Nuclear Physics 26

The Shell-Model &
The Spin-Orbit Force

RECAP

Separation energies, magical numbers

SHELL MODEL

1) mean field approximation

$$H = \sum \frac{p_i^2}{2m_i} + \sum V_{ij} + \sum V_{ijk} + \dots$$

$$\Rightarrow H = \sum \left(\frac{p_i^2}{2m_i} + V_i^{MF} \right) + \Delta V$$

2) nucleons are fermions

3) we fill shells (i.e. energy levels)

$\boxed{\text{SHELL MODEL}} \rightarrow \boxed{\text{Choice of an averaged potential}}$

$\boxed{\text{Fundamental piece of shell model}}$

Past lesson $\rightarrow$ $\boxed{\text{Really simple choice of avg potential}}$

$$V^{MF}(r) = \frac{1}{2} M_N \omega^2 r^2$$

"harmonic oscillator"

SHELL MODEL W/ HARMONIC OSCILLATOR

$\Downarrow$

LEVELS : (# = 2, 8, 20, 40, 70, ...)

$\frac{11}{2}\omega$ ——————— [30 p/n]

$n=2, \ell=0$
$n=1, \ell=2$
$n=0, \ell=4$

$\frac{9}{2}\omega$ ——————— [20 p/n]

$n=1, \ell=1$
$n=0, \ell=3$

$\frac{7}{2}\omega$ ——————— [12 p/n]

$n=1, \ell=0$
$n=0, \ell=2$

$\frac{5}{2}\omega$ ——————— [6 p/n]

$n=0, \ell=1$

$\frac{3}{2}\omega$ ——————— [2 p/n]

$n=0, \ell=0$

SHELL MODEL W/ HARMONIC OSCILLATOR

⇓

SHELL CLOSURES :

$N/Z = \boxed{2,\ 8,\ 20},\ \boxed{40,\ 70}$

MAGIC NUMBERS:

$N/Z = 2, 8, 20, 28, 50, 82$

$V^{MF}(r) = \frac{1}{2} M_N \omega^2 r^2$

NOT GOOD ENOUGH

something is missing

SHELL MODEL W/ SPIN-ORBIT FORCE

Maria Goeppert Mayer

$$V^{MF} = \frac{1}{2} M \omega^2 r^2 \boxed{- \xi \vec{\ell} \cdot \vec{s}}$$

$(\xi > 0)$

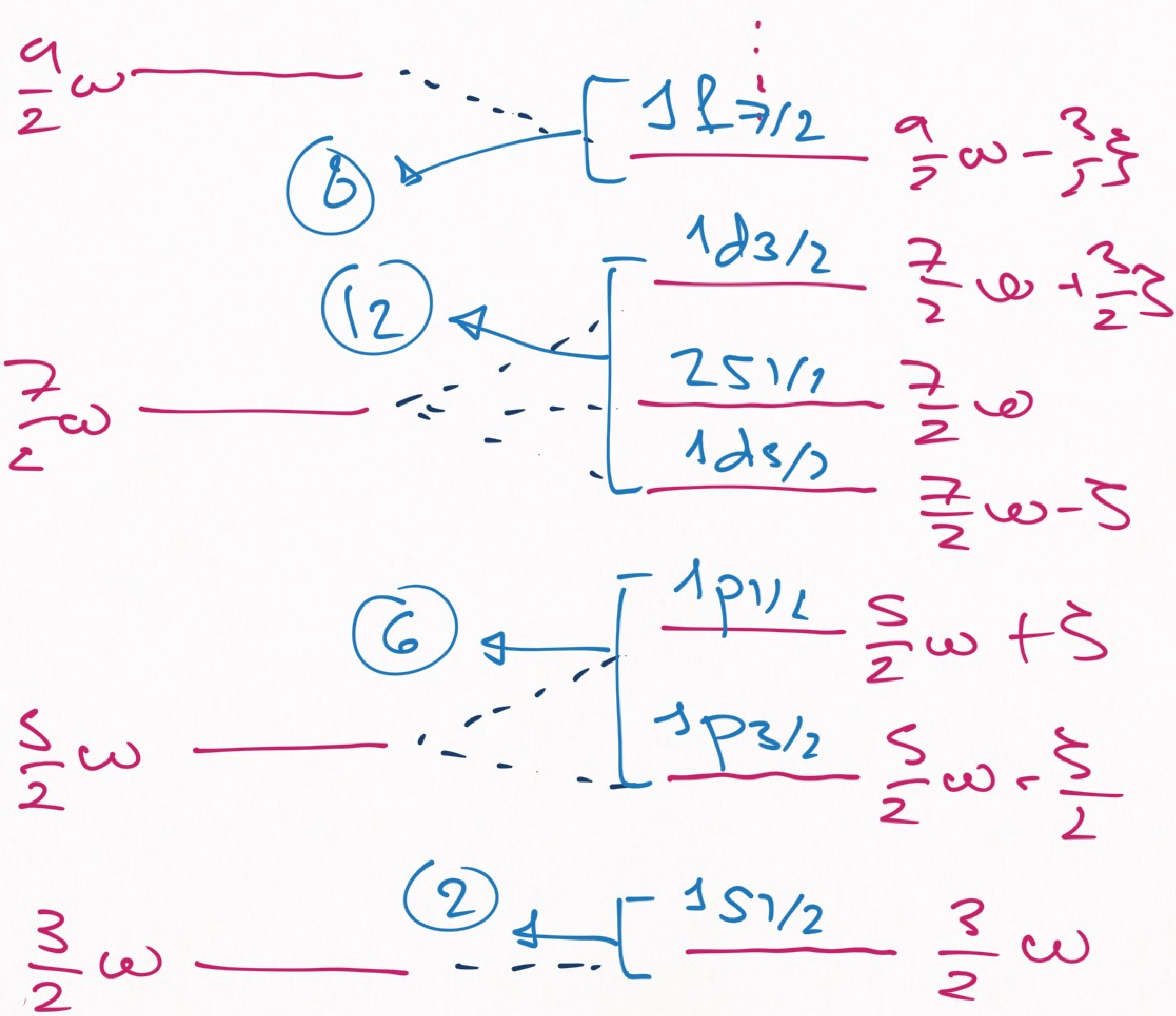
This detail fixes the ordering of the shells

SHELL MODEL w/ SPIN-ORBIT

LEVELS:

$$\left(\vec{\ell}\cdot\vec{s} = \frac{1}{2}\{j(j+1) - \ell(\ell+1) - s(s+1)\}\right)$$

without $\vec{\ell}\cdot\vec{s}$ with $\vec{\ell}\cdot\vec{s}$



SHELL MODEL w/o SPIN-ORBIT

$N = 2, 8, 20, 40, 70, \ldots$

SHELL MODEL w/ SPIN-ORBIT

$N = 2, 8, 20, 28, 50, 82, \ldots$

$\Rightarrow$ We get the correct order

Also notice the notation:

$1s_{1/2}, \; 1p_{3/2}, \; 1p_{1/2}, \; \ldots$

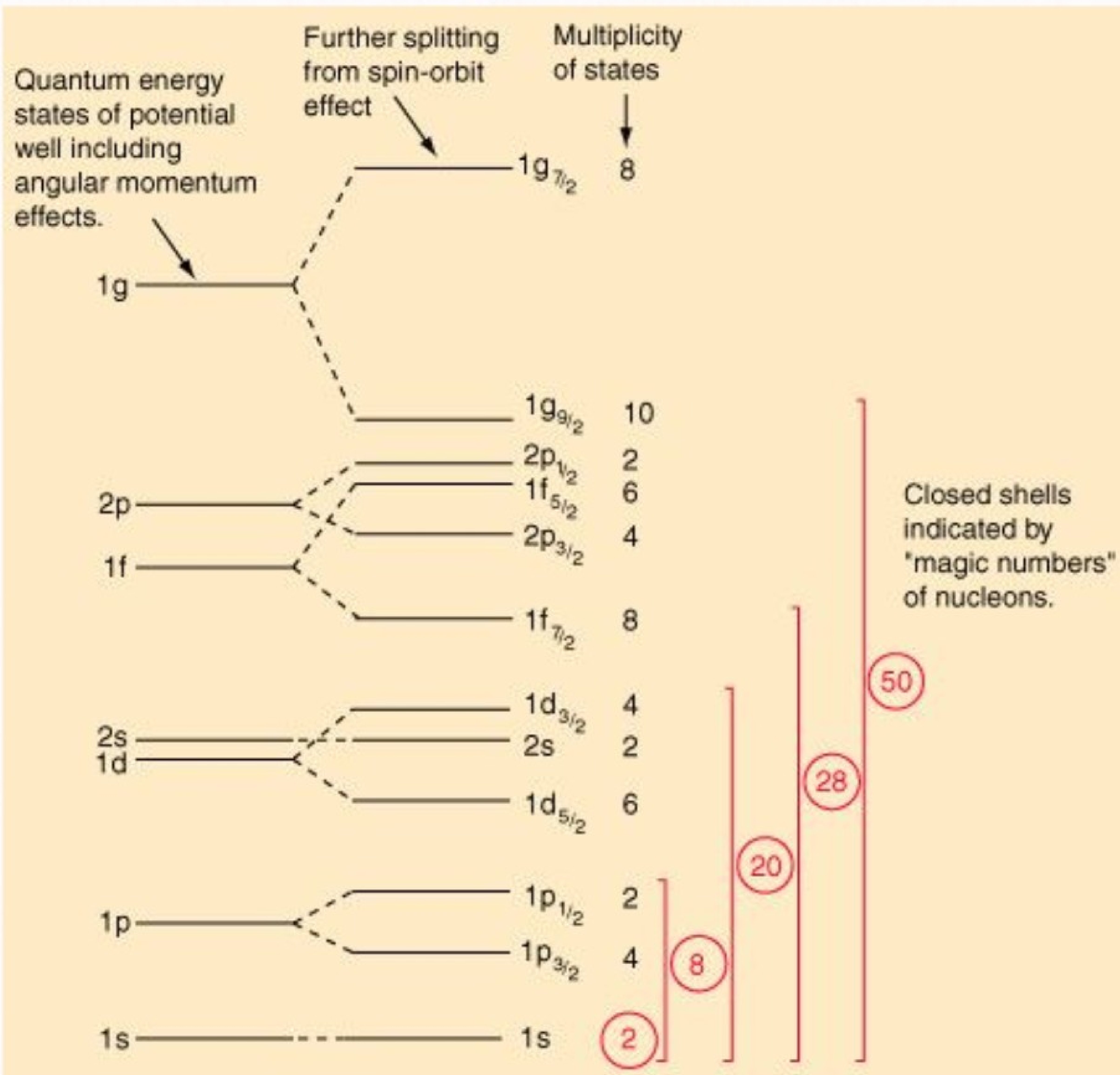
$[n \ell_j]$

energy level

orbital angular momentum

total angular momentum

SUMMARY OF THE ORDERING OF ORBITALS:



A good & simple model is:

$$V^{MF}(r) = \frac{1}{2} M_N \omega^2 r^2 - \kappa \vec{\ell}^2 - \beta \vec{\ell}\cdot\vec{s}$$

[SHELL MODEL APPLICATIONS:]

1) FILLING THE SHELLS:

^4He → Pretty simple

$1s_{1/2}$ ↑↓   ↑↓ $1s_{1/2}$

protons   neutrons

Pairing → ↑↓ → $\boxed{S=0}$

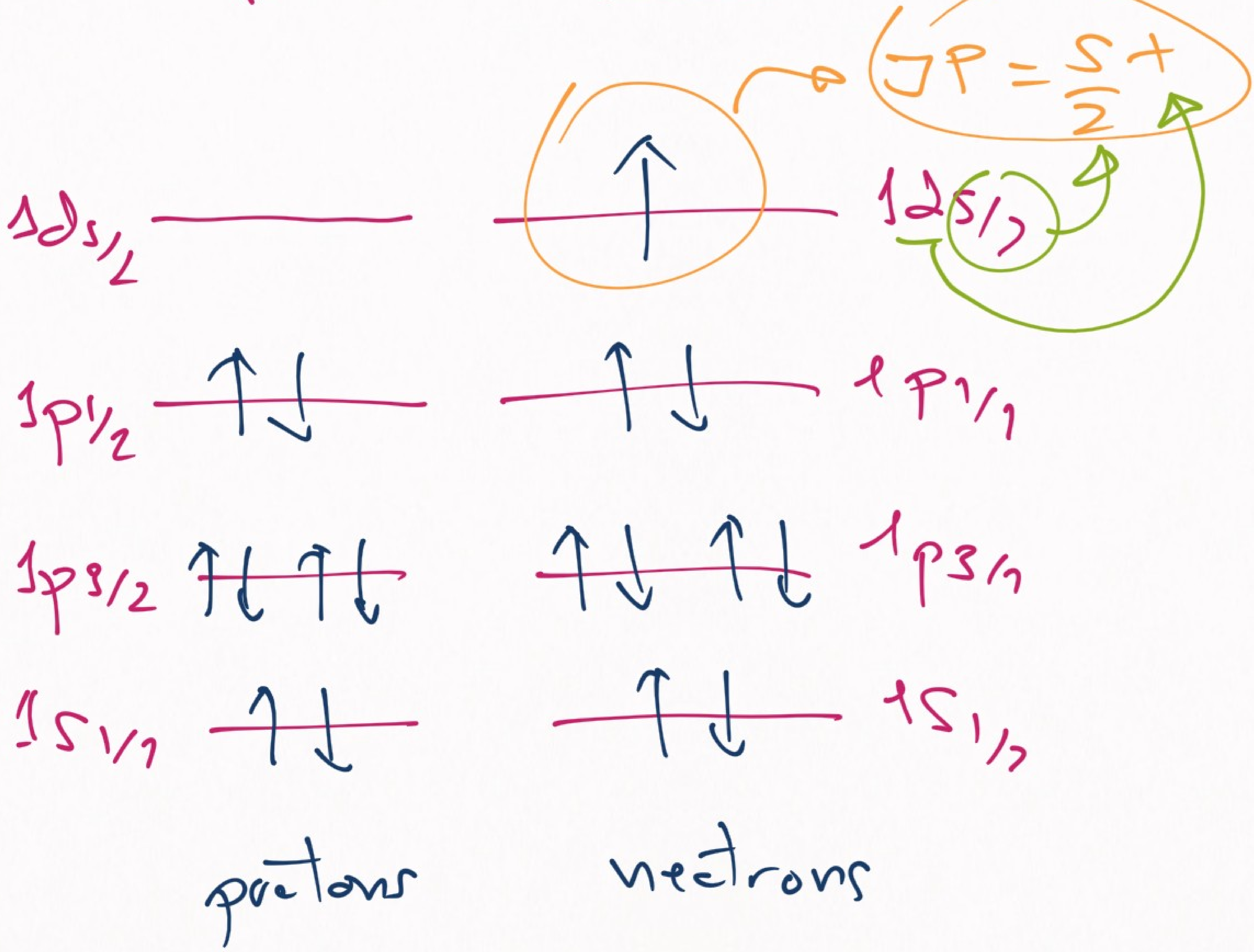
→ $\boxed{J^P(^4He) = 0^+}$

Prediction from the shell-model

2) PREDICTING J^P OF NUCLEI

→ works best w/ one proton/neutron away from a full shell

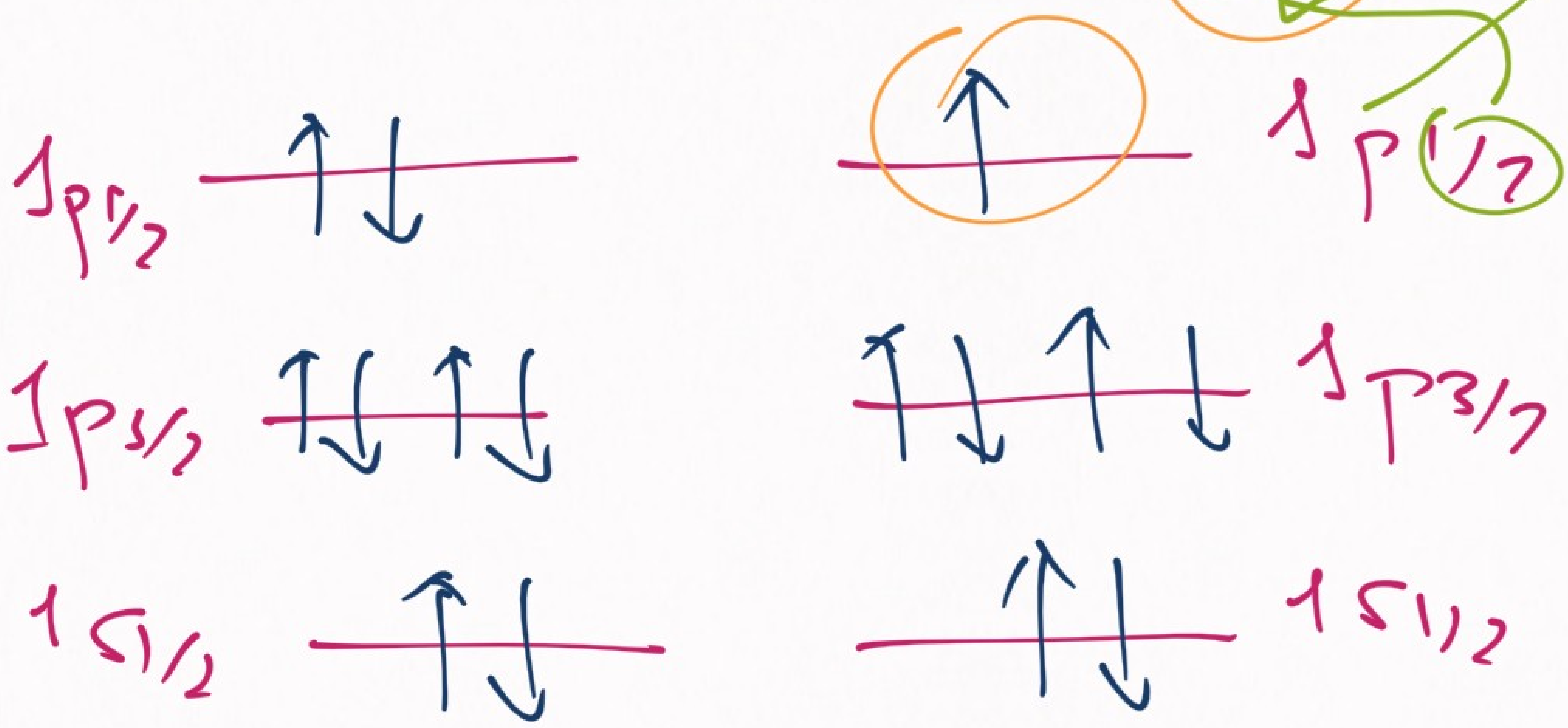
$\boxed{{}^{17}O \,/\, {}^{17}F}$ → $\boxed{\begin{array}{l} {}^{17}O \to {}^{16}O + n \\ {}^{17}F \to {}^{16}O + p \end{array}}$



→ If we lack a proton/neutron this is also possible:

$^{15}O / ^{15}N \longrightarrow (^{16}O - n/p)$

$J^P = \frac{1}{2}^-$

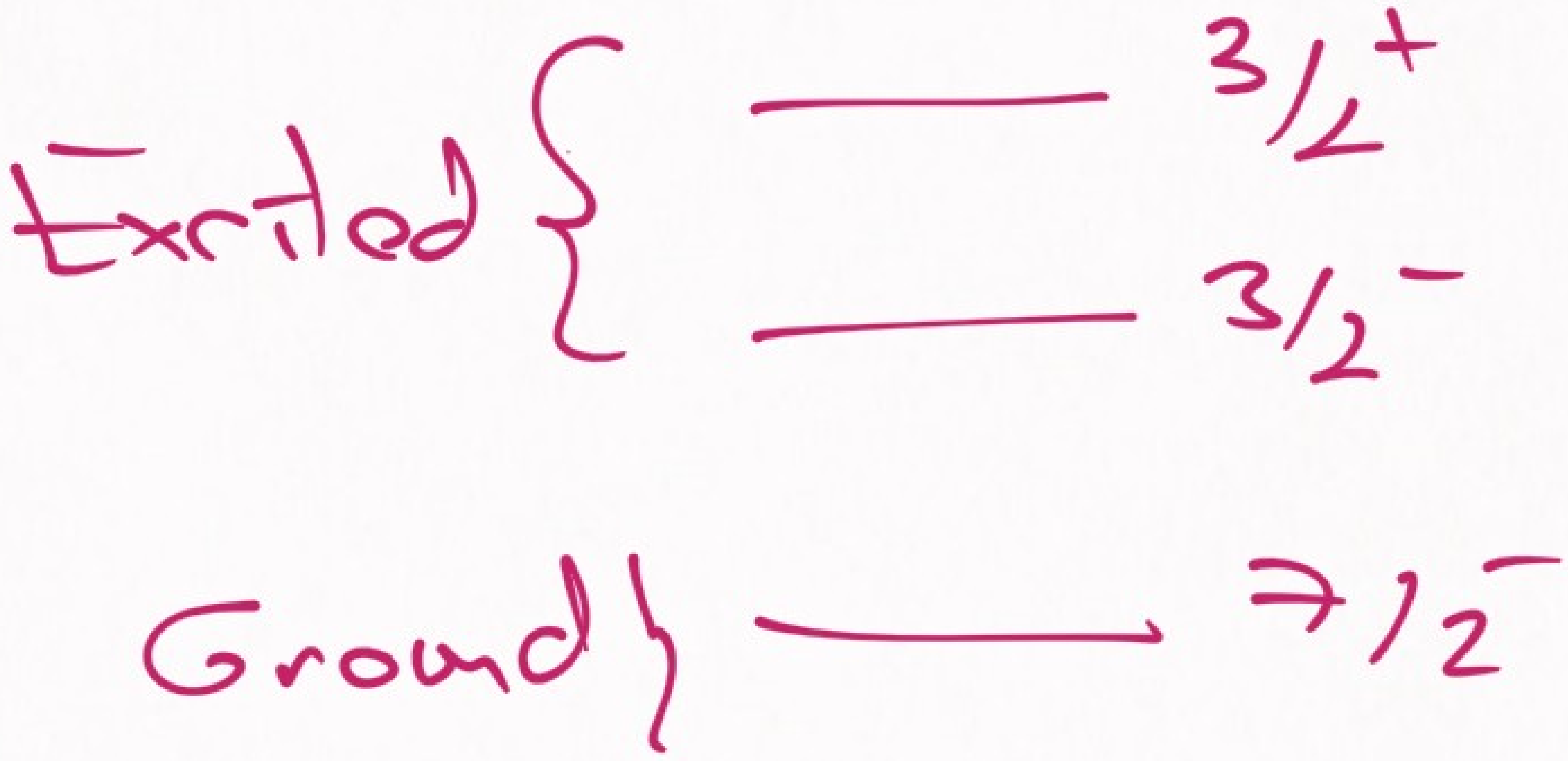


J^P given by unpaired nucleon

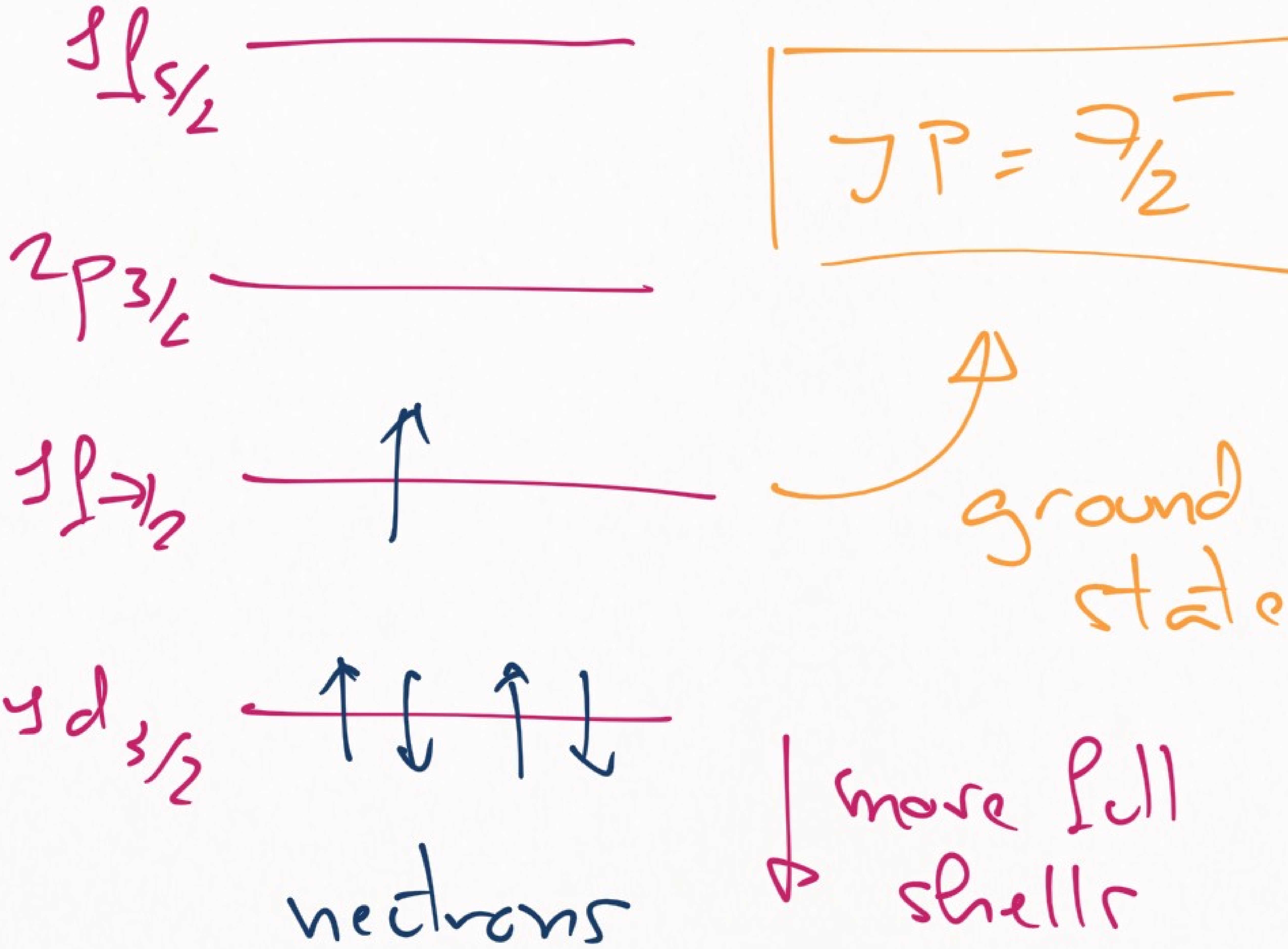
$$\left[\begin{array}{l} ^{16}O \longrightarrow J^P = 0^+ \\ ^{17}O / ^{17}F \longrightarrow J^P = \frac{5}{2}^+ \\ ^{15}O / ^{15}N \longrightarrow J^P = \frac{1}{2}^- \end{array}\right]$$

