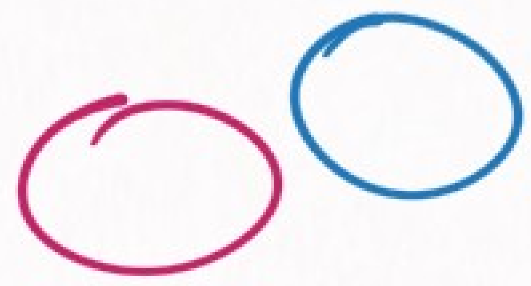
Nuclear Physics 23

Nuclear Structure 1:

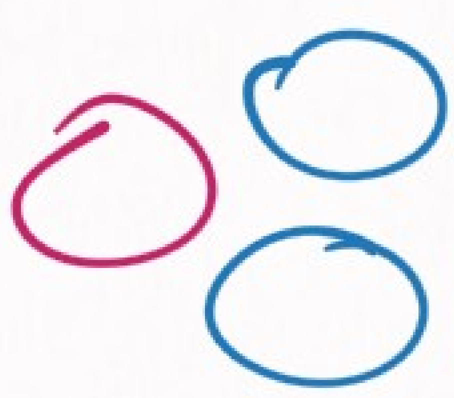
nuclear properties
(non-electromagnetic)

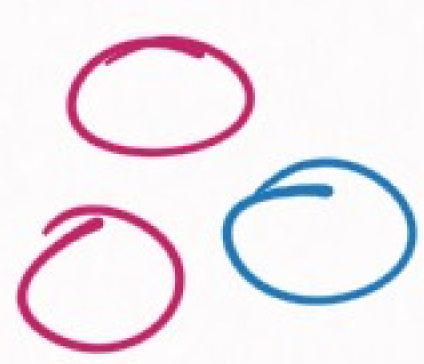
Nuclear Structure? WHY?



Deuteron (A=2)

Doable



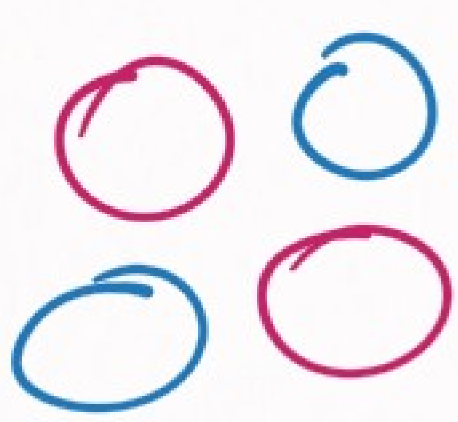


Triton &

Helium-3 (A=3)

Still doable

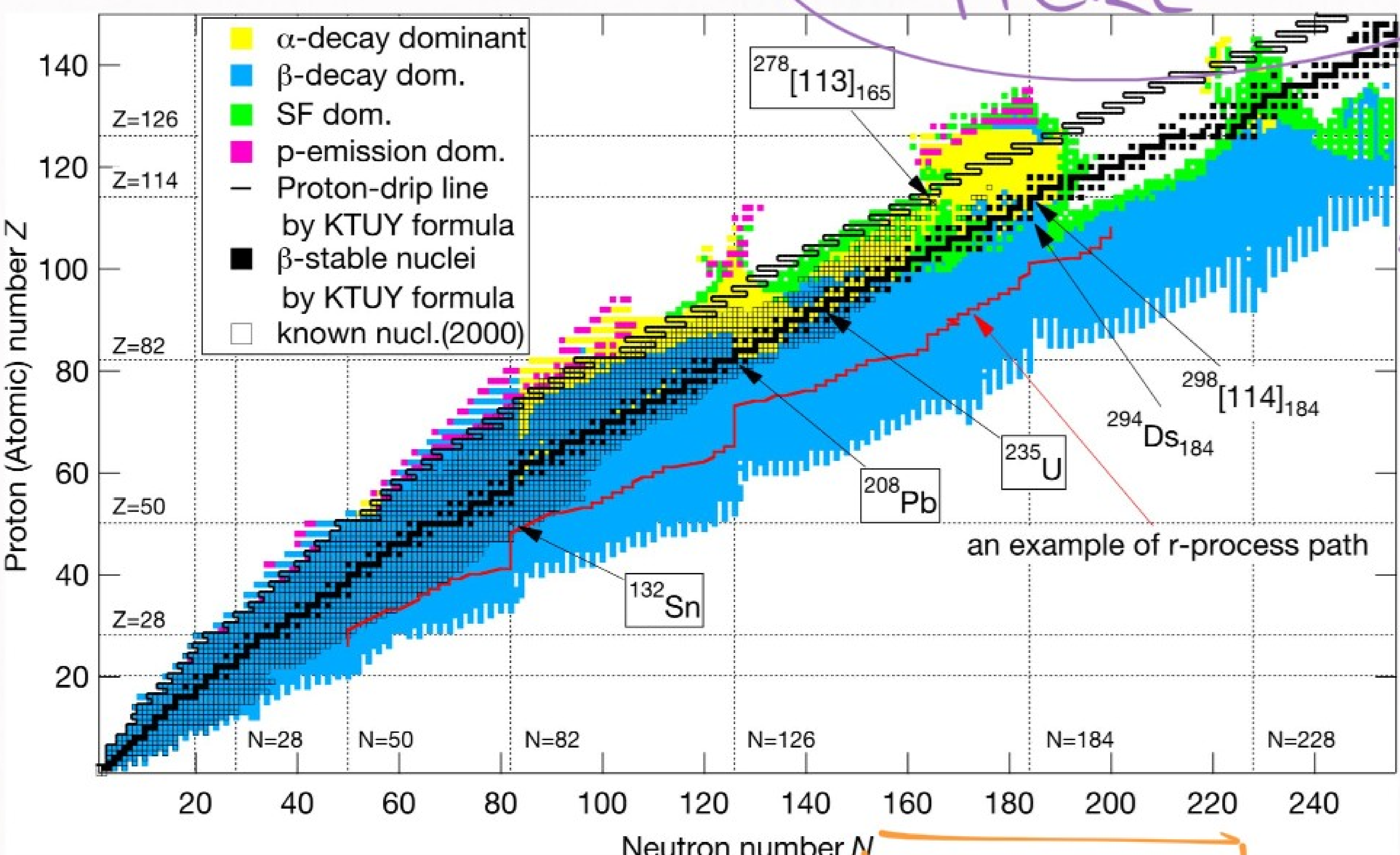
(FADDEEV)



Helium-4

Still...

