Nuclear Physics 22

The three-body problem

Part 2: the triton & the existence of three-body forces

3-nucleon bound states:

1) Triton $\boxed{nnp}$

$B(^3H) = 8.48$ MeV

2) ^3He $\boxed{npp}$

$B(^3He) = 7.72$ MeV

Related by isospin symmetry:

$|^3H\rangle = |nnp\rangle = |\tfrac{1}{2}\ -\tfrac{1}{2}\rangle_I$

$|^3He\rangle = |npp\rangle = \underbrace{|\tfrac{1}{2}\ +\tfrac{1}{2}\rangle_I}_{\text{isospin wave functions}}$

Besides, nucleons have also spin...

$\Rightarrow$ How to write the Faddeev equation?

$\downarrow$

We have to account for the additional quantum numbers

(spin, isospin)

1) Only spatial coordinates

$$\psi_{3B} = \psi^{(1)}(\vec{x}_{23}, \vec{p}_1) + \psi^{(2)}(\vec{x}_{13}, \vec{p}_2) + \psi^{(3)}(\vec{x}_{12}, \vec{p}_3)$$

2) Adding other quantum numbers

$$\psi_{3B} = \sum_{\alpha, j} \psi_\alpha^{(j)} |\alpha^{(j)}\rangle$$

$j = 1, 2, 3$

α some combination of new quantum numbers

(wave function)

(Faddeev equations)

1) Only spatial coordinates

$$\begin{pmatrix} \psi^{(1)} \\ \psi^{(2)} \\ \psi^{(3)} \end{pmatrix} = \begin{pmatrix} 0 & G_0 T_{23} & G_0 T_{23} \\ G_0 T_{13} & 0 & G_0 T_{13} \\ G_0 T_{12} & G_0 T_{12} & 0 \end{pmatrix} \begin{pmatrix} \psi^{(1)} \\ \psi^{(2)} \\ \psi^{(3)} \end{pmatrix}$$

2) Additional quantum numbers

$$\begin{pmatrix} \psi^{(1)}_{\alpha} \\ \psi^{(2)}_{\alpha} \\ \psi^{(3)}_{\alpha} \end{pmatrix} = \begin{pmatrix} 0 & G_0 T^{(\alpha)}_{23} & G_0 T^{(\alpha)}_{23} \\ G_0 T^{(\alpha)}_{13} & 0 & G_0 T^{(\alpha)}_{13} \\ G_0 T^{(\alpha)}_{12} & G_0 T^{(\alpha)}_{12} & 0 \end{pmatrix}$$

$$\times \begin{pmatrix} \sum_{\beta} \langle \alpha^{(1)} | \beta^{(1)} \rangle \psi^{(1)}_{\beta} \\ \sum_{\beta} \langle \alpha^{(2)} | \beta^{(2)} \rangle \psi^{(2)}_{\beta} \\ \sum_{\beta} \langle \alpha^{(3)} | \beta^{(2)} \rangle \psi^{(3)}_{\beta} \end{pmatrix}$$

(a bit more complex, yet doable)

How to apply this to the triton?

1) two nucleon system:

singlet $\rightarrow a_s = -27.4$ fm

triplet $\rightarrow a_t = 5.4$ fm

2) $m_\pi a_s \gg 1$

$m_\pi a_t \gg 1$

contact-range approximation possible

separable potentials

1) Wave functions

$$|N\rangle = |\tfrac{1}{2} m_S\rangle_S |\tfrac{1}{2} m_T\rangle_T \quad \Big] \; 1B$$

$$\otimes$$

$$\left.\begin{aligned} |NN(S)\rangle &= \psi_S(p)\, |00\rangle_S |1 m_T\rangle_T \\ |NN(T)\rangle &= \psi_T(p)\, |1 m_S\rangle_S |00\rangle_T \end{aligned}\right] 2B$$

$$\otimes$$

3B

$$|3N\rangle =$$

$$\psi_S(k_{23}, p_1)\, |0_{23} \otimes \tfrac{1}{2}\, 1\rangle_S |1_{23} \otimes \tfrac{1}{2}\, 1\rangle_T$$

$$+ (\text{permutations})$$

$$+ \psi_T(k_{23}, \bar{p}_1)\, |1_{23} \otimes \tfrac{1}{2}\, 1\rangle_S |0_{23} \otimes \tfrac{1}{2}\, 1\rangle_T$$

$$+ (\text{permutations})$$

$\longrightarrow$ general idea

Idea behind this:

$$\underbrace{\Pi_y |\psi_{3B}\rangle = -|\psi_{3B}\rangle}_{\text{antisymmetric (total) wave function}}$$

