Nuclear Physics 20

Nuclear forces from the Effective Field Theory perspective

Part 2 : pionful EFT

RECAP

DERIVATION OF NUCLEAR FORCES

QCD non-solvable at low energies

EFT approach

Expansion in $\frac{Q}{M}$ (depends on choice of Q & M)

Pionless
$Q \sim 1/a_0$
$M \sim m_\pi$

Pionful
$Q \sim m_\pi$
$M \sim m_\rho$

Pionful EFT

EFT(π)

1) Degrees of Freedom:

N, π (+ sometimes Δ's)

2) Symmetries: chiral symmetry

→ General form of the interaction

3) Renormalization

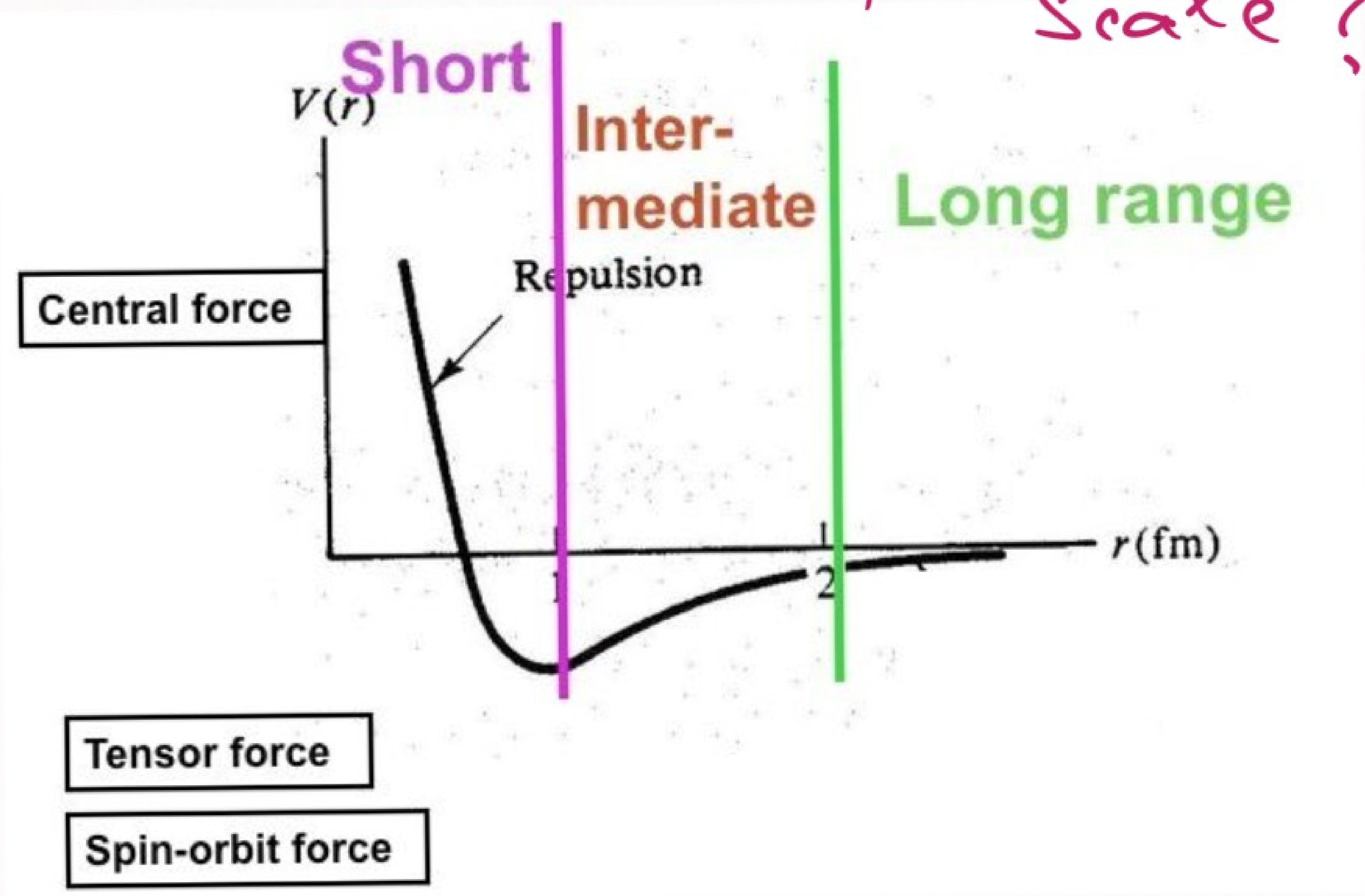
4) Power counting

(LO, NLO, N^2LO, ...)

→ The REALLY difficult part

(status: open problem)

$\boxed{\text{KEY POINT}}$ → Where is the separation scale?



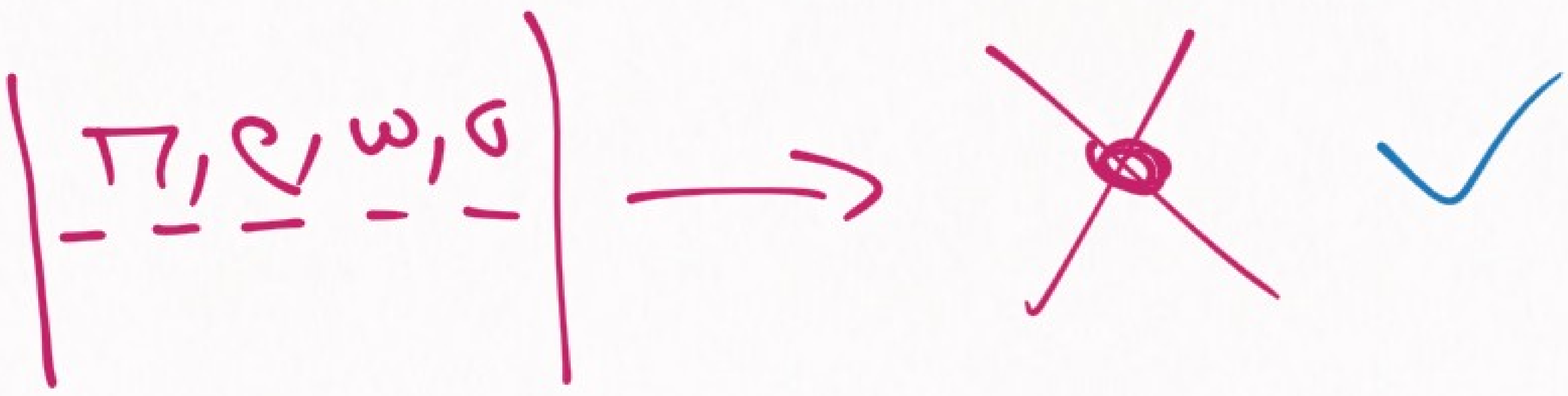
1) $R_s m_\pi \sim 1$ ⇒ $\boxed{\text{Pionless EFT}}$

⇒ $\boxed{\text{Only contact potential}}$

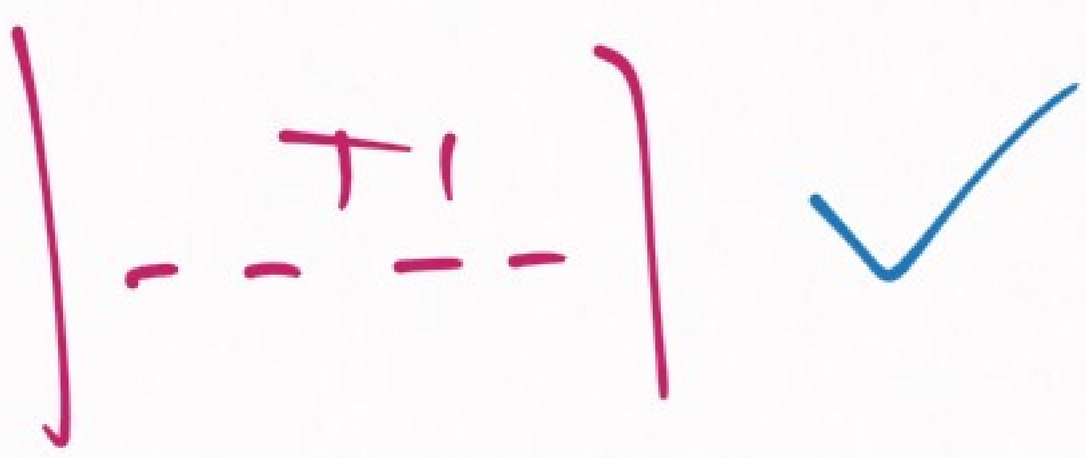
2) $R_s m_\rho \sim 1$ ⇒ $\boxed{\text{Pionful EFT}}$

