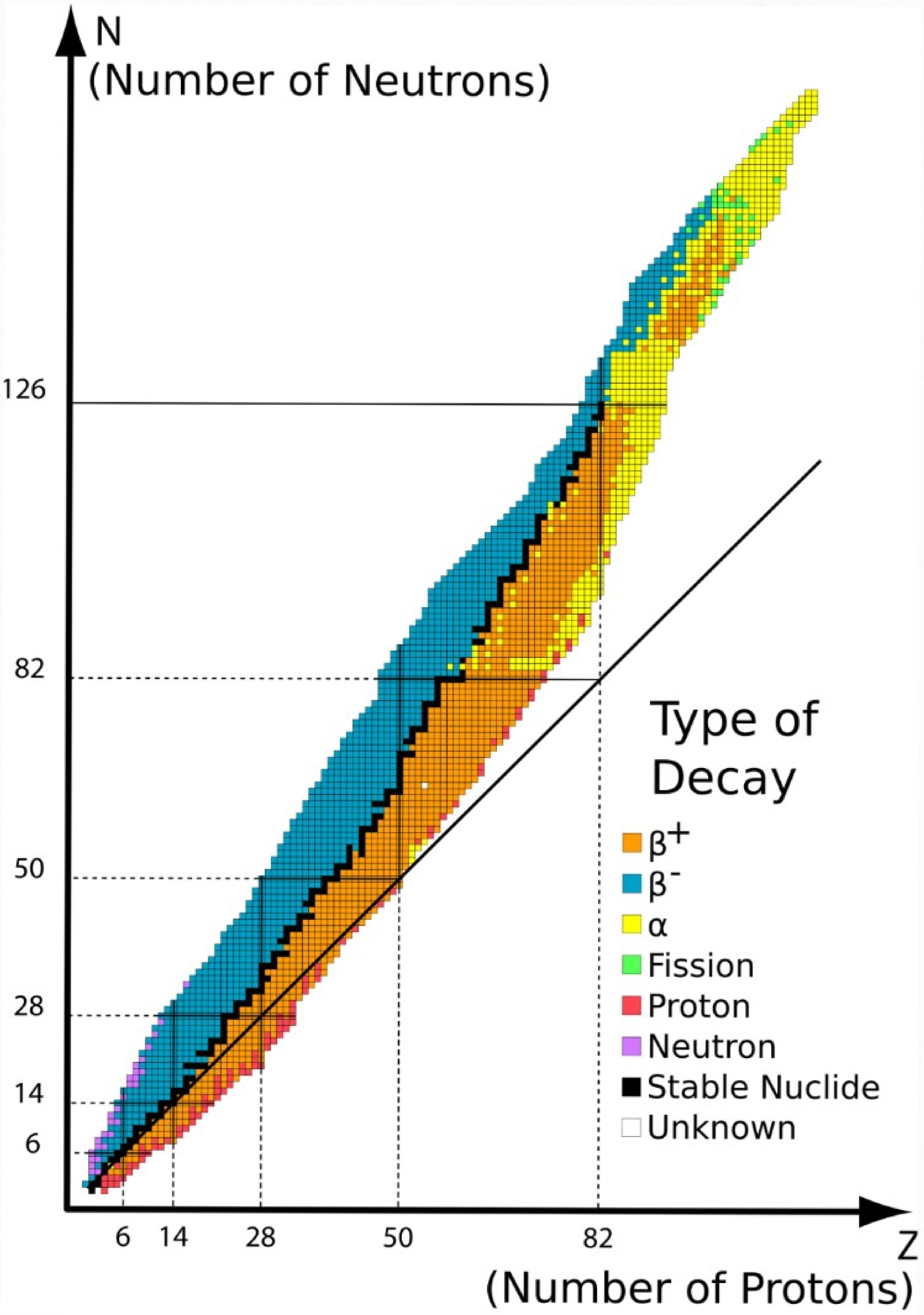
Nuclear Physics ㉕

Nuclear Stability

& The Liquid drop model

Nuclear Stability:



Very few nuclei are stable

↓

most will decay

Which nuclei will decay?

→ every nuclei for which decaying is allowed

Nuclear stability:

$$\boxed{E(\text{initial nuclei}) > E(\text{final nuclei} + \text{decay products})}$$

$$\Downarrow$$

$$\boxed{\text{Decay}}$$

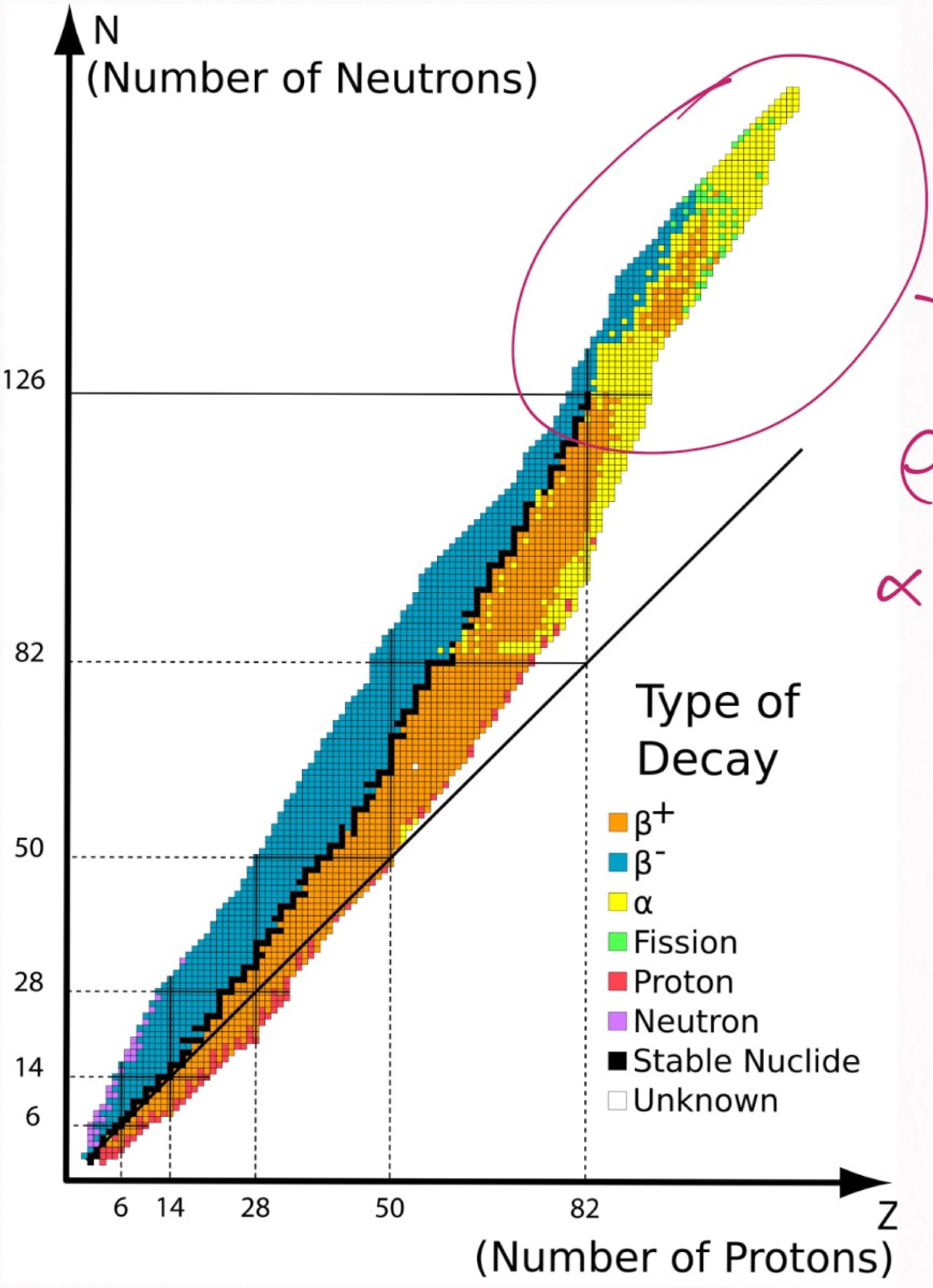
1) α-decay

$$^{A}_{Z}X_{N} \longrightarrow {}^{A-4}_{Z-2}X_{N-2} + \alpha$$

Energy:

$$S_\alpha(Z,N) = B(Z,N) - B(Z-2,N-2) - B(2,2)$$

Most heavy nuclei $\Rightarrow \boxed{S_\alpha > 0}$



Lots of α-decay

2) β^- and β^+ decay

(i.e. electron/positron emission)

$$\beta^-: {}^{A}_{Z}X_N \to {}^{A}_{Z+1}X_{N-1} + e^- + \bar{\nu}_e$$

$$\beta^+: {}^{A}_{Z}X_N \to {}^{A}_{Z-1}X_{N+1} + e^+ + \nu_e$$

Simplest example : the neutron

$$n \to p + e^- + \bar{\nu}_e$$

$$Q = m_p + m_e + m_{\bar{\nu}_e} - m_p$$

$$(\simeq 0)$$

$$\simeq 1.29 \text{ MeV}$$

$$\Gamma(n \to p e^- \bar{\nu}_e) = 2\pi G_v^2 (1 + 3 g_a^2) \Pi(Q)$$

$$\Pi(Q) = \frac{m_e^5}{4\pi^4} f(Q/m_e), \quad f(x) = \int_0^x (\ldots) dy$$

(not to be explained in detail here)

$$\tau = \frac{\hbar c}{\Gamma} \frac{1}{c} \simeq 930 \text{ seconds}$$

Next typical example: the Triton

$$^3H \to {}^3He + e^- + \bar{\nu}_e$$

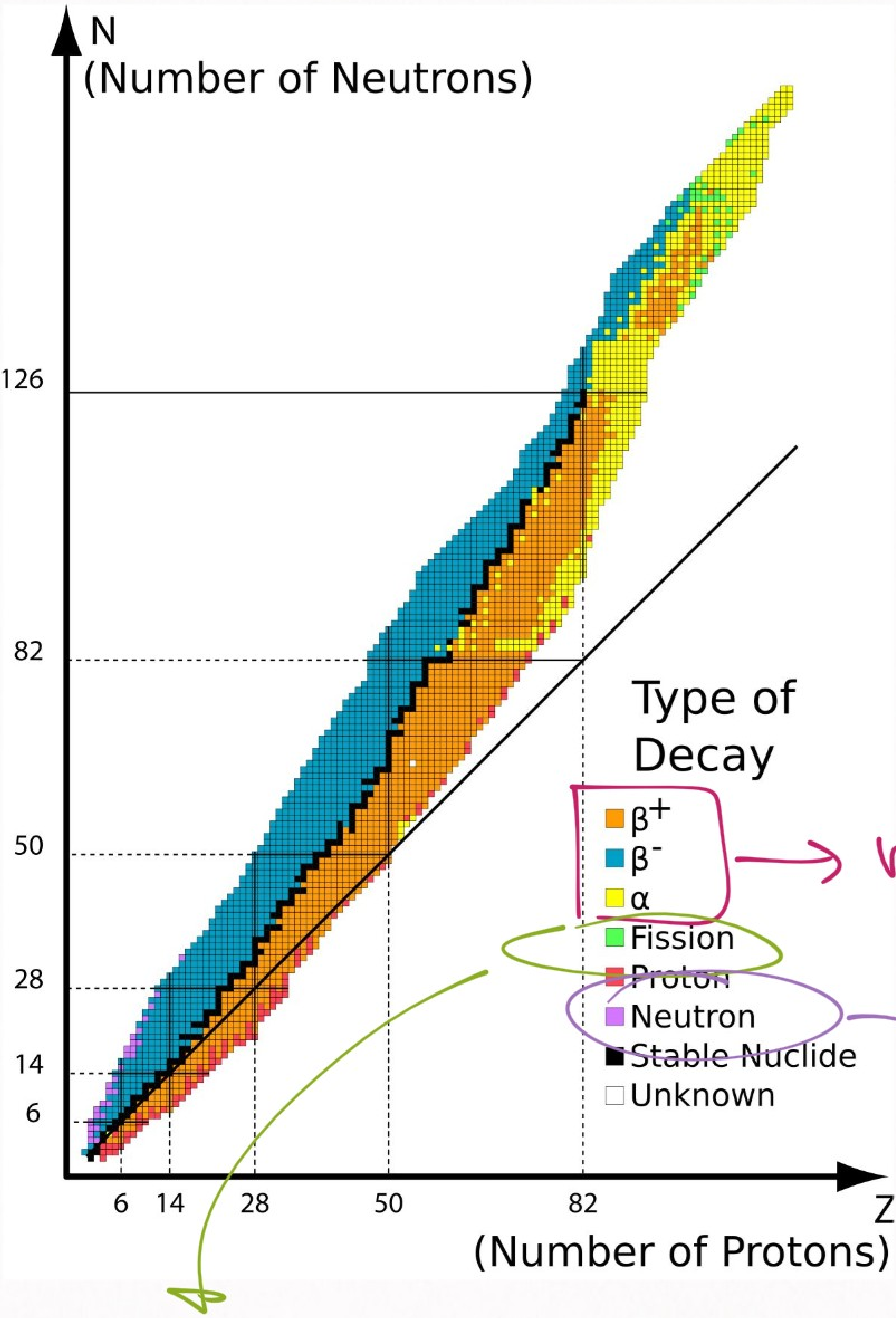
$$Q = -B(^3H) + (m_n - m_p) - B(^3He) - m_e \geq 0$$

($Q \simeq 0.0186\,\text{MeV}$, $\tau \sim 10$ years)

3) γ decay: $^A_Z X^*_N \to {}^A_Z X_N$

→ happen fast

→ from excited to ground state of **same** nucleus



most decays

Liquid drop model

Other modes of decay (less common)

