Nuclear Physics 19

Nuclear forces from the Effective Field Theory perspective

Part 1 : Pionless EFT

FUNDAMENTAL PROBLEM OF NUCLEAR PHYSICS

⇓

$\boxed{\text{Derivation of nuclear forces from first principles}}$

$\boxed{\text{QCD}}$

— ⊗ —

Meson theory → $\boxed{\text{pre QCD}}$

It's not the solution to the problem

$\boxed{\text{Post QCD}}$

↳ not a solvable theory
(at low energies)

alternatives

1) Lattice QCD

need supercomputer,
still not complete

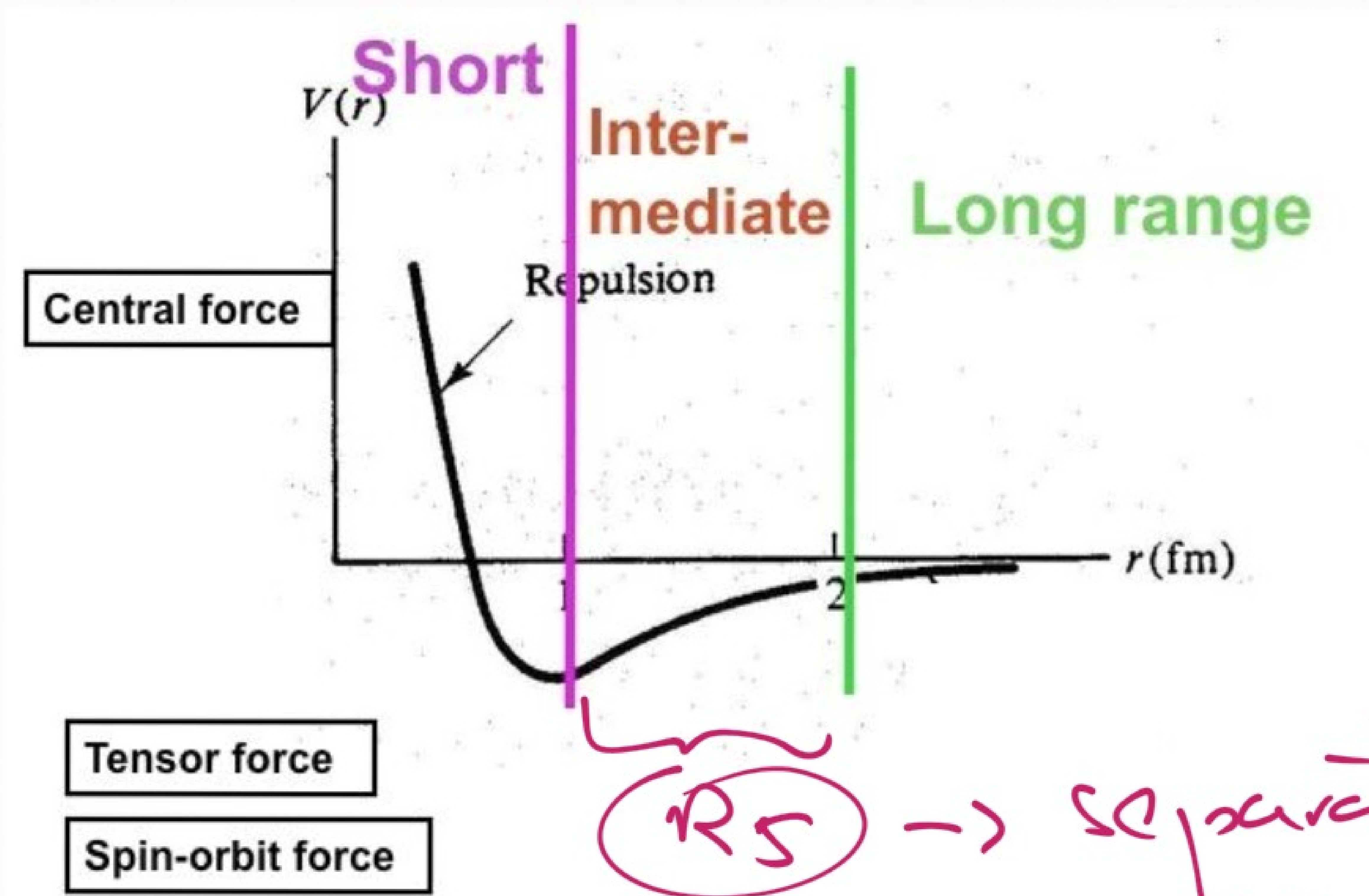
2) EFT

↳ the indirect method

Also, the modern method

Basic difference w/ OBE:

1) We can divide V in regions



R_S → separation scale

2) Different treatments:

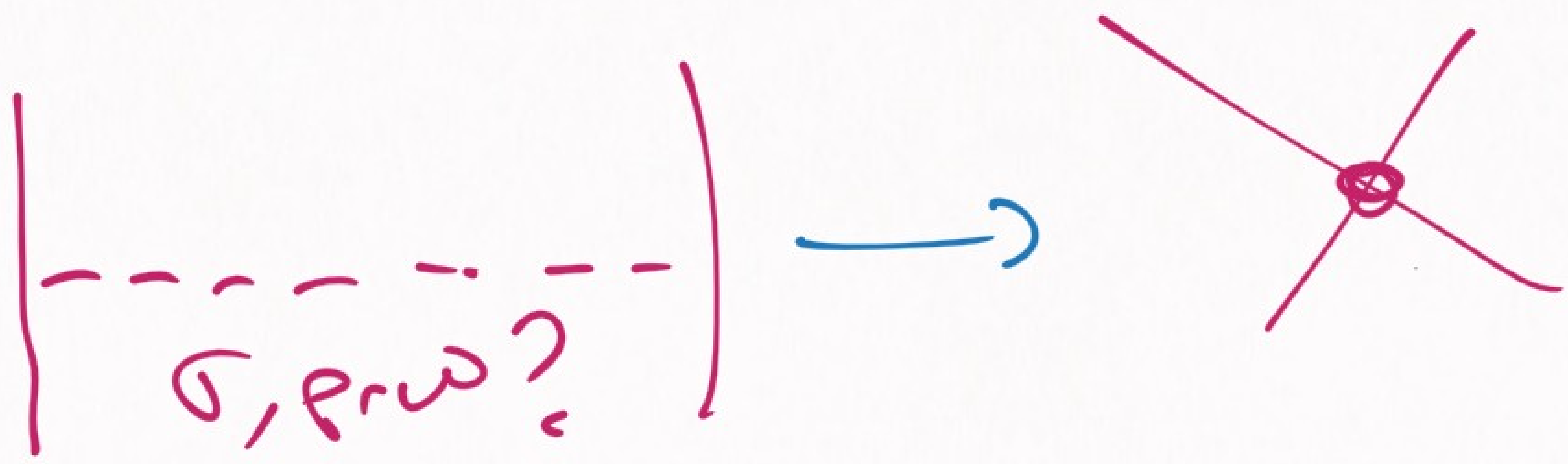
2.a) OBE → meson exchanges
(phenomenology)

2.b) EFT → choose R_S

$\boxed{r > R_S}$ → known

$\boxed{r < R_S}$ → unknown

Unknown part:



→ contact interactions

$$V_c(\vec q) = C_0 + C_2 \vec q^2 + \dots$$

or more generally:

$$\langle \vec p' | V_c | \vec p \rangle = C_0 + C_2 (p^2 + p'^2) + C_{1?}\, \vec p \cdot \vec p' + \dots$$

→ REMEMBER: depends on choice of R_S

1) $m_\pi R_s > 1 \longrightarrow \boxed{\text{pionless EFT}}$

EFT($\cancel{\pi}$)

→ only contact interactions

2) $m_\pi R_s < 1 \longrightarrow \boxed{\text{pionful EFT}}$

EFT(π)

