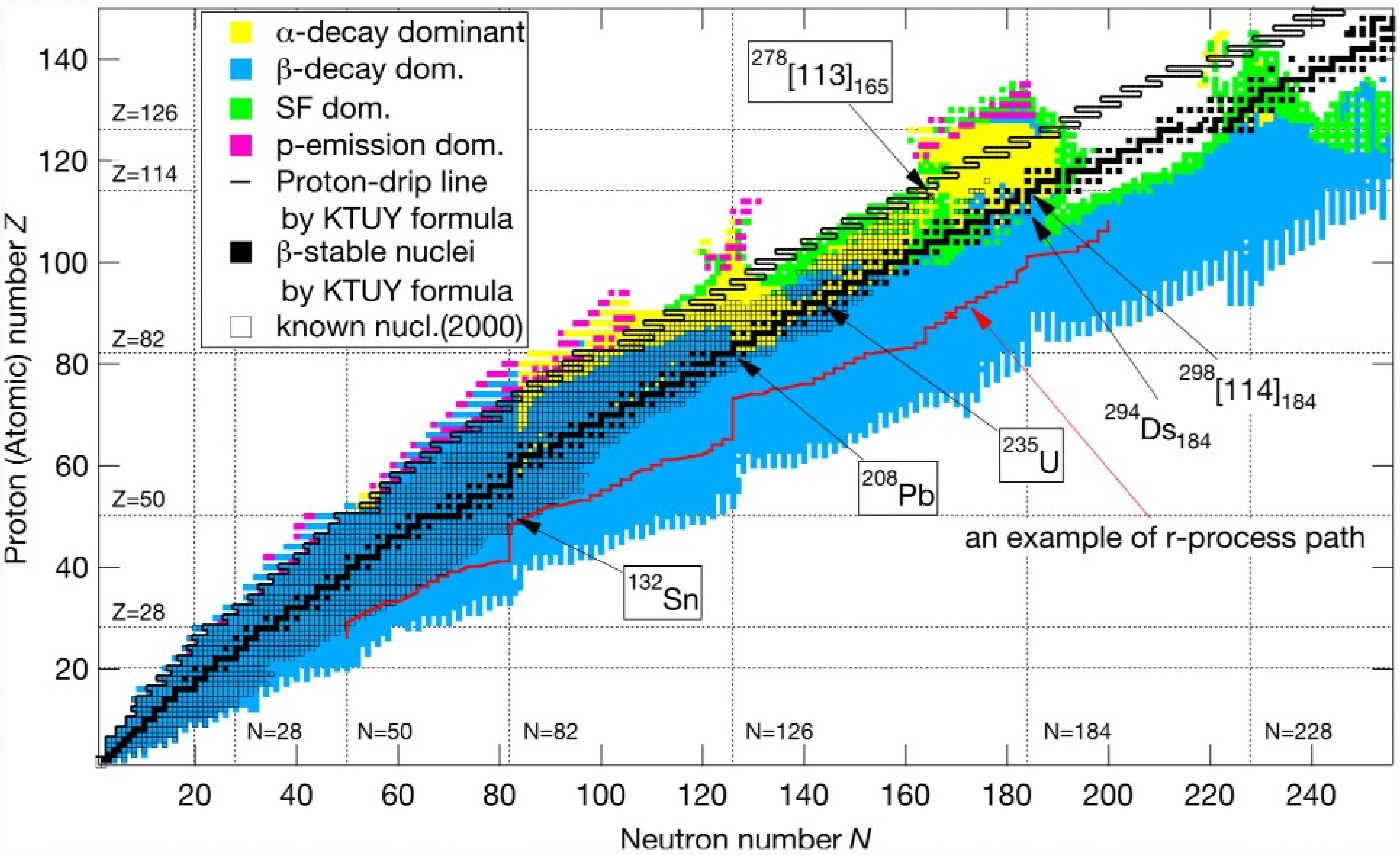
Nuclear Physics ㉔

Nuclear structure ②

Electromagnetic properties of nuclei

Nuclear structure

Necessary if we want to understand this mess



Not doable w/ ab-initio methods

Nuclear structure

↳ Nuclear properties

1) Binding energy
2) Size

Past lesson

3) J^P
4) Electromagnetic properties
(Electric quadrupolar / magnetic dipolar)
5) Stability / decays
(not a property per se)