3) EXPLAINING EXCITED STATES

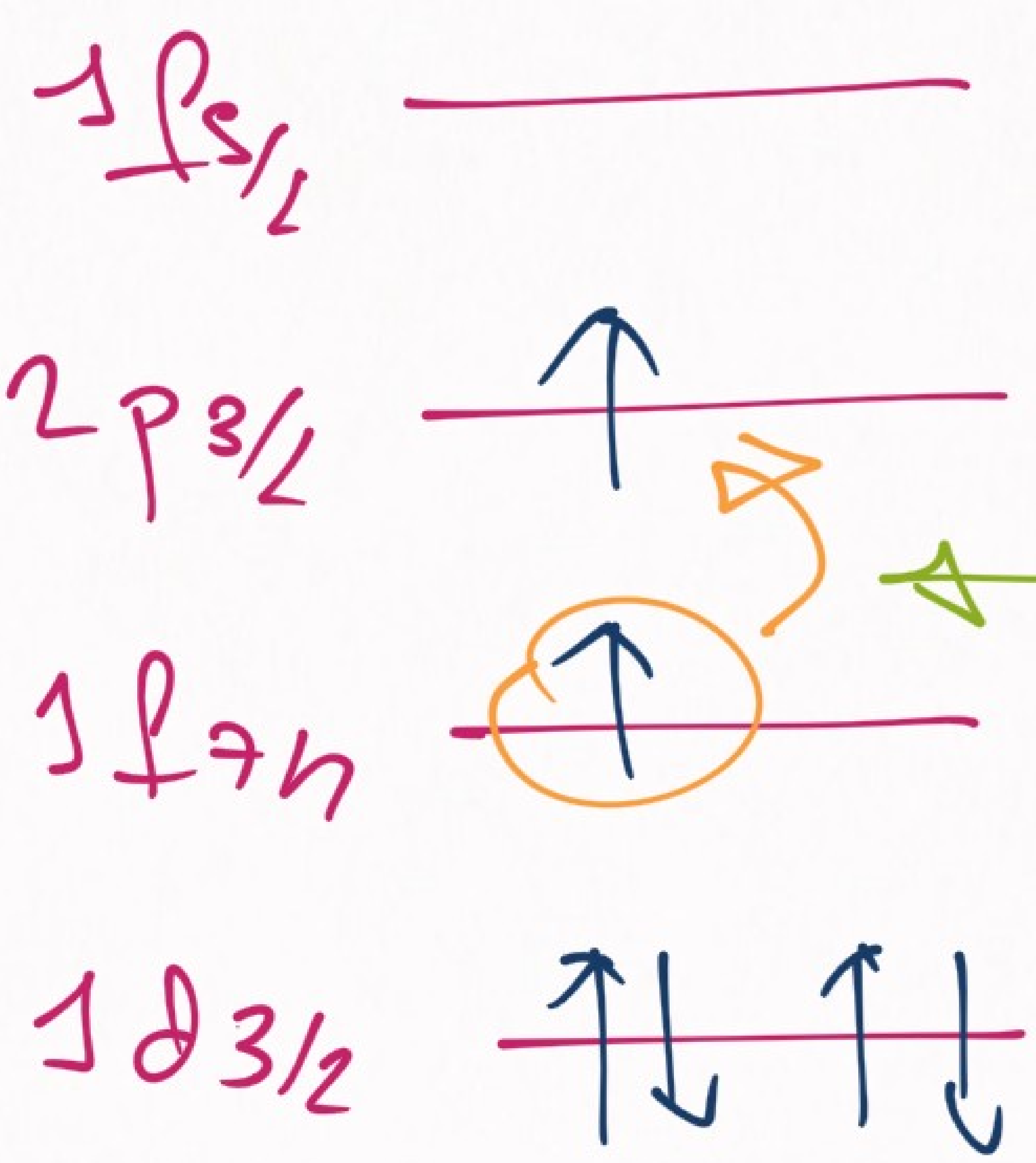
$^{41}Ca \rightarrow$ 20 protons, 21 neutrons



$\rightarrow$ Let's see how to explain them:



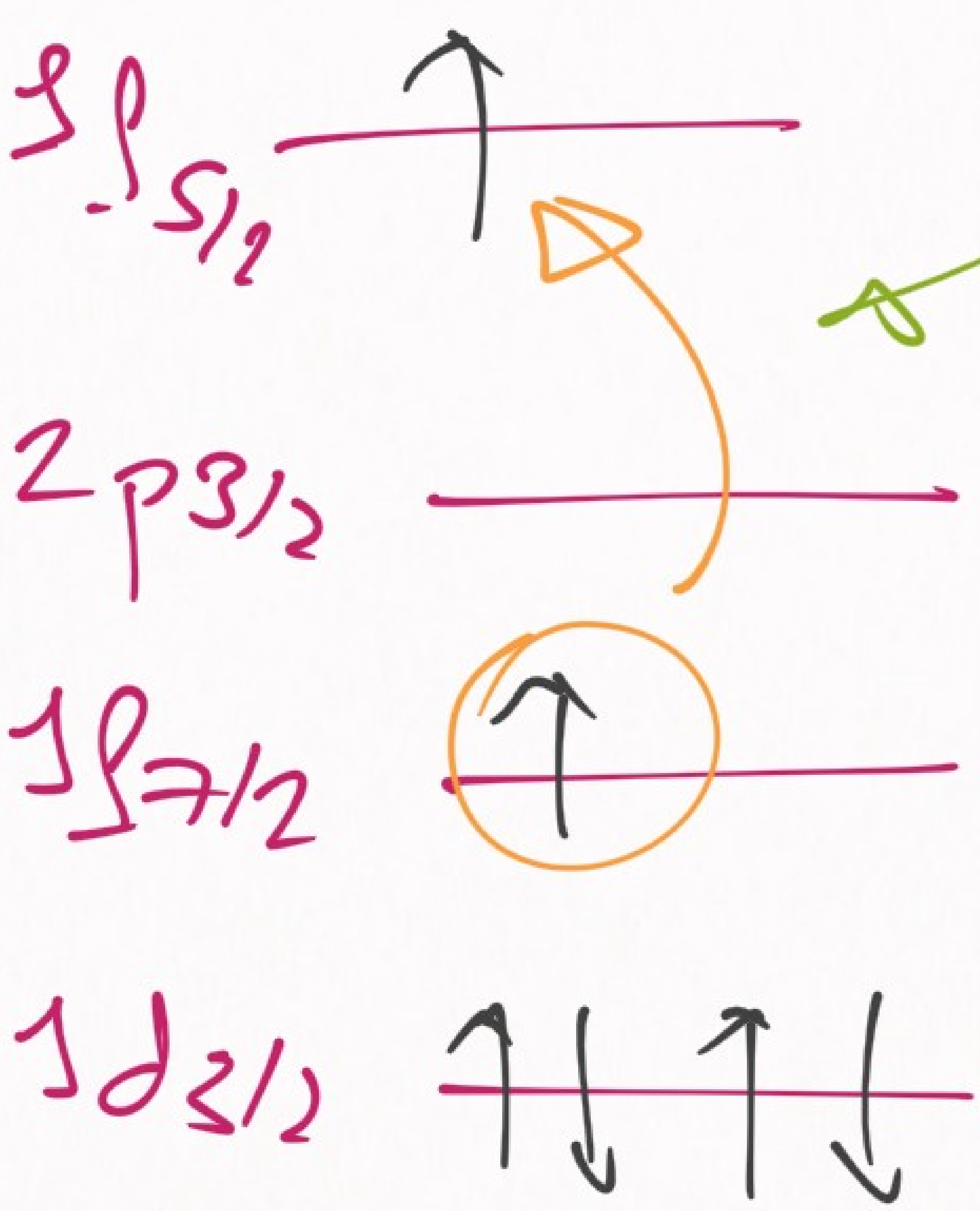
How do we get the excited states?



FIRST EXCITED STATE

$\boxed{J^P = \frac{3}{2}^-}$

RIGHT!



SECOND EXCITED STATE?

