Nuclear Physics 13

The One Boson Exchange (OBE) model

RECAP

{ FUNDAMENTAL PROBLEM OF NUCLEAR PHYSICS }

↓

What is the origin of the nuclear force?

1) Before QCD:

1.a) pion theories (50's)

1.b) one boson exchange model

2) Post QCD:

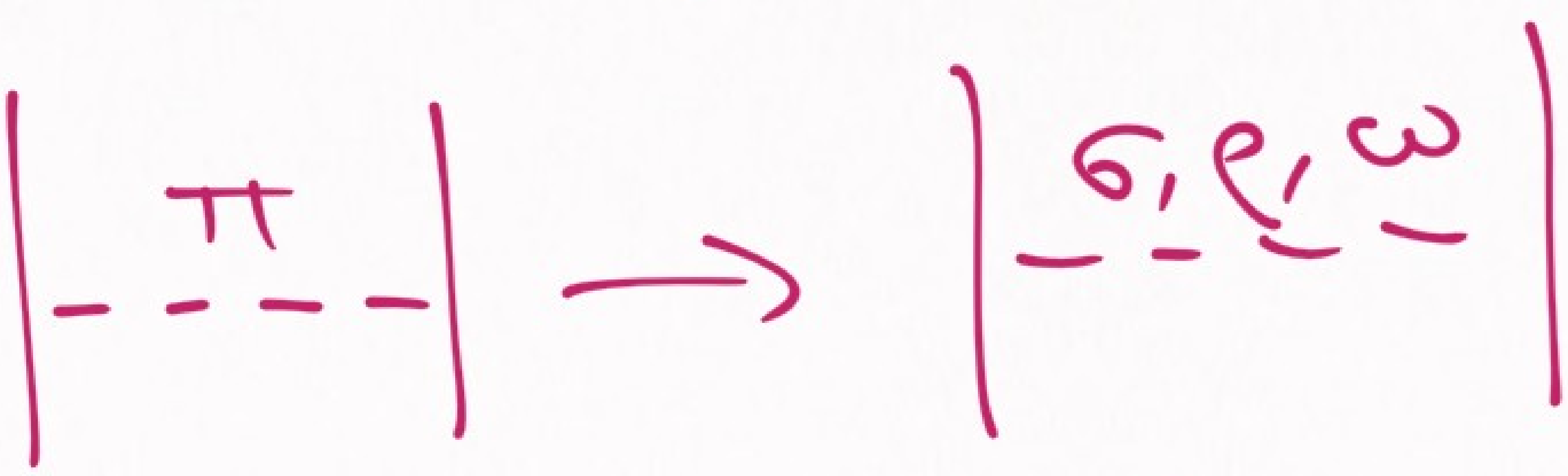
2.a) quark-model related

2.b) effective field theories

Today's lesson

OBE MODEL

1) Extension of the original idea by Yukawa



(+ other bosons)

2) Relatively simple

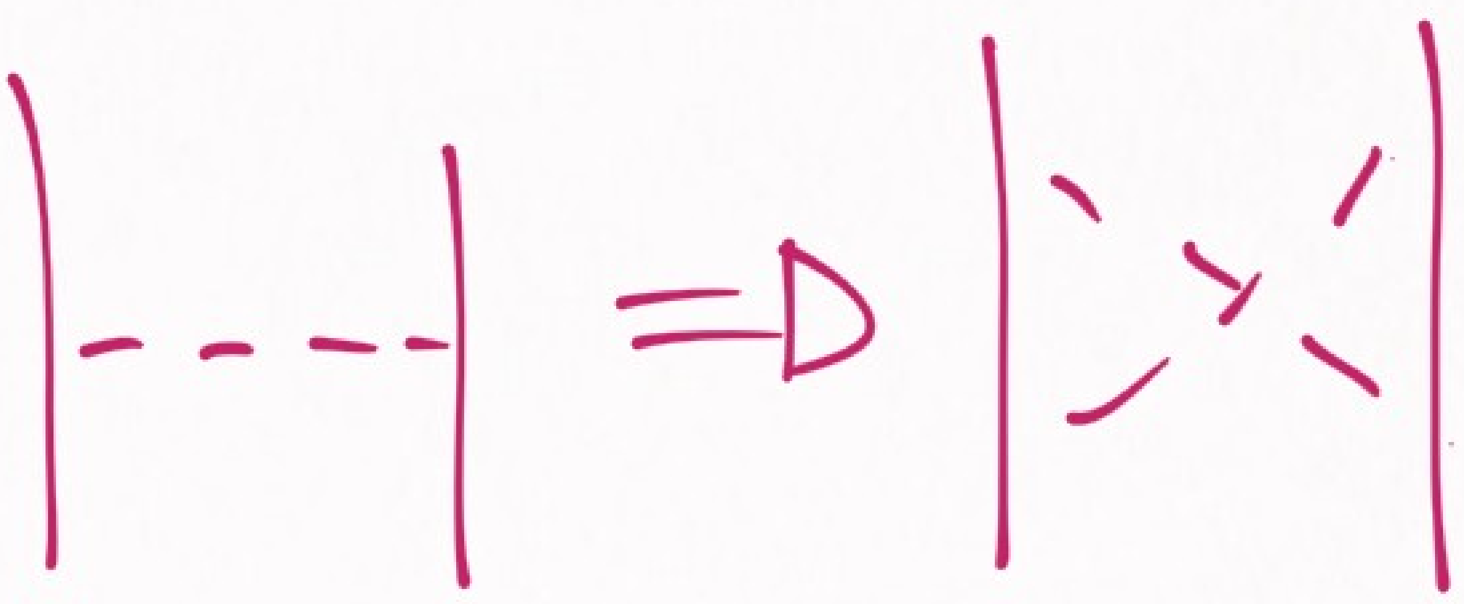
3) First quantitatively successful nuclear force