Today's lesson

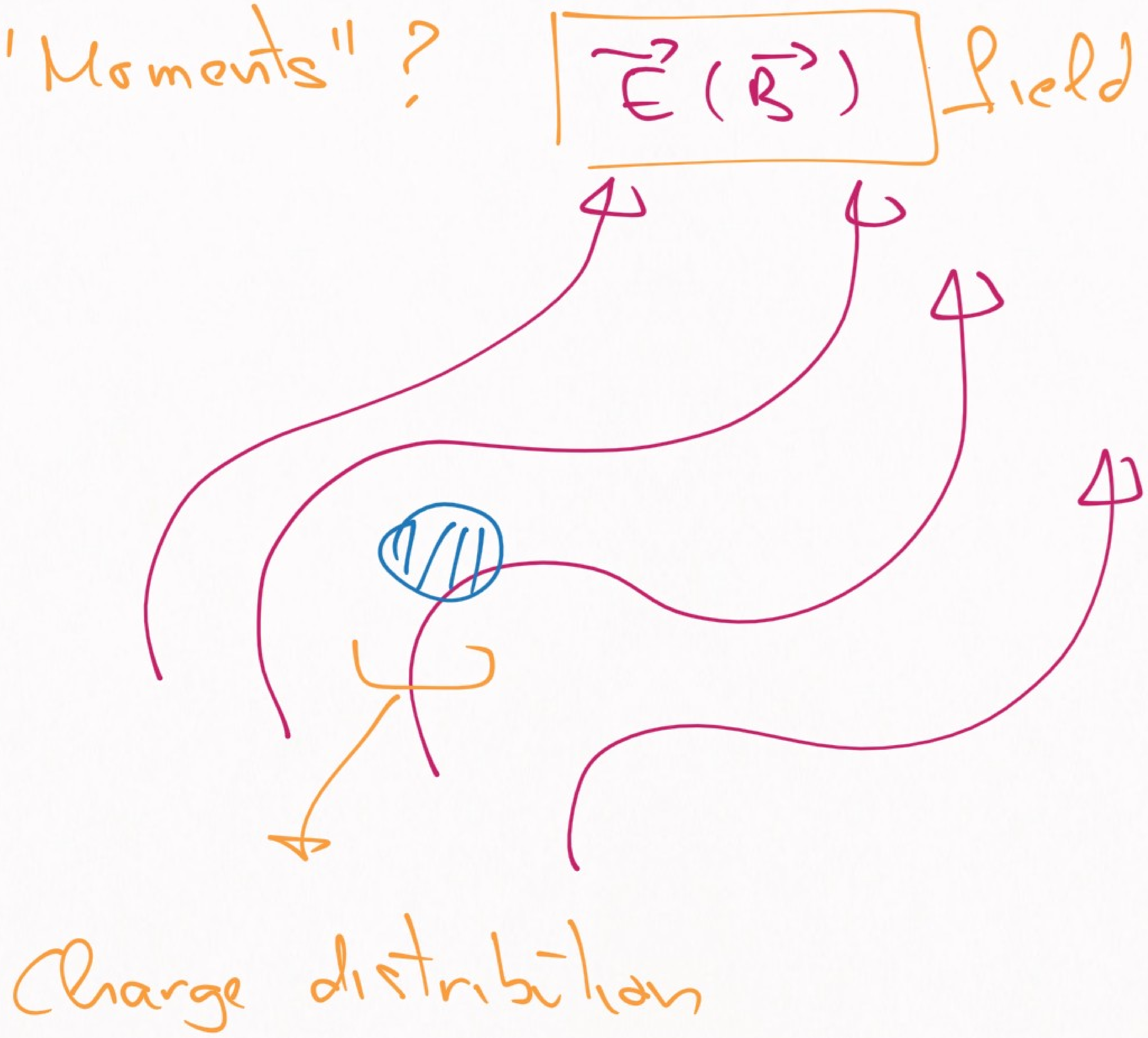
Nuclear Properties :

4) Electromagnetic properties

4.a) Electric Quadrupole moment

4.b) Magnetic Dipole moment

"Moments" ?