But:

$$\underbrace{\begin{aligned} \psi_S(-\bar{k},p) &= \psi_S(\bar{k},p) \\ \psi_T(-\bar{k},p) &= \psi_T(\bar{k},p) \end{aligned}}_{\text{symmetric spatial wave function}}$$

(S-wave solution)

(approximation)

2) Potential:

$$V_S = \lambda_S \, g(k') \, g(k)$$
$$V_T = \lambda_T \, g(k') \, g(k)$$

$$\Rightarrow T_S = \tau_S(z) \, g(k') \, g(k)$$
$$T_T = \tau_T(z) \, g(k') \, g(k)$$

$\Rightarrow$ In EFT ($\not\pi$) we can use $g(k) = e^{-(k/\Lambda)^{2n}}$,

$\lambda_S = C_S(\Lambda)$, $\lambda_T = C_T(\Lambda)$

$\Rightarrow$ But there are also pre-EFT separable potentials

The outcome:

$$a_S(p_3) = 2\tau_S(z_{1,2})\int \frac{d^3p_1}{(2\pi)^3} B_{31}(p_3,p_1)\left[+\frac{1}{4}a_S(p_1) - \frac{3}{4}a_T(p_1)\right]$$

$$a_T(p_3) = 2\tau_T(z_{1,2})\int \frac{d^3p_1}{(2\pi)^3} B_{31}(p_3,p_1)\left[-\frac{3}{4}a_S(p_1) + \frac{1}{4}a_T(p_1)\right]$$

Notation as in previous lesson:

$$\psi_S = \frac{g(k)\,a_S(p)}{\gamma^2+k^2+\frac{3}{4}p^2}$$ (separable potentials)

$$\tau_S(z) = \frac{2\pi}{\mu}\frac{1}{\frac{1}{a_S} - \sqrt{-mz} + \mathcal{O}(1/\Lambda)}$$

etcaetera ...

Triton & Efimov

1) $m_\pi a_s \simeq -16.6$
$m_\pi a_T \simeq 3.8$ } relatively large

approximation:

$m_\pi a_s \to \pm\infty$
$m_\pi a_T \to \pm\infty$] unitary limit

expected error:

$$\max\left\{\frac{1}{|m_\pi a_s|}, \frac{1}{|m_\pi a_T|}\right\} \sim 30\%$$

The point: in the unitary limit, the triton behaves exactly as a 3-boson system

[Triton too far from unitary limit to have Efimov excited states]

↳ not perfect match

↳ but still useful approximation

Effective theory of the triton

Paulo F. Bedaque (Washington U., Seattle), H.W. Hammer (TRIUMF), U. van Kolck (Caltech, Kellogg Lab and Washington U., Seattle)

Jun 15, 1999

14 pages

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↳ Check it here

→

This idea can be extended to 4He (and maybe beyond)

Nuclear Physics Around the Unitarity Limit

Sebastian König (Ohio State U. and Darmstadt, Tech. Hochsch. and Darmstadt, EMMI), Harald W. Grießhammer (George Washington U.), H.W. Hammer (Darmstadt, Tech. Hochsch. and Darmstadt, EMMI), U. van Kolck (Orsay, IPN and Arizona U.)