$$\underbrace{\left(\chi^2/\mathrm{d.o.f.} \simeq 1 \right)}_{\text{[Quality of the fit]}}$$

Original motivation:

FAILURE OF THE OLD PION THEORIES

1) Most direct extension of Yukawa

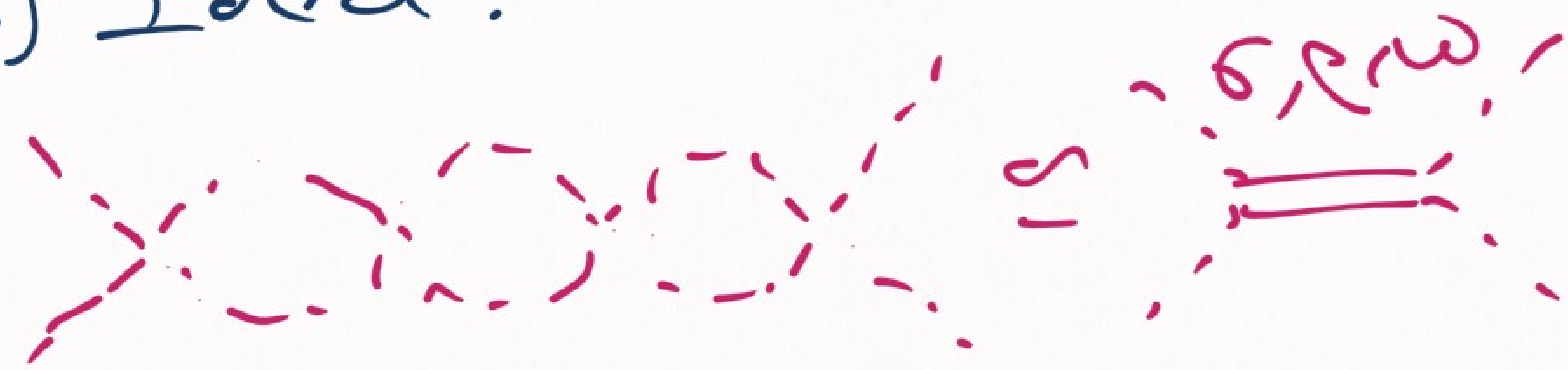


more pions!

2) But it did not work

→ no chiral symmetry /
no renormalization

3) Idea:



multi-pion exchange ≃ resonance

The major characters in this story :

1) The pion : $J^P = 0^-$, $I = 1$

$m_\pi = 140$ MeV

(explains quadrupole moment)

2) The sigma : $J^P = 0^+$, $I = 0$

$m_\sigma \simeq 500$ MeV

(strong mid-range attraction)

3) The rho : $J^P = 1^-$, $I = 1$

(counters the excesses of the tensor pion)

4) The omega : $J^P = 1^-$, $I = 0$

(provides the short-range repulsion)