WHAT ABOUT THESE GUYS HERE



Oups!

A small $\Rightarrow$ Exact methods

A big $\Rightarrow$ We have to think about the structure of a nucleus

GENERAL PROPERTIES OF NUCLEI

SHOPPING LIST

What to describe?

1) Binding energy
2) Size
3) Angular momentum & Parity
4) Magnetic dipole moment
5) Stability / Decays

Nuclear Properties :

1) Binding Energy

$$B_d = (m_p + m_n) - m_d \simeq 2.2\ \text{MeV}$$

$$B_t = (m_p + 2m_n) - m_t \simeq 8.5\ \text{MeV}$$

We can extend this

$$B(Z,N) = (Zm_p + Nm_n) - m(Z,N)$$

$Z \rightarrow$ # of protons

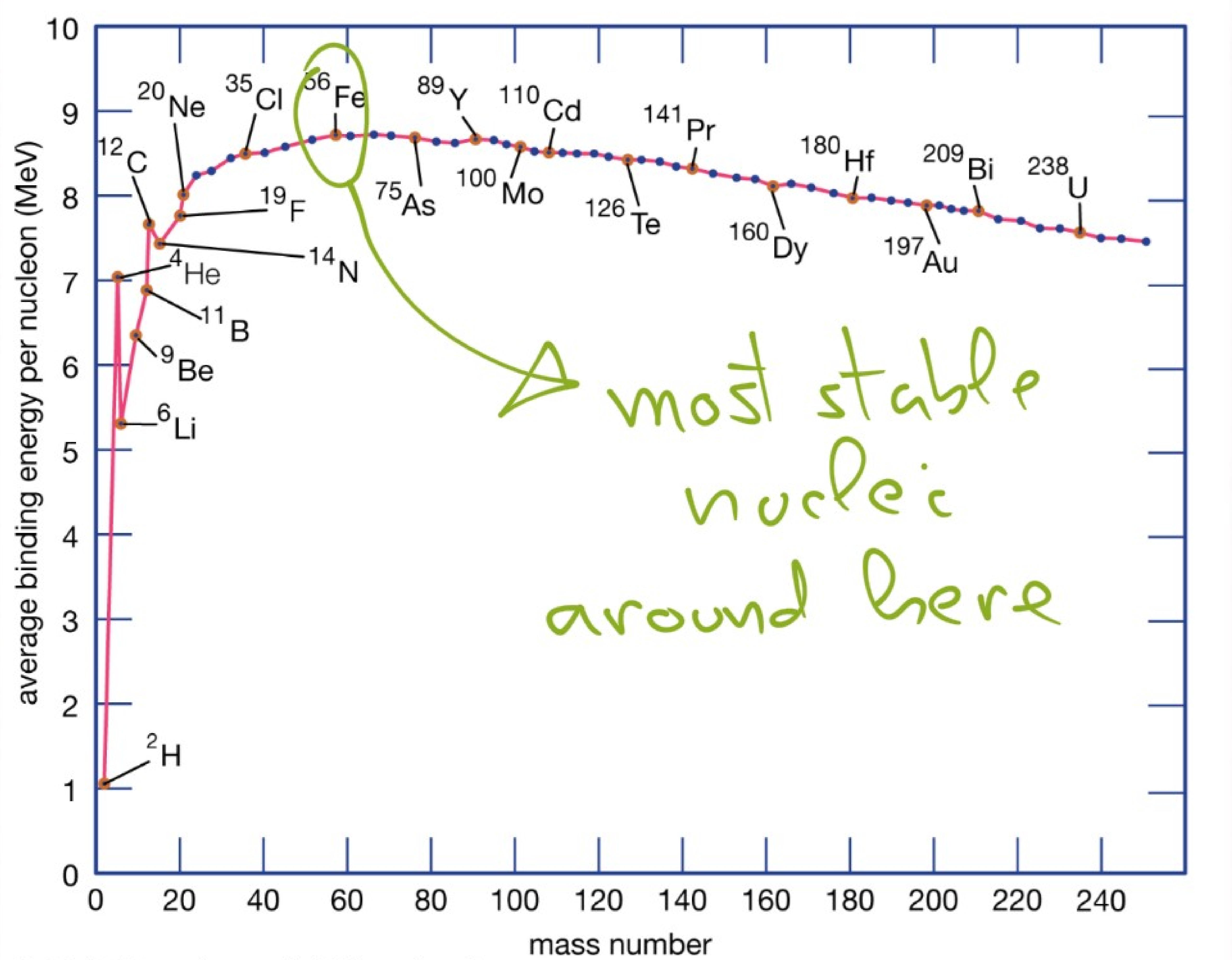
$N \rightarrow$ # of neutrons

$A = N + Z \rightarrow$ # of nucleons

We find $\boxed{\dfrac{B}{A} \sim 8\ \text{MeV/nucleon}}$

(SATURATION)

Here's a graphic of $\frac{B}{A}$



Interesting observation:

$A < 56$ → Fusion possible

$A > 56$ → Fission possible

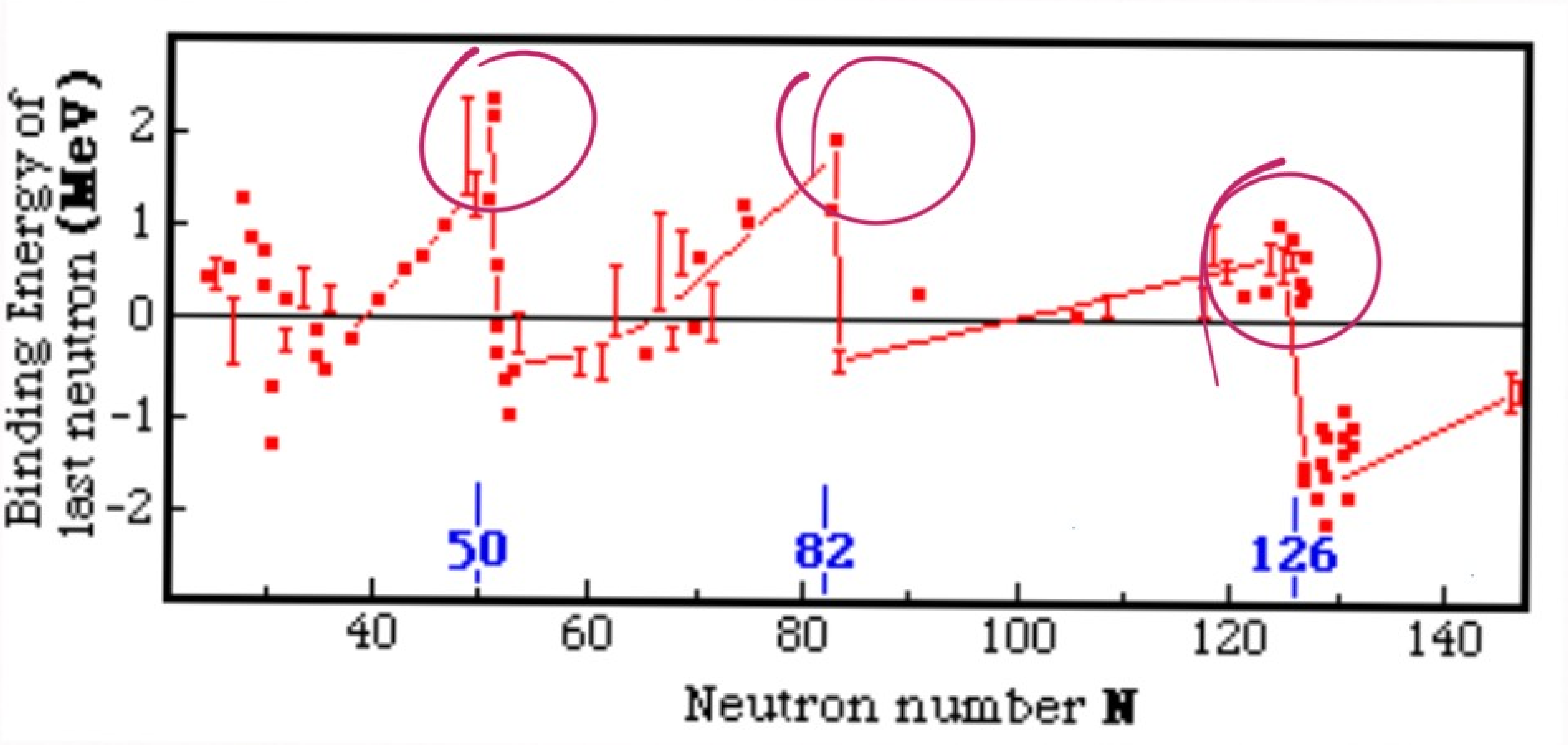
This curve gives us a fusion & fission region

Separation energy → another useful quantity

$$S_p(Z,N) = B(Z,N) - B(Z-1,N)$$

$$S_n(Z,N) = B(Z,N) - B(Z,N-1)$$

→ ∃ something curious here



→ For some (N,Z) the separation energy reaches a maximum

N, Z = 2, 8, 20, 28, 50, 82, ...
(magic numbers)

Another type of separation energy:

$$S_\alpha(N,Z) = B(Z,N) - B(Z-2,N-2) - B(2,2)$$

α-particle separation energy

$$\underbrace{A > 150 \Rightarrow S_\alpha < 0}_{\text{most heavy nuclei can decay by emitting an } \alpha\text{-particle}}$$

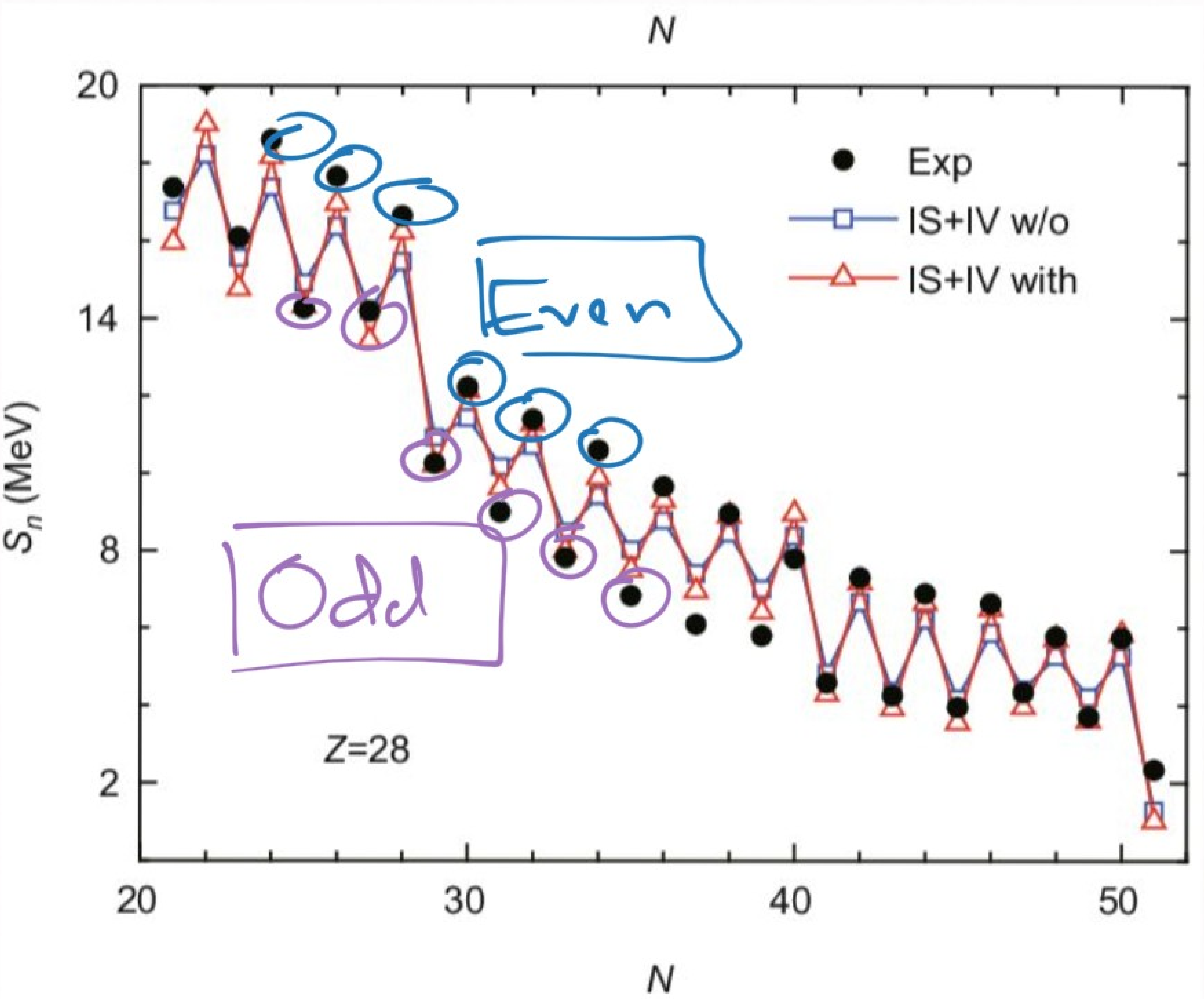
We have also a separation energy related to "pairs"