Jul 15, 2016

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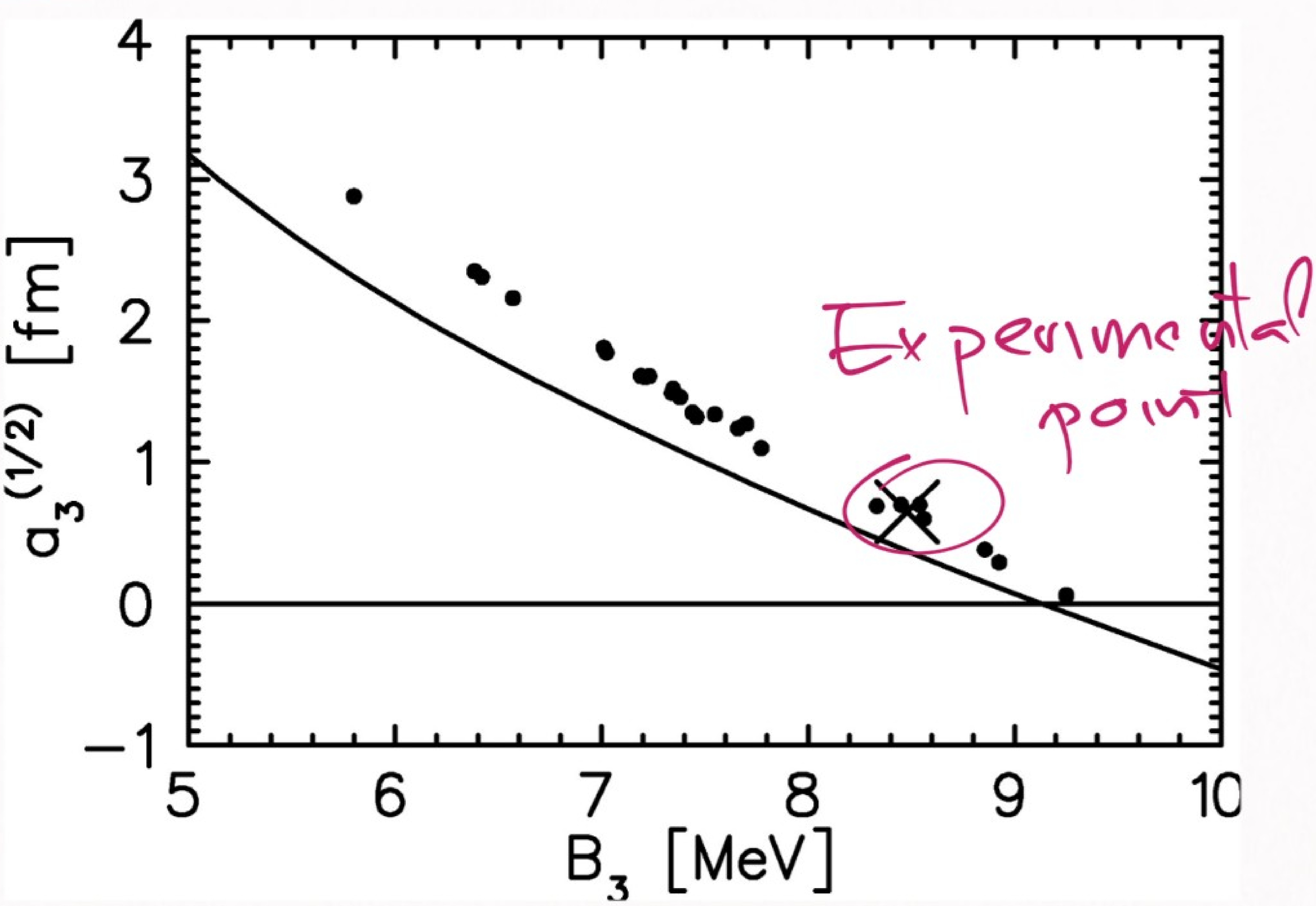
→ 2H; 3H; 3He; 4He explained w/ 1 parameter