The OBE potential:

$$V_{OBE} = \underbrace{V_\pi}_{} + V_\sigma + V_\rho + V_\omega$$

↳ We already know this one

$$V_\sigma(\vec{q}) = -\frac{g_\sigma^2}{q^2 + m_\sigma^2}$$

$$V_\rho(\vec{q}) = \vec{\tau}_1 \cdot \vec{\tau}_2 \Big[\frac{g_\rho^2}{q^2 + m_\rho^2} - \frac{(f_\rho + g_\rho)^2}{4M_N^2} \frac{(\vec{\sigma}_1 \wedge \vec{q}) \cdot (\vec{\sigma}_2 \wedge \vec{q})}{\vec{q}^2 + m_\rho^2} \Big]$$

$$V_\omega(\vec{q}) = \frac{g_\omega^2}{q^2 + m_\omega^2} - \frac{(f_\omega + g_\omega)^2}{4M_N^2} \frac{(\vec{\sigma}_1 \wedge \vec{q}) \cdot (\vec{\sigma}_2 \wedge \vec{q})}{q^2 + m_\rho^2}$$

Or in coordinate space:

$$V_\sigma(r) = -g_\sigma^2 m_\sigma W_Y(m_\sigma r)$$

$$V_\rho(r) = \vec{\tau}_1 \cdot \vec{\tau}_2 \Big[g_\rho^2 m_\rho W_Y(m_\rho r)$$

$$+ \frac{(f_\rho + g_\rho)^2}{4M_N^2} \Big(\frac{2}{3} \vec{\sigma}_1 \cdot \vec{\sigma}_2 m_\rho^3 W_Y(m_\rho r)$$

$$- \frac{1}{3} S_{12}(\hat{r}) m_\rho^2 W_T(m_\rho r) \Big]$$

$$V_\omega(r) = g_\omega^2 m_\omega W_Y(m_\omega r)$$

$$+ \frac{(f_\omega + g_\omega)^2}{4M_N^2} \Big(\frac{2}{3} \vec{\sigma}_1 \cdot \vec{\sigma}_2 m_\omega^2 W_Y(m_\omega r)$$

$$- \frac{1}{3} S_{12}(\hat{r}) m_\omega^2 W_T(m_\omega r) \Big)$$

$$W_Y(x) = \frac{e^{-x}}{4\pi x}, \quad W_T(x) = \frac{e^{-x}}{4\pi x} \Big(1 + \frac{3}{x} + \frac{3}{x^2} \Big)$$

Usual simplifications:

$$\left[\begin{array}{l} f_e \gg g_e \Rightarrow g_e = 0 \\ f_\omega \ll g_\omega \Rightarrow f_\omega = 0 \end{array}\right]$$

$\hookrightarrow$ This somewhat simplifies the expressions

Yet there's a problem...

$$W_T(x) = \frac{e^{-x}}{4\pi x}\left(1 + \frac{3}{x} + \frac{3}{x^2}\right)$$

$$\Rightarrow \left.\begin{array}{l} V_\pi \sim \dfrac{S_{12}(\hat{r})}{r^3} \\ V_\rho \sim \dfrac{S_{12}(\hat{r})}{r^3} \end{array}\right\} \boxed{\text{Singular Potential}}$$

$$\left(V_\omega \sim \frac{S_{12}(\hat{r})}{r^3}\right) \text{ for } f_\omega \neq 0$$

OBE Model $\rightarrow$ singular potentials

But renormalization not understood
when OBE model proposed

Proposed solution:

$\boxed{\text{finite-size of mesons will smear the potentials}}$

Form factors

What is a form factor:

$$\underbrace{V_M(\vec{q})}_{\text{meson-exchange potential}} \longrightarrow V_M(\vec{q})\,\underbrace{F_M^2(\vec{q};\Lambda)}_{\text{form factor}}$$

Also possible in r-space:

$$V_M(\vec{r}) \longrightarrow V_M(\vec{r})\,F_M^2(\vec{r};R_c)$$

$$\boxed{\text{form factor} \cong \text{regulator}}$$

Most standard choice:

$$F_M(\vec{q};\Lambda) = \left(\frac{\Lambda^2 - m^2}{\Lambda^2 + \vec{q}^{\,2}}\right)^{\alpha}$$

[Multipolar form factor]

Example : monopolar form factor

$$V_M(\bar{q}) \to V_M(\bar{q}) \left(\frac{\Lambda^2 - m^2}{\Lambda^2 + \bar{q}^2} \right)^2$$