$$\delta_n = S_n(Z,N) - S_n(Z,N-1)$$

w/ N even, N-1 odd

$\delta_n \sim 2 MeV$

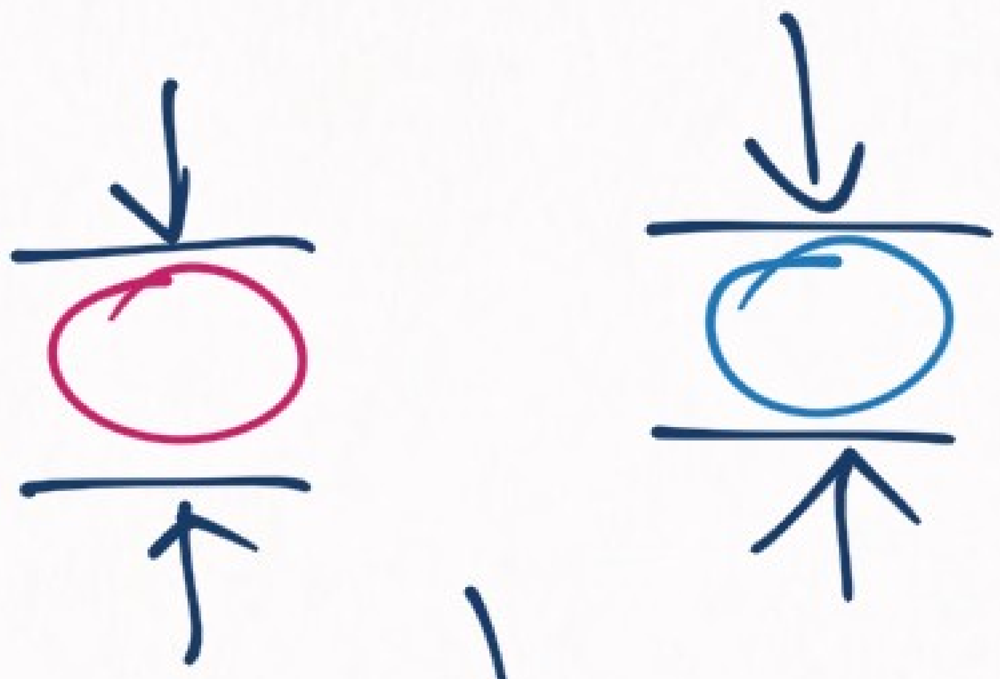
EXAMPLE



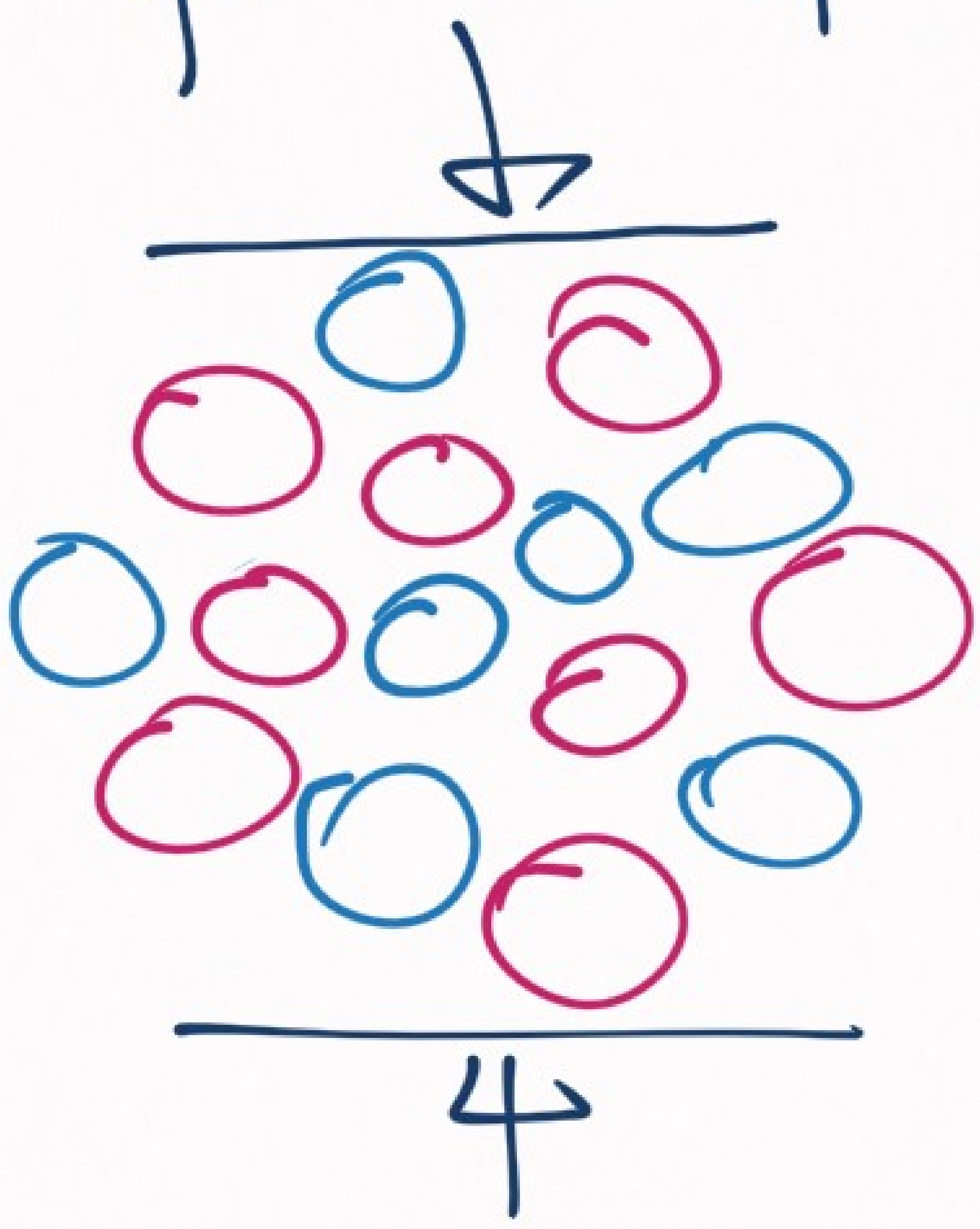
→ even # of neutrons / protons more stable