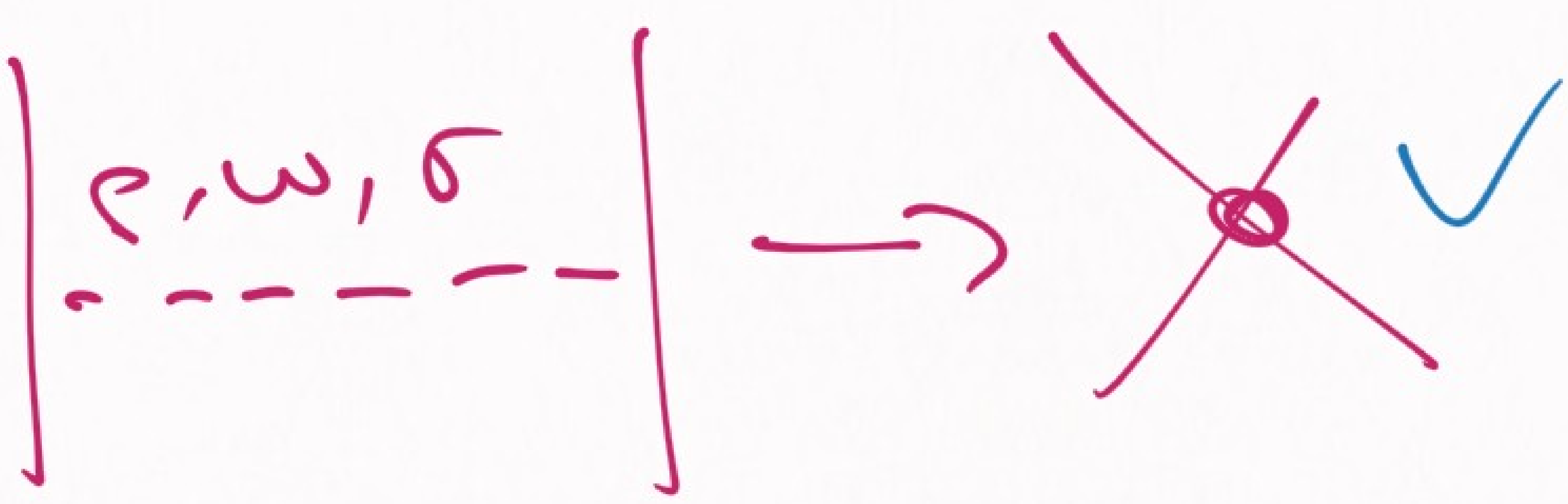
⇒ $\boxed{\text{Pions + contacts}}$

1) Pionless EFT:



2) Pionful EFT:





$\boxed{\text{1) \& 2)}}$ $\Rightarrow$

- ⊛ Same ideas
- ⊛ Different ingredients

A toor around EFT(π):

1) LO 

$$\boxed{V_{EFT} = \underbrace{V_c}_{\text{contact}} + \underbrace{V_F}_{\text{finite-range}}}$$

$$V_c^{LO} = C_S + C_T\, \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

$$V_F^{LO} = -\frac{g_a^2}{4 f_\pi^2}\, \vec{\tau}_1 \cdot \vec{\tau}_2\, \frac{\vec{\sigma}_1 \cdot \vec{q}\; \vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^{\,2} + m_\pi^2}$$

$$\hookrightarrow \boxed{V_{EFT}^{LO} = V_c^{LO} + V_F^{LO}}$$

Leading order EFT potential

→ $\boxed{\text{Regularization \& Renormalization}}$

→ Just as before

1) Momentum space :

$\langle \vec{p}' | V_{EFT}^{reg} | \vec{p} \rangle = \circledast$

$\circledast = \langle \vec{p}' | V_{EFT}^{unreg} | \vec{p} \rangle f\left(\frac{\vec{p}'}{\Lambda}\right) f\left(\frac{\vec{p}}{\Lambda}\right)$

$f(x) = e^{-x^{2n}}$ (popular choice)

$f(x) = \Theta(1-x)$

etc ...

$C_{2n} \rightarrow C_{2n}(\Lambda)$ for $\frac{d}{d\Lambda}[\ldots] \overset{!}{=} 0$

2) Coordinate space : $\boxed{\text{endless options}}$

2.a) Delta-shell

$$C_0 \delta(\vec{r}) \to C_0(R_c) \frac{\delta(r-R_c)}{4\pi R_c^2}$$

$$V_F(\vec{r}) \to V_F(\vec{r})\,\theta(r-R_c)$$

2.b) Gaussian

$$C_0 \delta(\vec{r}) \to C_0(R_c) \frac{e^{-(r/R_c)^{2n}}}{f(R_c)}$$

$$V_F(\vec{r}) \to V_F(\vec{r}) \left[1 - e^{-(r/R_c)^{2n}}\right]^m$$

Or any other choice ...

RECAP

Why do we renormalize?

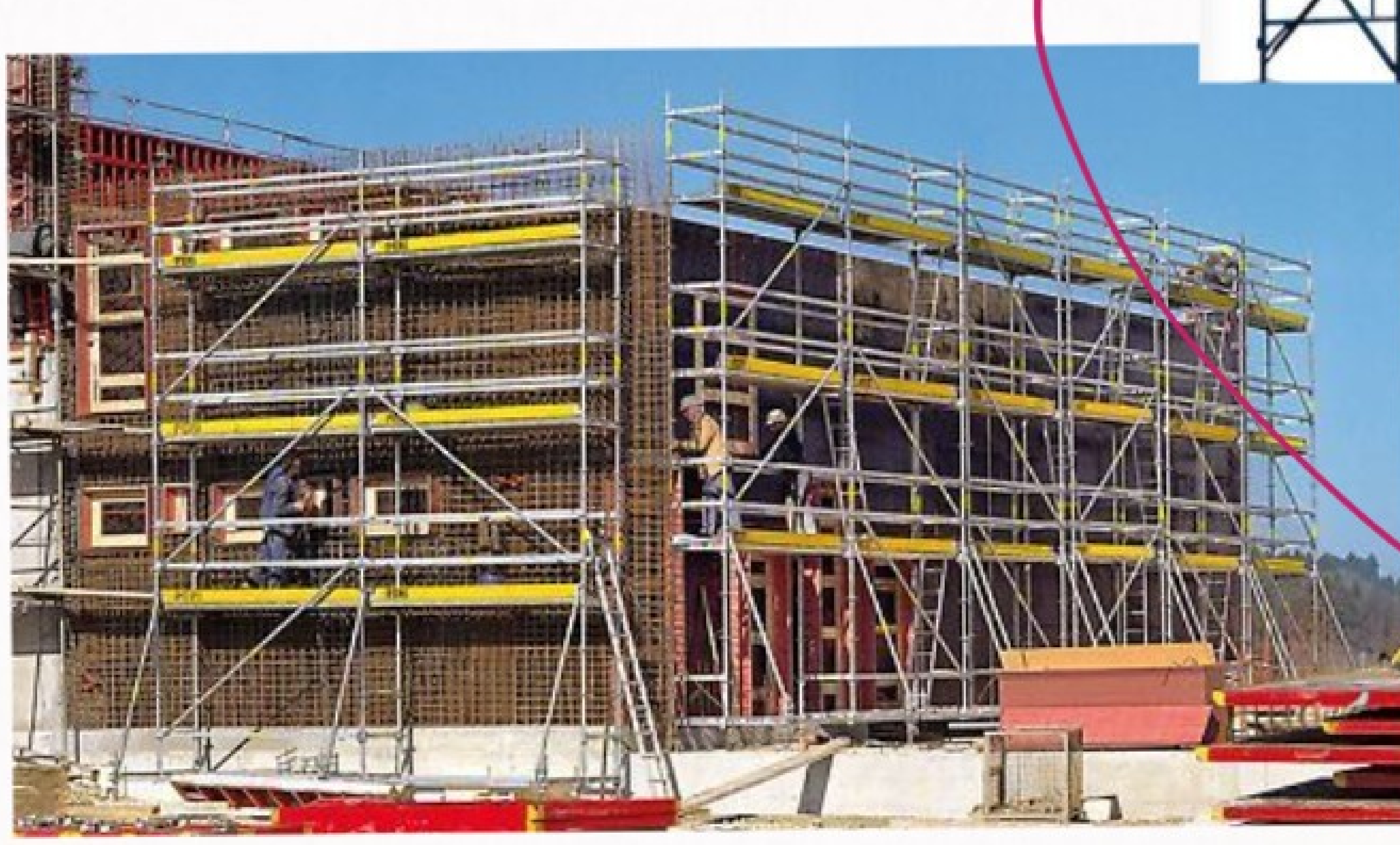
1) EFTs are based on a scale separation

1.a) $\boxed{Q, M}$ → M physical breakdow scale

1.b) $\boxed{\Lambda}$ (or R_c) → $\Lambda \neq M$, but a theoretical device (a tool)