$\Updownarrow$ Equivalent to

$$W_Y(x) \to W_Y(x, \lambda) = W_Y(x) - \lambda W_Y(\lambda x) - \frac{(\lambda^2 - 1)}{2x} \frac{e^{-\lambda x}}{4\pi}$$

$$W_T(x) \to W_T(x, \lambda) = W_T(x) - \lambda^3 W_T(\lambda x) - \frac{(\lambda^2 - 1)}{2\lambda} \lambda^2 \left(1 + \frac{1}{\lambda x}\right) \frac{e^{-\lambda x}}{4\pi}$$

with $\lambda = \frac{\Lambda}{m}$

m $\rightarrow$ mass of the meson

Now we are ready to use it

→ But there are still some extra improvements

1) more bosons

2) Relativistic corrections

↳ Spin-orbit force ($V_{LS}\vec{L}\cdot\vec{S}$)

↓

Necessary for a few P-waves (two-body system)

Or for the correct ordering of orbitals in the shell model (many-body system)

RELATIVISTIC CORRECTIONS

1) σ meson

original

$$V_\sigma = -\frac{g_\sigma^2}{m_\sigma^2 + |\vec{q}|^2}\left\{1 - \frac{\vec{k}^2}{2M^2} + \frac{\vec{q}^2}{8M^2} + \frac{\vec{L}\cdot\vec{S}}{2M^2}\right\}$$

$\oplus$

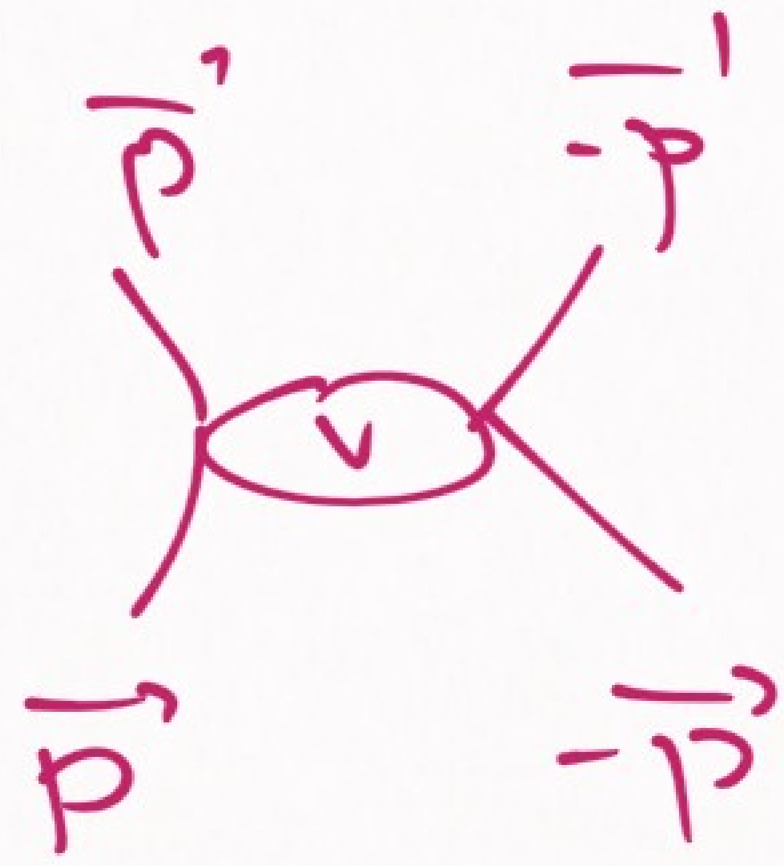
spin-orbit

2) ω meson

$$V_\omega = \frac{g_\omega^2}{m_\omega^2 + |\vec{q}|^2}\left\{1 - 3\frac{\vec{L}\cdot\vec{S}}{2M^2}\right\}$$

