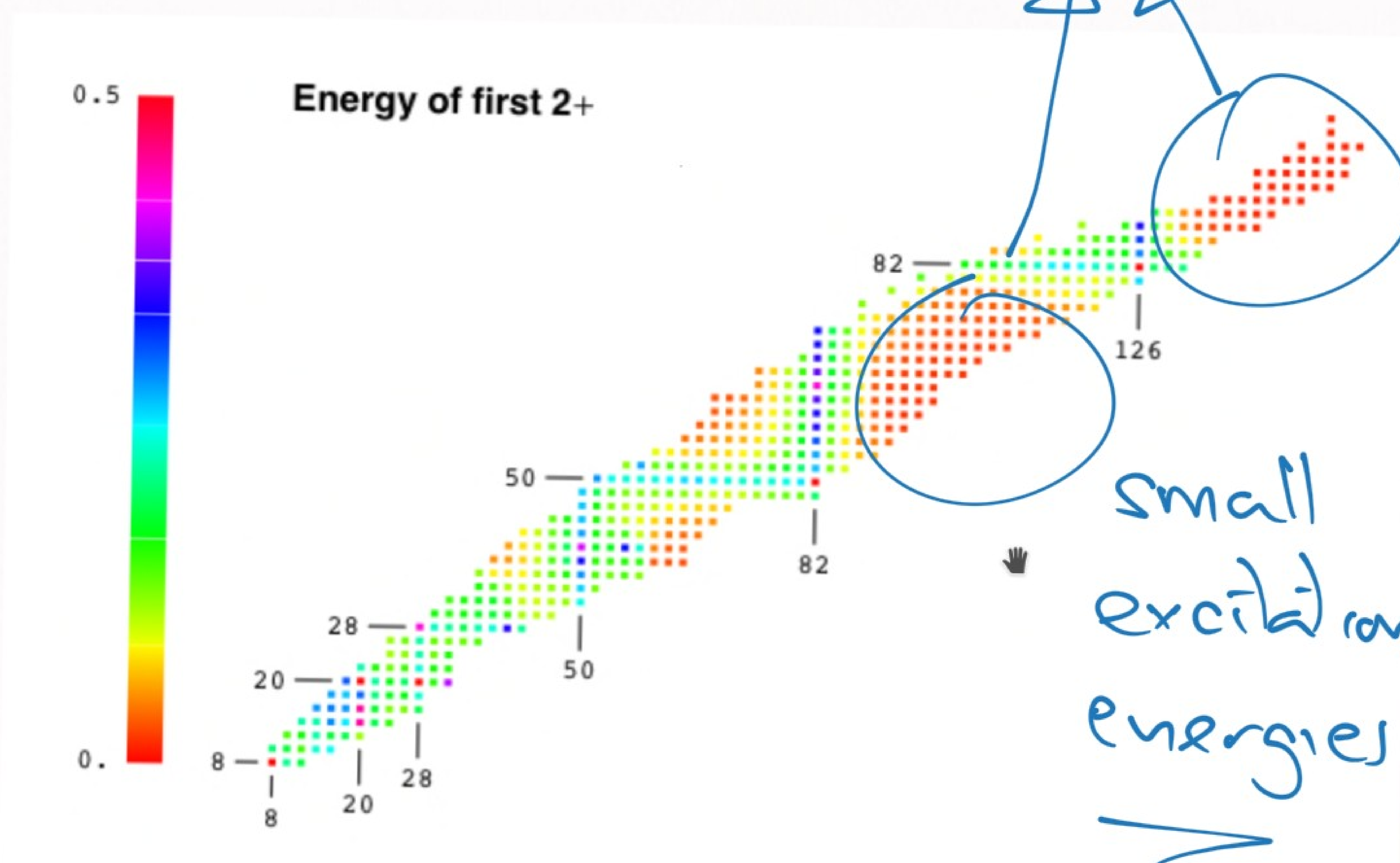
(as expected)

If we plot $\frac{E(4^+)}{E(2^+)}$, we can check which nuclei are rotational:



rotational

$\frac{E(4^+)}{E(2^+)} \simeq 3.33$



small excitation energies

Remember → [Physics is about scales]

1) Shell-model ~ (1-2) MeV

2) Rotational levels

~ 0.1 MeV (or less)

$E(4^+)/E(2^+) \sim 3.33$

Next lesson ↘

3) Vibrational levels

~ 0.5 MeV

$\frac{E(4^+)}{E(2^+)} \sim 2$

→ We will see why