$$\boxed{\frac{d}{d\Lambda}(\ldots) \simeq 0}$$

→ $\boxed{\Lambda}$



You don't want it once your theory is finished

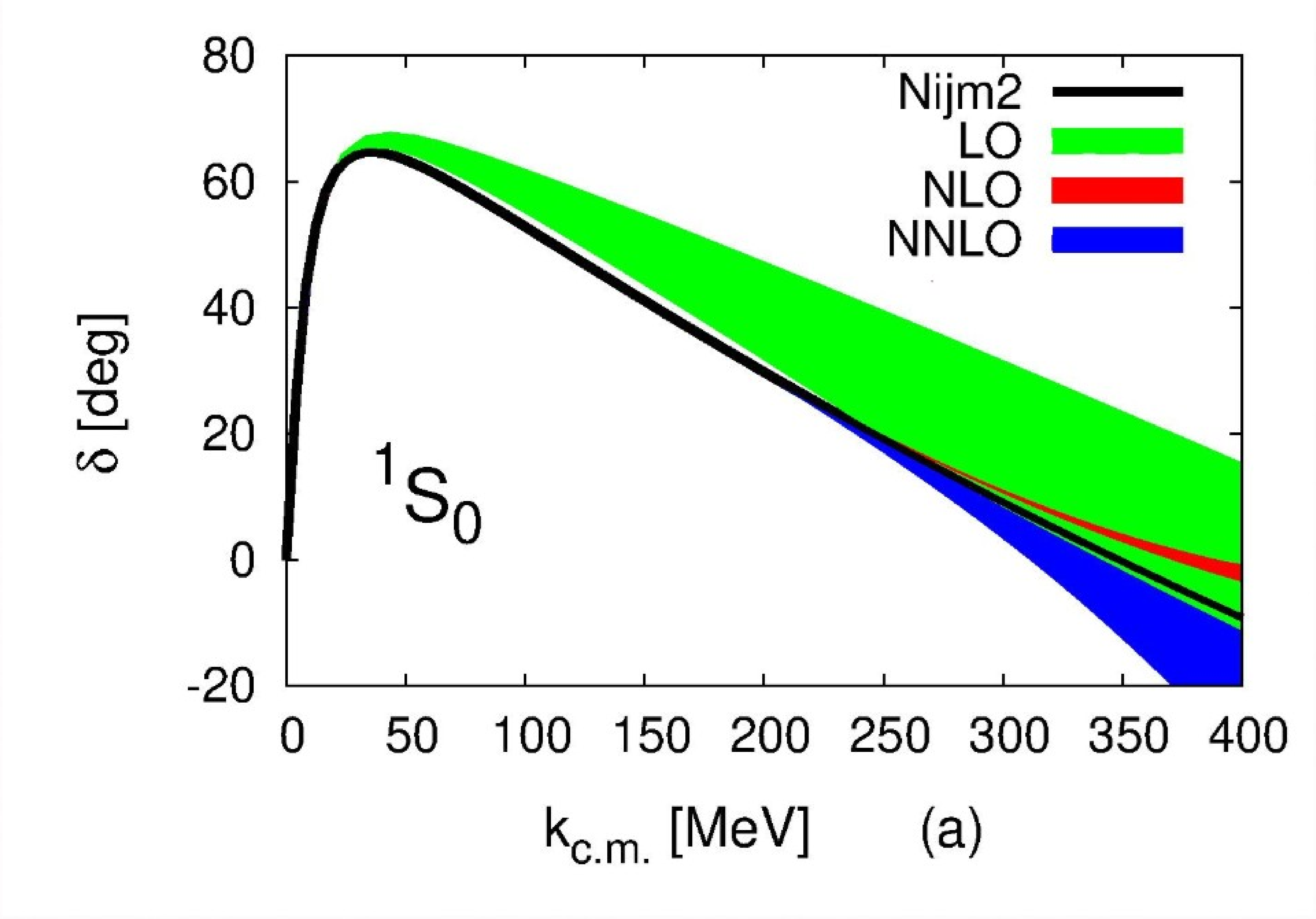
2) EFTs involve singular interactions

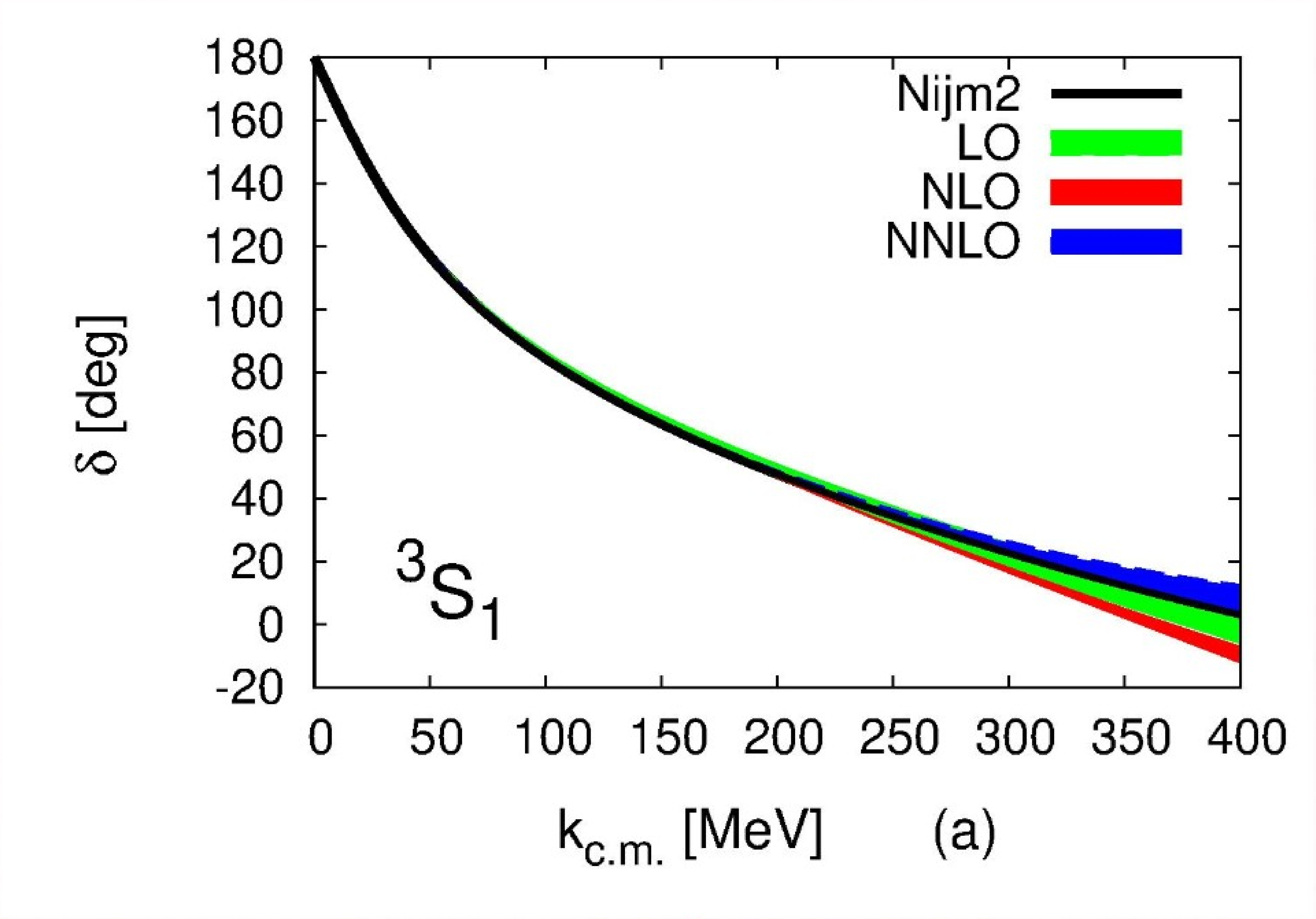
$$\underbrace{V_{EFT} = V_C + V_F}$$

singular ($\delta^{(3)}$-type) singular ($\frac{1}{r^n}$-type)