Really impressive

The Phillips line

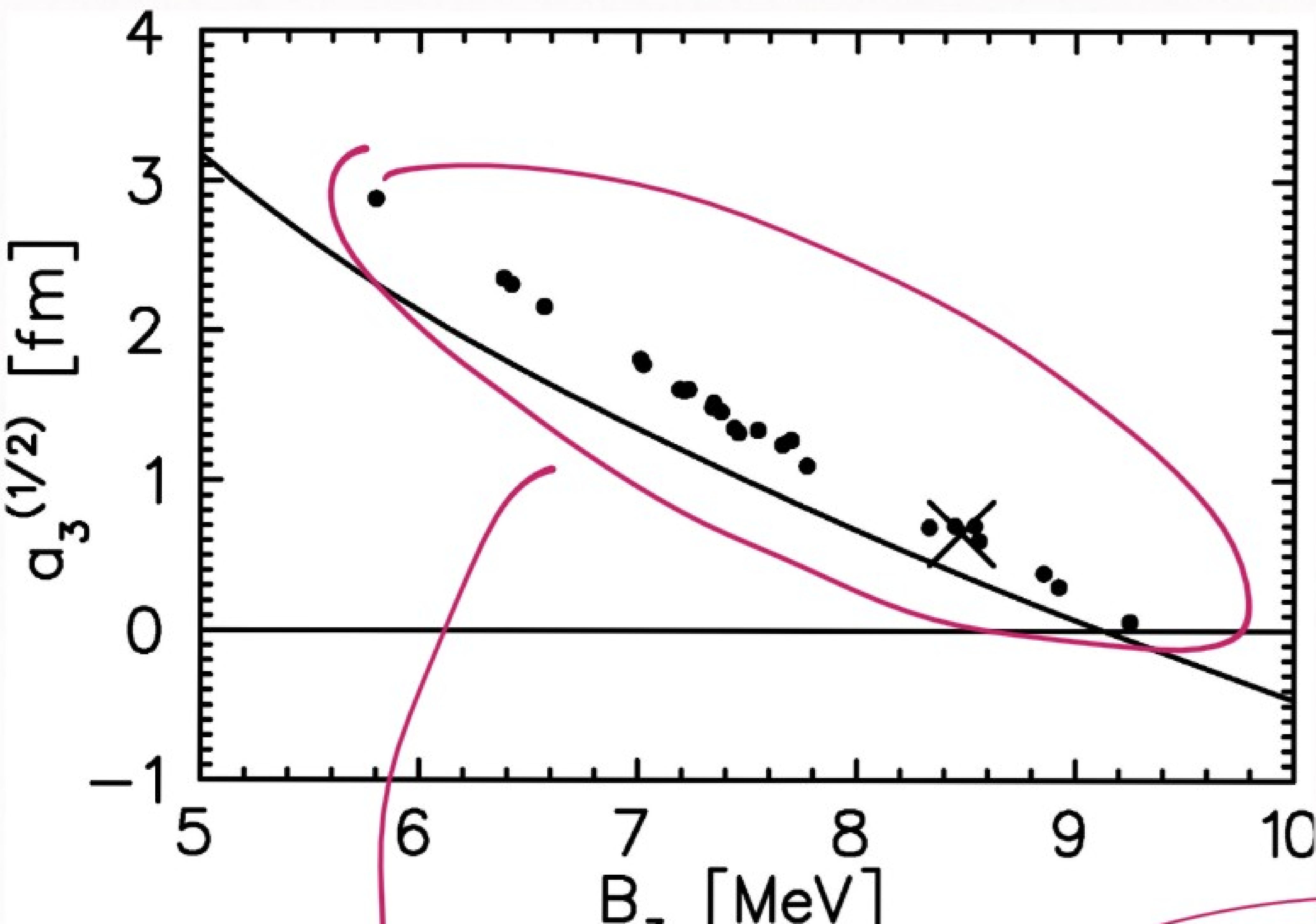
Idea: compute triton binding energy & n-deuteron scattering length w/ lots of potentials

Result:



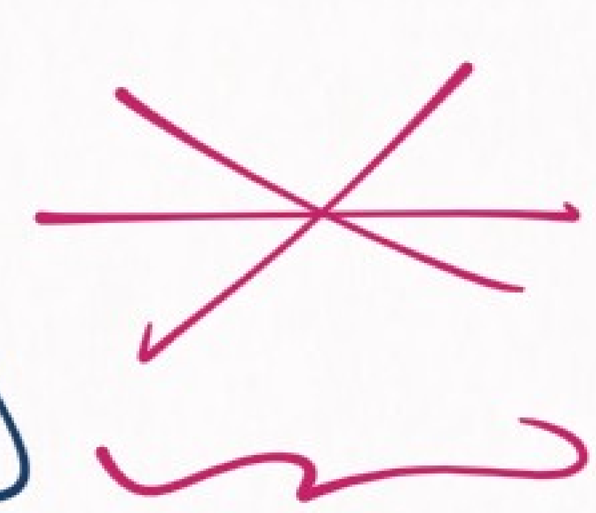
IMPORTANT → It's a line

But all these potentials get the two-body system right



Conclusion → something is missing

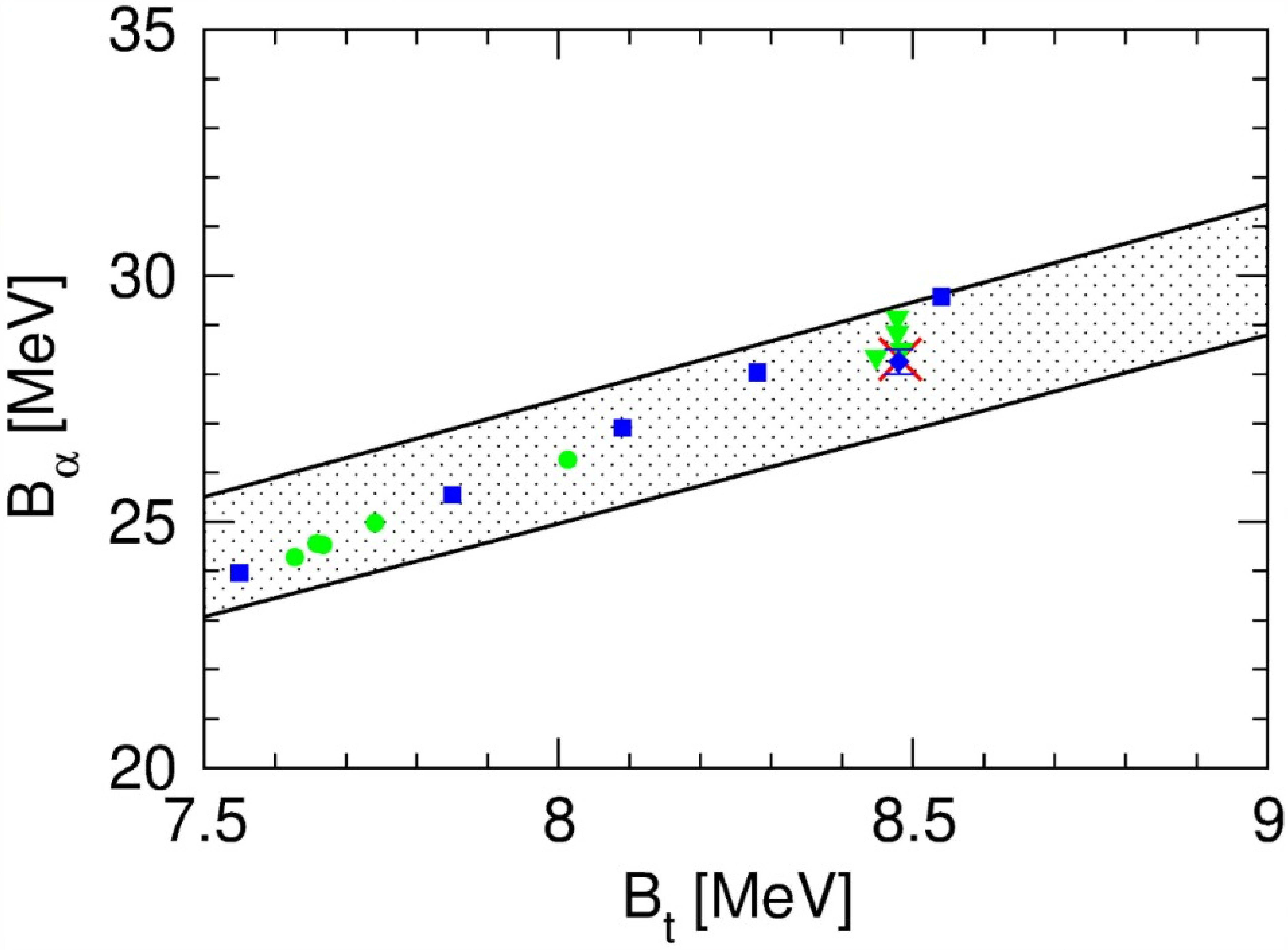
$\boxed{\exists \text{ 3-body force}}$



for example

(fixes the location in the line)

There's also a 4-body verson



The Tjon line

→ But also fixed by 3-body force

[4-body forces must be small]

RECAP

1) Triton close to unitary limit

→ Relation to Efimov
(but not Efimov effect)

→ Extends to ^4He

2) Phillips & Tjon lines

→ 3-body forces exist
and are necessary for
description of nuclei

→ 4-body forces much
less important than
3-body ones