→ pions + contacts

→ chiral symmetry

We will begin w/ pionless

$$\boxed{EFT(\not{\pi})}$$

→ valid for $k < m_\pi$

→ really simple & useful theory

→ only contact-interactions

$$\boxed{S\text{-waves}}$$

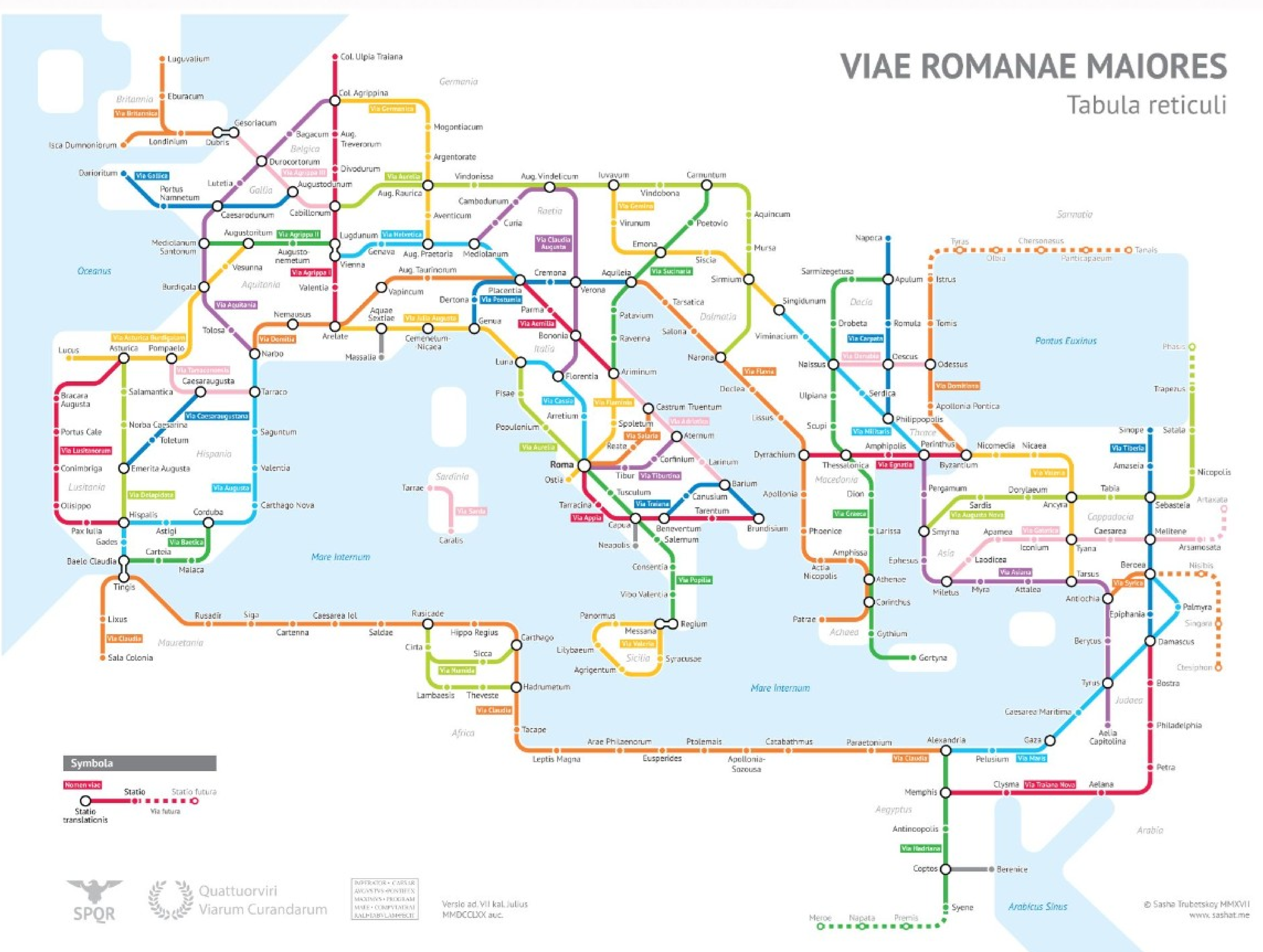
$$\langle p' | V_c | p \rangle = C_0 + C_2 (p^2 + p'^2) + C_4 (p^4 + p'^4) + C_4' p^2 p'^2 + \dots$$

We cut the expansion when we find it convenient

⇒

EFT trick

$\forall$ regularizations lead to ~~Rome~~ the same results



$\forall$ regularizations are equivalent

$\Downarrow$

We can use easier ones:

$$V_c(r;R_c) = \frac{C_k(R_c)}{4\pi R_c^2}\,\delta(r-R_c)$$

w/ $C_k = C_0 + C_2 k^2 + C_4 k^4 + \ldots$

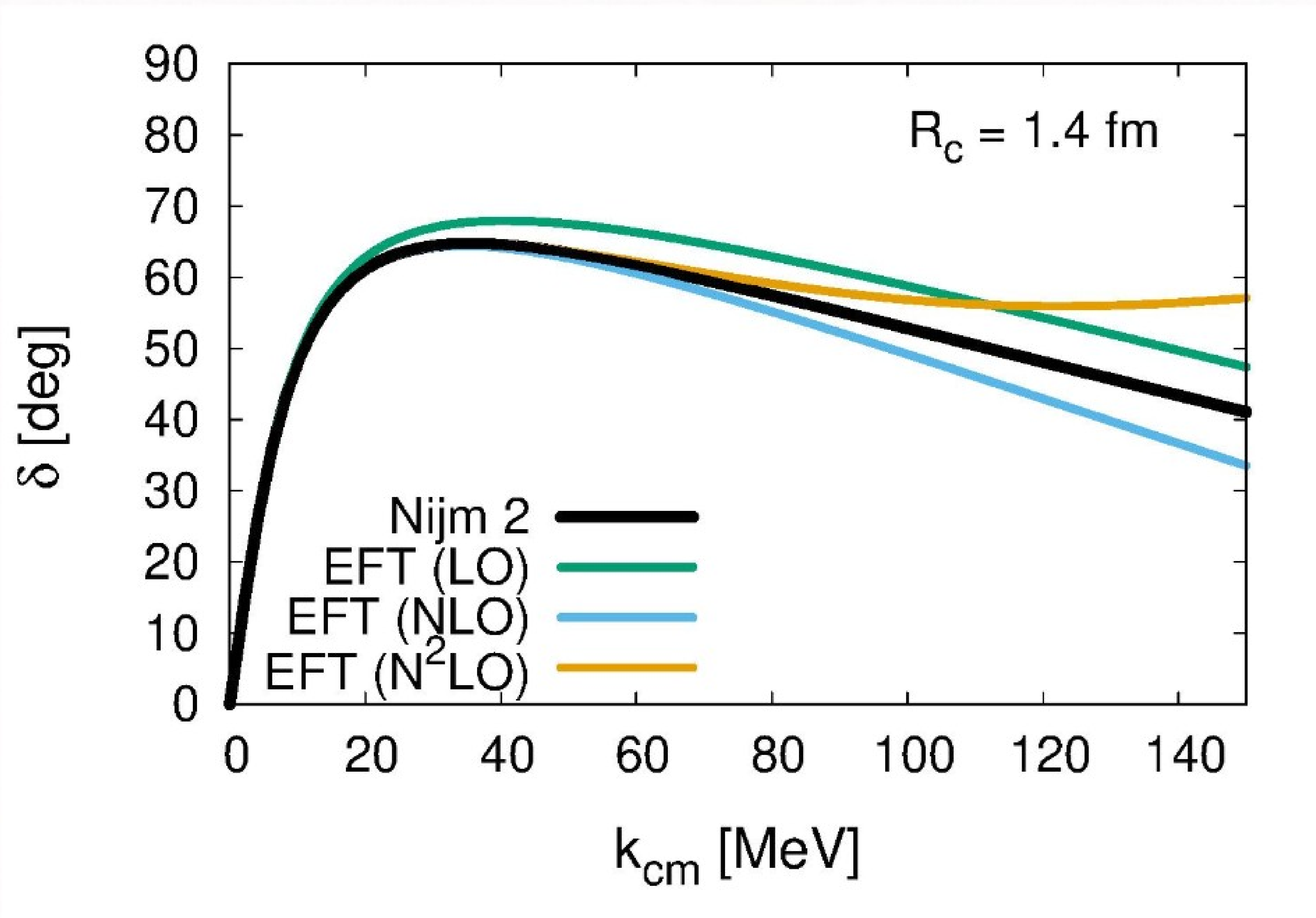
$\Downarrow$

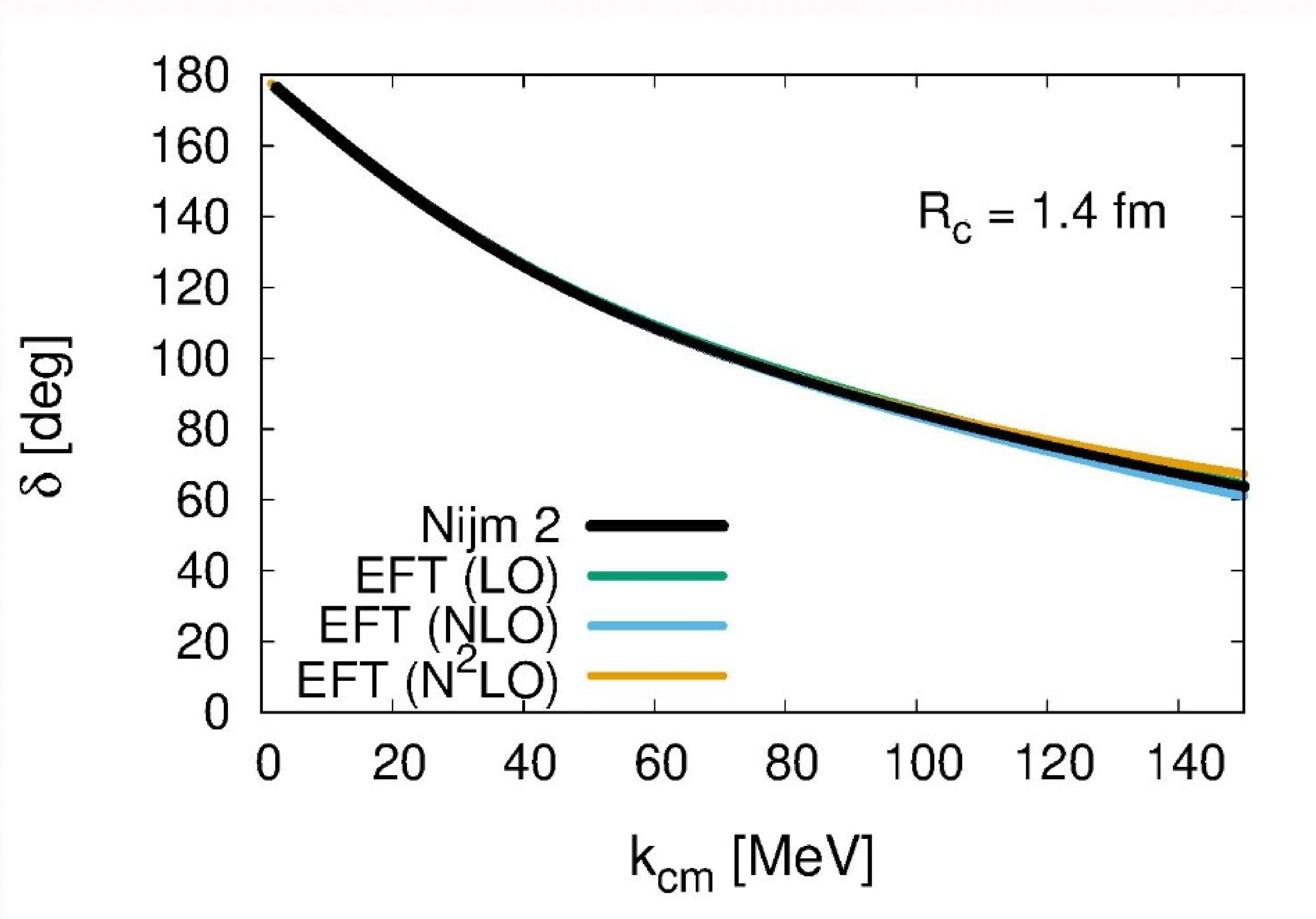
$$k\cot(kR_c+\delta) - k\cot kR_c$$

$$= 2\mu\,\frac{C_k(R_c)}{4\pi R_c^2}$$

(Explicit solution)

Singlet & Triplet in EFT($\not{\pi}$)





→ You can program it yourself in Mathematica

Comments:

1) EFT($\not\pi$) equivalent to ERE (in the two-body sector)

Nucleon-nucleon effective field theory without pions

Jiunn-Wei Chen (Washington U., Seattle), Gautam Rupak (Washington U., Seattle), Martin J. Savage (Washington U., Seattle and Jefferson Lab)

Feb 24, 1999

27 pages

Published in: *Nucl.Phys.A* 653 (1999) 386-412

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DOI: 10.1016/S0375-9474(99)00298-5

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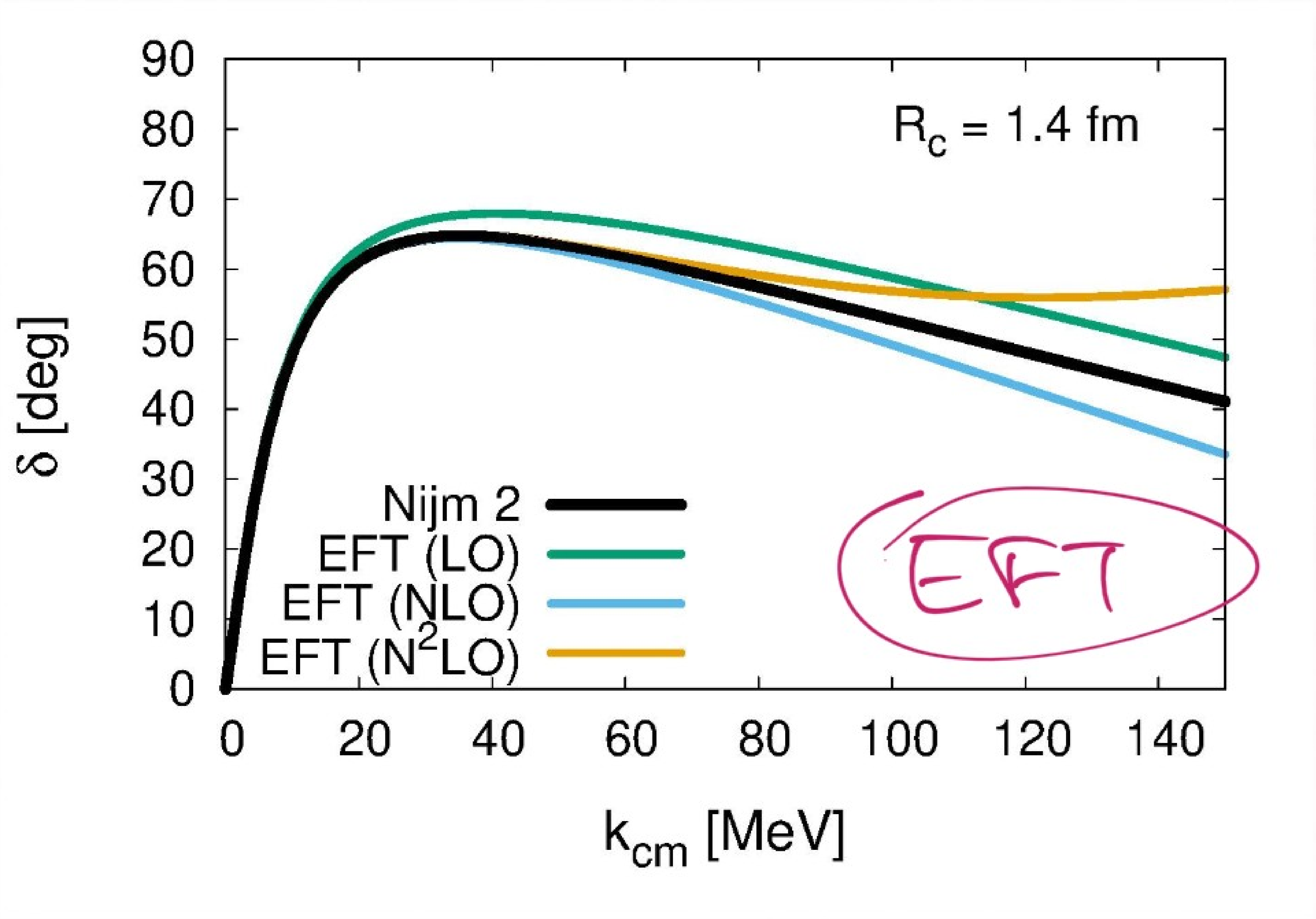
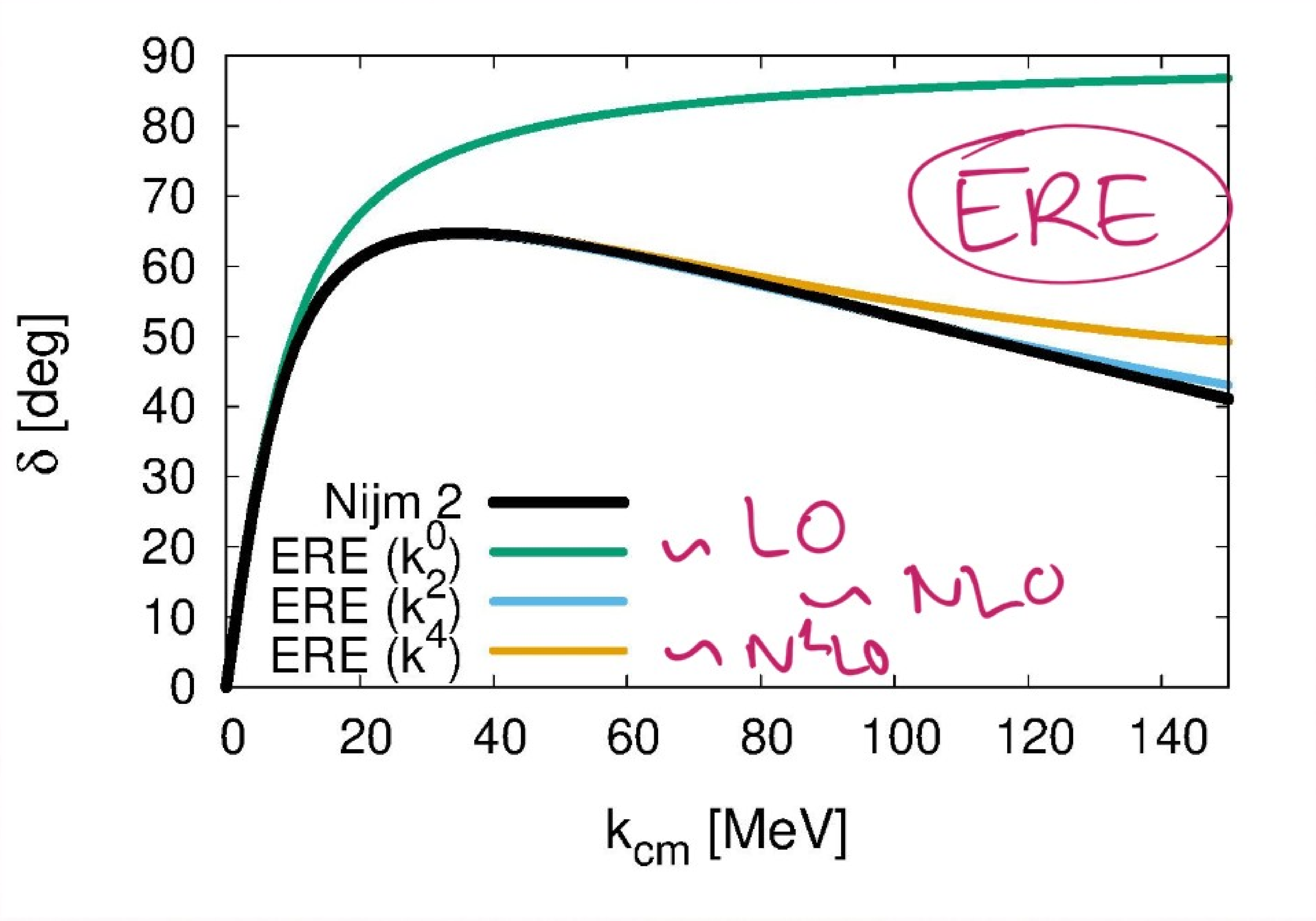
↳ Discussed in this paper