We want these two singular behaviours to cancel out

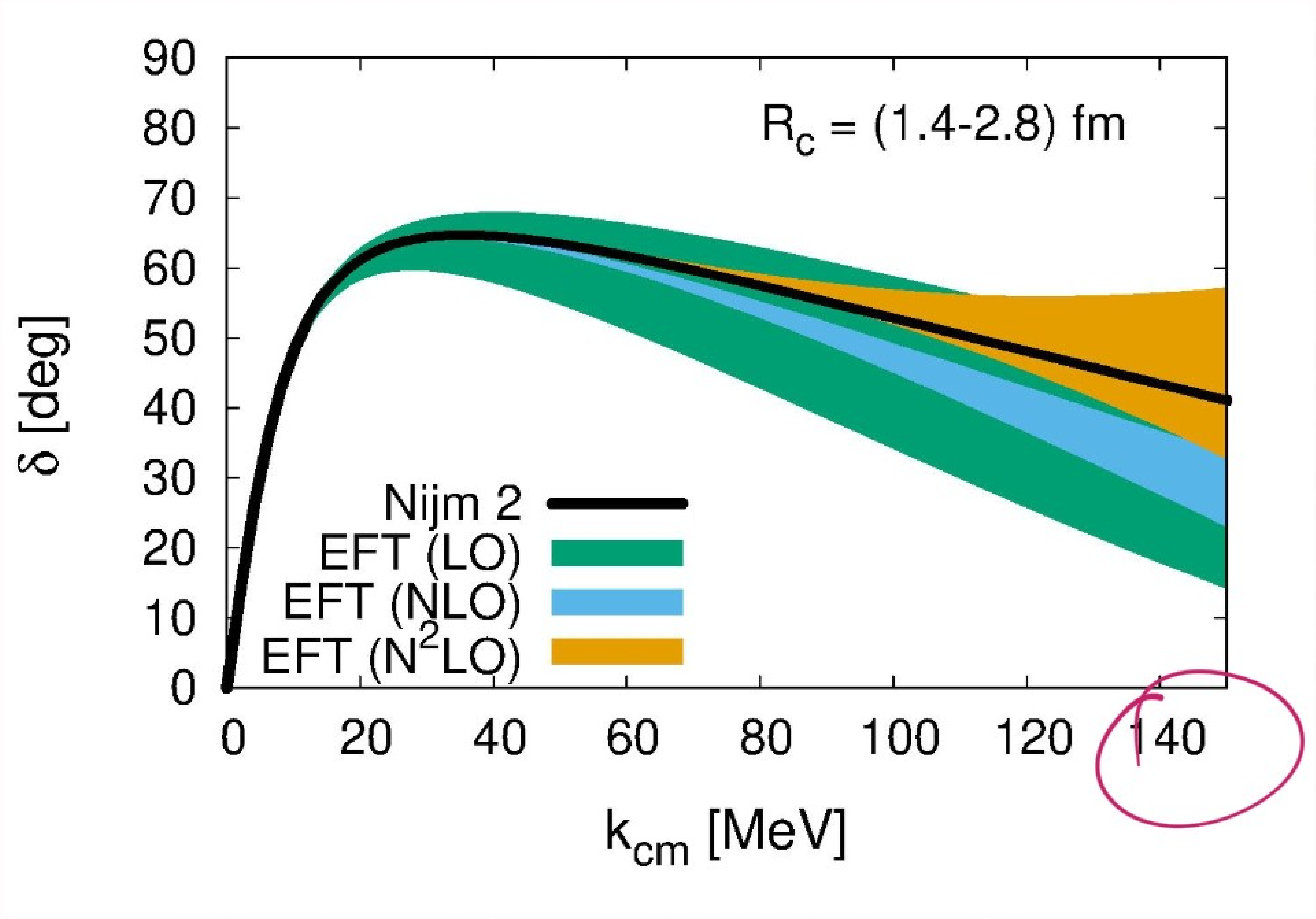
Let's see how this looks like

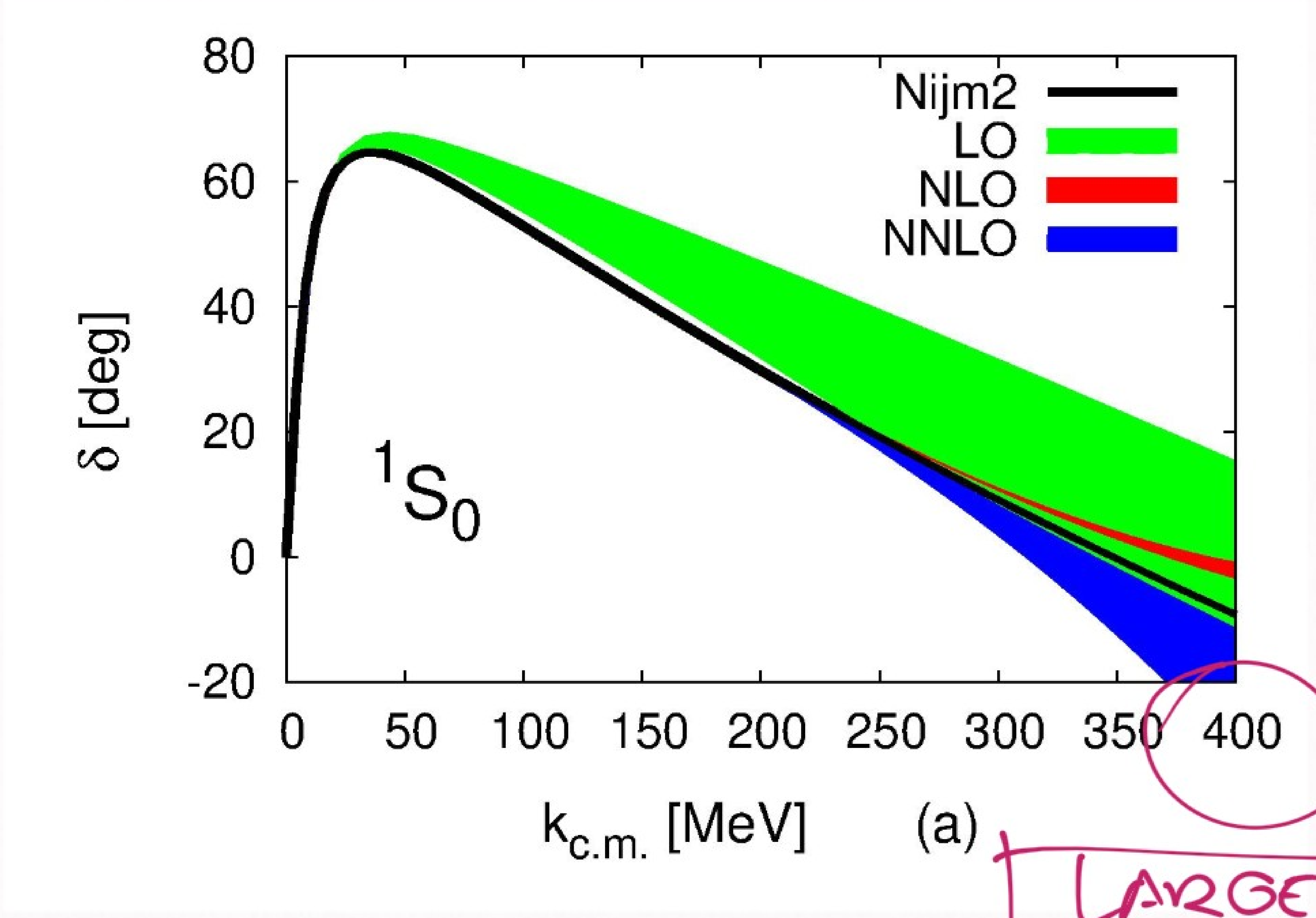




(R_c = 0.9 - 1.2 fm)

Comparison EFT(π̸) & EFT(π)





LARGER MOMENTA

But I have cheated:

LO, NLO, N^2LO, ...

Not explained yet (polemic topic)

only thing we have actually studied

Let's explain NLO, N^2LO, etc

Problem 1: $\underbrace{\text{NLO}, \text{N}^2\text{LO}}$?

not so clear

$\boxed{\text{Let's count powers!}}$

1) $V_{EFT} = V_C + V_F$

$g, m_\pi \sim Q$

2) $V_C^{LO} = C_0 \Rightarrow \boxed{Q^0}$

3) $V_F^{LO} = -\frac{g^2}{4f_\pi^2} \frac{\vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q}}{q^2 + m_\pi^2}$

$$\Rightarrow \boxed{\frac{Q^2}{Q^2} \sim Q^0}$$

Seems clear, right?