NUCLEAR MODELS

1) $A \leq 10-12$ → ab-initio

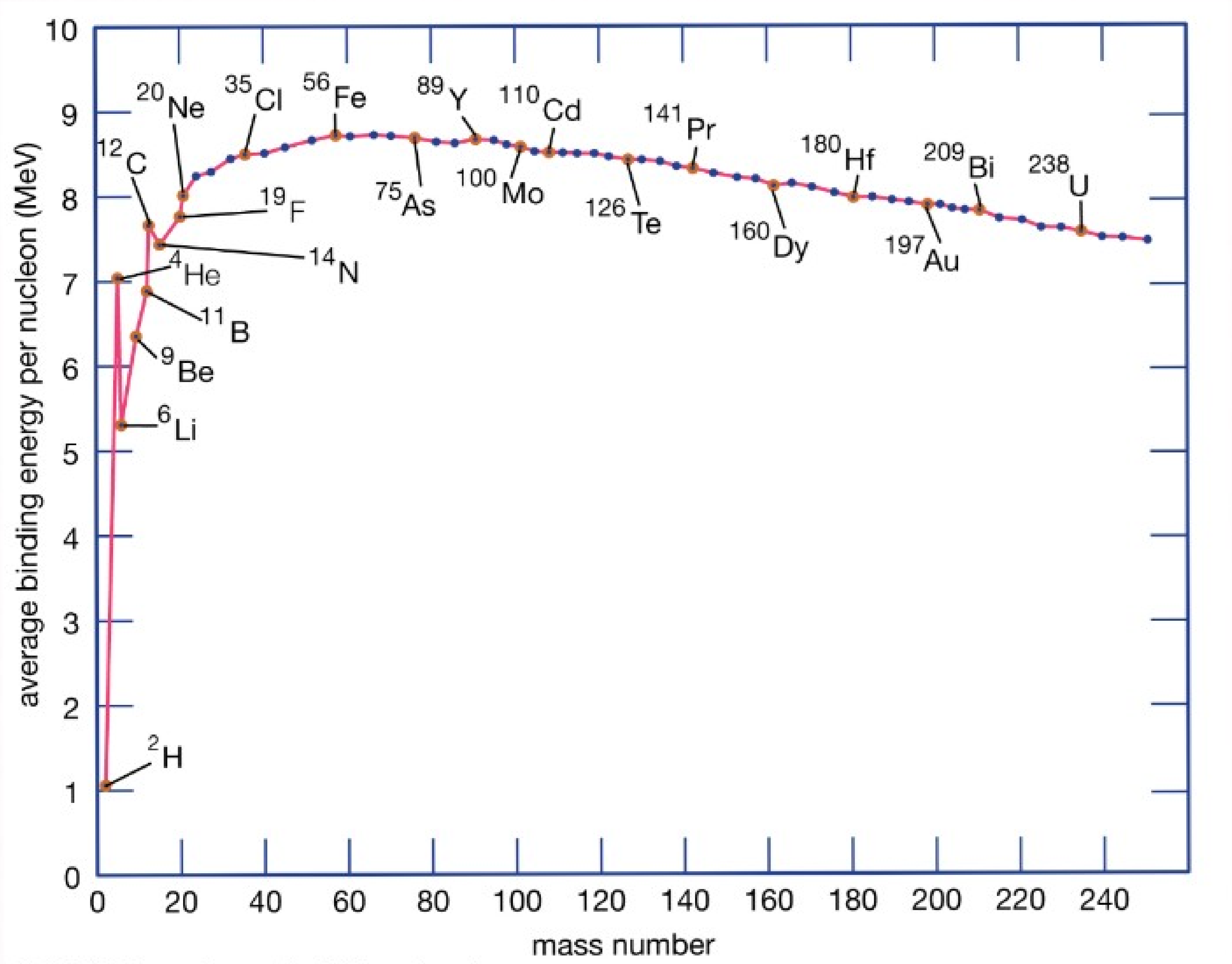
2) A big → we use a model

Most important models:

1) Liquid drop model
2) Shell-model
3) Collectivo model
    3.a) Vibrational
    3.b) Rotational

LIQUID DROP MODEL

$\frac{B}{A} \simeq 8$ MeV/nucleon (saturation)



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Why saturation?

1) Finite-range of nuclear force

2) Strong medium-range attraction

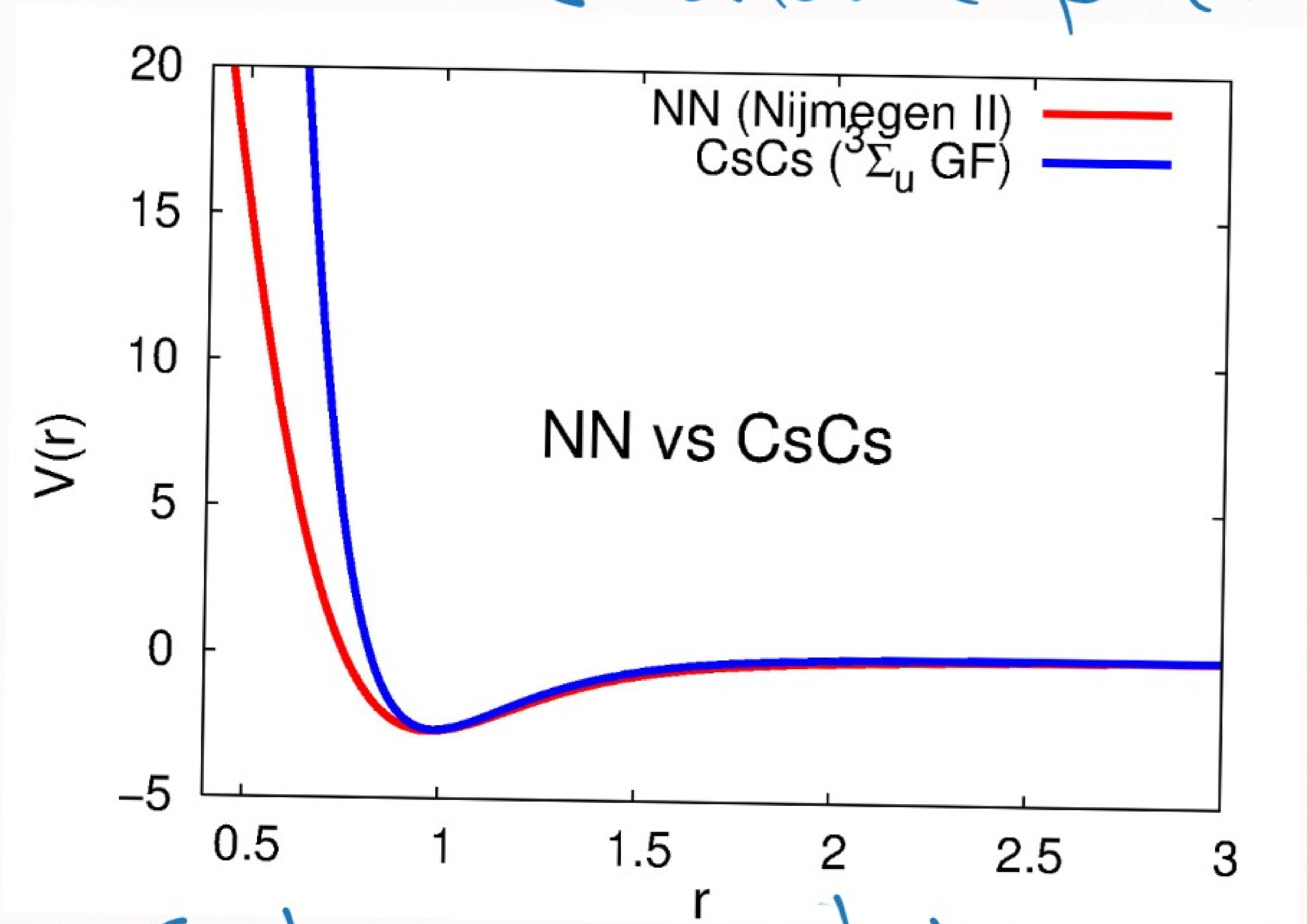
3) Stronger short-range repulsion

1) finite-range
2) medium attraction
3) short repulsion

} properties of
a) nuclear forces
b) atomic forces

[Analogy between atomic & nuclear physics]

Example: comparison of nuclear & atomic potentials



(not in same units)

$$\begin{tikzcd}
\text{Group of atoms} \arrow[r] \arrow[d] & \text{liquid drop} \\
\text{Group of nucleons} \arrow[ur, bend right] &
\end{tikzcd}$$