Electric moments :

$$V = q\Phi + \vec{d}\cdot\vec{\nabla}\Phi + \frac{1}{6}Q_{ij}\partial_i\partial_j\Phi$$

$+$ (higher moments)

$\Phi \rightarrow$ electric potential

$\vec{\nabla}\Phi = \vec{E} \rightarrow$ electric field

and so on...

$$q = \int d^3\vec{r}\,\boxed{\rho_c(\vec{r})}$$

$$\vec{d} = \int d^3\vec{r}\,\vec{r}\,\boxed{\rho_c(\vec{r})}$$

$$Q_{ij} = \int d^3\vec{r}\,(3r_ir_j - \delta_{ij}r^2)\,\boxed{\rho_c(\vec{r})}$$

→ Charge distribution of nucleus

Magnetic moments

$$V = \vec{\mu}\cdot\vec{B} + \frac{1}{6} Q_{\mu ij} \partial_i B_j$$

$$+ (\text{higher momenta})$$

$\vec{B}$ → magnetic field

$$\vec{\mu} = \int d^3\vec{r}\, \vec{\mu}(\vec{r})$$

$$Q_{\mu ij} = \int d^3\vec{r} \left(\frac{3}{2} r_i \mu_j(\vec{r}) \right.$$

$$\left. + \frac{3}{2} r_j \mu_i(\vec{r}) - \vec{r}\cdot\vec{\mu}(\vec{r})\, \delta_{ij} \right)$$

Notice that:

$\vec{d} = \vec{0}$ -> electric dipole

$Q_{\mu\nu} = \vec{0}$ -> magnetic quadrupole

$\Rightarrow$ Important moments:

1) Electric charge (trivial)

2) Electric quadropole

3) Magnetic dipole

(At higer orders we will have electric hexadecupole, magnetic octupole, but they are small)

Electric charge → eZ

Z = # of protons

(not difficult to understand)

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Quadrupole moment

(obviously electrical)

$$\rho(\vec{r}) = \langle \psi_A | \sum_{k=1}^{A} e \, | \psi_A \rangle$$

$$= \langle \psi_A | \sum_{k=1}^{A} e_k \, \delta(\vec{r} - \vec{r}_k) \, | \psi_A \rangle$$

$$Q_{ij} = \int d^3\vec{r} \, (3 r_i r_j - r^2 \delta_{ij}) \rho(\vec{r})$$

1) Deuteron: $Q_d = 0.256 \, e\,\mathrm{fm}^2$

$[Q_d > 0]$ → prolate

2) Triton, ^{3}He : $Q_d = 0$

3) ^{4}He : $Q_d = 0$

[Trick: $Q_d \neq 0$ requires $J \geq 1$]

But for most heavy nuclei:

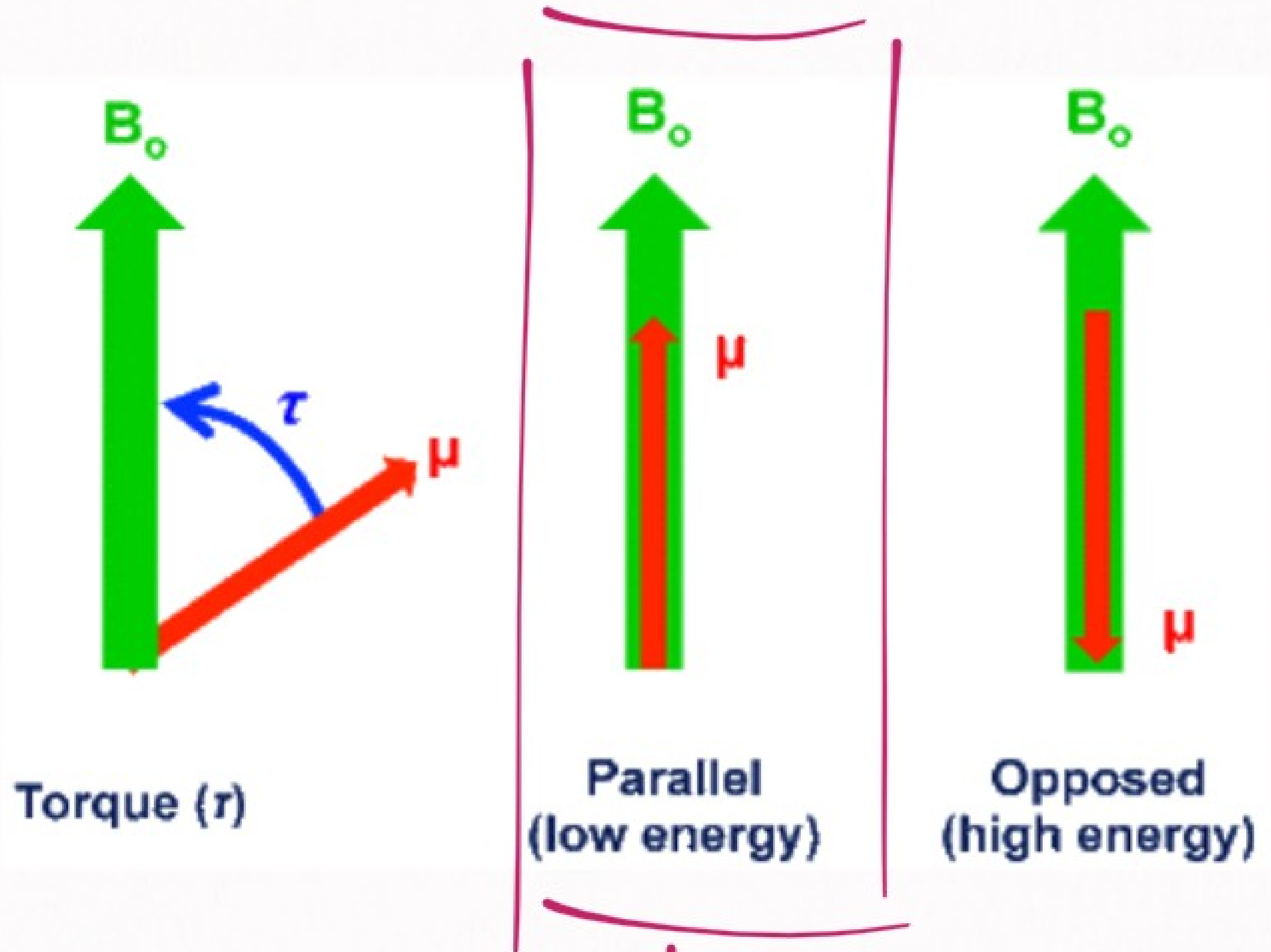
$$\boxed{Q_d < 0}$$

→ oblate

Like a 包子 or an orange

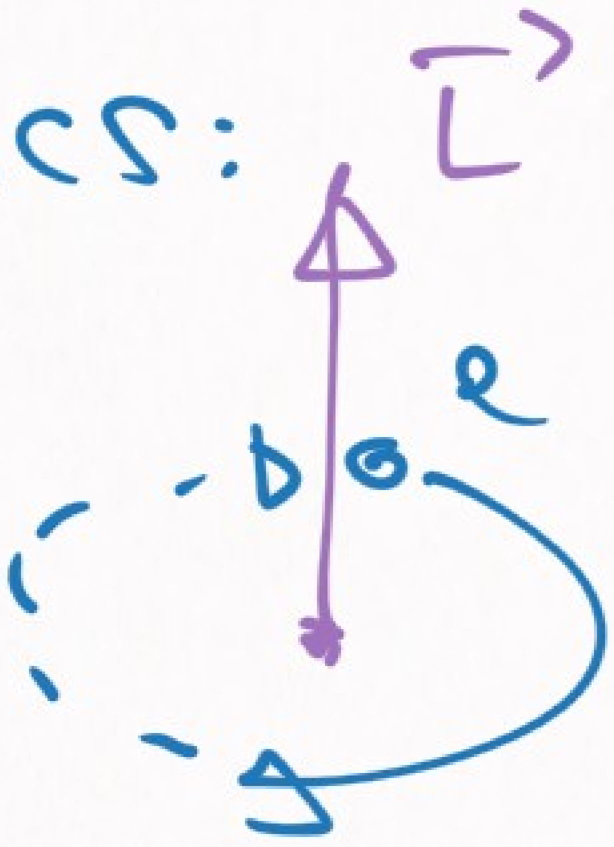
Magnetic moment

$H = -\vec{\mu}\cdot\vec{B}$ μ → magnetic dipole moment



Ground state

Classical mechanics:



$$\underbrace{\vec{\mu}_L = \frac{e}{2m}\vec{L}}_{\text{magnetic moment of spinning particle}}$$

Quantum mechanics: $\vec{J} = \vec{L} + \vec{S}$

$$\vec{\mu} = \vec{\mu}_L + \underbrace{\vec{\mu}_S}_{\downarrow}$$

Which is $\vec{\mu}_S$?