$\longrightarrow$ $\boxed{\text{Well, no!}}$

Let's see the contact:

1) $V_c = C_0(\Lambda)\,\delta_\Lambda(\vec{r})$

1.a) Perturbative system

$$C_0(\Lambda) = \frac{2\pi}{\mu} a_0$$

$a_0 \sim Q^0,\ \mu \sim Q^0$

$\boxed{C_0 \sim Q^0}$

1.b) Non-perturbative system

$$\frac{1}{C_0(\Lambda)} = \frac{\mu}{2\pi}\left(\frac{1}{a_0} - \frac{2}{\pi}\Lambda\right)$$

$a_0 \sim Q^{-1}$

$\boxed{C_0 \sim Q^{-1}}$

$\Rightarrow$ Power counting depends on the situation

Remember the Cup & Pot theory

Δ This was not a joke

$\boxed{\text{NDA}}$: $\underbrace{\text{naive dimensional analysis}}_{\text{Simplest counting possible}}$

$\downarrow$

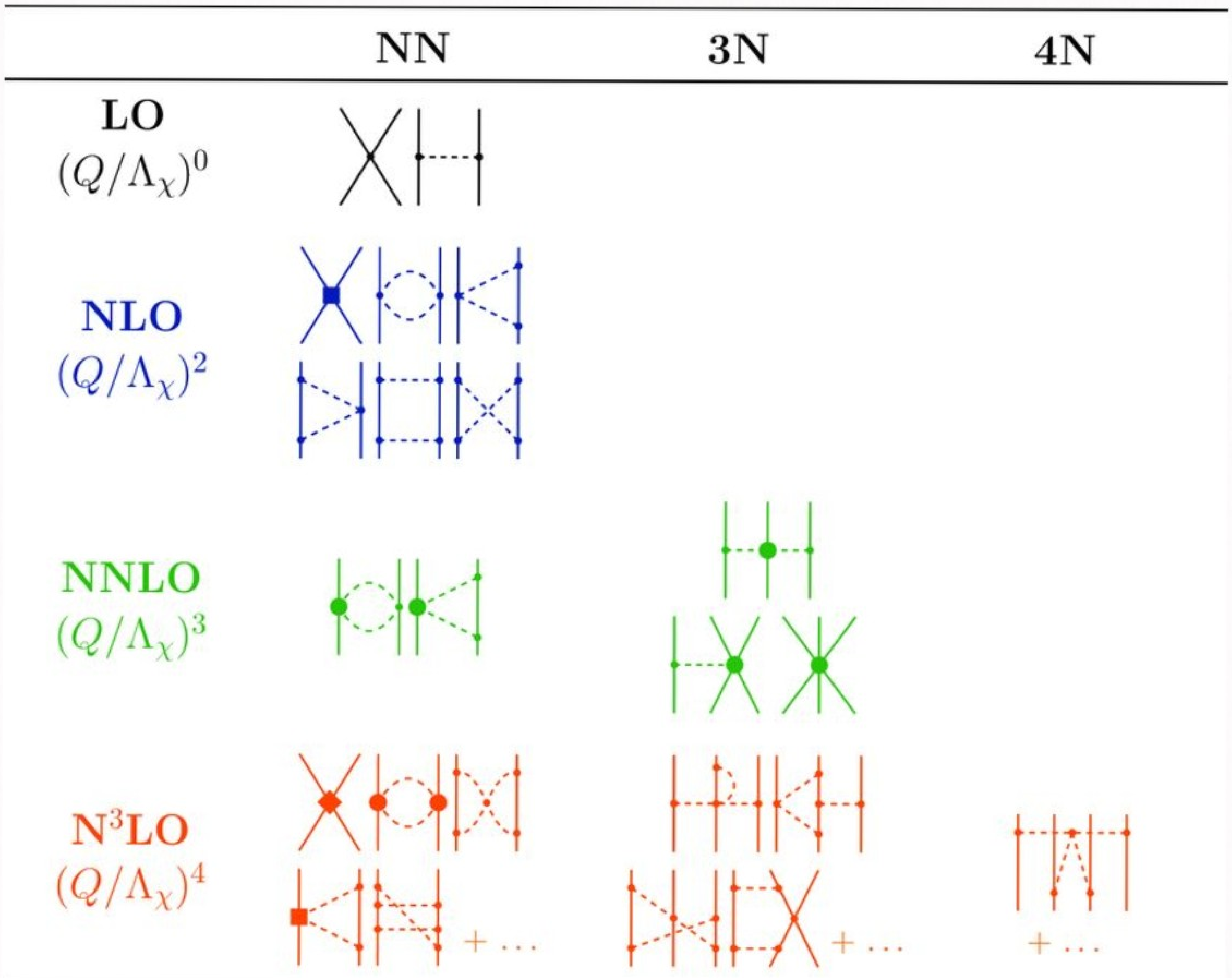
LO $\longrightarrow$ $\boxed{Q^0}$

 $\longrightarrow$ c_0 $\longrightarrow$ $\boxed{Q^0}$

 $\longrightarrow$ $\dfrac{\vec{\sigma}\cdot\vec{q}\;\vec{\sigma}\cdot\vec{q}}{q^2+m^2}$ $\longrightarrow$ $\boxed{Q^0}$

Now we continue to NLO,
N^2LO,
etc

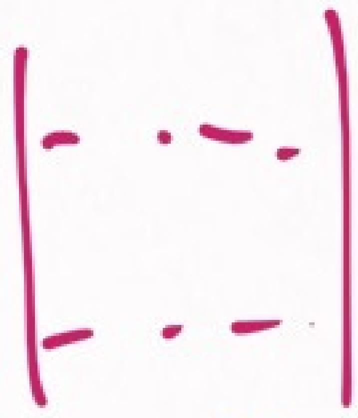
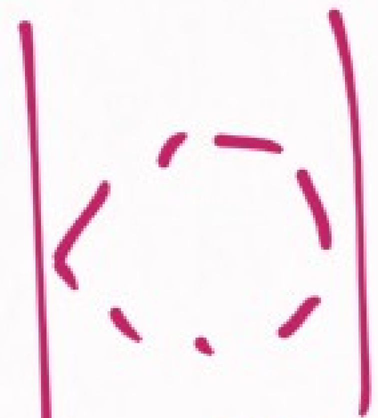
NDA looks like this :



↳ $\boxed{\text{NLO as example}}$

$$\boxed{\text{NLO}} \qquad V^{NLO}_{EFT} = V^{NLO}_{C} + V^{NLO}_{F}$$

$$V^{NLO}_{C} = \underbrace{C_2(p^2+p'^2)}_{\boxed{Q^2}} + \text{variations}$$

$V^{NLO}_{F} =$  +  +  + 

(= cross-box, box, football, triangle)

$$= W_C(\vec{q})\,\vec{\tau}_1\cdot\vec{\tau}_2 + V_S(\vec{q})\,\vec{\sigma}_1\cdot\vec{\sigma}_2$$

$$+ V_T(\vec{q})\,\vec{\sigma}_1\cdot\vec{q}\;\vec{\sigma}_2\cdot\vec{q}$$

W_C, V_S, V_T given by:

$$W_C(\boldsymbol{q}) = -\frac{L(q)}{192\pi^2 f_\pi^2}\left[4m_\pi^2\left(5g_A^4 - 4g_A^2 - 1\right) + q^2\left(23g_A^4 - 10g_A^2 - 1\right) + \frac{48g_A^4 m_\pi^2}{w(\boldsymbol{q})}\right],$$

$$V_T(\boldsymbol{q}) = -\frac{1}{q^2}V_S(\boldsymbol{q}) = -\frac{3g_A^3 L(q)}{32\pi^2 f_\pi^2},$$