Nuclear properties:

2) Nuclear size



$$\boxed{\sqrt{\langle r^2\rangle_N} \sim (0.5 - 1.0)\ \text{fm}}$$



$R^3 \sim A\, r_0^3$

w/ $r_0 \gtrsim \sqrt{\langle r^2\rangle_N}$

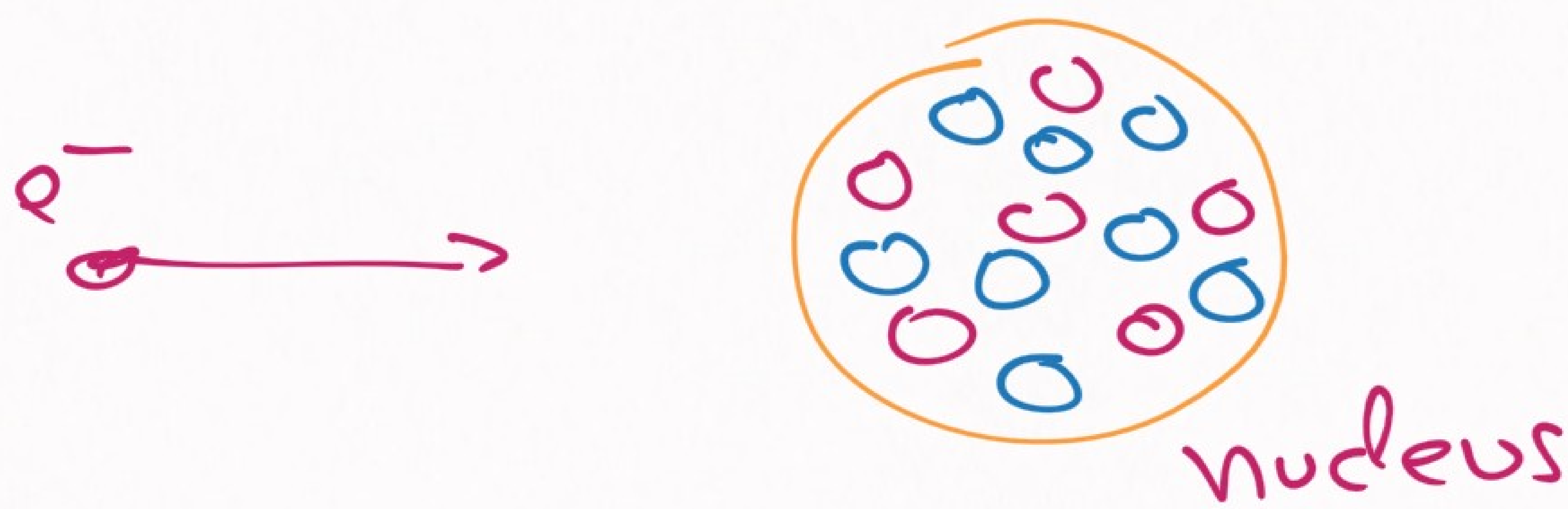
$r_0 \sim 1.2 - 1.3$ fm

$$\Rightarrow \boxed{R \sim A^{1/3}\, r_0}$$

But there is more ...

The Hofstadter experiment

→ Form factors



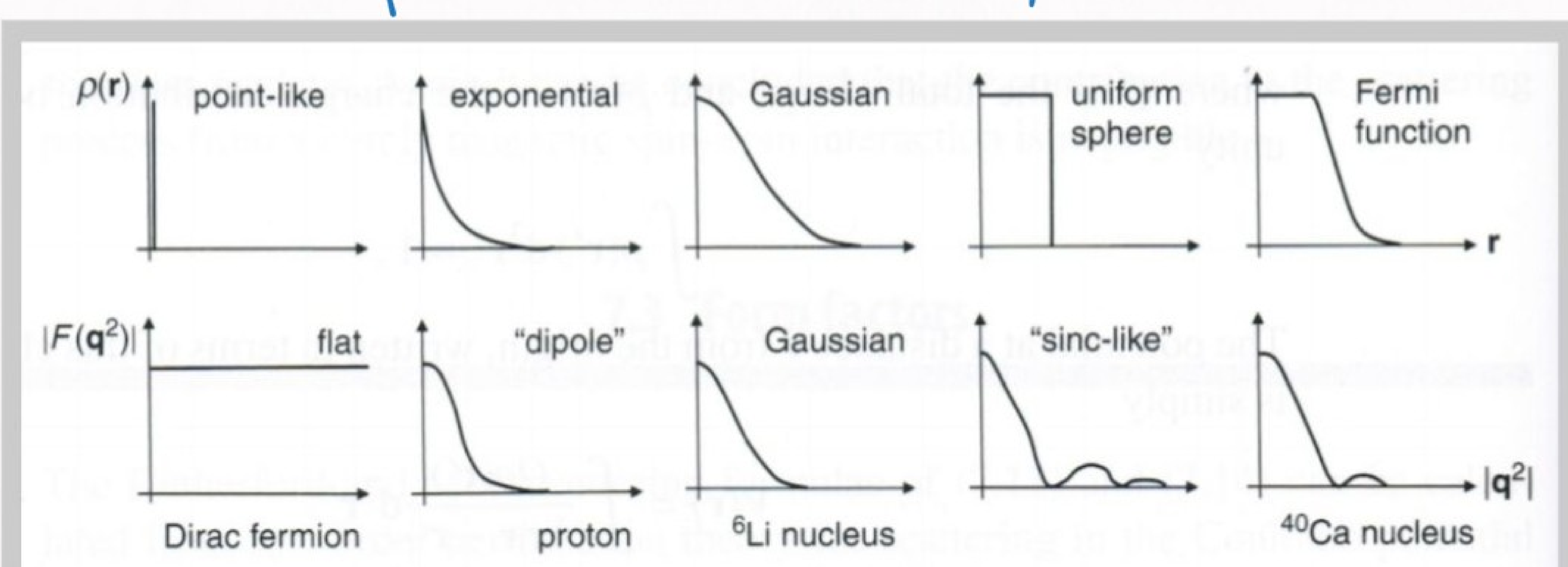
$$\frac{d\sigma}{d\Omega} = \underbrace{\left(\frac{d\sigma}{d\Omega}\right)_{\text{pointlike}}}_{\frac{d\sigma}{d\Omega} \text{ if nucleus was a point}} \underbrace{|F(\vec{q}^{\,2})|^2}_{\text{Form factor}}$$