We can make a direct comparison





2) Cutoff dependence:

2.a) Cutoff independence not exact

$$\frac{d}{dR_c}\langle\psi|\hat{O}|\psi\rangle \neq 0$$ (stricto sensu)

$$= 0$$ (modulo higher order corrections)

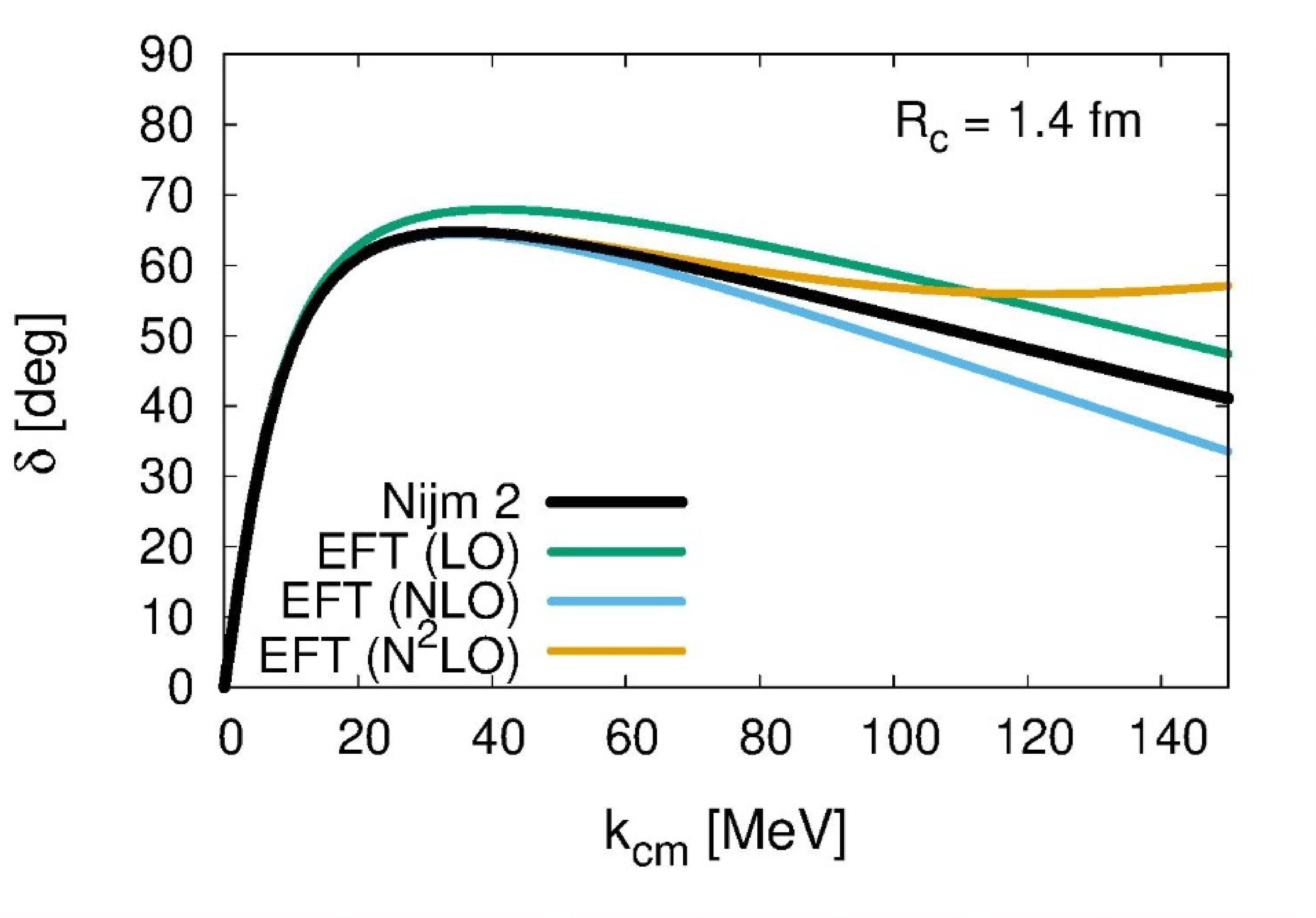
2.b) "$\neq 0$" called residual cutoff dependence

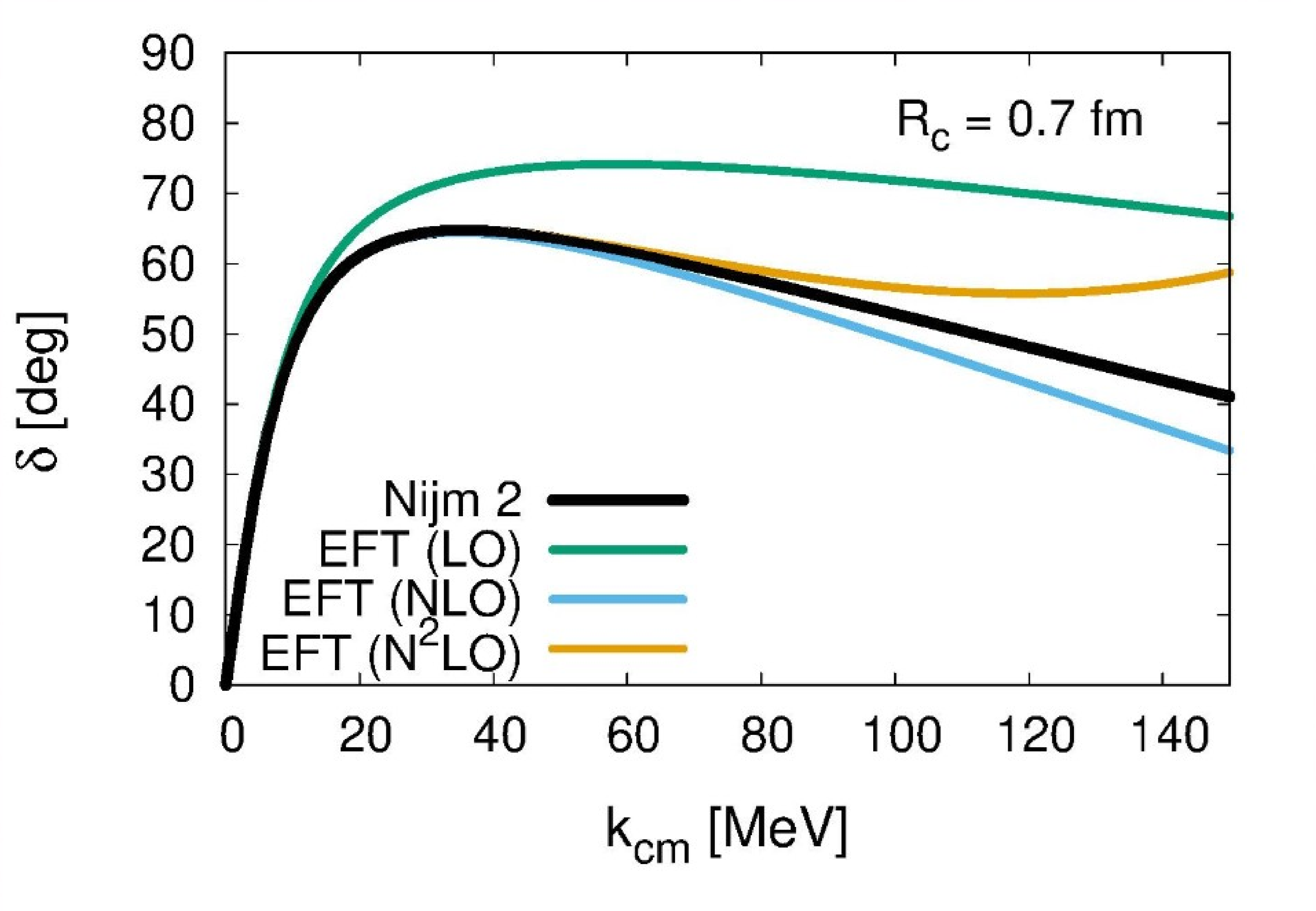
2.c) Cutoff independence recovered for $R_c \to 0$