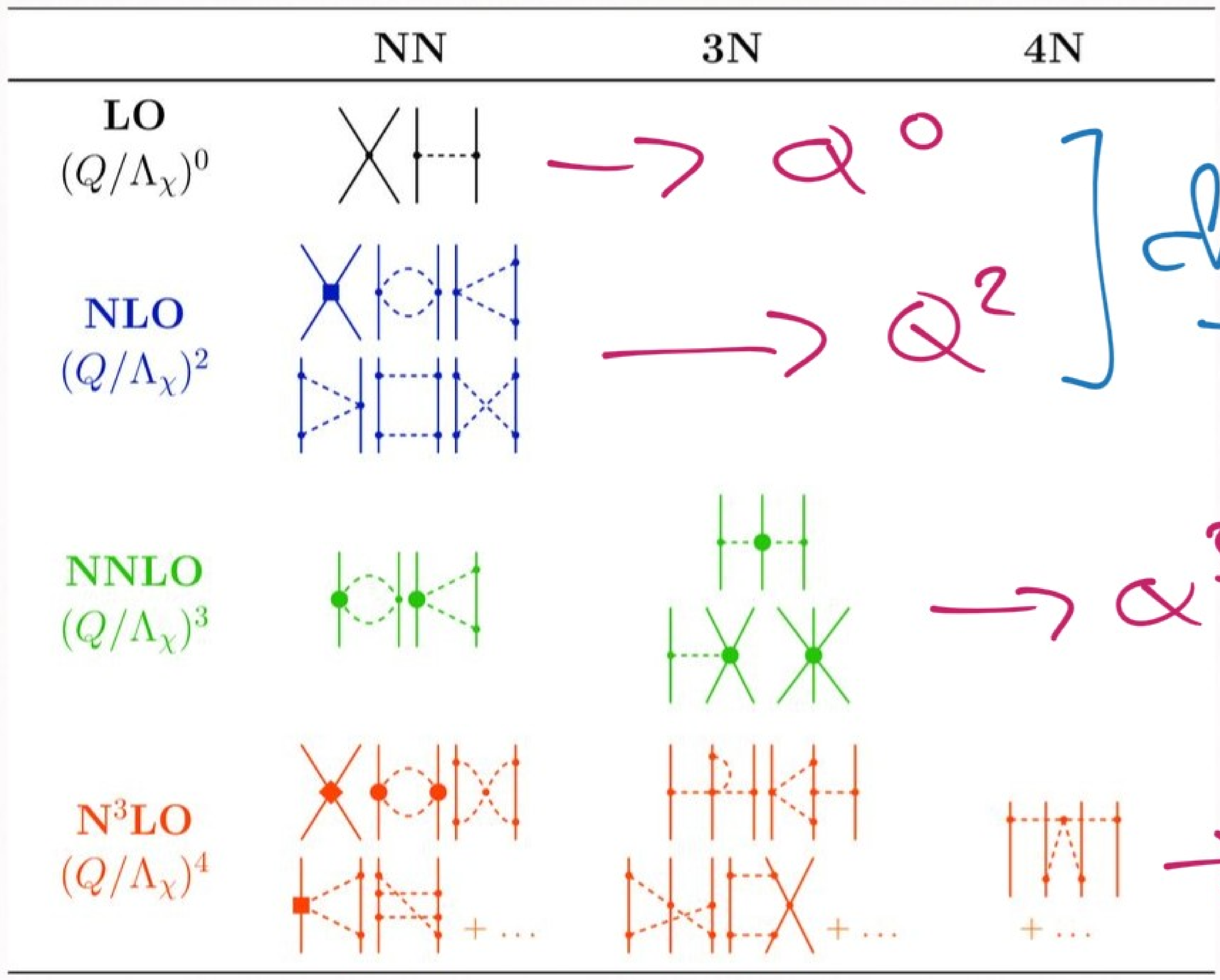
with $L(\underline{q}) = \frac{w(\underline{q})}{\underline{q}} \log\left[\frac{w(\underline{q}) + \underline{q}}{2m_\pi}\right]$

$w(\underline{q}) = \sqrt{4m_\pi^2 + \underline{q}^2}$

$\Rightarrow W_c\, \vec{\tau}_1 \cdot \vec{\tau}_2 \sim \boxed{Q^2}$

$V_S\, \vec{\sigma}_1 \cdot \vec{\sigma}_2 \sim \boxed{Q^2}$

$\underbrace{V_T}_{Q^0}\, \underbrace{\vec{\sigma}_1 \cdot \underline{q}\; \vec{\sigma}_2 \cdot \underline{q}}_{Q^2} \sim \boxed{Q^2}$



$\rightarrow Q^0$

$\rightarrow Q^2$

checked

$\rightarrow Q^3$

$\rightarrow Q^4$

($Q^1 \rightarrow$ no diagrams, because the possible diagrams are forbidden by symmetry)

$\boxed{\text{PROBLEM}}$

NDA fails for non-perturbative systems (e.g. nucleons)

⇓

If we use NDA, bad things will happen

⇓

$\boxed{\text{But most applications of EFT}(\not\pi)\text{ use NDA}}$

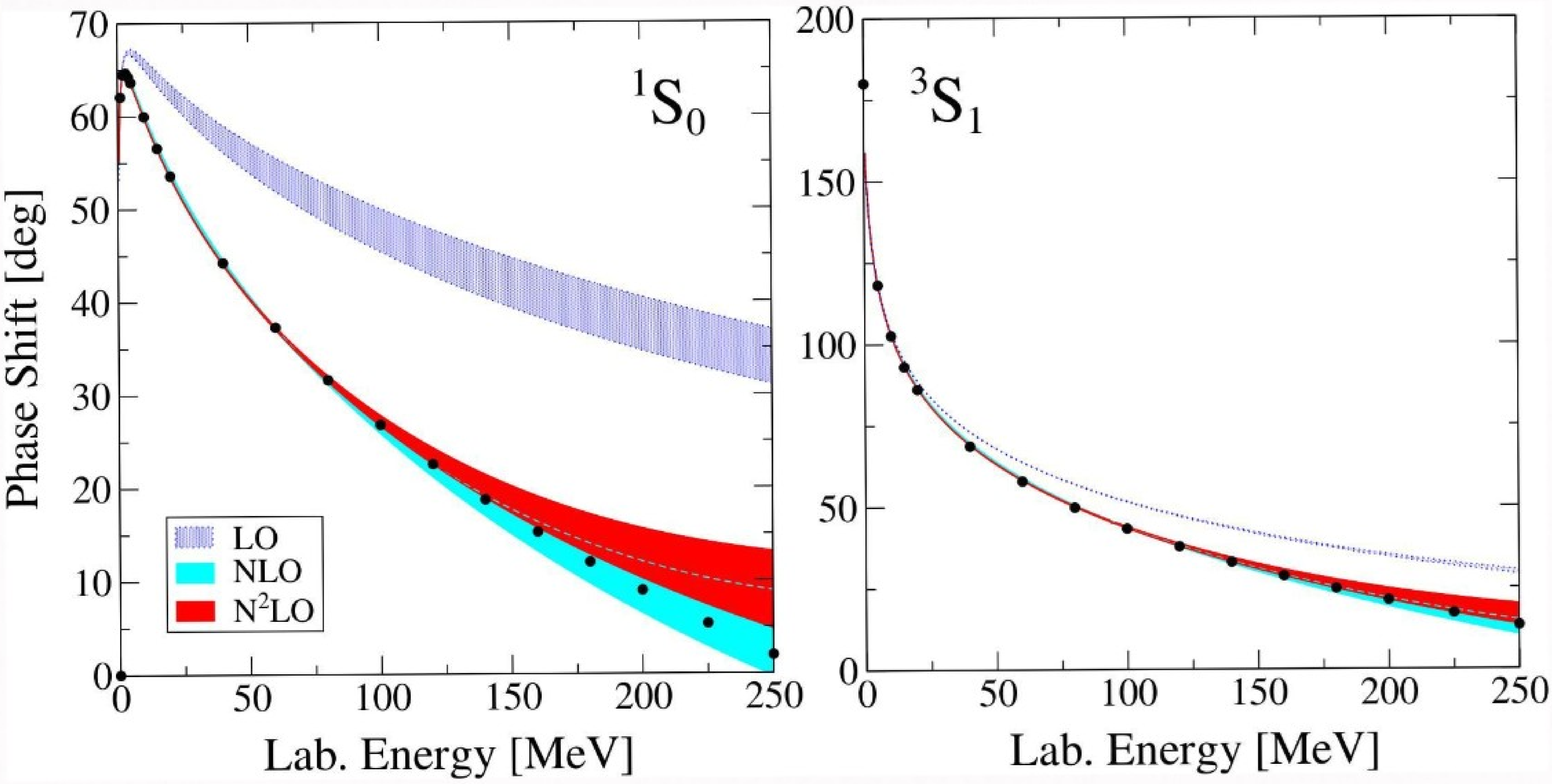
⇓

OOPS! → $\boxed{\text{Power Counting Wars}}$

1996 – TODAY

Standard EFT(π) applications based on NDA

↳ Easy & Convenient



But there is a price to pay:

$\boxed{\text{Can't take } R_c \to 0}$

↳ RG invariance broken

Problem discovered by these guys:

Renormalization of one-pion exchange and power counting

A. Nogga (Julich, Forschungszentrum), R.G.E. Timmermans (Groningen, KVI), U. van Kolck (Arizona U.)

Jun 2, 2005

19 pages

Published in: *Phys.Rev.C* 72 (2005) 054006

e-Print: nucl-th/0506005 [nucl-th]