1) First guess: WRONG!

$$\overline{\mu}_L = \frac{e}{2m}\vec{L} \Rightarrow \boxed{\vec{\mu}_S = \frac{e}{2m}\vec{S}}$$

2) Actually:

$$\vec{\mu}_S = g_s \frac{|e|}{2m}\vec{S}$$

↳ gyromagnetic factor

Gyromagnetic factor of electron

1) Dirac equation: $g_s(\bar{e}) = -2$

(QM + special relativity)

2) QED corrections:

$$g_s(e^-) = -2\left[1 + \frac{\alpha}{2\pi} + O(\alpha^2)\right]$$

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If neutron & proton fundamental:

$$g_p \simeq 2 \qquad g_n \simeq 0$$

but we know they are not.

Experimentally:

$$g_p = 5{,}586 \qquad g_n \simeq -3{,}826$$

To summarize:

$$\vec{\mu} = \mu_N (\vec{L} + g_s \vec{S})$$

w/ $\mu_N = \frac{e}{2M_N}$

But it's also common to write:

$$\vec{\mu} = \mu_N (\vec{L} + \mu_s \vec{\sigma})$$

w/ $\mu_s = \frac{g_s}{2}$ (a bit more handy)

$\mu_s(p) = +2.793$ $\mu_p = +2.793 \mu_N$

$\mu_s(n) = -1.913$ $\mu_n = -1.913 \mu_N$

→ more usual like this

EXAMPLE: Magnetic moment of the deuteron

Experiment: $\mu_d = 0.8573\,\mu_N$

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$$\mu({}^A_Z X_N) = \langle JJ|\hat{\mu}_3|JJ\rangle$$

$$\hat{\vec{\mu}} = \sum_{i=1}^{A} \hat{\vec{\mu}}_i$$

For the deuteron:

$$\hat{\vec{\mu}}_d = \hat{\vec{\mu}}_p + \hat{\vec{\mu}}_n$$

$$= \mu_N \vec{L}_p + \mu_p \vec{\sigma}_p + \mu_n \vec{\sigma}_n$$

$\Rightarrow \mu = \langle 11|\mu_3|11\rangle$ $(J = 1)$

If the deuteron was purely S-wave:

$$\mu_d = \langle 11 | \mu_p \sigma_{p3} + \mu_n \sigma_{n3} | 11 \rangle$$

$$= \mu_p + \mu_n \simeq 0.88 \, \mu_N$$

$$(\neq \mu_d(\exp) = 0.8573 \, \mu_N)$$

$\Rightarrow$ Discrepancy → Cause?

∃ some angular momentum

(deuteron D-wave component)

$$\vec{\mu} = \boxed{\mu_N \bar{L}_p} + \mu_p \vec{\sigma}_p + \mu_n \bar{\sigma}_n$$

here it is

Solution: include the D-wave

$$|\psi_d\rangle = a_s|^3S_1\rangle + a_d|^3D_1\rangle$$

$$|a_s|^2 + |a_d|^2 = 1$$

$$\mu(^3S_1) = \mu_p + \mu_n = 0.88\ \mu_N$$

$$\mu(^3D_1) = \frac{3}{4}\mu_N - \frac{1}{2}(\mu_p + \mu_n)$$

$$= 0.31\ \mu_N$$

$$\mu_d = |a_s|^2\mu(^3S_1) + |a_d|^2\mu(^3D_1)$$

$$= P_s\,\mu(^3S_1) + P_D\,\mu(^3D_1)$$

$$\boxed{\begin{aligned} P_s &\simeq 0.96 \\ P_D &\simeq 0.04 \end{aligned}} \rightarrow \text{The famous 4\% figure}$$

Now, if you are a purist of Quantum Mechanics...

[the wave function is not an observable]

$$\Downarrow$$

[P_S and P_D cannot be determined (they are arbitrary quantities)]