$$\lim_{R_c \to 0} \frac{d}{dR_c}\langle\psi|\hat{O}|\psi\rangle = 0$$

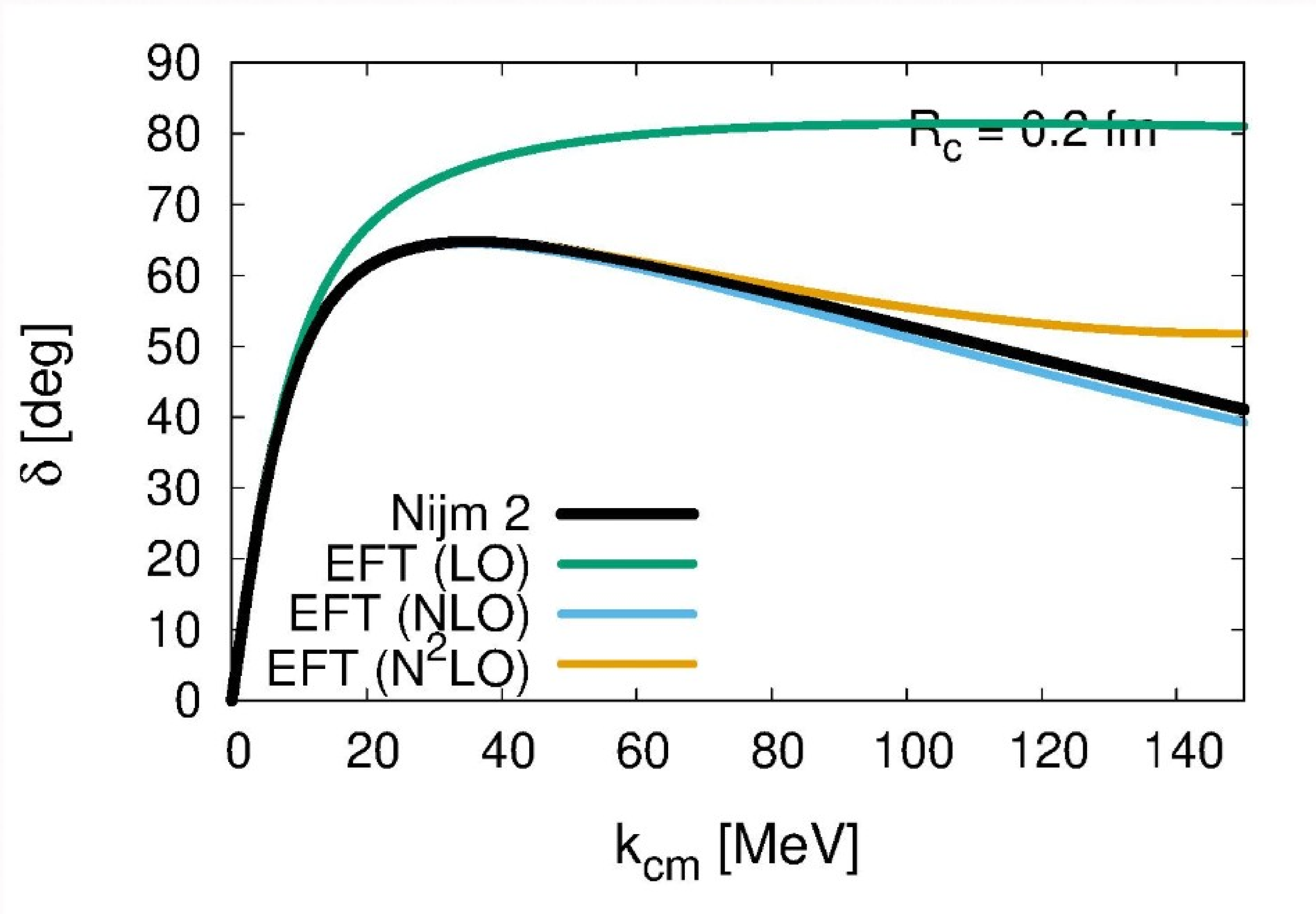
$$\boxed{\exists\ R_c \to 0 \text{ limit}}$$

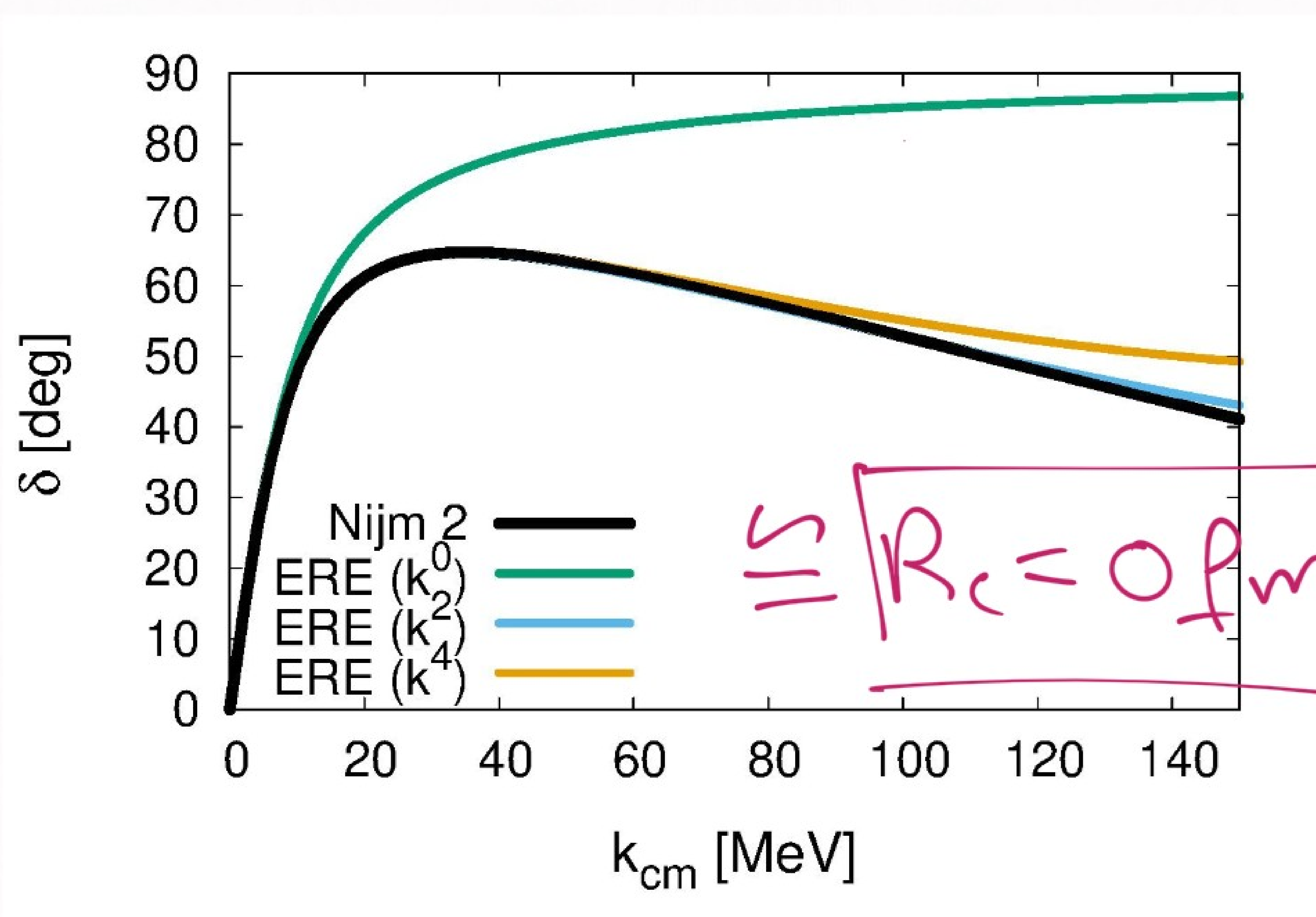
Example: $^1S_0$ phase shifts





And we continue:





$\cong \boxed{R_c = 0 \text{ fm}}$

We end up w/ ERE as the limit

$$\boxed{\text{CUTOFF DEPENDENCE}}$$

1) It's ok if UNDER CONTROL

$$\boxed{\lim_{R_c \to 0} \langle \Psi | \hat{O} | \Psi \rangle \to 0}$$

If not, you have a problem

2) Residual cutoff dependence

→ subleading error of the theory

$$\boxed{\text{Important point}}$$

REMINDER

TEAPOT & TEACUP THEORY

$$T = T_0 e^{-\lambda(t-t_0)}$$

$$\Downarrow$$

λ?

$$\lambda = S(c_0 + c_1 x + c_2 x^2 + \dots)$$

$$x = \left(\frac{S}{\rho\Sigma}\right) \quad \& \quad x < 1$$

EFT's are power series

The same is true for any EFT:

1) $$k\cot\delta|_{EFT} = k\cot\delta|_{LO} \times \left[1 + O\left(\frac{Q}{M}\right)\right]$$

$$\begin{bmatrix} Q \to \text{Light scale} \\ M \to \text{Heavy scale} \end{bmatrix}$$

$$k\cot\delta|_{EFT} = k\cot\delta|_{NLO} \times \left[1 + O\left(\frac{Q^2}{M^2}\right)\right]$$

$$k\cot\delta|_{EFT} = k\cot\delta|_{N^2LO} \times \left[1 + O\left(\frac{Q^3}{M^3}\right)\right]$$

→ as we go to higher orders, the error diminishes

The connection w/
residual cutoff dependence

1) $$k\cot\delta(R_c)\Big|_{LO} = -\frac{1}{a_0} + \frac{2}{3}k^2 R_c + \mathcal{O}(R_c^2)$$

2) $Q \sim a_0, k \qquad M \sim m_\pi$
(pionless)

3) $$k\cot\delta(R_c)\Big|_{LO} = -\frac{1}{a_0} \times \Big[1 - \frac{2}{3}\underbrace{(ka_0)}_{\left(\frac{Q}{M}\right)^0}\underbrace{(kR_c)}_{(QR_c)} + \dots\Big]$$

$\Rightarrow$ If $R_c \sim \frac{1}{m_\pi} \Rightarrow \boxed{(QR_c) \sim \left(\frac{Q}{M}\right)}$

Conclusion: If $MR_c \sim 1$ then residual cutoff dependence equivalent to EFT error

— ⊗ —

$$k\cot\delta\Big|_{LO} = -\frac{1}{a_0}\left[1 - \frac{2}{3}(a_0 k)(kR_c) + \mathcal{O}(R_c^2)\right]$$

If $m_\pi R_c \sim 1$ ⇒

$$k\cot\delta\Big|_{LO} = -\frac{1}{a_0}\left[1 + \mathcal{O}\left(\frac{Q}{M}\right)\right]$$

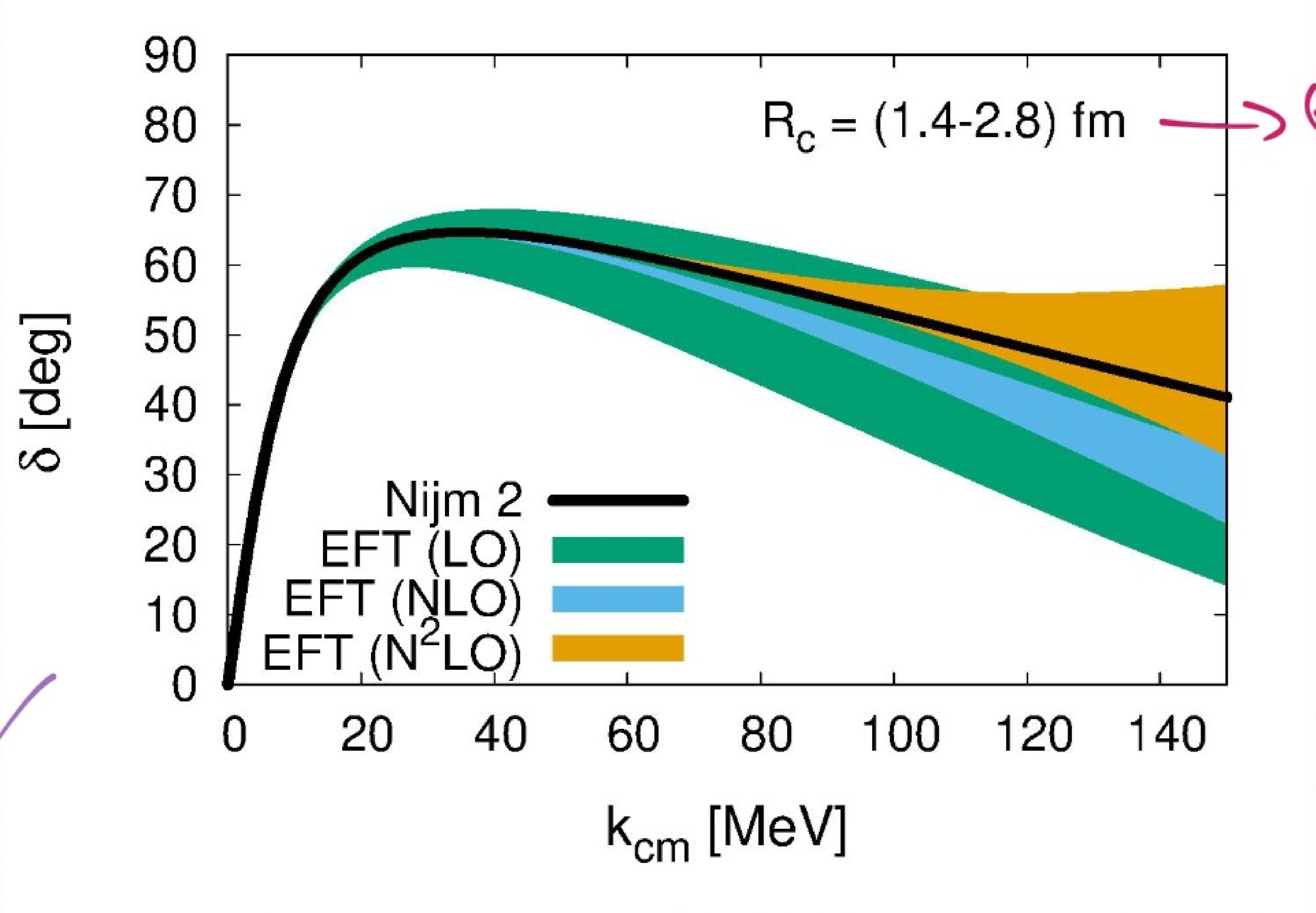
⇓

This is what we want

Residual cutoff Dependence

$\Rightarrow$ $\boxed{\text{proxy for EFT error}}$

Example $\longrightarrow$ $\boxed{\text{EFT}(\not\pi)}$



$\longrightarrow$ (*)