DOI: 10.1103/PhysRevC.72.054006

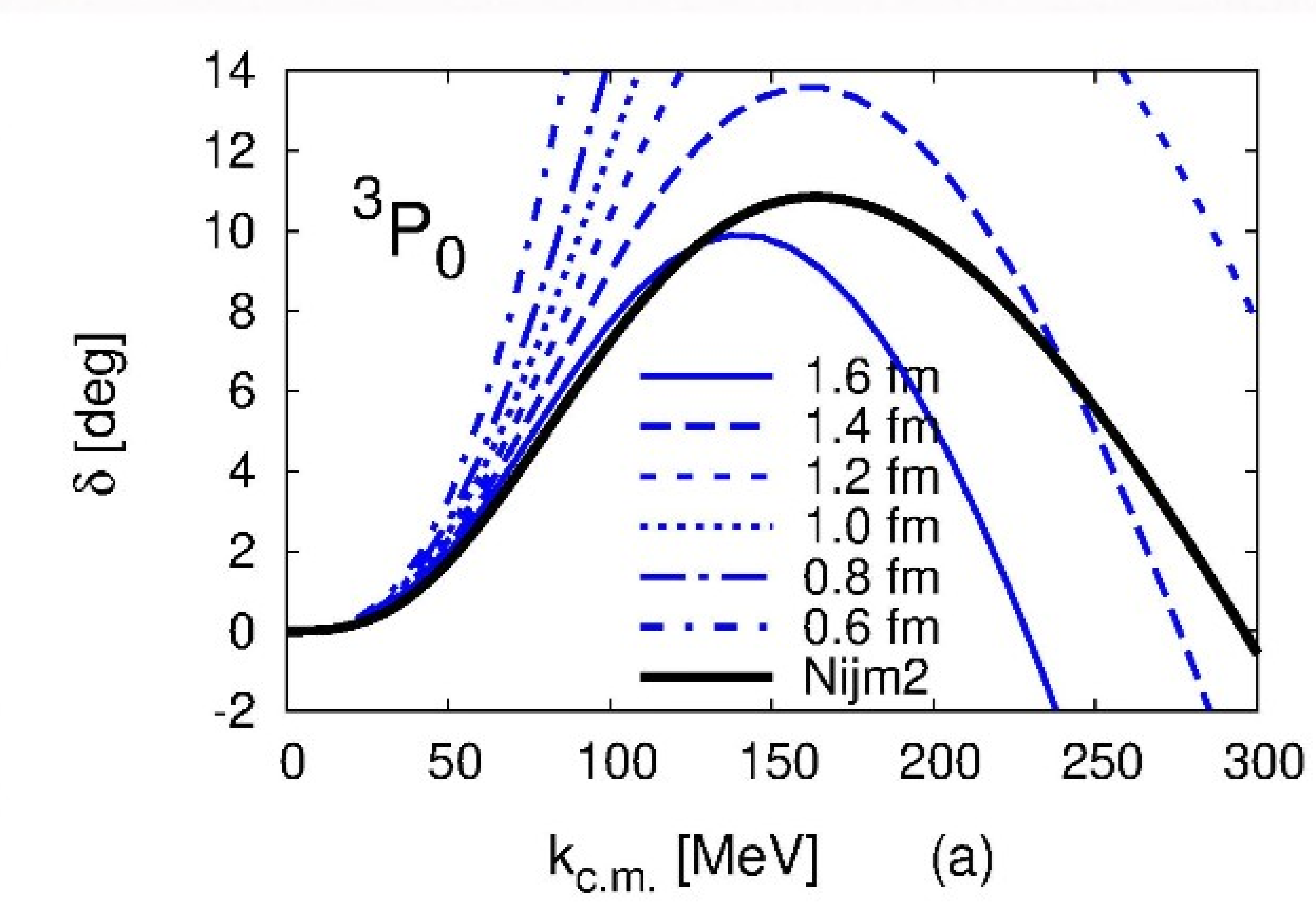
Report number: FZJ-IKP-TH-2005-19

View in: ADS Abstract Service

pdf   cite

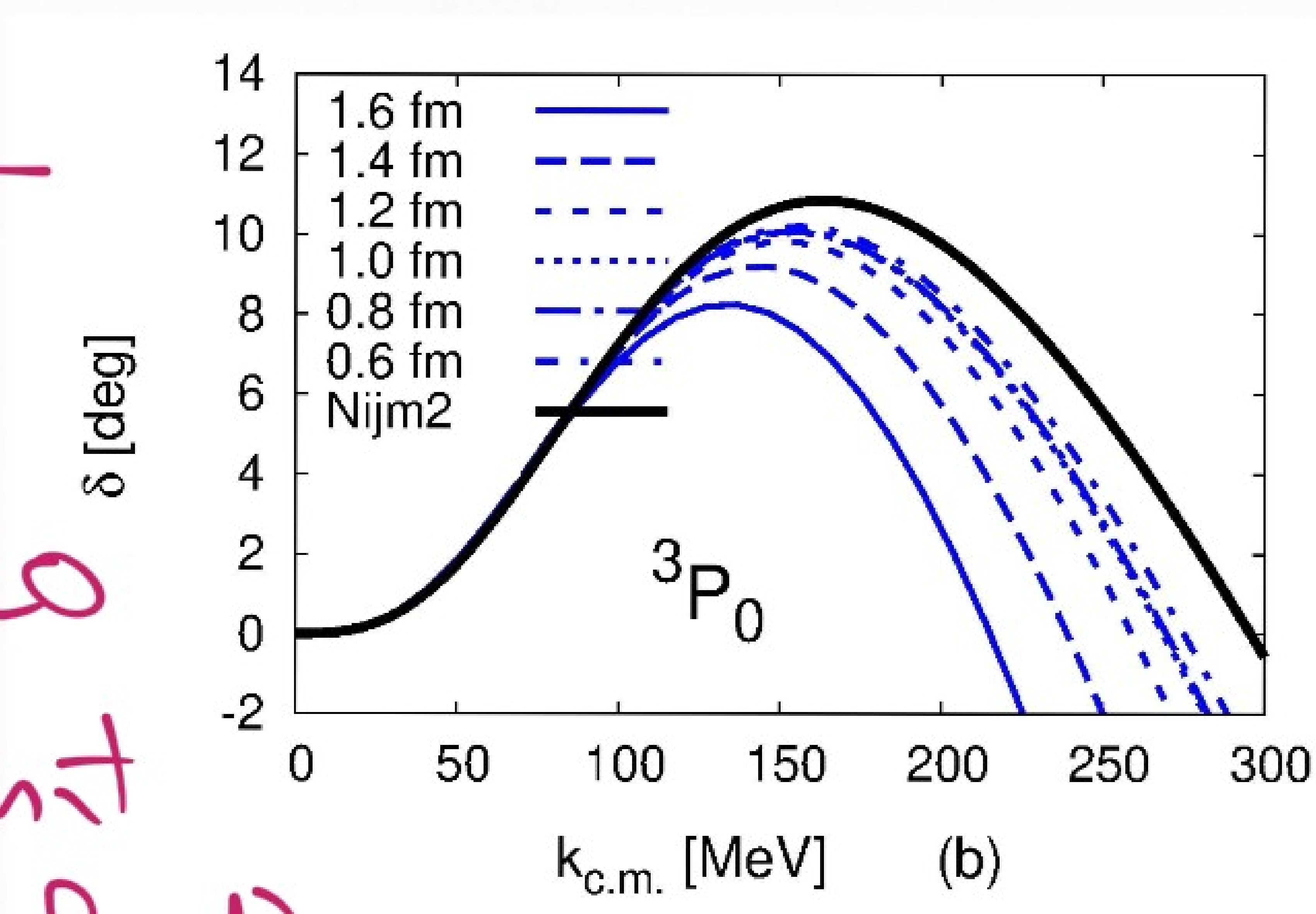
Classic paper in EFT(π)

EXAMPLE: $^3P_0$ phase shift at LO



→ NDA (no $R_c \to 0$ limit)

Include ← $C_1 \vec{p}\cdot\vec{p}\,'$ contact at LO ($R_c \to 0$ limit restablished)



$\boxed{\text{TAKE HOME MESSAGES}}$

1) Pionful theory is difficult

$\exists$ open problems

$\hookrightarrow$ good research topic

2) Power counting not unique

Pionless $\rightarrow$ $\frac{d}{dR_c}[C(R_c)] = 0$

$\frac{d}{dR_c}\left[\frac{C(R_c)}{R_c}\right] = 0$

from excercise set 1

3) Renormalization is not guaranteed to work

$\frac{d}{dR_c}[k \cot \delta_{EFT}] = 0$ requires hard work

CHALLENGES W/ RG INVARIANCE

$$V_{EFT} = V_{LO} + \underbrace{\sum_{\nu=0}^{\infty} V^{(\nu)}}_{\text{subleading corrections}}$$

1) Power counting implies:

$$\left(\sum_{\nu=0}^{\infty} V^{(\nu)}\right) << V_{LO}$$

subleading corrections are indeed subleading (small)