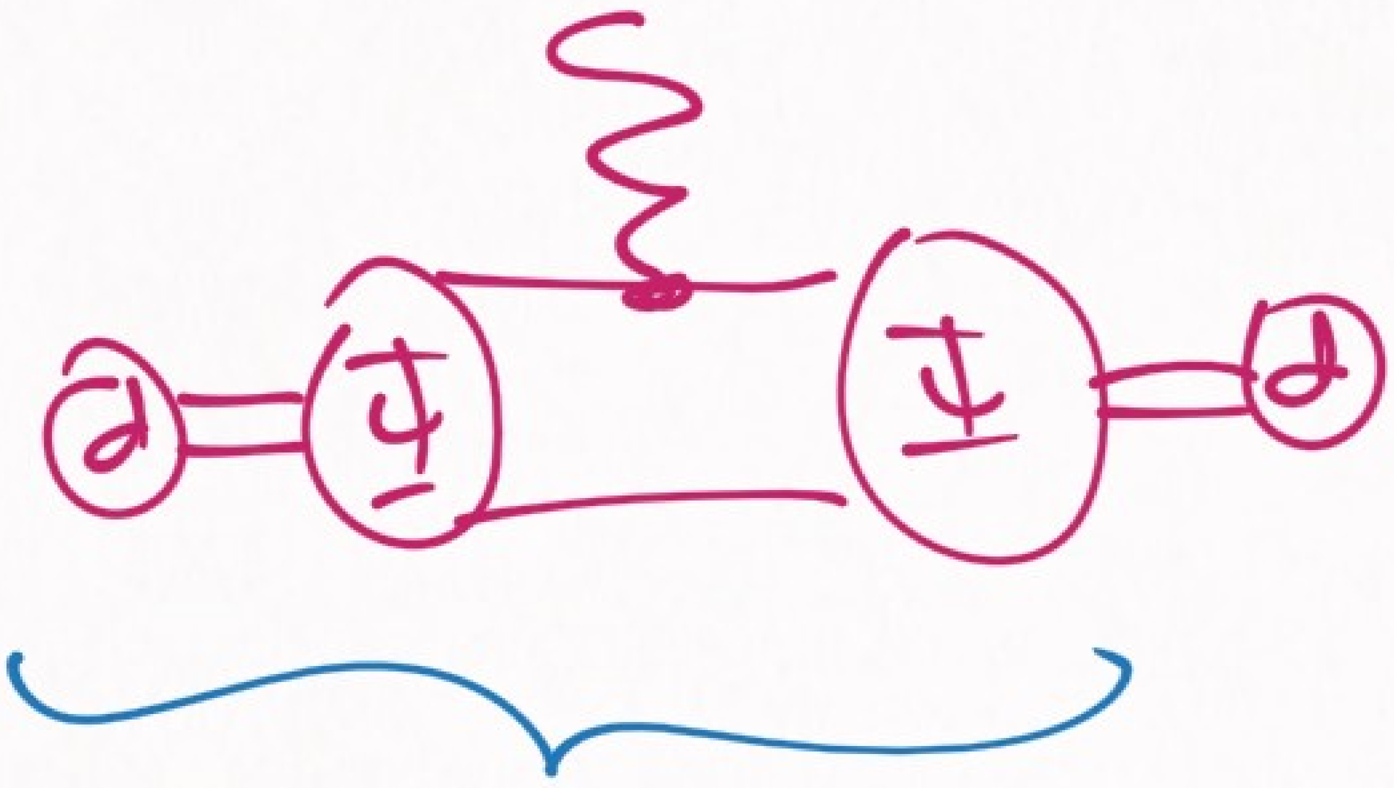
$$\Downarrow$$

[There's something missing in our previous calculation]

What is missing are:

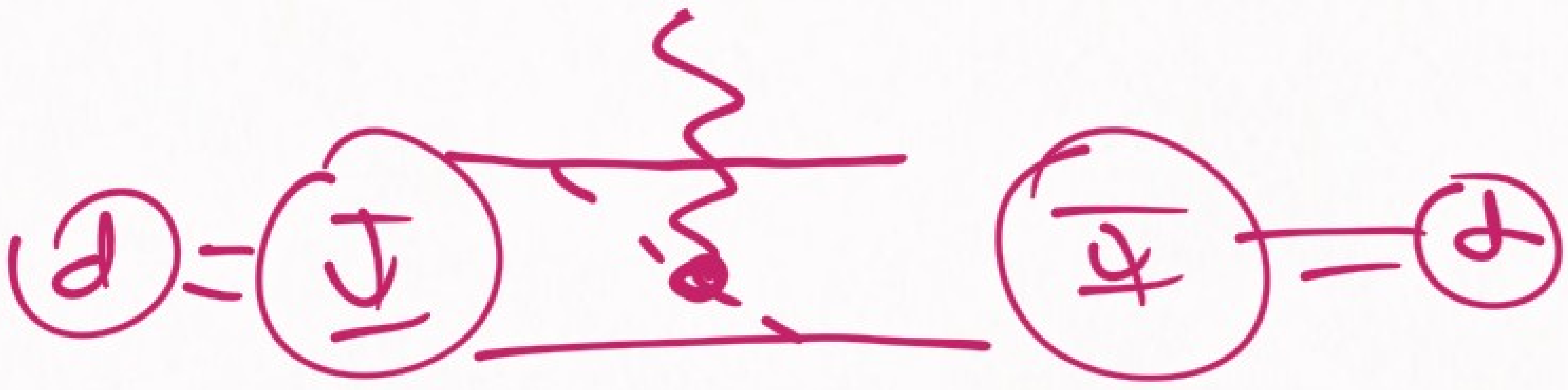
THE TWO-BODY CURRENTS

$$\left.\begin{array}{l} P_S \simeq 0.96 \\ P_D \simeq 0.04 \end{array}\right\} \Rightarrow$$