(*) $\longrightarrow$ $m_\pi R_c = (1-2)$

$\left(\frac{1}{m_\pi} \simeq 1.4\,\text{fm}\right)$

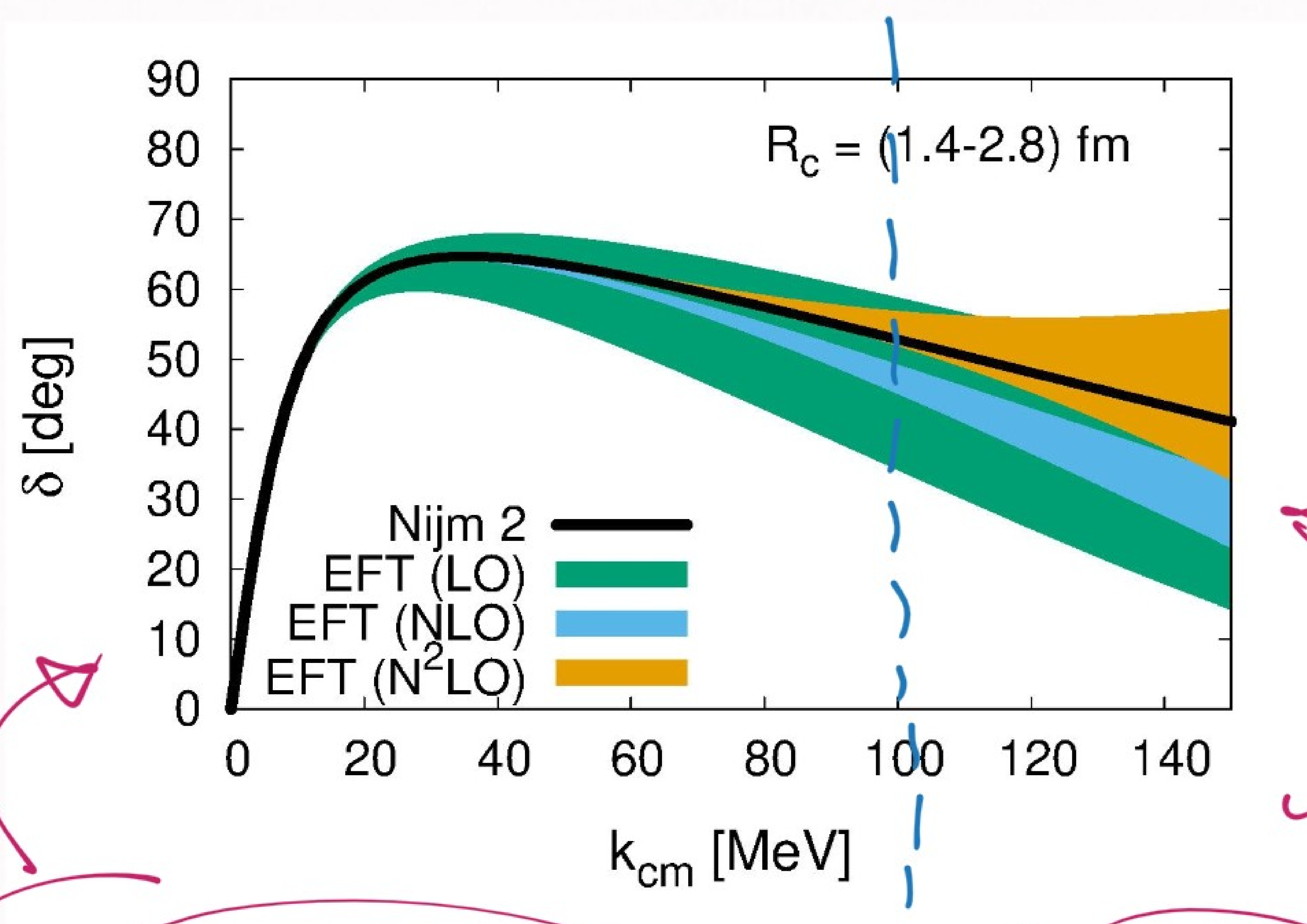
$\boxed{\text{Error bands! Hooray!}}$

1) Varying the cutoff

=> Cheap way of getting error bands

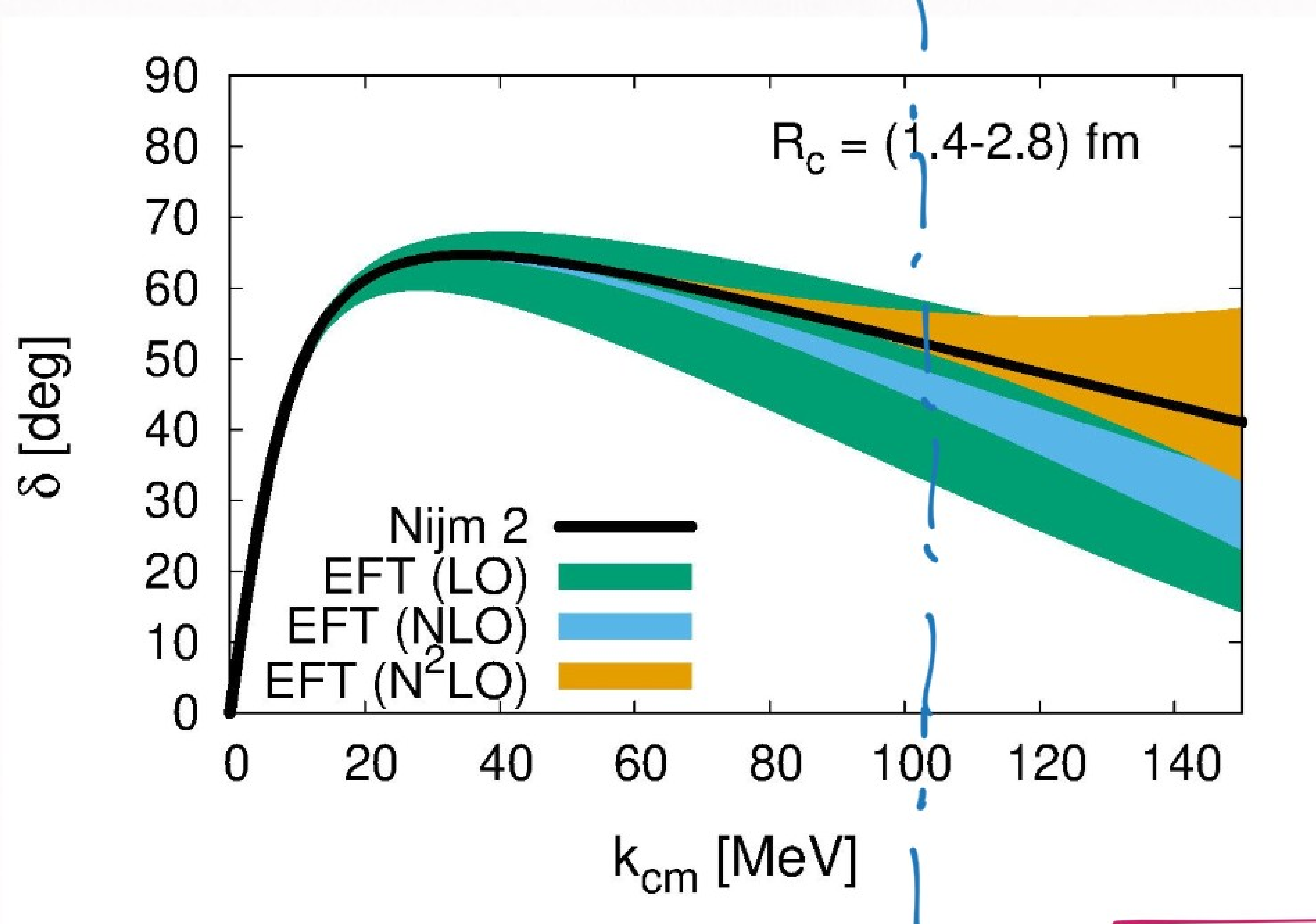
(Unfortunately, also prone to errors)