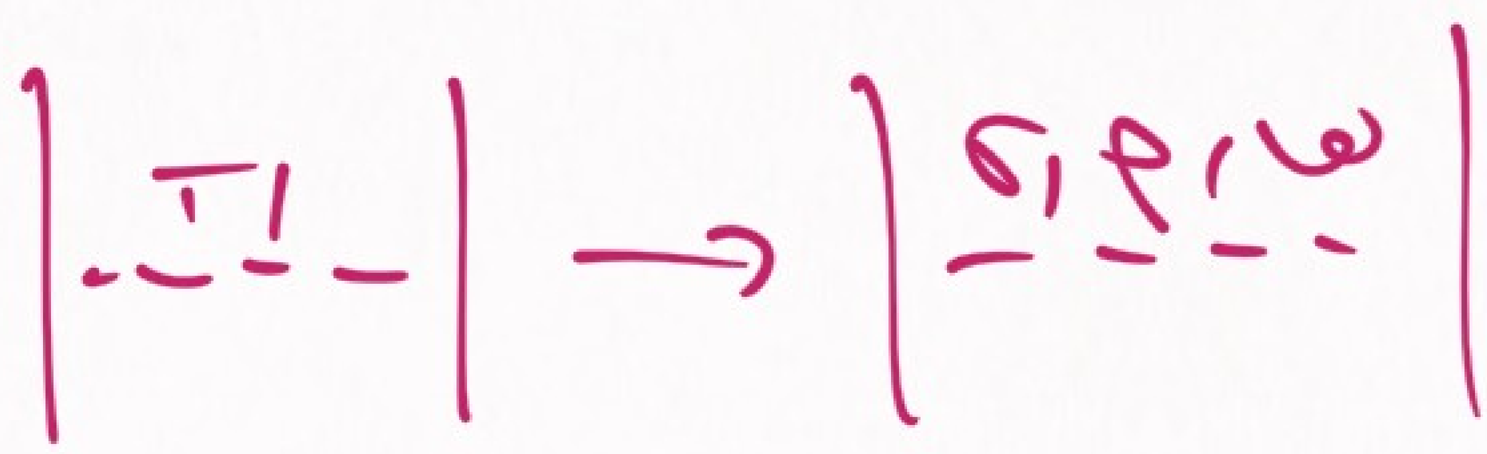
$\oplus \longrightarrow \vec{k} = \frac{1}{2}(\vec{p} + \vec{p}\,')$

$\vec{q} = \vec{p}\,' - \vec{p}$



FUNDAMENTAL IDEAS:

1) Extension of Yukawa's idea



2) Form-factors:

Otherwise we get $1/r^3$ factors

3) Relativistic corrections

Spin-orbit force

4) Each meson has a job

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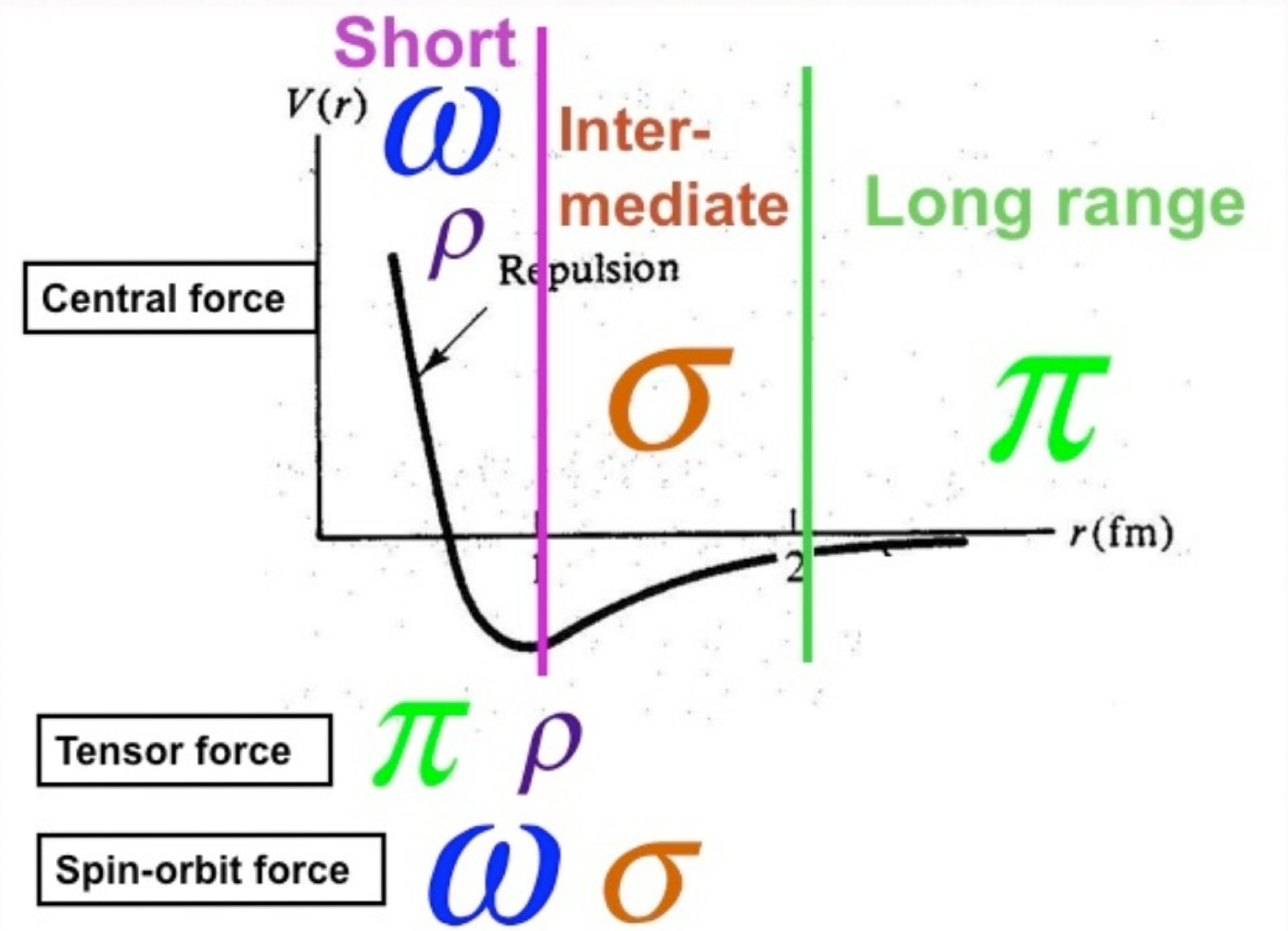
Let's check this one ...