assumes the photon directly interact with the nucleons (and w/ nothing else)

But in the Real World™



⊳ the photon can hit other things (e.g. an in-flight pion)

Two-body currents

Calculation of electric/magnetic moments more difficult than expected

[But luckily they are small]

→ Deuteron quadrupole moment

$$Q_d^{exp} = 0.2859(3)\ e\,fm^2$$

$$Q_d^{1B} \simeq 0.276\ fm^2$$

$$Q_d^{2B} \simeq 0.010\ fm^2$$

$$|Q_d^{1B}| \gg |Q_d^{2B}|$$

But two-body currents are
sort of an advanced topic

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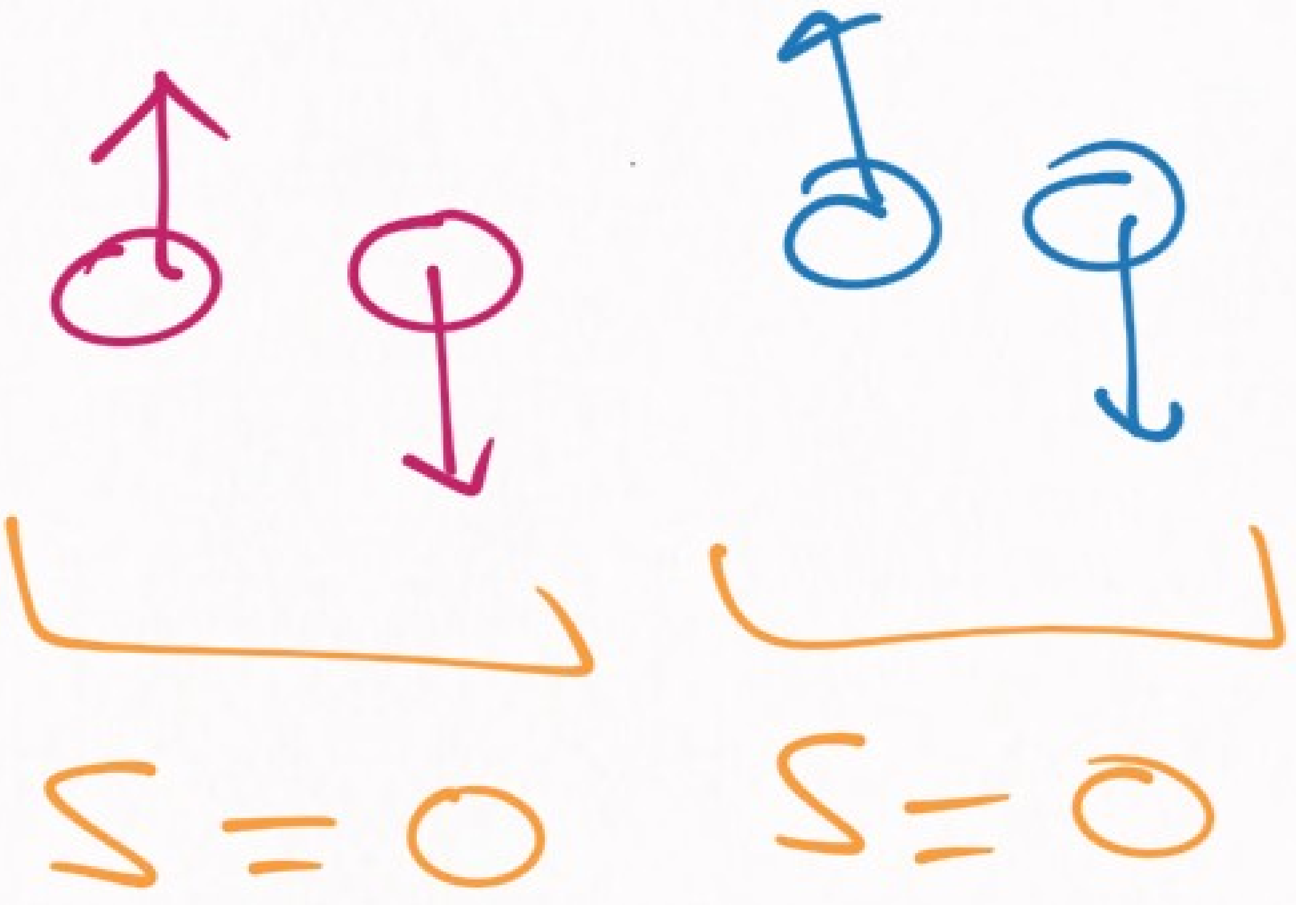
Back to nuclei...