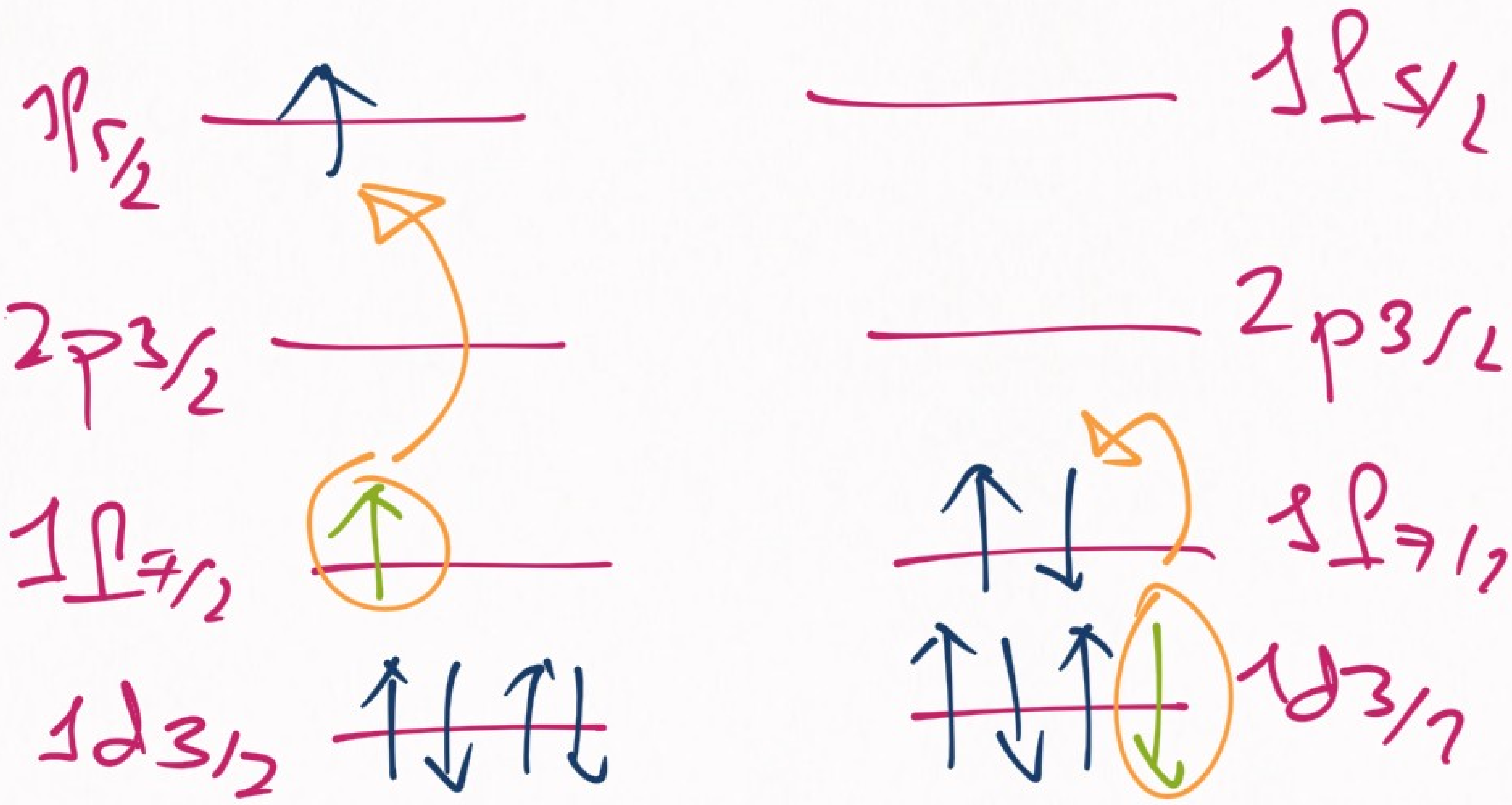
$\cancel{J^P = \frac{5}{2}^-}$

NOPE!

→ need a different explanation

Second excited state

→ more than one explanation is possible



$\boxed{J^P = 5/2^-}$

$\boxed{J^P = 3/2^+}$

↓

[The one that actually happens]

$\boxed{^{41}Ca}$

$3/2^+$

$3/2^-$

$7/2^-$

$\boxed{^{38}Ar}$ → a more complicated case

0^+ ground, 2^+ excited: Almost universal for even-even nuclei (0^+, 2^+)

$$\begin{array}{ll} \text{———} \; 2^+ & \text{excited} \\ \text{———} \; 0^+ & \text{ground} \end{array}$$

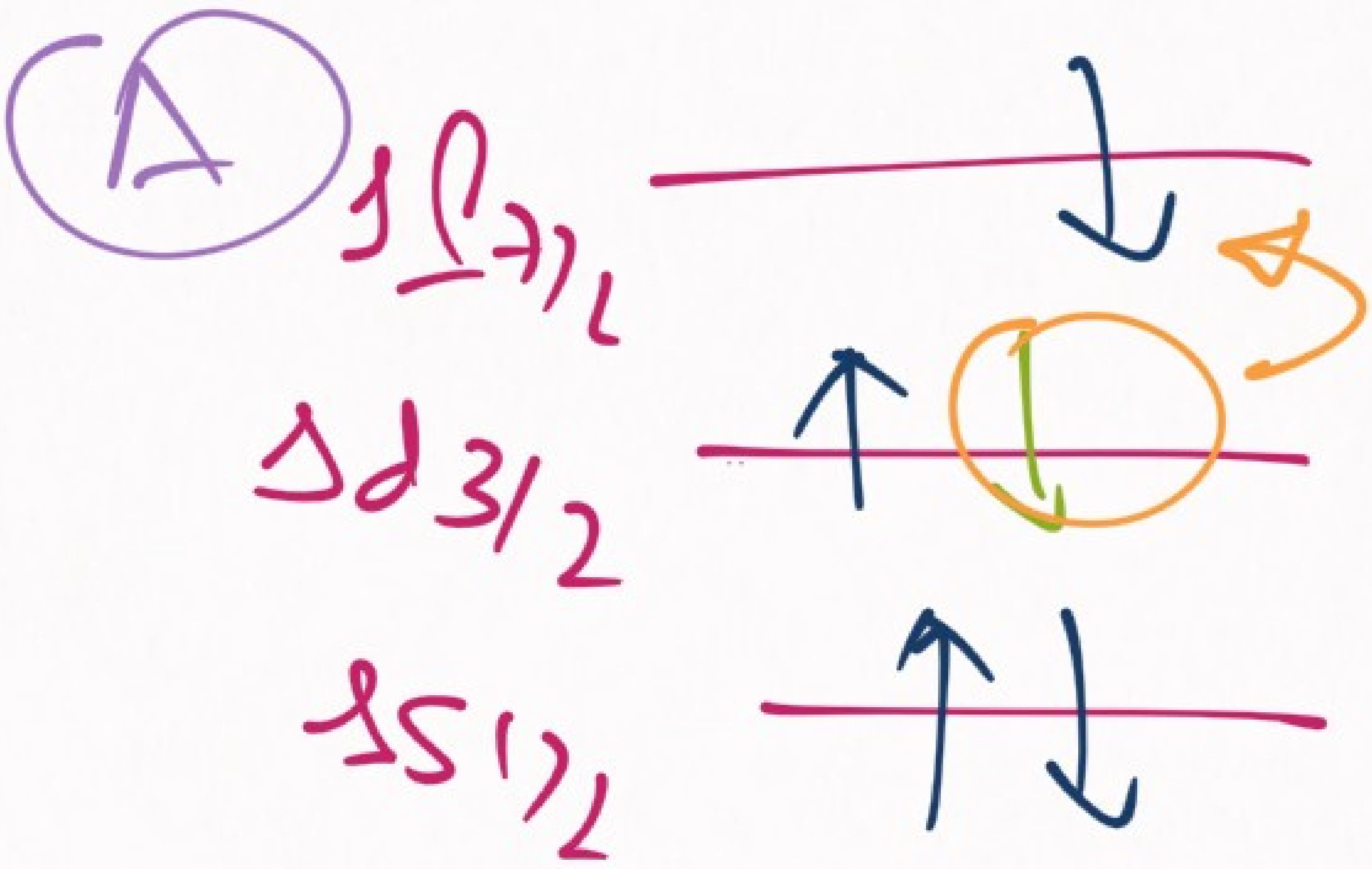
GROUND STATE:

$$\left.\begin{array}{ll} 1f_{7/2} & \text{———} \\ 1d_{3/2} & \uparrow\downarrow \\ 1s_{1/2} & \uparrow\downarrow \end{array}\right\} \rightarrow \boxed{J^P = 0^+}$$

protons

EXCITED STATE :