Role of the different mesons



Long/medium/short

Related to TNS

(Taketani-Nakamura-Sasaki)

idea of the 50's

Their jobs:

Table 1: The four most important mesons and the main characteristics of their contributions to components of the nuclear force.

Meson	Central	Spin-Spin	Tensor	Spin-Orbit
$\pi(138)$	---	weak, long-ranged	**strong**, long-ranged	---
$\sigma(500)$	**strong, attractive**, intermediate-ranged	---	---	moderate, intermediate-ranged
$\omega(782)$	**strong, repulsive**, short-ranged	---	---	**strong**, short-ranged, coherent with σ
$\rho(770)$	---	weak, short-ranged, coherent with π	moderate, short-ranged, **opposite to** π	---

The classical papers on the OBE model:

The Bonn Meson Exchange Model for the Nucleon Nucleon Interaction #1

R. Machleidt (Los Alamos and UCLA), K. Holinde (Bonn U.), C. Elster (Bonn U.) (Oct 18, 1987)

Published in: *Phys.Rept.* 149 (1987) 1-89, Phys. Rep. 149 (1987) 1-89

DOI cite 2,278 citations

The Meson theory of nuclear forces and nuclear structure #2

R. Machleidt (UCLA) (1989)

Published in: *Adv.Nucl.Phys.* 19 (1989) 189-376

cite 1,212 citations

↳ Now you can read them

A typical set of parameters for the OBE model:

R. Machleidt et al., *The Bonn meson-exchange model for the nucleon–nucleon interaction*

Table 5
Meson parameters used in the relativistic (energy-independent) momentum space one-boson-exchange potential (OBEPQ)

	$g_\alpha^2/4\pi$; $[f_\alpha/g_\alpha]$	$g_\alpha^2/4\pi(k^2=0)$	m_α [MeV]	Λ_α [GeV]	n_α
π	14.6	14.27	138.03	1.3	1
ρ	0.81; [6.1]	0.43	769	2.0	2
η	5	3.75	548.8	1.5	1
ω	20; [0.0]	10.6	782.6	1.5	1
δ	1.1075	0.64	983	2.0	1
σ	8.2797 [a]	7.07	550 [a]	2.0	1

Nuclear mass: $m = 938.926$ MeV. For notation and empirical values see table 4. Here, there are NN vertices only.

[a] The parameters for the σ-boson given in the table apply only to the $T=1$ NN potential. For $T=0$ we have: $m_\sigma = 720$ MeV, $g_\sigma^2/4\pi = 16.9822$ and $\Lambda_\sigma = 2$ GeV. The parameters for the other mesons in the table are the same for $T=0$ and $T=1$.

Next lesson:

$\boxed{\text{EFT Approaches}}$