Group of nucleons will behave just like a droplet of a

nuclear liquid

⇓

The idea is incredibly simple

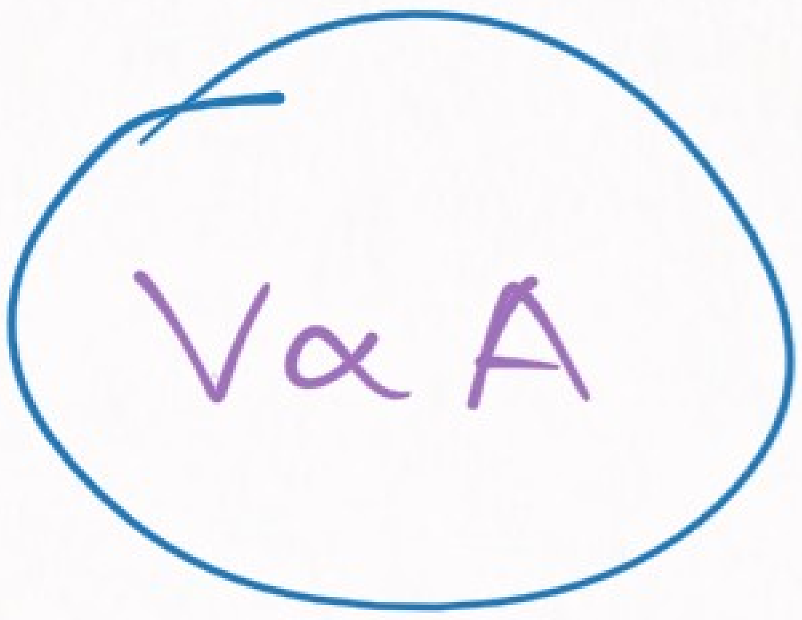
⇓

(Binding energy of nuclei)

LIQUID DROP MODEL

→ Calculate $B(Z,A)$

1) Volume term :



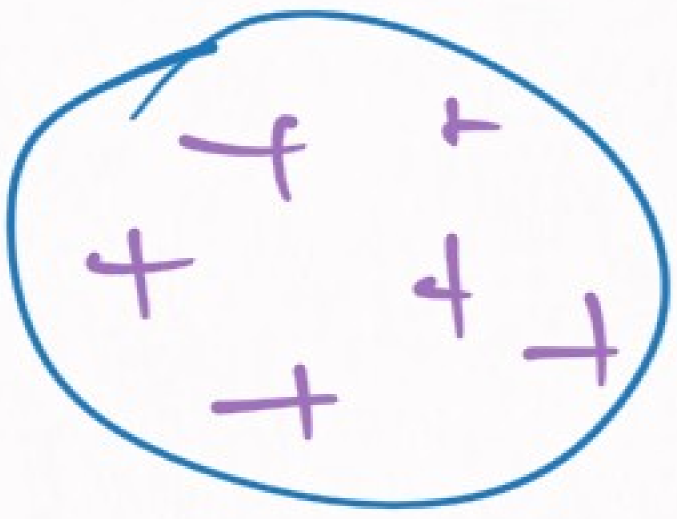
$$B(Z,A) = a_v A + \dots$$

2) Surface term :

$$B(Z,A) = a_v A - a_s A^{2/3} + \dots$$

$S \propto A^{2/3}$

3) Coulomb term



$$B(Z,A) = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} + \dots$$

1) + 2) + 3) easy to understand

4) + 5) → more nuclear-specific

4) Asymmetry term

(nuclei like to have same number of neutrons & protons)

$$B(Z,A) = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}}$$

$$- a_A \frac{(Z - A/2)^2}{A} + \dots$$

5) Pairing term

(even-even nuclei more stable)

$$B(Z,A) = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}}$$

$$- a_A \frac{(Z - A/2)^2}{A} + a_P \frac{((-1)^Z + (-1)^N)}{2 A^{1/2}}$$

1)+2)+3)+4)+5)