2) For k < M → error bands decrease



Decrease → Increase

3) Also a way to check the range of validity of an EFT

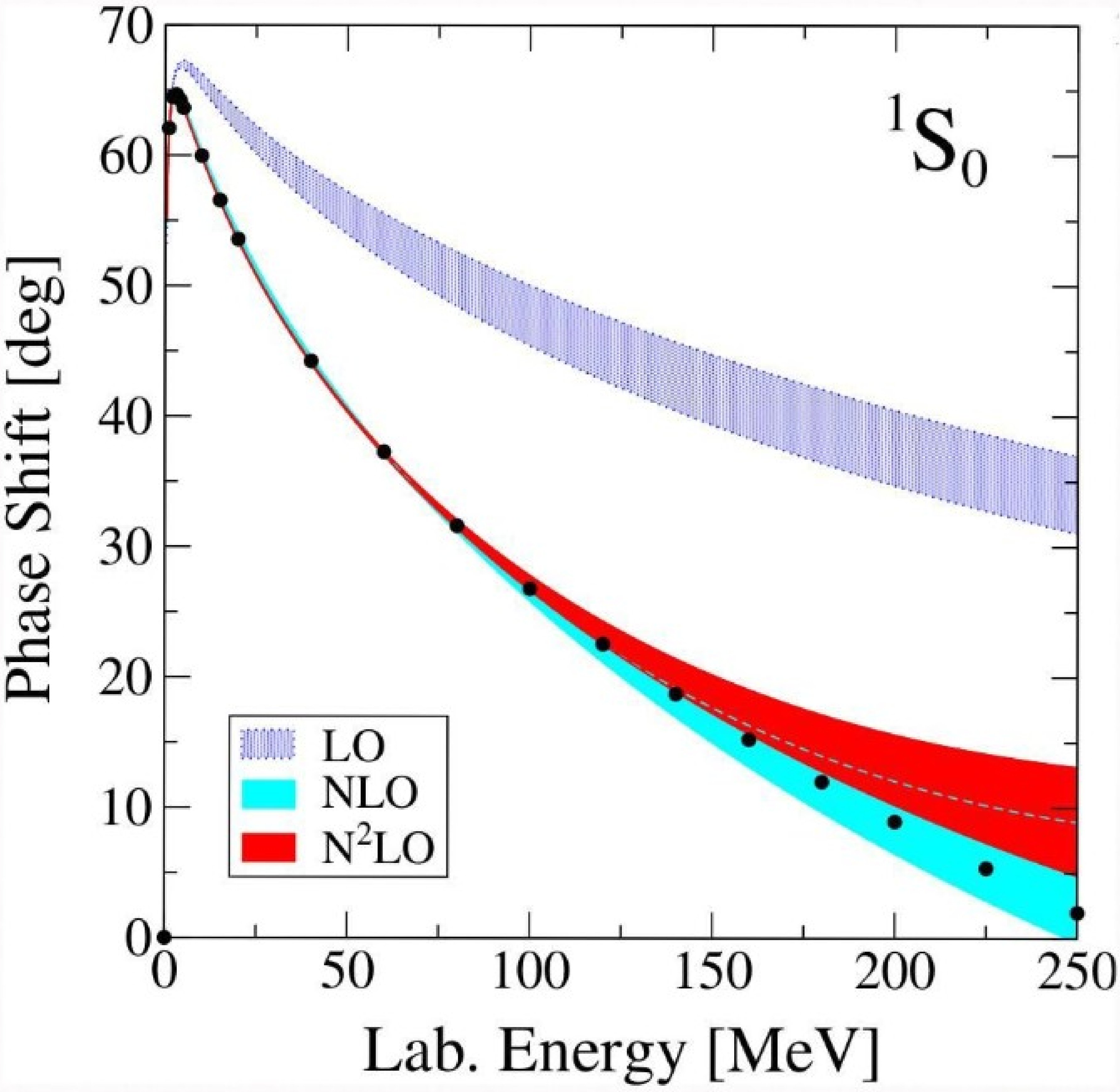


$\boxed{\text{EFT}(\not\pi)\ \text{converges}}$ ← → $\boxed{\text{EFT}(\not\pi)\ \text{maybe not valid}}$

$\boxed{\text{RECAP}}$

If $\frac{d}{dR_c}\langle\psi|\hat{O}|\psi\rangle$ under control, residual cutoff dependence a $\boxed{\text{useful tool}}$

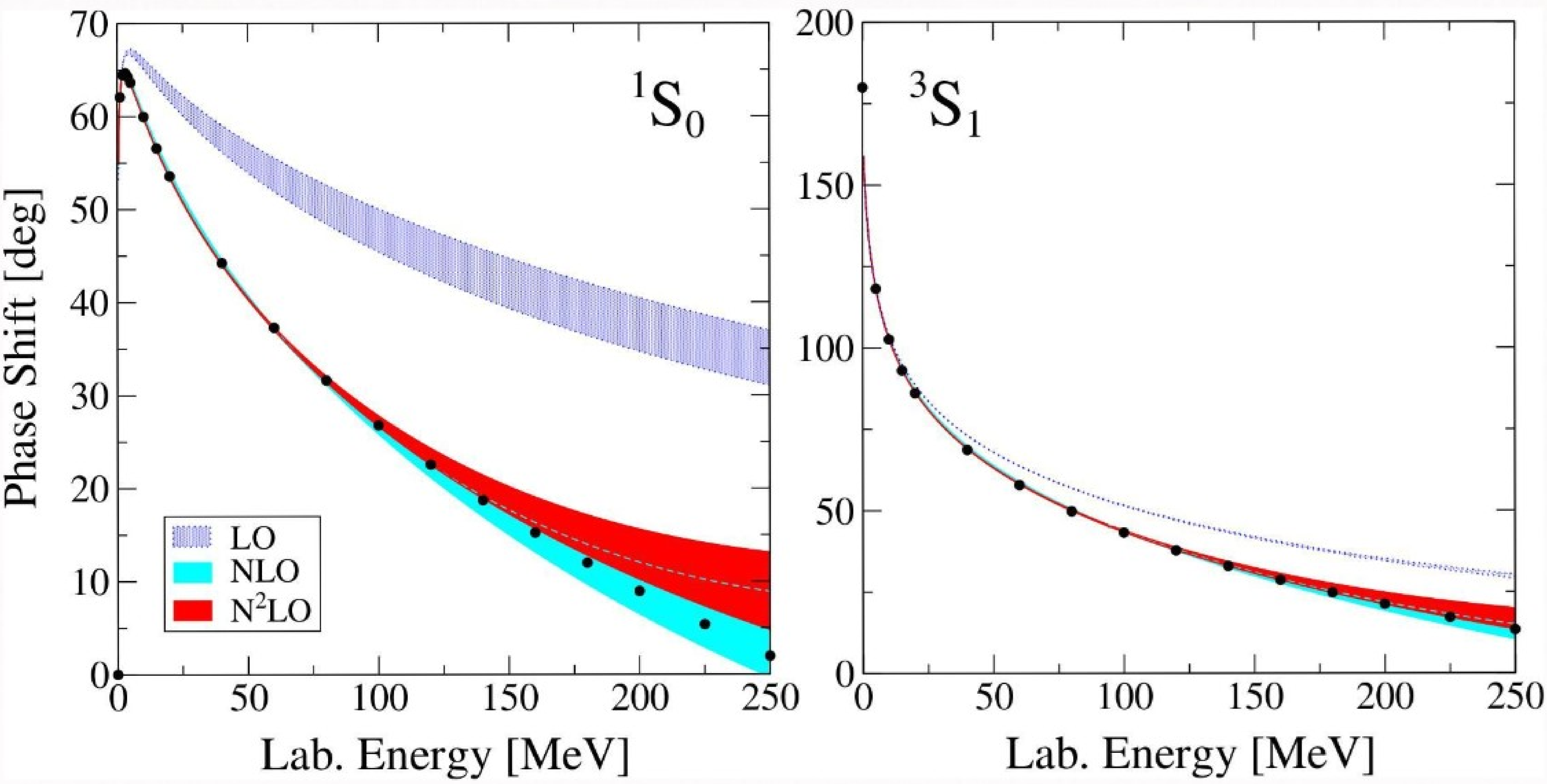
Is also used in pionful EFT



$\boxed{\text{LO band too small}}$

not ideal method (only quick & dirty)

A brief look at pionful:



+

Larger energy range

Local chiral effective field theory interactions and quantum Monte Carlo applications #4

A. Gezerlis (Guelph U.), I. Tews (Darmstadt, Tech. Hochsch. and Darmstadt, EMMI), E. Epelbaum (Ruhr U., Bochum), M. Freunek (Ruhr U., Bochum), S. Gandolfi (Los Alamos) et al. (Jun 2, 2014)

Published in: *Phys.Rev.C* 90 (2014) 5, 054323 • e-Print: 1406.0454 [nucl-th]

Pionful figures from here

However Pionful EFT is full of unsolved problems

$$\boxed{\text{NEXT LESSON}}$$

SUMMARY

1) EFTs $\rightarrow$ modern approach

2) Choice of separation scale

$\Rightarrow$ different EFT's

2.a) $m_\pi R_S \sim 1 \Rightarrow$ EFT($\cancel{\pi}$)

2.b) $m_\rho R_S \sim 1 \Rightarrow$ EFT(π)

3) RG invariance not automatic

$$\lim_{R_c \to 0} \frac{d}{dR_c} \langle \Psi | \hat{O} | \Psi \rangle \to 0$$

4) Residual d/dR_c proxy for EFT errors

(but not ideal)

5) EFT($\cancel{\pi}$) $\simeq$ ERE

Further reading :

Effective field theory of short range forces

U. van Kolck (Caltech, Kellogg Lab and Washington U., Seattle)
Aug 4, 1998

38 pages
Published in: *Nucl.Phys.A* 645 (1999) 273-302
e-Print: nucl-th/9808007 [nucl-th]
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Report number: KRL-MAP-230, NT-UW-98-01
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→ The two classic references about EFT($\not\pi$)