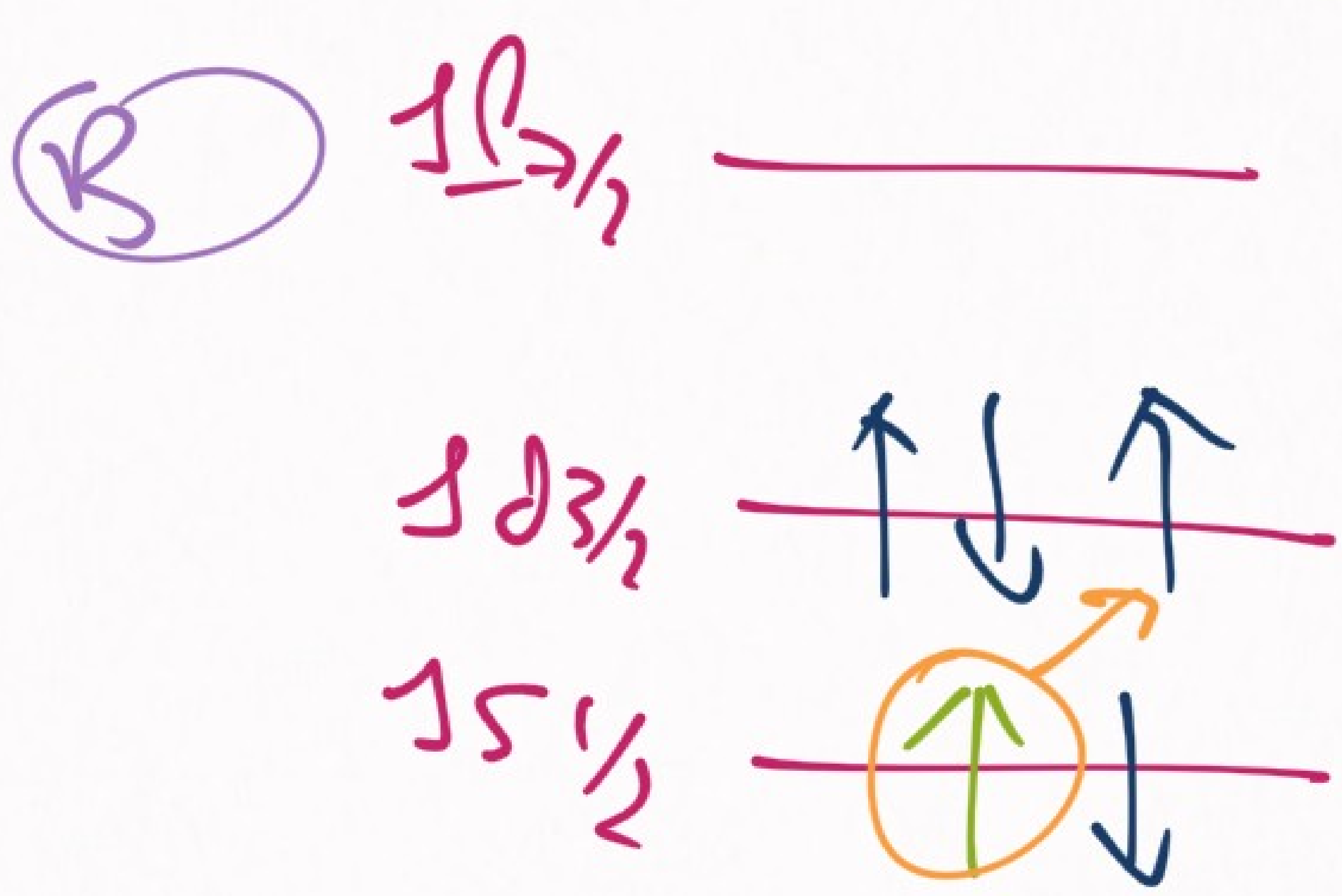
(A)



WRONG

$$J^P = \left(\tfrac{3}{2} \otimes \tfrac{7}{2}\right)^- = 2^-, 3^-, 4^-, 5^-$$

(B)



$$J^P = \left(\tfrac{1}{2} \otimes \tfrac{3}{2}\right)^+ = 1^+ \text{ or } 2^+$$

$\boxed{\text{RIGHT}}$

This is the choice nature makes

[SHELL MODEL w/ RESIDUAL INTERACTIONS]

$$H = \sum_i \frac{p_i^2}{2m_i} + \sum_{ij} V_{ij}^{2B} + \sum_{ijk} V_{ijk}^{3B} + \dots$$

$$\Rightarrow H = \sum_i \left(\frac{p_i^2}{2m_i} + V_i^{MF} \right) + \Delta V$$

Residual Interaction

["Pairing interaction" is the easiest example]

$$\langle jj(JM) | V_{pairing} | jj(JM) \rangle = -\frac{1}{2} g (J+1) \delta_{J0} \delta_{M0}$$

[SIMPLE & EASY APPLICATIONS]

Pairing interaction $\rightarrow$ $\boxed{\text{shells w/ high "j"}}$

(most often heavy nuclei)

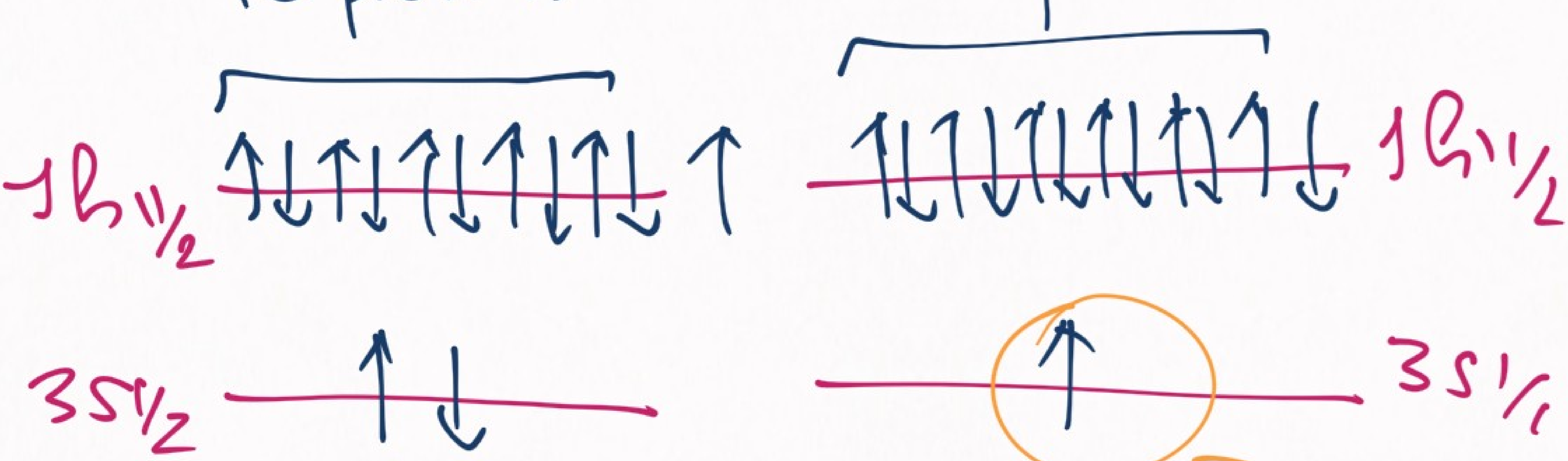
EXAMPLE 1:

$$\underbrace{^{203}Tl \ \& \ ^{205}Tl}$$

$$\underbrace{81 \text{ protons}}_{J^P \neq 0^+}, \quad \underbrace{122 \ \& \ 124 \text{ neutrons}}_{J^P = 0^+}$$

A $\rightarrow$ $\frac{11}{2}^-$ 　　　　 $\frac{1}{2}^+$ $\leftarrow$ B

10 protons 　　　　 12 protons



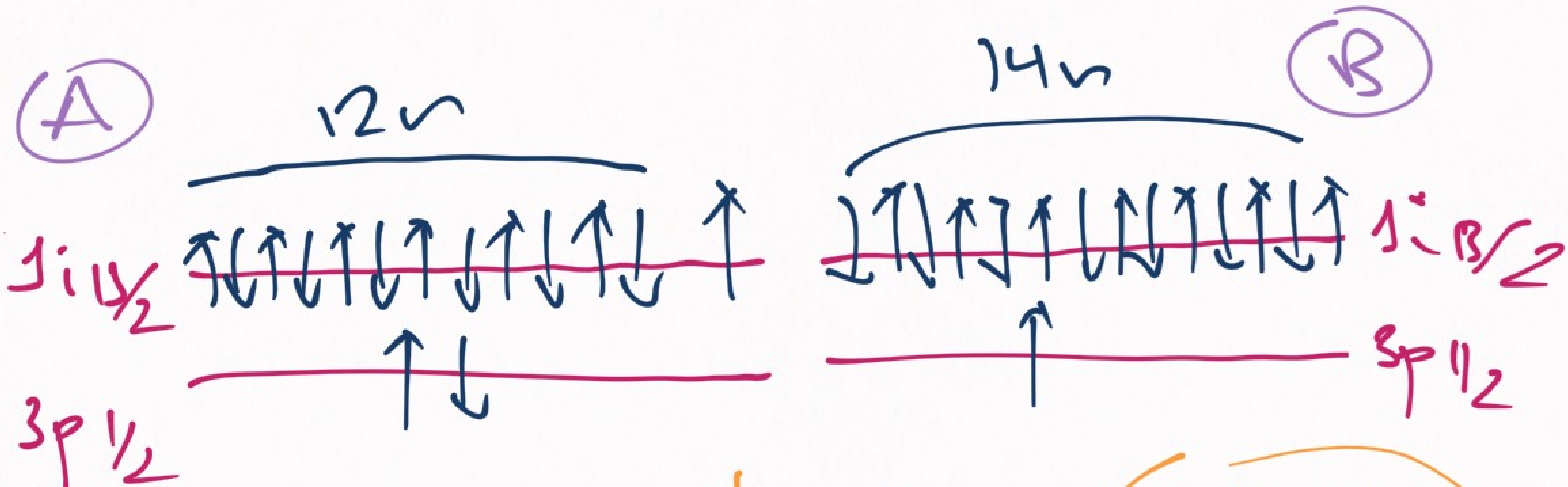
$E_B < E_A$ because pairing

$\Rightarrow$ B wins $\Rightarrow$ $J^P = \frac{1}{2}^+$

EXAMPLE 2 : ^{207}Pb

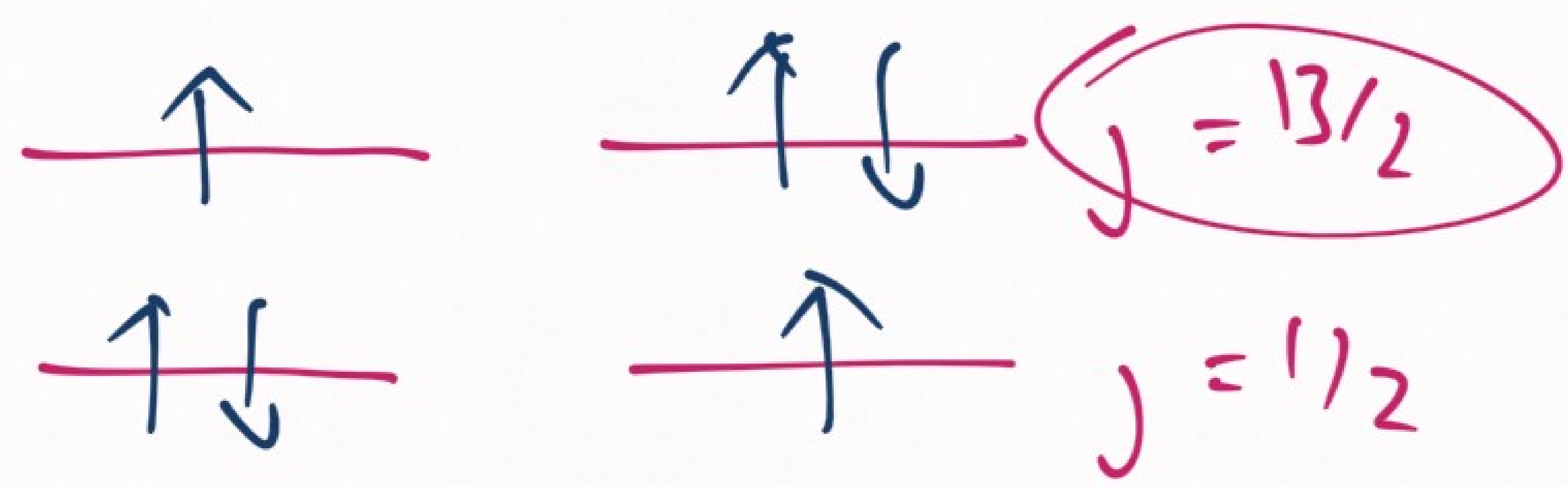
82 p, 125 n

$\boxed{J^P = \frac{1}{2}}$



(A) → $J^P = \frac{13}{2}^+$ (B) → $J^P = \frac{1}{2}^-$

Again:



$E_A > E_B$

Why? Because $j = \frac{13}{2}$ high enough as to change ordering

$\Rightarrow J^P = \frac{1}{2}^-$ for ^{207}Pb

That's all about the shell model for today

$\boxed{\text{NEXT LESSON}}$

The collective model