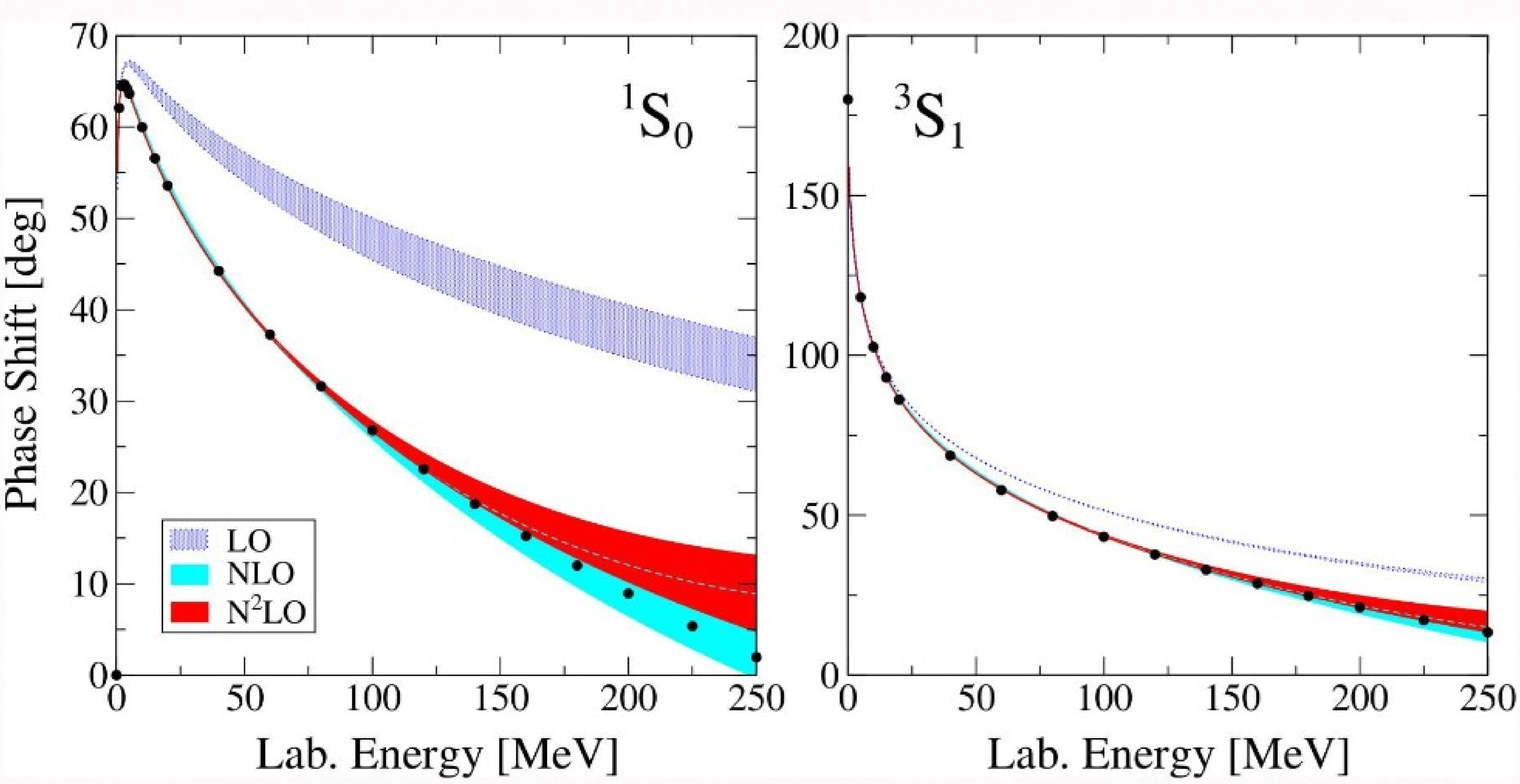
2) RG invariance requires that $\left(\sum_{\nu=0}^{\infty} V^{(\nu)}\right)$ is included as perturbations

$\rightarrow$ $\boxed{\text{DIFFICULT}}$

⇒ $\boxed{\text{Nowadays most implementations do not include subleading corrections as perturbations}}$

$\underbrace{\text{perturbations}}$ → technically difficult

like in this example →



Further reading: NDA (non-renormalizable)

1) The classic (Weinbergs counting)

Accurate charge dependent nucleon nucleon potential at fourth order of chiral perturbation theory #1

D.R. Entem (Idaho U. and Salamanca U.), R. Machleidt (Idaho U.) (Apr 8, 2003)

Published in: *Phys.Rev.C* 68 (2003) 041001 • e-Print: nucl-th/0304018 [nucl-th]

pdf DOI cite 1,194 citations

2) The other classic implementation

The Two-nucleon system at next-to-next-to-next-to-leading order #2

E. Epelbaum (Jefferson Lab), W. Glockle (Ruhr U., Bochum), Ulf-G. Meissner (Bonn U., HISKP and Julich, Forschungszentrum) (May 19, 2004)

Published in: *Nucl.Phys.A* 747 (2005) 362-424 • e-Print: nucl-th/0405048 [nucl-th]

pdf links DOI cite 596 citations

3) The easy-to-program potential

Local chiral effective field theory interactions and quantum Monte Carlo applications #2

A. Gezerlis (Guelph U.), I. Tews (Darmstadt, Tech. Hochsch. and Darmstadt, EMMI), E. Epelbaum (Ruhr U., Bochum), M. Freunek (Ruhr U., Bochum), S. Gandolfi (Los Alamos) et al. (Jun 2, 2014)

Published in: *Phys.Rev.C* 90 (2014) 5, 054323 • e-Print: 1406.0454 [nucl-th]

pdf DOI cite 139 citations

4) One of the latest implementations

Precision nucleon-nucleon potential at fifth order in the chiral expansion #8

E. Epelbaum (Ruhr U., Bochum), H. Krebs (Ruhr U., Bochum), U.G. Meißner (Bonn U., HISKP and IAS, Julich and JCHP, Julich and Julich, Forschungszentrum) (Dec 15, 2014)

Published in: *Phys.Rev.Lett.* 115 (2015) 12, 122301 • e-Print: 1412.4623 [nucl-th]

pdf DOI cite 244 citations

The problem w/ Weinberg counting
(i.e. NDA)

1) The first problem (1996)

Nucleon - nucleon scattering from effective field theory

David B. Kaplan (Washington U., Seattle), Martin J. Savage (Carnegie Mellon U.), Mark B. Wise (Caltech)
May 3, 1996

35 pages
Published in: *Nucl.Phys.B* 478 (1996) 629-659
e-Print: nucl-th/9605002 [nucl-th]
DOI: 10.1016/0550-3213(96)00357-4
Report number: DOE-ER-40561-257, INT-96-00-125, UW-PT-96-06, CMU-HEP-96-06, DOE-ER-40862-117, CALT-68-2047
View in: ADS Abstract Service

pdf cite

2) The big problem

Renormalization of one-pion exchange and power counting

A. Nogga (Julich, Forschungszentrum), R.G.E. Timmermans (Groningen, KVI), U. van Kolck (Arizona U.)
Jun 2, 2005

19 pages
Published in: *Phys.Rev.C* 72 (2005) 054006
e-Print: nucl-th/0506005 [nucl-th]
DOI: 10.1103/PhysRevC.72.054006
Report number: FZJ-IKP-TH-2005-19
View in: ADS Abstract Service

pdf cite

Renormalizable implementations:

1) Coordinate space (by me)

Perturbative renormalizability of chiral two pion exchange in nucleon-nucleon scattering

M.Pavon Valderrama (Julich, Forschungszentrum and JCHP, Julich) (Dec 3, 2009)

Published in: *Phys.Rev.C* 83 (2011) 024003 • e-Print: 0912.0699 [nucl-th]

pdf DOI cite 91 cita

Perturbative Renormalizability of Chiral Two Pion Exchange in Nucleon-Nucleon Scattering: P- and D-waves #12

M. Pavon Valderrama (Valencia U., IFIC) (Aug 3, 2011)

Published in: *Phys.Rev.C* 84 (2011) 064002 • e-Print: 1108.0872 [nucl-th]

pdf DOI cite 65 citations

2) Momentum space

Short-range nuclear forces in singlet channels #6

Bingwei Long (Jefferson Lab), C.J. Yang (Jefferson Lab) (Feb 20, 2012)

Published in: *Phys.Rev.C* 86 (2012) 024001 • e-Print: 1202.4053 [nucl-th]

pdf links DOI cite 54 citations

Renormalizing Chiral Nuclear Forces: Triplet Channels #7

Bingwei Long (Jefferson Lab), C.J. Yang (Arizona U. and Ohio U., Inst. Nucl. Part. Phys.) (Nov 17, 2011)

Published in: *Phys.Rev.C* 85 (2012) 034002 • e-Print: 1111.3993 [nucl-th]

pdf links DOI cite 68 citations

Renormalizing chiral nuclear forces: a case study of 3P0 #8

Bingwei Long (Jefferson Lab), C.J. Yang (Arizona U. and Ohio U., Inst. Nucl. Part. Phys.) (Aug 4, 2011)

Published in: *Phys.Rev.C* 84 (2011) 057001 • e-Print: 1108.0985 [nucl-th]

pdf links DOI cite 45 citatio