→ Internal structure of nucleus

Form factor & density:

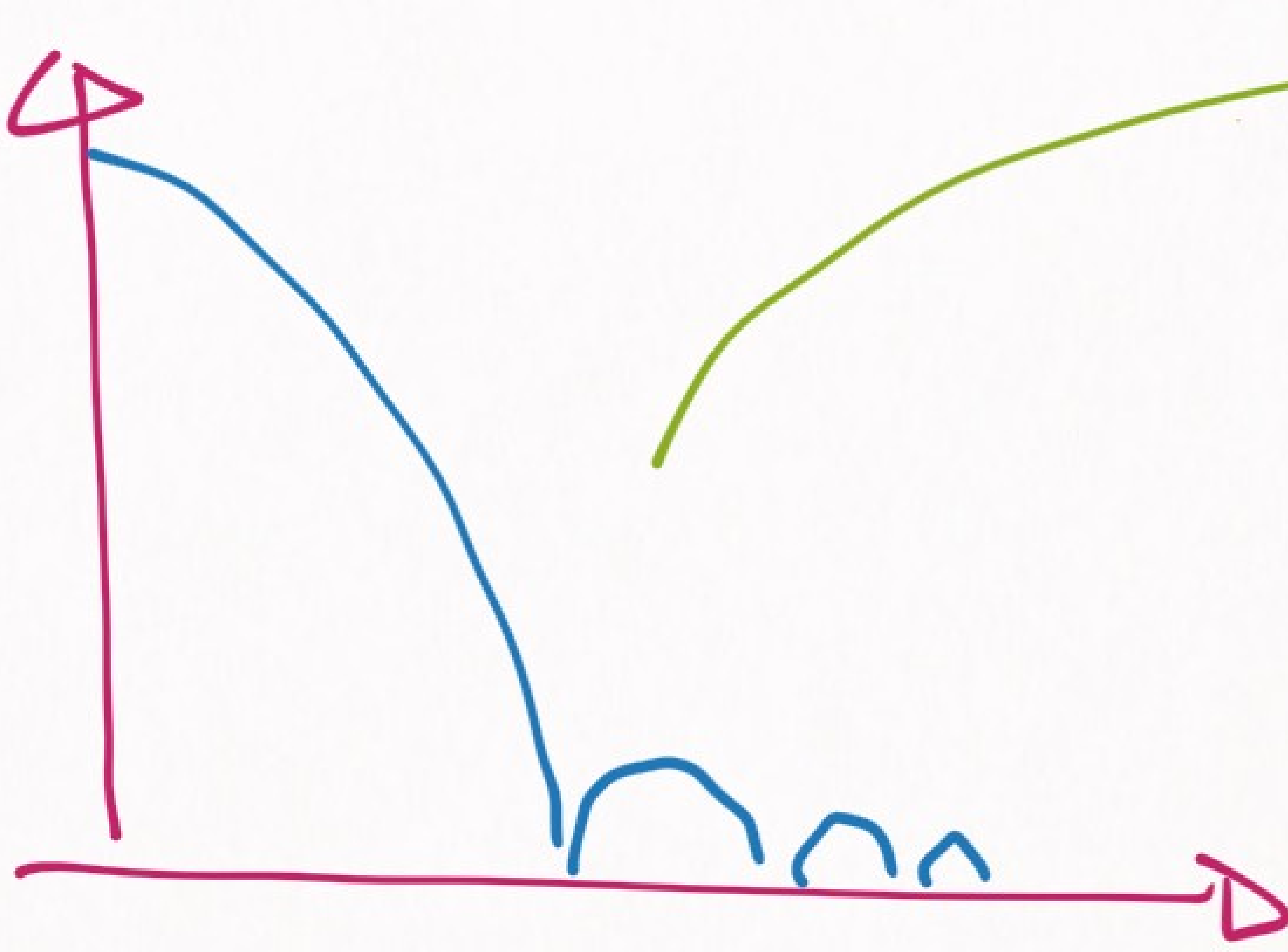
$$F(\vec{q}^{\,2}) = \int d^3\vec{r}\, \underbrace{|\psi(\vec{r})|^2}_{\rho(\vec{r})}\, e^{-i\vec{q}\cdot\vec{r}}$$

A few possible examples:

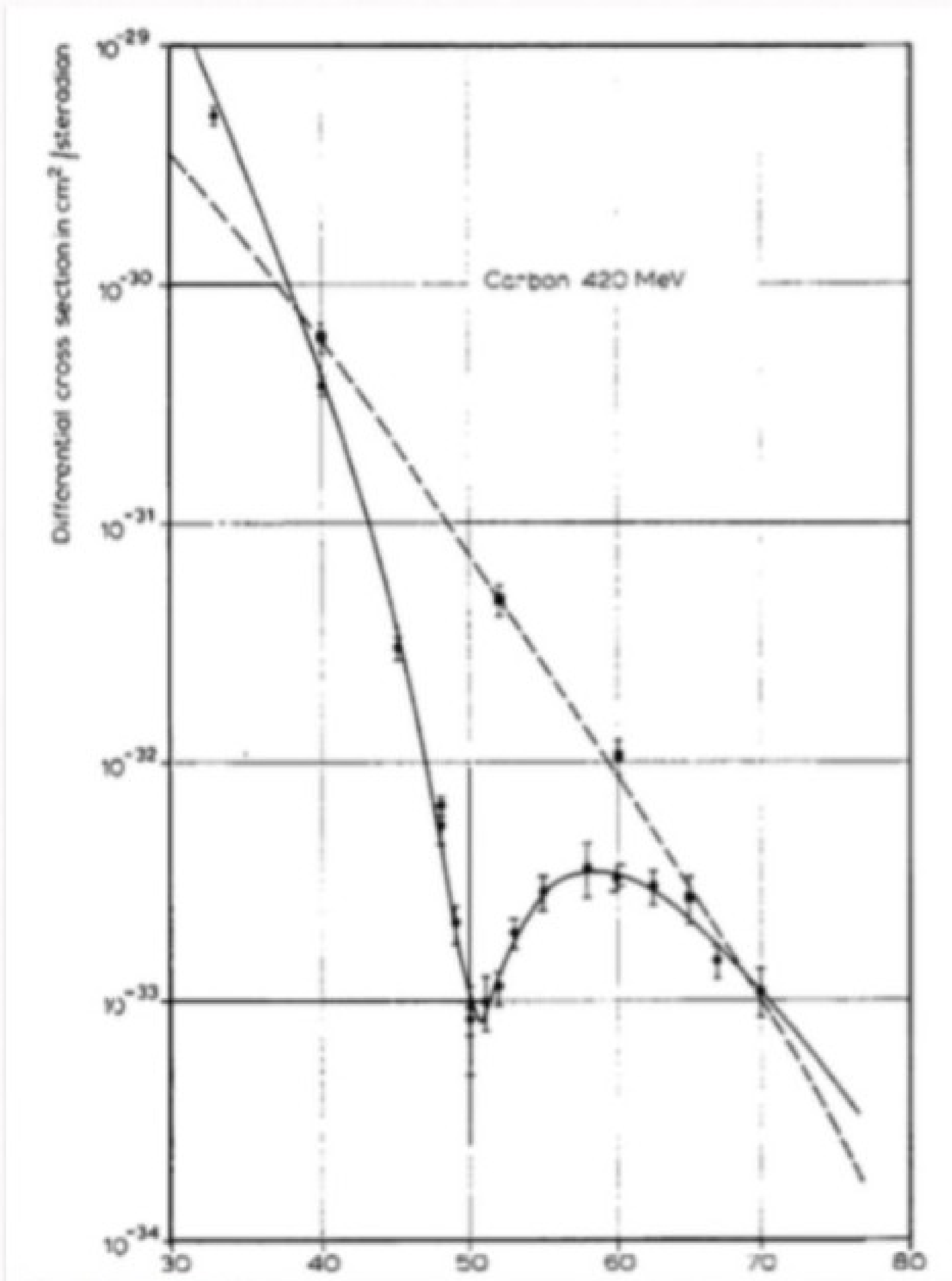


taken from ref. [2]

Experiment:

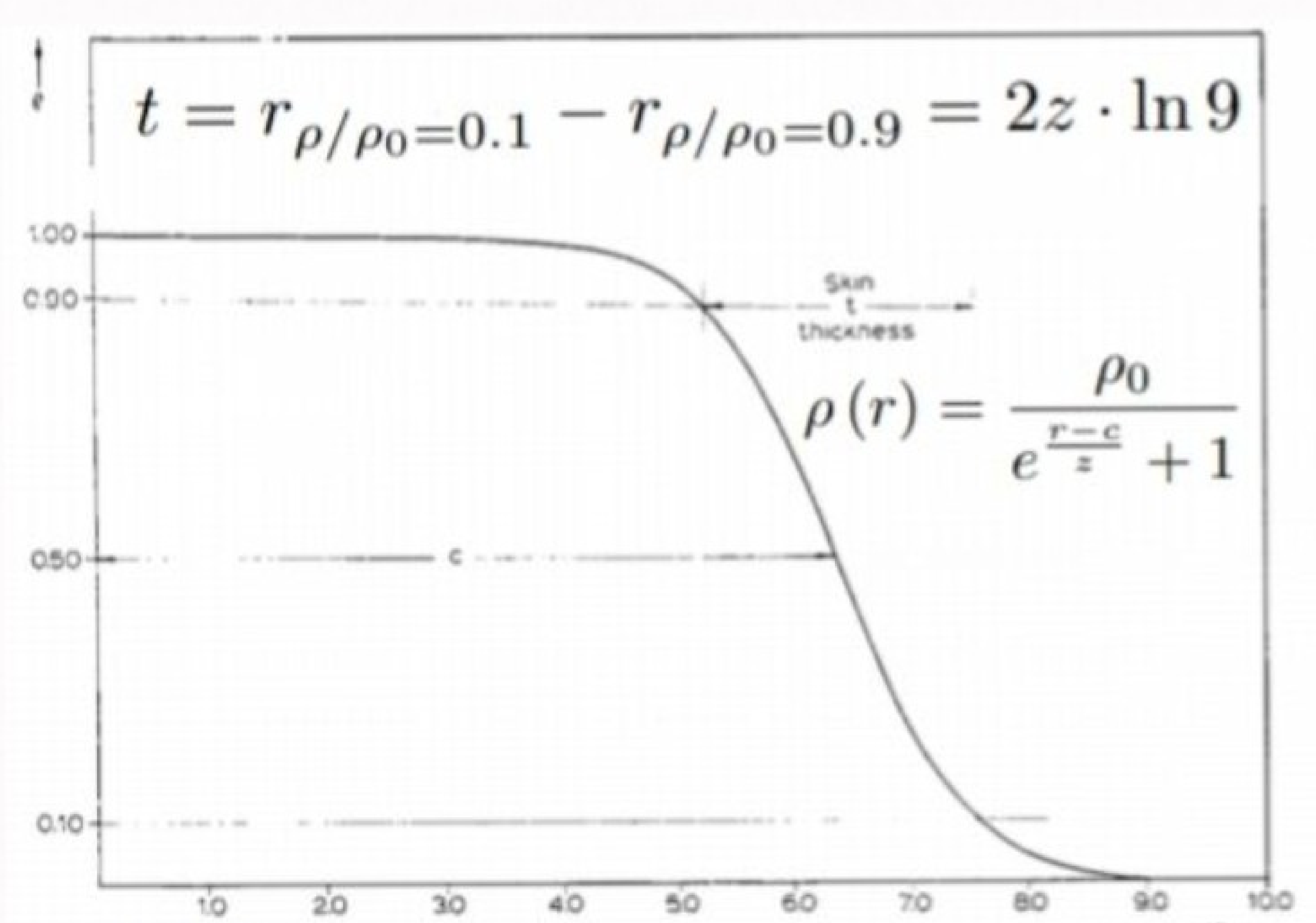


In particular Hofstadter found



taken from ref. [5]

from which he deduced



taken from ref. [5]