MAGNETIC MOMENT OF NUCLEI

even-even nucleus

→ even N, even Z

PAIRING →



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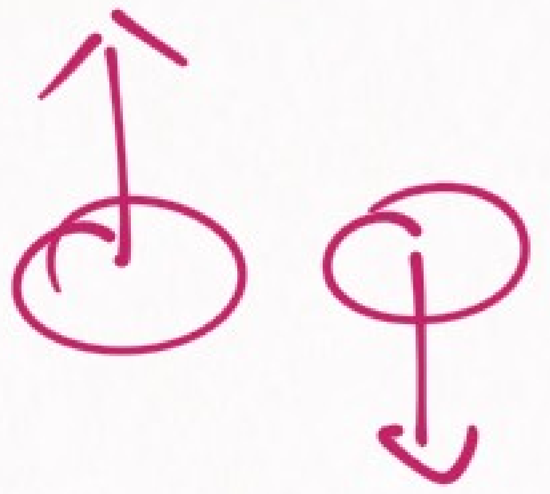
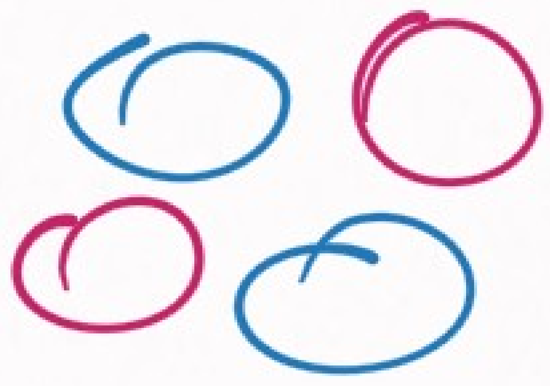
pair of neutrons & pairs of protons couple to spin-0



$$\mu_{pair} = \vec{\mu}_1 + \vec{\mu}_2$$

$$= \mu_{n/p}\underbrace{(\vec{\sigma}_1 + \vec{\sigma}_2)}_{=0}$$

⇒ Pairs of protons / neutrons result in $\vec{\mu}_{pair} = 0$



$$\boxed{\mu(^4He) = 0}$$

$$\underbrace{\boxed{\mu(\text{even-even}) = 0}}$$

$$\downarrow$$

$$\boxed{J^P(\text{even-even}) = 0^+}$$

Next case: even-odd nuclei

even-odd = (even-even) core + nucleon

$\downarrow$

$$\mu(A) = \underbrace{\mu_{core}}_{=0} + \mu_N = \boxed{\mu_N}$$

$$\downarrow$$

$$\boxed{\vec{J}, \vec{L}, S}$$

— ⊗ —

Unpaired nucleon:

$L, S = \frac{1}{2} \Rightarrow J = L \pm 1/2$

$$\mu_N(J = L + 1/2) = g_L \left(J - \frac{1}{2}\right) + \frac{1}{2} g_S$$

$$\mu_N(J = L - 1/2) = g_L \frac{J(J + 3/2)}{J+1} - g_S \frac{J}{2(J+1)}$$

SCHMIDT VALUES

RECAP (magnetic moments

1) Deuteron contains a D-wave

$\mu_d(^3S_1) = 0.88\,\mu_N$ (BK $\mu_d^{exp} = 0.8573\,\mu_N$)

2) Nuclei look complicated but there is pairing

2.a) Even-even

$\mu(\text{even-even}) = 0$

2.b) Even-odd

$\mu(A) = \overset{0}{\cancel{\mu_{core}}} + \mu_n = \mu_N$

2.c) Odd-odd (5 stable cases)

case-by-case

Next lesson :

Nuclear stability

& Liquid drop model