Liquid drop model

or

Semi-empirical mass formula

or

Bethe-Weizsäcker formula

$a_v \simeq 16$ MeV

$a_A \simeq 23$ MeV

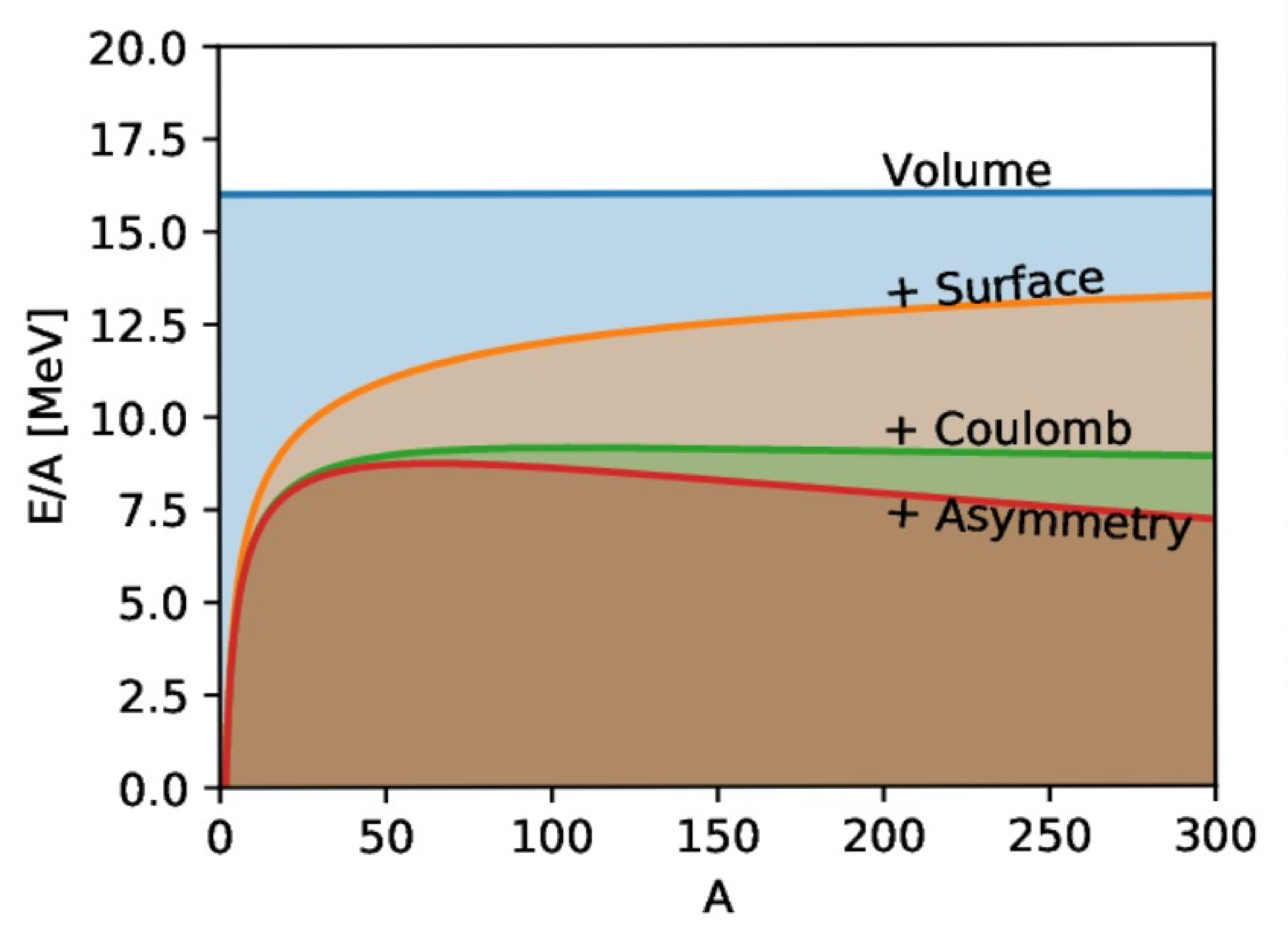
$a_s \simeq 18$ MeV

$a_p \simeq 11$ MeV

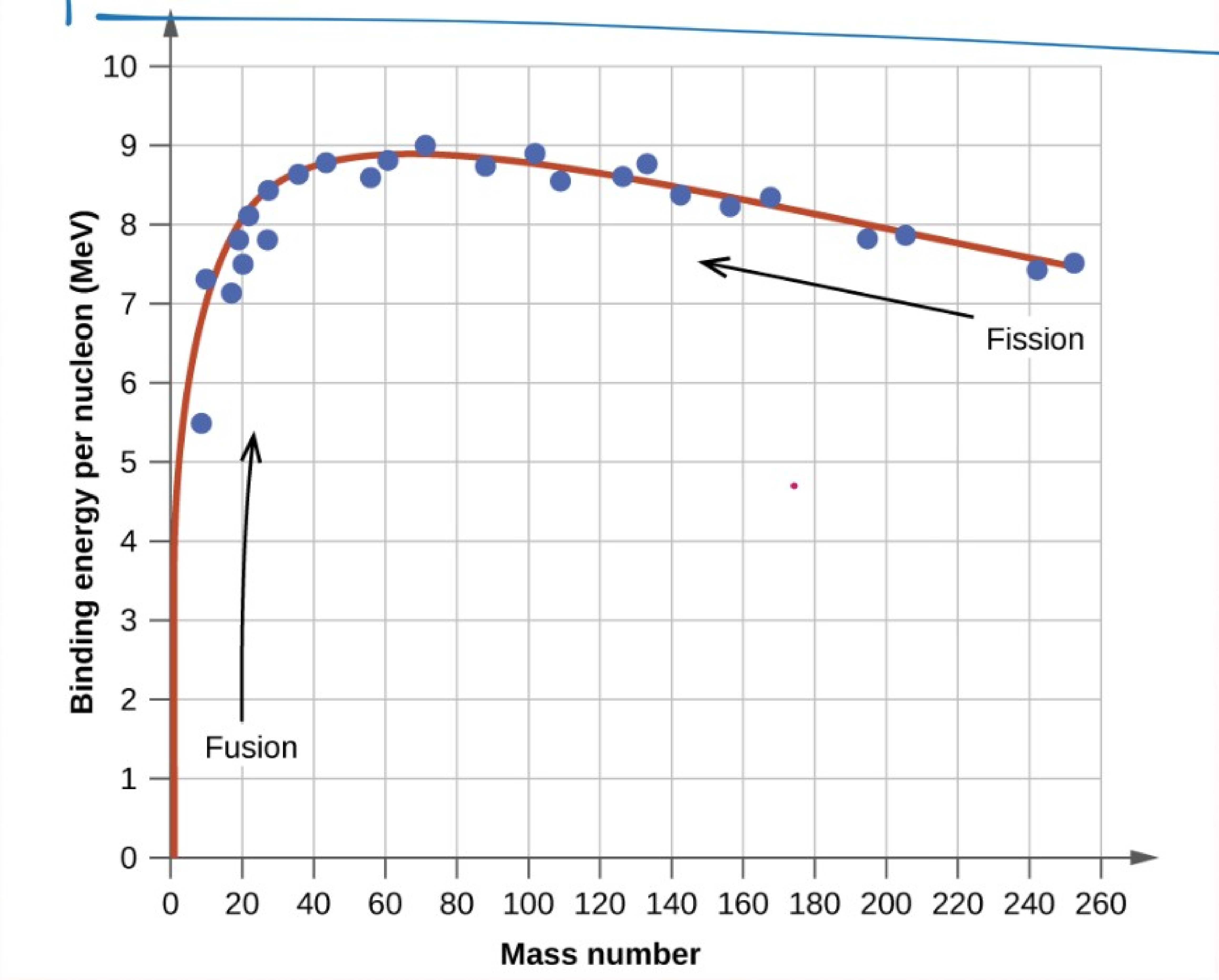
$a_c \simeq 0.7$ MeV

(there are a lot of fits)

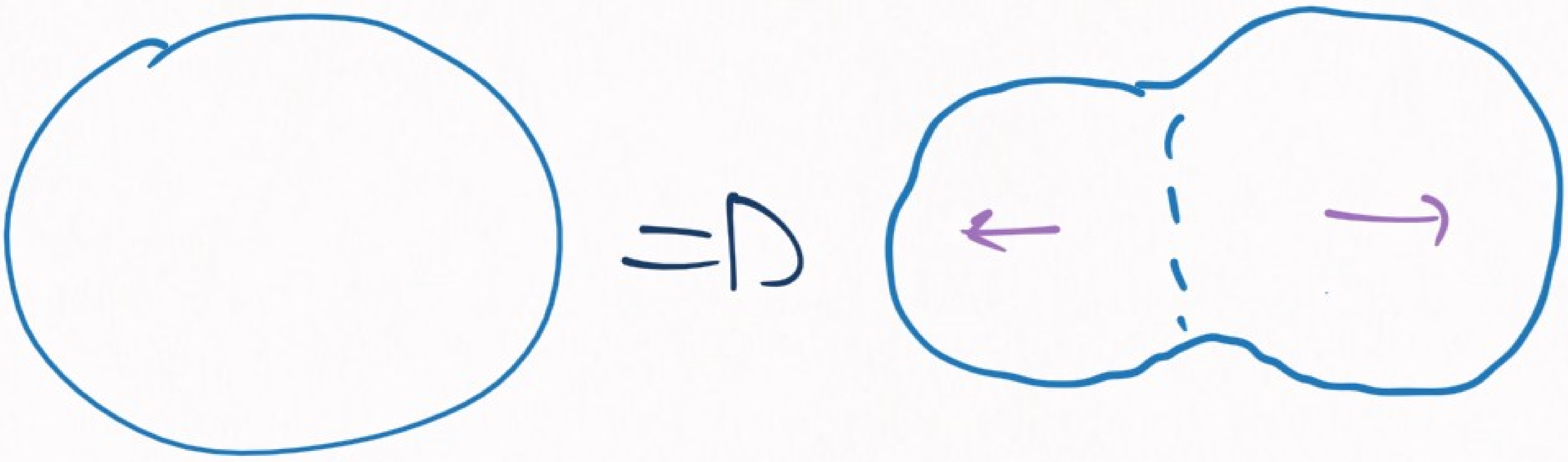
A graphical representation:

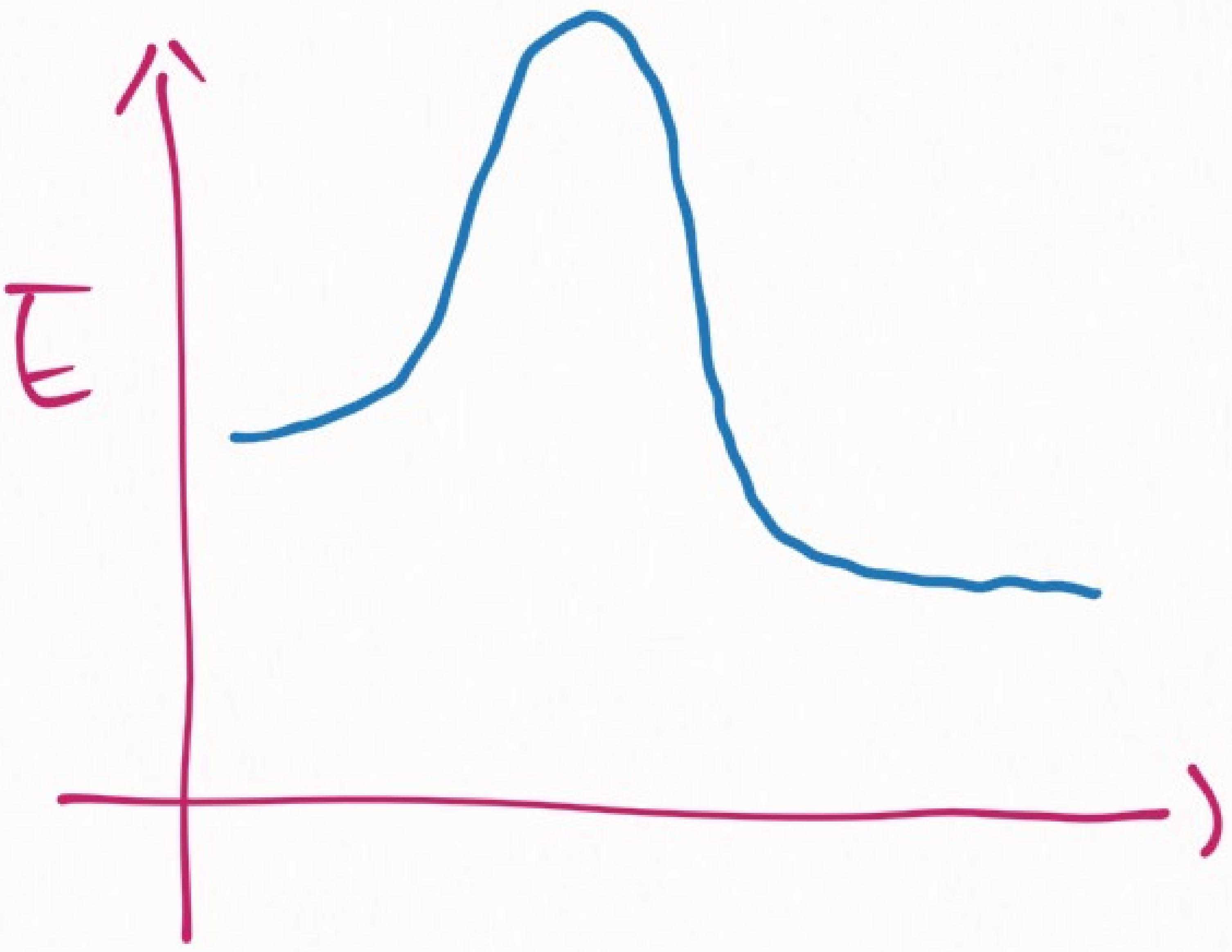


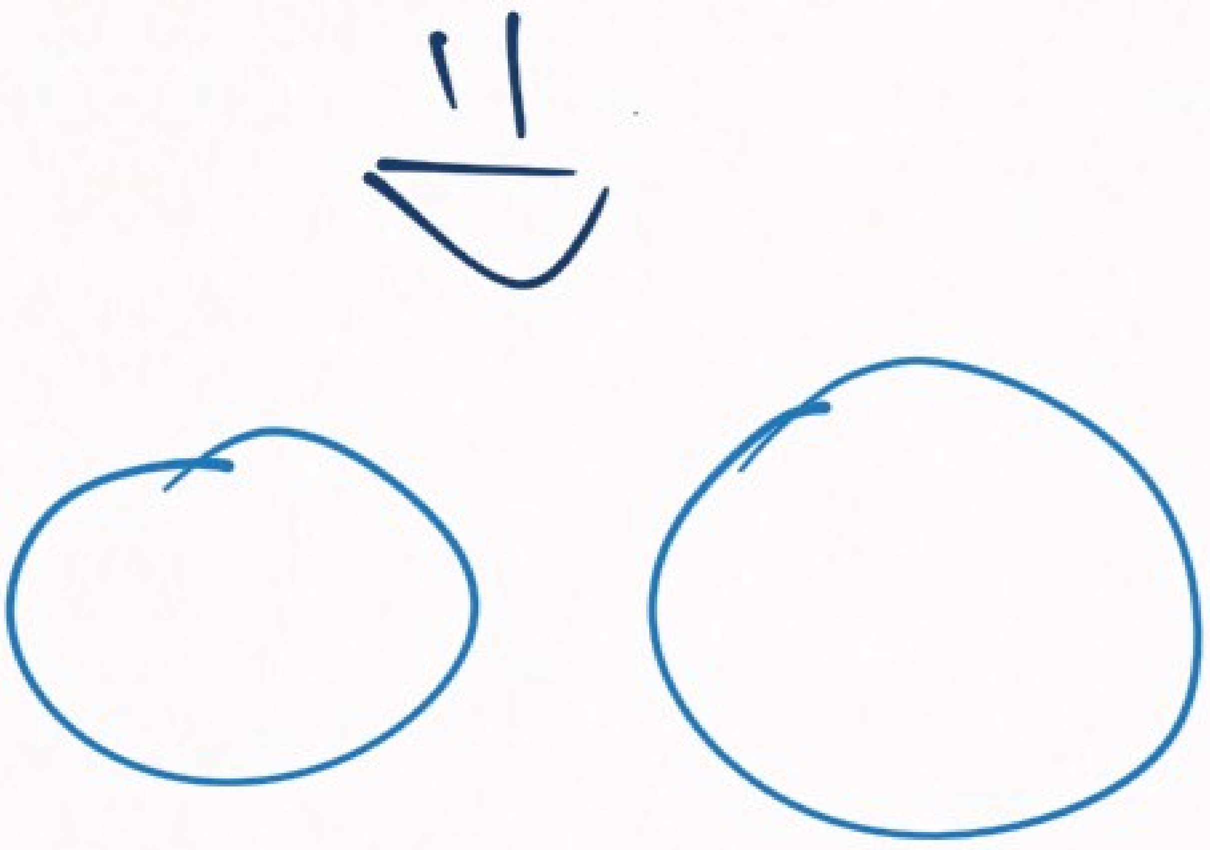
⇒ ALSO EXPLAINS FISSION



FISSION IN THE LIQUID DROP MODEL

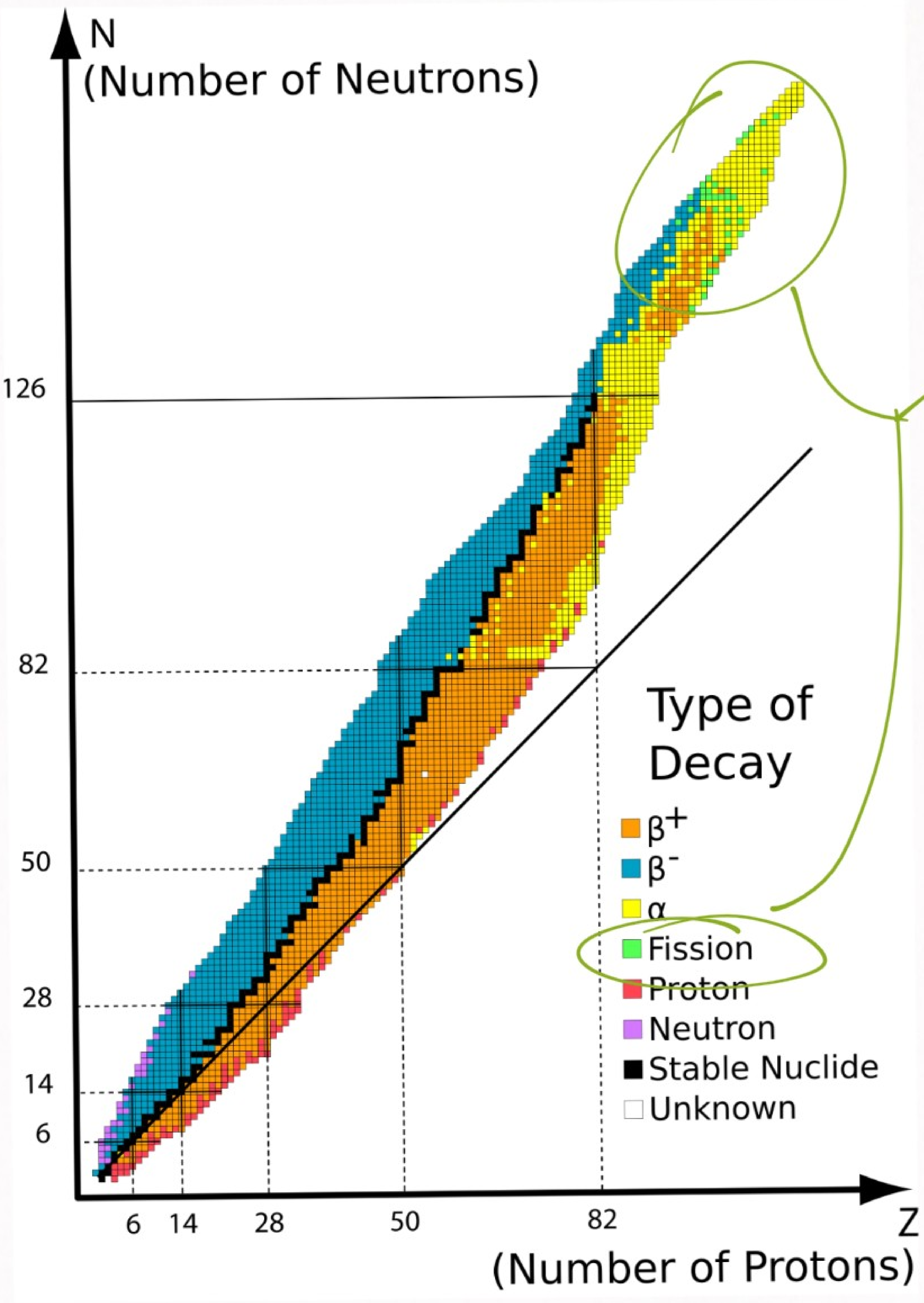






⇒ For really large Δ, this is energetically possible

(but will be a slow process)



LIQUID
DROP
MODEL

SHELL MODEL

1) Nucleons are Fermions

2) Nucleons generate sort of a "mean field"

[Let's check this idea]

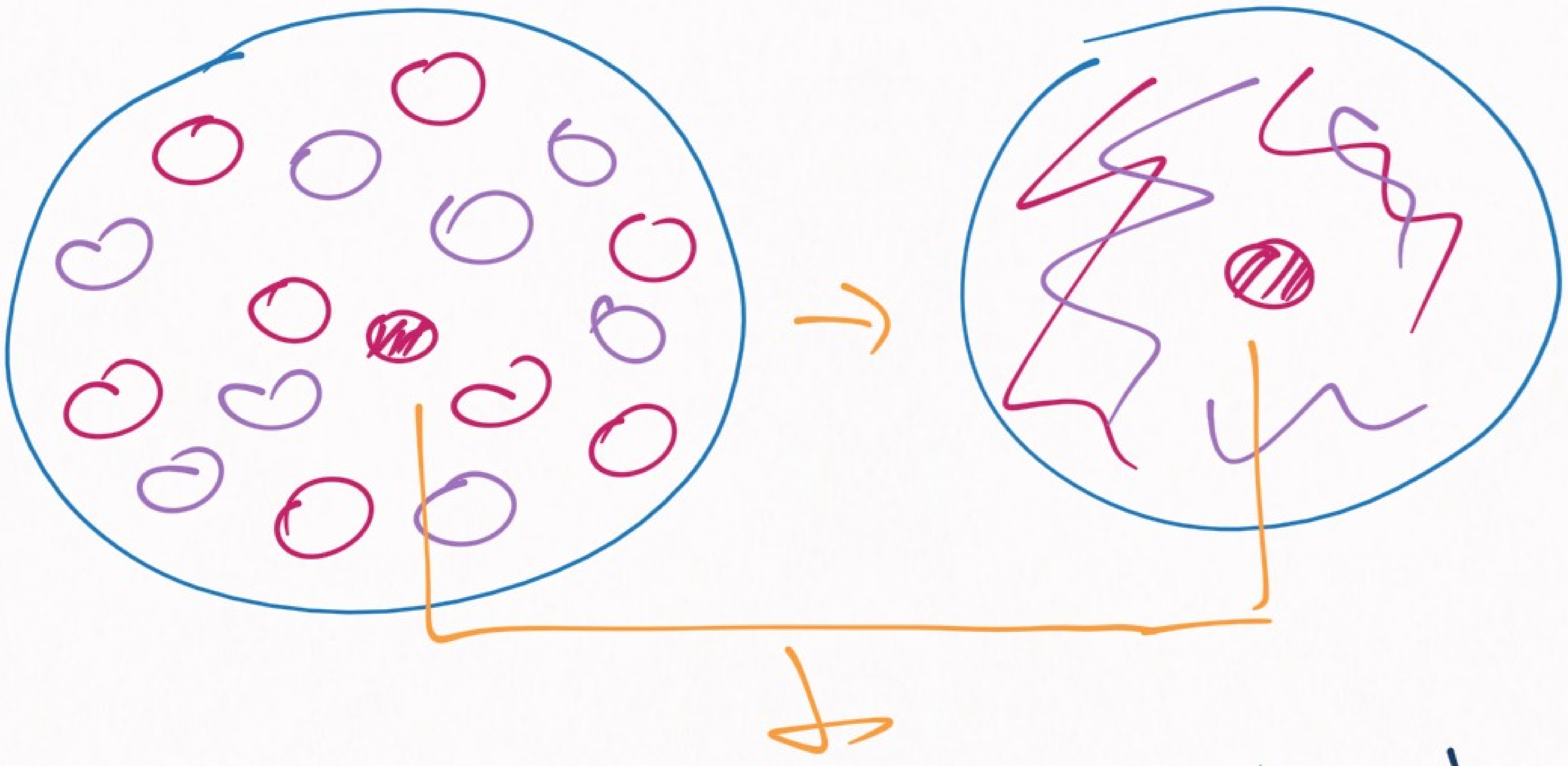
Microscopic (complete) description:

$$H = \frac{1}{2M_N}\sum_i p_i^2 + \sum_{i<j} V_{2B}(r_i - r_j) + \sum_{i,j,k} V_{3B}(r_i, r_j, r_k) + \dots$$

$$\Rightarrow \boxed{H|\Psi_A\rangle = E|\Psi_A\rangle}$$

Problem → microscopic description unmanageable for A large enough

Many solutions → Mean field/potential



Consider the averaged interactions of all nucleons over a particular nucleon

$$\sum_{ij} V^{2B}_{ij} + \sum_{ijk} V^{3B}_{ijk} + \dots \Rightarrow \sum_i V^{MF}_i$$

↓

The approximation we will be making

↓

$$\Rightarrow \boxed{H = \frac{1}{2M_N}\sum p_i^2 + \sum_i V^{MF}_i + \Delta V}$$

↓

"Residual interaction"

↓

$$\Delta V = \sum_{ij} V^{2B}_{ij} + \sum_{ijk} V^{3B}_{jk} + \dots - \sum_i V^{MF}_i$$

If V^{MF} well-chosen

$\Rightarrow$ ΔV should be a small correction

Let's assume we have a good mean field potential:

$$H = \sum_i \left(\frac{p_i^2}{2M_N} + V_i^{MF}\right) + \cancel{\Delta V}$$

Negligible

Then we solve it like this:

$$H = \sum_i H_i \Rightarrow H_i \phi_i(r_i) = E_i \phi_i(r_i)$$

$$\Rightarrow \boxed{\Psi_A = \prod_{i=1}^{A} \phi_i(r_i)}$$

(⚠ Except for a detail:

[Nucleons are fermions]

↓

Antisymmetric wave function

$\Rightarrow$ $\boxed{\text{We antisymmetrize}}$

$$\Psi_A = \prod_{i=1}^{A} \phi_i(r_i) \quad \text{(non-symm)}$$

$$\Downarrow$$

$$\Psi_A = \frac{1}{\underbrace{\sqrt{A!}}_{A!\text{ permutations of } A \text{ elements}}} \underbrace{\sum_{\sigma}}_{\text{permutation}} \prod_{i=1}^{A} (-1)^{\sigma} \phi_{\sigma(i)}(r_i)$$

Or, equivalently:

$$\underbrace{\Psi_A}_{\text{Slater determinant}} = \frac{1}{\sqrt{A!}} \begin{vmatrix} \phi_1(r_1) & \phi_1(r_2) & \dots & \phi_1(r_A) \\ \phi_2(r_1) & \phi_2(r_2) & \dots & \phi_2(r_A) \\ \vdots & \vdots & & \vdots \\ \phi_A(r_1) & \phi_A(r_2) & \dots & \phi_A(r_A) \end{vmatrix}$$

Bottom-line → every nucleon
in a different state

$$\boxed{\text{Let's understand it w/ an example}}$$

$$V_{MF}(r) = \frac{1}{2} M_N \omega^2 r^2$$

↳ mean-field potential is a
harmonic oscillator

$$H_i \phi_i = \epsilon_i \phi_i$$

$$\epsilon_i(n,l) = \omega(2n + l + \frac{3}{2})$$

energy
level
(n=0, 1, 2, ...)

angular
momentum
(l=0, 1, 2, ...)

What happens then?

0) Ground state of mean field

$n=0, \ell=0 \longrightarrow \epsilon = \frac{3}{2}\omega$

Pauli exclusion principle

→ we can put here

2 protons & 2 neutrons

(spin)

1) Next energy level:

$n=0, \ell=1 \longrightarrow \epsilon = \frac{5}{2}\omega$

Pauli exclusion: $2\times(2\ell+1)=6$

→ we can put here

6 protons & 6 neutrons

3) Next energy level:

$E = \frac{7}{2}\omega$ ← $n=1, \ell=0$

↖ $n=0, \ell=2$

↓

We can put here $2+10=12$ neutrons or protons

$\Rightarrow$ It's a game of filling energy shells

(shell-model)

Harmonic oscillator:

$\epsilon = \frac{3}{2} \rightarrow$ 2p / 2n

$\epsilon = \frac{5}{2} \rightarrow$ 6p / 6n

$\epsilon = \frac{7}{2} \rightarrow$ 12p / 12n

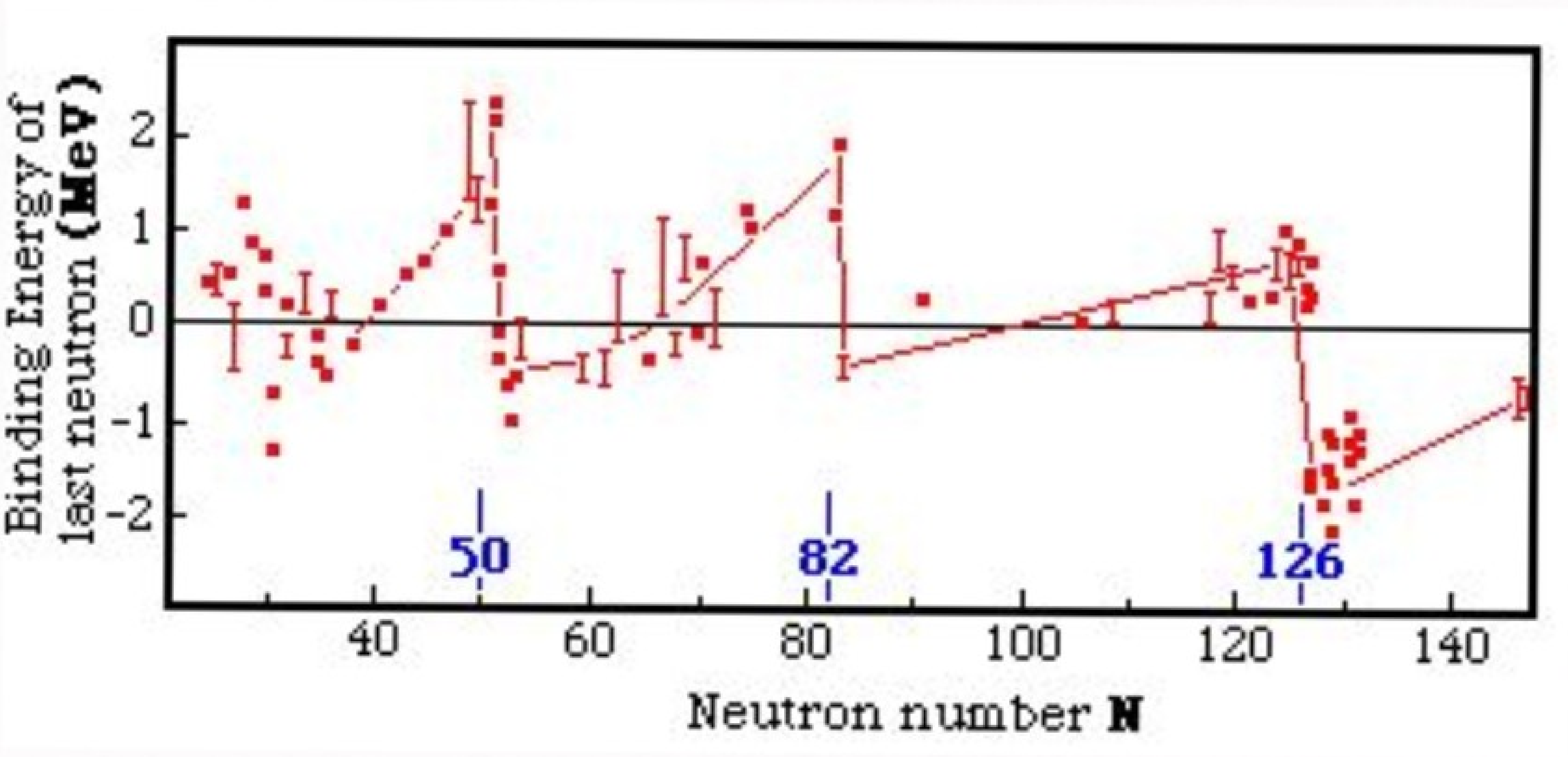
$$\Rightarrow \quad N = 2, 8, 20$$

$$Z = \underbrace{2, 8, 20}_{\downarrow}$$

Rise in the energy required to remove a nucleon

(separation energy, $S_p(N,Z)$, $S_n(N,Z)$)

RECAP → magic numbers



N = 2, 8, 20, 28, 50, 82, ...

Z = 2, 8, 20, 28, 50, 82, ...

ok! ok! ok! oups!

Harmonic oscillator:

N, Z = 2, 8, 20, 40, 70 ...

SHELL MODEL

1) Mean-field potential

2) Fill the shells,
get magic numbers

→ w/ harmonic oscillator:

$$N, Z = \underbrace{2, 8, 20}_{\text{right}}, \underset{\swarrow}{40}, \underset{\swarrow}{70}, \ldots \qquad \text{wrong}$$

→ We are close, but there's a missing ingredient

⇒ **FOR THE NEXT LESSON**