Woods-Saxon distribution

Typical mass distribution:

$$\rho(r) = \frac{\rho_0}{1 + e^{(r-R_0)/a_0}}$$

w/ $\rho_0 \simeq 0.17\ \mathrm{fm}^{-3}$

$a_0 \simeq 0.54\ \mathrm{fm}$

$R_0 \simeq 1.128\, A^{1/3}\ \mathrm{fm}$

Nuclear properties:

3) Angular momentum & Parity

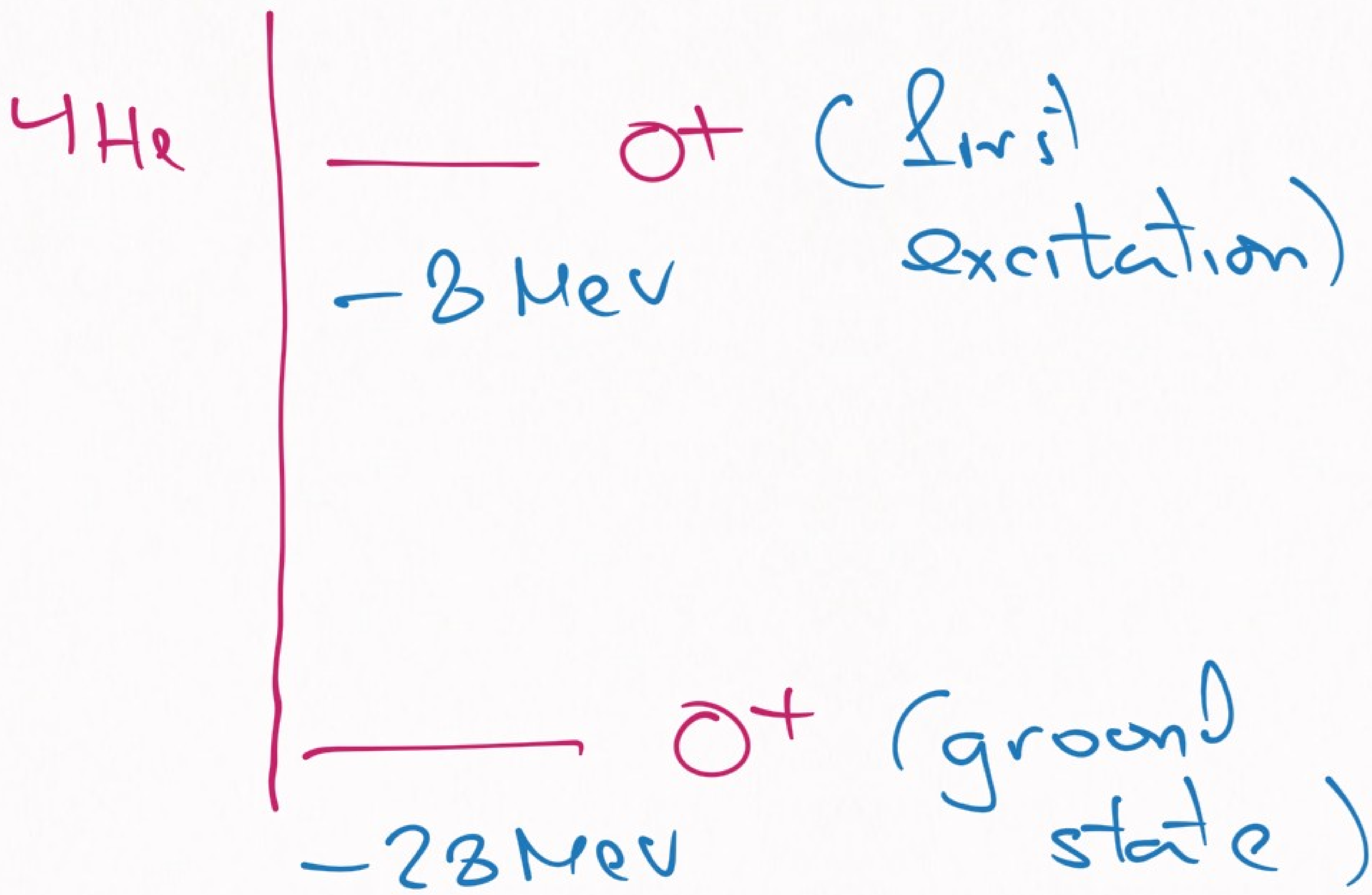
$J^P(\pi) = 0^-$ $J^P(\rho) = 1^+$

$J^P(\sigma) = 0^+$

$$\boxed{\begin{array}{l} J^P(N) = \frac{1}{2}^+ \quad N = n, p \\ J^P(d) = 1^+ \\ J^P({}^3H/{}^3He) = \frac{1}{2}^+ \\ J^P({}^4He) = 0^+ \end{array}}$$

And so on...

Most nuclei have excited states



$\Rightarrow$ Explaining J^P of excited states will be a very important part of nuclear models

Excited states J^π and energy
related to nuclear structure:

1) ^4He $(0^+/0^+)$

↳ "Breathing" mode of collective model

2) ^{41}Ca

— $3/2^+$
— $3/2^-$
— $7/2^-$

} Typical shell model spectrum

3) ^{122}Te

≡ $0^+, 2^+, 4^+$
— 2^+
— 0^+

} vibrational spectrum

4) ^{166}Er

— 6^+
— 4^+
— 2^+
— 0^+

} rotational spectrum

We will study all this
in the next few lessons

Next lesson:

→ Electromagnetic properties
of nuclei